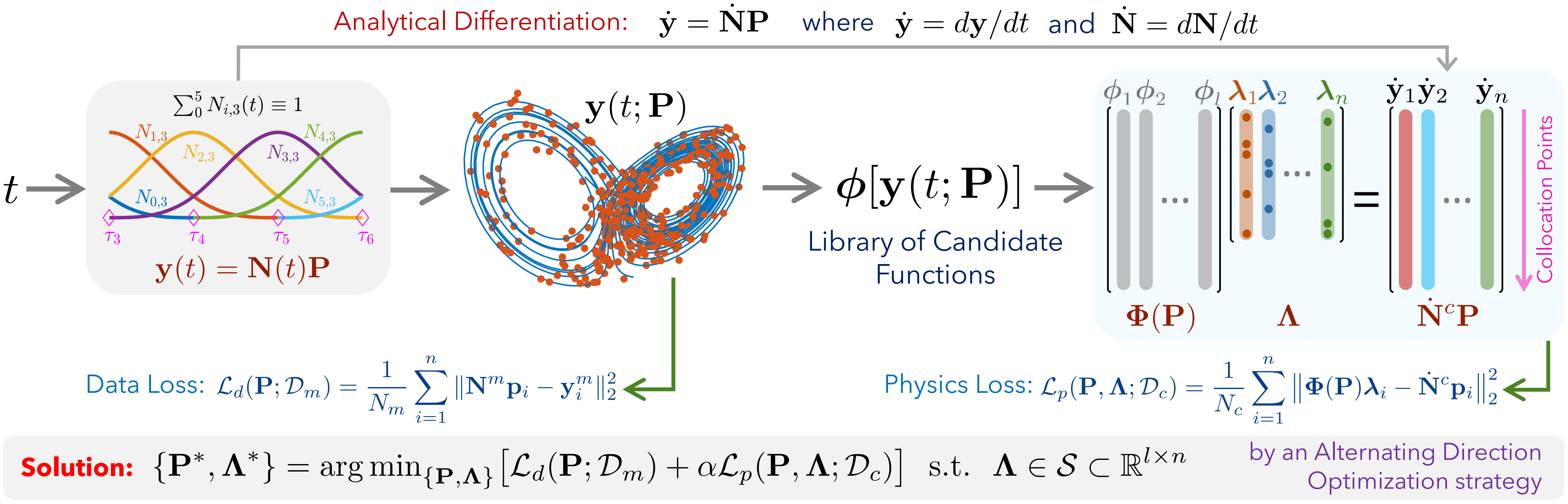

Analytical Differentiation: $\dot{\mathbf{y}} = \dot{\mathbf{N}}\mathbf{P}$ where $\dot{\mathbf{y}} = d\mathbf{y}/dt$ and $\dot{\mathbf{N}} = d\mathbf{N}/dt$
t
$\sum_0^5 N_{i,3}(t) \equiv 1$
$N_{1,3}$
$N_{2,3}$
$N_{3,3}$
$N_{4,3}$
$N_{0,3}$
$N_{5,3}$
τ_3
τ_4
τ_5
τ_6
$\mathbf{y}(t) = \mathbf{N}(t)\mathbf{P}$
$\mathbf{y}(t;\mathbf{P})$
$\phi[\mathbf{y}(t;\mathbf{P})]$
Library of Candidate Functions
$\phi_1\ \phi_2\ \phi_l$
$\boldsymbol{\lambda}_1\ \boldsymbol{\lambda}_2\ \boldsymbol{\lambda}_n$
$\dot{\mathbf{y}}_1\ \dot{\mathbf{y}}_2\ \dot{\mathbf{y}}_n$
Collocation Points
$\boldsymbol{\Phi}(\mathbf{P})$
$\boldsymbol{\Lambda}$
$\dot{\mathbf{N}}^c\mathbf{P}$
Data Loss: $\mathcal{L}_d(\mathbf{P};\mathcal{D}_m) = \frac{1}{N_m}\sum_{i=1}^{n} \|\mathbf{N}^m\mathbf{p}_i - \mathbf{y}_i^m\|_2^2$
Physics Loss: $\mathcal{L}_p(\mathbf{P},\boldsymbol{\Lambda};\mathcal{D}_c) = \frac{1}{N_c}\sum_{i=1}^{n} \left\|\boldsymbol{\Phi}(\mathbf{P})\boldsymbol{\lambda}_i - \dot{\mathbf{N}}^c\mathbf{p}_i\right\|_2^2$
Solution: $\{\mathbf{P}^*,\boldsymbol{\Lambda}^*\} = \arg\min_{\{\mathbf{P},\boldsymbol{\Lambda}\}}\left[\mathcal{L}_d(\mathbf{P};\mathcal{D}_m) + \alpha\mathcal{L}_p(\mathbf{P},\boldsymbol{\Lambda};\mathcal{D}_c)\right]$ s.t. $\boldsymbol{\Lambda} \in \mathcal{S} \subset \mathbb{R}^{l\times n}$
by an Alternating Direction Optimization strategy