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Integrating Slot Exchange, Safety, Capacity, and Equity Mechanisms Within an Airspace Flow Program

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In this paper, we study an airspace flow program in the context of weather-related disruptions by augmenting the airspace planning and collaborative decision-making model (APCDM). The proposed model selects among alternative flight plans for the affected flights while integrating slot exchange mechanisms induced by multiple ground delay programs (GDPs) to permit airlines to improve flight efficiencies through a mediated bartering of assigned slots, and simultaneously considering issues related to sector workloads and airspace conflicts, as well as overall equity concerns among the involved airlines in regard to accepted slot trades and flight plans. The APCDM is enhanced to include (a) the selection of slot exchange trade offers suggested by the airlines based on their allotted slots under a GDP; (b) connections between continuing flights; and (c) several alternative equity concepts. Both full and light versions of this model are developed and tested using realistic data derived from the enhanced traffic management system data provided by the Federal Aviation Administration.

Key words: airspace flow program; ground delay program; collaborative decision making; mediated bartering; slot exchanges; airline equity; air traffic management
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1. Introduction

The enhancements to the ground delay program (GDP) were the first major steps in the Federal Aviation Administration's (FAA) adoption of the collaborative decision-making (CDM) paradigm. The two key concepts developed under the enhancements were the ration-by-schedule (RBS) and compression procedures (see Ball et al. 2001; Chang et al. 2001). Under the RBS procedure, slots are allotted to airlines according to the order of the flight arrival times published in the *Official Airline Guide*. Hence, carriers own sets of slots, and thus, under the implementation of a GDP, have the rights associated with ownership to either utilize these slots or possibly transfer ownership to another carrier within the context of bartered exchanges. The compression algorithm is then applied as a means for closing the resulting gaps in the schedule, via a restricted set of one-for-one trades. In an extension of this concept, known as slot credit substitution, which was developed by the airline operational community and is in use today, airlines can explore offering the delay or cancellation of a flight conditioned on the reduction in delay of a subsequent flight.

Vossen and Ball (2006a, b) have further attempted to leverage the benefits of slot ownership and the foregoing existing intra-airline slot-exchange procedures by proposing new ideas for how airlines can mutually benefit by bartering assigned slots, with the FAA acting as a mediator or slot-exchange "broker" and ensuring that equity is maintained among airlines within this slot trading process. A particularly interesting and flexible trading mechanism, suggested by Vossen and Ball (2006b), whereby airlines are able to offer multiple slots in exchange for multiple slots in return and, central to our present work, is the *at-most*, *at-least* (*AMAL*) offer. In such an offer, an airline proposes to move a flight f_1 to a slot at most t_1 minutes later in time in exchange for moving another of its flights f_2 to a slot at least t_2 minutes earlier in time. This is a way of transferring delay from more critical to less critical flights. Note that such trades are explored only for flights that are still on the ground, as opposed to being airborne. The flexibility of an individual trade offer is reflected by the length of the additional delay an airline is willing to accept for some flight in return for a reduction in the delay of another. For example, if the "at-most" offer corresponds to simply an adjacent slot, then the offer has limited flexibility, whereas an offer that accepts any of the next several slots has increased flexibility. Note that based on the number of participating airlines, the set of available slots, and the flexibility of the individual trade offers, several viable compositions of acceptable trades can exist. A standing assumption in this trading environment (and in the present paper) is that, if an offer from an airline is not accepted, then that airline retains its current slot, and furthermore, that no flight for any airline is delayed without reducing the delay of some other designated flight for that airline. We refer to this as a *trade restriction* throughout the remainder of the paper.

Relevant to the present work is the development of the *airspace planning and collaborative decision-making model* (*APCDM*) by Sherali, Staats, and Trani (2003, 2006), based on the FAA's CDM initiative. The APCDM is a large-scale mixed-integer programming model designed to enhance the management of air traffic in the National Airspace System. Given a set of potential 4-D (space-time) trajectories (referred to as *surrogates*) for each flight, the objective of the APCDM is to select an optimal set of flight plans subject to sector workload, collision safety, and airline equity considerations. However, the current version of APCDM does not address slot exchanges or continuing flights in delineating surrogates for each flight, and it advocates a particular limited equity concept.

Another important air traffic management initiative sponsored by the FAA in 2006 is that of *airspace flow programs* (see FAA 2006 and Sud et al. 2009). Here, because of a severe convective weather pattern, certain affected *flow constrained areas* (FCAs) or subregions of the airspace are identified, and flights passing through these FCAs over a specified duration are given a controlled departure time and are efficiently routed in an equitable fashion, subject to FCA capacity constraints in terms of the maximum number of flights managed per hour.

This paper also focuses on an AFP but incorporates in more detail the foregoing AMAL type of slot-exchange mechanisms that arise from trading opportunities due to GDPs imposed at one or more destination airports, in concert with various equity concepts. It also considers sector workloads and safety issues with respect to conflict resolution restrictions and the overall equity achieved by the involved airlines. We accomplish this by adapting the APCDM to accommodate the foregoing features and enabling its use as a *collaborative tactical regional air traffic management tool*, spanning a duration of about two hours in advance, and run in a rolling horizon framework, say, every hour. The scenario addressed in this context might typically arise in the event of a dynamic severe convective weather system that has resulted in the partial closure or reduction in capacity of certain sections of the airspace covering a designated geographical region (FCAs) over a specified horizon. Flights traversing this region that are affected by the weather system might need to be delayed and/or rerouted according to one of several alternative surrogate flight plans. This might include rerouting airborne flights as well as setting ground delays and/or providing revised routes for not-yet-departed flights. The information regarding these alternative or surrogate flight plans for each flight would typically be provided by the flight operators. In addition, the FAA might have imposed a GDP at one or more destination airports. This would entail assigned slots at the GDP-imposed airports, for which the flight operators would file (alternative) corresponding flight plans, automatically inducing a ground delay at the originating airport. Even in cases when the GDP is imposed at a single airport, the flights associated with this airport would need to be considered in concert with other concurrent flights over the time horizon with respect to sector capacities and collision safety issues, where several of these other flights might also have alternative surrogate flight plans to select from based on diverted and/or delayed trajectories. In addition, as discussed in more detail in §§3 and 4, we also provide an opportunity for slot exchanges based on trade offers that are mutually beneficial to the involved carriers (which might include flight cancellation offers in return for better positioning for some other more critical flights). Alternative flight plans corresponding to the new slots in case such trade offers are accepted are also prespecified. The model ensures that the mix of flight plans selected for the different flights satisfy sector monitoring workload (or FCA capacity constraints) and conflict resolution restrictions and, moreover, achieve a measure of equity among the different carriers. As explained in greater detail in §6, this equity is governed by a relative performance ratio that is based on fuel and delay costs, on the average delay per passenger, or on the on-time operation of flights. Also, as the FAA moves away from the traditional sector definitions in the future, suitable alternative airspace segmentation schemes can be utilized in a likewise fashion for the purposes of representing congestion and air traffic control workload-based constraints within the model. The proposed model then selects among the alternative flight plans for each flight, subject to slot ownership and trade offer restrictions as applicable, as well as with respect to sector workload, conflict safety, and airline equity considerations. Although this paper focuses on developing a model for tactical operational use in managing regional air traffic, the developed model could be easily adapted to perform strategic planning exercises related to studying the impact of different types of trade restrictions, collaboration policies, equity concepts, and airspace sector configurations.

The principal contributions of this paper are threefold:

- Extend and adapt the APCDM model of Sherali, Staats, and Trani (2003, 2006) to incorporate slot-change mechanisms and continuing flight restrictions.
- Propose and provide insights into modeling different equity concepts.
- Examine different levels of detail of the developed model in support of AFPs, and design and test suitable solution methodologies and present related computational results and insights.

