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The European Air Traffic Flow Management Problem

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Air traffic flow management in Europe has to deal as much with capacity constraints in en route airspace as with the more usual capacity constraints at airports. The en route sector capacity constraints, in turn, generate complex interactions among traffic flows. We present a deterministic optimization model for the European air traffic flow management (ATFM) problem. The model designs flow management strategies involving combinations of ground and airborne holding. The paper illustrates the complex nature of European (EU) ATFM solutions, the benefits that can be obtained by purposely assigning airborne holding delays to some flights, and the issues of equity that arise as a result of the interactions among traffic flows. In particular, we show that, in certain circumstances, it is better, in terms of total delay and delay cost, to assign to a flight a more expensive airborne holding delay than a ground delay. We also show that in the EU ATFM context, fundamental conflicts may often arise between the objectives of efficiency and equity (or "fairness"). This finding may have profound implications for the possibility of developing a "collaborative decision-making" environment for air traffic flow management in Europe.

Key words: air traffic flow management; air traffic control; airport delays; air traffic delays
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1. Introduction

A critical practical difference between the current air traffic management (ATM) systems in the United States and in Europe is that the capacity constraints in the former occur primarily at major airports or in the terminal airspace around them, whereas in the latter the en route airspace poses an equally important and frequent capacity constraint. Moreover, the capacity constraints at the most congested airports in western and central Europe have been partly alleviated by treating these airports as "fully coordinated"—meaning, essentially, that the number of flights that can be scheduled there per hour (or other unit of time) is not allowed to exceed the "declared capacity" of the airports (de Neufville and Odoni 2003). As a result, scheduled demand at these airports exceeds capacity only on days when weather conditions are very unfavorable or other nonroutine events occur. By contrast, some en route sectors, especially in the central and western parts of Europe, experience congestion on an almost routine basis, especially during the summer months. While the capacity of these sectors has generally increased in very recent years as a consequence of measures taken on a local and continent-wide basis (e.g., increasing the number of high-altitude flight levels), the problem posed by the en route sector capacity constraints is persistent and may take at least another decade to resolve (EUROCONTROL Performance Review Commission 2004).

One of the implications of the almost constant presence of simultaneous airport and en route airspace constraints is that ATFM in Europe is, in some respects, a fundamentally more complicated task than in the United States. The overall European ATM system (which is composed of many different national ATM systems) provides limited flexibility when it comes to rerouting flights dynamically to avoid congested sectors on any given day. Thus, in developing ground-holding strategies and planning air traffic flows, European ATFM must consider a true network of capacitated elements (en route sectors and airports). We shall call this the European ATFM (EU ATFM) problem and explore some of its special characteristics by means of an optimization model.

This appellation does not mean that the work to be described is irrelevant to the U.S. ATFM environment. Its applicability, however, may be less common and straightforward. In the United States, traffic managers at the Federal Aviation Administration's (FAA's) Strategic Command Center in Virginia and at regional ATM centers around the country can usually concentrate on addressing congestion problems at *individual* airports, as they arise on a daily basis. These problems are addressed primarily through the device of ground delay programs (GDPs), as well as through other more tactical actions, such as "metering" and "miles-in-tail"—see, e.g., de Neufville and Odoni (2003) or Ball et al. (2007). This is done in the context of the highly successful collaborative decision making (CDM) initiative created by the FAA (Chang et al. 2001). Any problems that may arise on any given day due to en route capacity constraints or ground delay programs at other airports are typically addressed separately. This is achieved, in part, because the U.S. system is more flexible in its handling of en route traffic, which can often be rerouted around weather-affected sectors or reallocated among congested sectors. Thus, en route constraints in many—but by no means all—cases have only a secondary impact on airport GDPs.

Recent developments and research (e.g., Hoffman et al. 2005; Krozel, Jakobovits, and Penny 2006) make it likely that such flexibility will, if anything, increase in the near future. As a first approximation, GDPs in the United States usually deal with a simple "network" containing only a single capacitated element (the destination airport) at a time. This, of course, does not mean that models that deal simultaneously with en route and airport constraints are not important to the U.S. environment under the most critical conditions, such as adverse weather with widespread regional impacts.

The EU ATFM problem can now be stated as follows: Given an airspace system, consisting of a set of *airports*, *airways*, and *sectors*, each with its own capacity for each time period, t, over a time horizon of T periods, and given a schedule of flights through the airspace system during T, assign ground and airborne delays to the flights in a way that satisfies all the capacity constraints while minimizing a function of the cost of the total delay assigned.

This paper presents an approximate, low-level-of-detail model for the EU ATFM, similar in spirit to the conceptual "star network" model (Odoni 1987) for the single-airport GDP that motivated much of the early research on ATFM. The EU ATFM model is designed to highlight two points: (a) the intrinsic complexity of optimal ground and airborne holding strategies in the European context and (b) the fundamental conflict that may arise between the objectives of efficiency and equity (or "fairness"). Both characteristics stem from the underlying capacitated network structure.

Point (b) requires explanation. Efficiency (i.e., minimizing some function of total cost) and equity (i.e., the absence of a systematic bias against certain flights or airlines or origin/destination pairs) are generally thought of as the two overriding considerations of ATFM (Ball et al. 2007). In the United States "rationing by schedule" (RBS) is the cornerstone principle for GDPs and other CDM initiatives (Wambsganss 1996; Chang et al. 2001). Under RBS, resources—arrival slots at airports, in the case of GDPs—are assigned to flights in accordance with a first-scheduled, first-served (FSFS) priority discipline. All CDM stakeholders (the individual airlines and the FAA) have agreed that RBS provides a basis for GDP planning that is fair to all parties. It was this agreement that spurred the successful development of CDM during the mid-1990s. Indeed, FSFS not only is "fair" in the intuitive sense, but it can also be shown to have mathematical properties consistent with the notion of equity in delay allocation (Vossen and Ball 2006). In today's CDM environment, efficient solutions to ground-holding problems are sought subject to satisfying the RBS priorities. However, exemptions from RBS are made for flights that are already airborne, as well as for long-distance flights (e.g., of four or more hours), which must take off when the forecast of capacity for the airport of destination is still very tentative (Ball and Lulli 2004).

For the single-airport GDPs in the United States, the RBS constraint does not usually lead to a significant reduction of the efficiency of the selected ground-holding plan, especially when the GDP is supplemented by such CDM tools as flight substitution, cancellations, compression, and slot credit substitution—see Ball et al. (2005) for descriptions. It will be shown below that, in the context of the EU ATFM, the conflict between equity and efficiency can be far more severe. Adherence to simple principles, like RBS, may result in highly inefficient solutions with respect to cost. This implies that it may prove far more challenging to reach stakeholder consensus in Europe on what would constitute "fair and equitable" priority rules during periods of congestion.

Section 2 will present the EU ATFM model that has been developed to investigate these issues. Section 3 will use four generic cases, each involving a simple network configuration of airports and sectors, to illustrate the characteristics of solutions to EU ATFM problems. Finally, §4 will summarize the findings of this research and briefly discuss the practical implications.

