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Lorenzo Castelli, Raffaele Pesenti, Andrea Ranieri

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Lorenzo Castelli
DEEI - University of Trieste
Via A. Valerio 10, 31427 Trieste, Italy
castelli@units.it

Raffaele Pesenti
DMA - University Ca' Foscari of Venice
Dorsoduro 3825/E, 30123 Venice, Italy
pesenti@unive.it

Andrea Ranieri
DEEI - University of Trieste
Via A. Valerio 10, 31427 Trieste, Italy
aranieri@units.it

Abstract

In Europe, when an imbalance between demand and capacity is detected for air traffic network resources, Air Traffic Flow Management slots are allocated to flights on the basis of a First Planned First Served principle. We propose a market mechanism to allocate such slots in the case of a single constrained en-route sector or airport. We show that our mechanism provides a slot allocation which is economically preferable to the current one as it enables airlines to pay for delay reduction or receive compensations for delay increases. We also discuss the implementation of our mechanism through two alternative distributed approaches that spare airlines the disclosure of private information. Both these approaches have the additional advantage that they directly involve airlines in the decision making process. Two computational examples relying on real data illustrate our findings.

Keywords:Air Transportation, Market Mechanism Design, Air Traffic Flow Management slots, Collaborative Decision Making, SESAR.

JEL Classification Numbers: L93, C61, C71, L98

1   Introduction

In Europe, the Air Traffic Management (ATM) authority usually imposes a *regulation* when an imbalance between predicted traffic and available capacity is detected in either a sector or an airport on the day of operations. A regulation consists in forcing delays to the take-offs of some flights, the so-called Air Traffic Flow Management (ATFM) delays. ATFM delays are imposed on the basis of a concept of equity but without regarding the airlines' costs. The ATM authority allocates a departure time slot (ATFM slot) to flights through a First Planned First Served ($FPFS$) policy, i.e., according to the flights' Estimated Time Over (ETO) the specific sector or airport.

In this paper, we assume that airlines are interested in paying for delay reductions or receiving compensations for delay increases (Vossen and Ball, 2006). Then, we introduce a market mechanism that allows airlines to trade their $FPFS$-allocated slots. Our mechanism is distributed as a centralized policy based on the airlines' costs cannot be implemented unless the ATM authority knows the delay costs for each flight. Unfortunately, airlines consider these costs as private and valuable information, and are reluctant to disclose them even to the ATM authority. Our mechanism let airlines decide autonomously for each flight whether it is preferable to keep the slot obtained by the $FPFS$ policy or to exchange it at the market price.

Models for the optimal allocation of ATFM delays at a single airport were first conceptualized by Odoni (1987) for the deterministic case and successively by Andreatta and Romanin-Jacur (1987) for the stochastic case. Later, Vranas et al. (1994) generalized these models to the multi-airport case. Bertsimas and Stock Patterson (1998) and Bertsimas et al. (2008) proposed integer programming models considering also en-route constraints and re-routing options. Other models appeared in the 1990s (see, e.g., the overview in Hoffman and Ball, 1997). In all of them the ATM authority acts as the only decision maker optimizing a 'global' objective function that aggregates all the delay costs caused by the ATFM regulations.

In the recent years, the European authorities have recognized that airlines are in the best position to make decisions for their flights (see, e.g., EUROCONTROL EEC, 1999). This principle has induced both U.S. and European ATM regulators to introduce Collaborative Decision Making (CDM) programs (Chang et al., 2001; EUROCONTROL, 2006). The idea of CDM is to move away from a central decision maker to 'real-time' decision support tools that assist traffic managers and airline operational centers in making the best decisions. The CDM concept is a pillar of SESAR, the Single European Sky ATM Research program of the European Union. Our research falls within the scope of SESAR program, which states that airlines will be offered the possibility to indicate to an ATM authority a priority order for flights affected by delays (SESAR Joint Undertaking, 2007). This User Driven Prioritisation Process (UDPP) will be initiated by the ATM authority when an unexpected imbalance between capacity and demand is detected on a short notice. Since different airlines are in general competitors, market-based mechanisms seem natural ways of implementing the UDPP. A market mechanism suitable for the UDPP should satisfy the following properties to be acceptable (see e.g. Krishna (2009) for formal definitions):

- Individual rationality: the mechanism gives each airline a non-negative payoff for participating in it, otherwise no rational airline would participate without being enforced by the regulator. This property is sometimes also called participation constraint.

- Budget balance: the mechanism neither requires subsidization from outside nor produces profit to allocate outside the set of airlines involved. The overall amount of prices paid and received by participants sums up to zero. The central authority only objective is implementing the mechanism without injecting or receiving money to make it work.

We consider the above properties as hard constraints that the market mechanism must satisfy. In fact all airlines must derive a non-negative profit from the participation while the central authority objective is merely to facilitate the execution of the mechanism, without neither collecting nor injecting money. Thus all rational agents, whose objective is to maximize profits and the central authority, whose objective is the social welfare, will want to participate in the mechanism. Other desirable properties are:

- Allocative efficiency: the mechanism produces an allocation which optimizes the sum of the costs reported by the airlines.

- Incentive compatibility: the mechanism is designed so that no airline can increases its payoff by misrepresenting its costs, provided that all other airlines are truthful.

The minimal value of allocative efficiency can be achieved only if costs are reported truthfully, i.e. only in an incentive compatible mechanism. Unfortunately, the impossibility theorem by Myerson and Satterthwaite (1983) states that no mechanism can satisfy at the same time incentive compatibility, individually rationality, budget balance and efficiency at the equilibrium. The mechanism we propose thus relaxes the incentive compatibility constraint in order to guarantee individual rationality and budget balance and it implements the efficient allocation according to reported costs.

To the authors' knowledge, the literature on market mechanisms for slot allocation has been focusing so far on airport slots only. Airport slots are agreed between coordinated airports and airlines and define the scheduled times of arrivals or departures on a given date. Differently, the ATFM slots we deal with are used as a tactical tool to regulate air traffic. In Europe, airport slots are free and their allocation follows the 'grandfather' rule. Airlines can keep their slots year after year provided that their rate of utilization is above a given threshold (see, e.g., Sieg (2009) and Verhoef (2009) for recent analyses on different regulatory schemes for this market).

Rassenti et al. (1982) propose a sealed-bid type of combinatorial auction for the long-term strategic allocation of airport slots in the US. The slot prices are set by the ATM authority on the basis of bids submitted and in order to maximize the system surplus. After this initial slot allocation, a secondary market is allowed, in which airlines trade the slots previously received. This mechanism requires the revelation of airlines information to calculate the first allocation, and assigns the revenues from the auction to airports. Authors recognize that this latter point is debatable since airports with limited capacity would rise their revenues by imputing rents on scarce commodities. Fukui (2009) provides an empirical analysis of the current secondary market at four major US airports. Also Ball et al. (2006) observe that US airport slots are scarce commodities with both a private and a common value. They provide a deep analysis of the objectives and issues associated with slot auctions at different planning levels, from the strategic to the tactical one. They observe that one argument against auctions is that they can be seen as a way for the government to raise revenues, since airlines should pay to get the same slots that they now receive at no cost. Our market mechanism overcome this drawback since it takes the $FPFS$ allocated slots as initial endowment of each flight and enforces individual rationality and budget balance. However, we must require that no compensation is given for canceled flights that release their slot, to prevent the creation of 'ghost' flights just to make money.