The remainder of this paper is organized as follows. Section 2 provides a review of the conceptual structure of the APCDM model along with some relevant notation. Section 3 describes a hypothetical illustrative example to elucidate the slot-exchange concept. Section 4 develops the proposed slot-exchange mechanism for inclusion within the APCDM model, and §5 models continuing or connecting flight restrictions. Section 6 delineates various alternative approaches for modeling equity with respect to the selected trade offers. Section 7 discusses the resulting proposed full and light versions of the APCDM model, along with exact and heuristic solution procedures. Section 8 presents computational results and additional insights using test instances based on realistic data derived from the enhanced traffic management system (ETMS). Finally, §9 concludes the paper with a summary and some recommendations for follow-on research.

2. Structural Concept of the APCDM

In this section, we introduce some notation used in the APCDM model that is relevant to the present paper and provide a brief review of the basic structure of this model for the sake of completeness. We refer the reader to Sherali, Staats, and Trani (2003, 2006) for further details (additional discussion related to equity issues is in §6).

Airline and Flight Plan Related Notation

- $\alpha = 1, \ldots, \bar{\alpha}$: Airlines involved in the model analysis.
- $f = 1, \ldots, F$: Flights for the different airlines that pertain to the designated airspace region (FCA) and horizon (covering about two hours).
- A_α: Set of flights belonging to airline α.
- $p \in P_f$: Alternative (4-D or space-time) flight plans or *surrogates* for flight f. This is the primary input for the model. The various surrogate plans for a given flight are differentiated by flight trajectories (altitude and path) and departure and arrival times.
- $P_{f0} = P_f \cup \{0\}$: The set of flight plans for f augmented by the null flight plan ($p = 0$), which indicates that a flight has been cancelled (assuming that this is offered by the particular airline as a possible option). This cancellation surrogate is ascribed an inordinately high penalty if it is not a viable option (including the case of airborne flights).
- x_{fp}: *Principal binary decision variable*, which equals 1 if flight plan $p \in P_{f0}$ is selected for flight f and 0 otherwise, for $f = 1, \ldots, F$.
- c_{fp}: The cost to execute flight plan $p \in P_{f0}$ for flight f (prescribed as $c_{fp} = c_{fp}^{\text{fuel}} + c_{fp}^{\text{delay}}$, $\forall p \in P_f$, $f = 1, \ldots, F$, where c_{fp}^{fuel} and c_{fp}^{delay} are, respectively, the associated fuel and delay costs, as, for example, derived in Sherali, Staats, and Trani 2006). Note that the *delay* for a given flight plan p for flight f is determined as the difference between an originally scheduled arrival time and the arrival time as determined by the particular flight plan p.
- $c_f^* =$ Minimum $\{c_{fp}\colon p \in P_f\}$, for each flight f.

APCDM Model Structure

$$\text{Minimize}\ \Big[\text{Total system fuel, delay, and cancellation costs} = \sum_{f=1}^{F} \sum_{p \in P_{f0}} c_{fp} x_{fp}\Big]$$
$$+ \big[\text{Commensurate equity-related terms: see §6}\big]$$
$$+ \big[\text{Commensurate sector workload and conflict resolution penalties: see Sherali Staats, and Trani (2003, 2006)}\big] \quad (1)$$

subject to:

(a) Exactly one flight plan must be selected from among the proposed surrogates for each flight:

$$\sum_{p \in P_{f0}} x_{fp} = 1, \quad \forall f = 1, \ldots, F.$$

(b) Constraints related to sector occupancies and workload based on selected flight plans.

(c) Constraints prohibiting any irreconcilable violation of aircraft separation standards (denoted as *fatal conflicts*) and restricting the number of simultaneously existing resolvable conflicts occurring at any point in time within each sector, to be no more than a specified value depending on the capability of the particular sector.

(d) Constraints accounting for measures of equity (see §6 for more details).

(e) Additional valid inequalities to enhance the model solvability.

(f) Appropriate variable bounding and logical binary restrictions.

3. Illustrative Trade Offer Example and Insights

The APCDM model described in §2 selects an optimal set of flight plans, one from each surrogate set. Each flight plan conforms with an arrival time corresponding to a designated slot, where a given flight might have the option of arriving at one of possibly multiple slots. Accordingly, suppose that each airline delineates alternative arrival times for each of its flights that are later than the originally scheduled times, based on the slots it owns, as well as based on slots that it could possibly acquire through trade offers.

As an illustrative example, consider the following situation at a particular airport in the overall problem at which a GDP has been imposed. Suppose that the execution of the RBS procedure produces the arrival slot allocations for Airlines A, B, and C as depicted in the left-hand blocks of Figure 1. For example, Flight 1 for Airline A (designated as A1) has been allotted the 0800 arrival time slot, and so forth. Under the enhancements to the GDP, Airline A owns the 0800 slot and thus may either utilize it for another one of its flights through a swapping process or consider offering that slot to another airline in return for a slot that reduces the delay for one of its subsequent flights. In this spirit, consider the AMAL trade offers from Airlines A, B, and C as shown in Figure 1, where the flights operated by the particular airline are shaded for clarity in each case. For example, Airline A has offered to increase the delay of Flight 1 with an arrival time no later than the 0816 slot in return for moving Flight 3 up to an arrival time ranging between the 0824 and 0816 slots. It is insightful to note here the importance of considering interairline trading. Namely, airlines cannot always accomodate slot exchanges within their own operations. For example, Airline A cannot (or is not willing to) trade slots between Flights A1 and A3.

Using the slot times as nodes and the transition of flights from current to new slots as arcs, we can represent the slot offers as a directed network (see Figure 2). For notational simplicity, we designate slot time 0800 as Node 1, slot time 0804 as Node 2, and so on. The potential movements of a flight from its current slot to the newly proposed slots are designated by directed arcs. Acceptable trades, subject to the aforementioned trade restrictions, are in the form of directed cycles in this network. For example, referring to Figure 2, some resulting directed cycles that correspond to sets of possible trades that preserve feasibility to the trade restriction include $\{(1,4,8,6,2,5,1)\}$, $\{(4,6,4), (2,5,2)\}$, $\{(1,5,1), (2,4,8,6,2)\}$, and $\{(4,8,6,4), (1,2,5,1)\}$, where, for instance, the first foregoing set corresponds to the swaps A1 → B1, B1 → A3, A3 → B2, B2 → C1, C1 → C2, and C2 → A1. Observe also that the second set involves intra-airline swaps, which are also included within this modeling framework (any such enforced or previously declared exchange is assumed to be done a priori). Note that the final resulting

