OPTIMIZATION OF AIR TRAFFIC MANAGEMENT STRATEGIES AT AIRPORTS WITH UNCERTAINTY IN AIRPORT CAPACITY

Eugene P. Gilbo

Volpe National Transportation Systems Center
55 Broadway, Cambridge, MA 02142, U.S.A.

Abstract: This paper presents a stochastic optimization model for strategic management of arrival and departure traffic at a single airport with uncertainty in airport capacity. It is assumed that a set of possible capacity scenarios with probabilities of their realization are known. A concept of expected capacity is introduced. The average-case solutions based on the expected capacities provide a compromise between the strategies obtained separately for each of possible capacity scenarios. Illustrative numerical examples are presented.

Key words: optimization, queues, air traffic control, uncertainty, airport capacity

1. INTRODUCTION

Worldwide growth of air traffic, especially in the U.S., Europe and the Pacific rim requires more efficient utilization of airspace and airport operational resources. Limited airport capacity continues to be a major cause of costly delays. The role of strategic traffic flow management becomes increasingly important during periods of severe congestion.

Strategic decisions are made for a period of several hours in advance and are based on predicted traffic demand and capacities. Traffic managers resolve congestion problems by delaying a certain number of flights so that traffic flow meets capacity constraints. Some flights are delayed on the ground at the origin airports (ground delays) and some in the air (airborne delays). It is well known that airborne delay is much more costly than ground delay. Therefore, the cheapest strategy for resolving congestion problems would be to avoid airborne delays and accommodate all necessary delays on the ground only. This kind of strategy could be theoretically realizable only in a completely deterministic environment, when traffic demand and airport capacity are known exactly.

In reality, however, the airport capacity is not known exactly. Uncertainty in the weather is a major factor that causes uncertainty in airport capacity. In this case, a completely deterministic approach may not work, and airborne delays, along with ground delays should be considered for solving congestion problems. The stochastic nature of airport capacity risks having the actual, realized capacity scenario different from the one for which the traffic management strategy was calculated. In particular, there is a risk of underutilizing airport capacity if the strategy is based on a more conservative scenario than what actually occurs, or excessively overloading the airport, causing substantial arrival and departure delays if the strategy is based on a more optimistic scenario. In the latter case, some arriving flights must be delayed in the air.

A traffic flow management problem with uncertainty in airport capacity was addressed in recent publications, see (Terrab, *et al.*, 1993; Richetta, *et al.*, 1993; Vranas, *et al.*, 1994). In these publications, the amount of airborne delays in total arrival delays was determined for the ground-hold problem, using different costs for a unit of ground and airborne delay as well as probabilities of various capacity scenarios.

In this paper, we present a stochastic optimization model for strategic managing the arrival and departure traffic at a single airport with uncertainty in airport capacity based on the approach developed by the author, see (Gilbo, 1993). In that paper, a deterministic model was introduced to optimize arrival and departure traffic at an airport by optimal utilization of airport capacity via a dynamic, variable trade-off between arrival and departure capacities. In the present paper, the model is extended to a stochastic case. An optimization problem is formulated to determine traffic flow management strategies which are optimal on average for a set of airport capacity scenarios. A concept of expected airport capacity is introduced. Expected arrival/departure capacity curves are used to determine the optimal, average-case strategies of allocation of arrival and departure flows. As an optimization criterion, the minimum weighted sum of expected total arrival and departure delays is considered.

It is important for air traffic managers and controllers to have a decision support tool for determining the best arrival and departure strategies using quantitative estimations of various types of risk caused by uncertainty in airport capacity. These estimates would allow them to evaluate tradeoffs between positive and negative effects from implementing a chosen strategy under various capacity scenarios. One of the decisions is to predict and justify the amount of airborne and ground delays in case of decreasing airport capacity, and at the same time to have a surplus of arrival flights that can be accommodated without airborne delays if airport capacity increases. The latter determines so called Managed Arrival Reservoir. The approach developed in this paper will help traffic managers and controllers in this kind of decision making.

