Solving Optimally the Static Ground-Holding Policy Problem in Air Traffic Control*

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As air traffic congestion grows, ground-holding (or "gate-holding") of aircraft is becoming increasingly common. The "ground-holding policy problem" (GHPP) consists of developing strategies for deciding which aircraft to hold on the ground and for how long. In this paper we present a stochastic linear programming solution to the static GHPP for a single airport. The computational complexity of existing solutions requires heuristic approaches in order to solve practical instances of the problem. The advantage of our solution is that, even for the largest airports, problem instances result in linear programs that can be solved optimally using just a personal computer. We present a set of algorithms and compare their performance to a deterministic solution and to the passive strategy of no ground-holds (i.e., to the strategy of taking all delays in the air) under different weather scenarios.

The air traffic control (ATC) systems in the United States and in Western Europe are currently experiencing severe congestion during time periods when the weather is less than ideal and/or demand is at a peak. Long-term and medium-term solutions to this major problem are being pursued worldwide. In the short-term, however, the most effective way to deal with the situation is to adjust the flow of air traffic on a continuous basis so that it matches as well as possible the available capacity of the various components of the ATC network. This is known as the ATC Flow Management Problem (FMP).

This paper addresses the most important (in terms of the potential impact on the cost of air traffic delays) aspect of the FMP, namely the trade-off between delays taken on the ground before take-off ("ground-holding" delays) and delays suffered while airborne. We refer to this as the Ground-Holding Policy Problem (GHPP).

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The motivation for the GHPP is as follows: If a flight F is about to depart from airport A and is headed toward an airport Z and if it is known that Z will be congested at the time of arrival of F so that F would have to be delayed on landing, then it is both less expensive and safer to delay F on the ground at A before its take-off, rather than have F delayed in the air. The objective of the GHPP is to decide which flights should be delayed on the ground before take-off and by how much, in order to minimize the cost of delays. The ground-holding decision can be viewed as a *strategic* one in that it controls the flow of traffic into the ATC system, i.e., determines how many aircraft will be airborne at any given time. After aircraft become airborne, ATC can still adjust the flow of traffic through more "fine-tuning" *tactical* actions, such as speed-control, route adjustments, etc. (see also Section 2).

The GHPP is both stochastic and dynamic: stochastic because the available landing capacity at each airport cannot be predicted exactly even a few hours in advance; and dynamic because the available information regarding airport capacities,

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demand, etc., keeps changing over time on a daily basis.

Currently in the US, the ATC System Command Center (ATCSCC) in Washington, D.C. (known until recently as the Central Flow Control Facility), in coordination with the Traffic Management Units (TMUs) in key terminals and the Air Route Traffic Control Centers (ARTCCs), administers a ground-holding program (the EQF program.[4] The EQF program uses a deterministic landing capacity forecast for each of the major airports. It assigns forecasted available capacity on a first-come first-served basis with all expected delays exceeding 15 minutes assigned as ground-holds, and delays below this threshold assigned either as air delays or as ground-holds at the controller's discretion. The tendency at present is to assign most predicted delays as ground-holds.

Due to the probabilistic nature of airport capacities, even under optimal assignment of ground-holds, there will be instances in which airport landing capacity is lost while aircraft sit waiting on the ground. However, these occurrences may currently be more frequent than necessary when bad weather is expected, as airports tend to provide conservative capacity forecasts to protect their airspace from saturation. As a result, carriers are often advocating more "liberal" ground-holding strategies.[2] The trade-off in establishing ground-hold delays is between conservative policies that may at times assign excessive ground-holds and optimistic ones that may result in more expensive airborne delays.

An effective ground delay program should then: (i) consider the relative costs of ground and air delays; (ii) take into account uncertainty regarding airport capacities; and (iii) be able to respond to a constantly changing system. Research efforts in the GHPP are beginning to address these key issues.

The FMP and GHPP are discussed in detail by ODONI.[5] The first attempt to solve a probabilistic version of the GHPP is due to ANDREATTA and ROMANIN-JACUR[1] for the case of an airport whose capacity may be inadequate for only a single period of time. More recently, TERRAB[8] and TERRAB and ODONI[9] developed (i) a dynamic programming formulation for a general version of the GHPP which, however, is impractical for problems of size typically encountered in practice, as well as (ii) several heuristics for addressing the problem.

In this paper, we present a model for the general, multiperiod GHPP which, for the first time, can be used to obtain *optimal* solutions to practical instances of the problem, avoiding the use of heuristics. The model can be solved using stochastic linear programming with one stage, yielding linear programs of size solvable on a personal computer even for the largest airports in the US ATC network. The formulation allows for classification of aircraft into several cost classes and would work with probabilistic airport capacity forecasts that are in line with current weather forecasting technology. This major improvement in the "state of the art" on the GHPP is made possible by an approach that solves the problem at a somewhat more aggregate level, i.e., decides how many of the flights scheduled to arrive at an airport Z during a particular time period should be delayed on $0, 1, 2, \ldots$ time increments, without specifying which specific flights should be delayed. This paper is based on the dissertation of Richetta[7] which the reader may consult for a much more detailed discussion and additional related material.

In Section 1 we present the new model for the GHPP and discuss our assumptions, inputs to the model and decision variables. In Section 2 we present the stochastic programming formulations, starting from a deterministic linear programming model for a single class of aircraft, and discuss how to incorporate into the model constraints such as maximum ground-hold delay allowed, limits on air delay for particular times of the day and the possibility of choosing between "conservative" and "liberal" ground-hold policies by varying a single parameter in the model. In Section 3 we assess experimentally the performance of the stochastic linear programming model for one and three aircraft classes for a set of realistic problem instances based on data for Boston's Logan International Airport. Finally, in Section 4 we review our principal findings and motivate research on the dynamic probabilistic GHPP.