Recently Waslander et al. (2008) have considered the case in which competitor airlines have different private preferences over traffic control actions. They formalize a capacity resource market for the air traffic and they prove that the outcome of such market mechanism is to be preferred by all airlines over a solution that does not take into account preference information. We rather start our work from a formalization of the system currently adopted in Europe to allocate ATFM slots and then we suggest an implementation of a market mechanism, which can be employed to enhance this solution.

The remainder of this paper unfolds as follows. In Section 2, we provide an overview of the slot allocation mechanism currently used in Europe and we show that it minimizes the total flight delays. In Section 3, we introduce an individual rational and budget balanced mechanism that maximizes the efficiency. We show that the resulting slot allocation minimizes the cost of the overall delays of the airlines involved in the trading. To prevent the disclosure of private information and reduce computational and communication burdens, in Section 4, we suggest two different approaches to implement our market mechanism in a distributed fashion. Then, in Section 5, we provide two examples based on real data. In Section 6, we draw some conclusions.

2 The current European resource allocation system

In Europe, when an imbalance between demand and availability of air transport infrastructure is detected, the EUROCONTROL Central Flow Management Unit (CFMU) may employ a number of measures to avoid congestion such as: the reconfiguration of some sectors, the activation of mandatory routes for certain trajectories, and the creation of slot allocation regulations (Leal de Matos and Omerod, 2000). In this last case, the aim is to protect a certain resource of the system by limiting, through a regulation, the maximum number of flights that can enter it during an established period of time. On the day of operations all flights affected by regulations can either decide to re-route, in order to avoid the affected areas, or to be delayed on the ground by a controlled take-off time. This measure is based on the principle that delays on the ground are safer and less costly than those in the air. Any forecast delay somewhere in the system is anticipated at the departure airport prior to the take-off (Ball and Lulli, 2004). The ATFM slots are calculated by the Computer Assisted Slot Allocation (CASA) system at CFMU on a $FPFS$ policy (EUROCONTROL CFMU, 2007). To do so, initially, CASA creates for each resource (i.e., an en-route sector or an airport) a slot allocation list. This list is composed by a number of slots function on the rate of acceptance and the duration of the regulation. Then, CASA allocates the slots to flights as close to their ETO as possible. If two flights require the same slot, the slot is allocated to the one with the lower ETO. If a flight is affected by several regulations, the delay caused by the most penalizing one is forced in all the others. Delays caused by ATFM measures in 2007 amounted to 21.5M minutes, causing an estimated cost of €1300M to the airlines (EUROCONTROL PRC, 2008).

Even though a flight may cross several regulations during its trajectory, most of the ATFM delays are caused by just one regulation, meaning that there is a unique capacity constrained resource the flight planned to use. This paper focuses only on the latter scenario. Table 1 displays the number of regulations each regulated flight was affected in the 28 day period from 31st July 2008 to 27th August 2008.

Number of Regulations	Number of Flights affected	% of Flights affected
1	162991	68.730 %
2	50248	21.189 %
3	16865	7.112 %
4	5150	2.172 %
5	1432	0.604 %
6	358	0.151 %
7	86	0.036 %
8	13	0.005 %
9	3	0.001 %

Table 1: Flights delayed by number of regulations during AIRAC 311

The case of interest of this paper assumes that all the considered flights are affected by a single regulation on a unique resource and that each flight belongs to a different airline. Thus, in the following, we may use flight and airline as synonyms. Differently, it is out of the scope of this work to study the situation where each airline may own more than one flight and must take decisions on how to allocate its set of slots to its set of flights (see, e.g., Vossen and Ball, 2006).

We consider a single capacity-constrained resource s active from st_time to end_time with a fixed capacity K, expressed in number of entries per hour. In addition, we have a set of flights $F= \{1, ..., F\}$ that must enter s. The ATM authority must allocate a different slot to each flight $f \in F$ for the access to s. The N_s slots of resource s, with

$$N_s = \left\lfloor \frac{end\_time - st\_time}{\frac{60}{K}} \right\rfloor,$$

are defined as follows. Let $S = \{1, ..., N_s\}$ be the set of slots. Each slot $j \in S$ has capacity equal to one flight, begins at time I_j and ends at time U_j where:

$$I_j = \left\lfloor st\_time + (j-1) \cdot \frac{60}{K} \right\rfloor \quad \text{with } j \in \{2, ..., N_s\}$$
$$U_j = I_{j+1} - 1 \quad \text{with } j \in \{1, .., N_s - 1\},$$

and $I_1 = st\_time$, $U_N = end\_time$.

Each flight $f \in F$ has a published trajectory which contains the ETO E_f into s. Then, flight f can be allocated to any slot $j \in R^f$, where $R^f = \{j : E_f \leq U_j\} \subseteq S$ is the set of the feasible slots for flight f as they end after E_f. The allocation of flight f to slot j produces a delay d_{fj} equal to

$$d_{fj} = \max\{E_f, I_j\} - E_f \quad \forall f \in F, \forall j \in S.$$

Given the above data, the ATM authority decides the slot allocations. This operations is equivalent to deciding the value of the decision variables x_{fj}, for all $f \in F$ and $j \in R^f$, where x_{fj} is set equal to 1 if flight f is allocated to slot j, 0 otherwise.

The $FPFS$ slot allocation policy sorts flights $f \in F$ by the ascending value of E_f, then greedily allocates each flight to the first available slot, i.e., x_{fj} is set equal to 1 if

$$j = arg \min\{U_j : E_f \leq U_j \ \forall j \in S \text{ and } j \text{ is not allocated yet}\}$$

Hereafter, we refer x as the vector whose components are the variables x_{fj} and, in particular, we denote as x^F a slot allocation obtained by means of the $FPFS$ policy.

In the following, we show that the allocation x^F minimizes the total delay of the flights in F, as required by the European regulator (EUROCONTROL PRC, 2008). In other words, x^F is an optimal solution of the assignment problem:

$$\min \sum_{f \in F} \sum_{j \in R^f} d_{fj} x_{fj} \tag{1a}$$

$$\sum_{f \in F: j \in R^f} x_{fj} \leq 1 \quad \forall j \in S \tag{1b}$$

$$\sum_{j \in R^f} x_{fj} = 1 \quad \forall f \in F \tag{1c}$$

$$x_{fj} \geq 0 \quad \forall f \in F, j \in R^f. \tag{1d}$$

In (1), the objective function (1a) minimizes the total delay; constraints (1b) impose that at most one flight can enter s during each slot $j \in S$, whereas constraints (1c) require that all flights affected by the regulation must have a slot allocated; finally, constraints (1d) guarantee the integrality of the the decision variables x_{fj}. These last constraints can be expressed in their linearly relaxed form since (1) is an assignment problem.

Theorem 1 *A vector x^F obtained respecting the FPFS policy is optimal for problem (1).*

Proof. By closing paralleling the proof of Theorem 3.2 in Vossen and Ball (2006) where all slots are composed by only one unit of time, i.e., $I_j = U_j \, \forall j \in S$.

The $FPFS$ policy does not allow a flight f to make any choice or state any preference about the slots. On the other hand, the optimality of the $FPFS$ policy implies also its Pareto efficiency. Any other slot allocation policy that reduces the delay of a given flight f correspondingly increases the delay of at least another flight $g \neq f$, that consequently would not find convenient to move from its granted $FPFS$ slot.

A further perspective is available when we no longer consider delays only, but also the costs associated to them, and we allow monetary payments to be involved. In this case it is possible to introduce an appropriate reallocation policy such that all flights may prefer a slot allocation different from the $FPFS$ one, as we show in the next section.

3 A market mechanism for slot allocation

We consider the possibility of a slot trade between flights. We assume that flights are willing either to pay for earlier slots that reduce their delay or to receive a side payment to compensate possible increases of delays.