The paper has been organized as follows. Section 2 discusses uncertainty in airport capacity. A mathematical optimization model is presented in Section 3. Section 4 contains illustrative numerical examples.

2. UNCERTAINTY IN AIRPORT CAPACITY

For given weather conditions and runway configuration, the airport capacity can be represented by an arrival/departure capacity curve $v = \phi(u)$ that interconnects arrival capacity u and departure capacity v within the entire range of arrival/departure mix, see (Gilbo, 1993). An example of arrival/departure capacity curves for 15-minute capacities (number of arrivals and departures per fifteen minutes) is shown in Fig. 1.

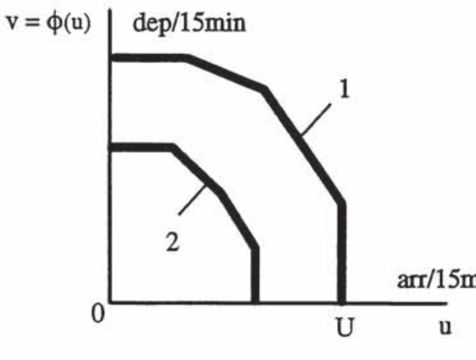

Fig. 1. Airport Arrival/Departure Capacity Curves

In Fig. 1, U denotes the upper bound for arrival capacity.

Weather conditions determine the operational flight rules such as VFR or IFR, as well as runway configurations that can be used. A weather forecast usually determines several capacity scenarios at the airport. A capacity scenario includes a sequence of capacity curves and time intervals applicable for each curve. In this paper, it is assumed that airport capacity scenarios and their probabilities are given.

3. OPTIMIZATION MODEL

3.1 Notation

T – time period of interest, consisting of N discrete time intervals Δ (e.g., $\Delta = 15$ min), $T = N\Delta$

$I = \{1, 2, ..., N\}$ – a set of consecutive time intervals

$S = \{1, 2, ..., s\}$ – a set of capacity scenarios at an airport

$\Phi = \{\phi^{(1)}(u), \phi^{(2)}(u), \dots, \phi^{(M)}(u)\}$ – a complete set of M arrival-departure capacity curves for various operational conditions at an airport

$\phi_i^j(u) =$ an arrival-departure capacity curve for the *j*th capacity scenario at the *i*th time interval; $\phi_i^j(u) \in \Phi$, $i \in I$, $j \in S$

p_j – probability of capacity scenario j, $j \in S$

X_i^j – arrival queue at the beginning of the *i*th time interval for the *j*th capacity scenario; $j \in S$, $i = 1, 2, \dots, N+1$

Y_i^j – departure queue at the beginning of the *i*th time interval for the *j*th capacity scenario; $j \in S$, $i = 1, 2, \dots, N+1$

u_i^j – airport arrival capacity at the *i*th time interval with scenario j, $i \in I$, $j \in S$

$v_i^j = \phi_i^j(u_i^j)$ – airport departure capacity at the *i*th time interval with scenario j, $i \in I$, $j \in S$

w_i^j – arrival flow at the *i*th time interval with capacity scenario j, $i \in I$, $j \in S$

z_i^j – departure flow at the ith time interval with capacity scenario j, $i \in I$, $j \in S$

3.2 Deterministic optimization model

A traffic flow management strategy determines the actual traffic flow at the airport compatible with the available airport capacity: the number of arrival flights that can land and number of flights that can depart at each 15-minute interval.

