

1. The worst case of the methods:

|            | isEmpty | Size   | insert      | findMin | deleteMin   |
|------------|---------|--------|-------------|---------|-------------|
| BinaryHeap | $O(1)$  | $O(1)$ | $O(\log n)$ | $O(1)$  | $O(\log n)$ |
| ThreeHeap  | $O(1)$  | $O(1)$ | $O(\log n)$ | $O(1)$  | $O(\log n)$ |
| MyPQ       | $O(1)$  | $O(1)$ | $O(n)$      | $O(1)$  | $O(n)$      |

2. Time

|            | Insert |           | deleteMin |          |
|------------|--------|-----------|-----------|----------|
|            | n      | Time(ns)  | n         | Time(ns) |
| BinaryHeap | 1000   | 85000     | 1000      | 90000    |
|            | 2000   | 220000    | 2000      | 260000   |
|            | 4000   | 500000    | 4000      | 560000   |
|            | 8000   | 800000    | 8000      | 1000000  |
|            | 16000  | 1370000   | 16000     | 2200000  |
| ThreeHeap  | 1000   | 74000     | 1000      | 120000   |
|            | 2000   | 150000    | 2000      | 260000   |
|            | 4000   | 290000    | 4000      | 680000   |
|            | 8000   | 750000    | 8000      | 1500000  |
|            | 16000  | 1400000   | 16000     | 3400000  |
| MyQueue    | 1000   | 3200000   | 1000      | 160000   |
|            | 2000   | 8000000   | 2000      | 290000   |
|            | 4000   | 42000000  | 4000      | 1400000  |
|            | 8000   | 170000000 | 8000      | 5900000  |
|            | 16000  | 640000000 | 16000     | 25000000 |

3. a. The asymptotic analysis for the three implementations are not very reliable. It show the right trend of time as n increases, but the results are not precise.

b. The runtime of insert and deleteMin should be log but both the results show more linearly. And for MyPQ, the expectation is linear relation, but the results are more likely to be quadratic. As for the reason, I believe that this is because the number n is not big enough. Since cases very so that the n should be large enough to get a precise trend. But if n were too large, my computer just could not run the results fast. Also, while inserting new items, the size will double when the original heap is full, which will also take some time.

c. Comparing the speed, MyQueue is obviously slower than the other two. BinaryHeap and ThreeHeap have basically the same speed. But I recommend the BinaryHeap since when percolate up or down, comparing the priority of parent with 2 children is much easy to operate than comparing with 3 children. And the algorithm of BinaryHeap is easier to understand than that of the ThreeHeap.

4. To test the implementation, I create a test file. I divide it by 3 parts. It first test the methods insert and size by inserting 10 random doubles and then report the size. The second part is to test findMin, isEmpty and deleteMin. The findMin found the min in the existing 10-double-PQ. Then insert the same double as min, and deleteMin for twice to see whether the two results of deleteMin are the same. This is to test whether the deleteMin run as expected when there are 2 same min. Then will go through deleteMin until the isEmpty becomes true. If all successful, the second part ends. The third part is to test makeEmpty. Firstly insert 100 random double and then execute makeEmpty. If the PQ becomes empty, third part succeeds.
5. a. For index of parent is  $i$   
Children:  
Binary:  $2i, 2i+1$   
Three:  $3i-1, 3i, 3i+1$   
Four:  $4i-2, 4i-1, 4i, 4i+1$   
Five:  $5i-3, 5i-2, 5i-1, 5i, 5i+1$   
  
b.  $i*d-d+2$