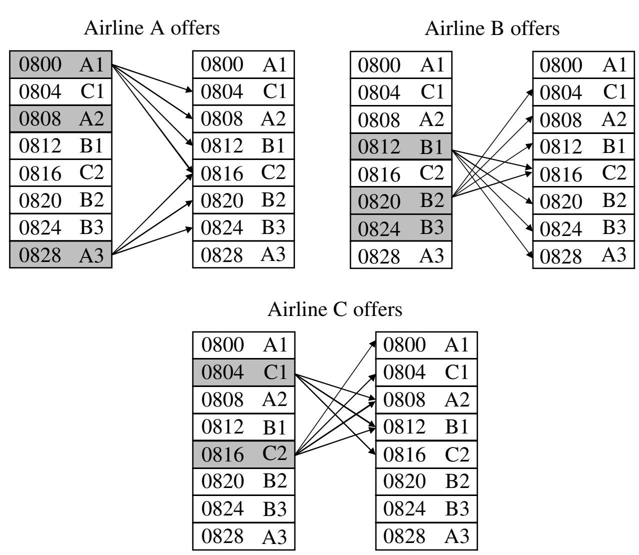

Figure 1 **Airline Slot Allocations and Slot Offers**

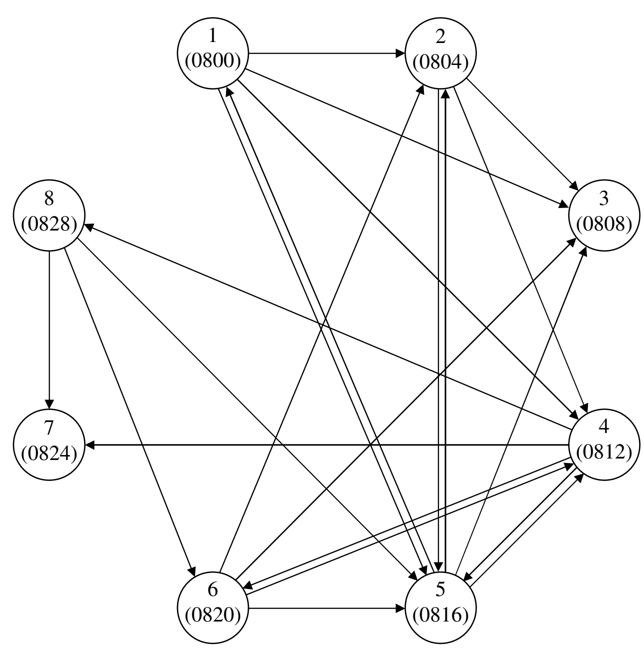

Figure 2 **Slot Offer Network**

mix of flight plans needs to be collectively compatible with respect to sector workload and conflict resolution restrictions, and also with the governing equity measures.

Finally, note that when offers are made as conditional queries that do not commit an airline to accept any trades until receiving feedback on the results of the exchange, this requires possibly multiple runs of the proposed model that would entail some additional acceptance conditions to ensure convergence of this process. To avoid this iterative process, we assume that airlines make firm trade offers that they commit to adopt if accepted as part of the overall exchange mechanism.

The following two sections respectively describe further modeling details for representing and automatically selecting feasible trades, and for enforcing equity, within the APCDM model.

4. Modeling Feasible Slot Exchanges Within APCDM

In this section, we propose an augmentation for the APCDM model to account for slot exchanges between participating airlines. Toward this end, consider any particular airport in the problem that is concerned with such trades under a GDP. For this airport, suppose that based on various offer schemes, we have a collection of connected *exchange graphs* G_k, $k = 1, \ldots, K$, where K represents the number of (separable, connected) components resulting from the trade offers. Each exchange graph G_k has node set N_k corresponding to the related slots with their currently occupying flights and has directed arcs, with each arc corresponding to an altered arrival time, and therefore corresponds to a proposed flight plan (f, p) for a flight f, which switches from its current tail-slot to the new head-slot. Accordingly, a variable x_{fp} is associated with each arc. For example, Figure 2 illustrates a particular exchange graph in which, for instance, arc $(1, 4)$ corresponds to Flight A1 adopting a designated flight plan that arrives at the GDP destination airport at time 0812. Hence, in effect, valid trades will be represented by directed circuits within any such graph G_k. Note that in case we wish to consider multiple flight plans corresponding to a given slot trade (or arc) that might differ in their trajectories and departure times, we will maintain multiple corresponding arcs, each representing a particular plan. Observe that we do not construct self-loop arcs pertaining to plans that retain the associated assigned slot for the corresponding flight in this approach. The x_{fp} variable associated with any such flight plan would be directly accommodated within Constraint (a) for the APCDM model; if set equal to one, it would therefore automatically imply the retention of the allotted slot for the particular flight.

Furthermore, we accommodate flight cancellation offers that, if accepted, would create vacant slots for other flights to possibly occupy as follows. Given any exchange graph G_k, suppose that some flight f that currently occupies slot s has an associated cancellation plan $p = 0$ as a possible option. We then create a dummy companion f_ϕ, say, for flight f and let it occupy a dummy blank slot s_ϕ, where s_ϕ now represents a new node in G_k. Next, we create a directed arc (with the associated variable x_{f0}) from the node representing slot s to that representing slot s_ϕ in G_k. Hence, node s_ϕ will be involved in an exchange circuit if and only if $x_{f0} = 1$, whence the dummy flight f_ϕ would end up moving to some newly vacated real slot, depending on the particular resulting exchange circuit (including possibly to slot s). Therefore, we also generate forward arcs going out of the slot s_ϕ node to all the other nodes corresponding to real slots in G_k; we also associate with each such arc a variable $x_{f_\phi p}$ for a corresponding distinct dummy plan $p \in P_{f_\phi}$. In addition, we model the case when there exist some real currently vacant slots by creating fictitious flights to occupy such slots; for each such slot (node in G_k), we then generate incident-directed arcs coming from, and going to, each of the real occupied slots within G_k, along with the appropriate designation of flight plan variables for each such arc, as before. Now, because the selected flight plans that constitute a valid exchange scheme within any graph G_k are represented by directed cycles (or *circuits*), with a node being involved in at most one such circuit, we formulate the following constraints:

$$\sum_{(f,p)\in \mathrm{RA}_n^k} x_{fp} = \sum_{(f,p)\in \mathrm{FA}_n^k} x_{fp}, \quad \forall\, n \in N_k,\ \forall\, k = 1, \ldots, K \tag{2a}$$

$$\sum_{(f,p)\in \mathrm{FA}_n^k} x_{fp} \le 1, \quad \forall\, n \in N_k,\ \forall\, k = 1, \ldots, K, \tag{2b}$$

where for each $k = 1, \ldots, K$, and $n \in N_k$, we have

$$\mathrm{RA}_n^k = \{(f,p)\colon x_{fp} \text{ corresponds to a "reverse" arc coming into node } n\},$$

$$\mathrm{FA}_n^k = \{(f,p)\colon x_{fp} \text{ corresponds to a "forward" arc going out of node } n\}.$$

Hence, (2a) requires flows to be circulatory, and (2b) enforces at most a unit flow in each circuit, with at most one circuit involving any particular node.

However, we also want to ensure that the trades prompted by the selected circuits satisfy the trade restrictions. Toward this end, let

$$D = \{(f, \#)\colon \text{flight } f \text{ is offered to be delayed in offer indexed by } \#\}$$

and for each $(f,\#) \in D$, let

$$H_{(f,\#)} = \{(f', p'):\text{ at least one of these designated flight plans } p' \text{ of flight } f' \text{ belonging to the same airline must be selected to accept delaying flight } f \text{ in offer } \#\}.$$

Also, let

$$DP_{(f,\#)} = \{p:\text{ plan } p \text{ corresponds to the delay of } f \text{ in offer } \#\}, \quad \forall (f,\#) \in D.$$

Note that by the foregoing notation, a given flight $f \in D$ can be possibly involved in multiple offers; i.e., we could have $(f,\#) \in D$ for more than one # value, where the different corresponding H sets might possibly intersect. However, in such a case, we assume that $DP_{(f,\#)} \cap DP_{(f,\#\#)} = \varnothing$ for distinct offers # and ##, so that implicitly, no more than one offer can be accepted for a given flight. Likewise, for any distinct flights f_1 and f_2, we assume that $H_{(f_1,\#)} \cap H_{(f_2,\#\#)} = \varnothing$, for any indices # and ##. Hence, a delay-reducing move cannot by itself compensate for more than one delay-increasing move. Accordingly, in any of the exchange graphs, if for some $(f,\#) \in D$ we have that $x_{fp} = 1$ for any $p \in DP_{(f,\#)}$, then at least one of $x_{f'p'}$ for $(f', p') \in H_{(f,\#)}$ must also be 1. This is enforced by the constraints

$$\sum_{p \in DP_{(f,\#)}} x_{fp} \le \sum_{(f',p') \in H_{(f,\#)}} x_{f'p'}, \quad \forall (f,\#) \in D. \tag{3}$$