1. THE SIMPLIFIED GHPP MODEL

THE AIR traffic network considered under the simplified GHPP can be described in reference to the single-destination network shown in Figure 1. The model is macroscopic in nature, yet it captures the essential elements needed to solve the GHPP:

(i) N aircraft (flights $F_1, \ldots, F_N$) are scheduled to arrive at the "arrivals" airport Z from the "departures" airports.
(ii) Airport Z is the only capacitated element of the network and thus the only source of delays. All other elements in the network (departure runways, airways, etc.) have unlimited capacity.
(iii) The departure and travel times of each aircraft are deterministic and known in advance.

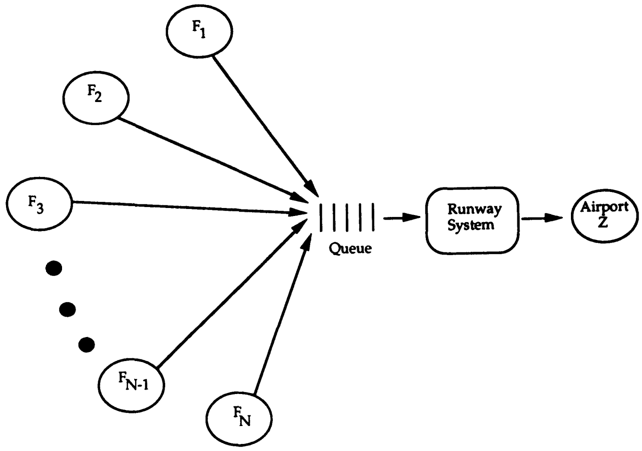


Fig. 1. "Star" configuration network.

(iv) The time interval of interest is $[0, B]$, with the earliest departure for Z scheduled at 0 and the latest arrival scheduled at B. The time interval $[0, B]$ is discretized into T equal time periods numbered $1, 2, \ldots T$.

(v) The capacity of airport Z for each time period t, M_t, becomes known with certainty at the beginning of t. However, at each t, we have access to the joint probability mass function (PMF) of future airport Z capacities: $P_{M_{t+1}-M_T}(t)$. The capacity at period $T+1$, M_{T+1}, is equal to N.

(vi) Ground and air delay cost functions for each flight are known.

Items (ii) and (iii) amount to an assumption of perfectly predictable travel times between the airport of departure of each flight and airport Z (for additional details see Odoni[5]. Thus, this ignores the effect of such tactical actions as speed control, "metering" and path-stretching that may sometimes take place during the "en route" portion of a flight in response to local ATC conditions. It is assumed that the impact of such actions on operating costs is entirely secondary to that of the delays due to congestion at the airport of destination Z—and, thus, to the ground-hold versus air delay trade-off. This is fully justified for the United States ATC system: although no specific statistics are collected on the matter, an overwhelming proportion of delays (possibly in the order of 95%) are undoubtedly due to airport, not en route, capacity limitations. (This may not be the case in Western Europe, where the en route airspace also imposes severe capacity constraints—primarily as a result of institutional and political factors).

Since we are dealing here with the static problem, we assume that the capacity forecasts (item (v)) are given (probabilistically) at the beginning of operations, i.e. at time $t = 0$. Specifically, it is assumed that at $t = 0$, there are Q alternative scenarios, each scenario providing a possible capacity forecast for the entire time interval of interest $[0, B]$ with the scenario q $(q = 1, 2, \ldots, Q)$ having a probability equal to p_q. Capacity forecast q is of the form $M_{q1}, \ldots M_{qT}$ (i.e., the landing capacity in period 1 will be equal to M_{q1}, in period 2 to M_{q2}, etc.) and we let $M_{qT+1} = N$ for all q in order to assure that all aircraft are able to land at Z within $T+1$ periods. Limiting the time horizon for the problem to $T+1$ time periods is reasonable since air traffic declines significantly towards the end of the day, so that all delayed aircraft eventually land.

While the formulation of our model is general, an implicit assumption has been that the number of alternative scenarios, Q, at the beginning of each day is quite small—probably 4 or less. This assumption is important for obtaining quick numerical solutions. A small value of Q is consistent with current weather forecasting technology which has advanced to the point where the type of weather conditions in a specific geographic area can be predicted with reasonable accuracy, but the exact timing of weather fronts and their local severity are uncertain. A typical example of the type of situation we wish to address is shown in Figure 5 of Section 4 where, at the beginning of a day, there is an expectation of some deterioration in weather conditions in early afternoon which may result in severe loss of landing capacity (profile 1) limited loss (profile 2) or no loss at all (profile 3). For this example, of course, $Q = 3$.

We now proceed to define notation and specify the model.

Aircraft are classified into K cost classes. This is in line with current ATC practice in the U.S. which defines three aircraft classes according to maximum take-off weight (MTOW): Small Aircraft with less than 12,500 pounds MTOW; Medium/Large Aircraft, between 12,500 and 300,000 pounds MTOW; and Heavy Aircraft with over 300,000 pounds MTOW. This leads to the definition of the following:

N_{ki}: The number of aircraft of class k scheduled to arrive at airport Z during period i $(k = 1, \ldots, K;$ $i = 1, \ldots, T)$ with:

$$\sum_{k=1}^{K} \sum_{i=1}^{T} N_{ki} = N \tag{1}$$

X_{qkij}: The decision variables indicating the number of aircraft of class k originally scheduled to arrive at Z during period i, and rescheduled to arrive during period j under capacity scenario q, due to a ground delay of $j - i$ time periods $(q = 1, \ldots, Q;$ $k = 1, \ldots, K;$ $i = 1, \ldots, T;$ $i \leqslant j \leqslant T+1)$.

