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Overview Paper

The design of a market mechanism to allocate Air Traffic Flow Management slots

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In Europe, when an imbalance between demand and capacity is detected for air traffic network resources, Air Traffic Flow Management slots are allocated to flights on the basis of a First Planned First Served principle. In the case of a single constrained en-route sector or airport, we propose a mechanism for slot allocation which is based on market principles as it enables airlines to pay for delay reduction or receive compensation for delay increase. The mechanism fulfills the properties of individual rationality, budget balance, coalitional rationality and Pareto efficiency, and it can be implemented through two alternative distributed approaches that do not require airlines to disclose confidential information. Both approaches have the additional advantage that they directly involve airlines in the decision making process. Two computational examples relying on real data indicate that the market-based solution allows the participating airlines to significantly decrease their overall delay-related costs with respect to the First Planned First Served allocation.

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1. Introduction

In Europe, when on the day of operations an imbalance between predicted traffic and available capacity is detected, either in the airspace or an airport, the Air Traffic Management (ATM) authority usually imposes a *regulation*. A regulation consists in capping the maximum rate of aircraft entries, a measure which is achieved by forcing delays to the take-offs of some flights, the so-called ground delays. Ground delays are assigned through a departure time slot, or Air Traffic Flow Management (ATFM) slot, on the basis of a First Planned First Served (FPFS) policy, i.e., according to the flights' Estimated Time Over (ETO) the specific sector or airport. In this paper, we assume that airlines are interested in paying for delay reductions or in receiving compensation for delay increase (Vossen and Ball, 2006a). This is achieved by introducing a market mechanism that first allocates ATFM slots according to the current FPFS policy, and successively allows airlines to trade these slots. This latter step of our mechanism is governed by a set of rules that guarantee that the final slot allocation has some suitable properties. Our paper first introduces these properties and then shows that they are actually fulfilled by the eventual slot allocation.

The first important property of the proposed mechanism is that it let airlines decide autonomously, for each flight, whether it is preferable to keep the slot obtained by the FPFS policy or to exchange it at the market price. Centralized market-based mechanisms would be more difficult to implement since it would require that the ATM authority has the delay cost data for each flight. Unfortunately, airlines might consider these costs confidential information, and are reluctant to disclose them even to the ATM authority.

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Following the seminal work of Odoni (1987), a number of researchers have focused their activity on the development of optimization models and algorithms for the assignment of ground delays as a short-term measure for traffic flow regulation. The problem of assigning ground delays to a set of flights in order to minimize an aggregated cost function, given airport capacity constraints, is known as Ground Holding Problem (GHP). In its basic version the GHP assumes that one airport in the system is subject to capacity constraints which are imposed only on arrival flights. This problem is referred to as the Single Airport Ground Holding problem (SAGHP) and has been formulated for different cases. Andreatta and Romanin-Jacur (1987) analyze and solve the problem with a dynamic programming algorithm for a single time period and one airport subject to limited capacity which is a random variable whose distribution is known. Terrab and Odoni (1993) formulate the problem as a minimum cost network flow for the multi-period deterministic case in which the future airport capacity profile is determined in advance. They also extend the algorithm proposed in Andreatta and Romanin-Jacur (1987) to the multi-period case with stochastic capacities and propose several heuristics to achieve approximate solutions due to the complexity of the exact dynamic programming approach. Richetta and Odoni (1993) extend these results by presenting a stochastic linear programming solution to the static version of SAGHP. Hoffman and Ball (2000) propose a variant of SAGHP by adding banking constraints that force sets of flights to be temporally grouped in order to allow the transfer of passengers, baggage and crews for aircraft operating under a hub-and-spoke system. Dell'Olmo and Lulli (2003) formulate a dynamic programming algorithm that optimally allocates airport capacity to both arrival and departure flights, using the concept of capacity envelope according to which arrival and departure capacities are interdependent and connected by a convex piecewise-linear functional relationship.

In all the aforementioned works the ATM authority acts as the only decision maker optimizing a 'global' objective function that aggregates all the delay costs caused by the ATFM regulations. Alternatively, our approach proposes to implement a distributed mechanism for the assignment of ATFM slots. This reflects the operational philosophy that emerged in recent years, that is, involving airspace users as much as possible in the decision making process, whenever this decision directly affects them (see, e.g., EUROCONTROL EEC, 1999). Thus both US and European ATM regulators have introduced Collaborative Decision Making (CDM) programs (Chang et al., 2001; EUROCONTROL, 2006). The idea of CDM is to move away from a central decision maker to 'real-time' decision support tools that assist traffic managers and airline operational centers in making the best collaborative decisions. The CDM concept is a pillar of SESAR, the Single European Sky ATM Research program of the European Union. Our research falls within the scope of SESAR program, which states that airlines will be offered the possibility to indicate to the ATM authority a priority order for flights affected by delays (SESAR Joint Undertaking, 2007). This User Driven Prioritisation Process (UDPP) will be initiated by the ATM authority when an unexpected imbalance between capacity and demand is detected on a short notice. Since different airlines are in general competitors, market-based mechanisms seem natural ways of implementing the UDPP. A market mechanism suitable for the UDPP should satisfy the following properties to be acceptable (see e.g. Krishna, 2009, for formal definitions):

- Individual rationality: the mechanism always gives each participant a non-negative payoff, otherwise no rational airline would participate without being enforced by the regulator. This property is sometimes also called participation constraint.
- Budget balance: the mechanism neither requires subsidization from outside nor produces profit to allocate outside the set of airlines involved. The overall amount of prices paid and received by participants sums up to zero. The ATM authority's unique objective is to guarantee the correct operation of the mechanism, without the need of financing it and without expecting a monetary utility out of it.

We consider the above properties as hard constraints that our market mechanism must satisfy. In fact all airlines must derive a non-negative profit from the participation while the ATM authority objective is merely to facilitate the implementation of the mechanism, without neither collecting nor introducing money. Thus all rational agents, whose objective is to maximize profits, and the ATM authority, whose objective is the social welfare, will want to participate in the mechanism. Other desirable properties are:

- Allocative efficiency: the mechanism produces a slot allocation which minimizes the sum of the costs reported by the airlines, among all the feasible allocations.
- Incentive compatibility: the mechanism is designed so that no airline can increase its payoff by misrepresenting its costs, provided that all other airlines are truthful.

Allocative efficiency can be achieved only if costs are reported truthfully and an allocation is computed with respect to true cost values, i.e. only in an incentive compatible mechanism. Unfortunately, the impossibility theorem by Myerson and Satterthwaite (1983) states that no exchange mechanism can satisfy at the same time incentive compatibility, individual rationality, budget balance and allocative efficiency at the equilibrium. The mechanism we propose thus relaxes the incentive compatibility constraint in order to guarantee individual rationality and budget balance and it maximizes the allocative efficiency according to reported costs.