2. The Mathematical Model

The mathematical model described here is designed to highlight the strategic aspects of the EU ATFM problem, while avoiding an excessive level of detail and "second-order" features. It is based on the following assumptions:

1. *Discrete time periods*. The time horizon, T, is fixed and subdivided into discrete, equal-sized, contiguous time periods, $t = 1, \dots, T$.

2. *Deterministic demand*. The schedule of arriving flights at all destination airports over T is known. The

schedule includes scheduled departure time, scheduled arrival time (assumed to also be the earliest arrival time that can be assigned to the flight), and the flight route as a set of en route sectors that must be traversed.

3. *Deterministic capacity*. Both airport and sector capacities are known in advance for every time period t.

4. *Location of airborne delay*. An airborne delay can be assigned to a flight only in the terminal airspace around its destination airport.

spiegato dopo il motivo.

5. *Equal speed of travel*. All aircraft fly at the same speed; i.e., flight times between any two elements of the network (airports and/or sectors) are the same for all the flights in the schedule.

6. *No rerouting*. Dynamic rerouting, in response to en route sector congestion, is not an available option.

7. *Sector capacity*. The capacity of an en route sector is equal to the average number of aircraft that are permitted to enter the sector in a single time period.

8. *Secondary*. Departures can take off when required from the airports of origin; no consideration is given to delay propagation from earlier flights to subsequent flights by the same aircraft; flights are not cancelled because of delays; there is no constraint on the number of aircraft that can be placed in airborne queues in terminal airspace.

Assumption 1 is typical of practically all ATFM models and agrees with practice. Assumptions 2 and 3 place the model in the "deterministic" category. Although stochastic considerations can be critical in ground-holding problems (Richetta and Odoni 1993; Ball et al. 2007), models with stochastic features have yet to be implemented even in support of the simpler GDPs in the United States. It is not necessary to include the additional complication of stochasticity in the EU ATFM model at this point, as the special characteristics highlighted by this paper are present even in the absence of uncertainty. Assumptions 4–7, as a group, simplify the model greatly by making it unnecessary to track individual aircraft (or types of aircraft) or to consider distances between elements of the network. At the same time, these assumptions are reasonable approximations to current practice. Traffic managers rarely, if ever, plan in advance on holding aircraft in en route airspace, and local air traffic controllers exercise en route holding only for tactical reasons, such as a brief period of excessive workload in a downstream sector. Thus, as in Assumption 4, practically all planned or unplanned airborne holding takes place in or near terminal airspace.

Assumption 5 is also a good approximation of current conditions when applied to turbojets, which, almost exclusively, populate high-altitude en route sectors and constitute the great majority of aircraft at the busiest commercial airports. Flight rerouting (Assumption 6), as already noted, is also unusual in Europe for technical reasons, as well as because of the "ripple effect" that the rerouting of any flight may have on other flights.

When the length of time units is selected appropriately, the definition of en route sector capacity in Assumption 7 is roughly equivalent to setting the capacity equal to the number of aircraft that can occupy a sector simultaneously, which is the more conventional definition. Indeed, in the examples that follow, the length of time periods has been set to 15 minutes, which is typical of the amount of time aircraft spend in any given sector. Although the model can be modified to accept sector occupancy times that vary from flight to flight, this would be unnecessary detail and would not add any insights to the issues of interest in the paper.

Finally, the simplifications in Assumption 8 are typical of those made (often without being explicitly stated) in models of a strategic scope similar to that of the EU ATFM model. If not made, these assumptions would produce only "second-order" perturbations to the results, while greatly complicating the analysis.

As a consequence of Assumptions 1–8, only two control variables are associated with each flight: the assigned amount of ground delay before takeoff from the airport of origin and the amount of airborne delay assigned to be taken in the airspace near the airport of destination.

The EU ATFM model, like most other ATFM models in the literature, minimizes a linear combination of the form $\alpha \cdot AH + GH$, of airborne-holding delay (AH) and ground holding delay (GH), with the "equivalence factor" α typically set to $\alpha > 1$. It will be convenient to rewrite the objective function as $TD + (\alpha - 1) \cdot AH$, i.e., as the sum of total delay ($TD = AH + GH$) and of the additional cost $(\alpha - 1) \cdot AH$, which is incurred if flight f is delayed in the air. To add flexibility, the model allows for the possibility of making the equivalence factor an increasing convex function, $\alpha = \alpha(AH)$, of the airborne-holding delay. This nonlinearity is handled computationally by means of a piecewise linear approximation, i.e., by introducing, for each flight f, an artificial variable z^f and a set I^f of constraints $z^f \geq \gamma^i \cdot AH_f + \beta^i$, $i \in I^f$ (where γ^i and β^i are appropriate coefficients) to the formulation.

A second feature of the model is designed to ensure equity in the assignment of delays to flights, and appears in other models as well (Kotnyek and Richetta 2006). This is achieved by including in the objective function cost coefficients that are a superlinear function of the tardiness of a flight of the form $c_t^f = (t - t_a^f)^\varepsilon$, with $\varepsilon > 1$, where t_a^f is the scheduled time of arrival of flight f. The use of these cost coefficients will favor the assignment of a moderate amount of total delay to each of two flights rather than the assignment of a small amount to one and a large amount to the other.

The following additional notation is now defined:

- K the set of airports
- F the set of flights
- S the set of sectors
- t_a^f the scheduled arrival time of flight f, which is also the earliest time flight f can arrive at its destination airport
- t_d^f the scheduled departure time of flight f
- $g(f)$ the subset of sectors that will be visited by flight f en route
- $a(f)$ the destination airport of flight f
- ft_f^s the flight time from the origin of flight f until it arrives at sector $s \in g(f)$
- FT^f the total scheduled airborne flight time for flight $f \in F$
- $A^k(t)$ the capacity (arrival acceptance rate) of airport $k \in K$ in time period t
- $C^s(t)$ the capacity of sector $s \in S$ in time period t.

The decision variables are

$$x_t^f = \begin{cases} 1 & \text{if flight } f \text{ lands at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_t^f = \begin{cases} 1 & \text{if flight } f \text{ takes off at time } t \\ 0 & \text{otherwise} \end{cases}$$

z^f an artificial variable which captures the airborne delay cost of flight f.

The EU ATFM model is the following:

$$\text{Min} \sum_{f \in F} \left[\sum_{t \in T} c_t^f \cdot x_t^f + z^f \right]$$

subject to

$$\sum_{t \in T} x_t^f = 1 \quad \forall f \in F. \tag{1}$$

$$\sum_{t \in T} y_t^f = 1 \quad \forall f \in F. \tag{2}$$

$$\sum_{f \in F: a(f)=k} x_t^f \le A^k(t) \quad \forall t \in T, \forall k \in K. \tag{3}$$

$$\sum_{f \in F: s \in g(f)} y_{t-ft_f^s}^f \le C^s(t) \quad \forall t \in T, \forall s \in S. \tag{4}$$

$$y_t^f \le \sum_{\tau = t+FT^f}^{T} x_\tau^f \quad \forall t \in T, \forall f \in F. \tag{5}$$

$$z^f \ge \gamma^i \cdot \left(-FT^f + \sum_{t \in T} t(x_t^f - y_t^f) \right) + \beta^i \quad \forall i \in I^f, \forall f \in F. \tag{6}$$

$$y_t^f, x_t^f = \{0, 1\} \quad \forall t \in T, \forall f \in F. \tag{7}$$

Constraints (1) and (2) are the assignment constraints that uniquely determine the arrival and the departure time of each flight $f \in F$. Constraints (3) and (4) are the airport and sector capacity constraints, respectively, that impose a bound on the number of flights that can be handled in each time period. Capacity may change over time because of many factors, most commonly meteorological conditions. Constraints (5) link the variables: The time of arrival of each flight is given by the departure time plus the flight time, which may include some airborne holding delay. Constraints (6) capture the nonlinearity of the airborne holding cost, as already discussed.

