

# Slot Trading Opportunities in Collaborative Ground Delay Programs

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The Federal Aviation Administration (FAA) and the major airlines in the United States have embraced a new initiative to improve air traffic flow management. This initiative, called collaborative decision making (CDM), is based on the recognition that improved data exchange and communication between the FAA and the airlines will lead to better decision making. In particular, the CDM philosophy emphasizes that decisions with a potential economic impact on airlines should be decentralized and made in collaboration with the airlines whenever possible.

The CDM paradigm has led to fundamental changes in the implementation of ground delay programs. A key component has been the introduction of the compression procedure, which allows for the exchange of arrival slots between airlines. In this paper, we consider opportunities for increased airline control by interpreting the compression procedure as a mediated slot trading mechanism. Based on this interpretation, we propose an extension that allows airlines to submit so-called at-least, at-most offers. We develop an efficient integer programming model to solve the mediator's problem, and show that the resulting mechanism can substantially improve the ability of airlines to optimize their internal cost functions.

*Key words:* air traffic flow management; ground delay programs; collaborative decision making; slot exchange

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## 1. Introduction

The Federal Aviation Administration (FAA) issues ground delay programs (GDPs) to manage temporary reductions in an airport's arrival capacity. In a GDP, flights bound for congested airports are delayed on the ground (prior to their departure), so as to balance the arrivals with the reduced capacity at the airport under consideration. The underlying motivation is that, as long as a delay is unavoidable, it is both safer and less costly for the flight to absorb this delay on the ground. The allocation of ground delays can be interpreted as a resource allocation problem in which available arrival capacity (i.e., a sequence of arrival slots) has to be distributed among the inbound flights. Given the allocation of arrival slots to flights, it is straightforward to determine the resulting ground delays. It is, however, important to note that the resulting allocation problem is complicated by a number of factors that are peculiar to the context in which GDPs are implemented. First, GDPs are intrinsically dynamic in nature because of the inherent uncertainty related to weather predictions. GDPs, for instance, are implemented anywhere from six to two hours before the flights are actually scheduled to land, leading to a very short time frame

for making decisions. In addition, the implementation of a GDP can have a severe negative impact on an airline's schedule integrity, given that capacity reductions of 50% for extended periods of time are not uncommon. Finally, it is worthwhile to note that GDPs, in fact, occur frequently. Between September of 1998 and April of 1999, for instance, there was an average of 1.6 GDPs per day (Hall 1999).

GDPs, as well as other traffic flow management initiatives, have been used by the FAA for nearly 20 years (see Nolan 1999 for their origins). The advent of the collaborative decision making (CDM) paradigm, however, has had a profound effect on the implementation of traffic flow management initiatives. CDM is based on the recognition that improved data exchange and communication between the FAA and the airlines will lead to better decision making (Wambsganss 1996). In particular, the CDM philosophy emphasizes that decisions with a potential economic impact on airlines should be decentralized and made in collaboration with the airlines whenever possible. While the CDM paradigm applies to a wide range of applications in air traffic flow management (ATFM), the primary focus so far has been the implementation of GDP enhancements.

The number of enhancements that have recently been implemented under CDM are numerous: examples include improved data exchange, better situational awareness tools, and increased flexibility for airlines (see Ball et al. 1998, 2000). However, the most significant improvements have been introduced by fundamental changes in the allocation of arrival capacity. This approach is based on the consensus recognition that airlines have claims on the available arrival capacity, according to their original flight schedules. This realization has led to a fundamental change in the way arrival capacity is allocated, as well as the introduction of a procedure for interairline slot exchange. Under CDM, arrival capacity is allocated to airlines by a procedure called *ration by schedule* (RBS). This procedure has removed disincentives airlines previously had to provide accurate information about delays and cancellations. In addition, CDM has led to the introduction of a new procedure for interairline slot exchange called *compression*. This procedure seeks to maximize utilization of the available arrival capacity in the presence of delays and cancellations and attempts to do so in a fair and equitable manner.

This paper is motivated by the interairline exchange of slots that currently occurs in the compression algorithm. The introduction of the compression algorithm has had a major impact, and is said to have enabled significant reductions in ground delay (Chang et al. 2001). Our main objective is to explore the potential benefits of enhancing the slot exchange capability. First, we interpret the exchange of slots as a form of bartering in which airlines may submit slot trading offers to the FAA, which acts as the central coordinator (see also Vossen 2002). Subsequently, we propose an optimization model for the FAA's mediation problem, which must determine the set of offers to accept. This approach generalizes current slot exchange procedures in that it allows airlines to submit so-called "at-least, at-most" offers; such offers may be viewed as trade-offs between pairs of flights and are motivated by differences in their marginal delay costs. To analyze the potential benefits of this approach, we consider two case studies that use different models of airline decision making: (1) airlines aim to maximize their on-time performance and (2) airlines aim to minimize passenger delay costs. We consider experimental results that are based on historical GDP data to show that, on the whole, slot trading may yield significant benefits over the current slot exchange mechanisms.

This paper is organized as follows. Section 2 provides background on GDPs, in particular, on the procedures used under CDM, and on the role of interairline slot exchange. Section 3 discusses our general approach to increasing slot trading opportunities

and describes the optimization models we propose to select a set trades to implement. Section 4 presents the results of our extensive computational experiments. Finally, §5 provides conclusions and avenues for further research.

## 2. Background

The use of ground holding to resolve air traffic congestion was first described systematically by Odoni (1987). The generic flow management problem defined by Odoni is extremely general in that it addresses congestion anywhere in the network. However, he provides a robust rationale for the practical relevance of models where the only capacitated element considered is the airport arrival mechanism. Under this assumption, the problem is commonly known as the ground holding problem (GHP). The basic version of the GHP (see Terrab 1990) assumes a discrete time horizon, deterministic demand, and deterministic capacity. Given these assumptions, the GHP can be formulated as an assignment problem. Most models that address the GHP, however, concentrate on the trade-off between airborne and ground delays in the presence of stochastic capacity. This version of the GHP was first studied by Odoni (1987) and Andreatta and Romanin-Jacur (1987). More efficient models, as well as several extensions, have also been proposed (Terrab and Odoni 1993; Richetta and Odoni 1993; Richetta 1995; Ball et al. 2003). A systematic review of some of these results may be found in Andreatta, Odoni, and Richetta (1993). Other related work has focused on different aspects of the GHP, in particular, on the effects of delay propagation through the air traffic network (Vranas, Bertsimas, and Odoni 1994a, b; Andreatta and Brunetta 1993) and on more general air traffic flow management problems (Bertsimas and Stock-Patterson 1998, 2000). The focus on aggregate trade-offs between airborne and ground delays limits the attention that can be given to airline-specific preferences. Even though airline-specific delay costs could, in principle, be incorporated into the decision problems, the "global optimization" perspective would likely introduce systematic biases against or in favor of individual airlines. That is, most ground holds would be assigned to aircraft with smaller per unit delay costs (e.g., regional aircraft), while aircraft with higher delay costs (e.g., wide-body aircraft) would be given priority (Odoni 1987). Consequently, the models described here are perhaps primarily suited for making aggregate decisions (e.g., determining overall flow rates per period (Ball et al. 2000)).

In contrast to the models proposed in the literature, which allocate delays to individual flights, the

allocation procedures instituted under CDM primarily address the distribution of delays among airlines. CDM has its origin in early efforts by the FAA to acquire up-to-date airline schedule information (Wambsganss 1996). Though human-in-the-loop experiments with ATFM specialists clearly showed the benefits of this information, airlines remained highly reluctant to submit this data. The reason for this was that the GDP procedures used at the time could actually penalize an airline for providing that information (see Vossen 2002). The resource allocation schemes implemented under CDM have addressed these issues through a fundamental change in the allocation of capacity. Rather than an assignment of individual flights to arrival slots, the central paradigm under CDM is that slots are allocated to airlines. This has led to the introduction of two new allocation mechanisms: RBS and compression.

The RBS algorithm creates an initial allocation of slots to airlines based on the consensus recognition that airlines have claims on the available arrival capacity through the original flight schedules. Under RBS, flights are assigned to slots according to their *original scheduled time of arrival* as opposed to the *most recent estimated time of arrival* that was used before. Consequently, airlines will not forfeit a slot by reporting a delay or a cancellation, which is what happened prior to CDM. It should be noted that the resulting flight schedule may be inefficient in its utilization of arrival capacity. Arrival slots may have been assigned to flights that have been cancelled or delayed, and therefore cannot use their assigned slot. However, the end result of RBS should not be viewed as an assignment of slots to flights but rather as an assignment of slots to airlines. After this initial allocation, airlines are free to reschedule flights according to their private objectives, through flight substitutions and cancellations.