To our knowledge, so far the literature on market mechanisms for slot allocation focused on airport slots only, resolving the airport capacity problems. Airport slots constitute an administrative approach to the strategic management of

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demand, since all the airlines that intend to schedule a flight movement to and from a congested airport need to be assigned an airport slot for this purpose. Differently, the ATFM slots we deal with are used as a tactical tool to regulate mainly en-route lack of capacity. In Europe, the allocation of airport slots follows the 'grandfather' rule. Airlines can keep their slots year after year provided that their rate of utilization is above the 80% threshold (see, e.g., Sieg (2010) and Verhoef (2010) for recent analyses on different regulatory schemes for this market). For the US context, Rassenti et al. (1982) propose a sealed-bid type of combinatorial auction for the long-term strategic allocation of airport slots. The slot prices are set by the ATM authority on the basis of bids submitted and in order to maximize the system surplus. After this initial slot allocation, a secondary market is allowed, where airlines trade the slots previously received. This mechanism requires the disclosure of airlines' information to calculate the first allocation, and assigns the revenue from the auction to airports. Authors recognize that this latter point is debatable since airports with limited capacity would increase their revenues by imputing rents on scarce commodities. Fukui (2010) provides an empirical analysis of the current secondary market at four major US airports. Also Ball et al. (2006) observe that US airport slots are scarce commodities with both private and common value. They provide a deep analysis of the objectives and issues associated with slot auctions at different planning levels, from the strategic to the tactical one. They observe that one argument against auctions is that auctions can be seen as a way for the government to raise revenues, since airlines should pay to get the same slots that they now receive at no cost. Our market mechanism overcomes this drawback since it takes the FPFS allocated slots as initial endowment of each flight and enforces individual rationality and budget balance. However, we set a rule that no compensation is given for canceled flights that release their slots, to prevent the creation of 'ghost' flights just to make money.

Recently, Waslander et al. (2008) considered the case in which competitor airlines have different preferences over traffic control actions. They formalize a capacity resource market for the air traffic and they demonstrate that the outcome of such market mechanism is preferred by all airlines over a solution that does not take into account preference information. The starting point of our work is a formalization of the system currently adopted in Europe to allocate ATFM slots and then we suggest an implementation of our market mechanism, which can be employed to enhance this solution.

Finally, our approach addresses the indications made by Vossen and Ball (2006a,b) who present slot trading opportunities for Ground Delay Program not based on market mechanisms and suggest "using exchanges that involve side payments as a possible way of compensating carriers for absorbing increases in delay".

The remainder of this paper unfolds as follows. In Section 2, we provide an overview of the slot allocation mechanism currently used in Europe and we show that it minimizes the total flight delays. In Section 3, we introduce individual rational and budget balanced mechanism that maximizes the allocative efficiency based on the reported costs. We show that the resulting slot allocation minimizes the cost of the overall delays of the airlines involved in the trading. To prevent the disclosure of confidential information and to reduce computational and communication burdens, in Section 4, we suggest two different approaches for implementing our market mechanism in a distributed fashion. Then, in Section 5, we provide two examples based on real data. In Section 6, we present our conclusions.

2. The current European resource allocation system

In Europe, when an imbalance between demand and available sector capacity is detected, the EUROCONTROL Central Flow Management Unit (CFMU) may employ a number of measures: the reconfiguration of some sectors, the activation of mandatory routes for certain trajectories, and the creation of slot allocation regulations (Leal de Matos and Omerod, 2000). In this last case, the aim is to protect a certain scarce resource of the system by limiting, through a regulation, the maximum number of flights that can enter it during an established period of time. On the day of operations all flights affected by regulations can either decide to re-route, in order to avoid the affected area, or to be delayed on the ground by a controlled take-off time, that is by adhering to the assigned slot. This measure is based on the principle that delays on the ground are safer and less costly than those in the air. Any forecasted delay somewhere in the system is anticipated at the departure airport prior to the take-off (Ball and Lulli, 2004). The ATFM slots are calculated by the Computer Assisted Slot Allocation (CASA) system at CFMU on a FPFS policy (EUROCONTROL CFMU, 2007). To do so, initially, for each resource (i.e., an en-route sector or an airport) CASA creates a slot allocation list. This list is composed of a number of slots which is a function of the rate of acceptance and the duration of the regulation. Then, CASA allocates identified slots to flights, as close to their ETO as possible. If two flights require the same slot, the slot is allocated to the one with the lower ETO. If a flight is affected by several regulations, the delay caused by the most penalizing one is forced on all the others. Delays caused by ATFM measures in 2009 amounted to 15.2M min, causing an estimated cost of €1000M to the airlines (EUROCONTROL PRC, 2010).

Here we present a case where all the considered flights are affected by a single regulation on a unique resource. We consider a single capacity-constrained resource s active from st_time to end_time with a fixed capacity K, expressed as number of entries per hour. In addition, we have a set of flights $F = \{1, \ldots, F\}$ that must enter s. The ATM authority must allocate a different slot to each flight $f \in F$ for the access to s. The N_s slots of resource s, with

$$N_s = \left\lfloor \frac{end\_time - st\_time}{\frac{60}{K}} \right\rfloor,$$

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are defined as follows. Let $S = \{1, \ldots, N_s\}$ be the set of slots. Each slot $j \in S$ has capacity equal to one flight and begins at time I_j where:

$$I_j = \left\lfloor st\_time + (j-1) \cdot \frac{60}{K} \right\rfloor \quad \text{with } j \in \{2, \ldots, N_s\},$$

and $I_1 = st\_time$. The same slot j ends at the last time unit U_j preceding the beginning of the next slot $j + 1$, i.e.,

$$U_j = I_{j+1} - 1 \quad \text{with } j \in \{1, \ldots, N_s - 1\},$$

and $U_{N_s} = end\_time$.

Each flight $f \in F$ has a published trajectory which contains the ETO E_f into s. Then, flight f can be allocated to any slot $j \in R^f$, where $R^f = \{j{:}E_f \leqslant U_j\} \subseteq S$ is the set of the feasible slots for flight f as they end after E_f. The allocation of flight f to slot j produces a delay d_{fj} equal to

$$d_{fj} = \max\{E_f, I_j\} - E_f \quad \forall f \in F, \ \forall j \in S.$$

Given the above data, the ATM authority allocates the available slots. This operation is equivalent to determining the value of the decision variables x_{fj}, for all $f \in F$ and $j \in R^f$, where x_{fj} is set equal to 1 if flight f is allocated to slot j, 0 otherwise.

The FPFS slot allocation policy sorts flights $f \in F$ by the ascending value of E_f, then greedily allocates each flight to the first available slot, i.e., x_{fj} is set equal to 1 if

$$j = arg\min\{U_j : E_f \leqslant U_j \quad \forall j \in S \quad \text{and} \quad j \text{ is not allocated yet}\}.$$

Hereafter, we refer to x as the vector whose components are the variables x_{fj} and, in particular, we denote as x^F a slot allocation obtained by the application of the FPFS policy.

In the following, we show that the allocation x^F minimizes the total delay of the flights in F, as required by the ATM authority (EUROCONTROL PRC, 2010). In other words, x^F is an optimal solution of the assignment problem:

$$\min \quad \sum_{f \in F} \sum_{j \in R^f} d_{fj} x_{fj}, \tag{1a}$$

$$\sum_{f \in F : j \in R^f} x_{fj} \leqslant 1 \quad \forall j \in S, \tag{1b}$$

$$\sum_{j \in R^f} x_{fj} = 1 \quad \forall f \in F, \tag{1c}$$

$$x_{fj} \geqslant 0 \quad \forall f \in F, j \in R^f. \tag{1d}$$

In (1), the objective function (1a) minimizes the total delay; constraints (1b) impose that at most one flight can enter s during each slot $j \in S$, whereas constraints (1c) require that all flights affected by the regulation must have a slot allocated; finally, constraints (1d) guarantee the integrality of the the decision variables x_{fj}. These last constraints can be expressed in their linearly relaxed form since (1) is an assignment problem.