For each capacity scenario j (j = 1, 2, ..., s), the following deterministic optimization problem is formulated to determine the best strategy at the airport [4]:

$$\min \sum_{i=1}^{N} [\alpha X_{i+1}^j + (1-\alpha) Y_{i+1}^j], \quad 0 \leqq \alpha \leqq 1 \quad (1)$$

subject to

$$X_{i+1}^j = X_i^j + a_i - w_i^j, \; i \in I, \; j \in S \quad (2)$$

$$Y^j_{i+1} = Y^j_i + d_i - z^j_i, \quad i \in I, \ j \in S \quad (3)$$

$$0 \leqq w^j_i \leqq u^j_i, i \in I, \ j \in S \quad (4)$$

$$0 \leqq z^j_i \leqq \phi^j_i(u^j_i), \ i \in I, \ j \in S \quad (5)$$

$$0 \leqq u^j_i \leqq U^j_i, \ i \in I, \ j \in S \quad (6)$$

w^j_i and z^j_i are integer, $i \in I$, $j \in S$

where (2) and (3) describe flow balance for arrivals and departures, respectively, with given initial conditions X^j_1 and Y^j_1 (4) and (5) are capacity constraints. The coefficient α in (1) can be interpreted as a priority rate for arrivals, see (Gilbo, 1993).

Expression (1) formalizes the optimization criterion: the minimum of a linear function of cumulative arrival and departure queues at the airport over a period T.

It is not difficult to show that if at the end of time period T there are no arrival and departure queues ($X_{N+1} = 0$ and $Y_{N+1} = 0$), then (1) minimizes also a weighted sum of total arrival and departure aircraft flight delay times.

3.3 Stochastic model

Suppose that according to the weather forecast for a time period T, several weather scenarios are predicted, each with a probability of its realization. Based on these probabilities, the probability p_j of each capacity scenario j (j = 1, 2, ..., s) is calculated.

It is not known with certainty which capacity scenario will be realized, but there are several alternative approaches for optimization of traffic management strategy. One approach can be based on using the deterministic solution for one of the capacity scenarios, e.g., the most probable scenario, or the most pessimistic scenario, or the most optimistic scenario, etc. Here, another approach is considered, which is based on constructing a strategy that is optimal on average for the set of capacity scenarios. Any approach in the uncertain environment is associated with a risk of uder–utilizing the airport capacity, or excessively overloading the airport if the implemented strategy does not match the realized capacity scenario. A stochastic approach based on the average-case strategy may alleviate the negative effects caused by the mismatch.

As an optimality criterion we will consider a minimum of the expected value of (1):

$$\min \sum_{j=1}^{S} \sum_{i=1}^{N} p_j [\alpha X^j_{i+1} + (1-\alpha) Y^j_{i+1}] \quad (7)$$

Henceforth, expected values of random values will be denoted by using their original notations with overbars, e.g.,

$$\overline{X}_i = \sum_{j=1}^{S} p_j X^j_i .$$

Criterion (7) minimizes a weighted sum of expected queues at the end of each time interval. Expected values of queues and constraints can be obtained by probabilistic averaging of (1) – (6).

As a result, the average-case optimization problem is formalized as follows:

$$\min \sum_{i=1}^{N} [\alpha \overline{X}_{i+1} + (1-\alpha) \overline{Y}_{i+1}], \ 0 \leqq \alpha \leqq 1 \quad (8)$$

subject to

$$\overline{X}_{i+1} = \overline{X}_i + a_i - \overline{w}_i, \ i \in I \quad (9)$$

$$\overline{Y}_{i+1} = \overline{Y}_i + d_i - \overline{z}_i, \ i \in I \quad (10)$$

$$0 \leqq \overline{w}_i \leqq \overline{u}_i, i \in I \quad (11)$$

$$0 \leqq \overline{z}_i \leqq \overline{\phi}_i(\overline{u}_i), \quad i \in I \quad (12)$$

$$0 \leqq \overline{u}_i \leqq \overline{U}_i, \ i \in I \quad (13)$$

Comparison of (1) – (6) with (8)– (13) shows that these models have the same structure. However, the stochastic model uses an expected curve $\overline{v}_i = \overline{\phi}_i(u)$, which may not coincide with any of the curves from a set of capacity scenarios used in the deterministic model.

The expected capacity curves for each 15-minute interval are calculated by probabilistic averaging of a complete set of capacity curves in the set of predicted scenarios. The computation is especially convenient if the curves are represented in polar coordinates. Then the radius-vector of the expected capacity curve is determined as a weighted sum of radius-vectors of all capacity curves from the set of scenarios with the weights equal to the probabilities of their realization.