The EU ATFM model can be seen as a "macroscopic version" of the model presented in Bertsimas and Stock Patterson (1998) in that it omits certain details of the latter. Its objective function is also different. Bertsimas and Stock Patterson are notable for using an alternative set of decision variables that lead to fast solutions of the resulting integer programs. The tightness of the formulation is not an issue in this paper, as the problems we solve are modest in size and solution times are insignificant. By adopting a low level of detail and a straightforward formulation, the EU ATFM model is better able to highlight the critical characteristics that could prove of overriding practical importance in the EU ATFM environment.

In a subsequent paper, Bertsimas and Stock Paterson (2000) extend their model to cover the possibility that flight routes may vary according to the level of congestion in the system. The decision space in this case becomes much larger, so that aggregate variables and approximate methods must be employed to solve problems of realistic size. By contrast, the EU ATFM model, per Assumption 6, does not allow flight routes to vary—a reasonable approximation for the ATM environment in Europe, as discussed earlier.

3. The Characteristics of Solutions to the EU ATFM

To better explore the characteristics of solutions to the EU ATFM and illustrate the consequences of sector capacity and the network structure, the following four generic cases (clockwise in Figure 1) will be examined:

(a) Single sector: All incoming flights to the destination airport must cross a single sector with capacity restrictions.

(b) Multiple sectors, in series: All incoming flights to the destination airport have to cross two or more sectors, in series, each with capacity restrictions.

(c) Two parallel sectors: Incoming flights to an airport from two different origins, Airport A and Airport B, cross two different and independent sectors, each with capacity restrictions.

(d) Multiairport: Two flows of flights with different origin-destination pairs (A, A') and (B, B') cross the same sector; the two flows are denoted as Flow AA' and Flow BB', respectively.

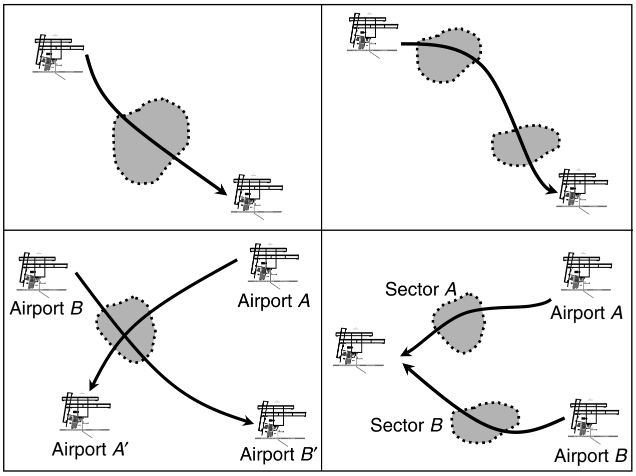

Figure 1 Instances of the EU ATFM Problem

Note. Clockwise from the upper left-hand corner the cases of single-sector, multiple sectors in series, two parallel sectors, and multiairport.

Case (a) involves the simplest conceivable "network" configuration and will be used primarily to illustrate the effects of the presence of a capacitated sector on the nature of the solutions to the EU ATFM. This case will demonstrate that the judicious use of airborne holding, in addition to ground holding, can generate large potential benefits, i.e., a substantial reduction in the total amount of delay and of delay cost. This is entirely unlike the case in which no en route capacity restrictions exist. In the latter case, it is well known that optimal solutions, *in the absence of uncertainty*, do not include any airborne holding—see, e.g., Terrab and Odoni (1993).

Case (b) will be shown to be reducible to Case (a) under the modelling assumptions of §2. Cases (c) and (d) are truer network configurations, as they involve interactions between different flows of traffic that compete for access to common resources. In addition to further exploring the characteristics of EU ATFM solutions, these two cases will be used to emphasize the strong possibility that such solutions may violate fundamental notions of "fairness" and of equitable treatment of all network users.

The EU ATFM model of §2 has been tested on instances based on demand at Milan's Malpensa (MXP) airport. The aircraft schedule data, provided by ENAV, Italy's Air Traffic Control Authority, represent a typical weekday of operations. A total of 368 arrivals are scheduled between 5 A.M. and 11 P.M., local time. This 18-hour span is subdivided into 72 15-minute time units. In Figure 2, the scheduled landings per time unit between 7:30 and 10:30 A.M., the airport's busiest 3-hour period, are shown. On average, 40 arrivals per hour are scheduled. Time unit 10 corresponds to the 15-minute period commencing at 7:30 A.M., and so on. The demand for landings at MXP reflects the hub-and-spoke operations at the airport, where periods with a large number of arrivals alternate with periods with a large number of departures. The demand pattern of Figure 2 is the one used throughout the experimental analysis in the paper.

For airport capacity, measured as the number of arrivals allowed per unit of time, several different values will be used, spanning a variety of weather conditions. MXP's maximum landing capacity is 46 aircraft per hour (or 11.5 per unit of time), but it can be considerably lower under less-than-ideal weather. Note that for the 7:30–10:30 A.M. period, the average arrival demand per hour is equal to 87% of the maximum landing capacity, so there is little "slack," even in good weather.

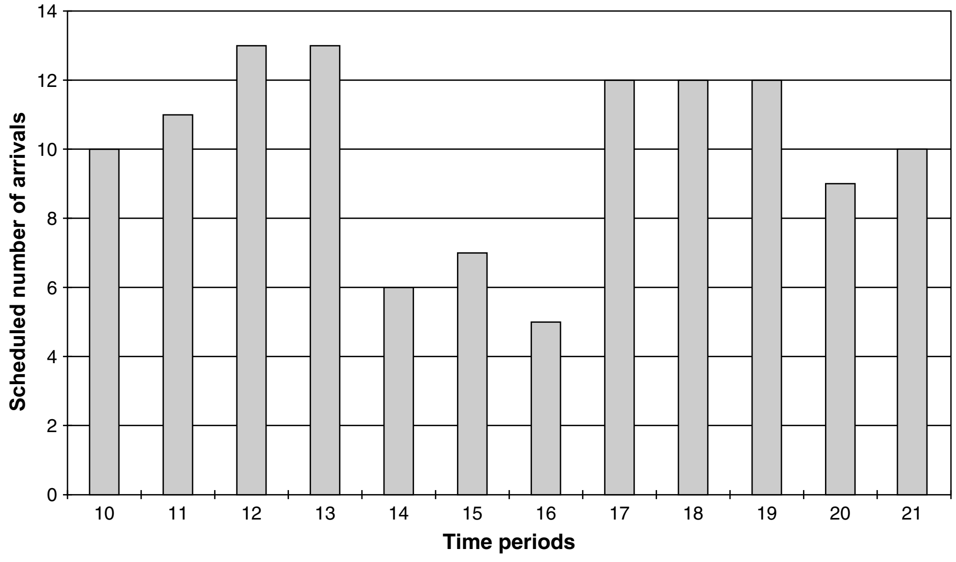

Figure 2 Demand for Landings at MXP (7:30–10:30 A.M. Local Time)

A range of en route sector capacity values will be used to illustrate various characteristics of the solutions to the EU ATFM problem for each of the four cases identified in Figure 1. All these values can be encountered in Italy's airspace, depending on weather conditions and sector location.