After a round of substitutions and cancellations, the utilization of slots can usually be improved. The reason for this is that the resultant schedule, which results after the airlines perform a round of flight substitutions and cancellations, may have “holes” because of the high degree of disruption to airline schedules that can occur on bad weather days. In other words, there will be arrival slots that have no flights assigned to them. The purpose of the compression algorithm is to move flights up in the schedule to fill these slots. The basic idea behind the compression algorithm is that airlines are “paid back” for the slots they release, so as to encourage airlines to report cancellations. To illustrate the compression algorithm, let us consider the example shown in Figure 1. The left-hand side of the figure represents the flight-slot assignment prior to the execution of the compression algorithm. Associated with each flight is an earliest

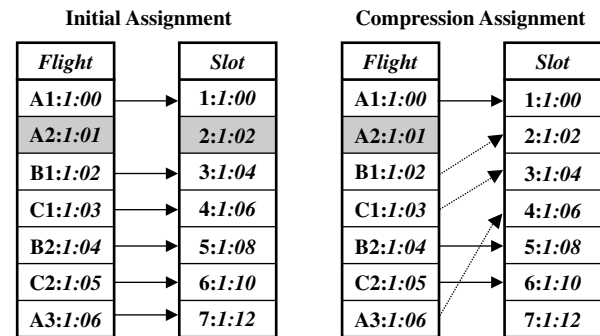


Figure 1 Compression Example

time of arrival, and each slot has an associated slot time. Note that there is one cancelled flight, A2. The right-hand side of the figure shows the flight schedule after execution of the compression algorithm: as a first step, the algorithm attempts to fill slot 2 owned by airline A. Because there is no flight from airline A that can use the slot, the slot is allocated to flight B1 and airline A takes ownership of slot 3. This process is repeated for slot 3, which results in the assignment of flight C1 to slot 3 and transfer of ownership of slot 4 to A. Now, slot 4 can be used by airline A and will be assigned to flight A3. The important features of the compression algorithm are that (1) arrival slots are filled whenever possible, (2) flights from the airline that owns the current open slot are considered before all others, (3) if the controlling airline cannot use a slot, then it is compensated by receiving control over the slot vacated by the flight that moves into its slot, and (4) airlines do not involuntarily lose slots they own and can use (see Vossen 2002 for a more detailed description).

From an airline standpoint, the ability to substitute and cancel flights is clearly the single most important aspect of a GDP. The increased control allows an airline to mitigate the disruptions to its flight schedule, and address the potential downstream effects of ground delays. A clear indication of their importance follows by considering Figure 2, which shows flight substitution and cancellation patterns, using empirical results from actual GDPs at O'Hare Airport during July 2000. The left-hand side of the graph in Figure 2 represents the cumulative number of flights (as a percentage of the total number of flights that have been allocated a slot) that are substituted during the course of a GDP day (note that percentages can be greater than 100 because a single flight can be involved in multiple substitutions). Time 0 corresponds to the first time instance at which each flight was first allocated a slot as a result of the RBS procedure, and each curve corresponds to one GDP day. It should be noted that flight substitutions because of the compression algorithm or GDP revisions were not included; substitutions of cancelled flights were

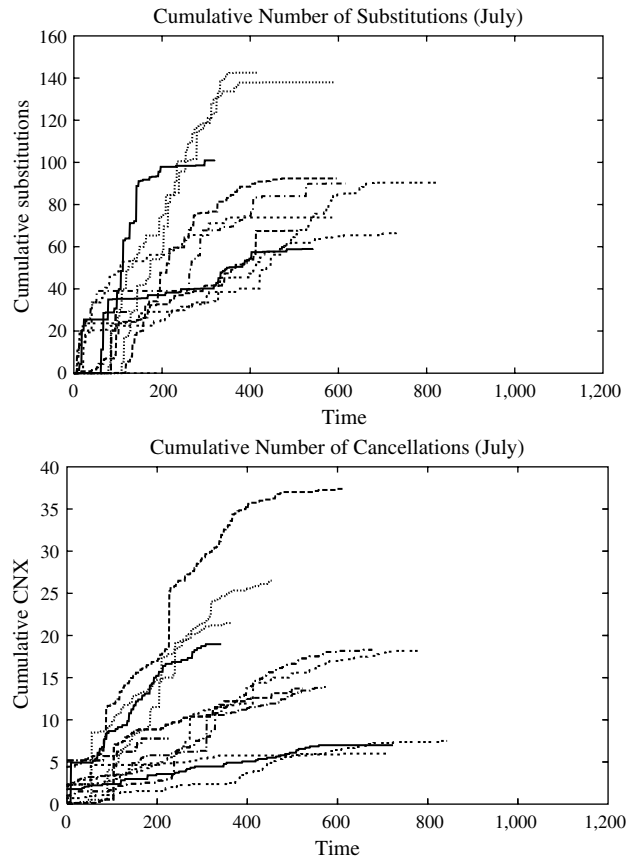


Figure 2 Airline GDP Behavior at O'Hare Airport (July 2000)

not included either. Similarly, the right-hand side of the graph in Figure 2 represents the cumulative percentage of flights that have been cancelled during the course of a GDP. The graphs in Figure 2 show first that airlines perform a large number of flight substitutions and cancellations, and second that airlines perform flight transactions throughout the course of GDP.

Given the sheer volume of flight substitutions, it is not difficult to imagine that potential benefits could be obtained by allowing the exchange of slots between *different* airlines. That is, by coordinating their flight schedule adjustments, airlines might be able to achieve mutual benefits that they would not be able to achieve by themselves. This, of course, is already inherent in the compression procedure: slots that an airline cannot use (e.g., because of flight cancellations) are exchanged in such a way that all parties involved will receive a reduction in their flight delays. Using the compression procedure and its reported benefits as a starting point, one could envision more general exchange mechanisms. In fact, a more dynamic form of slot exchange functionality, known as *slot credit substitutions* (SCS), has been defined by the CDM working group (Howard 2003) and is now operating. Under SCS, airlines submit

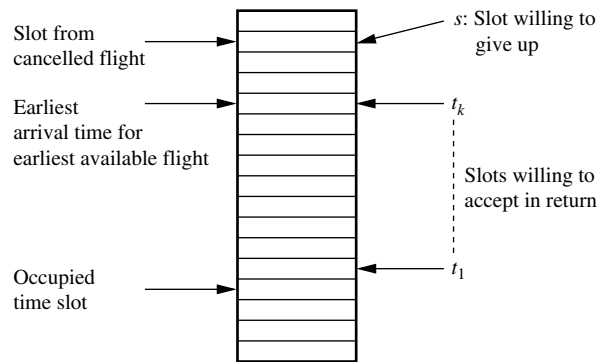
“conditional” cancellation requests of the form: “I am willing to cancel flight  $f_1$  (and release its currently assigned slot  $s_1$ ) if I can move flight  $f_2$  up into (a later) slot  $s_1$ .” The FAA monitors such requests on a continuous basis and, if possible, immediately implements the associated exchange(s) of slots. SCS can be viewed as a “real-time” version of the compression procedure, which is a “batch” process run periodically. The conditional nature of SCS requests as well as the real-time response provides increased trading opportunities over compression.

### 3. Slot Trading

The introduction of slot trading during the course of a GDP introduces a wide range of possibilities, in that a number of schemes could potentially be used to coordinate the exchange of slots. One approach, for instance, could be a market-based mechanism in which airlines would be able to buy and sell slots. Another approach could be a system where airlines would bargain amongst themselves (Adams et al. 1997). It is, however, difficult to envision the use of such highly decentralized mechanisms as near- or medium-term solutions within the context of GDPs. Among others, the high level of uncertainty, the very dynamic environment, and the potential impact on other ATFM initiatives all present significant barriers. In this paper, we therefore consider more modest generalizations of the compression and SCS framework. Under this framework, airlines may submit offers to exchange slots. The FAA, on the other hand, would act as a mediator that evaluates and selects possible trades. We note that, not only does this approach allow for more complex trade offers, but, by solving a formal mediation problem, it allows for explicit consideration of equity in determining the overall set of offers accepted.