$C_g(k, i)$: Cost of delaying one aircraft of class k for i periods on the ground ($k = 1, \ldots, K$; $i = 1, \ldots, T - 1$). Note that we allow for arbitrary ground-hold cost functions for each class of aircraft, and that this cost represents the total ground delay cost for one aircraft during i periods. The marginal cost for the ith period is: $C_g(k, i) - C_g(k, i - 1)$.

The marginal cost of air delay per aircraft is assumed to be a constant, c_a, identical for all aircraft. This assumption is based on the following "operational" characteristics of the ATC system:

(i) Aircraft which are already airborne are sequenced by ATC in an approximately first-come, first-served (FCFS) way; therefore, there is no need to draw distinctions among different classes of aircraft while airborne.
(ii) Within reasonable airborne delay levels (i.e., for up to the largest airborne delays observed in practice which are in the order of one hour) delay cost functions are approximately linear since "non-linearities," due to factors such as safety do not yet set in.

Since the marginal cost of air delay is assumed constant, we only need to consider the total number of aircraft that are unable to land during any period. Therefore, the airborne queueing process at airport Z can be described through the following variables:

W_{qi}: The number of aircraft unable to land at airport Z during period i under capacity scenario q (i.e., the number of aircraft incurring airborne delay during period i) for $q = 1, \ldots, Q$; $i = 1, \ldots, T$.

2. STOCHASTIC PROGRAMMING FORMULATION

IN THIS section, the static GHPP is formulated as a stochastic linear programming problem with one stage. We follow the framework presented in WAGNER,[10] which provides a basic introduction to stochastic programming (for a more advanced treatment the reader is referred to KALL[3]). For ease of presentation we start with the case of a single class of aircraft—thus omitting the subscript k—and then extend the formulation to cover the case of K aircraft classes.

2.1. The Deterministic Problem

Suppose capacity scenario q, with capacities $M_{q1}, \ldots, M_{qT+1}$, occurs with probability one. Since the optimization criterion is to minimize total delay cost (i.e., ground plus air delay costs) we have:

Minimize:

$$\sum_{i=1}^{T} \sum_{j=i+1}^{T+1} C_g(j - i) X_{qij} + \sum_{i=1}^{T} W_{qi} c_a \quad (2)$$

Subject to:

(i) All aircraft scheduled to land during i must be rescheduled to arrive during $i, i + 1, \ldots, T + 1$:

$$\sum_{j=i}^{T+1} X_{qij} = N_i \qquad i = 1, 2, \ldots, T \quad (3)$$

(ii) The flow balance at airport Z at the end of each period yields:

$$W_{qi} \geqslant \sum_{j=1}^{i} X_{qji} + W_{qi-1} - M_{qi} \qquad i = 1, \ldots, T + 1 \quad (4)$$

(with $W_{q0} = W_{qT+1} = 0$).

(iii) $X_{qij}, W_{qi} \geqslant 0$ and integer (5)

The formulation above is an integer programming problem with linear cost function. Note how constraints (4) model the airborne queueing process at airport Z: If the RHS of (4) is less than or equal to zero (i.e., capacity M_{qi} is adequate to land all aircraft waiting to land during i), then $W_{qi} = 0$ by constraint (5) and the positive cost coefficient c_a in the objective function.

2.1.1. *Reduction to a Minimum Cost Flow Problem*

Figure 2 shows how the formulation (2)–(5) is transformed to a minimum cost flow problem in an uncapacitated network (for clarity, subscript q has been omitted in Figure 2). An additional node S_q with supply $\sum_{i=1}^{T} M_{qi}$, and arcs (S_q, i); $i = 1, \ldots, T + 1$; is introduced. The flow S_{qi} on arc (S_q, i) has

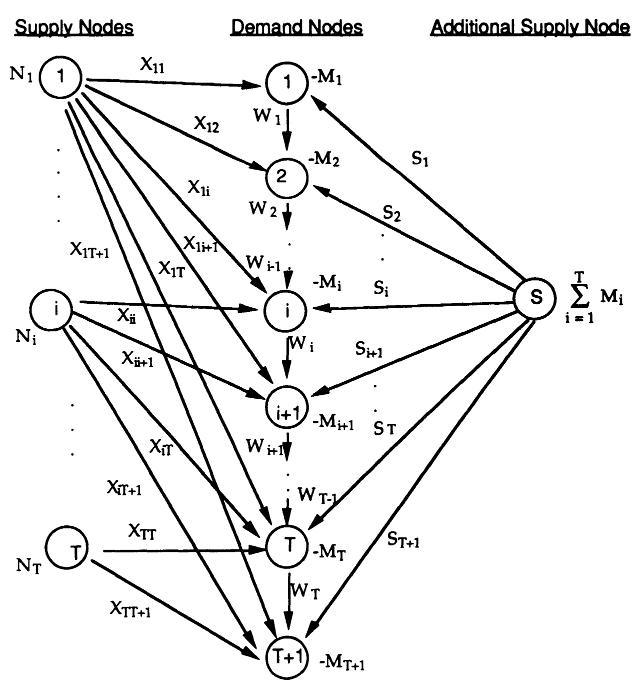


Fig. 2. Network diagram for minimum cost flow problem.

zero cost and can be interpreted as surplus capacity. The resulting problem has objective function (2) and the following constraint set:

Supply nodes:

(i) $X_{qii} + X_{qii+1} \cdots + X_{qiT+1} = N_i$;
$i = 1, \ldots, T$. (6)

(ii) $\Sigma_{i=1}^{T+1} S_{qi} = \Sigma_{i=1}^{T} M_{qi}$; (7)

Demand nodes:

(iii) $W_{qi} - (W_{qi-1} + \Sigma_{j=1}^{i} X_{qji} + S_{qi}) = -M_{qi}$;
$i = 1, \ldots, T+1$ (8)
(with $W_{q0} = W_{qT+1} = 0$).