The allocation of flights to slots has already been formulated as an assignment problem through the OPTIFLOW model proposed by Vossen and Ball (2006a). Differently from (1), the OPTIFLOW model minimizes the overall delay costs and considers all slots composed by only one unit of time, i.e., $I_j = U_j \ \forall j \in S$.

Theorem 1. A vector x^F obtained respecting the FPFS policy is optimal for problem (1).

Proof. By closing paralleling the proof of Theorem 3.2 in Vossen and Ball (2006a). □

The FPFS policy does not allow a flight f to make any choice or state any preference about the slots. On the other hand, the optimality of the FPFS policy also implies its Pareto efficiency. Any other slot allocation policy that reduces the delay of a given flight f correspondingly increases the delay of at least another flight $g \neq f$, that consequently would not find convenient to move from its granted FPFS slot.

A different solution becomes available when we no longer consider delays only, but also the costs associated with delays, and we allow monetary payments to be involved. In this case it is possible to introduce an appropriate reallocation policy such that all flights may prefer a slot allocation different from the FPFS one, as we show in the next section.

3. A market mechanism for slot allocation

We consider the possibility of a slot trade between flights. We assume that airlines are willing either to pay for earlier slots that reduce their delay or to receive a side payment to compensate possible increase of delays.

Let x^F be the vector representing the FPFS slot allocation and let x represent an alternative feasible slot allocation obtained by the market mechanism. In addition, let c_{fj} be the delay cost of flight f allocated to slot j, for all $f \in F$ and $j \in R^f$. Costs c_{fj} vary depending on, e.g., the time of the day, the type of flight (connecting or not) and the type of airline. This cost parameter is a non-linear function of the length of delay, since costs per minute are considerably higher for longer delay durations (Cook

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et al., 2004). Finally, for all $j \in S$, let $v_j \geqslant 0$ be the price of the slot j, i.e., the amount of money that the airline that sells the slot receives from the airline that buys the same slot.

We define

$$p_f = \sum_{j \in R^f} c_{fj} \cdot \left(x^F_{fj} - x_{fj}\right) + \sum_{j \in R^f} v_j \cdot \left(x^F_{fj} - x_{fj}\right), \tag{2}$$

as the *profit* that flight $f \in F$ obtains by exchanging the slot assigned through the FPFS allocation with the slot assigned by the alternative allocation. The former term of p_f represents the difference in the cost of delay between the allocation of flight f from the FPFS slot to the alternative one. The latter term represents the economic payoff between the trade of its FPFS slot and the purchase of the alternative slot.

The first property that a market mechanism must fulfill is the individual rationality: each flight f participating in the trade must obtain a non-negative p_f, i.e.,

$$p_f \geqslant 0, \quad \forall f \in F. \tag{3}$$

Another property is the budget balance: the trade should neither require payments from outside (i.e., there is no subsidization) nor generate a surplus, but just redistribute the payments between flights, i.e.,

$$\sum_{f \in F} \sum_{j \in R^f} v_j \cdot \left(x^F_{fj} - x_{fj}\right) = 0. \tag{4}$$

Finally, a market mechanism may be required to maximize the allocative efficiency according to the costs reported by flights. From the society's point of view, it is desirable to minimize the overall cost of delay. This is equivalent to identifying a new slot allocation which maximizes the decrease of the total delay cost from the granted FPFS allocation. On the other hand, airlines may wish to use the trading not only to minimize their costs of delay, but also to maximize their profits. However, from Eqs. (2) and (4) the two objectives coincide as

$$\sum_{f \in F} p_f = \sum_{f \in F} \sum_{j \in R^f} c_{fj} \cdot \left(x^F_{fj} - x_{fj}\right). \tag{5}$$

Solving the following nonlinear problem we obtain a slot allocation that satisfies the above described desirable characteristics of a market mechanism:

$$\max \quad \sum_{f \in F} \sum_{j \in R^f} c_{fj} \cdot \left(x^F_{fj} - x_{fj}\right), \tag{6a}$$

$$\sum_{f \in F : j \in R^f} x_{fj} \leqslant 1 \quad \forall j \in S, \tag{6b}$$

$$\sum_{j \in R^f} x_{fj} = 1 \quad \forall f \in F, \tag{6c}$$

$$\sum_{j \in R^f} c_{fj} \cdot \left(x^F_{fj} - x_{fj}\right) + \sum_{j \in R^f} v_j \cdot \left(x^F_{fj} - x_{fj}\right) \geqslant 0 \quad \forall f \in F, \tag{6d}$$

$$\sum_{f \in F} \sum_{j \in R^f} v_j \cdot \left(x^F_{fj} - x_{fj}\right) = 0, \tag{6e}$$

$$x_{fj} \in \{0, 1\} \quad \forall f \in F, \; j \in R^f, \tag{6f}$$

$$v_j \geqslant 0 \quad \forall j \in S. \tag{6g}$$

Next we show that constraints (6d), (6e) and (6g) are redundant, thus problem (6) reduces to the simpler linear problem:

$$\min \quad \sum_{f \in F} \sum_{k \in R^f} c_{fk} x_{fk}, \tag{7a}$$

$$\sum_{f \in F : k \in R^f} x_{fk} \leqslant 1 \quad \forall k \in S, \tag{7b}$$

$$\sum_{k \in R^f} x_{fk} = 1 \quad \forall f \in F, \tag{7c}$$

$$x_{fk} \geqslant 0 \quad \forall f \in F, \; k \in R^f. \tag{7d}$$

We denote as x^C the optimal solution of (7). As problems (1) and (7) share the same constraints, x^C is also feasible for (1), but not necessarily optimal. Analogously, x^F is feasible for (7), but not necessarily optimal. When all slots are composed of only one unit of time, i.e., $I_j = U_j \; \forall j \in S$, (7) corresponds to the OPTIFLOW model proposed by Vossen and Ball (2006a). In this case, the optimal solution x^C of problem (7) is also optimal for problem (1) (see Theorem 3.1 in Vossen and Ball (2006a)).

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Definition 1. We define as MINCOST the policy that implements the slot allocation according to problem (7).

Definition 2. We define as MM the market mechanism where airlines, for each flight, sell its granted FPFS slot j at price υ_j and purchase the MINCOST slot k at price υ_k.

When the slot trade occurs between two flights belonging to two different airlines we may assume a monetary flow between these airlines. However, nothing prevents to apply MM between two flights of the same airline. In this case, we may consider virtual monetary compensations within the airline.

The next theorem proves that MM fulfills the individual rationality property.

Property 1. MM is individually rational if, for all $k \in S$, the slot costs are $\upsilon_k = -\xi_k$ where ξ_k are the nonpositive dual variables associated with constraints (7b).