Let x^F be the vector representing the $FPFS$ slot allocation and let x represent an alternative feasible slot allocation obtained by the market mechanism. In addition, let c_{fj} be the delay cost of flight f allocated to slot j, for all $f \in F$ and $j \in R^f$. Costs c_{fj} vary depending on, e.g., the time of the day, the type of flight (connecting or not) and the type of airline. In general, they do not depend linearly on the amount of delay: the cost of a few minutes of delay is proportionally much lower than, e.g., the cost of a 30-minute delay (Cook et al., 2004). Finally, for all $j \in S$, let $v_j \geq 0$ be price of the slot j, i.e., the amount of money that the seller flight receives from the buyer flight for this resource.

We define

$$p_f = \sum_{j \in R^f} c_{fj} \cdot (x_{fj}^F - x_{fj}) + \sum_{j \in R^f} v_j \cdot (x_{fj}^F - x_{fj}) \tag{2}$$

as the *profit* that flight $f \in F$ obtains by exchanging the slot assigned through the $FPFS$ allocation with the slot assigned by the alternative allocation. The former two terms of p_f represent the difference in the cost of delay between the allocation of flight f from the $FPFS$ slot to the alternative one. The latter two terms represent the economic payoff between the selling of its $FPFS$ slot and the purchase of the alternative slot.

The first property that a market mechanism must fulfill is the individual rationality: each flight f participating to the trade must obtain a non negative p_f, i.e.,

$$p_f \geq 0, \quad \forall f \in F. \tag{3}$$

Another attractive property is the budget balanced: the trade should neither require payments from outside (i.e., there is no subsidization) nor generate a surplus, but just redistribute the payments among the flights, i.e.,

$$\sum_{f \in F} \sum_{j \in R^f} v_j \cdot (x_{fj}^F - x_{fj}) = 0. \tag{4}$$

Finally, a market mechanism may be required to allocate the resource efficiently. From the society's point of view, it is desirable to minimize the overall cost of delay. This is equivalent to identify a new slot allocation which maximizes the decrease of the total delay cost from the granted $FPFS$ allocation. On the other side, airlines may wish to use the trading not only to minimize their costs of delay, but also to maximize their profits. However, from Equations (2) and (4) the two objectives coincide as

$$\sum_{f \in F} p_f = \sum_{f \in F} \sum_{j \in R^f} c_{fj} \cdot (x_{fj}^F - x_{fj}). \tag{5}$$

Solving the following nonlinear problem we can obtain a slot allocation that satisfies the above desirable characteristics of a market mechanism:

$$\max \sum_{f \in F} \sum_{j \in R^f} c_{fj} \cdot (x_{fj}^F - x_{fj}) \tag{6a}$$

$$\sum_{f \in F : j \in R^f} x_{fj} \leq 1 \quad \forall j \in S \tag{6b}$$

$$\sum_{j \in R^f} x_{fj} = 1 \quad \forall f \in F \tag{6c}$$

$$\sum_{j \in R^f} c_{fj} \cdot (x_{fj}^F - x_{fj}) + \sum_{j \in R^f} v_j \cdot (x_{fj}^F - x_{fj}) \geq 0 \quad \forall f \in F \tag{6d}$$

$$\sum_{f \in F} \sum_{j \in R^f} v_j \cdot (x_{fj}^F - x_{fj}) = 0 \tag{6e}$$

$$x_{fj} \in \{0,1\} \quad \forall f \in F, j \in R^f \tag{6f}$$

$$v_j \geq 0 \quad \forall j \in S. \tag{6g}$$

Next we show that constraints (6d), (6e) and (6g) are redundant, thus problem (6) reduces to the simpler following linear problem:

$$\min \sum_{f \in F} \sum_{k \in R^f} c_{fk} x_{fk} \tag{7a}$$

$$\sum_{f \in F : j \in R^f} x_{fk} \leq 1 \quad \forall k \in S \tag{7b}$$

$$\sum_{k \in R^f} x_{fk} = 1 \quad \forall f \in F \tag{7c}$$

$$x_{fk} \geq 0 \quad \forall f \in F, k \in R^f. \tag{7d}$$

We denote as x^C the optimal solution of (7). As problems (1) and (7) share the same constraints, x^C is also feasible for (1), but not necessarily optimal. Analogously, x^F is feasible for (7), but not necessarily optimal. The optimal solution x^C of problem (7) is also optimal for problem (1) when all slots are composed by only one unit of time, i.e., $I_i = U_i$, for all $i \in S$ (see Theorem 3.1 in Vossen and Ball (2006)).

Definition 1 *We define as MINCOST the policy that implements the slot allocation according to problem (7).*

Definition 2 *We define as MM the market mechanism where each flight sells its granted FPFS slot j at price v_j and purchases the MINCOST slot k at price v_k.*

The next theorem proves that MM fulfills the individual rationality property.

Property 1 *MM is individual rational if, for all $k \in S$, the slots costs are $v_k = -\xi_i$ where ξ_k are the nonpositive dual variables associated to constraints (7b).*

Proof. Let ξ_k be the nonpositive dual variables associated to constraints (7b) for all $k \in S$, and μ_f be the dual variables associated to constraints (7c) for all $f \in F$. The slackness conditions impose $\mu_f + \xi_k \leq c_{fk}$ where the equality holds if $x^C_{fk} = 1$. As a consequence, for $k \in S$ such that $x^C_{fk} = 1$, we obtain

$$c_{fi} - c_{fk} + \xi_k - \xi_i \geq 0, \quad \forall i \in S \setminus \{k\}. \tag{8}$$

Finally, observe that condition (8) becomes $p_{fk} = c_{fj} - c_{fk} + v_j - v_k \geq 0$, that is condition (3), when we let $v_i = -\xi_i$ for all $i \in S$ and we denote as j the slot allocated to f under the $FPFS$ policy.

The following two lemmas are necessary to prove that MM is also budget balanced.

Lemma 1 *For every pair of slot allocation x^F and x^C, respectively defined by the $FPFS$ and by the $MINCOST$ policy, a slot is empty in x^F if and only if it is empty in x^C.*

Proof. By contradiction. Let $\hat{j}$ be the first empty slot in one slot allocation but not in the other one. We initially assume that $\hat{j}$ is empty in x^F, but is allocated in x^C. Let F^F, respectively F^C, be the set of the flights allocated to the slots $1, \ldots, \hat{j}$ in x^F, respectively in x^C. The minimality of $\hat{j}$ guarantees that $F^C \setminus F^F \neq \emptyset$. Then consider a flight $\hat{f} \in F^C \setminus F^F$. In x^F, flight $\hat{f}$ is allocated to a slot $\hat{l} > \hat{j}$. Define $\hat{x}^F$ such that

$$\hat{x}^F_{fj} = \begin{cases} 0 & \text{if } j = \hat{l} \text{ and } f = \hat{f} \\ 1 & \text{if } j = \hat{j} \text{ and } f = \hat{f} \\ x^C_{fj} & \text{otherwise} \end{cases}.$$

That is, $\hat{x}^F$ induces the same slot allocations of x^F except for flight $\hat{f}$ which is allocated to slot $\hat{j}$ instead of $\hat{l}$. The feasibility of x^F and the fact that $\hat{f}$ is allocated to slot $\hat{j}$ in x^C imply the feasibility of $\hat{x}^F$. Also the

cost (7a) of x^F is strictly greater than the corresponding cost of $\hat{x}^F$ as $E_{\hat{f}} \leq U_{\hat{j}} < I_{\hat{i}}$. Hence, we obtain the contradiction that x^F is not optimal for (7) and cannot be defined by a $MINCOST$ policy. As a consequence, we cannot assume that $\hat{j}$ is empty in x^F, but not in x^C. An analogous argument proves that we cannot assume that $\hat{j}$ is empty in x^C, but is allocated in x^F. Then, we must conclude that $\hat{j}$ cannot exist.

