Software Model Checking

Nicholas Kidd, nkidd@purdue.edu
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Resources Used

- Edmund Clarke's *course*
    - And his *Turing Award Lecture*
- Online tutorial papers/slides:
    - Clarke, Merz, Schmidt, Muller-Olm, Visser, Rushby, Jhala,...
- Others slides from Akash Lal

Outline

- Model checking in 20 minutes
- Software Model Checking
  ... what's the big deal

Temporal Logic Model Checking
[Clarke]

- Model checking is an *automatic verification technique* for finite-state concurrent systems
- Developed independently by Clarke and Emerson and by Queille and Sifakis in early 80's
- Specifications are written in *propositional temporal logic*
- Verification procedure is an *exhaustive search* of the state space

Model Checking of Hardware Systems

- By definition a *finite* system
- Verification successes
  - Cache-coherence protocols
- Important bugs
  - *Intel Pentium FDIV bug* ($500 million)
  - See [Clarke] for more

Advantages of Model Checking

[Clarke]

- No proofs
- Fast
- Counterexamples
- Partial specifications OK
- Logics can express *concurrency properties*

Mutual Exclusion

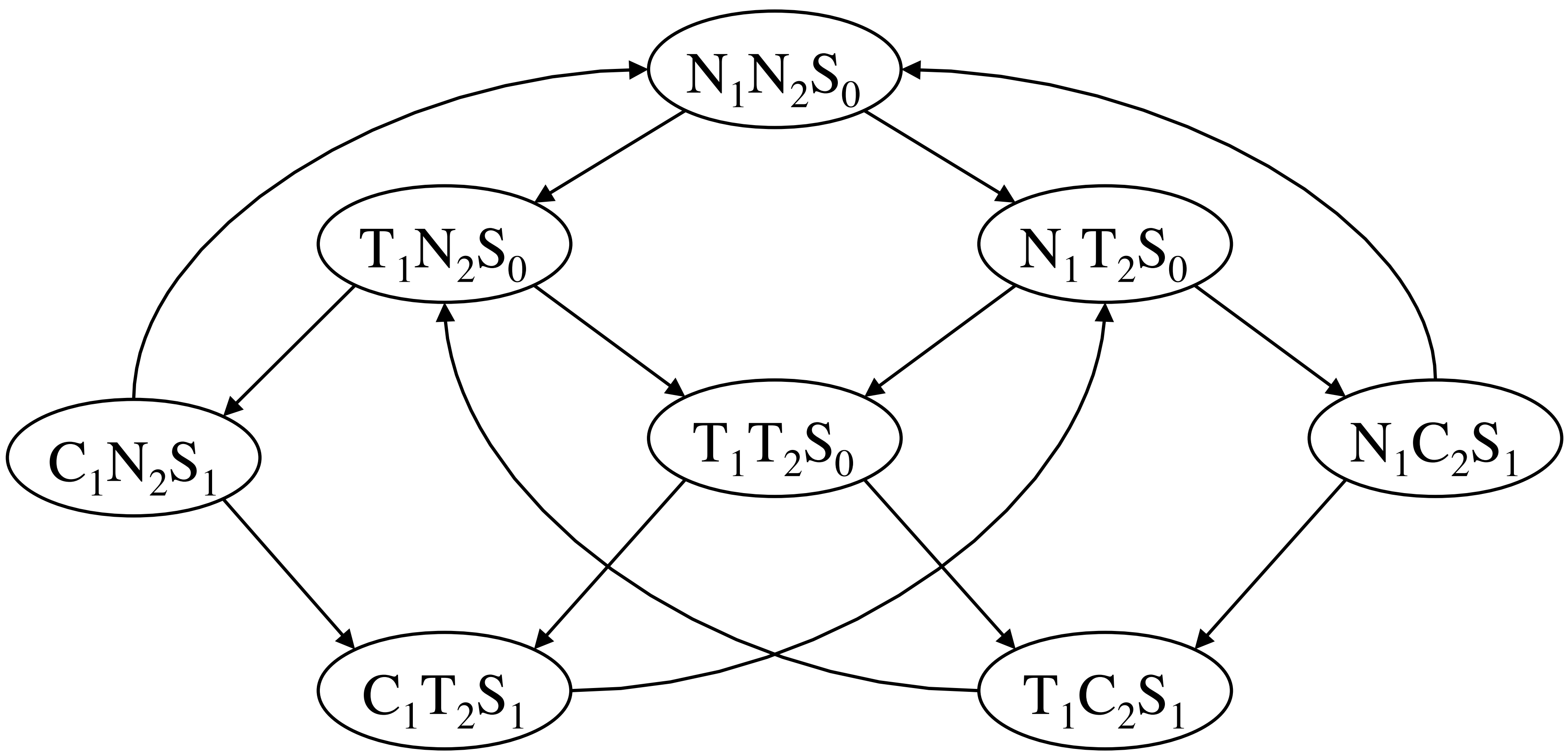
[Visser, ASE 2000]

- Two process mutual exclusion
- Each process has 3 states:
  - Non-critical (N), Trying (T), Critical (C)
- Semaphore is free (S_0) or taken (S_1)

$$
\begin{array}{lcl}
N_1 & \rightarrow & T_1 \\
T_1 \,\&\, S_0 & \rightarrow & C_1 \& S_1 \\
C_1 & \rightarrow & N_1 \& S_0
\end{array}
\quad || \quad
\begin{array}{lcl}
N_2 & \rightarrow & T_2 \\
T_2 \,\&\, S_0 & \rightarrow & C_2 \& S_1 \\
C_2 & \rightarrow & N_2 \& S_0
\end{array}
$$

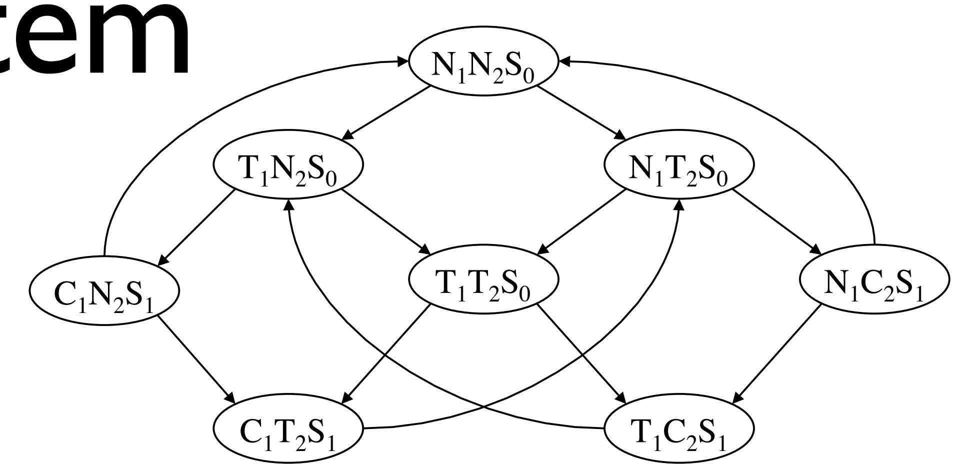
Mutual Exclusion

[Visser, ASE 2000]



Finite-State System

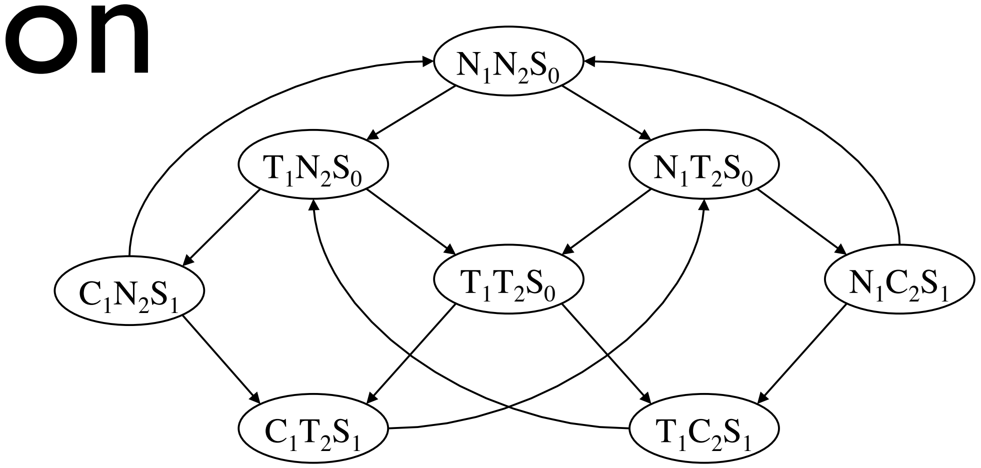
[Clarke]



- State-transition graph (*Kripke* structure)
  - Set of states S
  - Set of atomic propositions AP
  - Labeling function $L : S \to 2^{AP}$
  - Transition relation $R \subseteq S \times S$

Mutual Exclusion

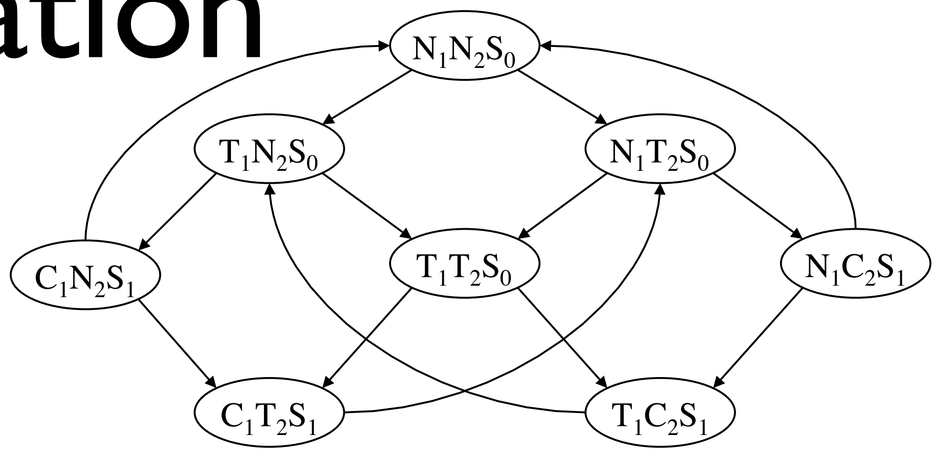
[Visser, ASE 2000]



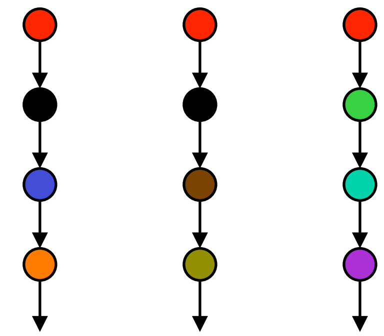
- *Goal*: Show that from any state, it is always possible to get to the initial state
- Need way to specify property of *execution traces*!
  - *Liveness* -- Something good eventually happens (infinitely often)
  - Safety -- Something bad does not happen

Property Specification

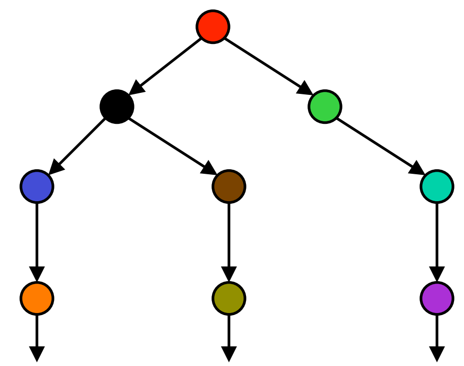
[Visser, ASE 2000]