Proof. Let ξ_k be the nonpositive dual variables associated to constraints (7b) for all $k \in S$, and μ_f be the dual variables associated to constraints (7c) for all $f \in F$. The slackness conditions impose $\mu_f + \xi_k \leqslant c_{fk}$, where the equality holds if $x^C_{fk} = 1$. As a consequence, for $k \in S$ such that $x^C_{fk} = 1$, we obtain

$$c_{fi} - c_{fk} + \xi_k - \xi_i \geqslant 0, \quad \forall i \in S \setminus \{k\}. \tag{8}$$

Finally, observe that condition (8) becomes $p_{fk} = c_{fj} - c_{fk} + \upsilon_j - \upsilon_k \geqslant 0$, that is condition (3), when we let $\upsilon_i = -\xi_i$ for all $i \in S$ and we denote as j the slot allocated to f under the FPFS policy. □

The following two lemmas are necessary to prove that MM is also budget balanced.

Lemma 1. *For every pair of slot allocation x^F and x^C, respectively defined by the FPFS and by the MINCOST policy, a slot is empty in x^F if and only if it is empty in x^C.*

Proof. By contradiction. Let $\hat{j}$ be the first empty slot in one slot allocation but not in the other one. We initially assume that $\hat{j}$ is empty in x^F, but is allocated in x^C. Let F^F, respectively F^C, be the set of the flights allocated to the slots $1, \ldots, \hat{j}$ in x^F, respectively in x^C. The minimality of $\hat{j}$ guarantees that there exists at least one flight $\hat{f} \in F^C$ which does not belong to F^F. In x^F, flight $\hat{f}$ is allocated to a slot $\hat{l} > \hat{j}$. Define $\hat{x}^F$ such that

$$\hat{x}^F_{fj} = \begin{cases} 0 & \text{if } j = \hat{l} \text{ and } f = \hat{f}, \\ 1 & \text{if } j = \hat{j} \text{ and } f = \hat{f}, \\ x^C_{fj} & \text{otherwise}. \end{cases}$$

That is, $\hat{x}^F$ induces the same slot allocations of x^F except for flight $\hat{f}$ which is allocated to slot $\hat{j}$ instead of $\hat{l}$. The feasibility of x^F and the fact that $\hat{f}$ is allocated to slot $\hat{j}$ in x^C imply the feasibility of $\hat{x}^F$. Also the cost (7a) of x^F is strictly greater than the corresponding cost of $\hat{x}^F$ as $E_{\hat{f}} \leqslant U_{\hat{j}} < I_{\hat{l}}$. Hence, we obtain the contradiction that x^F is not optimal for (7) and cannot be defined by a MINCOST policy. As a consequence, we cannot assume that $\hat{j}$ is empty in x^F, but not in x^C. An analogous argument proves that we cannot assume that $\hat{j}$ is empty in x^C, but is allocated in x^F. Then, we must conclude that $\hat{j}$ cannot exist. □

The above lemma implies that any slot allocated under the FPFS policy is also allocated, possibly to a different flight, under the MINCOST policy. Hence constraints (7b) can be rewritten as:

$$\sum_{f \in F: j \in R^f} x_{fj} = \sum_{f \in F: j \in R^f} x^F_{fj} \quad \forall j \in S. \tag{9}$$

Another important consequence of Lemma 1 is that the asymmetric assignment problem (7) can be decomposed into a set of smaller symmetric assignment problems. To prove this last statement, we need to introduce the following notation. Let i and l be two empty slots in x^F such that $U_i < I_l$. We define as $B^F_{il} \subseteq S$ (respectively $B^C_{il} \subseteq S$) the set of slots allocated in x^F (respectively in x^C) between the empty slots i and l. Let $F^F_{il} \subseteq F$ (respectively $F^C_{il} \subseteq F$) be the corresponding set of flights assigned to the B^F_{il} (respectively B^C_{il}) slots.

Lemma 2. *For every pair of slot allocations x^F and x^C, respectively defined by the FPFS and by the MINCOST policy, if i and l are two empty slots in x^F such that $U_i < I_l$ and no other empty slots exists between i and l, then, a flight is allocated to a slot between i and l in x^F if and only if it is allocated to a slot between the same empty slots i and l also in x^C, that is $B^C_{il} = B^F_{il}$ and $F^C_{il} = F^F_{il}$.*

Proof. From Lemma 1, it follows that each element of B^F_{il} is allocated to a flight also in the solution x^C, i.e., $B^C_{il} = B^F_{il}$. Let $f \in F^F_{il}$ be a flight allocated to slot h in B^F_{il}, i.e., $I_h > U_i$ and $U_h < I_l$. It follows that $E_f > U_i$, otherwise slot i would be occupied by flight f. Hence also in x^C flight f cannot be anticipated to slot i or to any other slot earlier than slot i. Similarly no flight assigned to a slot later than slot l can receive slot h, earlier than l, hence flight f cannot be postponed in x^C to any free slot later than slot l. Thus, if in x^C slot h is allocated to a flight preceding slot i in x^F, it follows that there must be an empty slot in x^C that was not empty in x^F, and this contradicts Lemma 1. Hence, we must have that $F^C_{il} = F^F_{il}$. A symmetric argument holds for the 'only if' part of the statement. □

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The above lemma implies that the optimal MINCOST allocation x^C is achieved by minimizing the cost of delay of each separate subset of consecutive allocated slots between two empty slots given by the FPFS policy. Or, the solution of the asymmetric assignment problem (7) is found by solving a sequence of smaller symmetric assignment problems, one for each set B_{il}^F. A further consequence of Lemma 2 is that MM is budget balanced.

Property 2. MM is budget balanced.

Proof. The statement is an immediate consequence of the fact that $F_{il}^C = F_{il}^F$ for any pair of slots i and l considered in Lemma 2. Indeed, $F_{il}^C = F_{il}^F$ implies that no flight $f \in F_{il}^F$ must sell back its FPFS assigned slot to the ATM authority, nor any flight $f \in F_{il}^F$ must buy an unassigned FPFS slot from the ATM authority. The flights just sell and buy slots to and from each other. □

Another consequence of Lemma 2 is that, from a game theory perspective, we can see MM as a permutation game and then we can derive further characteristics.

A cooperative game with transferable utility, or TU game, is described (see, e.g., Borm et al., 2001) by a pair (F,θ) that is, by the set of players $F = \{1,\ldots,n\}$ and a characteristic function $\theta{:}2^F \to R$, being 2^F the set of subsets (usually referred to as coalitions) of F. In these games side payments between the players are allowed and the characteristic function assigns to every coalition $A \subseteq F$ a value $\theta(A)$, representing the maximal total monetary reward that the members of this group can obtain when they cooperate. By convention, $\theta(\emptyset) = 0$. In this context, a *permutation game* (see, e.g., Curiel and Tijs, 1986) is a TU game describing a situation in which the n players in F all have one job to be processed and one machine on which each job can be processed, but no machine is allowed to process more than one job. If player f processes his job on the machine of player j, the processing costs are a_{fj}. Then, the permutation game characteristic function is defined as

$$\theta(A) = \begin{cases} \sum\limits_{f\in A} a_{ff} - \min\limits_{\pi\in\Pi_A} \sum\limits_{f\in A} a_{f\pi(i)} & \text{if } \emptyset \neq A \subseteq F, \\ 0 & \text{if } A = \emptyset, \end{cases} \tag{10}$$

where Π_A is the class of all A-permutations. Here, the value of $\theta(A)$ denotes the maximal cost savings a coalition A can obtain by processing their jobs according to an optimal schedule compared to the situation in which every player processes his job on his own machine.