(iv) $X_{qij}, W_{qi}, S_{qi} \geqslant 0$ (9)

Note that the constraint matrix for the network formulation is totally unimodular and supply/demands are integer. Thus, the integrality constraints can be relaxed.

2.2. The Distribution Problem

By introducing the probabilities p_q the above can be transformed into a distribution problem. This distribution problem assumes that airport capacities for all periods, $M_{q1}, \ldots, M_{qT}$, become known *before* the ground-hold decisions are made. Therefore, the optimal solution to the distribution problem is a complete policy consisting of optimal ground-holds X_{qij}^* with associated W_{qij}^* for each one of the capacity scenarios $q = 1, \ldots, Q$. The distribution problem is as follows:

Minimize

$$\Sigma_{q=1}^{Q} p_q \Big\{ \Sigma_{i=1}^{T} \Sigma_{j=i+1}^{T+1} C_g(j-i) X_{qij} + c_a \Sigma_{i=1}^{T} W_{qi} \Big\} \quad (10)$$

Subject to:
For each $q = 1, \ldots, Q$:

(i) $X_{qii} + X_{qii+1} \ldots X_{qiT+1} = N_i$;
$i = 1, 2, \ldots, T$ (11)

(ii) $\Sigma_{i=1}^{T+1} S_{qi} = \Sigma_{i=1}^{T} M_{qi}$; (12)

(iii) $W_{qi} - (W_{qi-1} + \Sigma_{j=1}^{i} X_{qji} + S_{qi}) = -M_{qi}$;
$i = 1, \ldots, T+1$ (13)
(with $W_{q0} = W_{qT+1} = 0$).

(iv) $X_{qij}, W_{qi}, S_{qi} \geqslant 0$. (14)

The solution to the distribution problem, (10)–(14), is equivalent to solving Q separate minimum cost flow problems (i.e., one for each $q = 1, \ldots, Q$), since the constraint matrix consists of Q separate network components.

2.3. The Static GHPP

In practice, ground-hold decisions must be made before knowing airport capacities. Thus, we need to modify the distribution problem formulation by introducing the following set of constraints:

(v) $X_{1ij} = X_{2ij} = \cdots = X_{Qij}$; $i = 1, \ldots, T$;
$i \leqslant j \leqslant T+1$ (15)

Figure 3 shows that after adding these constraints the structure of the constraint matrix becomes block angular. Elements outside the rectangles are equal to zero.

It is worth noting that the network structure of the constraint matrix is lost since variable X_{qij} now appears with additional $+1$ or -1 coefficients in the equations of type (15). We could still have integer solutions if the constraint matrix were unimodular. This is because total unimodularity (i.e., network structure) is a sufficient condition, while unimodularity is a necessary condition for integrality of basic feasible solutions to linear programming relaxations of integer programming problems with arbitrary integer RHS.[6]

Since solutions to the linear programming relaxation of our models have yielded integer solutions so far, we conjecture the constraint matrix may be unimodular. We have not been successful in either proving unimodularity (which requires verifying that the inverse, B^{-1}, for all basis matrices B of the constraint matrix are integer) or generating a counterexample with non-integer solutions.

Although unimodularity of the constraint matrix remains an open question, this is not critical to the results in this paper. Even if the solutions were not all-integer, rounding would still produce good sol-

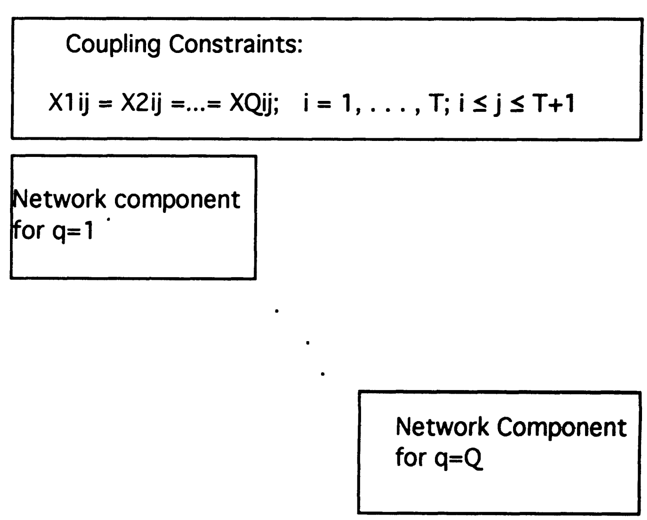


Fig. 3. Constraint matrix structure.

utions, from a practical point of view, because constraints (13) can be violated, within reason, as airport capacities, M_{qi}, are somewhat approximate. Thus, it would not be catastrophic if, in assigning flights to time periods, the capacity constraints were exceeded occasionally due to rounding.

We also note that the structure of the constraint matrix makes it suitable for application of decomposition techniques. The key advantage of decomposing the problem is that the subproblems have network structure and can be solved faster than generic linear programs. Since the subproblems can be solved independently, a decomposition algorithm would also lend itself to parallel computation.