#### 3.1. Model Concepts

The general idea behind this approach is based on interpreting the compression procedure as a form of mediated bartering. This interpretation relies on the observation that all slot exchanges are instigated by a slot that is made available through a cancelled or a delayed flight. Such a slot leads to a series of slot exchanges in which flights are repeatedly moved up in a way that maximizes the return for the releasing airline. Each flight cancellation may be therefore viewed as an offer to trade the resulting open slot for a later slot that will reduce the delay on one of the releasing airline's subsequent flights. This type of offer is depicted in Figure 3. In addition, we assume that airlines are willing to offer a slot currently occupied by one of its flights in return for an earlier slot,

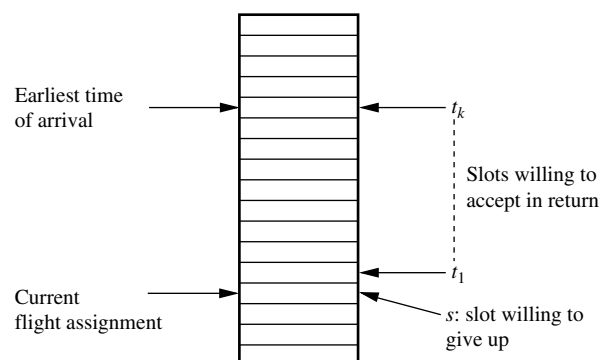


**Figure 3** Offer Associated with Cancelled or Delayed Flights

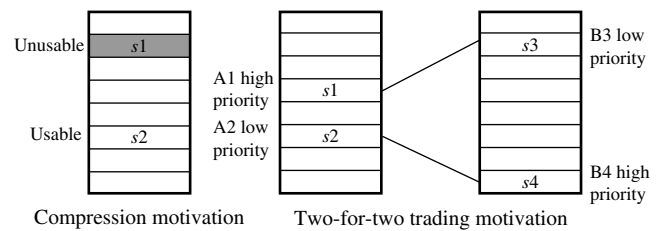
as long as the new slot is not earlier than the earliest time of arrival for the flight. This offer type is depicted in Figure 4.

Given a resulting set of offers, the FAA (in its role as mediator) will have to determine which offers to select and execute. In the case of compression, all exchanges are one for one (i.e., a single slot owned by one airline is exchanged for a single slot owned by another airline). As a result, the problem of finding a feasible set of exchange sequences is equivalent to finding a set of nonintersecting trade cycles, which correspond to the solutions of an assignment problem. Several criteria could be used to select the actual trades that are executed: one possibility, for instance, is to use a bilevel programming approach in which offers to move down are given priority. This approach yields solutions that are similar to the compression algorithm.

Under the interpretation of compression as slot trading, only one-for-one trades are allowed. More generally, one might consider  $k$ -for- $n$  trades. To illustrate the potential benefits, consider the two cases illustrated in Figure 5. On the left-hand side of the figure, we see an example of a slot state that would motivate one-for-one trading (compression). Such trading must always start with a case where an airline is willing to give up an earlier slot for a later one. The economic motivation for such a move is that an airline



**Figure 4** "Default" Offers



**Figure 5** Value Proposition for One-for-One and Two-for-Two Trades

cannot use the earlier slot, i.e., to that airline it has a zero value. On the other hand, as illustrated in the right-hand side of the figure, airlines A and B might both have a pair of usable slots, which are occupied by flights with differing priority (delay cost) levels. It could easily be that the value to A of slots  $s_1$ ,  $s_2$ ,  $s_3$ , and  $s_4$  are \$1,500, \$500, \$2,000, and \$300 and the value to B of the same four slots are \$500, \$2,500, \$800, and \$1,800. Thus, the value proposition to A of trading  $s_1$  and  $s_2$  for  $s_3$  and  $s_4$  is  $2,000 - 1,500 + 300 - 500 = \$300$  and the value proposition to B of the same trade is  $500 - 800 + 2,500 - 1,800 = \$400$ . Thus, both airlines place positive values on all of the slots, but because of the manner in which the slots can be mapped to the airlines' flights, both airlines benefit from the trade. Such trade opportunities should be much more common than the ones motivated by the compression algorithm, which arise mainly as a result of cancelled flights.

The remainder of this section describes trading mechanisms and models to support trading of the type described above. Section 3.2 introduces a general approach and mediator model that supports general  $k$ -for- $n$  trades. Subsequently, in §3.3, we discuss a specific model of airline decision making within GDPs that leads to a restricted but more practical approach. Finally, in §3.4, we propose a mediator optimization model that incorporates these restrictions. The experimental results presented in §4 show that the trading model we propose is capable of providing significant performance improvements over those achieved by the compression procedure, indicating that it represents a practical new approach to slot trading that is relevant in the current system.

### 3.2. Model Overview

The general slot trading framework we envision consists of the following basic "iteration" between the airlines and the mediator.

- Periodically (say every 15 or 30 minutes), airlines submit a list of trade offers that they would desire.
- Subsequently, the mediator (FAA) chooses a set of offers to accept. There are potentially many alternate approaches to formulating and solving this mediator decision problem.

The model we are proposing can be viewed as a “direct” extension of the one-for-one trading model described in the previous section in the sense that each offer will be considered independently of the others. Specifically, the mediator has the ability to accept or reject any individual offer or combination of offers. Thus, the approach does not support offers that might be contingent on the acceptance of other offers or other complex transactions. While it might be tempting to include such complexities, we wish to maintain a certain level of simplicity to achieve a practical system. The notation introduced below allows us to specify the precise nature of the trade offers and facilitates the development of a rational framework for the type of offers proposed by the airlines, as well as the mediators’ decision on which offers to accept.

- $F = \{f_1, \dots, f_n\}$  denotes the flights affected by the GDP; for each flight  $f_i \in F$ , we let  $erta_i$  be its earliest runway time of arrival.

- $S = \{s_1, \dots, s_n\}$  denotes the sequence of arrival slots in the GDP. (We note that there are several conditions that can lead to differences in the number of flights and slots, but for notational convenience we assume that they are the same ( $n$ ).) For each slot  $s_j \in S$ , we let  $t_j$  represent the slot time.

- $A$  denotes the airlines involved in the GDP.

- $O: F \rightarrow A$  denotes a mapping that defines the flight-airline relation. For each airline  $a \in A$ ,  $F_a$  represents the flights from that airline, i.e.,  $F_a = \{f \in F \mid O(f) = a\}$ . At the start of a period of trading, all flights have been assigned a slot; we assume that flight  $f_i$  is assigned to slot  $s_i$  for all  $f_i \in F$ . This assignment specifies each airline’s allotment of slots, that is,  $S_a = \{s_i \in S: f_i \in F_a\}$  represents the set of slots owned by airline  $a$ .

Given these initial allotments, we can associate with each airline  $a$  a set of offers  $T_a \subseteq 2^{S_a} \times 2^{S-S_a}$ . That is, each offer  $\tau = (U_\tau; V_\tau) \in T_a$  specifies that airline  $a$  would be willing to offer the slots in  $U_\tau$  in return for the slots in  $V_\tau$ . We assume that  $U_\tau$  and  $V_\tau$  have the same cardinality. In addition to the offers proactively provided by the airlines, we also assume the availability of default offers,  $D$ , which specify that an airline would always be willing to reduce the delay of any of its flights,

$$D = \{(s_i, s_j): 1 \leq i, j \leq n, erta_i \leq t_j \leq t_i\},$$

i.e., an airline is willing to trade the current slot  $s_i$  assigned to a flight for an earlier slot  $s_j$  up to the flight’s earliest feasible arrival slot. Note that the airline-provided information does not include any information about its relative value for these trades. We assume implicitly that airlines only create offers that “make sense” for their flights, i.e., if any offer

is executed then, afterward, there will be a feasible assignment of an airline’s flights to the slots it owns.

Given these offers, the mediator’s task is to choose a compatible set of offers to accept. Of course, one could evaluate a set of offers along several dimensions in trying to choose the best set to accept. Here, we use the relatively simple criterion of accepting the maximum number of compatible offers. This problem can be formulated as a set-partitioning problem, which has a variable associated with both the offers submitted by the airlines and the default offers; that is,

- $y_\tau \in \{0, 1\}$  for all  $a \in A, \tau \in T_a$ ;  $y_\tau = 1$  if and only if offer  $\tau$  is selected.