We can interpret MM as a permutation game since there is an immediate correspondence between 'machine' and 'slot', between 'job' (or 'player') and 'flight', and between 'utility' and 'allocative efficiency'. Accordingly, a subset of flights forms a coalition A if they limit the slot trading only to the slots assigned by the FPFS policy to the members of the coalition. Hence, imposing $a_{fj} = c_{fj}$ for all $f \in F$ and $j \in S$, the characteristic function $\theta(A)$ is equal to the difference between the cost of delay due to the FPFS allocation and the MINCOST allocation when limited to flights in A. Under this latter hypothesis, condition (5) holds even when its sums are restricted only to the members of the coalition A, then $\theta(A)$ is also equal to the maximum sum of profits that the flights of the coalition can obtain by exchanging their slots.

In a TU game (F,θ) a payoff allocation $p = (p_1, p_2,\ldots,p_n)$ is the vector of the amounts of utility allocated to each player. A payoff allocation is individually rational if $p_f \geqslant \theta(\{f\})$ for all $f \in F$, it is efficient if $\sum_{f\in F} p_f = \theta(F)$. In this context, the imputation set $\mathscr{I}(F,\theta)$ of the game is the set of the payoff allocations that are efficient and individually rational. Finally, the core $\mathscr{C}(F,\theta) \subseteq \mathscr{I}(F,\theta)$ of the game, if it exists, is the set of the payoff allocations that are efficient and coalition rational, that is $\sum_{f\in A} p_f \geqslant \theta(A)$, for all $A \subseteq F$.

Then if we deal with MM as a permutation game, the results in Curiel and Tijs (1986) state that the core of MM exists and that there is a bijective relation between the elements of the core and the optimal solutions of the dual problem of (7), provided that the objective function (7a) is replaced by the equivalent function (6a). In particular, the profits p_f as in Eq. (2) define a core payoff allocation. Given the core characteristics, the next property follows.

Property 3. MM is coalition rational and defines Pareto optimal profits the overall value of which is equal to $\sum_{f\in F} p_f = \sum_{f\in F}\sum_{k\in R^f} c_{fk}(x_{fk}^F - x_{fk}^C)$.

The coalition rationality implies that, under the MM policy, no coalition $A \subseteq F$ of flights has an incentive to part company with the remaining flights of F and establish cooperation on its own.

In general, the dual of problem (7) may present multiple solutions and then multiple slot prices v_j, and hence profits p_f, can be determined. If the ATM authority centrally solves problem (7), it could select particular values of v_j to make the payoff allocations/profits enjoy further properties such as being pairwise-monotonic (Miquel, 2009) to try to obtain non-null profits for all flights.

It is important to note that there are some situations where an airline may find convenient to misrepresent the true cost of a flight delay to increase its payoff, at the expense of other flights, as shown in Appendix A. This is not surprising due to the impossibility theorem of Myerson and Satterthwaite (1983), since we require individual rationality and budget balance constraints to hold.

4. Distributed market-based mechanisms

The practical implementation of the MINCOST policy requires from airlines, for each flight f, to communicate to the ATM authority complete set of values c_{fj} for each slot $j \in R^f$, in order to determine the optimal allocation and the related payments.

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The optimal exchange is then calculated centrally according to this information, thus implementing a single-round, sealed-bid type of exchange (de Vries and Vohra, 2003). This type of mechanism suffers from (a) high computational effort for airlines for calculating the complete sets of requests and the associated values, (b) high communication cost of sending the complete set of values over a communication network, (c) complete disclosure of airlines' confidential information, which might not be well-received in a highly competitive environment as commercial aviation (Martin et al., 1998), and (d) the lack of dynamism, since all bids from participants must be communicated before a deadline. To overcome these drawbacks, we propose a distributed market mechanism to reallocate the slots in accordance with the MINCOST policy. This mechanism directly involves each airline in the decision process of the slot reallocation and does not require the disclosure of the delay costs. As a positive side effect, it also puts the cost misrepresentation under a different perspective. When a centralized market mechanism as MM is implemented, an airline misrepresenting its costs is actually lying to an authority that pursues a common objective and then should be possibly sanctioned. On the contrary, when a distributed market mechanism is implemented, the same cost misrepresentation can be seen as a part of a bargaining process where each airline pursues its own interest. In the following, we present two approaches for a distributed implementation of MM.

4.1. Lagrangian Relaxation

A classical dual decomposition can be used to transform the centralized allocation problem into a distributed market mechanism. In fact, by dualizing constraints (9) the corresponding Lagrangian formulation of problem (7) is:

$$\max_{\lambda}\min_{x} \quad \sum_{f\in F}\sum_{j\in R^f} c_{fj}x_{fj} + \sum_{j\in S}\lambda_j\left(\sum_{f\in F:E_f\leqslant U_j}\left(x_{fj}-x^F_{fj}\right)\right), \tag{11a}$$

$$\sum_{j\in R^f} x_{fj} = 1 \quad \forall f, \tag{11b}$$

$$x_{fj} \geqslant 0 \quad \forall f\in F,\ j\in R^f, \tag{11c}$$

$$\lambda_j \geqslant 0 \quad \forall j\in S. \tag{11d}$$

As problem (7) is a continuous linear problem, the optimal Lagrangian multipliers λ^*_j in problem (11) are equal to the optimal dual variables υ^*_j associated to constraints (9), i.e., $\lambda^*_j = \upsilon^*_j\ \forall j\in S$. Hence when the optimal Lagrangian multipliers are identified, the optimal value of function (11a) is equal to the optimal value of function (7a) due to the complementary slackness conditions, and also all constraints (9 or 7b), (7c) and (7d) are satisfied. Then the optimal solution to problem (11) simultaneously allocates slots in accordance with the MINCOST policy and computes all the optimal slot values υ^*_j. We further notice that the objective function (11a) is separable into F functions, one for each flight, and that constraints (11b) and (11c) are also separable in terms of flights, thus allowing the decomposition of problem (11) into F subproblems. We now exploit the above properties to show how to implement the market mechanism without requiring from the airlines to disclose flights' delay costs to the ATM authority.

The following iterative procedure, referred to as LR algorithm, determines an optimal solution for the problem (11). Given some tentative values λ_j, for all the slots $j\in S$, to each flight f a solution of its own subproblem is assigned:

$$x^*_f = \arg\min_{x} \quad \sum_{j\in R^f} c_{fj}x_{fj} + \sum_{j\in R^f}\lambda_j\left(x_{fj}-x^F_{fj}\right), \tag{12a}$$

$$\sum_{j\in R^f} x_{fj} = 1, \tag{12b}$$

$$x_{fj} \geqslant 0 \quad \forall j\in R^f. \tag{12c}$$

Then, for each flight f, the airline communicates to the ATM authority its slot request $x^*_f = \{x^*_{fj},\ \forall j\in R^f\}$ and the ATM authority checks for compatibility of the set of requests $x^* = \{x^*_f,\ \forall f\in F\}$ with a feasible slot allocation, where at most one flight is allocated to each slot (constraints (7b)). If no feasible solution is found, the ATM authority updates the slot tentative values λ_j. In particular, at each iteration the ATM authority sets slot prices by increasing the value obtained at the previous iteration for an over-demanded slot and by decreasing it for an unused one, always assigning a positive value, according to a subgradient algorithm (see, e.g., Bertsekas, 1999). If constraints (7b) on slot capacity are respected, slots are allocated according to the MINCOST policy. Otherwise the current prices are communicated to airlines, which in turn solve again local optimization model (12) for each flight and answer with a new slot request $x^*_f = \left\{x^*_{fj},\ \forall j\in R^f\right\}$ for all $f\in F$. If a feasible solution cannot be achieved after a pre-defined number of iterations, then slots are allocated according to the FPFS policy. However, our computational experiments based on real instances easily converge, as described in the next Section 5. Note that this procedure allows the ATM authority to allocate the slots without the delay costs. In fact, the knowledge of the delay costs is needed only for the solution of the problem (12) that is solved by an airline for each flight.