This is illustrated in Fig. 2 where two capacity curves $\phi^1(u)$ and $\phi^2(u)$ for two capacity scenarios with probabilities $p_1 = 0.7$ and $p_2 = 0.3$, respectively, are averaged. The bold curve shows the expected capacity curve, each point of which is calculated as a linear combination of two radius-vectors: $\overline{\mathbf{r}}(\theta) = 0.7\,\mathbf{r}^1(\theta) + 0.3\,\mathbf{r}^2(\theta)$.

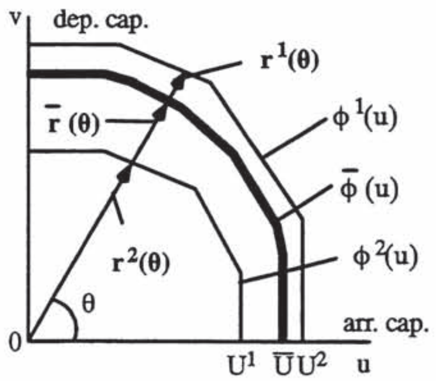

Fig. 2. Expected Airport Capacity Curve for Two Capacity Scenarios

A traffic management strategy based on the expected airport capacity represents the average-case solution, which is a compromise among the strategies obtained separately for each of the possible capacity scenarios. The level of the compromise depends on the distribution of probabilities among the forecasted scenarios. In particular, the average-case strategy may justify a rate of trade-off between airborne and ground delays. Moreover, within the stochastic model several alternative average-case strategies with different relations between airborne and ground delays can be generated by varying the parameter α (arrival priority rate) in the objective function (1).

4. NUMERICAL EXAMPLES

Suppose that according to the weather forecast for a three-hour period from 7:00 to 10:00, one of two capacity scenarios at an airport can be realized: a high-level capacity scenario (see capacity curve 1 in Fig. 3) with probability $p_1 = 0.5$, and a low-level capacity scenario (curve 2 in Fig. 3) with probability $p_2 = 0.5$. Let us say that curves 1 and 2 represent optimistic and pessimistic scenarios, respectively, with approximately 20% difference in airport capacity. In this example, both scenarios are equally probable. Curve 3 in Fig. 3 shows the expected capacity curve.

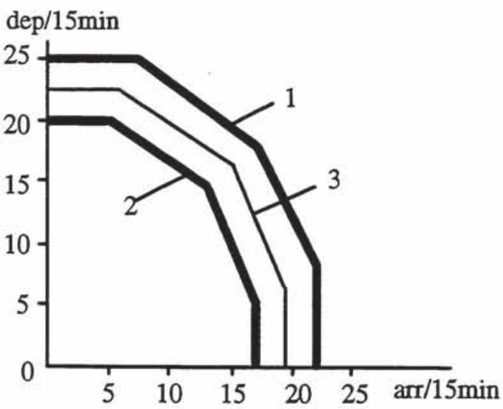

Fig. 3. Airport Capacity Curves

The arrival and departure demands predicted for each 15-minute interval of the three-hour period are shown in Table 1.Because the original traffic demand exceeds available capacity at some intervals, a certain number of flights will always be delayed.

Table 1 Predicted Traffic Demand

Time	Demand arr	Demand dep	Time	Demand arr	Demand dep
7:00–7:15	23	7	8:30–8:45	10	22
7:15–7:30	18	19	8:45–9:00	24	11
7:30–7:45	9	24	9:00–9:15	8	14
7:45–8:00	20	13	9:15–9:30	10	8
8:00–8:15	13	21	9:30–9:45	7	7
8:15–8:30	20	16	9:45–10:00	3	4
			Total:	165	166

The optimal strategy should utilize the airport capacity to the fullest extent to reduce queues and delays. The problem, however, is that airport capacity is not exactly known, so that it may turn out that a strategy has been chosen for the capacity scenario that was not realized. Below we present several numerical examples calculated for various capacity scenarios.