- $x_{ij} \in \{0, 1\}$  for all  $(s_i, s_j) \in D$ ;  $x_{ij} = 1$  if and only if slot  $s_i$  is exchanged for slot  $s_j$  (or equivalently, flight  $f_i$  is assigned to slot  $s_j$ ).

The objective function equals

$$\text{Max} \sum_{a \in A, \tau \in T_a} y_\tau, \quad (1a)$$

although, as stated before, other objective functions could also be used. The constraints can be represented as

$$\sum_{s_j: (s_i, s_j) \in D} x_{ij} + \sum_{a \in A, \tau \in T_a: s_i \in U_\tau} y_\tau = 1 \quad \text{for all } s_i \in S, \quad (1b)$$

$$\sum_{s_i: (s_i, s_j) \in D} x_{ij} + \sum_{a \in A, \tau \in T_a: s_j \in V_\tau} y_\tau = 1 \quad \text{for all } s_j \in S. \quad (1c)$$

The right-hand side of this formulation represents the set of slots twice and the constraints ensure that each slot is assigned to both an offer that gives it up and an offer that receives it. Constraint (1b) states that each slot is assigned to some offer (default or airline provided) that proposes to give up that slot. Constraint (1c) states that each slot is assigned to some offer that requires that slot in return. The situation where slot  $s_i$  is not traded corresponds to selecting the default offer  $(s_i, s_i)$ .

### 3.3. Airline Decision Making and Offer Restrictions

While the set-partitioning formulation provides a general framework for slot trading, the arbitrary complexity of the offers that can be submitted reduces its practical relevance. To enable a more realistic approach, we first consider a basic model of airline decision making within GDPs, in which airline preferences are represented by an assignment model (see Brennan 2001; Niznik 2001; Luo and Yu 1998).

In formulation (2) below,  $x_{ij}$  and  $y_i$  are binary variables. Variable  $x_{ij}$  takes value 1 if and only if flight  $f_i$  is assigned to slot  $s_j$ ;  $y_i$  is a slack variable that takes value 1 if and only if  $f_i$  is cancelled. Expression (2a) defines  $u_a(S')$ , the optimal value to airline  $a \in A$  of

any subset of slots  $S' \subseteq S$ . The objective function coefficients  $w_{ij}$  and  $c_i$  represent the value of assigning flight  $f_i$  to slot  $j$  and the cost of cancelling flight  $f_i$ , respectively.

$$u_a(S') = \text{Max} \sum_{f_i \in F_a, s_j \in S} w_{ij} x_{ij} - \sum_{f_i \in F_a} c_i y_i \quad (2a)$$

subject to the constraints

$$\sum_{s_j \in S': t_j \geq \text{erta}_i} x_{ij} + y_i = 1 \quad \text{for all } f_i \in F_a, \quad (2b)$$

$$\sum_{f_i \in F_a: t_i \geq \text{erta}_i} x_{ij} \leq 1 \quad \text{for all } s_j \in S', \quad (2c)$$

$$x_{ij}, y_i \geq 0, \quad (2d)$$

where  $w_{ij}$  represents the value of assigning flight  $f_i$  to slot  $s_j$  and  $c_i$  represents the cost of cancelling flight  $f_i$ . Note the following two important special cases of this model.

*Intra-Airline Optimization*— $S' = S_a$ . Here, formulation (2) can be interpreted as the intra-airline optimization airlines perform in the substitution/cancellation process, i.e., it seeks to find the best assignment of that airline's flights to the slots it "owns."

*Systemwide Optimization*—Let  $a$  Represent All Airlines in  $A$  and  $S_a = S$ . The formulation can be viewed as a global interairline optimization model. The model seeks a solution that achieves systemwide efficiency. Although employing such a model and "forcing" implementation of such a solution is incompatible with the CDM paradigm, the resulting value will provide an upper bound on the benefits that can be obtained by slot trading.

In trying to understand the possible motivation for slot trading, it is instructive to consider the structure of a flight's delay costs. Delay costs are oftentimes reasonably approximated by a staircase structure as shown in Figure 6 (Brennan 2001). This structure is motivated by operationally significant delay levels within each carrier. For instance, the industry standard for an on-time arrival is 0–15 minutes delay beyond scheduled arrival time. Thus, the difference between four and nine minutes of delay is not nearly as significant as the difference between 14 and 19 minutes of delay. Similarly, between 15 and 25 minutes,

the rate of missed baggage connections begins to increase, and between 25 and 45 minutes of delay, passengers begin to miss connections. With delays more than 45 minutes, crews begin to miss connections. Of course, the exact times and significance of these classes may differ on a flight-to-flight basis.

The intra-airline optimization is actually implemented via the CDM substitution process in which airlines interchange the slot assignment of pairs of flights. For instance, an airline could decide to delay a flight with few passengers, while reducing the delay on a more heavily loaded flight that would allow its passengers to make their connections. The current use of pairwise exchanges as well as their potential benefits suggest a possible restriction to the general slot trading framework, in which airlines are allowed to propose only "two-for-two" trade offers (i.e., an offer consists of an exchange of two slots for two other slots). Any two-for-two trade offer involves two flights, whose assigned slots are offered for two other slots. As such, these offers can be separated into three classes: (1) the offer expresses a trade for two earlier slots (i.e., both flights are moved up), (2) the offer expresses a trade for two later slots (i.e., both flights are moved down), or (3) the offer expresses a trade for one earlier slot and one later slot (i.e., one flight is moved up while the other is moved down). It is safe to dismiss the first two classes: the first class is subsumed by the default offers, while it is hard to imagine why an airline would submit an offer in the second class. As such, we can safely interpret (a class of) two-for-two trades as an at-least, at-most offer, which indicates that an airline demands a certain minimum delay reduction on one flight in return for a maximum amount of additional delay imposed on another flight. It is worthwhile to note the resulting generalization of SCS requests: whereas as an SCS request states, "I am willing to cancel flight  $f_i$  in return for a reduction in the delay of flight  $f_j$ ," a two-for-two trade offer states, "I am willing to delay (but not necessarily cancel) flight  $f_i$  in return for a reduction in the delay of flight  $f_j$ ."

While these restrictions in theory limit the potential exchanges during the course of a GDP, our experiments show that two-for-two trades yield substantial benefits over current procedures. Moreover, the restriction to two-for-two trades significantly reduces the complexity of the resulting framework. This reduction in complexity not only applies to the mediator's problem, but also to the evaluation and generation of potential offers by each individual airline. This observation is crucial as experience in other CDM initiatives has shown that it is essential to simplify airline information requirements, as well as operational processes to achieve acceptance and eventual implementation.