Two values of the equivalence factor α have been used in the objective function: 1.2 for airborne holding delays shorter than 3 time units (45 minutes) and 3 for longer airborne delays. These values, although arbitrarily selected, are in a range similar to that often used in the literature. They reflect the fact that airlines and air traffic controllers may be willing to accept reasonably small amounts of airborne delay for individual flights but are strongly averse to long airborne holding. For the exponent in the super-linear coefficients for the tardiness of a flight we have used $\varepsilon = 1.05$.

3.1. The Single-Sector Case

In applying the EU ATFM model to an example illustrating the single-sector case, we first consider an instance in which the airport arrival capacity is 10 per time unit (solid line in Figure 3), marginally below its largest possible value at MXP. The capacity of the hypothetical capacitated single sector is 12 per time unit, with the exception of time units 14 and 15, when only 7 aircraft per time unit are allowed into the sector (dashed line in Figure 3). The sector's capacity "curve" (dashed line) has been shifted in time by a number of time units equal to the flight time from the sector to the airport. This allows a direct visual comparison of sector capacity, airport capacity, and demand.

Figure 3 also displays the solution obtained through the EU ATFM model. For each time unit, the number of flights assigned a delay of one time unit, either on the ground or in the air, is shown in gray and in black bars, respectively.

The following will help in interpreting the solution. Consider time unit 12. One of the flights (gray bar) scheduled to land at the airport in time unit 12 is delayed on the ground for one time unit; i.e., it departs one time unit later than its scheduled departure time. Moreover, three flights (black bar) are delayed in the air, for a total of four delayed flights. This means that the total demand for landings in time unit 12 will be 13 (the original demand) minus 4 (the delayed flights) plus 1 (the flight from unit 11—see Figure 3—that was delayed in the air) for a total of 10, exactly matching airport capacity in time unit 12. The four flights delayed in unit 12 will increase the demand for landings at time unit 13, which will now total 17—the scheduled 13 flights plus the 4 delayed flights. As a result, 7 of the 13 flights originally scheduled to land in time unit 13 will be delayed by one time unit—5 in the air and 2 on the ground, as shown in Figure 3. (The reason that the 4 flights delayed in time unit 12 obtain "priority"—and 7 flights scheduled to land in time unit 13 are delayed—is the super-linearity of the objective function.) The solution shown beyond time unit 13 can now be understood in similar fashion.

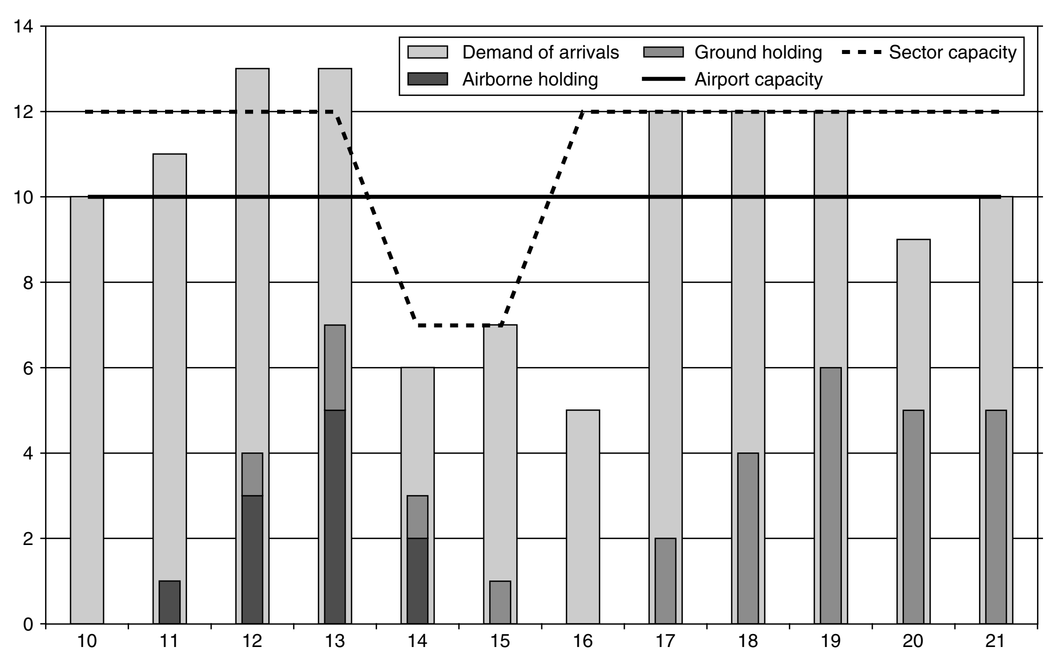

Figure 3 A Single-Sector Example and Its Solution

Note that en route sector demand is affected only by the flights delayed on the ground, not those delayed in the air, due to Assumption 4 (that postulates that airborne delay may occur solely in the terminal airspace of the destination airport). For example, the three flights delayed in the air in time unit 12 will not be part of sector demand in time unit 13 because that airborne delay is taken after the flights have traversed the sector. However, these three flights *are* part of sector demand in time unit 12. The sector demand at time unit 13 will consist of 14 flights—the 13 scheduled originally plus the one flight that was delayed on the ground in time unit 12.

Note that at least one unit of ground delay is assigned in each of the time units 12 through 21, with the exception of time unit 16, while airborne holding occurs only in time units 11 through 14. The fact that there is any airborne delay at all, in the absence of uncertainty about future capacity and demand values, is due to the combined effect of capacity limitations at the airport and the en route sector. For instance, in time unit 11, one flight is assigned one unit of airborne holding because of the combination of an airport capacity constraint in time unit 11 and a sector capacity constraint in time unit 12. The flight in question will leave its airport of origin on time, so it can cross the en route sector in the equivalent of time unit 11 (thus avoiding the sector capacity overload that begins at unit 12) and arrive at the terminal airspace of the airport in time unit 11, thus necessitating one period of airborne holding because of the airport capacity restriction in time unit 11.

To further illustrate the effect of sector capacity constraints and the possible advantage of assigning airborne holding delays in the more complex EU ATFM environment, we compare solutions provided by three different models: the EU ATFM model; a model, Only GH, that relies solely on ground holding to address both airport and sector restrictions; and a ground-holding model, No Sect, in which sector capacity constraints are removed. Note that the No Sect model provides a lower bound for solutions of the EU ATFM model with respect to total assigned delay and the Only GH model an upper bound. The difference between the upper and lower bounds will generally increase as sector capacity becomes more restrictive. Table 1 reports the number of flights delayed in each time period according to each of the proposed models.