2.4. Restating the Model

The simple structure of (15) suggests elimination of these coupling constraints through substitution into the sub-problem constraints (11)–(14). Although the resulting linear program lacks the structure that would make it suitable for solution via faster decomposition algorithms, substituting the coupling constraints reduces significantly the number of constraints and variables, resulting in problem sizes that can be solved efficiently using generic linear programming codes on a personal computer. Extending our formulation to K aircraft classes, we obtain:

Minimize

$$\sum_{k=1}^{K} \sum_{i=1}^{T} \sum_{j=i+1}^{T+1} C_g(k, j-i) X_{kij} + c_a \Big\{ \sum_{q=1}^{Q} p_q \sum_{i=1}^{T} W_{qi} \Big\} \tag{16}$$

Subject to:

(i) $\sum_{j=i}^{T+1} X_{kij} = N_{ki}$; $k = 1, \ldots, K$; $i = 1, \ldots, T$ (17)

(ii) $\sum_{i=1}^{T+1} S_{qi} = \sum_{i=1}^{T} M_{qi}$, for $q = 1, \ldots, Q$ (18)

(iii) $W_{qi} - W_{qi-1} - \sum_{k=1}^{K} \sum_{j=1}^{i} X_{kji} - S_{qi} = -M_{qi}$; $q = 1, \ldots, Q$; $i = 1, \ldots, T+1$ (19) (with $W_{q0} = W_{qT+1} = 0$).

(iv) $X_{kij}, W_{qi}, S_{qi} \geqslant 0$ and integer. (20)

The subscript for the capacity scenario, q, has been omitted from the ground-hold variables X_{qkij}, since we make a unique set of ground-hold decisions (i.e., $X_{1kij} = X_{2kij} = \cdots = X_{Qkij} = X_{kij}$).

2.5. Additional Model Features

A few closing remarks are in order regarding the static GHPP model:

By changing a single parameter, c_a, in the objective function, we can adjust the bias of the model towards conservative (liberal) ground-holding policies. A higher value for c_a will result in a greater emphasis on ground-holds since ground delays become less expensive vis-a-vis air delays. Conversely a lower c_a will result in more liberal ground-hold times. This point will be illustrated in Section 4. Note also that it is possible to let the marginal air delay cost depend on the time of the day.

Planners may also be interested in more than just minimizing expected costs. Due to safety-related considerations, they may, for instance, wish to limit the amount of airborne delay at the arrivals airport. The model easily allows for constraints of this type. For example, suppose the duration of the discrete time intervals is 15 minutes, and we wish to limit airborne queueing delay under capacity scenario q to at most 30 minutes at the end of time period i. The corresponding constraint is:

$$W_{qi} \leqslant M_{qi+1} + M_{qi+2}.$$

The above constraint can also be interpreted as limiting the queue length at airport Z at the end of period i, under capacity scenario q, to the total capacity for the next two periods, $i+1$ and $i+2$.

Suppose now that we wish to limit ground delays for a given aircraft class and during a certain period i to at most P periods. Then, the constraint (11) in the original model becomes:

$$\sum_{j=i}^{i+P} X_{kij} = N_{ki}.$$

Obviously by introducing limits on the duration of airborne queueing delays at Z and ground delays at the departure airports simultaneously, one could generate infeasible problems and may then need to relax some of these additional constraints or even cancel some flights in order to render the problem feasible.

3. EXPERIMENTAL RESULTS

FIRST, WE describe the instances of the GHPP to be solved for Logan airport (i.e., the aircraft schedule, the airport capacity forecasts, and the ground/air cost functions) and the different algorithms that will be evaluated. Then, we discuss the performance of the stochastic programming algorithms for one and three classes of aircraft compared to a deterministic solution and the "passive" strategy of no ground-holding.

3.1. Instances of the GHPP for Logan Airport

The aircraft schedule data represent a typical weekday of operations at Boston's Logan Airport during the Fall of 1988 based on information taken from the November 1988 issue of the *Official Airline Guide*, including scheduled direct international flights (a total of 5 flights). It is worth noting that, there are approximately 50 unscheduled daily flights into Logan (less than 10% of the total) which are not subject to CFCF ground-holding. Aircraft are classified into the three classes defined by the MTOW's described previously, with aircraft classes 1, 2, and 3 corresponding to small, medium/large, and heavy aircraft, respectively. There is a total of 551 scheduled arrivals between 6 a.m. and midnight yielding 72 fifteen minute intervals and a 73rd representing the "dummy" final period $T + 1$. (Detailed data regarding all aspects of the computational experiments are in Richetta.[7])

Figure 4 shows hourly landings by aircraft type for scheduled flights. Small and medium/large aircraft, each with approximately 45% of scheduled flights, account for about 90% of Logan traffic. During the busiest periods (8 to 11 and 16 to 19 hours) landing demand for scheduled flights averages 36 and 43.5 landings per hour respectively, representing 60% and 74% of the "good weather" maximum landing capacity of 60 aircraft per hour. Thus, in good weather this schedule yields little congestion. However, as discussed below, bad weather can significantly reduce landing capacity—and it is precisely during bad weather days that uncertainty about landing capacity is the greatest. This is the motivation for pursuing probabilistic approaches to the GHPP.

A total of 10 different capacity cases, consisting of three capacity profiles each, were studied. Four different probability scenarios were explored for capacity cases 1–3, and a single probability scenario for capacity cases 4–10, for a total of 19 different capacity forecasts. The capacity forecasts cover a wide variety of conditions in regard to the levels, timing and duration of periods of restricted capacity, and reflect operating conditions typically prevailing at Logan during bad weather days. Figure 5 shows the three capacity profiles for capacity case 1 and Table I the corresponding four probability scenarios. For example, for capacity case 1, under probability scenario 1, profiles 1, 2, and 3 have probabilities of 0.5, 0.3, and 0.2, respectively.