Furthermore, although some slot exchanges might result in a significant improvement in terms of reducing the overall net delay in passenger minutes, the structure of the individual trade offers could possibly result in a net increase in passenger minutes of delay for one or more airlines. In some instances, this might be acceptable for the corresponding airline, when the airline's motivation was something other than passenger delay reduction, such as reducing the downstream effects of airframe or crew delays in the airline's network of flights. Nonetheless, we also impose an additional restriction that for each airline, the realized net reduction in passenger minutes of delay at the GDP-restricted airport because of slot exchanges should be nonnegative. Note that whereas the information regarding the actual number of passengers on any flight is not readily available, and is in fact guarded by the airlines, we could use a fixed (agreed-on) value to estimate this quantity (referred to as PAX_f below) based on applying an assumed load factor to the type of aircraft, perhaps using historical data. To model this (optional) restriction, we define the following additional entities, where all delays are measured with respect to the published schedule:

- A^{trade}: The set of airlines involved in the trade offers.
- PAX_f: The estimated number of passengers associated with flight f.
- δ_{fp}: The delay (minutes) for plan p of flight f relative to the published schedule (note that $\delta_{fp} \ge 0$, $\forall (f,p)$).
- DL_f^{GDP}: The delay (minutes) for flight f based on the GDP assigned slot.

The restriction on achieving a nonnegative *net reduction in passenger minutes* (NRPM) of delay at the GDP airport for each airline can be formulated as follows:

$$\mathrm{NRPM}_\alpha \equiv \sum_{(f,\#)\in D: f \in A_\alpha} \Bigg\{ \sum_{p \in DP_{(f,\#)}} \mathrm{PAX}_f [DL_f^{\text{GDP}} - \delta_{fp}] x_{fp} + \sum_{(f',p') \in H_{(f,\#)}} \mathrm{PAX}_{f'} [DL_{f'}^{\text{GDP}} - \delta_{f'p'}] x_{f'p'} \Bigg\} \ge 0, \quad \forall \alpha \in A^{\text{trade}}. \tag{4}$$

Alternatively, to complement (3), we could require that the net reduction in passenger minutes of delay corresponding to each trade, given by $\{\cdot\}$ in (4), should be nonnegative. However, this might be too restrictive, although it would permit the corresponding airline that is making the offer to specify additional delay-increasing move slots for the related $f \in D$, while being assured that any such accepted delay will be adequately compensated by an associated delay-reducing move in terms of the resulting net reduction in passenger minutes of delay.

To summarize, we formulate slot exchanges within the APCDM model by incorporating (2) and (3), and optionally, (4). Related flight connection constraints and equity constraints in the model are discussed in §§5 and 6, respectively.

5. Continuing or Connecting Flights

In this section we address the issue of *continuing* or *connecting flights*. Given a pair of flights f_1 and f_2, where flight f_2 must necessarily follow flight f_1 (designated by $f_1 \to f_2$), we need to ensure that the departure time (including turnaround time) of flight f_2 is no earlier than the arrival time of flight f_1 at the connecting airport. (This is also relevant when considering multiple GDP-restricted airports.) Toward this end, let us define

$\tau_{fp}^{\text{dep}}, \tau_{fp}^{\text{arr}}$: Respectively, the departure and arrival times of flight plan (f,p), $p \in P_f$, $f = 1, \ldots, F$.

Observe that if for some pair of continuing flights f_1 and f_2, where $f_1 \to f_2$, we have that

$$\max_{p \in P_{f_1}} \{\tau_{f_1 p}^{\text{arr}}\} \le \min_{p \in P_{f_2}} \{\tau_{f_2 p}^{\text{dep}}\}, \tag{5}$$

then we do not need to further restrict the choices of flight plans for f_1 and f_2, except to tie their cancellation surrogate plans by equating $x_{f_10} = x_{f_20}$. Hence, let us define the set

$$F^{\text{cont}} = \{(f_1, f_2)\colon f_1 \to f_2, \text{ where } f_1, f_2 \in \{1, \ldots, F\} \text{ are such that (5) does not hold}\}.$$

Accordingly, we incorporate the following set of constraints within APCDM:

$$\sum_{p\in P_{f_1}} \tau^{\text{arr}}_{f_1p} x_{f_1p} \le \sum_{p\in P_{f_2}} \tau^{\text{dep}}_{f_2p} x_{f_2p}, \quad \forall (f_1, f_2) \in F^{\text{cont}} \tag{6a}$$

$$x_{f_10} = x_{f_20}, \quad \forall (f_1, f_2) \in F^{\text{cont}}. \tag{6b}$$

Remark 1. Note that in lieu of (6a) and (6b), hereafter referred to as (6) as we could have combined f_1 and f_2 into a single "block flight" f_{12}, say, having $|P_{f_1}||P_{f_2}| + 1$ surrogate flight plans (where the "+1" represents the cancellation surrogate). However, this might entail a substantial increase in flight plans, and moreover, could preclude (or complicate) the consideration of multiple jointly connecting flights such as $f_1 \to f_2$ and $f_3 \to f_2$, besides inconveniencing data manipulations. Hence, we utilize (6).

Remark 2. We can tighten the model representation by incorporating the following set of valid inequalities, which are implied by (6) and Constraint (a) of APCDM in §2:

$$x_{f_1p} \le \sum_{\substack{p'\in P_{f_2}:\\ \tau^{\text{dep}}_{f_2p'} \ge \tau^{\text{arr}}_{f_1p}}} x_{f_2p'}, \quad \forall p \in P_{f_1}, \ \forall (f_1, f_2) \in F^{\text{cont}}, \tag{7a}$$

$$x_{f_2p} \le \sum_{\substack{p'\in P_{f_1}:\\ \tau^{\text{arr}}_{f_1p'} \le \tau^{\text{dep}}_{f_2p}}} x_{f_1p'}, \quad \forall p \in P_{f_2}, \ \forall (f_1, f_2) \in F^{\text{cont}}. \tag{7b}$$

Observe that any one of the constraint sets (6a), or (7a), or (7b) by itself correctly enforces the precedence relationship $f_1 \to f_2$ in concert with (6b). Jointly, however, they assist in tightening the continuous relaxation of the model representation. Hence, we incorporate (7) into the model to accompany (6).

6. Equity Constraints

When addressing equity issues, Vossen et al. (2003), Tadenuma (2002), and others agree that efficiency and equity habitually conflict, in that attempts to improve equity among the participants vying for a resource typically result in a reduction in the efficient distribution of that resource. The original APCDM model attempts to trade off efficiency and equity through the objective function (1), where the equity-related terms are based on certain collaboration efficiency and collaboration equity functions defined for each airline as described below. The general framework given below will be used as a foundation for the original APCDM equity concept as well as for proposing two alternative equity formulations in this section.