4.2. Bertsekas' Auction algorithm

An alternative approach to implement the MINCOST policy in a distributed way is the Bertsekas' Auction algorithm for the Assignment Problem (see, e.g., Bertsekas, 1990), referred to in the following text as BA algorithm. The BA algorithm

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implements an iterative ascending type of auction, which employs both bid and ask prices since bidders can increase slot prices proposed by the ATM authority, according to their valuation functions. Here we sketch the main principles of this algorithm as described in Bertsekas (1991). The algorithm starts with an empty assignment, i.e., all flights are unallocated, proceeds iteratively and terminates when a feasible assignment is reached. At each iteration the ATM authority proposes to one unallocated flight f the current slot prices b_j for all $j \in S$. Then f announces the slot j_f that minimizes its cost function, i.e.,

$$j_f = \arg\min_{j \in R^f} \{c_{fj} + b_j\},$$

and proposes for this slot a higher bid price $\hat{b}_{j_f} = b_{j_f} + \gamma_i$ where $\gamma_i \geqslant 0$ is the cost caused by being allocated the second best slot rather than j_f, i.e., $\gamma_i = \min_{j \in R^f : j \neq j_f} \{c_{fj} + b_j\} - \min_{j \in R^f} \{c_{fj} + b_j\}$. The price of each slot is set equal to the highest bid and it is allocated to the corresponding bidder. If another flight was already allocated to this slot at the beginning of the iteration, it becomes again unallocated. This procedure iteratively continues until all flights $f \in F$ have a slot allocated. When this condition holds, it has been proven (Bertsekas, 1991) that this allocation is also optimal for problem (7) and that the associated prices b_j^* are equal to the optimal dual solutions v_j^* for all $j \in S$.

The BA algorithm allows to simultaneously allocate slots according to the MINCOST policy and to compute the optimal slot values v_j^* in a distributed way as was the case for the LR algorithm. In addition, the BA algorithm gives airlines a greater degree of control of the course of the auction, as it provides the possibility for airlines to actively set slot prices through their bids. However, if on one hand, the BA algorithm allows for a price discovery more actively driven by users, on the other, it requires that airlines are able to calculate their γ_i values. In fact, for each flight, an airline calculates internally the bid price relying on the current slot values and on its own cost of delay. Then from all unallocated flights the final bid prices only are communicated to the ATM authority. In turn, the ATM authority determines the new slot values relying only on these bid prices, and transfers them back to the remaining unallocated flights. Differently, no internal computations are required in the LR setting.

5. Examples of application

In this section, we consider two case studies based on real operational data with the aim of analyzing the performances of the LR and the BA algorithms introduced in the previous section. Both algorithms take advantage of the fact that the asymmetric assignment problem (7) can be decomposed into a set of smaller symmetric assignment problems as shown in Lemma 2.

In the case of the BA algorithm, we solved the real instances presented next using the ϵ-Scaling Forward version for a symmetric assignment problem which allows avoiding degeneracy and increases the computational performance of the basic Auction Algorithm (see Section 4.1 in Bertsekas, 1991). Degeneracy occurs when a flight f is indifferent in exchanging its FPFS slot as this exchange gives no profit to it, i.e., $p_f = 0$. In such a situation, to avoid cycling, the BA algorithm sets the profit p_f equal to $-\epsilon$ with $0 < \epsilon < \frac{1}{|B_{jk}^F|}$ (see Section 1.2.1 in Bertsekas, 1991).

For sake of simplicity and without loss of generality, we assume here that delay costs c_{fj} are proportional to the amount of delay, i.e., $c_{fj} = w_f * d_{fj}$ for all $f \in F$ and $j \in R^f$, where $w_f > 0$ is the cost of 1 min of delay. In particular, in both cases we assume that, for each flight f, the tactical cost of one minute of ground delay w_f is a stochastic variable uniformly distributed between €5 and €20 per minute, in line with the figures provided in Cook et al. (2004).

5.1. Case A: en-route regulation

This case is representative of the situation in which several flights are affected by the same and unique regulation limiting the maximum capacity of an en-route sector. This was the case, for instance, on the 2nd August 2008 between 04:00 AM and 06:00 AM, when a regulation set the number of flights entering the French sector LFEERESMI to 14 per hour, for ATC capacity reasons. There were 18 flights originally planned to enter this sector at different points in time between 04:18 AM and 05:51 AM. Given the baseline schedule obtained by applying the FPFS policy, we simulated the two distributed market mechanisms: by tuning the appropriate parameters the LR algorithm converges to an optimal solution after 25 iterations, and the BA algorithm after 69 iterations.

Under the FPFS policy, the overall delay is 91 min and the overall cost of delay is €1175. The MINCOST policy provides a lower cost of delay (€736), but a slightly higher overall delay (93 min).

Table 1 shows the different slots and their associated values for both algorithms. The slots in boldface are the ones allocated to a flight under both the FPFS and the MINCOST policies. Table 2 displays the main information associated with each flight f. The second and the third column report the cost of the delay w_f and the estimated time of entry E_f into sector LFEERESMI respectively. The fourth and the fifth column respectively show the optimal slot allocations x^F and x^C along with the exact time of entry into the sector. Finally, the sixth and the seventh column respectively present the profit obtained by each flight using the LR and the BA algorithm. We notice that the profit of an individual flight may change depending on the algorithm, as the problem (7) may have multiple optimal dual solutions. However, the sum of all the profits is constant (€439 = €1175 − €736) as shown by Eq. (5). In Table 2 the flights in boldface are the ones that do

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Table 1
Slot table – Case A (LFEERESMI).

Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}	Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}
S1	04:00–04:03	0	0	**S15**	**05:00–05:03**	222.78	392.52
S2	04:04–04:07	0	0	**S16**	**05:04–05:07**	168.32	326.87
S3	04:08–04:11	0	0	**S17**	**05:08–05:11**	126.49	271.47
S4	04:12–04:16	0	0	**S18**	**05:12–05:16**	90.20	234.47
S5	**04:17–04:20**	0	105.93	**S19**	**05:17–05:20**	45.28	194.07
S6	**04:21–04:24**	0	74.10	**S20**	**05:21–05:24**	21.11	170.15
S7	**04:25–04:29**	57.96	64.77	**S21**	**05:25–05:29**	0	159.23
S8	**04:30–04:33**	33.71	24.93	S22	05:30–05:33	0	0
S9	**04:34–04:37**	11.00	0.77	**S23**	**05:34–05:37**	0	0
S10	04:38–04:41	0	0	S24	05:38–05:41	0	0
S11	**04:42–04:46**	338.21	554.47	S25	05:42–05:46	0	0
S12	**04:47–04:50**	336.11	544.03	S26	05:47–05:50	0	0
S13	**04:51–04:54**	308.40	467.95	**S27**	**05:51–05:54**	0	0
S14	**04:55–04:59**	276.87	435.87	S28	05:55–05:59	0	0

Table 2
Flight table – Case A (LFEERESMI).