The capacity levels used in preparing the forecasts were 60, 40, and 28 landings per hour, corresponding to VFR1, VFR2/IFR1, and IFR2/IFR3 conditions respectively. VFR stands for visual flying rules and IFR for instrument flying rules. Airport landing capacity under IFR conditions decreases versus VFR conditions as aircraft minimum separation rules are enforced strictly, increasing the time between landings. As well, some landing runways available for VFR operations may not be equipped with an instrument landing system (ILS), further

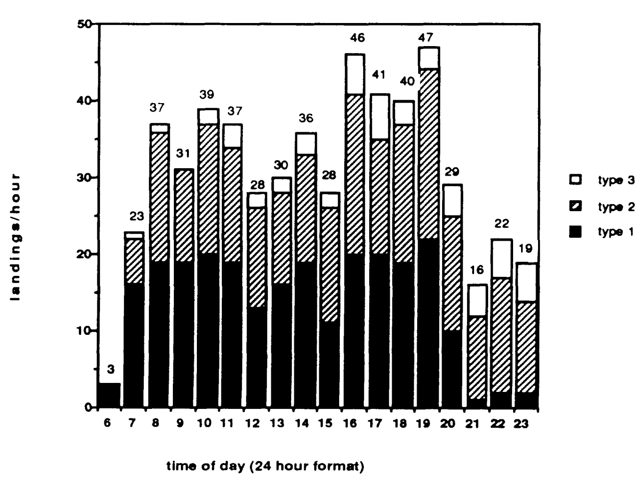
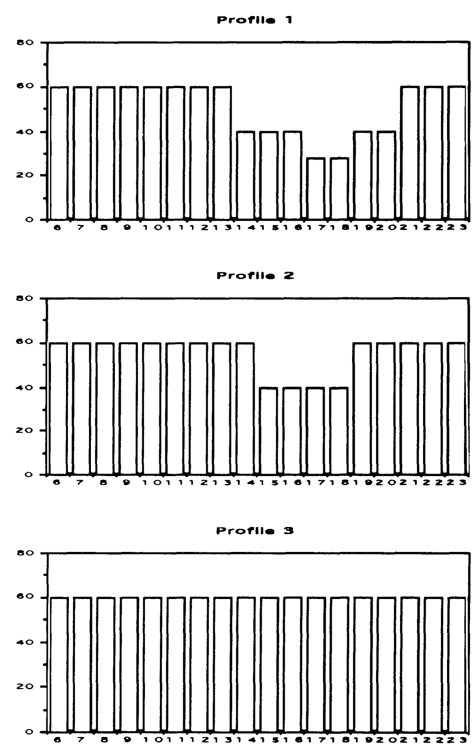


Fig. 4. Hourly landings by aircraft type for scheduled flights.



(Vertical axis: landings/hour. Horizontal axis: time of day - 24 hour format)

Fig. 5. Capacity profiles for landing capacity case 1.

TABLE I
Probability Scenarios for Capacity Case 1

Profile No.	Probability Scenario 1	2	3	4
1	0.5	0.3	0.3	0.34
2	0.3	0.5	0.2	0.33
3	0.2	0.2	0.5	0.33

reducing capacity during IFR conditions. Logan historical data indicate that VFRI weather conditions prevail about 80% of the time, VFR2/ IFR1, 12% of the time, and IFR2 and worse during the remaining 8%. We have not included extreme cases such as shut down of operations, that are likely to require flight cancellations due to unacceptable delay levels.

In order to assure FCFS within each aircraft class, the ground delay cost functions used are slightly increasing. For the case of a single aircraft class the ground-hold costs per aircraft are $250/period (i.e., $1000/hour) for the first period of ground-holding and then increase by $10/period. In the case of three aircraft classes, the ground-hold cost function used yields the same average ground-hold cost of $1,000/hour for a single class of aircraft, based on a 45%–45%–10% aircraft class split. The marginal rate of ground-hold cost increase is $10/period. The cost for the first period of ground delay by aircraft class is shown in Table II and is typical of the true direct costs to operators of these types of aircraft.

As discussed in Section 2.5 we can adjust the bias of our models towards conservative (liberal) ground-holding strategies by increasing (decreasing) the cost of air delay, c_a. For single aircraft class algorithms we explore marginal air delay costs of $1,200, $1,600, $2,000, and $3,000 per hour, representing cost premiums of approximately 20%, 60%, 100%, and 200% vs. the average cost of ground delays. We solved the GHPP for each one of the problems defined in Table III using the algorithms described below (except for algorithms with tree aircraft classes for which we limit ourselves to problems with marginal air delay cost of $3000/hour so that the cost of air delays is higher than the cost of ground delays for all aircraft classes).

3.2. The Algorithms

We evaluated the performance of four algorithms, including the passive "no action" strategy of no ground-holds, in the solution of the GHPP's defined above.

1. *Deterministic*. This algorithm provides a static deterministic solution to the GHPP by selecting the most likely capacity profile for each probability scenario (see, e.g., Table II) as the only capacity profile and disregarding completely all other profiles. Available capacity is then assigned on a FCFS basis with all delays assigned as ground-holds. This algorithm is labeled DETERM and corresponds approximately to current practice, as described in the Introduction.
2. *Passive*. This algorithm allows aircraft to depart according to the original schedule and calculates expected air delays using the probabilistic landing capacity forecast. This algorithm is labeled PASSIVE.
3. *Static*. This algorithm provides the optimal probabilistic static solution for the GHPP with a single aircraft class using stochastic linear programming with one stage as described in Section 2. This algorithm is labeled STATIC.
4. *Static Algorithm for Three Aircraft Classes*. This algorithm provides the optimal probabilistic static solution for the GHPP with three aircraft classes using stochastic linear programming with one stage as described in Section 3. This algorithm is labeled STATIC3C.