Each of these equity measures is based on a specifically defined *relative performance ratio* $d_\alpha(x)$ (e.g., delay per passenger) for each airline $\alpha \in \{1, \ldots, \bar{\alpha}\}$, as a linear function of the binary flight plan choice variables x (vector of the x_{fp} variables). Associated with this we define a particular *collaboration efficiency function* $E_\alpha(x) \in [0, 1]$, which is also linear in x, where $E_\alpha(x) = 1$ represents the best possible outcome (a 100% efficiency). Contingent on these functions, we then define the *collaboration equity function* as the deviation of the efficiency value from its (weighted) mean as given by

$$E^{\text{equity}}_\alpha(x) = E_\alpha(x) - \left(\sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha E_\alpha(x)\right), \tag{8}$$

where $\omega_\alpha > 0$ is a weight factor ascribed to airline α, $\forall \alpha = 1, \ldots, \bar{\alpha}$, with $\sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha = 1$ (e.g., ω_α might be taken in proportion to the number of flights in A_α). For efficiency and equity, we would like $E_\alpha(x)$ to be close to 1 and $E^{\text{equity}}_\alpha(x)$ to be close to 0, $\forall \alpha$. Accordingly, the "equity-related term" in the objective function (1) is given by

$$\mu\left[\sum_\alpha \omega_\alpha[1 - E_\alpha(x)] + \sum_\alpha \omega_\alpha \left|E^{\text{equity}}_\alpha(x)\right|\right], \tag{9}$$

where μ is designated as $0.1\sum_{f=1}^{F} c^*_f$, as motivated by Sherali, Staats, and Trani (2006). In addition, for a specified constant $E^{\text{equity}}_{\max}$, (designated as $0.07/\bar{\alpha}$ by Sherali, Staats, and Trani 2006), Constraint (f) of APCDM also restricts

$$\omega_\alpha \left|E^{\text{equity}}_\alpha(x)\right| \le E^{\text{equity}}_{\max}, \quad \forall \alpha = 1, \ldots, \bar{\alpha}. \tag{10}$$

The three alternative equity concepts proposed in §§6.1–6.3 specify different relative performance ratios $d_\alpha(x)$ and their related efficiency functions $E_\alpha(x)$. Constraint set (d) of APCDM is then embodied by the respective formulas for $d_\alpha(x)$ and $E_\alpha(x)$ and their stated bounding relationships, along with (8) and (10). The first of these equity measures is from Sherali, Staats, and Trani (2006), whereas the other two are new and focus on delays, assuming that the wind/weather optimized trajectory costs are compatible among the different surrogate plans for each flight.

6.1. Equity Method 1 (EM1)

This method, adopted by Sherali, Staats, and Trani (2006), defines the *relative performance ratio* as

$$d^1_\alpha(x) = \frac{\sum_{f\in A_\alpha} \sum_{p\in P_{f0}} c_{fp} x_{fp}}{\sum_{f\in A_\alpha} c^*_f}, \quad \forall \alpha = 1, \ldots, \bar{\alpha}, \tag{11}$$

where (11) measures the total fuel and delay costs incurred by airline α as a multiple of the best possible (unconstrained) cost. Note that by virtue of the scale-invariance of the ratio in (11), airlines could internally prioritize their own flights in this regard by appropriately weighting or adjusting the relative c_{fp} values. Sherali, Staats, and Trani (2006) restrict $d^1_\alpha(x) \le d^1_{\max}$, $\forall\,\alpha$, where $d^1_{\max} = 1.2$ is selected based on some empirical sensitivity analyses. Based on this ratio, we define the *collaboration efficiency* as a linear function for each airline α according to

$$E^1_\alpha(x) = \frac{d^1_{\max} - d^1_\alpha(x)}{d^1_{\max} - 1}, \quad \forall\,\alpha = 1, \ldots, \bar{\alpha}, \tag{12}$$

so that the efficiency $E^1_\alpha(x) = 1$ if $d^1_\alpha(x) = 1$, and $E^1_\alpha(x) = 0$ if $d^1_\alpha(x) = d^1_{\max} \equiv 1.2$ (i.e., exceeds the minimal possible total cost by 20%).

6.2. Equity Method 2 (EM2)

The *relative performance ratio* in this case measures the total *average delay realized per passenger* and is given by

$$d^2_\alpha(x) = \frac{\sum_{f\in A_\alpha} (\mathrm{PAX}_f) DL^{\mathrm{CDM}}_f(x)}{\sum_{f\in A_\alpha} (\mathrm{PAX}_f)}, \quad \forall\,\alpha = 1, \ldots, \bar{\alpha}, \tag{13}$$

where $DL^{\mathrm{CDM}}_f(x) \equiv \sum_{p\in P_{f0}} \delta_{fp} x_{fp}$ is the CDM-realized delay function for flight f, and PAX_f represents the aforementioned passenger count estimate, $\forall\, f$. In lieu of using such passenger count estimates based on historical load factors, which might be controversial for airlines, and also—because FAA subscribes to the general policy of treating all aircraft uniformly—we could assume that $\mathrm{PAX}_f = 1$, $\forall\, f$, so the focus in this equity measure would then be on the *average aircraft delays*, as opposed to the average delay per passenger. As another option, we could treat PAX_f as a normalized ordinal weight provided by airlines for prioritizing their different flights based on criticality with respect to downstream operations (and/or profits), if this information is available. In this lattermost case, any relative scale can be used, because the constraints described below are all scale invariant, and $d^2_\alpha(x)$ would then represent a *weighted average delay per flight* for each airline α. We note here that this could also be problematic in that it requires typically guarded information, but might be workable if such information regarding priority weights is provided to the FAA in a confidential parameter data file to be perceived and utilized by the model only. Using any of these alternative interpretations, we define the collaboration efficiency in this method as the linear function

$$E^2_\alpha(x) = \frac{d^2_{\max} - d^2_\alpha(x)}{d^2_{\max}}, \quad \forall\,\alpha = 1, \ldots, \bar{\alpha}, \tag{14}$$

where $d^2_{\max}$ is given as follows, based on some preliminary empirical analysis (see McCrea 2006):

$$d^2_{\max} = 1.5\Big(\min_{\alpha=1,\ldots,\bar{\alpha}} \{d^2_\alpha(x)\colon x \text{ corresponds to selecting the highest delay surrogate option for each flight}\}\Big).$$

Hence, the efficiency $E^2_\alpha(x) = 1$ if $d^2_\alpha(x) = 0$, and $E^2_\alpha(x) = 0$ if $d^2_\alpha(x) = d^2_{\max}$, where we also restrict $d^2_\alpha(x) \le d^2_{\max}$.

6.3. Equity Method 3 (EM3)

This method is inspired by the discussion in Vossen and Ball (2006b), which indicates that a feature of important relevance to airlines is *on-time performance* (e.g., delays not exceeding 15 minutes). In this case, we define a binary on-time performance index $\delta^{\text{on-time}}_{fp}$ for each flight plan $p \in P_{f0}$ of flight $f \in \{1, \ldots, F\}$ based on the estimated delay δ_{fp} and a permissible tolerance δ^{tol} (e.g., the 15-minute allowance mentioned above) according to

$$\delta^{\text{on-time}}_{fp} = \begin{cases} 1 & \text{if } \delta_{fp} \le \delta^{\mathrm{tol}} \\ 0 & \text{otherwise.} \end{cases}$$

Accordingly, the relative performance ratio and collaboration efficiency functions for this method are defined as follows:

$$d^3_\alpha(x) = \frac{\sum_{f\in A_\alpha} \sum_{p\in P_{f0}} \delta^{\text{on-time}}_{fp} x_{fp}}{|A_\alpha|} \tag{15}$$

and

$$E^3_\alpha(x) = d^3_\alpha(x), \quad \forall\,\alpha = 1, \ldots, \bar{\alpha}. \tag{16}$$

7. Enhanced Formulation of APCDM, APCDM-Light, and Algorithmic Issues

The enhanced version of APCDM inherits the following additional constraints to augment the model delineated in §2:

(g) Constraints representing slot exchanges, given trade offers (Equations (2), (3), and (4)).

(h) Constraints for tying in continuing or connecting flights (Equations (6) and (7)).

For the sake of simplicity and computational expediency from the viewpoint of airspace flow programs, we also study a reduced version of APCDM, which we refer to as *APCDM-Light*. This model has the following characteristics: it retains within the objective function (1) only the first term (total system fuel, delay, and cancellation costs) along with the efficiency-based first term from (9); it imposes only the basic sector (or FCA) capacity restrictions in Constraints (b); it eliminates the above-mentioned slot-exchange constraints (g) and connection constraints (h) (assuming that, barring slot exchanges, the delays associated with the different alternative flight plans are compatible with flight connections; otherwise, the relevant constraints (h) could be incorporated); and it retains Constraints (a) and (c)–(f).

