(8) $4x^2 < 1 \Rightarrow x^2 < 1/4 \Rightarrow |x| < 1/2$

$R = -1/2 < x < 1/2$

$$\sum_{n=1}^{\infty} \frac{4^n x^{2n}}{n} = \sum_{n=1}^{\infty} \frac{4^n(-1/2)^{2n}}{n} = \sum_{n=1}^{\infty} \frac{4^n\left(\frac{1}{2^{2n}}\right)}{n} = \sum_{n=1}^{\infty} \frac{4^n\left(\frac{1}{4^n}\right)}{n} = \sum_{n=1}^{\infty}$$

$p = 1 \leq 1$ divergence so no values at x converge

(9) $x = 0$ e^{-x}

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$$

5) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{4/3}}$

Direct compare $0 < \frac{\sin^2 n}{n^{4/3}} \leq \frac{1}{n^{4/3}} = b_n$

Since $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ p-series $(p = 4/3 > 1)$ converges

So $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{4/3}}$ converges ∎

6) $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$ Find $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{n!}{(n+2)^2}}{\frac{(n-1)!}{(n+1)^2}}\right| = \lim_{n\to\infty} \left|\frac{n!(n+1)^2}{(n-1)!(n+2)^2}\right| =$

$\lim_{n\to\infty} \left|\frac{n(n+1)^2}{(n+2)^2}\right| = \infty$ Limit is $\infty > 1$
ratio test series diverges ∎

7) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ $u_n = \frac{n}{n^2+1}$ u_n positive for all n

$f(n) = \frac{n}{n^2+1} = f'(n) = \frac{n'(n^2+1) - n(n^2+1)'}{(n^2+1)^2}$

$= \frac{n^2+1-n(2n)}{(n^2+1)^2} = \frac{n^2+1-2n^2}{(n^2+1)^2} = \frac{-n^2+1}{(n^2+1)^2} \leq 0$

$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{n}{n^2+1} = 0$ Converges ∎

2) $\sum_{n=1}^{\infty} \frac{4^n x^{2n}}{n} = \sum_{n=0}^{\infty} u_n$

$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \left|\frac{4^{n+1} x^{2(n+1)}}{n+1} \Big/ \frac{4^n}{n} x^{2n}\right|$

$\lim_{n\to\infty} \left|\frac{4^n 4 x^{2n} x^2}{n+1} \cdot \frac{n}{4^n x^{2n}}\right| = 4|x^2| \lim_{n\to\infty} \left|\frac{n}{n+1}\right|$

$= 4|x^2| \lim_{n\to\infty} \frac{1}{1+1/n} = 4x^2$ $\lim_{n\to\infty} \frac{1}{n} = 0$ Absolutely convergence

MATH 142 EXAM 2

① $\int_{-\infty}^{0} xe^x\,dx$

$\lim_{a\to-\infty}\int_a^0 xe^x\,dx$ $\qquad u = x$, $dv = e^x\,dx$

$= xe^x - e^x + C$

$= e^x(x-1) + C$

$\lim_{a\to-\infty}\left[e^x(x-1)\right]_a^0 = \lim_{a\to-\infty}\left(e^0(0-1) - (e^a(a-1))\right)$

$\lim_{a\to-\infty}\left(-1(e^a(a-1))\right)$

$-1 - \lim_{a\to-\infty}\left(e^a(a-1)\right)$

$-1 - \lim_{a\to-\infty} e^a(1-a)$

$-1 - \lim_{a\to-\infty} e^a(1-a) = -1 - 0 = -1$

$\lim_{a\to-\infty}\frac{(1-a)'}{(e^{-a})'}$

$\lim_{a\to-\infty}\frac{-1}{-e^{-a}}$

$= 0$

② $\sum_{n=0}^{\infty}\frac{(-2)^n}{5^n}$ $\quad -\sum_{n=1}^{\infty}\frac{2^n}{5^n} = -\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n = -\left(\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n - \left(\frac{2}{5}\right)^0\right)$

$\left(-\frac{5}{3} - \left(\frac{2}{5}\right)^0\right) = -\frac{2}{3}$ convergent

③ $0.\overline{34} = 0.343434$

$= .34 + .0034 + .000034 + \dots$

$\frac{34}{100}\left[1 + \frac{1}{100} + \left(\frac{1}{100}\right)^2 + \dots\right]$ $\qquad a = 1,\ r = \frac{1}{100}$

$\frac{34}{100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{34}{99}$

④ $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ $\quad f(x) = \frac{\ln x}{x}$ $\quad \int_1^{\infty}\frac{\ln x}{x}\,dx = \lim_{n\to\infty}\int_1^{b}\frac{\ln x}{x}\,dx \dots$

$u = \ln x$, $du = 1/x\,dx$ $\quad \int_1^{\infty}\frac{\ln x}{x}\,dx = \lim_{b\to\infty}\int_1^{b}\frac{\ln x}{x}\,dx = \lim_{b\to\infty}\int_0^{\ln b} u\,du$

$= \lim_{b\to\infty}\left(\frac{u^2}{2}\right)_0^{\ln b} = \frac{1}{2}\lim_{b\to\infty}\left(\ln b^2 - 0^2\right) = \infty$

divergent