The above lemma implies that any slot allocated under the $FPFS$ policy is also allocated, possibly to a different flight, under the $MINCOST$ policy. Hence constraints (7b) can be rewritten as:

$$\sum_{f \in F: j \in R^f} x_{fj} = \sum_{f \in F: j \in R^f} x_{fj}^F \quad \forall j \in S. \tag{9}$$

Another important consequence of Lemma 1 is that the asymmetric assignment problem (7) can be decomposed into a set of smaller symmetric assignment problems. To prove this last statement, we need to introduce the following notation. Let i and l be two empty slots in x^F such that $U_i < I_l$. We define as $B_{il}^F \subseteq S$ (respectively $B_{il}^C \subseteq S$) the set of slots allocated in x^F (respectively in x^C) between the empty slots i and l. Let $F_{il}^F \subseteq F$ (respectively $F_{il}^C \subseteq F$) be the corresponding set of flights assigned to the B_{il}^F (respectively B_{il}^C) slots.

Lemma 2 *For every pair of slot allocation x^F and x^C, respectively defined by the $FPFS$ and by the $MINCOST$ policy, if i and l are two empty slots in x^F such that $U_i < I_l$ and no other empty slots exists between i and l, then, a flight is allocated to a slot between i and l in x^F if and only if it is allocated to a slot between the same empty slots i and l also in x^C, that is $B_{il}^C = B_{il}^F$ and $F_{il}^C = F_{il}^F$.*

Proof. From Lemma 1, it follows that each element of B_{il}^F is allocated to a flight also in the solution x^C, i.e., $B_{il}^C = B_{il}^F$. Let $f \in F_{il}^F$ be a flight allocated to slot h in B_{il}^F, i.e., $I_h > U_i$ and $U_h < I_l$. It follows that $E_f > U_i$, otherwise slot i would be occupied by flight f. Hence also in x^C flight f cannot be anticipated to slot i or to any other slot earlier than slot i. Similarly no flight assigned to a slot later than slot l can receive slot h, earlier than l, hence flight f cannot be postponed in x^C to any free slot later than slot l. Thus, if in x^C slot h is allocated to a flight preceding slot i in x^F, it follows that there must be an empty slot in x^C that was not empty in x^F, and this contradicts Lemma 1. Hence, we must have that $F_{il}^C = F_{il}^F$. A symmetric argument holds for the only if part of the statement.

The above lemma implies that the optimal $MINCOST$ allocation x^C is achieved by minimizing the cost of delay of each separate subset of consecutive allocated slots between two empty slots given by the $FPFS$ policy. Or, the solution of the asymmetric assignment problem (7) is found by solving a sequence of smaller symmetric assignment problems, one for each set B_{il}^F. A further consequence of Lemma 2 is that MM is budget balanced.

Property 2 *MM is budget balanced.*

Proof. The statement is an immediate consequence of the fact that $F_{il}^C = F_{il}^F$ for any pair of slots i and l considered in Lemma 2. Indeed, $F_{il}^C = F_{il}^F$ implies that no flight $f \in F_{il}^F$ must sell back its $FPFS$ assigned slot to the ATM authority, nor any flight $f \in F_{il}^F$ must buy an unassigned $FPFS$ slot from the ATM authority. The flights just sell and buy slots to and from each others.

Another consequence of Lemma 2 is that, from a game theoretic perspective, we can see MM as a permutation game and then we can derive further characteristics.

A cooperative game with transferable utility, or TU game, is described (see, e.g., Borm et al., 2001) by a pair (F, θ) that is, by the set of players $F = \{1, ..., n\}$ and a characteristic function $\theta : 2^F \to R$, being 2^F the set of subsets (usually referred as coalitions) of F. In these games side payments between the players are allowed and the characteristic function assigns to every coalition $A \subseteq F$ a value $\theta(A)$, representing the maximal total monetary reward that the members of this group can obtain when they cooperate. By convention, $\theta(\emptyset) = 0$. In this context, a *permutation game* (see, e.g., Curiel and Tijs, 1986) is a TU game describing a situation in which the n players in F all have one job to be processed and one machine on which each job can be processed,

but no machine is allowed to process more than one job. If player f processes his job on the machine of player j, the processing costs are a_{fj}. Then, the permutation game characteristic function is defined as

$$\theta(A) = \begin{cases} \sum_{f \in A} a_{ff} - \min_{\pi \in \Pi_A} \sum_{f \in A} a_{f\pi(i)} & \text{if } \emptyset \neq A \subseteq F \\ 0 & \text{if } A = \emptyset \end{cases} \tag{10}$$

where Π_A is the class of all A-permutations. Here, the value of $\theta(A)$ denotes the maximal cost savings a coalition A can obtain by processing their jobs according to an optimal schedule compared to the situation in which every player processes his job on his own machine.

We can interpret MM as a permutation game since there is an immediate correspondence between 'machine' and 'slot' on one side, and between 'job' (or 'player') and 'flight' on the other side. Accordingly, a subset of flights forms a coalition A if they limit the slot trading only to the slots assigned by the $FPFS$ policy to the members of the coalition. Hence, imposing $a_{fj} = c_{fj}$ for all $f \in F$ and $j \in S$, the characteristic function $\theta(A)$ is equal to the difference between the cost of delay due to the $FPFS$ allocation and the $MINCOST$ allocation when limited to flights in A. Under this latter hypothesis, condition (5) holds even when its sums are restricted only to the members coalition A, then $\theta(A)$ is also equal to the maximum sum of profits that the flights of the coalition can obtain by exchanging their slots.

In a TU game (F, θ) a payoff allocation $p = (p_1, p_2, \ldots, p_n)$ is the vector of the amounts of utility allocated to each player. A payoff allocation is individually rational if $p_f \geq \theta(\{f\})$ for all $f \in N$, it is efficient if $\sum_{f \in N} p_f = \theta(N)$. In this context, the imputation set $\mathcal{I}(F, \theta)$ of the game is the set of the payoff allocations that are efficient and individually rational. Finally, the core $\mathcal{C}(F, \theta) \subseteq \mathcal{F}(N, \theta)$ of the game, if it exists, is the set of the payoff allocations that are efficient and coalition rational, that is $\sum_{f \in A} p_f \geq \theta(A)$, for all $A \subseteq F$.

Then if we deal with MM as a permutation game, the results in Curiel and Tijs (1986) state that the core of the game exists and that there is a bijective relation between the elements of the core of the permutation game and the optimal solutions of the dual problem of (7) provided that the objective function (7a) is replaced by the equivalent function (6a). In particular, the profits p_f as in Equation (2) define a core payoff allocation. Give the core characteristics, the next property follows.

Property 3 *MM is coalition rational and defines Pareto optimal profits whose overall value is equal to* $\sum_{f \in F} p_f = \sum_{f \in F} \sum_{k \in R^f} c_{fk}(x_{fk}^F - x_{fk}^C)$.

The coalition rationality implies that, under the MM policy, no coalition A of flights has an incentive to part company with $N \backslash A$ and establish cooperation on its own.

In general, the dual of problem (7) may present multiple solutions and then multiple slot prices v_j, and hence profits p_f, can be determined. If the ATM authority centrally solves problem (7), it could select particular values of v_j to make the payoff allocations/profits enjoy further properties such as being pairwise-monotonic (Miquel, 2008) to try to have that all flights obtain non-null profits.