Flight f	w_f	E_f	FPFS policy slot (time)	MINCOST policy slot (time)	p_f^{LR}	p_f^{BA}
F1	16	04:18	S5 (04:18)	S5 (04:18)	0	0
F2	17	04:24	S6 (04:24)	S6 (04:24)	0	0
F3	8	04:25	S7 (04:25)	S7 (04:25)	0	0
F4	6	04:26	S8 (04:30)	S8 (04:30)	0	0
F5	6	04:36	S9 (04:36)	S9 (04:36)	0	0
F6	14	04:44	S11 (04:44)	S11 (04:44)	0	0
F7	9	04:45	S12 (04:47)	S18 (05:12)	20.91	84.56
F8	6	04:46	S13 (04:51)	S20 (05:21)	107.29	117.80
F9	19	04:47	S14 (04:55)	S12 (04:47)	92.76	43.84
F10	10	04:48	S15 (05:00)	S17 (05:08)	16.29	41.05
F11	16	04:53	S16 (05:04)	S13 (04:53)	35.92	34.92
F12	13	04:54	S17 (05:08)	S14 (04:55)	18.62	4.60
F13	17	05:00	S18 (05:12)	S15 (05:00)	71.42	45.95
F14	15	05:04	S19 (05:17)	S16 (05:04)	71.96	62.20
F15	7	05:12	S20 (05:21)	S19 (05:17)	3.83	4.08
F16	11	05:24	S21 (05:25)	S21 (05:25)	0	0
F17	18	05:37	S23 (05:37)	S23 (05:37)	0	0
F18	8	05:51	S27 (05:51)	S27 (05:51)	0	0

not exchange the slots allocated under the FPFS policy. In particular, holding Lemma 2, flights 5, 17 and 18 cannot move from their FPFS slots.

5.2. Case B: arrival airport regulation

This case is representative of the situation in which there is a limit on the number of flights arriving at an airport in a certain time period, i.e. it is an instance of the SAGHP. We found that London City Airport (EGLC) imposed a regulation on the 4th August 2008, limiting the arrival rate to 18 flights per hour, between 06:00 AM and 07:30 AM. This regulation was the only regulation affecting 24 flights. In this case, the LR algorithm converges to an optimal solution after 39 iterations, and the BA algorithm after 38 iterations.

Under the FPFS policy, the overall delay is 73 min and the overall cost of delay is €957. The MINCOST policy provides a lower cost of delay (€631), but a slightly higher overall delay (77 min).

Table 3 shows different slots and their associated values for both algorithms. The slots in boldface are the ones allocated to a flight under both the FPFS and the MINCOST policies. In Table 4 the second and the third column respectively report the cost of the delay w_f and the estimated time of entry E_f at EGCL. The fourth and the fifth column respectively show the optimal slot allocations x^F and x^C along with the exact time of entry. Finally, the sixth and the seventh column respectively present the profit obtained by each flight using the LR and the BA algorithm. Clearly, the sum of all the profits is constant (€326 = €957 − €631) as shown by Eq. (5). In Table 4 the flights in boldface are the ones that do not exchange the slot allocated to them under the FPFS policy. In particular, holding Lemma 2, flights 15 and 24 cannot move from their FPFS slots. Finally, this example shows a case where degeneracy occurs. According to the slot values computed with the BA algorithm, flight F3 does not obtain any economic profit by moving from slot S3 to S4. The resulting negative profit $p_3 = -0.06$ is due to the introduction of the the perturbation constant $\epsilon > 0$ to prevent cycling, and should be interpreted as zero, i.e., $p_3 = 0$.

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Table 3
Slot table – Case B (EGCL).

Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}	Slot j	$[I_j, U_j]$	v_j^{LR}	v_j^{BA}
S1	**06:00–06:02**	0	437.41	**S15**	**06:46–06:49**	27.78	125.51
S2	**06:03–06:05**	0	448.23	S16	06:50–06:52	0	0
S3	**06:06–06:09**	311.83	428.33	**S17**	**06:53–06:55**	133.04	34.56
S4	**06:10–06:12**	284.36	410.39	**S18**	**06:56–06:59**	116.96	20.67
S5	**06:13–06:15**	264.71	422.35	**S19**	**07:00–07:02**	105.26	46.56
S6	**06:16–06:19**	276.66	428.25	**S20**	**07:03–07:05**	73.31	16.67
S7	**06:20–06:22**	259.94	393.41	**S21**	**07:06–07:09**	105.23	209.56
S8	**06:23–06:25**	239.69	366.41	**S22**	**07:10–07:12**	88.46	200.67
S9	**06:26–06:29**	200.50	313.41	**S23**	**07:13–07:15**	71.61	187.78
S10	**06:30–06:32**	157.10	273.48	**S24**	**07:16–07:19**	46.60	173.89
S11	**06:33–06:35**	126.57	239.41	S25	07:20–07:22	0	0
S12	**06:36–06:39**	90.80	204.41	**S26**	**07:23–07:25**	0	0
S13	**06:40–06:42**	60.11	164.51	S27	07:26–07:29	0	0
S14	**06:43–06:45**	31.50	143.58	–	–	–	–

Table 4
Flight table – Case B (EGCL).

Flight f	w_f	E_f	FPFS policy slot (time)	MINCOST policy slot (time)	p_f^{LR}	p_f^{BA}
F1	7	06:01	S1 (06:01)	S1 (06:01)	0	0
F2	10	06:03	S2 (06:03)	S2 (06:03)	0	0
F3	10	06:08	S3 (06:08)	S4 (06:10)	9.47	−0.06
F4	7	06:08	S4 (06:10)	S13 (06:40)	14.25	35.88
F5	14	06:08	S5 (06:13)	S3 (06:08)	22.88	64.02
F6	16	06:15	S6 (06:16)	S5 (06:15)	27.95	21.90
F7	19	06:18	S7 (06:20)	S6 (06:18)	21.28	3.16
F8	14	06:19	S8 (06:23)	S7 (06:20)	21.75	15.00
F9	6	06:21	S9 (06:26)	S14 (06:43)	67.00	67.83
F10	19	06:22	S10 (06:30)	S8 (06:23)	50.41	40.07
F11	11	06:22	S11 (06:33)	S9 (06:26)	3.07	3.00
F12	20	06:28	S12 (06:36)	S10 (06:30)	53.70	50.93
F13	10	06:30	S13 (06:40)	S12 (06:36)	9.31	0.10
F14	12	06:33	S14 (06:43)	S11 (06:33)	24.93	24.17
F15	15	06:46	S15 (06:46)	S15 (06:46)	0	0
F16	18	06:55	S17 (06:55)	S17 (06:55)	0	0
F17	14	06:55	S18 (06:56)	S18 (06:56)	0	0
F18	11	07:00	S19 (07:00)	S19 (07:00)	0	0
F19	10	07:00	S20 (07:03)	S20 (07:03)	0	0
F20	17	07:09	S21 (07:09)	S21 (07:09)	0	0
F21	9	07:09	S22 (07:10)	S22 (07:10)	0	0
F22	13	07:12	S23 (07:13)	S23 (07:13)	0	0
F23	14	07:15	S24 (07:16)	S24 (07:16)	0	0
F24	8	07:23	S26 (07:23)	S26 (07:23)	0	0