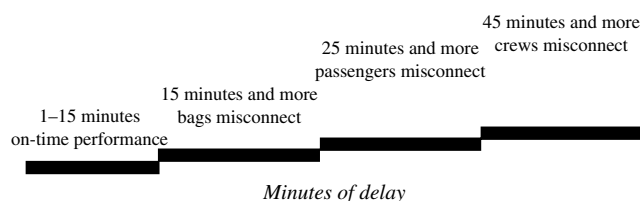


Figure 6 Delay Cost Structure

### 3.4. Model Formulation Under Offer Restrictions

Even though the set-partitioning formulation of the mediation problem can be used to find compatible trades in the case of at-least, at-most offers, the large number of variables makes this approach intractable for all but the smallest cases. When only two-for-two trades are allowed, the resulting set of offers can be defined more succinctly, motivating the alternate formulation of the mediation problem presented in this section. The formulation below models the mediation problem as a network flow problem with side constraints. Defining  $T$  as the set of all possible trades, the two-for-two offers to trade are characterized by a tuple  $(f_d, s_{d'}; f_u, s_{u'}) \in T$ , which states that the airline is willing to move down flight  $f_d$  to a slot no later than  $s_{d'}$  in return for moving up flight  $f_u$  to a slot that is no later than  $s_{u'}$ . In the remainder of this section we discuss an alternate formulation of the mediation problem, which takes into account the underlying offer structure. This formulation may be viewed as a network flow problem with side constraints. Flow proceeds from a flight source node to a class node and finally to a slot node. The slot-defined classes represent either the maximum amount of delay or the minimum delay reduction requested for that flight if the offer is executed. For instance, if the offer  $(f_d, s_{d'}; f_u, s_{u'})$  is executed, flight  $f_d$  will be assigned to class  $s_{d'}$  and  $f_u$  will be assigned to class  $s_{u'}$ . This assignment will ensure that  $f_u$  will receive a slot no later than  $s_{u'}$  and that  $f_d$  will receive a slot no later than  $s_{d'}$ . The side constraints are needed to ensure that only assignments corresponding to proposed offers are selected. To illustrate this idea, we start by considering a single flight,  $f_i$ , and examine all the offers in which it occurs. These offers determine a sequence of classes, which we can represent by the indices of the corresponding slots. That is, the offers specify a sequence  $i_1, \dots, i_{k_i}$ , where  $t_{i_k} < t_{i_{k'}}$  if  $k < k'$ . Thus, if  $i_k < i$ , there exists a trade offer that requests that flight  $f_i$  is moved up (earlier in time) to at least slot  $s_{i_k}$  (from its current assignment to slot  $s_i$ ). If, on the other hand,  $i_k > i$ , there exists a trade offer to move  $f_i$  down (later in time) to a slot no later than  $s_{i_k}$ . We assume that there is one  $k$ :  $1 \leq k \leq k_i$ , such that  $i_k = i$ , corresponding to the default offer. Moreover, we use  $i_0$  to represent the index of the earliest slot that flight  $f_i$  can be assigned to, i.e.,  $i_0 = \min_{j: t_j \geq \text{erta}_i} j$ . Consequently, if flight  $f_i$  is assigned to class  $i_k$ , it will receive a slot in the range  $s_{i_0}, \dots, s_{i_k}$ .

To represent the integer programming (IP) formulation, we partition the set of offers to trade as follows:  $D_T = \bigcup_{(f_d, s_{d'}; f_u, s_{u'}) \in T} (f_d, s_{d'})$ , which contains the classes that correspond to downward moves for each flight  $f_d \in F$ ;  $U_T = \bigcup_{(f_s, s_{d'}; f_u, s_{u'}) \in T} \{(f_u, s_{u'})\}$ , which contains the classes that correspond to upward moves for each flight  $f_d \in F$ ; and  $N_T = \bigcup_{f_i \in F} \{(f_i, s_i)\}$ , which contains

the classes corresponding to default offers for each flight  $f_i \in F$ . The resulting IP formulation uses the following variables.

- $x_{ik} \in \{0, 1\}$  for  $f_i \in F$ ,  $1 \leq k \leq k_i$ ;  $x_{ik} = 1$  iff  $f_i$  is assigned to class  $i_k$ .
- $y_{ij} \in \{0, 1\}$  for  $f_i \in F$ ,  $s_j \in S$ ;  $y_{ij} = 1$  iff flight  $f_i$  is assigned to slot  $s_j$ .
- $z_{ik} \in \{0, 1\}$  for  $f_i \in F$ ,  $1 \leq k \leq k_i$ ;  $z_{ik} = 1$  iff  $f_i$  has been assigned to a class with an index lower than  $k$  in the sequence of classes for  $f_i$  but receives at least slot  $s_{i_k}$ . The use of this variable is explained in more detail below.

- $\hat{x}_{dd'uu'} \in \{0, 1\}$  for  $(f_d, s_{d'}; f_u, s_{u'}) \in T$ .  $\hat{x}_{dd'uu'} = 1$  iff offer  $(f_d, s_{d'}; f_u, s_{u'})$  is executed.

The following constraints represent the network flow component of the IP formulation in which the flights are assigned to classes and classes are assigned to slots:

$$\sum_{k=1}^{k_i} x_{ik} = 1 \quad \text{for all } f_i \in F, \quad (3a)$$

$$x_{i1} + z_{i1} - \sum_{j=i_0}^{i_1} y_{ij} = 0 \quad \text{for all } f_i \in F, \quad (3b)$$

$$x_{ik} + z_{ik} - z_{i(k-1)} - \sum_{j=i_{k-1}+1}^{i_k} y_{ij} = 0 \quad \text{for all } f_i \in F, 1 < k \leq k_i, \quad (3c)$$

$$\sum_{f_i \in F: i_0 \leq j \leq i_k} y_{ij} = 1 \quad \text{for all } s_j \in S. \quad (3d)$$

Constraint (3a) represents the assignment of flights to classes, while constraints (3b) and (3c) represent the subsequent assignment of classes to slots. Finally, constraint (3d) represents the restriction that each slot is assigned exactly once. The resulting network flow structure is shown in Figure 7 for a single flight  $f_i$  with  $k_i = 5$ . The left-hand nodes in Figure 7 correspond to flights, which are first assigned to classes. After a flight has been assigned to class  $i_k$ , it can either be assigned to a slot with an index in the range  $i_{k-1} + 1, \dots, i_k$  or it can be assigned to a higher class. The latter is achieved with the  $z_{ik}$  variables, which ensure that when flight  $f_i$  is assigned to class  $i_k$ , it can also receive a slot in the range  $i_0, \dots, i_{k-1}$ .

In addition to the network flow constraints, the IP formulation also contains the following side constraints:

$$x_{dd'} = \sum_{(f_d, s_{d'}; f_u, s_{u'}) \in T} \hat{x}_{dd'uu'} \quad \text{for all } (f_d, s_{d'}) \in D_T, \quad (3e)$$

$$x_{uu'} = \sum_{(f_s, s_{d'}; f_u, s_{u'}) \in T} \hat{x}_{dd'uu'} \quad \text{for all } (f_u, s_{u'}) \in U_T. \quad (3f)$$

Constraints (3e) and (3f) ensure that the resulting trades only include offers proposed by the airlines.



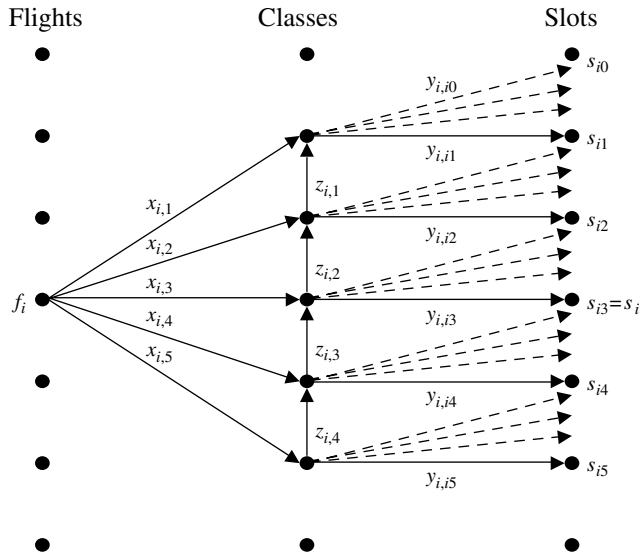


Figure 7 Structure of Network Flow Constraints

In other words, the constraint states that a flight is moved down if and only if another flight is moved up, in accordance with one of the proposed trades.

The constraints in formulation (3) define the set of feasible trades. An optimal set of trades is achieved by selecting an appropriate objective function. The objective function may be as simple as maximizing the number of trades that are executed. However, one may also consider objective functions that incorporate systemwide performance measures such as maximizing on-time performance or minimizing passenger delays, which we examine in the experiment described in §4. It is important to note that, even though it is beyond the scope of our work in this paper, our model is also capable of incorporating equity considerations via the objective function and/or additional constraints.

#### 4. Experimental Results

In this section, we analyze the slot trading approach presented in §§3.3 and 3.4 using two case studies. The objective of these case studies is threefold: (1) to analyze potential benefits of increased coordination, (2) to consider the impact of airline behavior in the trading process, and (3) to analyze the efficiency of the underlying optimization models. The IP formulations in our experiments were solved using CPLEX 7.1 on a Sun Ultra 10 workstation.

In the first case study, the airlines' objective is to maximize on-time performance; in the second, the airlines' objective is to minimize passenger delays. These objectives represent two of the most important factors that airlines consider in their decision-making process. The importance of on-time performance is discussed in Luo and Yu (1998). The minimization

of passenger delays, an equally important objective, has been used by American Airlines in their GDP decision-support tools (Vasques-Marquez 1991; Niznik 2001).