Table 1 **Number of Delayed Flights per Time Unit per the Solutions of the Only GH, EU ATFM, and No Sect Models**

	Time periods										
Model	11	12	13	14	15	16	17	18	19	20	21
Only GH	1	4	7	6	6	1	3	5	7	6	6
EU ATFM											
GH	—	1	2	1	1	—	2	4	6	5	5
AH	1	3	5	2	—	—	—	—	—	—	—
No Sect	1	4	7	3	—	—	2	4	6	5	5

Figure 4 displays the results of Table 1, i.e., the number of delayed flights in each time period for each model. Note that the EU ATFM model manages to assign the minimum possible amount of total delay (as indicated by the No Sect model) in every time unit but 15—when No Sect assigns no delay, while EU

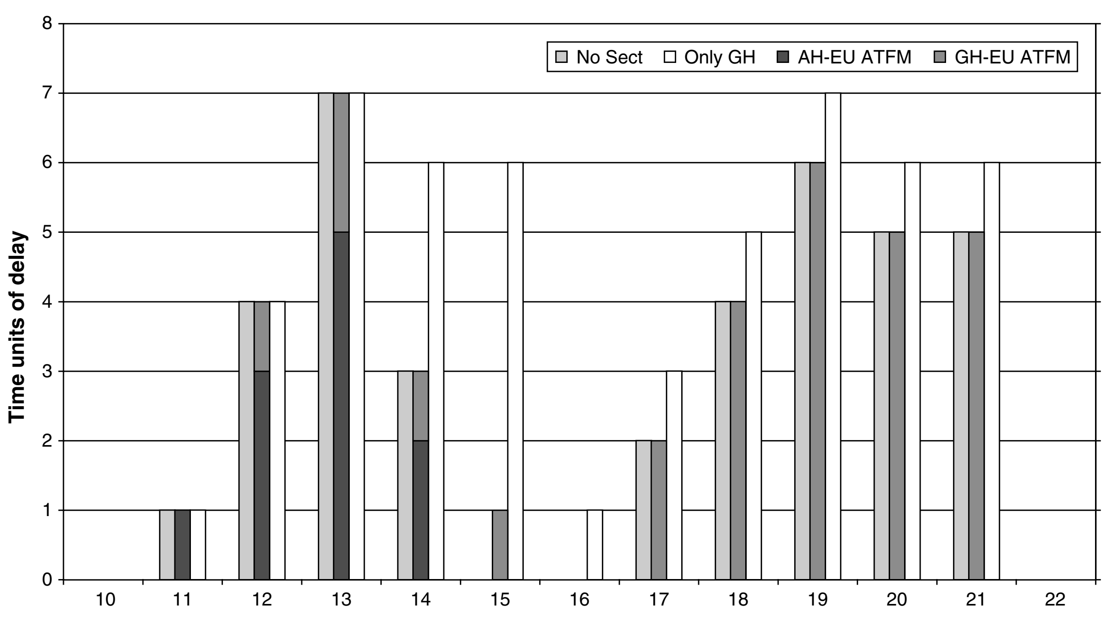

Figure 4 **Comparison of Solutions of EU ATFM, Only GH, and No Sect Models**

ATFM assigns one unit of delay. Essentially, EU ATFM assigns airborne holding "strategically" to mitigate the effects of the sector capacity constraints. In this particular instance, a total of 11 units of airborne holding during time units 10 through 13 in the EU ATFM offset 25 units of ground holding in the Only GH solution. For the value of α (=1.2) that applies to this example, the advantage of the EU ATFM over the Only GH solution is substantial not only in terms of total delay, but also of total "cost." This is achieved primarily by having airplanes take off before an en route constraint sets in, thus avoiding long ground-holding delays. For example, the effect of the one flight delayed on the ground in time unit 16 in the solution of the Only GH model (Figure 4) propagates throughout all the remaining time units, contributing 6 time units to the total delay.

3.2. The Multiple Sectors In-Series Case

The EU ATFM model can handle without difficulty the generalization of the single-sector case of the previous section to the case in which a flight between a given origin-destination pair of airports has to traverse multiple en route sectors arranged in series. It can be observed, however, that this particular case can be easily reduced to the single-sector case shown in §3.1, because of Assumptions 4 and 5. All one has to do is merge all in-series sectors into a single "equivalent sector." First, the capacity profiles of all the sectors must be shifted to a single time reference (e.g., the local time of the destination airport) by transposing the time axis by an amount equal to the flight time between each of the sectors and the reference location—as was done in Figure 3 for the single capacitated sector. The capacity profile of the single equivalent sector is then determined by the envelope defined by the minimum of the sector capacities available in each time period. The destination airport must be treated as a separate entity because of the availability of airborne holding space between the "equivalent sector" and the airport.

3.3. The Case of the Two Parallel Sectors

Turning next to the "two parallel sectors" case (Figure 1), our interest lies primarily in illustrating and discussing the inherent tendency of EU ATFM toward solutions that would be viewed by most as "unfair" in some important respect.

The same demand profile as in Figure 2 is used, but now each flight is associated randomly with one of the two different airports of origin, Airport *A* and Airport *B*. The resulting demand for arrivals from each airport is shown in Figure 5, with the total demand as in Figure 2. The capacity of the destination airport (not shown in Figure 5) is kept constant throughout at 10 per time unit, as in §3.1 (Figure 2). The capacity of the two sectors is also constant at 7 each per time unit, with the important exception that in units 14, 15, and 16, the capacity of Sector *A* alone (Figure 5) is reduced to 0, presumably because of bad weather. Note that Sector *B* has adequate capacity to serve all traffic through it, except for time units 18 and 19, when its capacity falls short by only one flight in each.

Figure 6 shows the solution to the EU ATFM model for this example. Consider the four time units 10 through 13. During this period there is no sector capacity problem at either of the parallel sectors, but the overall demand (10, 11, 13, and 13 in the four consecutive time units) exceeds airport capacity by 1 in unit 11