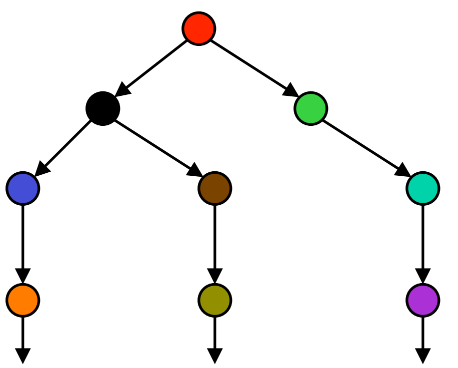
- *Temporal Logic*: Express properties of event orderings in time
- Linear Time (LTL)
  - Every moment has a unique successor
  - Infinite sequences

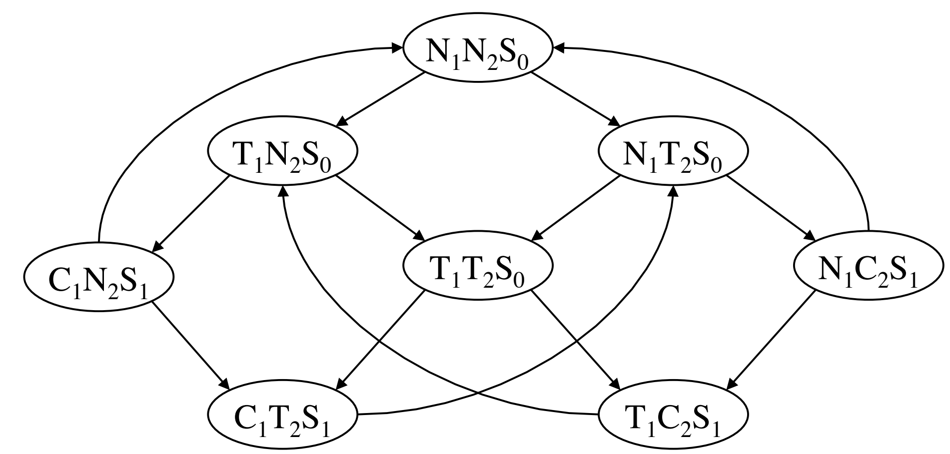


- Branching Time (CTL)
  - Every moment has >= 1 successors
  - Infinite tree



CTL
[Clark]

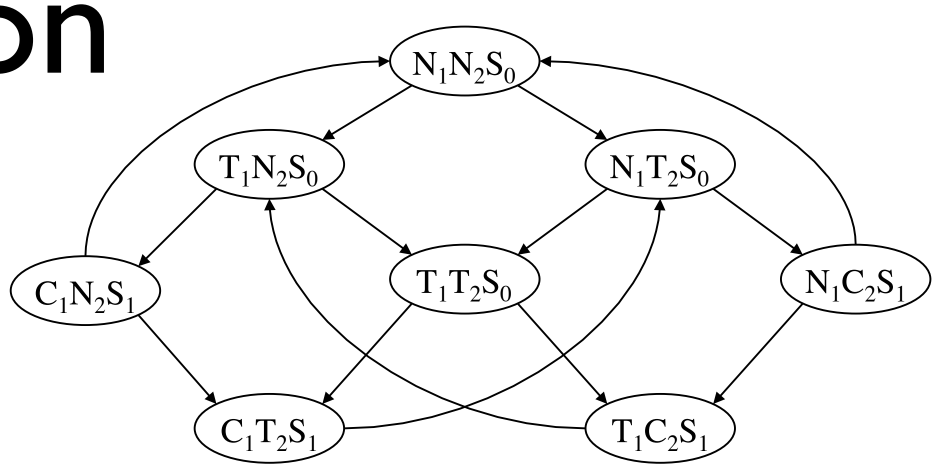




- Formula built from *path quantifiers* and *temporal operators*
- *Path quantifier*
  - A ≡ "On *All* paths" (all my children)
  - E ≡ "*Exists* some path" (>= 1 of my children)
- Temporal Operator
  - (X|F|G) p ≡ p holds (*neXt* | *Globally* | *Future*)

Mutual Exclusion

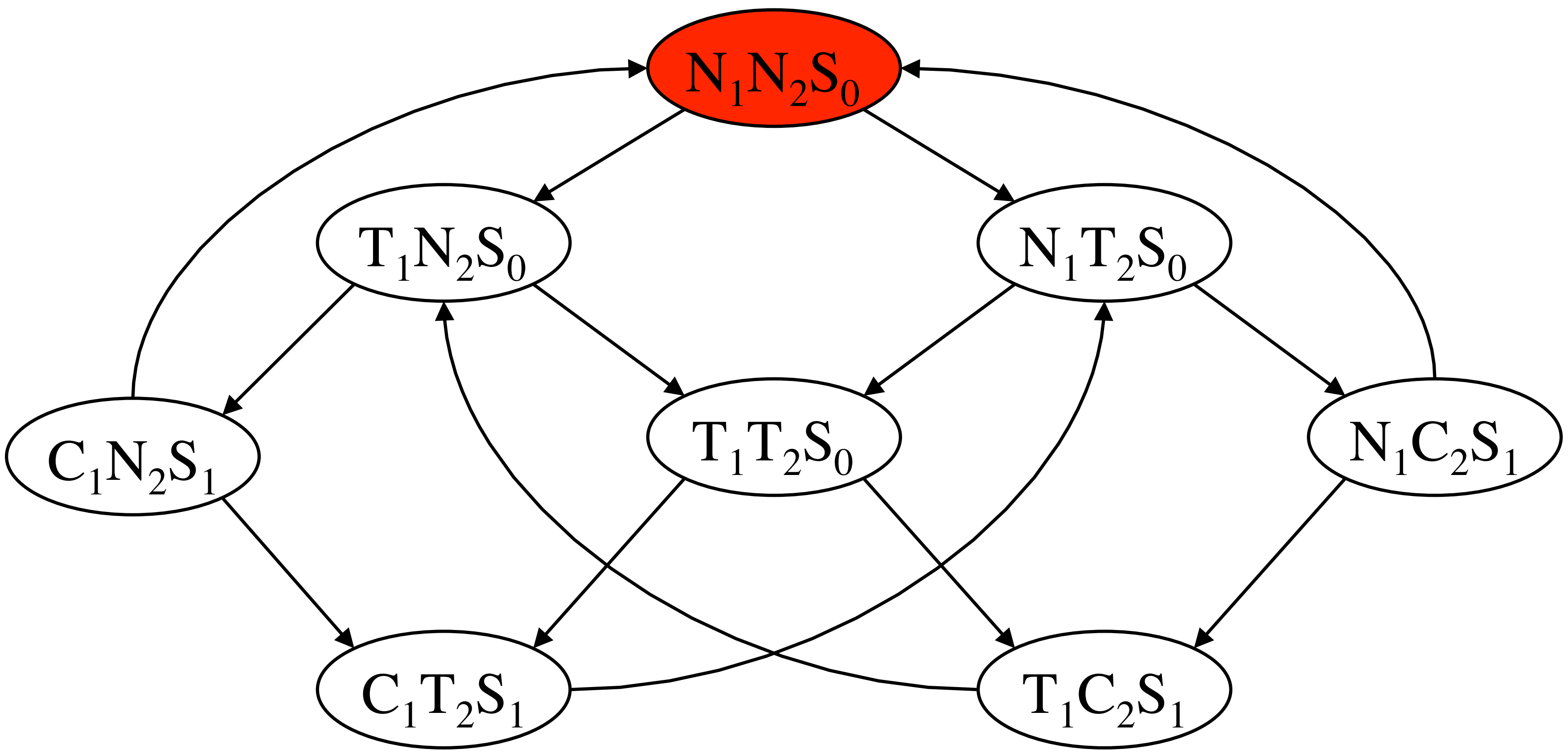
[Visser, ASE 2000]



- *Goal*: Show that from any state, it is always possible to get to the initial state
- AG EF (N_0 & N_1 & S_0)

Mutual Exclusion

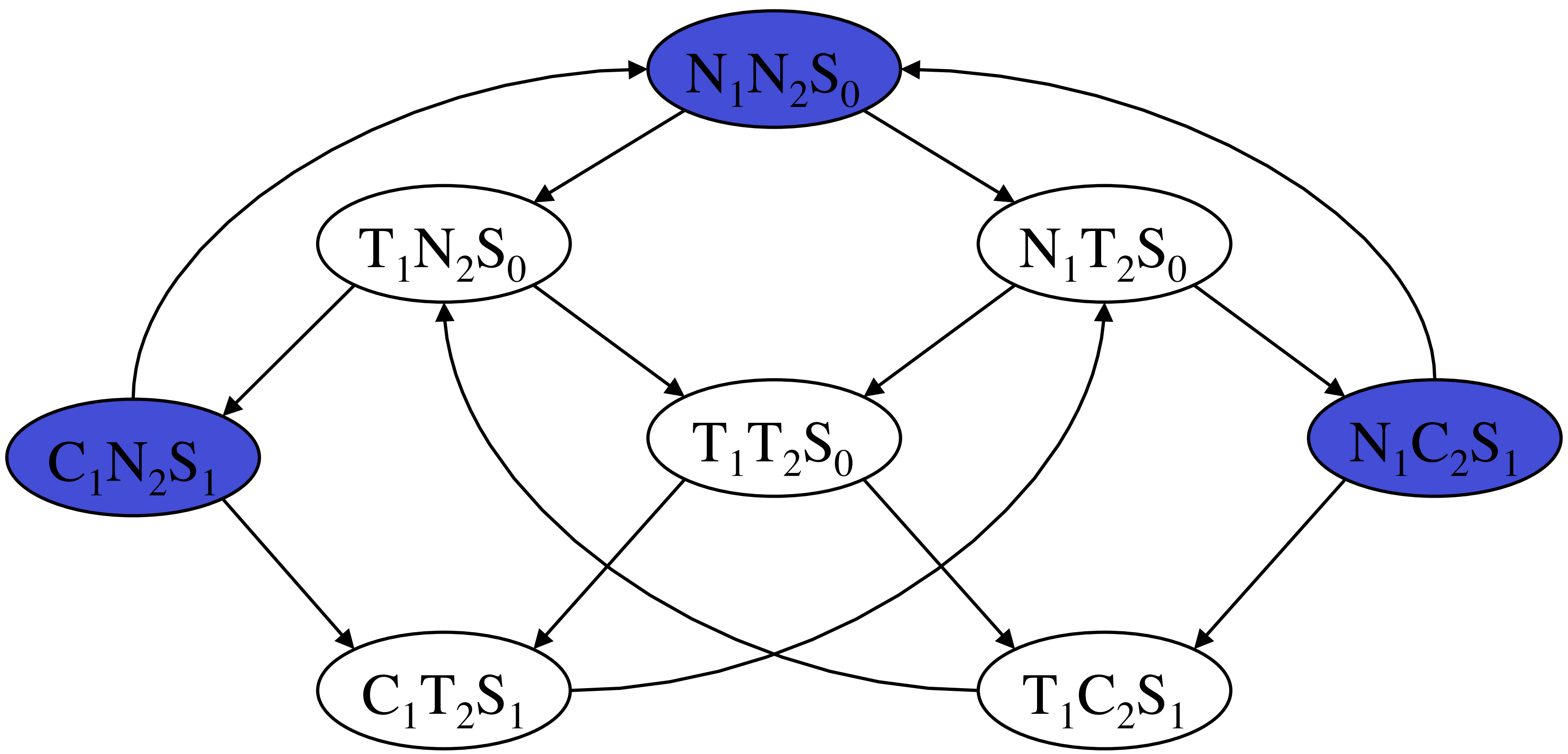
[Visser, ASE 2000]



$$K \models AG\ EF\ (N_1 \wedge N_2 \wedge S_0)$$

Mutual Exclusion

[Visser, ASE 2000]