TABLE II

First Period Ground-Hold Delay Cost

	Aircraft Type		
	Class 1	Class 2	Class 3
Aircraft split	45%	45%	10%
Ground delay cost (first period)	$430/hour	$1,300/hour	$2,225/hour

TABLE III

Problems Generated by the Different Capacity Forecast-Air Cost Combinations

Capacity Case	No. of Probability Scenarios	No. of Forecasts	Air Delay Costs	No. of Problems[a]
1–3	4	12	1200, 1600 2000, 3000	48
4–9	1	6	1600	6
10	1	1	3000	1
		19		55

[a] A problem is defined as a capacity forecast-air delay cost combination.

PASSIVE was implemented using C programming language code that utilizes the input schedule to calculate expected air delays using the probabilistic landing capacity forecast. STATIC and STATIC3C are stochastic linear programming algorithms. Stochastic linear programs of the size we have described can be solved on a personal computer. DETERM was implemented as the deterministic linear program presented in Section 2.1.

To solve the linear programs resulting from DETERM, STATIC, and STATIC3C we used LINGO on a 386 (16 MHz) machine with a 80387 coprocessor and 4mb of RAM memory. LINGO combines an exterior point linear programming algorithm (LINDO) and a modeling language for problem input. Our version of LINGO is able to solve systems with a constraint matrix size of up to $5000 \times 15{,}000$, for moderately dense matrices. Running times for STATIC were for more cases

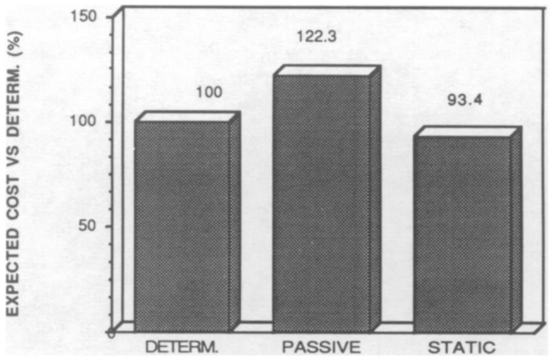


Fig. 6. Average cost performance (% basis).

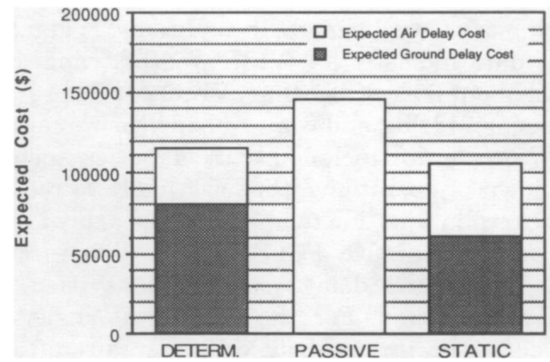


Fig. 7. Average cost performance ($ basis).

under 10 minutes. This is remarkable considering that we used a "generic" linear programming algorithm and we made no attempt to optimize the software/hardware combination. LINGO running times can be improved by 50% to 60% by using a Weitek co-processor and a 486 machine.

3.3. Performance Evaluation

We have evaluated the performance of the different algorithms with respect to the following statistics for each solution: total expected costs, expected cost of ground and air delays, expected ground delay and expected air delay measured in aircraft-periods. Recall that STATIC3C was applied only to problem instances with marginal air delay cost of $3000/hour.

Overall Performance

STATIC3C is not included here since it was not applied to all problems. Figures 6 and 7 show the average performance of DETERM, PASSIVE, and STATIC. Figure 6 is on a percentage basis (i.e., equal weight for each of the 55 problems shown in table 4) while Figure 7 shows the average expected total delay cost of the solutions.

In Figure 6, we see that STATIC provides a 6.6% savings vs. DETERM. We also see that both ground-holding algorithms, DETERM and STATIC, perform significantly better than the passive strategy of no ground-holds.

Figure 7 shows that the average cost savings versus DETERM are traceable to lower ground-holding costs for STATIC. In the case of PASSIVE, we see that the higher cost of air delays results in significantly higher expected delay cost (despite the fact that PASSIVE minimizes expected delay times by allowing all aircraft to take off on time and thus not "wasting" any available airport capacity).

Figure 8 shows the expected number of aircraft-periods of air and ground delay averaged for all the problems solved (each period is 15 minutes long). We see that total expected delays for STATIC are below those for DETERM and within 15% of the absolute minimum given by PASSIVE.

Effect of Marginal Air Delay Costs

Figure 9 shows the average relative performance of the algorithms for each marginal air delay cost value tested for capacity cases 1–3 (i.e., the capacity cases for which we tested each marginal air delay cost as shown in Table III). STATIC provides savings versus DETERM in the 3% to 12% range, with air delay cost of $1200/hour showing the best relative performance.

The advantage of STATIC decreases, as expected, as the cost of air delays increases. For air delay cost of $1,200/hour STATIC provides 12% savings while for $3,000/hour savings are reduced to 4%. This is because as we increase the marginal air delay cost—particularly for cases with the most pessimistic capacity profile as the most likely profile—the solution for DETERM improves (i.e., for high enough marginal air delay cost the optimal STATIC solution is the strategy of assigning available capacity for the most pessimistic capacity profile on a FCFS basis with all delays taken in the form of ground-holds as in DETERM).