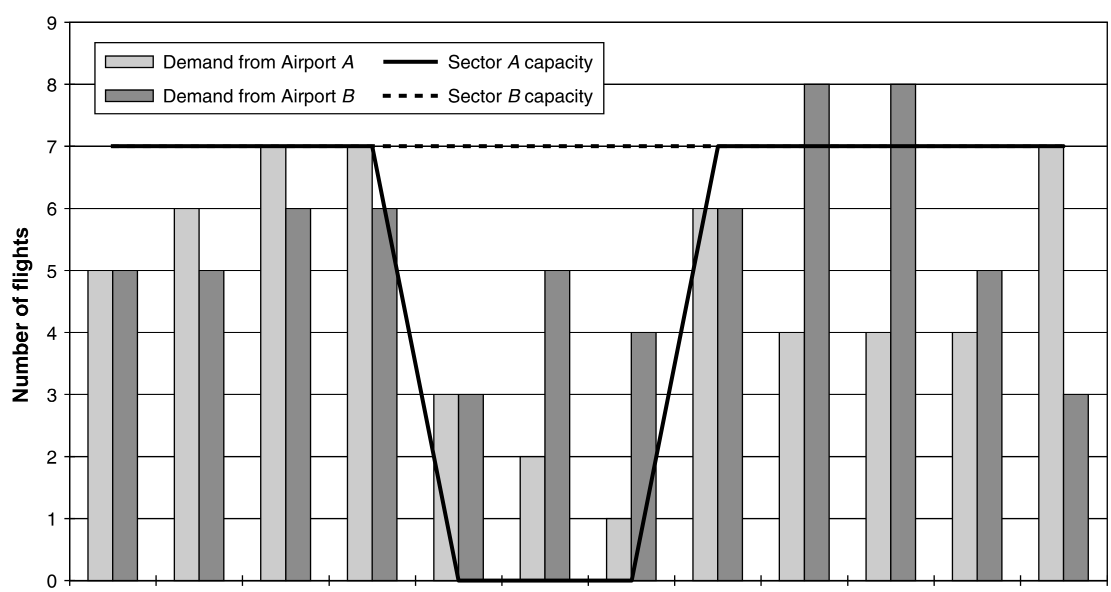

Figure 5 **The "Two-Sectors, Parallel" Example**

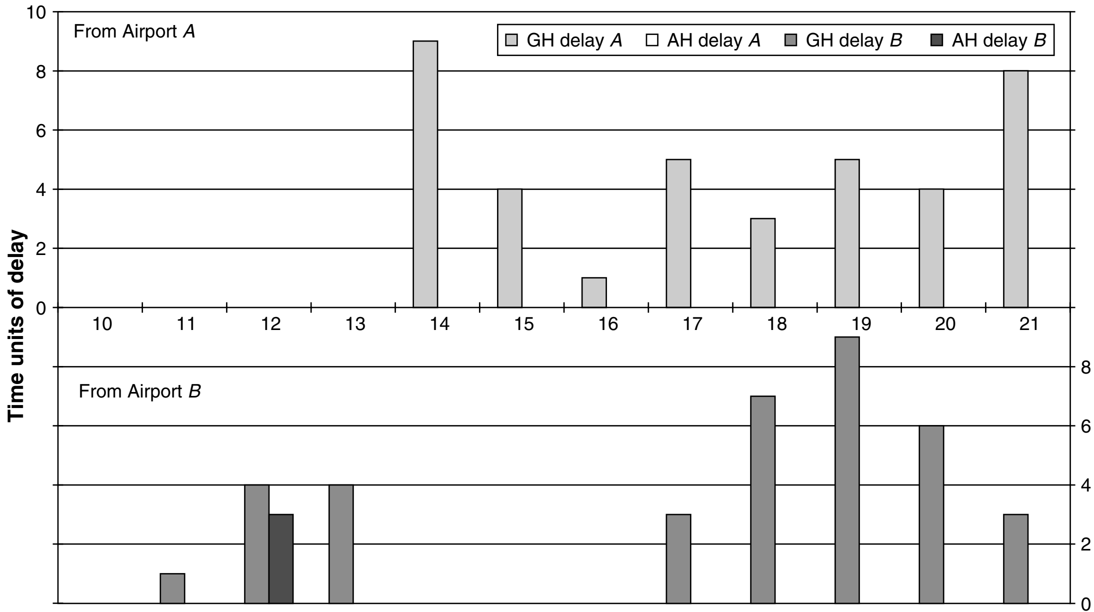

Figure 6 The Solution to the "Two-Sectors, Parallel" Example

and by 3 in units 12 and 13. Note that the solution assigns the *entire* "burden" of the congestion in these four time units to flights from Airport *B* in the form of 9 units of ground delay and 3 of airborne delay. This happens despite the fact that Sector *B* is not affected by *any* capacity reductions during the day, while Sector *A* is severely affected! The explanation is that, in the interest of minimizing total delay cost, the solution to the EU ATFM "pushes" through the network as many flights from Airport *A* as possible during time units 10–13, to the detriment of flights from Airport *B*. This is done because all access from Airport *A* to the destination airport will be cut off during time units 14–16, as a result of the capacity of Sector *A* being zero. Thus, the solution essentially takes maximum advantage of the window of opportunity available to flights from Airport *A* during time units 10–13. The net effect is that the capacity reduction in Sector *A* "harms" only flights departing from Airport *B*, which do not fly through Sector *A* at all!

The next example is purposely constructed to reinforce this point. Let $T = 7$ time units and assume that the capacity of the destination airport, which usually equals 6 arrivals per time unit, is only 3 per time unit on a particular day. Moreover, Sector *B* will have a capacity of 0 for time units 4–6 that day, while Sector *A* will experience partial reduction of its normal capacity during that period. The exact situation is summarized in Table 2.

The detailed solution of the EU ATFM in this instance is shown in Table 3. Flights have been numbered 1–18, with 1–9 originating from Airport *A*. Flights from Airport *A* that pass through the relatively uncongested Sector *A* bear the brunt of the delays and delay costs (21 units of ground delay and 3 of airborne delay), whereas flights from Airport *B* are "pushed" through the network to avoid the times when Sector *B* has very low capacity and thus suffer no ground delay and only 3 units of airborne delay.

The solution shown in Table 3 is for $\alpha = 1.2$. If, instead, $\alpha = 1.4$, then flights 16, 17, and 18 would not be delayed, and flights from Airport *A* would suffer *all* the delays for the day.

3.4. The Multiairport Case

In this last case (Figure 1), a more extensive example will illustrate the complex nature of EU ATFM solutions, the benefits that can be obtained by assigning airborne holding delays, and, most important, the equity issues arising in the EU ATFM context as a result of the network structure of the problem. The example involves two flows of flights, from *A* to *A*′ (Flow *AA*′) and from *B* to *B*′ (Flow *BB*′) that share a common en route sector.

The demand pattern for *each* of the airports *A*′ and *B*′ (Figure 1) will be assumed to be identical to that depicted in Figure 2. However, the capacity at each of the airports, *A*′ and *B*′, is set to 13 flights

Table 2 Inputs for the Second Example of the "Two Parallel Sectors" Case

Time unit	1	2	3	4	5	6	7
Demand from Airport *A*	3	3	3	0	0	0	0
Capacity of Sector *A*	4	4	4	2	2	2	10
Demand from Airport *B*	3	3	3	0	0	0	0
Capacity of Sector *B*	3	3	3	0	0	0	10
Capacity of destination airport	3	3	3	3	3	3	10

Table 3 Solution of the Second "Two-Sectors, Parallel" Example

Flight/Airport of departure	Scheduled arrival time	Assigned arrival time	Ground holding	Airborne holding
1/*A*	1	3	2	—
2/*A*	1	3	2	—
3/*A*	1	3	2	—
4/*A*	2	5	2	1
5/*A*	2	5	2	1
6/*A*	2	5	3	—
7/*A*	3	6	2	1
8/*A*	3	6	3	—
9/*A*	3	6	3	—
10/*B*	1	1	—	—
11/*B*	1	1	—	—
12/*B*	1	1	—	—
13/*B*	2	2	—	—
14/*B*	2	2	—	—
15/*B*	2	2	—	—
16/*B*	3	4	—	1
17/*B*	3	4	—	1
18/*B*	3	4	—	1

per time unit, i.e., equal to or greater than demand in every time unit, with the exception that Airport A' is assumed to have zero capacity in time units 14, 15, and 16 because of bad weather. The capacity of the shared en route sector is set to a constant value of 20 flights per time unit throughout the interval of interest (Figure 7). From knowledge of the flight time, $ft^{s,k}$, between the shared sector and the corresponding destination airport, one can obtain the sector demand (Figure 7) by simply left-shifting the demand at the two airports by the number of time periods equal to that flight time. In this specific example, $ft^{s,A'}$ and $ft^{s,B'}$ are assumed equal to 5 and 4 time units, respectively. For instance, sector demand at time unit 6 is obtained by adding demand for Airport A' in time unit 11 and for Airport B' in time unit 10, giving a total demand of 21 flights (Figure 7). It can be seen that there is an excess of demand at the sector in time units 6, 7, and 8, as well as in 13, 14, and 15.