It is important to note that there are some situations where a flight may find convenient to misrepresent its true cost of delay to increase its payoff, at the expense of other flights, as shown in Appendix A. This is not surprising due to the impossibility theorem of Myerson and Satterthwaite (1983), since we require individual rationality and budget balance constraints to hold.

4   Distributed market-based mechanisms

The practical implementation of the $MINCOST$ policy requires each flight f to communicate to the ATM authority its complete set of values c_{fj} for each slot $j \in R^f$, in order to determine the optimal allocation and the related payments. The optimal exchange is then calculated centrally according to this information, thus implementing a single-round, sealed-bid type of exchange (de Vries and Vohra, 2003). This type of mechanism

suffers from a) the high computational effort for Airlines for calculating the complete sets of requests and the value associated, b) the high communication cost of sending the complete set of values over a communication network, c) the complete disclosure for Airlines of private information, which might be considered as private values in a highly competitive environment as commercial aviation (Martin et al., 1998), and d) the lack of dynamism, since all bids from participants must be communicated before a deadline. To overcome these drawbacks, we propose a distributed market mechanism to reallocate the slots in accordance with the $MINCOST$ policy. This mechanism directly involves each airline in the decision process of the slot reallocation and does not require the disclosure of the delay costs. As a side a effect, it also puts the cost misrepresentation under a different perspective. When a centralized market mechanism as MM is implemented, an airline misrepresenting its costs is actually lying to an authority that pursues a common objective and then should be possibly sanctioned. On the contrary, when a distributed market mechanism is implemented, the same cost misrepresentation can be seen as a part of a bargaining process where each airline pursues its own interest.

4.1 Lagrangian Relaxation

A classical dual decomposition can be used to transform the centralized allocation problem into a distributed market mechanism. In fact, by dualizing constraints (9) the corresponding Lagrangian formulation of problem (7) is:

$$\max_{\lambda} \min_{x} \sum_{f \in F} \sum_{j \in R^f} c_{fj} x_{fj} +$$

$$\sum_{j \in S} \lambda_j \left(\sum_{f \in F: E_f \leq U_j} (x_{fj} - x_{fj}^F) \right) \tag{11a}$$

$$\sum_{j \in R^f} x_{fj} = 1 \qquad \forall f \tag{11b}$$

$$x_{fj} \geq 0 \qquad \forall f \in F, j \in R^f \tag{11c}$$

$$\lambda_j \geq 0 \qquad \forall j \in S \tag{11d}$$

As problem (7) is a continuous linear problem, the optimal Lagrangian multipliers λ_j^* in problem (11) are equal to the optimal dual variables v_j^* associated to constraints (9), i.e., $\lambda_j^* = v_j^* \; \forall j \in S$. Hence when the optimal Lagrangian multipliers are identified, the optimal value of function (11a) is equal to the optimal value of function (7a) due to the complementary slackness conditions, and also all constraints (9) (or 7b), (7c) and (7d) are satisfied. Then the optimal solution to problem (11) simultaneously allocates slots in accordance with the $MINCOST$ policy and computes all the optimal slot values v_j^*. We further notice that the objective function (11a) is separable into F functions, one for each flight, and that constraints (11b) and (11c) are also separable in term of flights, thus allowing the decomposition of problem (11) into F subproblems. We now exploit the above properties to show how to implement the market mechanism without requiring the flights to disclose their delay costs to the ATM authority.

The following iterative procedure, referred as LR algorithm, determines an optimal solution for problem (11). Given some tentative values λ_j, for all the slots $j \in S$, each flight f solves its own subproblem:

$$x_f^* = \operatorname*{argmin}_{x} \sum_{j \in R^f} c_{fj} x_{fj} + \sum_{j \in R^f} \lambda_j (x_{fj} - x_{fj}^F) \tag{12a}$$

$$\sum_{j \in R^f} x_{fj} = 1 \tag{12b}$$

$$x_{fj} \geq 0 \quad \forall j \in R^f. \tag{12c}$$

Then, each flight f communicates to the ATM authority its slot request $x_f^* = \{x_{fj}^*, \forall j \in R^f\}$ and the ATM authority checks for compatibility of the set of requests $x^* = \{x_f^*, \forall f \in F\}$ with a feasible slot allocation, where at most one flight is allocated to each slot (constraints (7b)). If no feasible solution is found, the ATM

authority updates the slot tentative values λ_j. In particular, at each iteration the ATM authority sets slot prices by increasing the value obtained at the previous iteration for an over-demanded slot and by decreasing it for an unused one, always assigning a positive value, according to a subgradient algorithm (see, e.g., Bertsekas, 1999). If constraints (7b) on slot capacity are respected, slots are allocated according to the $MINCOST$ policy. Otherwise the current prices are communicated to flights, which in turn solve again their local optimization model (12) and answer with a new slot request $x_f^* = \{x_{fj}^*, \forall j \in R^f\}$ for all $f \in F$. If a feasible solution cannot be achieved after a pre-defined number of iterations, then slots are allocated according to the $FPFS$ policy. However, our computational experiments based on real instances easily converge, as described in the next Section 5. Note that this procedure allows the ATM authority to allocate the slots without knowing the delay costs. In fact, the knowledge of the delay costs is needed only for the solution of problem (12) that is solved privately by each flight.

4.2   Bertsekas' Auction Algorithm

An alternative approach to implement the $MINCOST$ policy in a distributed way is the Bertsekas' Auction Algorithm for the Assignment Problem (see, e.g., Bertsekas (1990)), referred in the following as BA algorithm. The BA algorithm implements an iterative ascending type of auction, which employs both bid and ask prices since bidders can increase slot prices proposed by the ATM authority, according to their valuation functions. Here we sketch the main principles of this algorithm as described in Bertsekas (1991). The algorithm starts with an empty assignment, i.e., all flights are unallocated, proceeds iteratively and terminates when a feasible assignment is reached. At each iteration the ATM authority proposes to one unallocated flight f the current slot prices b_j for all $j \in S$. Then f announces the slot j_f that minimizes its cost function, i.e.,

$$ j_f = \underset{j \in R^f}{\operatorname{argmin}} \{c_{fj} + b_j\}, $$

and proposes for this slot a higher bid price $\hat{b}_{j_f} = b_{j_f} + \gamma_i$ where $\gamma_i \geq 0$ is the cost caused by being allocated the second best slot rather than j_f, i.e., $\gamma_i = \min_{j \in R^f : j \neq j_f}\{c_{fj} + b_j\} - \min_{j \in R^f}\{c_{fj} + b_j\}$. The price of each slot is set equal to the highest bid and it is allocated to the correspondent bidder. If another flight was already allocated to this slot at the beginning of the iteration, it becomes again unallocated. This procedure iteratively continues until all flights $f \in F$ have a slot allocated. When this condition holds, it has been proven (Bertsekas, 1991) that this allocation is also optimal for problem (7) and that the associated prices b_j^* are equal to the optimal dual solutions v_j^* for all $j \in S$.

The BA algorithm allows to simultaneously allocate slots in accordance with the $MINCOST$ policy and to compute the optimal slot values v_j^* in a distributed way as was the case for the LR algorithm. In addition, the BA algorithm gives airlines a greater degree of control on the course of the auction, as it provides the possibility for the airlines to actively set slot prices through their bids. However, if on one side, the BA algorithm allows a price discovery more actively driven by users, on the other, it requires that the airlines are able to calculate their γ_i values. In fact, each flight internally calculates the bid price relying on the current slot values and on its own cost of delay. Then from all unallocated flights the final bid prices only are communicated to the ATM authority. In turn, the ATM authority only relying on these bid prices determines the new slot values and passes them back to the remaining unallocated flights. Differently, no internal computations are required in the LR setting.