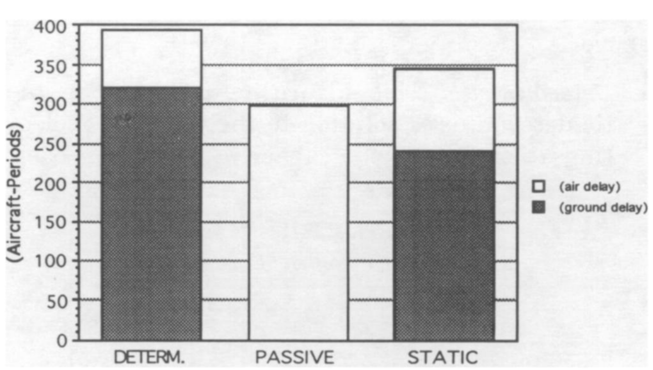


Fig. 8. Average expected ground and air delay.

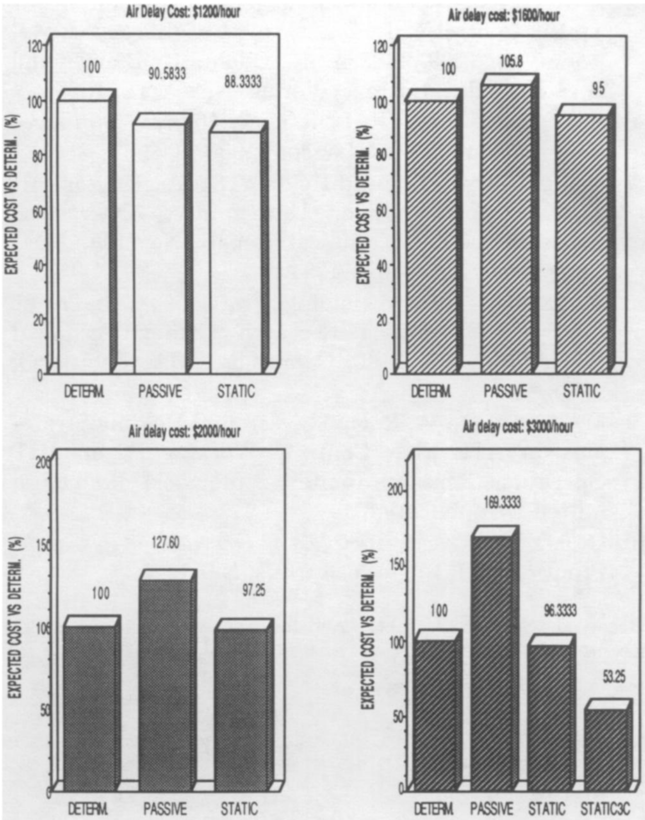


Fig. 9. Average cost performance by air delay cost: capacity cases 1–3.

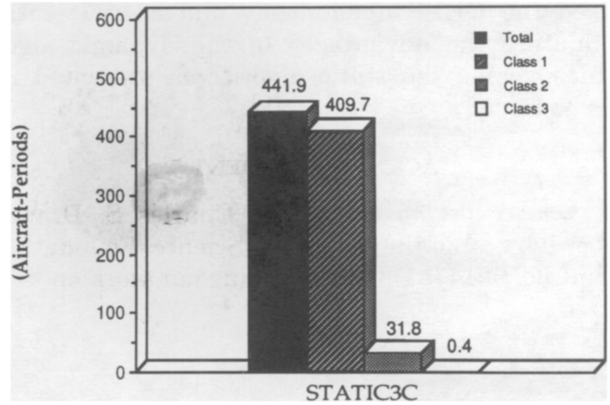


Fig. 10. Average expected ground delay by aircraft class.

The very significant savings achieved for STATIC3C (see air delay costs of $3000/hour) are due to the assignment of ground-holds to the lowest cost aircraft eligible for delay. We also see that the performance of PASSIVE deteriorates with increasing air delay cost as we would expect.

Distribution of Ground Delays among the Three Aircraft Classes

Figure 10 shows the average expected ground delay for each aircraft class for STATIC3C. As might be expected, STATIC3C, which discriminates among the three classes of aircraft, assigns a disproportionate amount of ground delays to the lowest cost class. Here we see that for the STATIC3C small aircraft (class 1) absorb over 90% of the ground delays, while representing only 45% of the traffic. It is because of this type of inequitable distribution of delays that implementation of (unconstrained versions of) algorithms that discriminate among aircraft classes may be difficult.

4. CONCLUSION AND FUTURE RESEARCH

IN THIS paper the Static Probabilistic GHPP has been formulated as a stochastic linear program with a single stage. The main feature of the stochastic programming model is that it simplifies the structure of the control mechanism by making ground-hold decisions on groups of aircraft (i.e., on aircraft classified according to cost class, and schedule) rather than individual flights. It considers few rather than many airport capacity profiles, in line with current weather forecasting technology. The model provides solutions to realistic instances of the GHPP using just a personal computer. Another important aspect is that additional constraints, such as limiting the maximum acceptable ground-holds and airborne delays, are easily introduced.

The computational experiments attempted to quantify the potential advantages that may be obtained by trying to account for uncertainty in ground-hold planning. Although the specific estimates of savings reported in Section 3, clearly depend on capacity scenarios selected, the results are nonetheless encouraging. For perspective, just considering the reported [2] current ground delays of approximately 2000 hours/day within the US ATC network, each 1% savings for the case of a single aircraft class would result in annual savings of approximately $5 million (based on 250 weekdays in the year and ground delay cost of $1000/hour).

Another important finding came from comparing the algorithms tested to the "passive" policy of no ground-holds, which minimizes total delay time. The stochastic programming solution for a single aircraft class performed remarkably well compared to this passive strategy. Total expected delays were within 15% of the minimum expected delay, with the advantage that over 70% of the delays are in the form of less expensive ground delays.

Further improvement in the solution of the probabilistic GHPP could be achieved by considering the dynamic nature of the problem. We have recently solved the GHPP dynamically and are currently evaluating the advantages of the dynamic algorithms versus the static algorithms presented in this paper.

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