Figure 8 shows the EU ATFM solution to this example, with the top diagram referring to Flow AA' and the bottom one to Flow BB'. As noted above, the sector first becomes congested in time units 6, 7, and 8. This will have an impact on flights scheduled to arrive during the time interval 11–13 at Airport A' and time interval 10–12 at Airport B'. Under any "fair" allocation of the sector's capacity, one might expect that a certain number of flights to each of the two airports would be delayed to accommodate the imbalance between sector capacity and sector demand. However, as shown in Figure 8, flights bound to Airport B' (Flow BB') are the *only ones* affected in the EU ATFM solution: 18 units of ground delay are assigned to flights scheduled to arrive at Airport B' during time units 10–12 (bottom diagram of Figure 8), yet no delay is assigned to flights scheduled to arrive at Airport A' during time units 11–13. Moreover, this happens despite the fact that there are no capacity restrictions at Airport B'. In effect, the combination of congestion at the en route sector and the capacity shortfall at Airport A' during time units 14–16 forces the imposition of ground delays on flights to

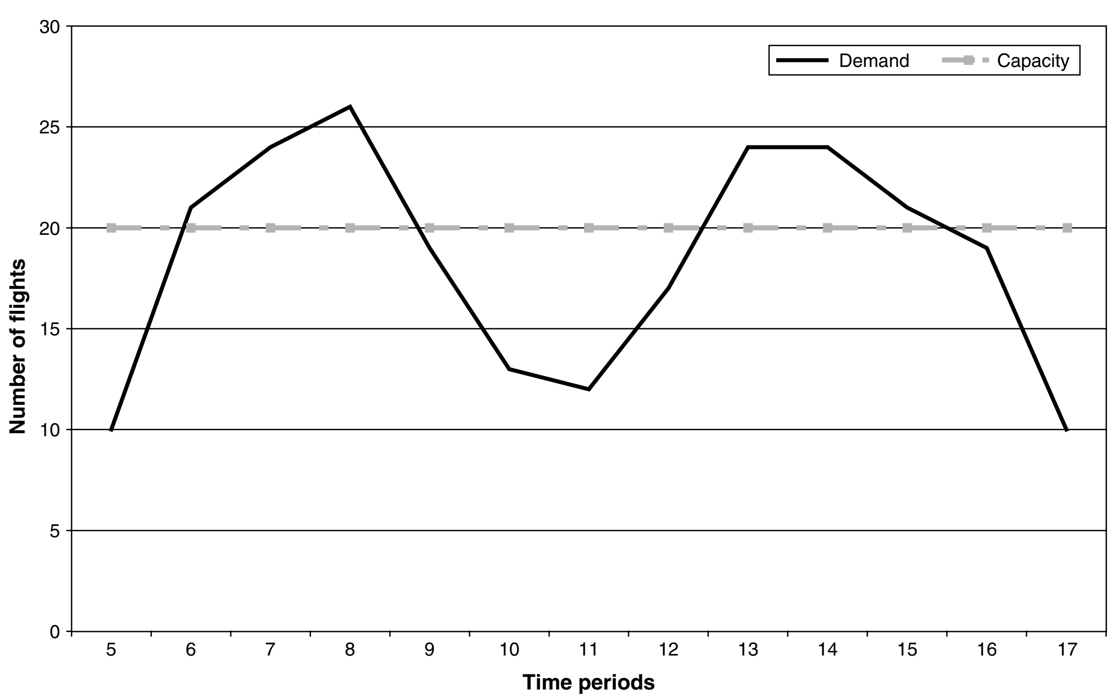

Figure 7 Sector Demand and Capacity for the Multiairport Example

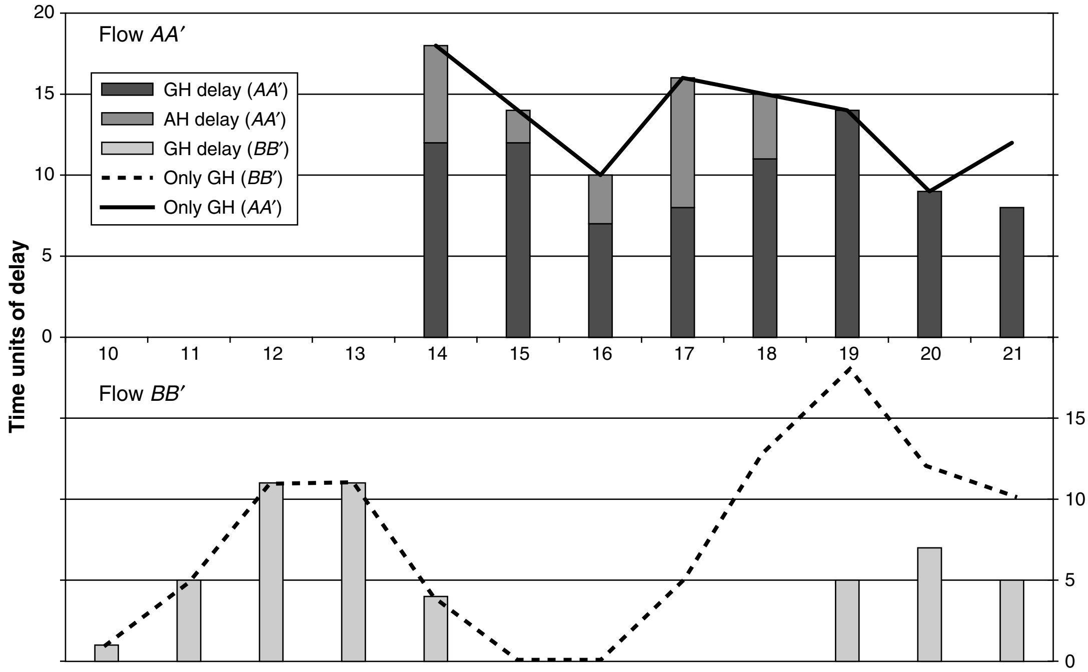

Figure 8 The Solution to the Multiairport Example

Airport B' so that flights to Airport A' can take full advantage of the available en route sector capacity to arrive at A' prior to its temporary shut-down. The *network effect*, in this case, means that some flights to an uncongested airport (B') will be delayed by congestion at another airport (A') that these flights do not use as either origin or destination. The overall results are summarized in the top third of Table 4.