5   Examples of application

In this section, we consider two case studies based on real operational data with aim of analyzing the performances of the LR and the BA algorithms introduced in the previous section. Both algorithms takes advantage of the fact that the asymmetric assignment problem (7) can be decomposed into a set of smaller symmetric assignment problems as shown in Lemma 2.

In the case of the BA algorithm, we solved the real instances presented next using the ϵ-Scaling Forward version for a symmetric assignment problem which allows to avoid degeneracy and increase the computational performance of the basic Auction Algorithm (see Section 4.1 in Bertsekas, 1991). Degeneracy occurs when a flight f is indifferent in exchanging its $FPFS$ slot as this exchange gives no profit to it, i.e., $p_f = 0$. In such a situation, to avoid cycling, the BA algorithm sets the profit p_f equal to $-\epsilon$ with $0 < \epsilon < \frac{1}{|B_{jk}^F|}$ (see Section 1.2.1 in Bertsekas, 1991).

For sake of simplicity and without loss of generality, in the following we assume that delay costs c_{fj} are proportional to the amount of delay, i.e., $c_{fj} = w_f * d_{fj}$ for all $f \in F$ and $j \in R^f$ where $w_f > 0$ is the cost of one minute of delay. In particular, in both cases we assume that, for each flight f, the tactical cost of one minute of ground delay w_f is a stochastic variable uniformly distributed between €5 and €20 per minute, in line with the figures provided in Cook et al. (2004).

5.1   Case A: en-route regulation

This case is representative of the situation in which several flights are affected by the same and unique regulation limiting the maximum capacity on ATC en-route sector. This was the case, for instance, on the 2nd August 2008 between 04:00AM and 06:00AM, when a regulation was limiting the number of flights entering the French sector LFEERESMI to 14 per hour, for ATC capacity reasons. There were 18 flights originally planned to enter this sector at different points in time between 4:18AM and 5:51AM. Given the baseline schedule obtained by applying the $FPFS$ policy, we simulated the two distributed market mechanisms: by tuning the appropriate parameters the LR algorithm converges to an optimal solution after 25 iterations, and the BA algorithm after 69 iterations.

Under the $FPFS$ policy, the overall delay is 91 minutes and the overall cost of delay is €1175. The $MINCOST$ policy provides a lower cost of delay (€736), but a slightly higher overall delay (93 minutes).

Table 2 shows the different slots and their associated values for both algorithms. The slots in boldface are the ones allocated to a flight under both the $FPFS$ and the $MINCOST$ policies. Table 3 displays the main pieces of information associated to each flight f. The second and the third column respectively report the cost of the delay w_f and the estimated time of entry E_f into sector LFEERESMI. The fourth and the fifth column respectively show the optimal slot allocations x^F and x^C along with the exact time of entry into the sector. Finally, the sixth and the seventh column respectively present the profit obtained by each flight using the LR and the BA algorithm. We notice that the profit of an individual flight may change depending on the algorithm as problem (7) may have multiple optimal dual solutions. However, the sum of all the profits is constant (€439 = €1175 - €736) as shown by Equation (5). In Table 3 the flights in boldface are the ones that do not exchange the slot allocated to them under the $FPFS$ policy. In particular, holding Lemma 2, flights 5, 17 and 18 cannot move from their $FPFS$ slots.

Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}	Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}
S1	04:00 - 04:03	0	0	**S15**	**05:00 - 05:03**	222.78	392.52
S2	04:04 - 04:07	0	0	**S16**	**05:04 - 05:07**	168.32	326.87
S3	04:08 - 04:11	0	0	**S17**	**05:08 - 05:11**	126.49	271.47
S4	04:12 - 04:16	0	0	**S18**	**05:12 - 05:16**	90.20	234.47
S5	**04:17 - 04:20**	0	105.93	**S19**	**05:17 - 05:20**	45.28	194.07
S6	**04:21 - 04:24**	0	74.10	**S20**	**05:21 - 05:24**	21.11	170.15
S7	**04:25 - 04:29**	57.96	64.77	**S21**	**05:25 - 05:29**	0	159.23
S8	**04:30 - 04:33**	33.71	24.93	S22	05:30 - 05:33	0	0
S9	**04:34 - 04:37**	11.00	0.77	**S23**	**05:34 - 05:37**	0	0
S10	04:38 - 04:41	0	0	S24	05:38 - 05:41	0	0
S11	**04:42 - 04:46**	338.21	554.47	S25	05:42 - 05:46	0	0
S12	**04:47 - 04:50**	336.11	544.03	S26	05:47 - 05:50	0	0
S13	**04:51 - 04:54**	308.40	467.95	**S27**	**05:51 - 05:54**	0	0
S14	**04:55 - 04:59**	276.87	435.87	S28	05:55 - 05:59	0	0

Table 2: Slot Table - Case A (LFEERESMI)

Flight f	w_f	E_f	$FPFS$ Policy Slot (Time)	$MINCOST$ Policy Slot (Time)	p_f^{LR}	p_f^{BA}
F1	16	04:18	S5 (04:18)	S5 (04:18)	0	0
F2	17	04:24	S6 (04:24)	S6 (04:24)	0	0
F3	8	04:25	S7 (04:25)	S7 (04:25)	0	0
F4	6	04:26	S8 (04:30)	S8 (04:30)	0	0
F5	6	04:36	S9 (04:36)	S9 (04:36)	0	0
F6	14	04:44	S11 (04:44)	S11 (04:44)	0	0
F7	9	04:45	S12 (04:47)	S18 (05:12)	20.91	84.56
F8	6	04:46	S13 (04:51)	S20 (05:21)	107.29	117.80
F9	19	04:47	S14 (04:55)	S12 (04:47)	92.76	43.84
F10	10	04:48	S15 (05:00)	S17 (05:08)	16.29	41.05
F11	16	04:53	S16 (05:04)	S13 (04:53)	35.92	34.92
F12	13	04:54	S17 (05:08)	S14 (04:55)	18.62	4.60
F13	17	05:00	S18 (05:12)	S15 (05:00)	71.42	45.95
F14	15	05:04	S19 (05:17)	S16 (05:04)	71.96	62.20
F15	7	05:12	S20 (05:21)	S19 (05:17)	3.83	4.08
F16	11	05:24	S21 (05:25)	S21 (05:25)	0	0
F17	18	05:37	S23 (05:37)	S23 (05:37)	0	0
F18	8	05:51	S27 (05:51)	S27 (05:51)	0	0

Table 3: Flight Table - Case A (LFEERESMI)

5.2 Case B: arrival airport regulation

This case is representative of the situation in which there is a limitation on the maximum number of flights arriving at an airport during a certain time period. We found that London City Airport (EGLC) was imposing a regulation on the 4th August 2008, limiting the arrival rate to 18 flight per hour, between 06:00AM and 07:30AM. This regulation was the only one affecting 24 flights. In this case, the LR algorithm converges to an optimal solution after 39 iterations, and the BA algorithm after 38 iterations.

Under the $FPFS$ policy, the overall delay is 73 minutes and the overall cost of delay is €957. The $MINCOST$ policy provides a lower cost of delay (€631), but a slightly higher overall delay (77 minutes).