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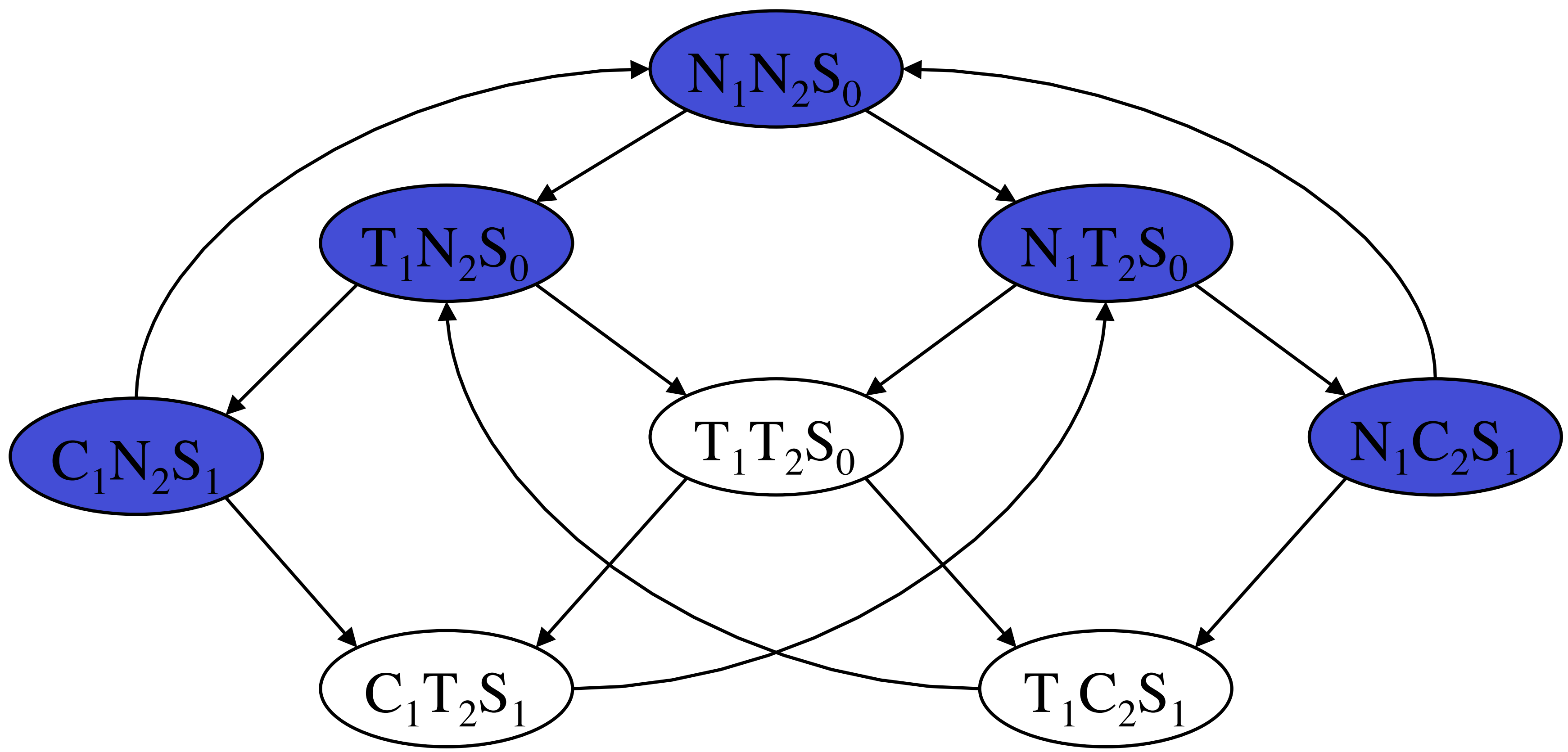
Mutual Exclusion

[Visser, ASE 2000]



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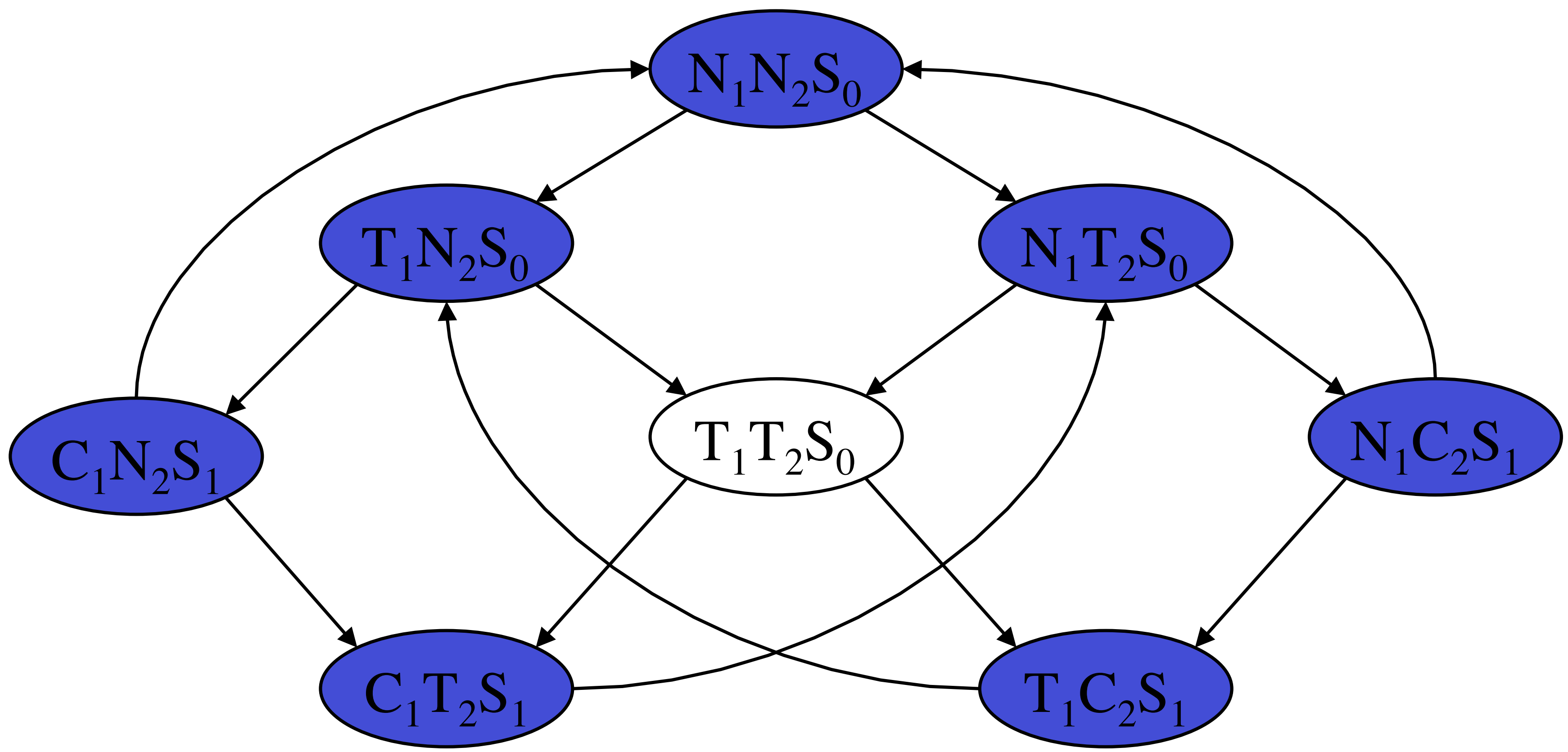
Mutual Exclusion

[Visser, ASE 2000]



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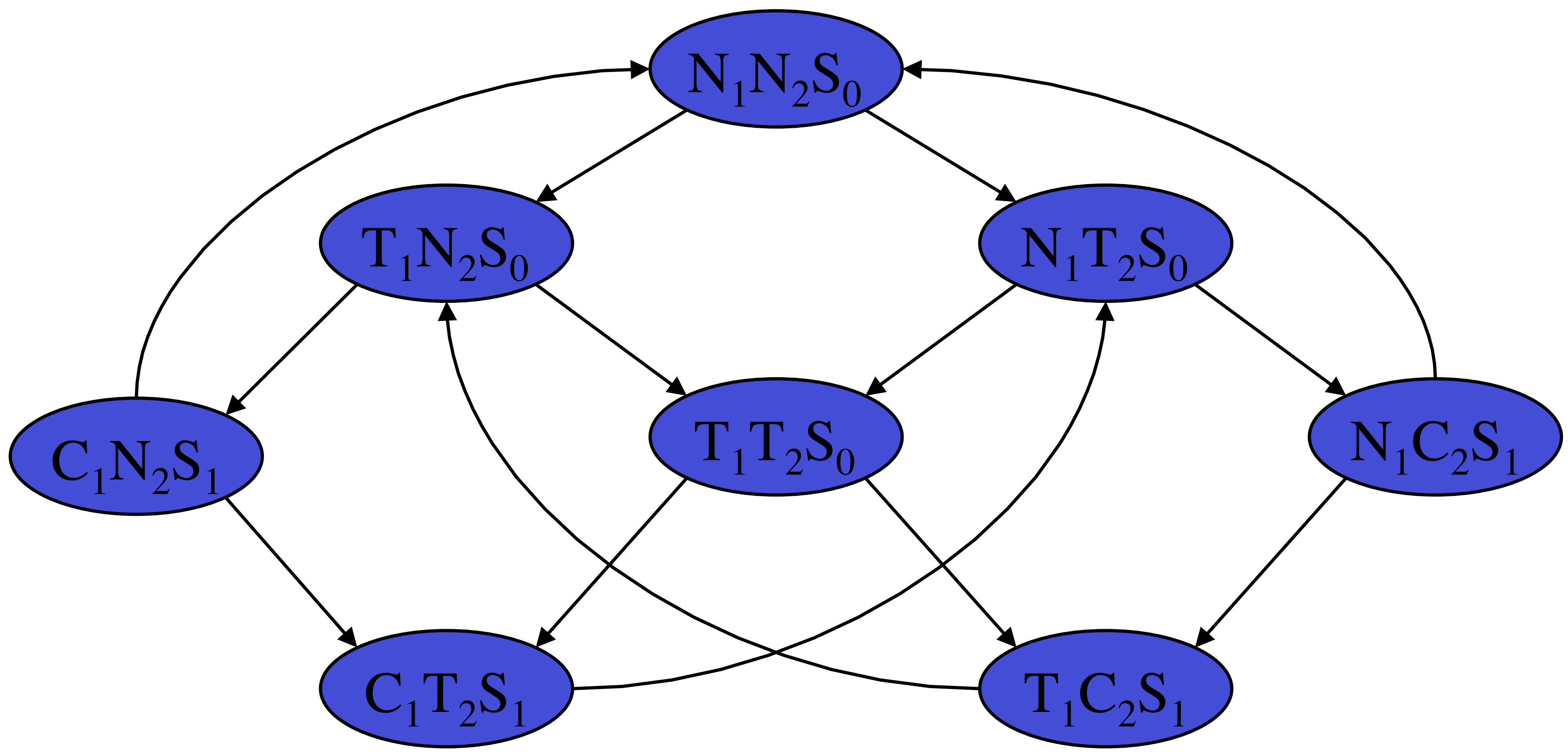
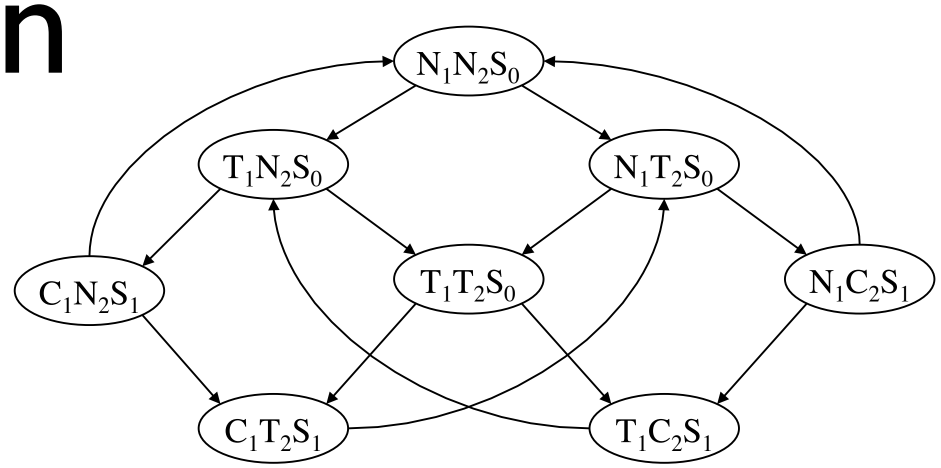