In a direct implementation, we employ the mixed integer program (MIP) option in CPLEX 11.1 to solve APCDM to (ϵ) optimality, where we explore $0 \le \epsilon \le 5\%$ in §8 to assess the effect of ϵ on the quality of the solution produced and the required effort. As an alternative, we also propose and implement the following *sequential variable-fixing heuristic* (*SFH*) to solve APCDM (similar comments apply to APCDM-Light).

Given the estimated arrival time data (τ_{fp}^{arr}) for the different flight plans (f, p), $p \in P_f$, $f = 1, \ldots, F$, let

$$\tau_{f*}^{\text{arr}} \equiv \min_{p \in P_f}\{\tau_{fp}^{\text{arr}}\}, \quad \forall f = 1, \ldots, F,$$

and accordingly, construct a list L of the flights $f = 1, \ldots, F$ arranged in nondecreasing values of τ_{f*}^{arr}. Partition L (in its given order) into subsets $L_1, \ldots, L_U$, $U \ge 1$, such that $|L_u|$ is roughly equal to F/U, $\forall u = 1, \ldots, U$ (we advocate $F/U \approx 30$). Denote

$$L_u^* = \bigcup_{u'=1}^{u} L_{u'}, \quad \forall u = 1, \ldots, U.$$

Now, let $\mathbf{Alg}[L_u^*, \epsilon]$ connote an application of CPLEX-MIP to solve APCDM to ϵ% of optimality, while restricting x_{fp}, $\forall p \in P_f$, $f \in L_u^*$ to be binary valued (with x_{fp} fixed at designated binary values as described below, $\forall p \in P_f$, $f \in L_{u-1}^*$, where $L_0^* \equiv \varnothing$) and while relaxing the remaining x variables x_{fp}, $\forall p \in P_f$, $f \in L \backslash L_u^*$ to be continuous (nonnegative) variables. Note that by including the cancellation surrogate (possibly a dummy one if this is not a real option) and adjusting the right-hand side of (10) if necessary, we ensure that each updated version of APCDM solved below is feasible.

Heuristic SFH:

Initialization. Set $u = 1$ and choose $0 \le \epsilon \le 5\%$.

Step 1. Run $\mathbf{Alg}[L_u^*, \epsilon]$, and let $\bar{x}$ be the solution obtained.

Step 2. If $\bar{x}$ is binary valued, then stop with this as the prescribed solution.

Step 3. Else, update APCDM by permanently fixing $x_{fp} \equiv \bar{x}_{fp}$, $\forall p \in P_f$, $f \in L_u^*$. Replace $u \leftarrow u + 1$, and return to *Step* 1.

Note that once we get $u = U$ in Heuristic SFH, we shall necessarily stop at *Step* 2. Of course, it is possible for this procedure to terminate earlier than this juncture.

8. Computational Experience

We now provide results for several computational experiments. In §8.1 we present a realistic test case derived from the FAA's ETMS, and we use this test set to investigate the effects of the different equity methods given in §6, including the consequences with respect to solution equity when slot exchanges are considered. We also demonstrate the impact of the sector workload and conflict resolution constraints on the permissible slot exchanges. Section 8.2 studies the computational effort required to solve APCDM and APCDM-Light for randomly generated problems of various sizes. It also presents insights into the effect of utilizing different equity measures. Finally, we comment on our experience with the proposed SFH discussed in §7. All computations were performed on a Dell Precision T7400 workstation with an Intel Xeon E5410 2.33 GHz CPU having 3.25 GB of RAM and using CPLEX 11.1 to solve the different optimization problems.

8.1. Illustrative Test Case

We constructed a test scenario using data obtained from the FAA based on the ETMS flight information pertaining to the Miami and Jacksonville Air Route Traffic Control Centers. From this data, we selected flight trajectories that traverse an FCA comprised of a subset of the 88 sectors that define the two aforementioned Air Route Traffic Control Centers. Some of the arrival times were modified slightly to account for specific slot allocations.

The FAA Benchmark Report for Miami International Airport (MIA) (2004) details the airport's reported capacity under optimal, marginal, and Instrument Flight Rules operating conditions. The test set was generated to study the case in which MIA is operating within Instrument Flight Rules maximum capacity rates (40 arrivals and 40 departures per hour). Each of the 80 flights in this data set was ascribed six surrogate flight plans that represent alternative routes as well as revised arrival and departure times as necessary, based on imposed ground delays. The test set also features eight AMAL trade offers among the six involved airlines that we use to demonstrate the effects of slot exchanges on overall system cost and solution equity. The details of these trade offers, including passenger (PAX) estimates, are presented in Table 1. Note that "Flight" represents a simplified flight number for a particular airline.

The APCDM model was solved using different options, with a run-time limit of 20 minutes and an optimality tolerance of 0.01%. Table 2 displays the results obtained. The columns (a)–(o) in Table 2 represent the 15 different options under which APCDM was executed, where the top section of the table specifies the particular constraint sets or modeling features that were activated (indicated by ✓) for each of these options. These features are, in order, the consideration of: sector workload (sect. wkld.) restrictions, conflict resolution constraints, slot exchange mechanisms,

Table 1 APCDM Trade Offers

	Induce delay				Reduce delay			
Airline	Flight	Scheduled arrival	Delay until at most	PAX	Flight	Scheduled arrival	Move up to at least	PAX
AAL	1	600.0	610.5	98	3	616.5	615.0	160
AAL	8	642.0	649.5	111	10	652.5	651.0	200
COA	2	630.0	637.5	118	3	636.0	630.0	170
DAL	1	606.0	613.5	122	2	610.5	609.0	179
NWA	1	601.5	612.0	142	3	621.0	619.5	230
NWA	4	645.0	654.0	155	5	651.0	646.5	320
UAL	1	607.5	615.0	95	2	609.0	606.0	215
USA	1	612.0	621.0	112	2	618.0	616.5	217

the incorporation of nonnegativity restrictions on the NRPM of delay at the GDP airport for each airline, and the particular equity method implemented (EM1, EM2, or EM3 from §6). The middle section of Table 2 provides the resulting collaboration efficiencies ($E_\alpha(x) \in [0, 1]$) achieved by each of the six airlines for each case (a)–(o). Also displayed in bold in the final row of this section is the mean collaboration equity across all airlines as given by the second term in (9), where we used equal weights $w_\alpha = 1/6$, $\forall \alpha = 1, \ldots, \bar{\alpha} \equiv 6$. The bottom section of Table 2 presents the NRPM values (Equation (4)) for each airline, along with the system total given in bold in the final row, for each of the runs (a)–(o). Sections 8.1.1 and 8.1.2 below discuss some specific details of these results.

8.1.1. Comparison of Equity Methods. Columns (a)–(c) in Table 2 display collaboration efficiencies and NRPM values for the different airlines at the optimal solution under each of the three equity methods when applying Constraint (4). Columns (d)–(f) provide the corresponding information when Constraint (4) is not applied. The collaboration efficiency values are normalized to allow for more direct comparisons. EM2 consistently yielded the most equitable solutions across all airlines, both with and without Constraint 4. Applying Constraint 4 ensures airline acceptability within the slot exchanges by way of nonnegative reduction in overall passenger delays for each airline; however, it is noteworthy that the overall collaboration equity measure and the collaboration efficiencies for most airlines improve when this constraint is ignored. In addition, the objective value decreased by 1.04% under EM1, which is the emphasis of EM1, with decreases of 0.21% and 0.30% under EM2 and EM3, respectively (not shown in Table 2).