Tables 2, 3, and 4 show the optimal strategies of managing arrival and departure traffic for optimistic, pessimistic, and expected capacity scenarios, respectively.

Table 2 Optimal Optimistic Strategy (α = 0.5)

Time	Traffic Flow Arr	Traffic Flow Dep	Queue Arr	Queue Dep	Airport Capacity Arr	Airport Capacity Dep
7:00–7:15	22	7	1	0	22	8
7:15–7:30	17	18	2	1	17	18
7:30–7:45	11	22	0	3	11	22
7:45–8:00	18	16	2	0	18	16
8:00–8:15	15	19	0	2	15	19
8:15–8:30	17	18	3	0	17	18
8:30–8:45	13	21	0	1	13	21
8:45–9:00	20	12	4	0	20	12
9:00–9:15	12	14	0	0	19	14
9:15–9:30	10	8	0	0	22	8
9:30–9:45	7	7	0	0	22	8
9:45–10:00	3	4	0	0	22	8
Total	165	166	12	7		

Table 3 Optimal Pessimistic Strategy ($\alpha = 0.5$)

Time	Traffic Flow		Queue		Airport Capacity	
	Arr	Dep	Arr	Dep	Arr	Dep
7:00–7:15	16	7	7	0	16	7
7:15–7:30	13	14	12	5	13	14
7:30–7:45	13	14	8	15	13	14
7:45–8:00	13	14	15	14	13	14
8:00–8:15	13	14	15	21	13	14
8:15–8:30	13	14	22	23	13	14
8:30–8:45	13	14	19	31	13	14
8:45–9:00	13	14	30	28	13	14
9:00–9:15	13	14	25	28	13	14
9:15–9:30	13	14	22	22	13	14
9:30–9:45	13	14	16	15	13	14
9:45–10:00	13	14	6	5	13	14
Total	159	161	197	207		

Table 4 Optimal Average-Case Strategy ($\alpha = 0.5$)

Time	Traffic Flow		Queue		Airport Capacity	
	Arr	Dep	Arr	Dep	Arr	Dep
7:00–7:15	19	7	4	0	19	7
7:15–7:30	15	16	7	3	15	16
7:30–7:45	15	16	1	11	15	16
7:45–8:00	15	16	6	8	15	16
8:00–8:15	15	16	4	13	15	16
8:15–8:30	15	16	9	13	15	16
8:30–8:45	15	16	4	19	15	16
8:45–9:00	15	16	13	14	15	16
9:00–9:15	15	16	6	12	15	16
9:15–9:30	15	16	1	4	15	16
9:30–9:45	8	11	0	0	17	11
9:45–10:00	3	4	0	0	19	7
Total:	165	166	55	97		

In these tables, the Traffic Flow columns show optimal traffic flow management strategies for each 15-minute interval, i.e., number of flights that can land (arrival flights) and depart without violation of capacity constraints. The Queue columns show numbers of flights in arrival and departure queues at the end of each 15-minute interval. The arrival queues shown in these tables, are ground queues, i.e., the delayed flights are delayed on the ground at origin airports. The Airport Capacity columns show optimal allocation of arrival and departure capacities during each 15-minute interval.

Alternative strategies can be obtained by varying the parameter α in (1).

If a strategy has been implemented then all arrival flights from the Traffic Flow column Tables 2, 3, and 4 are considered airborne and constitute the actual arrival demand at the airport. In the situations when the optimistic or average-case strategy has been implemented but the low-level capacity scenario (the pessimistic scenario) has been realized, the airport may experience congestion for both arrivals and departures. During periods of congestion airborne delays become inevitable.

Series of computational experiments have been performed for the optimistic and the average-case strategies with pessimistic capacity scenario realized. In these situations, the average-case strategy absorbed more arrival delays on the ground and provided less airborne delays than the optimistic strategy.