$$K \models AG\ EF\ (N_1 \wedge N_2 \wedge S_0)$$

Property Verification
[Visser, ASE 2000]

- For Kripke structure $M = (S,R,L)$, find set of states satisfying TL formula f
  - $\{ s \in S \mid M,s \models f \}$
- Property verified if initial states $I \subseteq S$ are in the set
- If M,i $\not\models$f, for some i$\in$I, give witness trace (tree)
  - Also called counterexample

State Explosion



- n states
- m threads
- $O(n^m)$ is size of total state space

$N_1 \rightarrow T_1$		$N_2 \rightarrow T_2$
$T_1 \,\&\, S_0 \rightarrow C_1 \& S_1$	$\|\|$	$T_2 \,\&\, S_0 \rightarrow C_2 \& S_1$
$C_1 \rightarrow N_1 \& S_0$		$C_2 \rightarrow N_2 \& S_0$

Addressing State Explosion

- Symbolic Model Checking
  - Binary Decision Diagrams (BDDs)
- Partial order reduction
- Bounded Model Checking

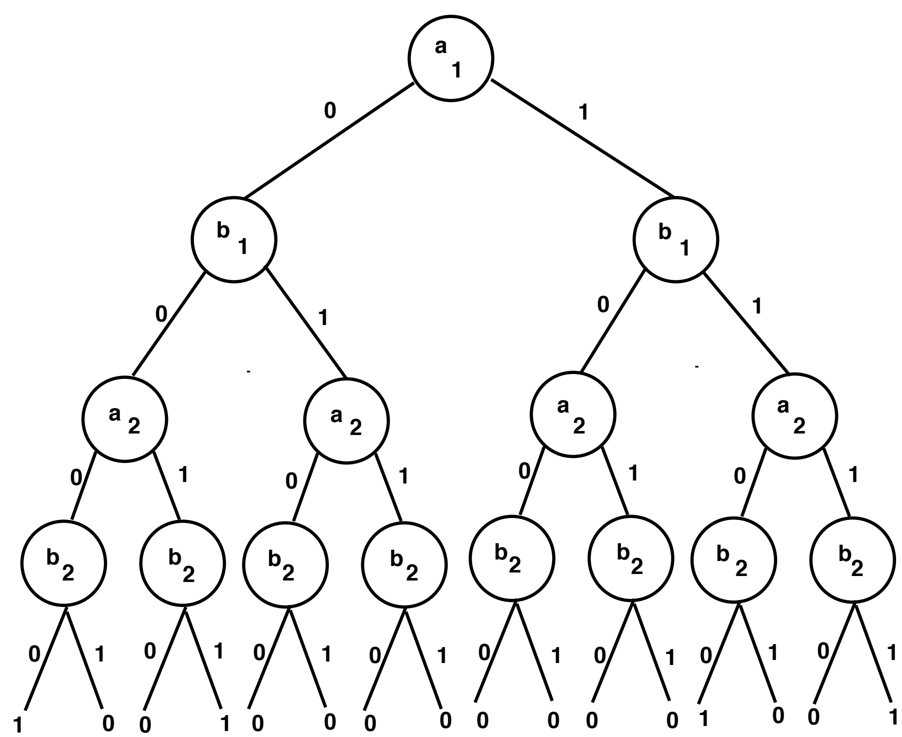
BDDs

Binary Decision Trees (Cont.)

A BDD for the two-bit comparator given by the formula

$$f(a_1, a_2, b_1, b_2) = (a_1 \leftrightarrow b_1) \wedge (a_2 \leftrightarrow b_2),$$

is shown in the figure below:

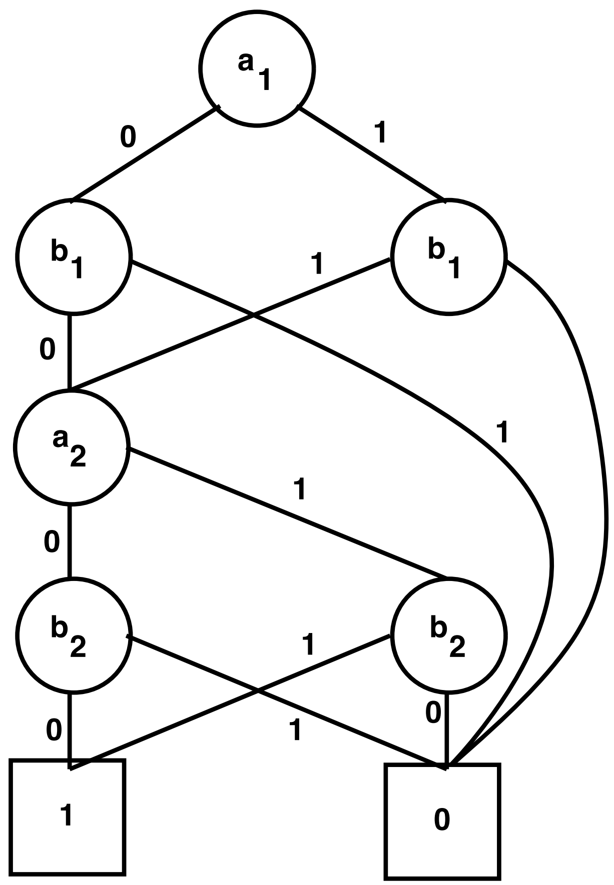


- http://www.cs.cmu.edu/afs/cs.cmu.edu/usr/emc/www/15817-f09/lecture1.pdf

BDDs

OBDD for Comparator Example

If we use the ordering $a_1 < b_1 < a_2 < b_2$ for the comparator function, we obtain the OBDD below:



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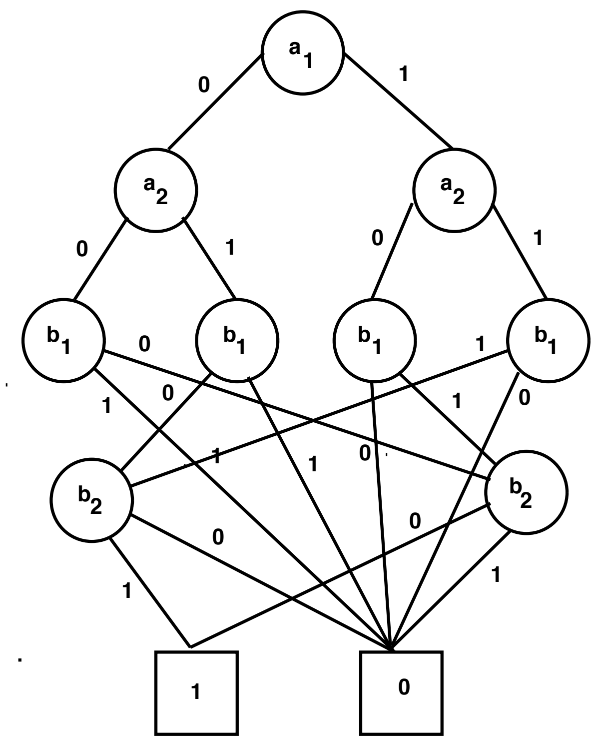
BDDs

Variable Ordering Problem

The size of an OBDD depends critically on the variable ordering.

If we use the ordering $a_1 < a_2 < b_1 < b_2$ for the comparator function, we get the OBDD below:



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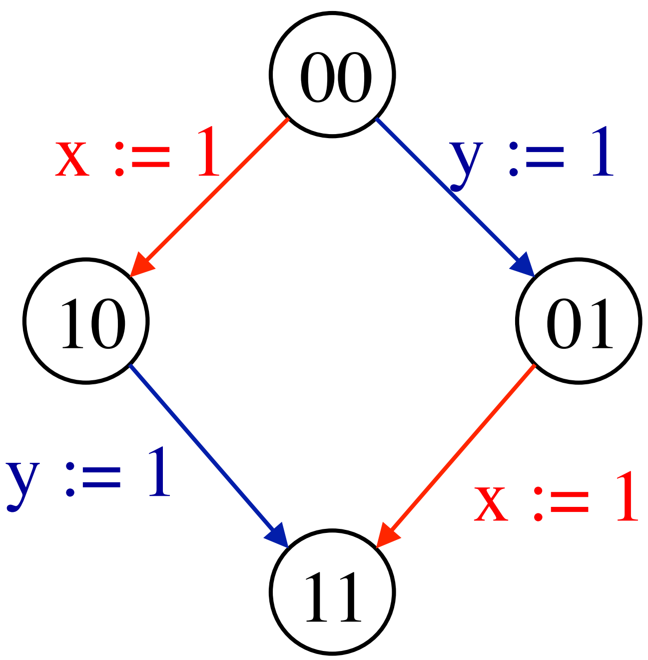
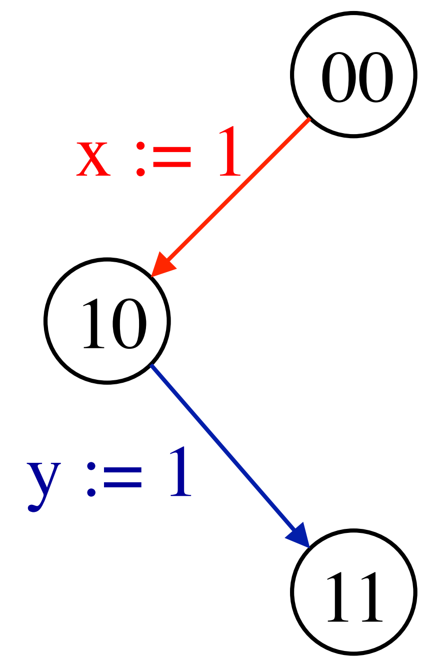
Partial Order Reduction