Table 4 shows the different slots and their associated values for both algorithms. The slots in boldface are the ones allocated to a flight under both the $FPFS$ and the $MINCOST$ policies. In Table 5 the second and the third column respectively report the cost of the delay w_f and the estimated time of entry E_f at EGCL. The fourth and the fifth column respectively show the optimal slot allocations x^F and x^C along with the exact time of entry. Finally, the sixth and the seventh column respectively present the profit obtained by each flight using the LR and the BA algorithm. Clearly, the sum of all the profits is constant (€326 = €957 - €631) as shown by Equation (5). In Table 5 the flights in boldface are the ones that do not exchange the slot allocated to them under the $FPFS$ policy. In particular, holding Lemma 2, flights 15 and 24 cannot move from their $FPFS$ slots. Finally, this example shows a case where degeneracy occurs. According to the slot values computed with the BA algorithm, flight F3 does not have any economic profit in moving from slot S3 to S4. The resulting negative profit $p_3 = -0.06$ is due to the introduction of the the perturbation constant $\epsilon > 0$ to prevent cycling, and should be interpreted as zero, i.e., $p_3 = 0$.

6 Conclusions

This paper deals with the slot allocation problem for a single constrained Air Traffic Network resource, e.g., an en-route sector or an airport. The market mechanism proposed shows several characteristics that make it preferable to the First Planned First Served policy, currently adopted in Europe to sequence flights whenever an imbalance between planned traffic and resource capacity occurs on the day of operations. First, it produces a slot allocation that makes every flight economically better off. High-valued flights can reduce their delays by acquiring resources while low-priority flights may be reimbursed for the resources they relinquish. Second, the ATM authority is neutral with respect to the final outcome of the slot exchanges as it is not economically

Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}	Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}
S1	**06:00 - 06:02**	0	437.41	**S15**	**06:46 - 06:49**	27.78	125.51
S2	**06:03 - 06:05**	0	448.23	S16	06:50 - 06:52	0	0
S3	**06:06 - 06:09**	311.83	428.33	**S17**	**06:53 - 06:55**	133.04	34.56
S4	**06:10 - 06:12**	284.36	410.39	**S18**	**06:56 - 06:59**	116.96	20.67
S5	**06:13 - 06:15**	264.71	422.35	**S19**	**07:00 - 07:02**	105.26	46.56
S6	**06:16 - 06:19**	276.66	428.25	**S20**	**07:03 - 07:05**	73.31	16.67
S7	**06:20 - 06:22**	259.94	393.41	**S21**	**07:06 - 07:09**	105.23	209.56
S8	**06:23 - 06:25**	239.69	366.41	**S22**	**07:10 - 07:12**	88.46	200.67
S9	**06:26 - 06:29**	200.50	313.41	**S23**	**07:13 - 07:15**	71.61	187.78
S10	**06:30 - 06:32**	157.10	273.48	**S24**	**07:16 - 07:19**	46.60	173.89
S11	**06:33 - 06:35**	126.57	239.41	S25	07:20 - 07:22	0	0
S12	**06:36 - 06:39**	90.80	204.41	**S26**	**07:23 - 07:25**	0	0
S13	**06:40 - 06:42**	60.11	164.51	S27	07:26 - 07:29	0	0
S14	**06:43 - 06:45**	31.50	143.58	-	-	-	-

Table 4: Slot Table - Case B (EGCL)

involved. In fact, it is not requested to subsidize the market players neither it can make any profit from the trading process. Third, under the above conditions the allocative efficiency is maximized both from the social welfare and market players' perspectives. Finally, the mechanism is distributed as it actively involves airlines in the slot allocation process. They express their preferences over different feasible solutions to the slot allocation problem in the spirit of Collaborative Decision Making, as advocated by the SESAR operational concept.

This research may be generalized to the definition of a market-based policy for slot trading when flights are concerned with multiple regulated resources. The use of combinatorial exchanges seems to be a promising approach to deal with this last complex problem.

References

Andreatta, G., Romanin-Jacur, G., 1987. Aircraft flow management under congestion. Transportation Science 21 (4), 249–253.

Ball, M. O., Donohue, G. L., Hoffman, K., 2006. Combinatorial Auctions. MIT Press, Cambridge, MA, Ch. Auctions for the Safe, Efficient and Equitable Allocation of Airspace System Resources, pp. 507–538.

Ball, M. O., Lulli, G., 2004. Ground delay programs: optimizing over the included flight set based on distance. Air Traffic Control Quarterly 12, 1–25.

Bertsekas, D. P., 1990. The auction algorithm for assignment and other network flow problems: A tutorial. Interfaces 20 (4), 133–139.

Bertsekas, D. P., 1991. Linear Network Optimization, Algorithms and Codes. The MIT Press, Cambridge, MA.

Bertsekas, D. P., 1999. Nonlinear Programming, 2nd Edition. Athena Scientific, Belmont, MA.

Bertsimas, D., Lulli, G., Odoni, A. R., 2008. An integer optimization approach to large-scale air traffic flow management. Operations Research In press.

Bertsimas, D., Stock Patterson, S., 1998. The air traffic flow management problem with enroute capacities. Operations Research 46 (3), 406–422.

Borm, P., Hamers, H., Hendrickx, R., 2001. Operations research games: A survey. TOP 9, 139–216.

Chang, K., Howard, K., Oiesen, R., Shisler, L., Tanino, M., Wambsganss, M. C., 2001. Enhancements to the FAA Ground-Delay Program Under Collaborative Decision Making. Interfaces 31 (1), 57–76.

Cook, A., Tanner, G., Anderson, S., 2004. Evaluating the True Cost to Airlines of One Minute of Airborne or Ground Delay. University of Westminster.

Curiel, I. J., Tijs, S. H., 1986. Assignment Games and Permutation Games. Methods of Operations Research 54, 323–334.

Flight f	w_f	E_f	$FPFS$ Policy Slot (Time)	$MINCOST$ Policy Slot (Time)	p_f^{LR}	p_f^{BA}
F1	7	06:01	S1 (06:01)	S1 (06:01)	0	0
F2	10	06:03	S2 (06:03)	S2 (06:03)	0	0
F3	10	06:08	S3 (06:08)	S4 (06:10)	9.47	-0.06
F4	7	06:08	S4 (06:10)	S13 (06:40)	14.25	35.88
F5	14	06:08	S5 (06:13)	S3 (06:08)	22.88	64.02
F6	16	06:15	S6 (06:16)	S5 (06:15)	27.95	21.90
F7	19	06:18	S7 (06:20)	S6 (06:18)	21.28	3.16
F8	14	06:19	S8 (06:23)	S7 (06:20)	21.75	15.00
F9	6	06:21	S9 (06:26)	S14 (06:43)	67.00	67.83
F10	19	06:22	S10 (06:30)	S8 (06:23)	50.41	40.07
F11	11	06:22	S11 (06:33)	S9 (06:26)	3.07	3.00
F12	20	06:28	S12 (06:36)	S10 (06:30)	53.70	50.93
F13	10	06:30	S13 (06:40)	S12 (06:36)	9.31	0.10
F14	12	06:33	S14 (06:43)	S11 (06:33)	24.93	24.17
F15	15	06:46	S15 (06:46)	S15 (06:46)	0	0
F16	18	06:55	S17 (06:55)	S17 (06:55)	0	0
F17	14	06:55	S18 (06:56)	S18 (06:56)	0	0
F18	11	07:00	S19 (07:00)	S19 (07:00)	0	0
F19	10	07:00	S20 (07:03)	S20 (07:03)	0	0
F20	17	07:09	S21 (07:09)	S21 (07:09)	0	0
F21	9	07:09	S22 (07:10)	S22 (07:10)	0	0
F22	13	07:12	S23 (07:13)	S23 (07:13)	0	0
F23	14	07:15	S24 (07:16)	S24 (07:16)	0	0
F24	8	07:23	S26 (07:23)	S26 (07:23)	0	0

Table 5: Flight Table - Case B (EGCL)

de Vries, S., Vohra, R. V., 2003. Combinatorial auctions: A survey. INFORMS Journal on Computing 15 (3), 284–309.