In Figure 8, the solution given by the Only GH model is also shown using continuous and hashed lines, rather than bars. Comparing the solutions of the EU ATFM and the Only GH models, one can clearly identify the benefits obtained through the assignment of airborne holding delay by the EU ATFM model. As shown in the middle portion of Table 4, the Only GH model increases the total units of delay assigned by 31% (198 versus 151) and replaces the 23 units of airborne delay assigned by the EU ATFM model with 70 units of ground delay. Note that most of this additional ground delay (41 units) is assigned to flights to Airport B': Absent the possibility of airborne holding, most of the capacity of the en route sector is allocated by the Only GH model to flights to A' during the critical period of congestion, thus forcing large ground delays on flights to B' that are scheduled to arrive in time unit 17 and beyond.

Table 4 Summary of Results for the Multiairport Case

	Ground delay	Airborne delay	Total delay	Total delay, both flows
EU ATFM solution				
Flow AA'	79	23	102	151
Flow BB'	49	0	49	
Only GH solution				
Flow AA'	108	0	108	198
Flow BB'	90	0	90	
"Equitable allocation of sector capacity" solution				
Flow AA'	161	18	179	216
Flow BB'	37	0	37	

However, the most striking aspect of this case emerges when one attempts to enforce a "fair" allocation of scarce shared resources between the two flows of flights. Suppose, for example, that in the interest of fairness and because the two flows are equal, it is decided to allocate half of the sector's capacity (10 flights per time unit) to each flow. The understanding is that any capacity that is not used by one of the flows during any particular time unit can be assigned to the other flow. One can then optimize each flow separately, subject to the constraints imposed by this "fair" allocation of sector capacity. The resulting solution is shown in Figure 9 and is summarized in the bottom third of Table 4.

It can be seen that this new solution requires a 43% (216 versus 151) increase in total delay, compared with the EU ATFM solution, a 28% increase in airborne delay and a 55% increase in ground delay. The only beneficiaries are flights to B', whose total ground delay is reduced by 12 units. It is doubtful that a consensus of ATFM stakeholders could be reached on the right choice between the large aggregate efficiency benefits achieved by the EU ATFM model and

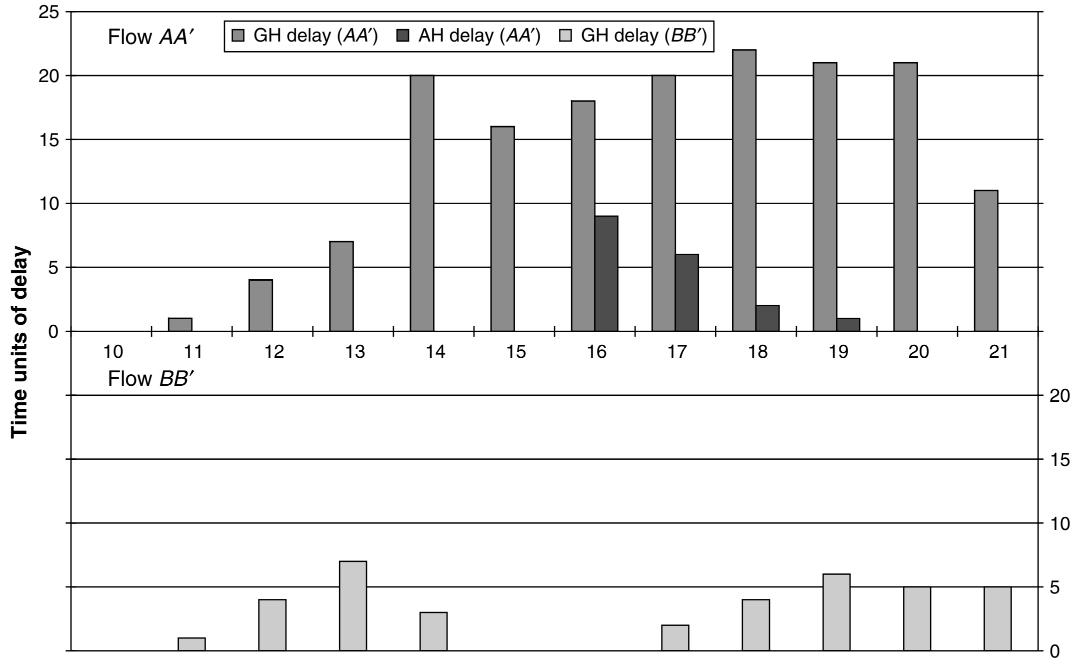

Figure 9 Solution with an Equal Distribution of the Scarce Resources

the more equitable but possibly far less efficient allocation of available capacity implied by approaches that emphasize fairness. Note as well that, despite the possibility of using airborne delay as an additional instrument for minimizing delay costs, the "equitable allocation of sector capacity" solution is less efficient in every aggregate respect (more total delay, more airborne delay, and more ground delay) than even the Only GH solution.

4. Conclusions and Discussion

This paper has presented a deterministic optimization model appropriate for the European ATFM environment. The model takes into consideration airport and en route sector constraints and assigns both ground and airborne delays to flights. It can accommodate delay costs that increase nonlinearly with the amount of airborne holding. It also "spreads" delays among similar affected flights, in the sense of a preference for assigning one unit of delay to each of two flights instead of two units to a single flight.

The model has been applied to a number of examples based on four generic network configurations to demonstrate three important characteristics of solutions to the EU ATFM problem:

(a) The solutions typically involve complex combinations of ground and airborne holding, the complexity arising from the conflicting capacity constraints imposed by the en route sectors and destination airports when traffic flows compete for access to the same set of resources.

(b) In many cases, total delay and/or total delay cost can be reduced significantly by assigning airborne holding delays to selected flights, instead of ground delays, despite the fact that the latter are less expensive per unit of time. This is true even with perfect knowledge of future capacity ("deterministic" case) and is in sharp contrast to the situation in which only airport capacity constraints exist.

(c) As a result of its network structure, the EU ATFM problem gives rise to fundamental conflicts between efficient solutions (in the sense of minimizing some function of total delay costs) and equitable ones (in the sense of handling network users in a FSFS order, or other similar priority discipline, with respect to access to resources). Despite the fact that the EU ATFM model presented here is designed to distribute delay costs evenly among users with similar characteristics (e.g., flights traveling between the same two airports on identical routes), efficient solutions may end up discriminating consistently against, for example, aircraft traveling on certain routes, originating from or terminating at certain airports, or flying at certain times. Moreover, these biases may often be of a "perverse" nature, such as assigning the overwhelming majority of delays to flights traveling on routes with uncongested sectors and airports, while assigning minimal delays to flights using certain congested resources in the network.

All three of the above findings have important practical implications. First, simple, manually developed ATFM strategies may fall far short of being efficient: Well-designed, computer-based decision support systems might improve ATFM performance significantly.

Second, ATFM strategies that rely exclusively on ground holding and assign airborne delays only under extreme circumstances may be missing opportunities to substantially and regularly reduce ATFM delay and associated costs. Finally, it may prove far more difficult than in the United States to reach agreement among the various stakeholders in Europe on a fundamental set of "rules of the game," such as FSFS or RBS. Thus, the development of a successful CDM program for ATFM in Europe may require considerable ingenuity and time.

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