Special Triangles in 3-Colorings of Complete Graphs

Andy Cui

Montgomery Blair High School
51 University Blvd E
Silver Spring, MD 20901, USA

William Gasarch

Department of Computer Science
University of Maryland
College Park, MD 20742, USA

Abstract. Ramsey theory is a combinatorial field of mathematics that investigates how graphs contain ordered substructures. We study triangles in 3-edge-colorings of complete graphs K_n, introducing the notion of a *special triangle*: a triangle that is either rainbow (all three edge colors distinct) or monochromatic (all edges the same color). Although both rainbow and monochromatic configurations appear in classical Ramsey-type questions, their combined behavior has not been previously studied. Using a Java-based computational counting algorithm, we generate and evaluate 3-colorings of graphs K_n and measure the occurrences of special triangles as n grows. We show that K_9 is the smallest graph for which every 3-coloring contains at least two special triangles, and we provide computational data for K_n up to $n = 13$.

Keywords: Ramsey theory; 3-edge-coloring; complete graph; rainbow triangle; monochromatic triangle; anti-Ramsey theory; computational combinatorics.