Table 2 Collaboration Efficiency and NRPM by Airline for 15 Model Run Options

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)
Applied constraints:															
—Sect. Wkld.	✓	✓	✓	✓	✓	✓				✓	✓	✓	✓	✓	✓
—Conflict Res.	✓	✓	✓	✓	✓	✓	✓	✓	✓				✓	✓	✓
—Slot Exch.	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			
—NRPM ≥ 0	✓	✓	✓				✓	✓	✓	✓	✓	✓			
—Eq. Method	EM1	EM2	EM3	EM1	EM2	EM3	EM1	EM2	EM3	EM1	EM2	EM3	EM1	EM2	EM3
Collaboration efficiency:													(infeas)		
—AAL	0.960	1.000	0.866	0.998	1.000	0.912	1.000	1.000	0.866	0.963	1.000	0.866	—	0.985	0.898
—COA	0.917	1.000	1.000	0.947	1.000	0.955	0.955	1.000	1.000	0.978	1.000	1.000	—	0.987	1.000
—DAL	1.000	0.992	0.955	0.998	0.992	0.929	0.963	0.992	0.883	0.898	0.992	0.955	—	0.990	0.974
—NWA	0.937	1.000	0.909	0.991	1.000	0.955	0.954	1.000	0.909	1.000	1.000	0.909	—	1.000	1.000
—UAL	0.982	1.000	0.921	1.000	1.000	0.967	0.962	1.000	0.988	0.922	1.000	0.921	—	0.971	0.945
—USA	0.894	0.990	0.955	0.924	0.990	1.000	0.883	0.990	0.955	0.961	1.000	0.955	—	0.987	0.962
MCE	**0.032**	**0.004**	**0.035**	**0.027**	**0.004**	**0.022**	**0.023**	**0.004**	**0.047**	**0.029**	**0.002**	**0.035**	**—**	**0.006**	**0.028**
Net reduction in passenger minutes:															
—AAL	0	501	501	−621	−48	99	0	0	0	501	501	246			
—COA	1,863	1,332	1,332	1,863	1,332	1,332	1,332	1,863	1,863	312	0	0			
—DAL	879	86	879	−12	0	86	342	86	86	1,428	86	1,428			
—NWA	51	1,769	1,769	1,717	1,718	1,718	0	51	51	917	1,718	1,769			
—UAL	1,650	1,185	0	1,507	1,793	1,650	360	1,185	1,185	503	360	1,793			
—USA	1,449	620	620	1,449	−504	−504	1,449	620	620	1,124	0	1,124			
System	**5,892**	**5,492**	**5,100**	**5,904**	**4,290**	**4,380**	**3,483**	**3,804**	**3,804**	**4,784**	**2,664**	**6,359**			

Note. NRPM = net reduction in passenger minutes.

8.1.2. Effect of Slot Exchanges. We next studied the impact of including slot exchanges in the APCDM modelling construct by solving variants of APCDM with and without certain constraint sets and examining the results (see Columns (g)–(o) in Table 2). Ignoring sector workloads (Columns (g)–(i)) rendered slot exchanges unnecessary for American Airlines (AAL), i.e., no AAL trades were accepted while the model generally prescribed more overall efficient and equitable solutions in this capacity-free environment. (Recall that the objective function compromises among cost, efficiency, and equity.) Omitting the conflict resolution constraints also impacted the solution with respect to slot trades (Columns (j)–(l)). One of the traded flight plans in this case that belongs to Continental Airlines (COA) is actually involved in a fatal conflict, which goes undetected without imposing the conflict resolution constraints. This underscores the importance of considering conflict risks and conflict resolution workload limits in selecting a mix of flight plans. Optimizing the test case without permitting slot exchanges (Columns (m)–(o)) somewhat affected the aforementioned compromise between cost, efficiency, and equity under EM2 and EM3, but that rendered the test problem infeasible when using EM1 because the value of $d^1_{\max} = 1.2$ was too stringent to permit a feasible solution in the absence of slot exchanges. In other words, the average cost to at least one airline would have been more than 20% higher than the minimum possible cost for that airline. Moreover, the solutions obtained using EM2 and EM3 resulted in objective values that were 3.44% and 4.16% higher, respectively, than the corresponding values when slot exchanges were permitted. Because of the compromise between the different objective terms, allowing slot exchanges between airlines resulted in more overall cost effective, although not necessarily more equitable, solutions.

8.2. Computational Effort Analysis

To examine the computational effort required to solve APCDM and APCDM-Light, we generated 120 random problems. All these problems included flights between the 40 busiest airports in the United States through a notional airspace comprising 26 sectors. In an effort to make these random problems as realistic as possible with respect to actual traffic patterns, we sampled ETMS data for flights between these airports and scheduled the flights accordingly. The 40 busiest airports were considered in this process to allow a wide variety of potential flights. The relatively low number of sectors when compared to the actual National Airspace System was chosen for two reasons: (a) to emulate a smaller FCA region and (b) to increase the likelihood that sector capacity constraints would be active. We varied the number of flights considered as 125, 250, 500, or 1,000 (with 30 problems each), and each flight was assigned a random number of surrogate flight plans (20% with two surrogates, 50% with three, and 30% with four). We formulated the models APCDM and APCDM-Light and obtained solutions using different optimality tolerances (0.01%, 1%, and 5%).

Table 3 provides details on the computational effort required (averaged over the 30 instances of each size) to optimize APCDM to different optimality tolerances, along with the average quality of the resulting solutions. Here, we refer to *solution quality* as the percentage difference between the final objective function value produced and the best known objective function value for that problem instance. The average time required to generate the constraints is also listed for each problem size. Note that the runs in Tables 3 and 4 use equity method EM2. This was the only equity method for which there existed a feasible solution to each of the randomly generated problems; as seen in §8.1, it was the most robust with respect to slot exchanges. EM1 was the most restrictive, producing 15 infeasible instances (these instances could be made feasible by suitably increasing the value of $d^1_{\max}$). EM3 produced three infeasible instances, which are more difficult to resolve because of the discrete manner in which the collaboration efficiency ($E^3_\alpha(x)$) is computed in this case. Because on-time performance is a discrete value, there might exist no solution for which the resulting collaboration equity lies within the imposed bounds. A possible resolution might be to increase (or relax) $E^{\text{equity}}_{\max}$ and depend primarily on the objective

Table 3 APCDM Solution Time and Quality (EM2)

		Optimality tolerance (%)					
		0.01		1		5	
No. of flights	Constraint generation time (sec)	Solution time (sec)	Exceeded time limit	Solution time (sec)	Solution quality (%)	Solution time (sec)	Solution quality (%)
125	0.184	4.204	0	1.256	0.161	0.349	1.904
250	0.511	217.999	2	3.320	0.166	0.586	2.309
500	1.518	859.028	18	5.620	0.196	1.771	2.465
1,000	5.427	1,200.000	30	28.696	0.267	9.281	2.213

Table 4 APCDM-Light Solution Time and Quality (EM2)

No. of flights	Constraint generation time (sec)	Optimality tolerance (%) 0.01 Solution time (sec)	1 Solution time (sec)	1 Solution quality (%)	5 Solution time (sec)	5 Solution quality (%)
125	0.134	0.032	0.034	0.001	0.038	0.001
250	0.365	0.050	0.049	0.000	0.063	0.126
500	1.082	0.081	0.090	0.004	0.092	0.335
1,000	4.088	0.174	0.173	0.024	0.185	0.503

function penalties to achieve equity. Each run was terminated after 20 minutes, and the number that exceeded this time limit (which occurred only with the tight optimality tolerance of 0.01%) is provided in the table. Nonetheless, the solution for every problem that reached the 20-minute time limit under EM2 was within 1% of optimality. (Results using all three equity methods are compared subsequently).