EUROCONTROL, 2006. Airport CDM Operational Concept Document.

EUROCONTROL CFMU, 2007. Air Traffic Flow and Capacity Management Operations Users Manual.

EUROCONTROL EEC, 1999. Report of the Ad-hoc Expert Group on CDM.

EUROCONTROL PRC, 2008. An Assessment of Air Traffic Management in Europe during the Calendar Year 2007 - Performance Review Report.

Fukui, H., 2009. An empirical analysis of airport slot trading in the United States. Transportation Research Part B In press.

Hoffman, R., Ball, M. O., 1997. A comparison of formulations for the single-airport ground holding problem with banking constraints. Tech. rep., Institute for System Research - University of Maryland.

Krishna, V., 2009. Auction Theory, 2nd edition. Academic Press.

Leal de Matos, P., Omerod, R., 2000. The application of operational research to European air traffic flow management - understanding the context. European Journal of Operational Research 123 (1), 125–144.

Martin, P., Hudgell, A., Bouge, N., Vial, S., 1998. Investigation of improved information distribution for the realisation of collaborative decision making. 2nd USA and Europe ATM R&D Seminar, Orlando, FL.

Miquel, S., 2008. A pairwise-monotonic core selection for permutation games. Mathematical Methods of Operations Research In press.

Myerson, R., Satterthwaite, M., 1983. Efficient mechanisms for bilateral trading. Journal of Economic Theory 29 (2), 265–281.

Odoni, A. R., 1987. Flow Control of Congested Networks. Springer-Verlag, Berlin, Germany, Ch. The flow management problem in air traffic control.

Rassenti, S., Smith, V., Bulfin, R., 1982. A combinatorial auction mechanism for airport time slot allocation. Bell Journal of Economics 13 (2), 402–417.

SESAR Joint Undertaking, 2007. SESAR Milestone Deliverable D3 - The ATM Target Concept.

Sieg, G., 2009. Grandfather rights in the market for airport slots. Transportation Research Part B In press.

Verhoef, E. T., 2009. Congestion pricing, slot sales and slot trading in aviation. Transportation Research Part B In press.

Vossen, T., Ball, M. O., 2006. Optimization and Mediated Bartering Models for Ground Delay Programs. Naval Research Logistics 53, 75–90.

Vranas, P. B., Bertsimas, D., Odoni, A. R., 1994. The multi-airport ground-holding problem in air traffic control. Operations Research 42 (2), 249–261.

Waslander, S. L., Raffard, R. L., Tolmin, C. J., 2008. Market-Based Air Traffic Flow Control with Competing Airlines. Journal of Guidance, Control and Dynamics 31, 148–161.

A Discussion on Incentive Compatibility

In the following we provide a simple example where a flight may find convenient to misrepresent its true cost of delay to increase its payoff, at the expense of other flights.

Consider two flights, f_1 and f_2, and two slots, a and b. Both flights request slot a. Then we set $c_{1a} = c_{2a} = 0$. We assume that $0 < c_{1b} < c_{2b}$ and that $E_1 < E_2$. The $FPFS$ policy allocates flight f_1 at slot a and flight f_2 at slot b. Let p_1 and p_2 be the profits of flight f_1 and f_2, respectively, obtained by selling its $FPFS$ slot and purchasing the other slot. We finally assume that flights do not know the costs of delay of each other and that the $MINCOST$ policy is centrally implemented by the ATM authority through problem (7). We want to investigate the opportunity for flight f_1 to cheat about the true value of its delay to get a higher profit. Let c'_{1b} be the false value of the delay cost displayed by flight f_1 for slot b, and v'_a, v'_b and p'_1 be the corresponding modified slot prices and profit for flight f_1, respectively.

Flight f_1 (f_2) has a nonnegative profit in selling its slot a (b) and purchasing the other slot b (a) at prices v_a (v_b) and v_b (v_a), respectively. In particular,

$$p_1 = c_{1a} - c_{1b} + v_a - v_b$$
$$p_2 = c_{2b} - c_{2a} + v_b - v_a$$

where v_a and v_b are the optimal solutions of the following problem, the dual of problem (7):

$$
\begin{aligned}
\max Z \quad = \quad & m_1 + m_2 - v_a - v_b \\
m_1 - v_a \quad \leq \quad & 0 \\
m_1 - v_b \quad \leq \quad & c_{1b} \\
m_2 - v_a \quad \leq \quad & 0 \\
m_2 - v_b \quad \leq \quad & c_{2b} \\
& m_1, m_2, v_a, v_b \geq 0
\end{aligned}
\tag{13}
$$

In the (v_a, v_b) space the optimal region is $c_{1b} \leq v_a - v_b \leq c_{2b}$. This optimal region has only two finite vertices, i.e., $(v_a = c_{1b}, v_b = 0)$ and $(v_a = c_{2b}, v_b = 0)$. Using a standard algorithm, as the simplex or the dual simplex algorithm, to solve problem (13), the optimal solution is always point $(v_a = c_{1b}, v_b = 0)$. Hence the profit of flight f_1 in selling its slot a at price $v_a = c_{1b}$ and purchasing the slot b at price $v_b = 0$ is $p_1 = 0 - c_{1b} + c_{1b} - 0 = 0$. As long as $c'_{1b} \leq c_{2b}$ the slot allocation remains the same. Hence the optimal region is $c'_{1b} \leq v'_a - v'_b \leq c_{2b}$. Then the optimal solution obtained by the simplex algorithm is $(v'_a = c'_{1b}, v'_b = 0)$. Hence the modified profit

is $p'_1 = c_{1a} - c_{1b} + v'_a - v'_b = 0 - c_{1b} + c'_{1b} - 0$. Then $p'_1 - p_1 = c'_{1b} - c_{1b}$. Then if $0 \leq c'_{1b} < c_{1b}$ it follows that $p'_1 - p_1 < 0$, and if $c_{1b} < c'_{1b} < c_{2b}$ we have $p'_1 - p_1 > 0$. When $c'_{1b} > c_{2b}$, the slot allocation changes and becomes identical to the $FPFS$ allocation. Hence in this case $p'_1 = 0$.

As the value c_{2b} is not known to flight f_1, we conclude that the flight taking the first slot under the $FPFS$ allocation knows that it does not have to display a false delay cost lower that the true one because this choice may lead to a profit p'_1 lower than the true profit p_1. On the other side, if this flight f_1 communicates to the ATM authority a false delay cost c'_{1b} higher than the true one, the profit p'_1 it gets is higher than or equal to the true profit p_1. Then our mechanism is not incentive compatible because there is no disadvantage for the first flight in the $FPFS$ allocation to appropriately misrepresent its delay costs.

We remind that these findings assume that flights know in advance which is the vertex of the optimal region of problem (13) chosen by the solving algorithm. On the contrary, when we consider as slot prices a generic pair of optimal values (v_A, v_B) the corresponding profit p_1 can be strictly positive, as MM is by construction individual rational. In this situation, it can be risky for flight f_1 to cheat about its cost of delay c_{1b}. In fact, if it sets its false value c'_{1b} strictly larger than c_{2b} its profit p'_1 is equal to 0. Since c_{2b} is unknown to flight f_1, a misrepresentation of its cost of delay may produce a profit p'_1 lower than the true p_1.

We conclude that when we do not know in advance which are the optimal slot values, we are unable to identify up to which limit a false value of the delay cost does not lead to a profit $p'_1 < p_1$.