6. Conclusions

This paper deals with the slot allocation problem for a single constrained Air Traffic Network resource, e.g., an en-route sector or an airport. The market mechanism proposed shows several characteristics that make it preferable to the First Planned First Served policy, currently adopted in Europe to sequence flights whenever an imbalance between planned traffic and resource capacity occurs on the day of operations. First, it produces a slot allocation that makes every flight economically better off. High-valued flights can reduce their delays by acquiring resources while low-priority flights may be reimbursed for the resources they relinquish. Second, the ATM authority is neutral with respect to the final outcome of the slot exchanges as it is not economically involved. In fact, the mechanism neither requires an external subsidization to work, nor produces an economical surplus to be distributed outside the set of participants. Third, under the above conditions the allocative efficiency is maximized both from the social welfare and market players' perspectives. Finally, the mechanism is distributed as it actively involves airlines in the slot allocation process. They express their preferences over different feasible solutions to the slot allocation problem in the spirit of Collaborative Decision Making, as advocated by the SESAR operational concept.

This research may be extended to the definition of a market-based policy for slot trading when flights need to use multiple regulated resources. The use of combinatorial exchanges seems to be a promising approach to deal with this complex problem.

Appendix A. Discussion on incentive compatibility

In the following we provide a simple example where an airline may find convenient to misrepresent the true cost of a flight delay to increase its payoff, at the expense of other flights.

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Consider two flights, f_1 and f_2, and two slots, a and b. Both flights request slot a. Then we set $c_{1a} = c_{2a} = 0$. We assume that $0 < c_{1b} < c_{2b}$ and that $E_1 < E_2$. The FPFS policy allocates flight f_1 at slot a and flight f_2 at slot b. Let p_1 and p_2 be the profits of flight f_1 and f_2, respectively, obtained by selling their FPFS slot and purchasing the other slot. We finally assume that the airlines do not know the costs of delay for the involved flights and that the MINCOST policy is centrally implemented by the ATM authority through problem (7). We want to investigate the opportunity for an airline that operates flight f_1 to cheat about the true value of its delay to get a higher profit. Let c'_{1b} be the false value of the delay cost displayed by flight f_1 for slot b, and v'_a, v'_b and p'_1 be the corresponding modified slot prices and profit for flight f_1, respectively.

Flight f_1 (f_2) has a non-negative profit in selling its slot a (b) and purchasing the other slot b (a) at prices v_a (v_b) and v_b (v_a), respectively. In particular, $p_1 = c_{1a} - c_{1b} + v_a - v_b$ and $p_2 = c_{2b} - c_{2a} + v_b - v_a$, where v_a and v_b are the optimal solutions of the following problem, the dual of problem (7):

$$\begin{aligned}
&\max Z = m_1 + m_2 - v_a - v_b,\\
&m_1 - v_a \leqslant 0,\\
&m_1 - v_b \leqslant c_{1b},\\
&m_2 - v_a \leqslant 0,\\
&m_2 - v_b \leqslant c_{2b},\\
&m_1, m_2, v_a, v_b \geqslant 0.
\end{aligned} \tag{A.1}$$

In the (v_a,v_b) space the optimal region is $c_{1b} \leqslant v_a - v_b \leqslant c_{2b}$. This optimal region has only two finite vertices, i.e., ($v_a = c_{1b}, v_b = 0$) and ($v_a = c_{2b}, v_b = 0$). Using a standard algorithm, as the simplex or the dual simplex algorithm, to solve problem (A.1), the optimal solution is always point ($v_a = c_{1b}, v_b = 0$). Hence the profit for flight f_1 obtained by selling its slot a at price $v_a = c_{1b}$ and purchasing the slot b at price $v_b = 0$ is $p_1 = 0 - c_{1b} + c_{1b} - 0 = 0$. As long as $c'_{1b} \leqslant c_{2b}$ the slot allocation remains the same. Hence the optimal region is $c'_{1b} \leqslant v'_a - v'_b \leqslant c_{2b}$. Then the optimal solution obtained by the simplex algorithm is $(v'_a = c'_{1b}, v'_b = 0)$. Hence the modified profit is $p'_1 = c_{1a} - c_{1b} + v'_a - v'_b = 0 - c_{1b} + c'_{1b} - 0$. Then $p'_1 - p_1 = c'_{1b} - c_{1b}$. Then if $0 \leqslant c'_{1b} < c_{1b}$ it follows that $p'_1 - p_1 < 0$, and if $c_{1b} < c'_{1b} < c_{2b}$ we have $p'_1 - p_1 > 0$. When $c'_{1b} > c_{2b}$, the slot allocation changes and becomes identical to the FPFS allocation. Hence in this case $p'_1 = 0$.

As the value c_{2b} is not known to flight f_1, we conclude that the flight taking the first slot under the FPFS allocation knows that it does not have to display a false delay cost lower that the true one because this choice may lead to a profit p'_1 lower than the true profit p_1. On the other hand, if this flight f_1 communicates to the ATM authority a false delay cost c'_{1b} higher than the true one, the profit p'_1 it gets is higher than or equal to the true profit p_1. Then our mechanism is not incentive compatible because there is no disadvantage for the first flight in the FPFS allocation for misrepresenting its delay costs.

We emphasize that these findings assume that for the involved flights airlines know in advance what is the vertex of the optimal region of problem (A.1) chosen by the solving algorithm. On the contrary, when we consider as slot prices a generic pair of optimal values (v_A, v_B) the corresponding profit p_1 can be strictly positive, as MM is by construction individual rational. In this situation, it can be risky for flight f_1 to cheat about delay cost c_{1b}. In fact, if it sets its false value c'_{1b} strictly larger than c_{2b} its profit p'_1 is equal to 0. Since c_{2b} is unknown to flight f_1, a misrepresentation of its cost of delay may produce a profit p'_1 lower than the true p_1.

We conclude that when we do not know in advance which are the optimal slot values, we are unable to identify up to which limit a false value of the delay cost does not lead to a profit $p'_1 < p_1$.

Appendix B. Table of abbreviations

See Table B.1.

Table B.1
Table of abbreviations.

ATFM	Air Traffic Flow Management
ATM	Air Traffic Management
BA	Bertsekas' Auction
CASA	Computer Assisted Slot Allocation
CDM	Collaborative Decision Making
CFMU	Central Flow Management Unit
EGLC	London City Airport
ETO	Estimated Time Over
FPFS	First Planned First Served
GHP	Ground Holding Problem
LR	Lagrangian Relaxation
MM	market mechanism
SAGHP	Single Airport Ground Holding Problem
SESAR	Single European Sky ATM Research
TU	transferable utility
UDPP	User Driven Prioritization Process

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