For each case study, we assess the potential improvement to the airlines' objective under consideration that could be obtained by trading slots among airlines. Our analysis is based on historical data from a set of GDPs at Boston's Logan Airport between January and April 2001. For each of these GDPs, we collected the program data after RBS was first executed. The resulting data provided us with the sets of flights, airlines, and slots (based on the actual airport acceptance rates), parameter values such as the earliest runway arrival times and slot times, as well as the initial allocation of flights to slots. Given these parameters, we fixed the assignment of all the flights that were exempted from the program, and the resulting flight-slot assignment of the remaining (nonexempt) flights was used to determine each airline's initial allotment of slots,  $S_a$ .

In each case study, we analyze the results of solving the following optimization problems:

(1) We solved formulation (2) for each airline using the objective function under consideration in the case study. The resulting slot allocation represents an optimal allocation without coordination. This is the equivalent of the substitution process in the current system before compression occurs.

(2) We also solved formulation (2) without considering slot ownership. In this case, the resulting slot allocation represents an optimal allocation with coordination, i.e., the solution that could be obtained if a centralized optimization approach were possible, which, of course, is incompatible with the decentralized planning approach introduced by CDM. However, obtaining this solution is important as it provides an upper bound to the benefits that could be achieved by interairline trading of slots in the current system.

(3) To analyze the potential benefits of the two-for-two slot trading mechanism, we started with the flight-slot allocation that maximized internal airline performance, i.e., the allocation described in (1) above using formulation (3). Then, for each airline, we generated all two-for-two trades that improved the airline objective of interest and solved optimization model (3) to find a mutually compatible set of trades to implement.

(4) For the on-time performance case, we also assessed the benefit provided by the compression algorithm in place in the current system. We made the assumption that airlines would cancel flights with excessive delays, i.e., delays of two hours or more, prior to the execution of compression. We did not perform this step in the passenger delay cost case study

because there was no natural way to represent airline cancellation behavior or to assign a cost penalty to passengers on cancelled flights. We should also note that the analysis of this case for the on-time performance case study demonstrated significantly better performance for two-for-two trading.

We conclude each study with a high-level assessment on running times that suggests the trading models we propose are suitable for decision making in real time.

#### 4.1. On-Time Performance

In the first case we consider, we assume that each airline's objective is to maximize on-time performance. A flight is said to be on time if it arrives within 15 minutes of its arrival time as scheduled in the official airline guide,  $oag_i$  (we note that  $erta_i$ , defined earlier, and  $oag_i$  can differ; in general,  $erta_i \geq oag_i$ , but  $erta_i$  can be strictly greater than  $oag_i$  most typically because of "upstream" delays that induce unavoidable delay on flight  $f_i$ ). For the on-time performance case, an airline's objective is to maximize the number of flights that are delayed at most 15 minutes. Restricting the airlines' objectives to maximizing on-time performance offers a substantial simplification of the trading model. To illustrate this, we first observe that in this case, we can represent the airline's performance function using formulation (2) with coefficients

$$w_{ij} = \begin{cases} M & \text{if } (t_j - oag_i) < 15, \\ 0 & \text{otherwise} \end{cases}$$

with  $M \gg 0$ . A slightly different approach will be discussed in §4.1.1.

Let us now consider applying the trading model introduced in the previous section, under a scenario where airlines submit any offers that would improve on-time performance. An offer of this type would state that an airline is willing to move down any flight not arriving on time in the current allocation in return for a delay reduction that makes another flight arrive on time. Note that such offers would never involve a flight that is already arriving on time in the current allocation. In this case, we can distinguish three possible classes for each flight: (1) a flight will be assigned at least a slot corresponding to an on-time arrival, (2) a flight will be assigned at least the slot it currently occupies (i.e., a default offer), and (3) a flight will be assigned at most the last slot in the GDP (i.e., the flight is delayed).

As before, the resulting classes can be represented by a sequence of slot indices. If a flight  $f_i$  is already arriving on time, only the default offer applies; therefore, we have  $k_i = 1$  and  $i_1 = i$ . For all other flights, we have  $k_i = 3$  and the slot indices for the three classes outlined above can be defined as  $i_1 = \max_{s_j \in S: t_j - oag_i < 15} j$ , corresponding to on-time arrivals,  $i_2 = i$ , corresponding to the default offers, and  $i_3 = n$ , corresponding to

the flight delays (recall that  $n$  is the index of the latest slot in the GDP). These class definitions suggest replacing constraints (3e) and (3f) with

$$\sum_{f_i \in F_a} x_{i3} \leq \sum_{f_i \in F_a} x_{i1} \quad \text{for all } a \in A, \quad (4)$$

which ensures that for each airline, only offers that improve on-time performance are executed. Thus, because airlines are willing to arbitrarily trade-off additional delays for an improvement in on-time performance, it is possible to represent the acceptable offers with a single constraint for each airline.

Finally, we note that the choice of an objective function comprises several possibilities. The obvious choice is to maximize the number of flights that have been moved on time (i.e., Maximize  $\sum_{f_i \in F} x_{i1}$ ). However, because each delayed flight will induce another flight to arrive on time, one can also consider maximizing the number of flights that have been delayed (i.e., Maximize  $\sum_{f_i \in F} x_{i3}$ ). Finally, we can consider minimizing the number of default offers executed (i.e., Minimize  $\sum_{f_i \in F} x_{i2}$ ) as this maximizes the execution of offers in which flights are moved from their current slot. The impact of these first two possibilities is discussed in §4.1.1.

**4.1.1. Slot Trading Benefits Under the On-Time Performance Objective.** Figures 8 and 9 give the results of our experiments under the on-time performance objective. Figure 8 shows the improvement in on-time performance relative to the best that could be obtained without coordination (as a percentage of the total number of nonexempted flights in the GDP). Here, the solid line represents the upper bound on the increase that could be obtained by coordination, i.e., as a result of solving the optimization model described in (2) above. The dashed line represents the relative improvements that would be obtained by two-for-two slot trading, i.e., as a result of solving the optimization problem described in (3) above, using an objective function of maximizing the number of

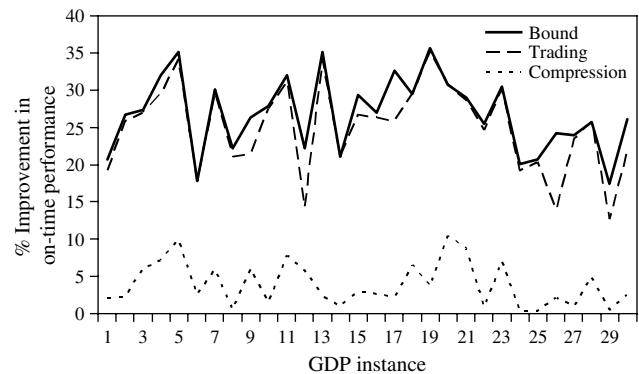


Figure 8 On-Time Performance Improvements from Slot Trading

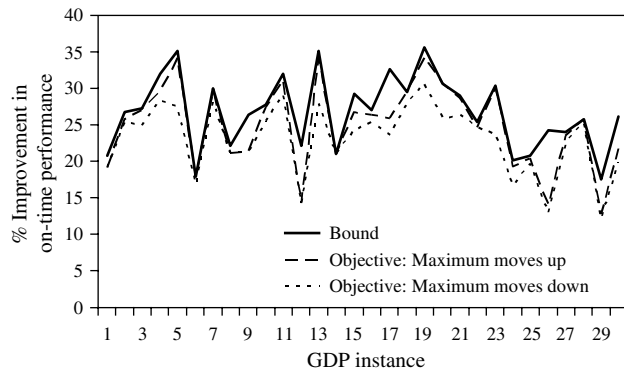


Figure 9 Impact of Objective Function Choice

flights moved up and constraint (4). The dotted line represents the improvements obtained by the compression algorithm, i.e., as a result of solving the optimization model described in (4) above. On average, the potential increase in on-time performance would be 26.8% (the average number of flights in a GDP was 216.1, while the average number of flights arriving on time without coordination was 100.1). The average relative improvement obtained by slot trading was 24.9%, while the average improvement with the compression algorithm was 3.9%. These results clearly indicate that slot trading could yield substantial benefits, while the compression algorithm would only lead to modest improvements in on-time performance. The use of slot trading nearly always yielded improvements close to the theoretical maximum.