[Visser, ASE 2000]

x := 1 || y := 1 where initially x = y = 0



No Reductions



States Reduced

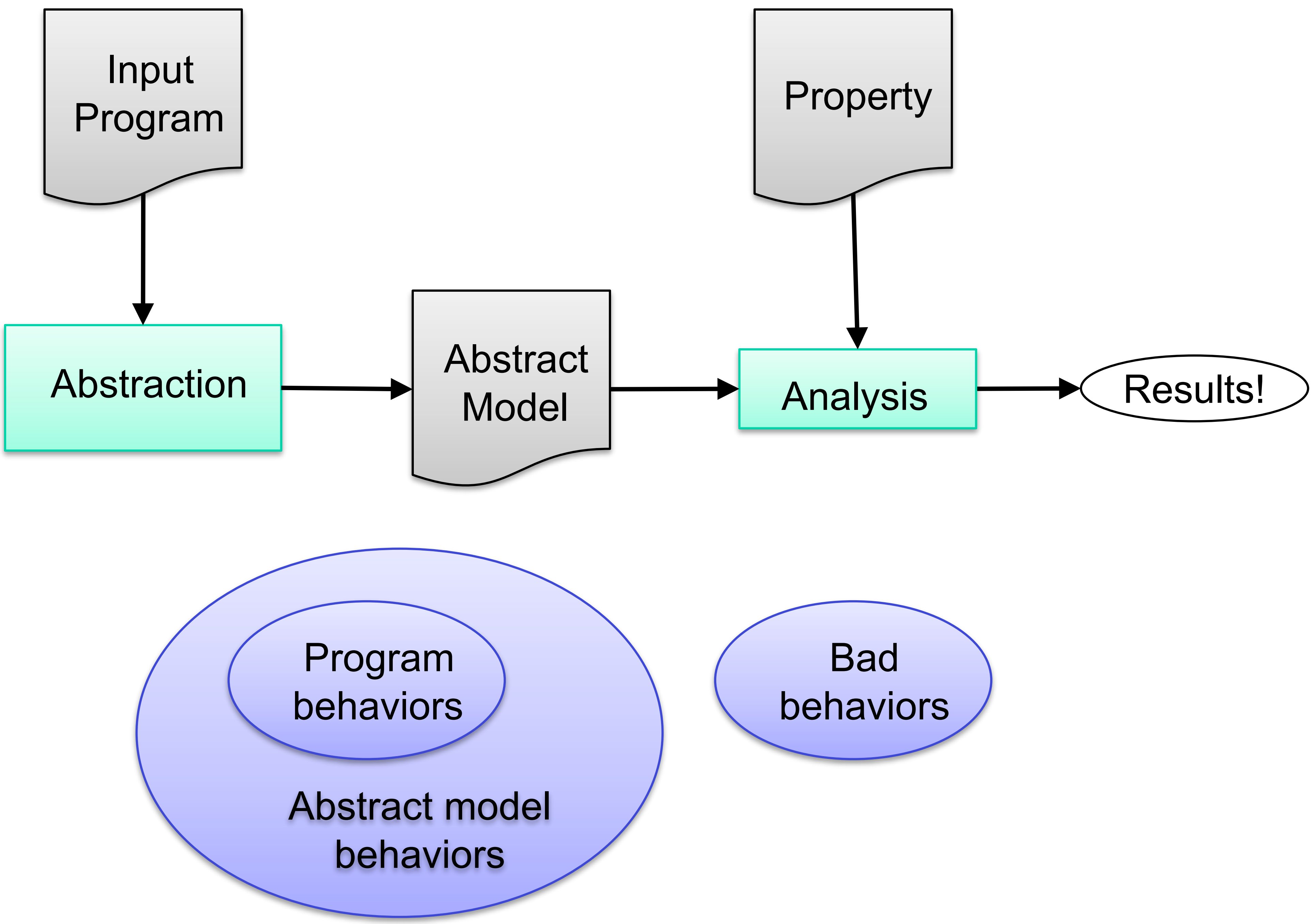
Bounded Model Checking

- Don't do *exhaustive* exploration of state space
- Search *bounded-depth* execution traces
- Encode as SAT to use fast solvers
- http://www.cprover.org/cbmc/

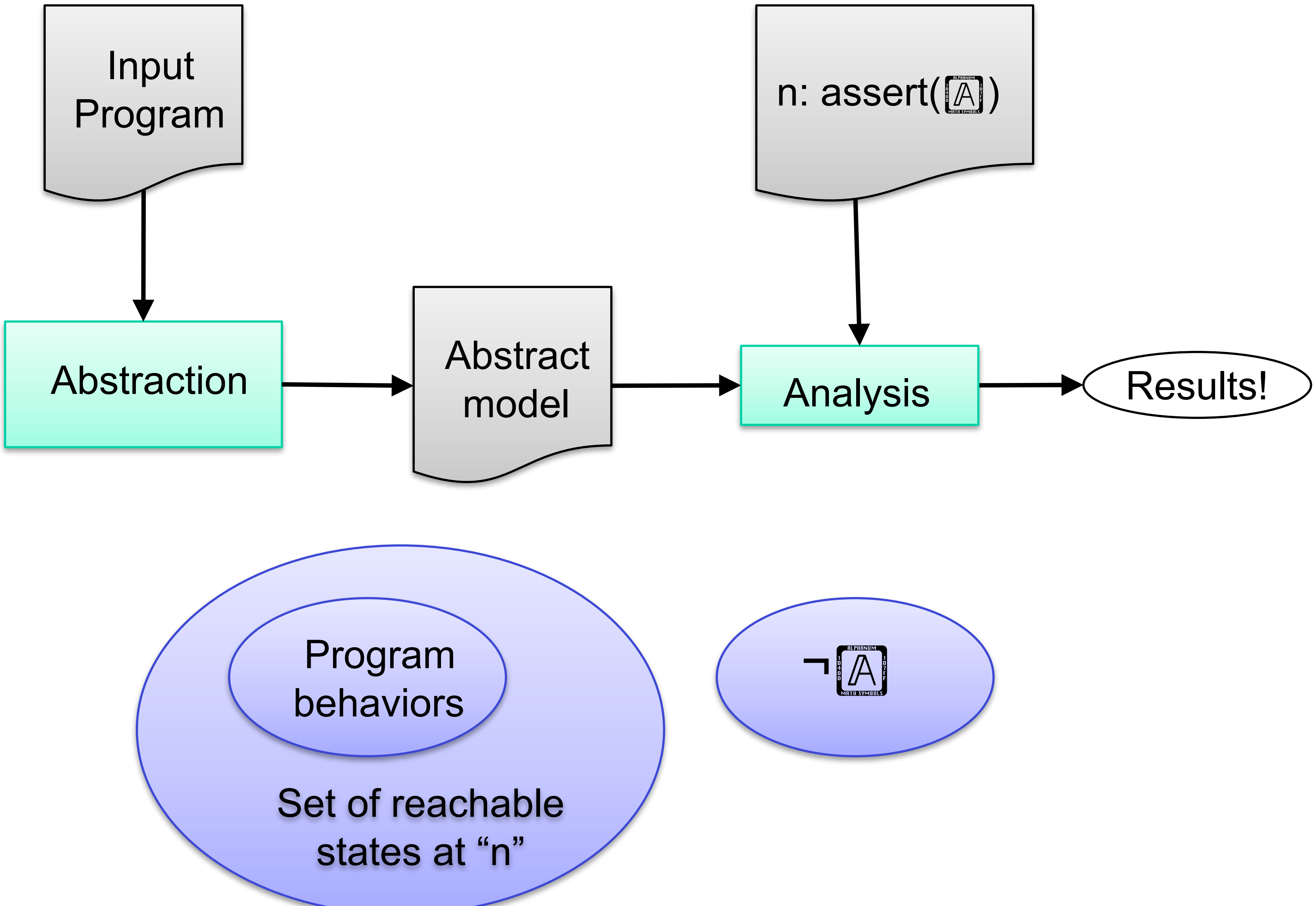
Software Model Checking

- *Idea*: apply same techniques to software verification (automatic, no proofs, counterexamples, ...)
- Programs are *infinite* state!
  - Memory allocation, recursion, memory values, input/output, ...
  - Use *abstraction*
  - Use *infinite* Kripke structures