It is noteworthy that the solutions obtained for APCDM when CPLEX exits within the specified tolerance of 1% of optimality are actually of a relatively high quality (ranging from 0.161%–0.267% of the best known solution value, on average) and are generally obtained much faster than the results derived using CPLEX's default optimality tolerance of 0.01%. However, although increasing the optimality tolerance to 5% substantially reduces the solution effort, the solution quality is significantly degraded.

Table 4 provides similar average solution time and quality results for APCDM-Light using different optimality tolerances. Here again, *solution quality* refers to the percentage of deviation in the objective function value with respect to the best known objective value for solving that particular instance of APCDM-Light. In this case, solutions to within 0.01% of optimality are obtained expeditiously.

Tables 5 and 6 present comparisons of the computational effort required to solve APCDM and APCDM-Light, respectively, using each of the three equity methods and employing the specified optimality tolerances (1% for APCDM and 0.01% for APCDM-Light). In addition to the average solution times, these tables include the minimum and maximum central processing unit (CPU) times required to solve each problem size. As before, the CPU times are reported only for feasible problem instances. Note that some problem instances modeled using EM3 were particularly challenging; even three of the APCDM-Light runs using EM3 reached the 20-minute time limit. Perhaps, because of the discrete nature of EM3, there exist several alternative (near-) optimal solutions that the branch-and-bound solver enumerates in the search process before being able to verify optimality.

8.3. Experience with Heuristic SFH

Finally, we provide some computational experience for implementing the SFH described in §7 as applied to model APCDM. We attempted partitioning the list of flights L into subsets where $L/U \approx 30$. Fixing variables in these relatively small subsets sometimes

Table 5 APCDM Solution Time by Equity Method (1% Opt. Tolerance)

No. of flights	EM1 Min.	EM1 Max.	EM1 Avg.	EM2 Min.	EM2 Max.	EM2 Avg.	EM3 Min.	EM3 Max.	EM3 Avg.
125	0.062	0.125	0.082	0.218	8.625	1.256	0.859	233.258	16.769
250	0.094	0.625	0.273	0.375	21.187	3.320	4.171	1,118.090	107.578
500	0.156	16.844	3.422	0.921	25.375	5.620	1.265	1,200.000	215.550
1,000	0.390	1,200.000	342.598	2.625	221.400	28.696	3.469	1,200.000	159.817

Table 6 APCDM-Light Solution Time by Equity Method (0.01% Opt. Tolerance)

No. of flights	EM1 Min.	EM1 Max.	EM1 Avg.	EM2 Min.	EM2 Max.	EM2 Avg.	EM3 Min.	EM3 Max.	EM3 Avg.
125	0.031	0.094	0.045	0.015	0.062	0.032	0.031	0.047	0.034
250	0.047	0.125	0.074	0.031	0.109	0.050	0.046	0.234	0.068
500	0.078	74.170	2.782	0.062	0.110	0.081	0.093	1,200.000	80.103
1,000	0.188	65.467	2.632	0.140	0.235	0.174	0.188	1,200.000	80.208

Table 7 **APCDM Solution Time and Quality Using SFH (EM2)**

	Optimality tolerance (%)					
	0.01		1		5	
No. of flights	Solution time (sec)	Solution quality (% above optimal)	Solution time (sec)	Solution quality (% above optimal)	Solution time (sec)	Solution quality (% above optimal)
125	0.442	0.451	0.300	0.702	0.156	4.406
250	1.591	0.502	0.584	0.837	0.408	7.590
500	4.932	0.387	1.531	1.577	1.336	11.073
1,000	20.519	0.358	46.321	3.523	6.124	19.601

induced infeasibilities in later iterations of the heuristic, when currently relaxed variables were subsequently enforced to be binary valued. This phenomenon occurred most often when the optimality tolerance for the subproblems was larger (1% or 5%), but it also occurred when subproblems were solved to within 0.01% of optimality. Increasing the size of the subsets resulted in fewer infeasibilities, but processing times increased. EM2 seemed to be the most robust with respect to SFH. Under EM2, the heuristic was unsuccessful only once (with 1,000 flights and $\epsilon = 0.01$). The CPU times and solution quality metrics when optimizing APCDM using SFH are provided in Table 7. Comparing Tables 3 and 7, we see that using the heuristic SFH with a 0.01% optimality tolerance yields results that are comparable to solving APCDM with a 1% optimality tolerance. However, we can expect the utility of SFH to become more prominent when problem size increases further.

9. Summary and Conclusions

The notion of slot ownership, formalized via the FAA's CDM initiative under the enhancements to the GDP, has spawned new research efforts pertaining to slot-trading opportunities that could provide additional benefits in terms of flight efficiencies and desirable airline schedules. Given that airlines submit AMAL trade offers, we have significantly expanded the functionality of the original APCDM model of Sherali, Staats, and Trani (2003, 2006) to automatically accommodate the consideration of such trades in terms of related slot exchanges, along with appropriate flight connection constraints, while simultaneously considering the impact of the resultant overall mix of flight plans on sector workloads, safety with respect to conflict risk, and equity among the involved airlines. We have also proposed two alternative equity formulations that produce comparable solutions to those of the original APCDM model's equity concept. One of the new methods, EM2, which is based on average delay per passenger (or weighted average delay per flight), seems to be a particularly robust way to model equity considerations in conjunction with sector workloads, conflict resolution, and slot exchanges. Moreover, employing EM2 allowed us to solve the relatively more difficult 1,000 flight problem instances for APCDM within 30 seconds on average using a 1% optimality tolerance, and produced comparable solutions within about 20 seconds on average using the SFH with an optimality tolerance of 0.01%. The actual solutions obtained for these largest problem instances were well within 1% of the best known solution. In addition, we have investigated a simplified variant, APCDM-Light, which is more amenable for use in an airspace flow program. This model was readily optimized to a 0.01% tolerance within about 5 seconds on average for the largest set comprising 1,000 flights. The augmented APCDM model therefore offers a viable tool that can be used for tactical air traffic management purposes as an airspace flow program (particularly, APCDM-Light). This model could also be used for strategic applications to study the impact of different types of trade restrictions, collaboration policies, equity concepts, and airspace sector configurations.

The development of this model is part of ongoing research related to NASA's Next Generation Air Transportation System, which envisages the sequential use of a variety of models pertaining to three tiers. The Tier 1 models, such as the one described by Bertsimas, Lulli, and Odoni (2009), are more strategic in scope and attempt to identify potential problematic areas, e.g., areas of congestion resulting from a severe convective weather system over a given time frame, and provide aggregate measures of sector workloads and delays. The affected FCAs highlighted by the results from these Tier 1 models would then be analyzed by more detailed Tier 2 models, such as APCDM, which consider more specific alternative flight plan trajectories through the different sectors along with related sector workload, aircraft conflict, and airline equity issues. Finally, Tier 3 models are being developed to dynamically examine smaller-scale, localized fast-response readjustments in air traffic flows within the time frame of about an hour prior to departure (e.g., to take advantage of a break in the convective weather system).

Future research areas include a two-stage stochastic extension of the APCDM model. In such a scenario, the imminent flight plan selection decisions could be considered as first-stage variables, and other future time period flight plan selections could be conditioned on several alternative scenarios pertaining to the stochastic dynamic weather pattern and predicted sector capacities, and would constitute second-stage variables. It would be interesting to examine differences in the nature of solutions produced under single-versus-multiple weather pattern scenarios, and to investigate if some equivalent aggregate weather pattern scenario can capture the observed effects of considering multiple scenarios in such a two-stage stochastic programming framework. Other future research may include the evaluation of side payments as part of the trade offer and equity structure, as suggested by Vossen (2002), and using S-shaped value functions (Kirkwood 1997) to more generally reflect airlines' assessments of delay impacts within the defined relative performance ratios (see McCrea 2006 for some results in this vein).

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