We note, however, that the objective function choice impacts the performance of the resulting IP model. Figure 9 shows the relative improvements in on-time performance under two objective function alternatives, maximizing the number of flights moved up and maximizing the number of flights moved down. The solid line again represents the theoretical maximum, the dashed line represents the relative improvements when the number of flights moved up is maximized, while the dotted line represents the improvements when the number of flights moved down is maximized. Both objectives yield substantial improvements; however, the use of the first objective leads to slightly better results and yields significantly faster running time. This indicates that, in addition to the differences in on-time performance, the objective function choice can also lead to substantial differences in the efficiency of the resulting IP formulations. The average solution time when the number of flights moved up is maximized was 8.3 seconds. However, the number of flights moved down is maximized and the average solution time increased to 163 seconds; a most significant difference. It is important to note that we limited the maximum number of nodes visited in the branch-and-bound tree to 1,000, while the node limit was reached occasionally when

the number of flights moved down was maximized. The removal of this restriction did not impact the solution values shown in Figure 9. That is, the optimal solution was found within the node limit in all cases.

**4.1.2. Slot Trading Dynamics.** In this case study, we have so far assumed that an airline would agree to any amount of additional delay for a flight in return for a reduction in delay that would make another flight arrive on time. In other words, an airline would submit all at-least, at-most offers that would improve its on-time performance. As a final step in our analysis, we now consider the benefits that may be obtained if airlines only submit smaller sets of offers.

To analyze this situation, we consider cases in which airlines may no longer accept an arbitrary increase in delay in return for an increase in on-time performance. The approach we follow is based on the use of a slightly more complex value  $w_{ij}$  of assigning flight  $f_i$  to slot  $s_j$  in formulation (2). More specifically, we define the coefficients  $w_{ij}$ , whose units are minutes, as

$$w_{ij} = \begin{cases} M & \text{if } (t_j - oag_i) \leq 15, \\ \max(120 - (t_j - oag_i), 0) & \text{otherwise.} \end{cases}$$

As before, a large value is associated with on-time performance. However, in this case, additional delays to a flight that is not arriving on time will incur a penalty cost (reduction in value). We assume that the value decreases linearly in the additional delay up to a maximum delay of 120 minutes, i.e., there is no benefit associated with reducing the delay of flights if the delay remains more than 120 minutes. Note that  $M$  should be set to a value much greater than 120.

Given this value function for each flight, the number of offers submitted by an airline  $a \in A$  can be limited by the specification of aspiration levels  $v_a \geq 0$ . An aspiration level  $v_a$  signifies that an airline will only agree to delay a flight if the increase in delay is no more than the aspiration level  $v_a$ . Of course, an airline would only accept this delay if another of its flights were to arrive on time. As in §4.1, therefore we still recognize three classes for flights that are not arriving on time. In the presence of aspiration levels, however, we define the class  $i_3$  corresponding to flight delays for a flight  $f_i \in F_a$  as  $i_3 = \max_{s_j \in S: w_{ij} - w_{ij} \leq v_a} j$ . Note that aspiration levels consider increases in delay cost with respect to a flight's current arrival time; that is, the arrival time after RBS and an airline's internal schedule adjustments have been executed.

With aspiration levels, an airline will no longer accept an arbitrary increase in delay in return for an increase in on-time performance. As such, aspiration levels provide a simple, yet intuitive way to analyze

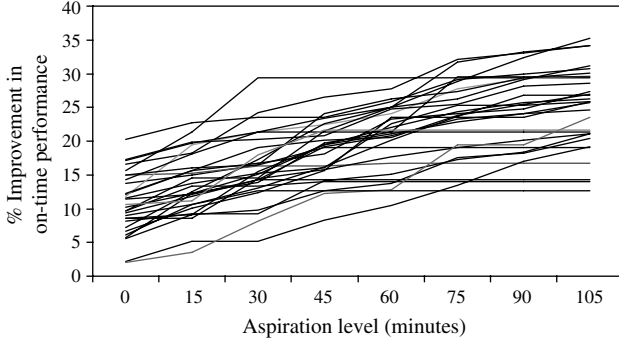


Figure 10 On-Time Performance Improvements by Aspiration Levels

the impact of limiting the number of offers proposed; by varying the aspiration levels, the number of proposed offers can be adjusted. Figure 10 shows the relative improvements in on-time performance as a function of the aspiration level for the same set of GDPs in Boston that was used before. Each curve in Figure 10 corresponds to a single GDP. These results show that even for small aspiration levels (e.g., 0–30 minutes), considerable improvements in on-time performance can be obtained. It is interesting to note that even with aspiration levels of 0 (in this case, an airline will only allow flights with excessive delays to be moved down), the improvements can be substantially higher than those obtained by the compression algorithm. Overall, these results indicate that slot trading may be beneficial even when the number of offers is limited.

#### 4.2. Passenger Delay Costs

The second case study considers the situation in which each airline's objective is to minimize (total or average) passenger delays. A flight's passenger delay is defined as the total number of passengers on that flight, multiplied by the delay assigned to that flight. As such, the trade-offs that occur in this case will typically consider the overall benefits of delaying flights with few passengers, while reducing the delays of more heavily loaded flights. There are several possibilities to incorporate passenger delay minimization into the airlines' performance function shown in formulation (2). A straightforward approach is to define the value  $w_{ij}$  of assigning flight  $f_i$  to slot  $s_j$  in formulation (2) as

$$w_{ij} = -p_i(t_j - oag_i),$$

where  $p_i$  represents the number of passengers on flight  $f_i$  (note that the coefficient is negative so as to represent a cost). We will refer to an objective function based on these coefficients as a standard passenger delay objective. Another, somewhat more involved approach, is based on the staircase cost structure shown in Figure 6. This applies in situations where the exact slot assignment is less important than the

specific interval in which the flight is assigned, based on the occurrence of operationally significant events at certain time epochs (see Brennan 2001). In this approach, we define for each flight  $f_i$  a set of *critical times*,  $\bar{t}_{i1}, \dots, \bar{t}_{ik_i}$ , which define slot classes,  $1, \dots, k_i$ , corresponding to the interval boundaries of the cost structure shown in Figure 6. Given these critical times, the value  $w_{ij}$  of assigning flight  $f_i$  to slot  $s_j$  in formulation (2) can be defined as

$$w_{ij} = \begin{cases} 0 & \text{if } (t_j - oag_i) \leq \bar{t}_{i1}, \\ -p_i \bar{t}_{ik} & \text{if } \bar{t}_{ik} < (t_j - oag_i) \leq \bar{t}_{i(k+1)}, \quad 1 < k < k_i. \end{cases}$$

We will refer to an objective function based on these coefficients as a critical times passenger delay objective.

Recall that the on-time performance objective allowed us to represent the offers proposed using a single constraint. On the other hand, because of the increase in the number of flight classes that can be considered in the airlines' offers when minimizing passenger delay costs, it is no longer feasible to represent the set of offers that may be proposed by means of a single constraint. If airlines use passenger delay minimization as their objectives, the potential number of offers can be substantial: in principle, an airline may submit any at-least, at-most offer that will reduce its passenger delay cost. That is, for any two flights  $f_d, f_u \in F_a$  and any two slots  $s_{d'}, s_{u'}$  such that  $t_d < t_{d'}, t_u < t_{u'}$  and  $w_{d,d'} - w_{d,d''} < w_{u,u'} - w_{u,u''}$ , the offer  $(f_d, s_{d'}; f_u, s_{u'})$  might be considered desirable. However, many of these offers are redundant, and thus there is no need to consider them.

Suppose, for instance, that an airline submits two offers  $(f_d, s_{d_1}; f_u, s_{u'})$  and  $(f_d, s_{d_2}; f_u, s_{u'})$  with  $t_{d_1} < t_{d_2}$ . That is, an airline would be willing to accept at-most slot  $s_{d_1}$  or at-most slot  $s_{d_2}$  for flight  $f_d$ , in return for giving flight  $f_u$  at-least slot  $s_{u'}$ . Clearly, the first offer,  $(f_d, s_{d_1}; f_u, s_{u'})$ , will be redundant. A similar situation occurs with two offers  $(f_d, s_{d'}; f_u, s_{u_1})$  and  $(f_d, s_{d'}; f_u, s_{u_2})$  with  $t_{u_1} < t_{u_2}$ . That is, an airline would be willing to accept at-most slot  $s_{d'}$  for flight  $f_d$ , in return for giving flight  $f_u$  at-least slot  $s_{u_1}$  or at-least slot  $s_{u_2}$ . Any of these redundant trades can be removed from consideration. In addition, the critical arrival times further reduce the number of potential offers. It is safe to assume that trades will only involve slots corresponding to critical times (i.e., we could define an at-least class corresponding to all slots with a delay less than 15 minutes, etc.). To reduce the number of offers considered, we also applied the definition of classes just presented when considering the standard passenger delay objective (of course, the relative value of these classes was different under the two objectives). Thus, the potential offers evaluated in formulation (3) were the same for both the

standard and critical times passenger delay objectives. When evaluating both the passenger delay objectives, we employed the same objective function in formulation (3), i.e., to maximize the number of flights moved up.