Software Model Checking



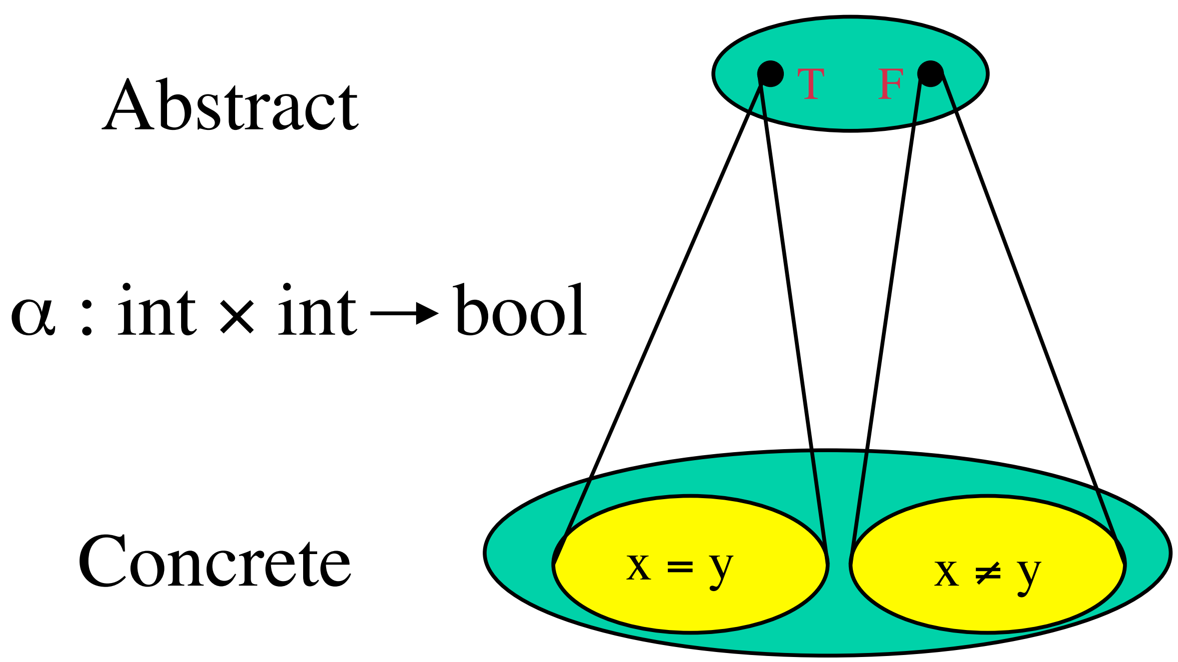
Software Model Checking

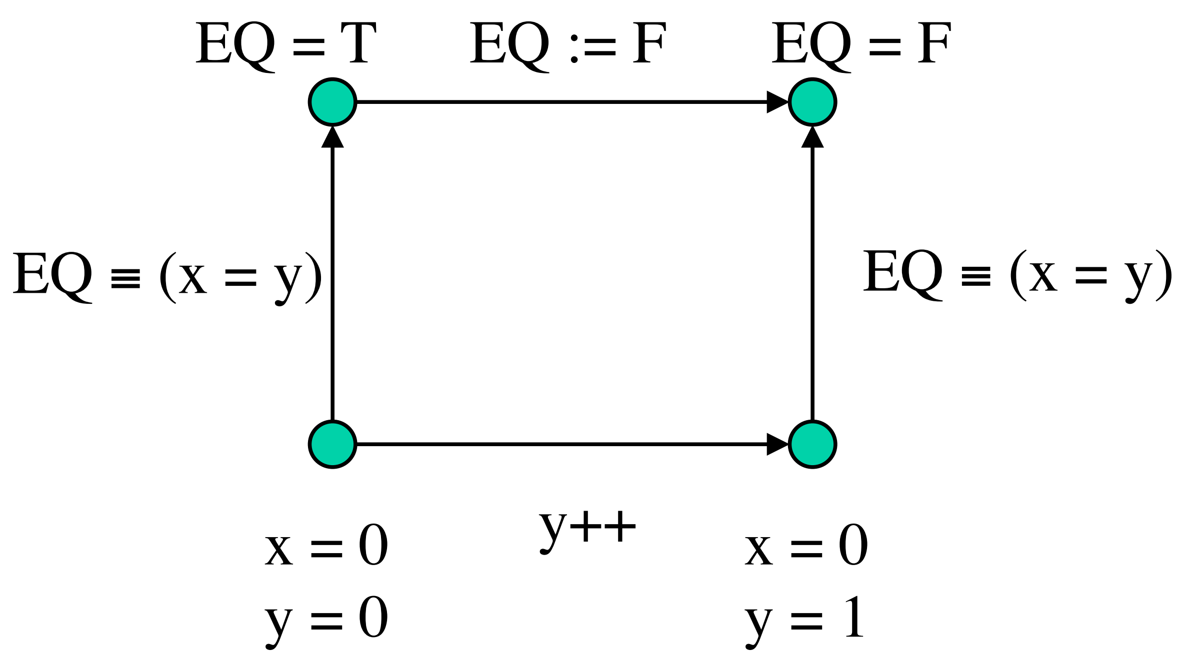


Predicate Abstraction

[Graf & Saidi] [Slide: Visser, ASE 2000]

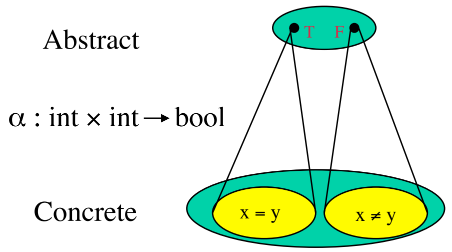
- Map *concrete* system to *abstract* system, whose states correspond to truth values of set of predicates

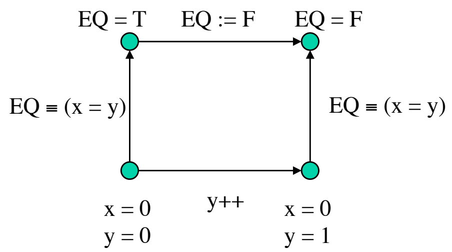




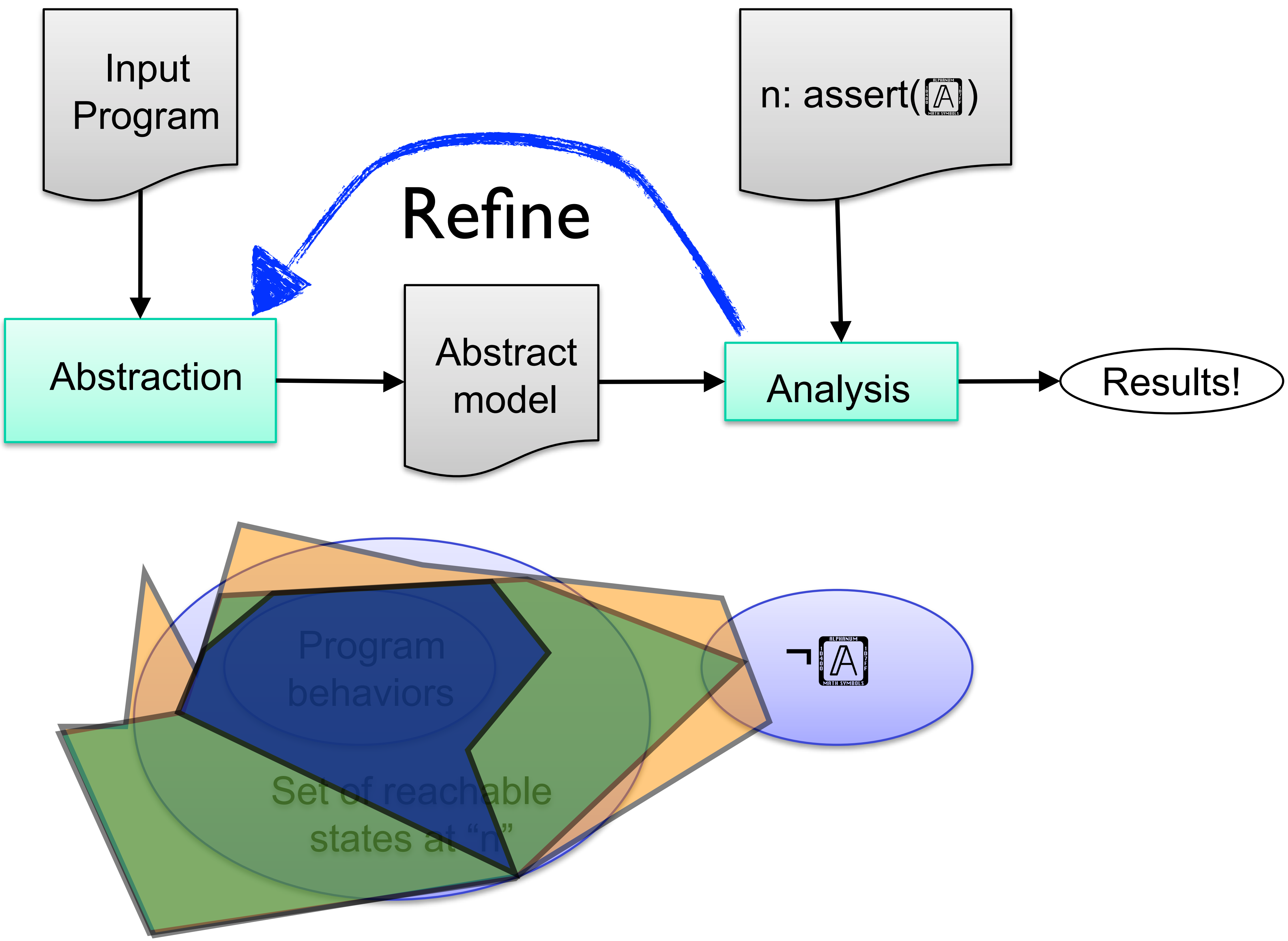
Predicate Abstraction
[Visser, ASE 2000]

- Map *concrete* system to *abstract* system, whose states correspond to truth values of set of predicates
- Adds *spurious* behaviors to abstract system
- Requires *verifying* counterexample validation if bug found