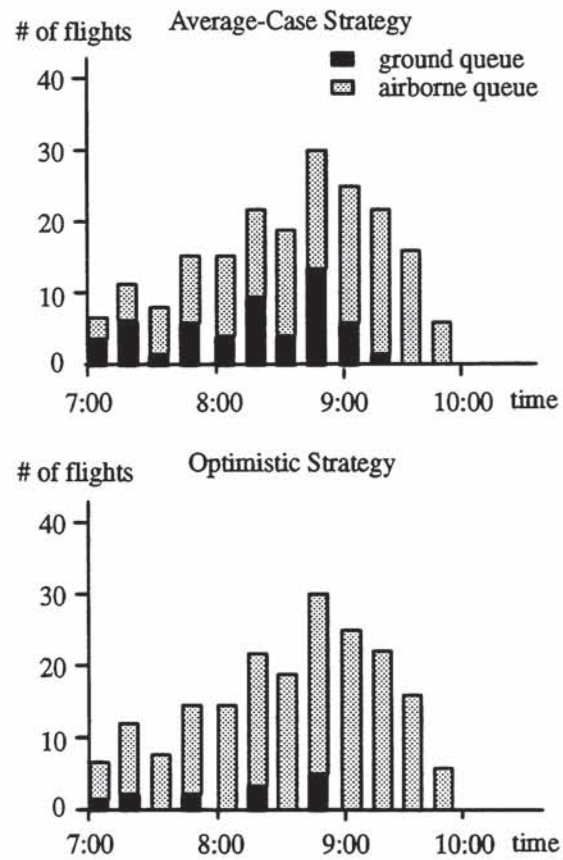

Fig. 4. Airborne and Ground Queues

Fig. 4 illustrates this effect showing airborne and ground queues for arrival flights under pessimistic capacity scenario with optimistic and average-case strategies implemented.

If the optimistic capacity scenario realized, then both average-case and optimistic strategies would be implemented without any airborne delay.

A strategy, based on pessimistic capacity scenario (see Table 3), is very conservative and deliberately keeps the traffic flow at its lowest level. When implemented, it does not allow any chance to use additional arrival capacity at the airport if a scenario with increased capacity has been realized. However, in this case it is possible to improve the departure process by reallocating unused arrival capacity (due to small arrival demand) to increase departure capacity. It would reduce departure queues and delays.

5. CONCLUSION

In this paper we have discussed strategic management of arrival and departure traffic at airports with uncertainty in airport capacity. A stochastic optimization problem has been formulated to obtain strategies that are optimal on average for a set of forecasted capacity scenarios. A concept of expected airport capacity was introduced. The latter was applied to generate average-case strategies that optimize the allocation of arrival and departure flows at airports under uncertainty in airport capacity forecast. The proposed model can be used as a decision support tool for air traffic managers and controllers. In particular, based on the rate of uncertainty, the average-case strategies could justify a magnitude and proportion of airborne and ground delays in situations of unexpected reduction of airport capacity. This also would suggest a size of arrival reservoir (a surplus of arrival flights) that could be accepted at the airport without airborne delays if airport capacity unexpectedly increased. This paper does not discuss decisions about whether the duration of airborne delays or the size of the arrival reservoir are acceptable. However, the model presented will allow traffic managers to adjust a particular strategy to achieve an acceptable solution.

REFERENCES

Gilbo, E.P. (1993). Airport Capacity: Representation, Estimation, Optimization. *IEEE Transactions on Control Systems Technology,*, **1**, pp. 144–154.

Richetta, O. and A.R. Odoni (1993). Solving Opti–mally the Static Ground-Holding Policy Problem in Air Traffic Control. *Transportation Science*, **27**, pp. 228–238.

Terrab, M. and A.R. Odoni (1993). Strategic Flow Management for Air Traffic Control. *Operations Research*, **41**, pp. 138–152.

Vranas, P.B., D.J. Bertsimas, and A.R. Odoni (1994). Dynamic Ground-Holding Policies for a Net–work of Airports. *Transportation Science*, **28**, pp. 275 – 291.