**4.2.1. Slot Trading Benefits Under the Passenger Delay Cost Objective.** To analyze the benefits of our slot trading approach when the objective is to minimize passenger delay costs, we again considered the potential benefits that could be obtained by coordination. For our analysis, we used the same set of GDPs at Boston's Logan Airport, and collected the program data after RBS was first executed. In addition, we estimated the number of passengers per flight by considering capacities of the various aircraft types (Bhogadi 2002), assuming a load factor of 75%. The initial flight-slot assignment of the remaining (nonexempt) flights was used to determine each airline's initial allotment of slots,  $S_a$ .

Subsequently, we first considered the slot trading benefits that could be achieved when airlines use the standard passenger delay objective. As before, we started by minimizing the passenger delay cost for each individual airline using formulation (2), corresponding to the airlines' internal substitution process, i.e., model (1) described in the introduction to §4. After that, we generated trade offers and applied the mediation problem to the allocation that would have resulted after each airline minimized its individual passenger delay costs, i.e., model (3) from the introduction to §4. In addition, we obtained a bound on the overall benefits by minimizing passenger delay cost without considering slot ownership corresponding to the centralized optimization approach, i.e., model (2) from the introduction to §4. The results are shown in Figure 11. Here, the dashed line shows the relative improvement achieved by the slot trading model, while the solid line represents the bounds on potential improvements that are possible. The potential reductions in passenger delay costs are substantial, ranging from 33% to 67% relative to the passenger delay

costs produced after each airline minimized its individual delay costs (the average reduction equals 52%). The average reduction in passenger delay obtained by slot trading is approximately 7.7%. This represents approximately 15% of the average potential reduction.

The same analysis was also performed for the critical times passenger delay objective and the results are shown in Figure 12. The critical times we used in the objective function for formulation (2) are 15, 30, 45, 75, and 120 minutes. Again, the dashed line shows the relative improvements achieved by the slot trading model, while the solid line represents the bounds on potential improvements that are possible. In this case, the bounds on the reduction in passenger delay costs range from 37% to 84%, and the average reduction equals 67%. The average reduction in passenger delay costs produced by the two-for-two slot trading model, i.e., the dashed line in Figure 12, equals 36%. This represents approximately 54% of the average improvement achieved by the bounds solution.

We see that for the standard passenger delay objective, the trading model was only able to achieve a very small percentage of the benefits obtained by the centralized optimization model, whereas for the critical times passenger delay objective, the trading model achieved more than 50% of the centralized benefits. In addition to the improvement in passenger delay costs, we also determined the number of flights that were moved up (i.e., assigned to an earlier class) for both alternatives. For the standard passenger delay case, an average of 83 flights were moved up, compared to an average of 93.5 flights moved up for the critical times case. Because the number of flights moved up is a measure of trading activity, we can see that under both airline objectives, there was substantial activity. Finally, it is important to note that the bounds produced by the centralized optimization approach are infeasible in a system that operates under the CDM paradigm. For both passenger delay objectives, globally optimal allocations are usually achieved by assigning the majority of delay to

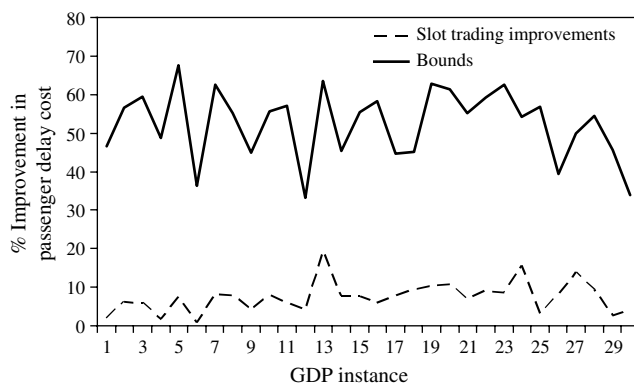


Figure 11 Slot Trading Benefits with Standard Passenger Delay Objective

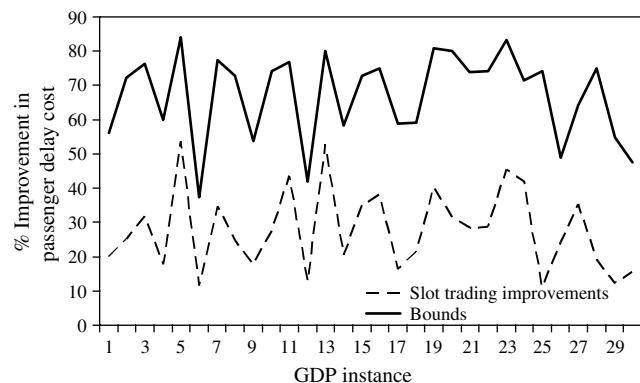


Figure 12 Slot Trading Benefits with Critical Times Passenger Delay Objective

flights employing smaller aircraft. Thus, the centralized approach penalized airlines that operate smaller aircraft in favor of airlines operating larger aircraft. This violates the FAA's equal access policy, a crucial component of CDM and the FAA's air traffic management philosophy.

With respect to the efficiency of solutions under the passenger delay cost alternatives considered, we note that using the critical times version of the objective function of formulation (2) had little impact on the performance of formulation (3) in the on-time performance study experiments. Specifically, the average solution time for the critical times passenger delay case was 12.5 seconds. This compares well to the average solution time of 8.3 seconds achieved for the on-time performance case when the objective was to maximize the number flights moved up. However, the average running time under the standard passenger delay costs objective increased significantly to 120 seconds. We have been unable to explain this increase in running time but note that even 120 seconds is a relatively modest time, which would be acceptable within an application setting.

## 5. Conclusions

This paper explores the potential benefits of increased coordination during GDPs. We introduced a general bartering framework in which airlines may engage in slot trading by submitting offers to the FAA, which acts as the central coordinator. We proposed an optimization model for the FAA's mediation problem, which generalizes current slot exchange procedures in that it allows airlines to submit so-called at-least, at-most offers. These offers to trade slots may be viewed as trade-offs between pairs of flights, and are motivated by operationally significant delay levels. To analyze the potential benefits of this approach, we considered two case studies that use different models of airline decision making. Empirical results using historical GDP data showed considerable promise. Under a basic model of airline decision making in which airlines aim to maximize their on-time performance, slot trading yielded benefits that approached the theoretical bound. Moreover, the level of performance achieved remained even when the number of proposed offers was limited. Significant benefits from trading were also obtained for the case where airlines use passenger delay minimization as their objectives, in particular, if passenger delay costs increased according to a stepwise functional form.

We feel that the results of this paper provide strong evidence of the practical usefulness of the slot trading models defined. We are now working on simplifying data input requirements, so that these capabilities can be incorporated into the flight schedule monitor,

the CDM decision-support tool. Further research in this area is warranted to support this move toward implementation and also the development of more advanced models.

While the IP formulation for the mediation problem is quite efficient, initial results show that alternative formulations may substantially improve IP performance (Vossen 2002), suggesting that further research into more efficient models seems merited. In any implementation of the trading models discussed, airlines would have substantial flexibility in defining offers. Thus, it would be worthwhile to study more complex models of decision making at the airline level with the goal of developing airline trading strategies and evaluating how these strategies affect overall system performance. Finally, the results obtained with the passenger delay cost objectives suggest using exchanges that involve side payments as a possible way of compensating carriers operating smaller aircraft for absorbing unilateral increases in delay. Even though adding side payments increases the complexity of trading models significantly and also represents a major change in the underlying business paradigm, such an approach is consistent with recent moves toward market mechanisms on the part of the air traffic management community (see Ball, Donohue, and Hoffman 2005).

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