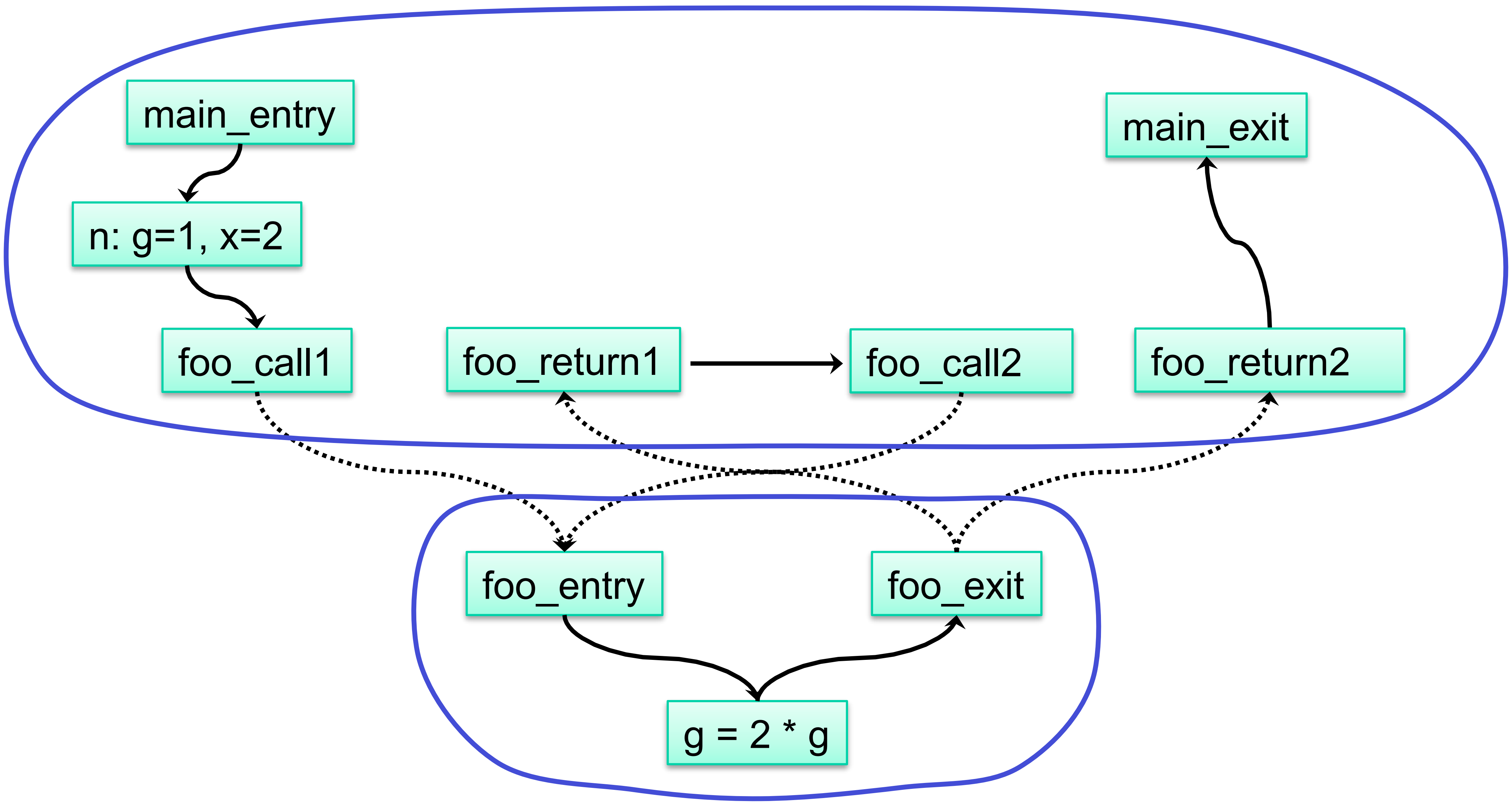
Software Model Checking



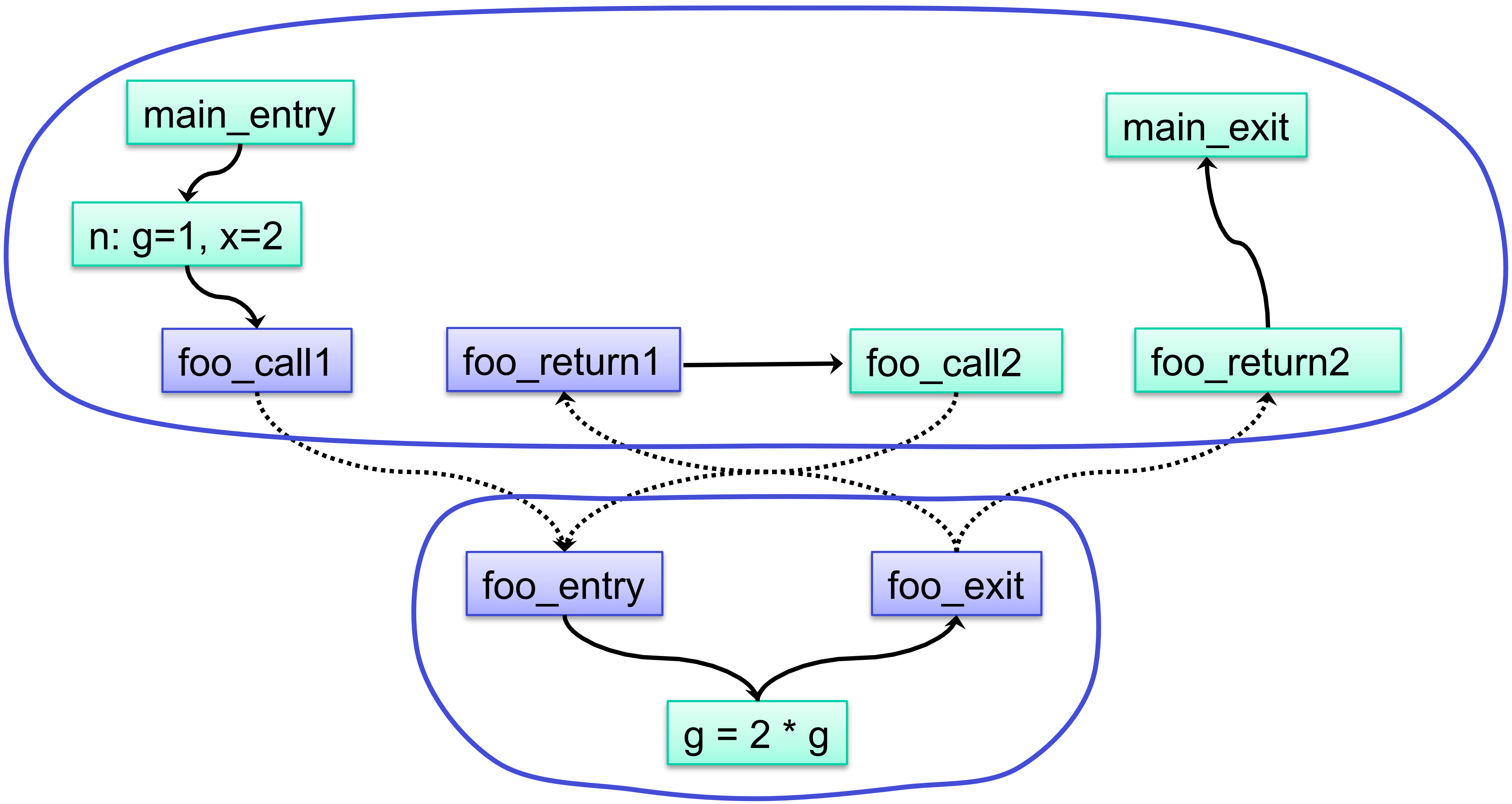
What about the stack

- Programs have a stack
    - Support component reuse and *modularity*
    - Support *recursion*
- *Context-sensitive analysis*
    - Precise with respect to matched calls and returns

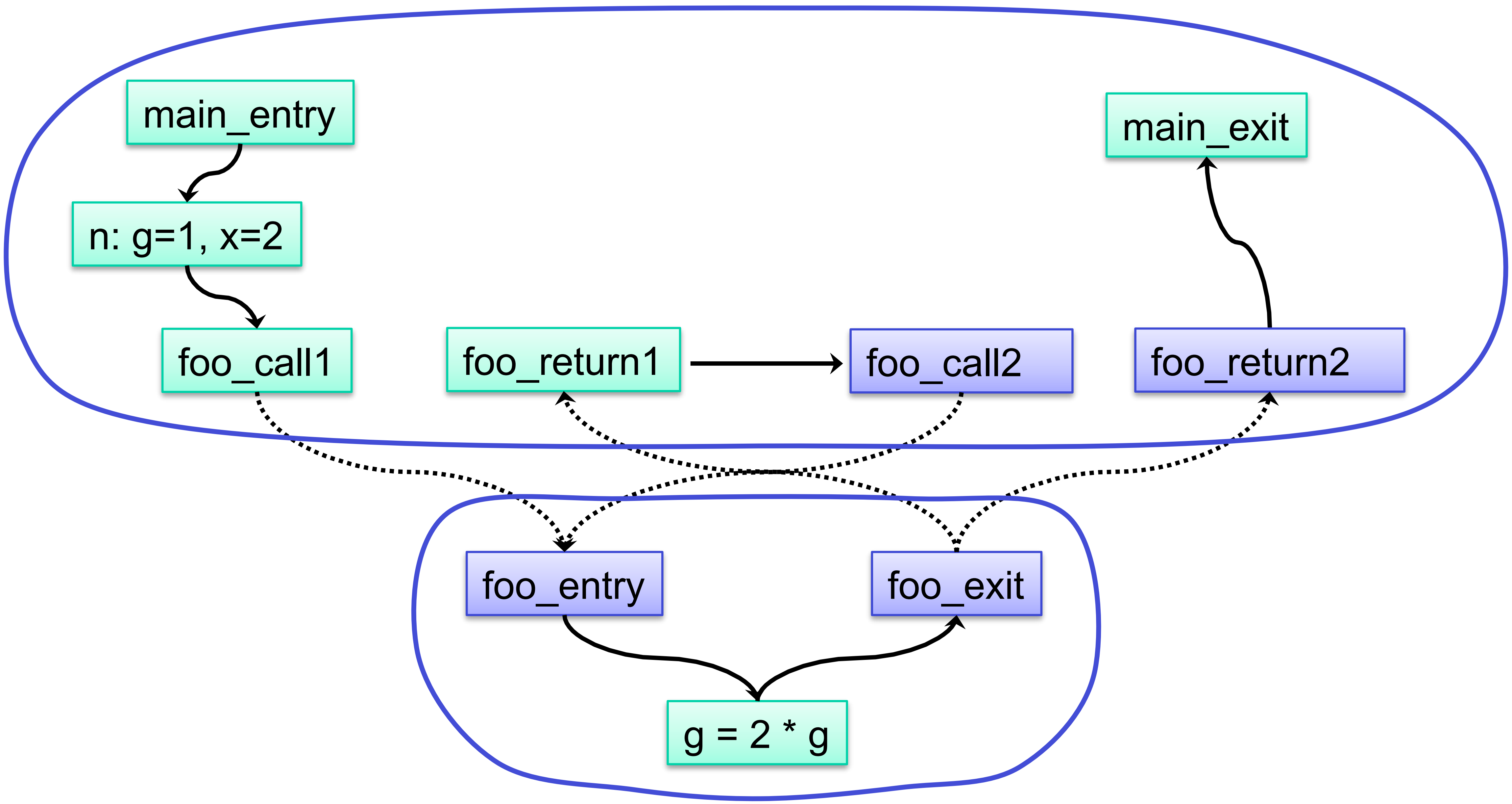
Matched Calls and Returns



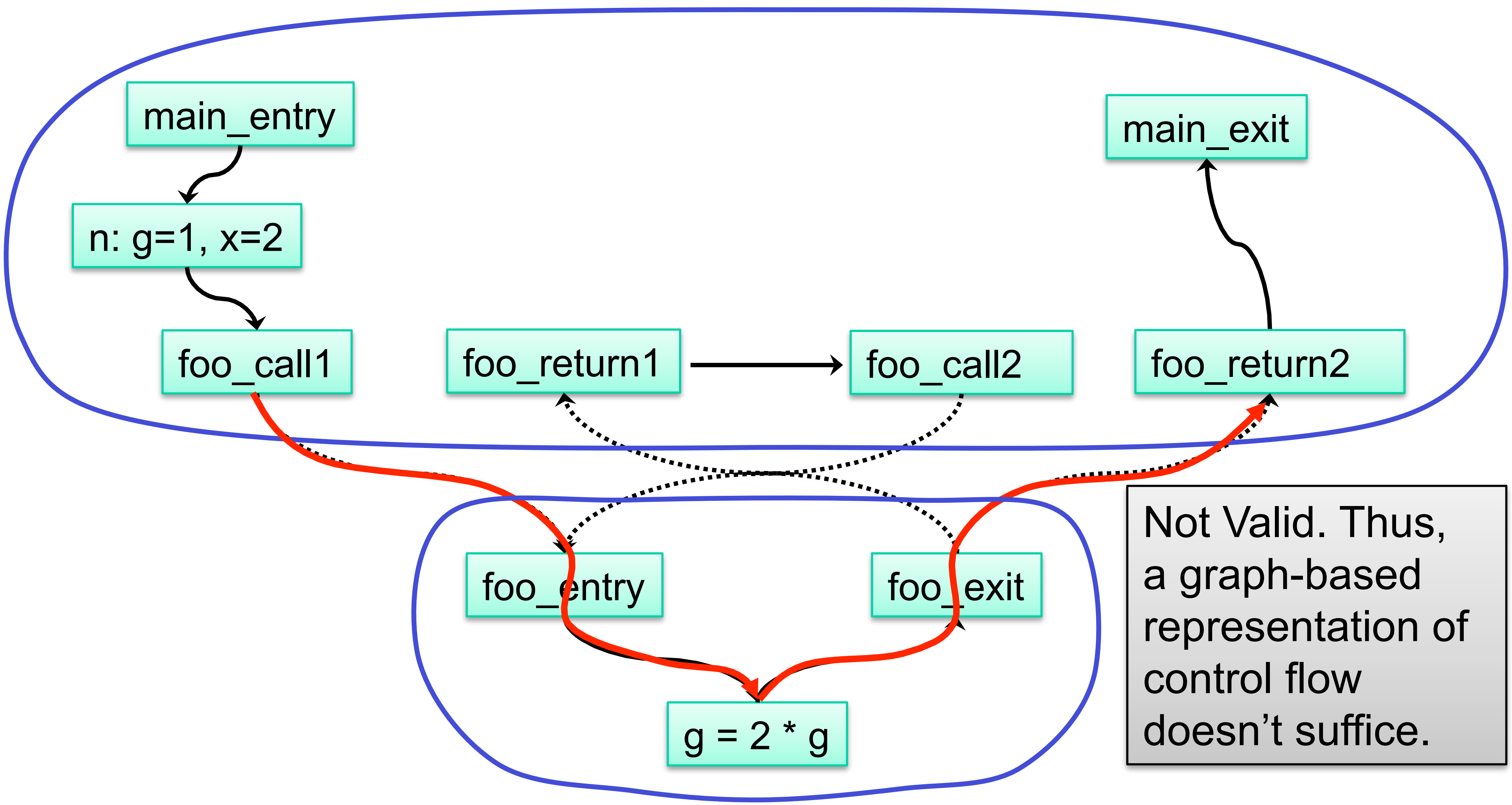
Matched Calls and Returns



Matched Calls and Returns

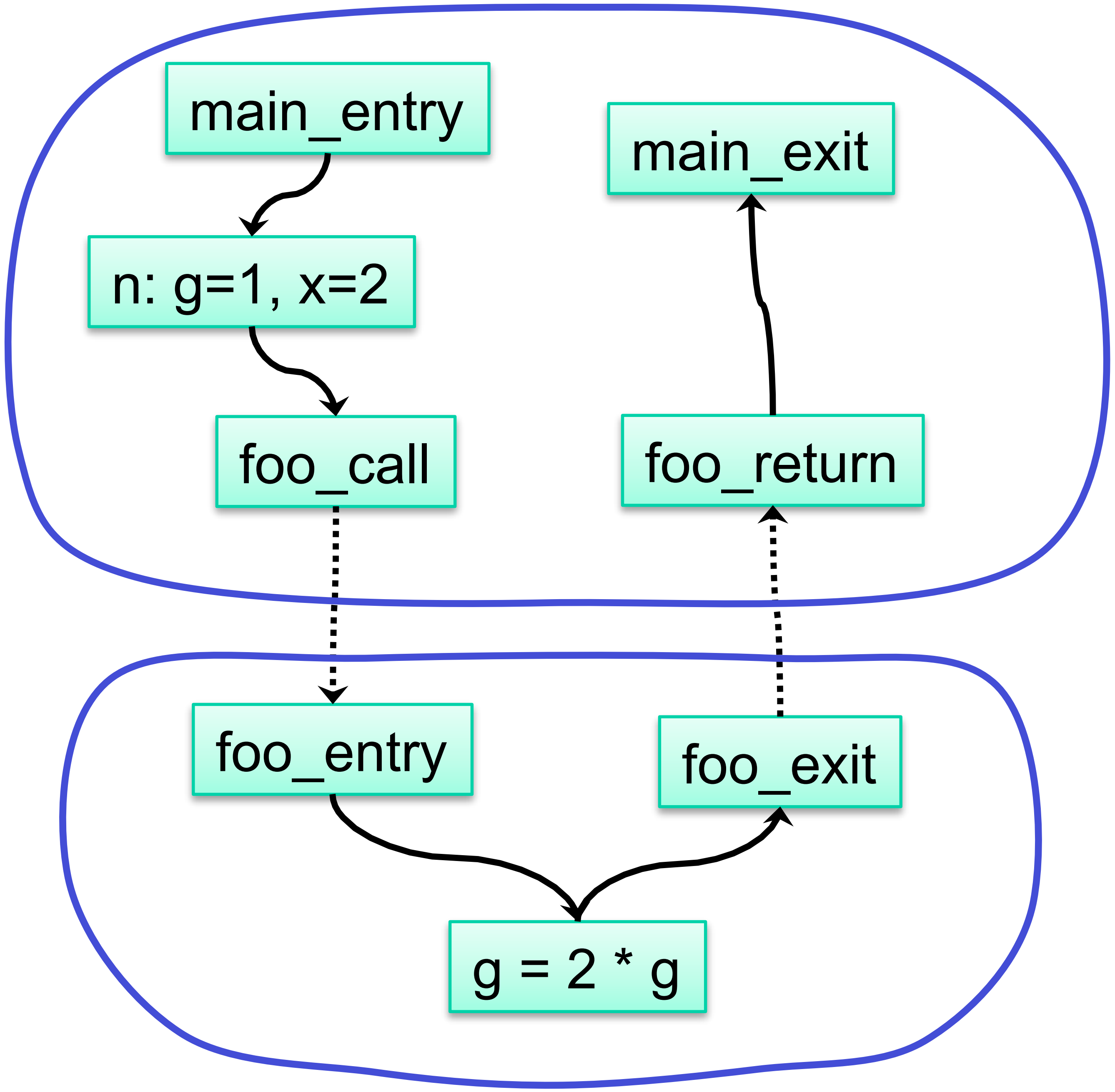


(Un)Matched Calls and Returns

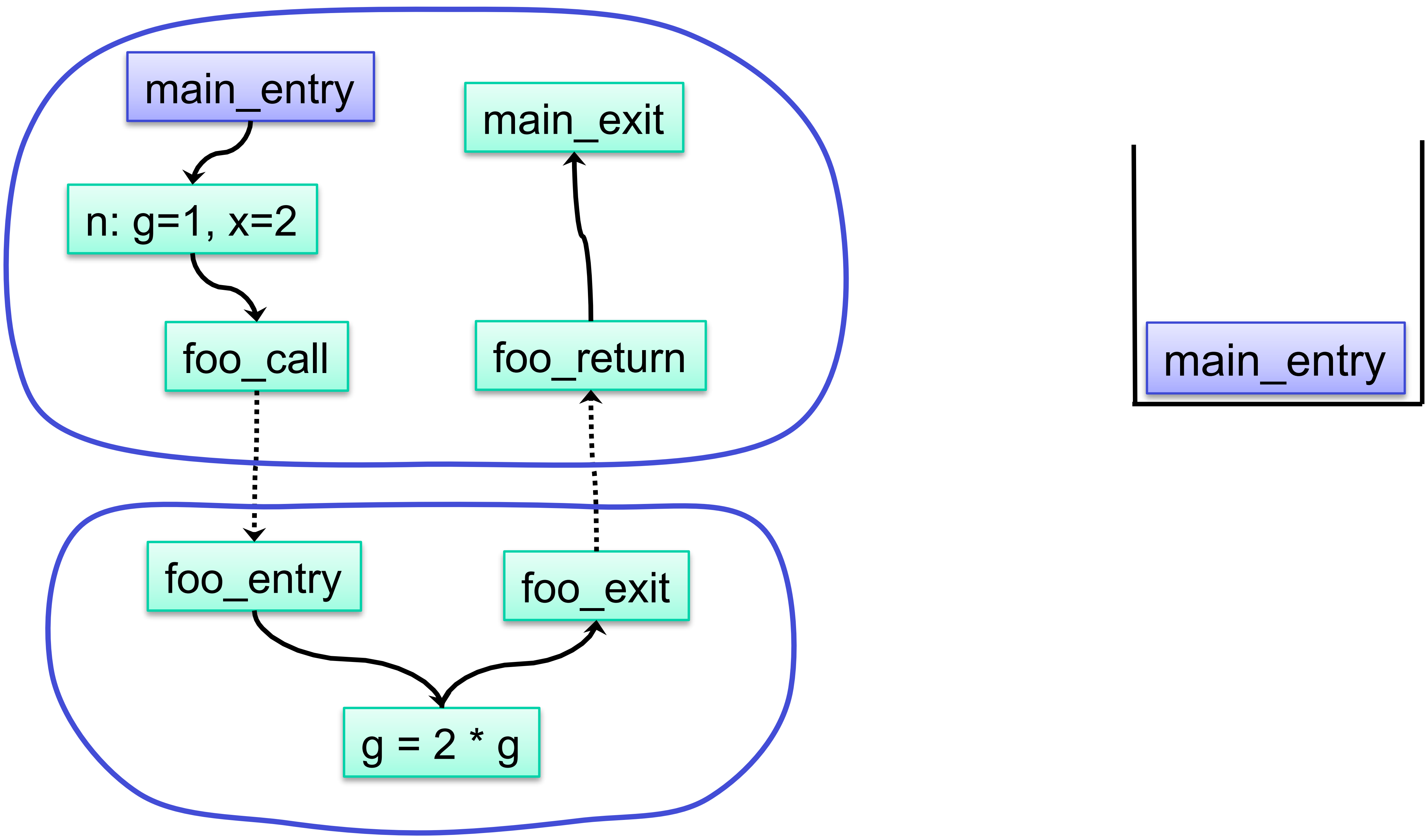


Not Valid. Thus, a graph-based representation of control flow doesn't suffice.

Modeling the Stack



Modeling the Stack



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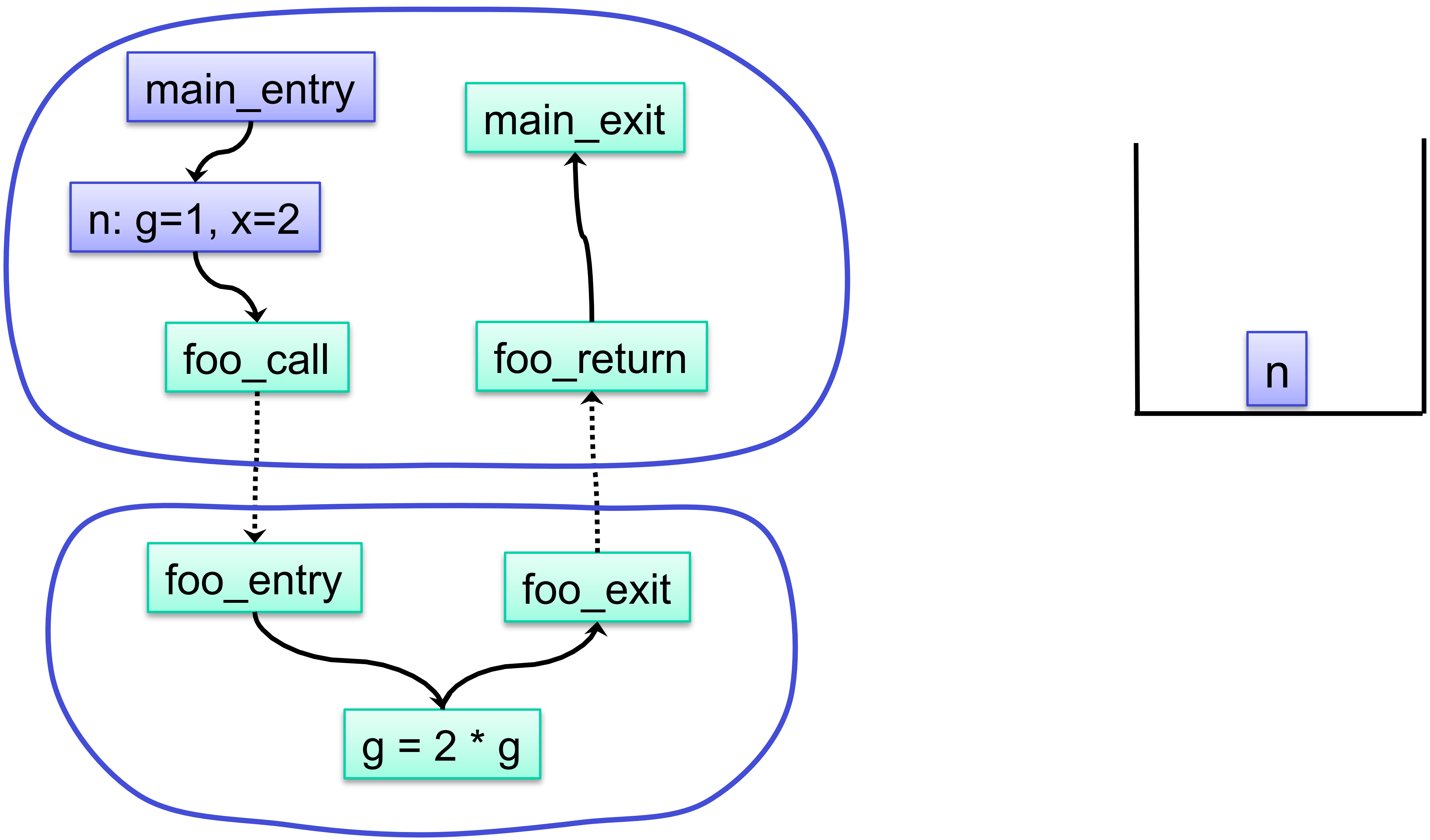

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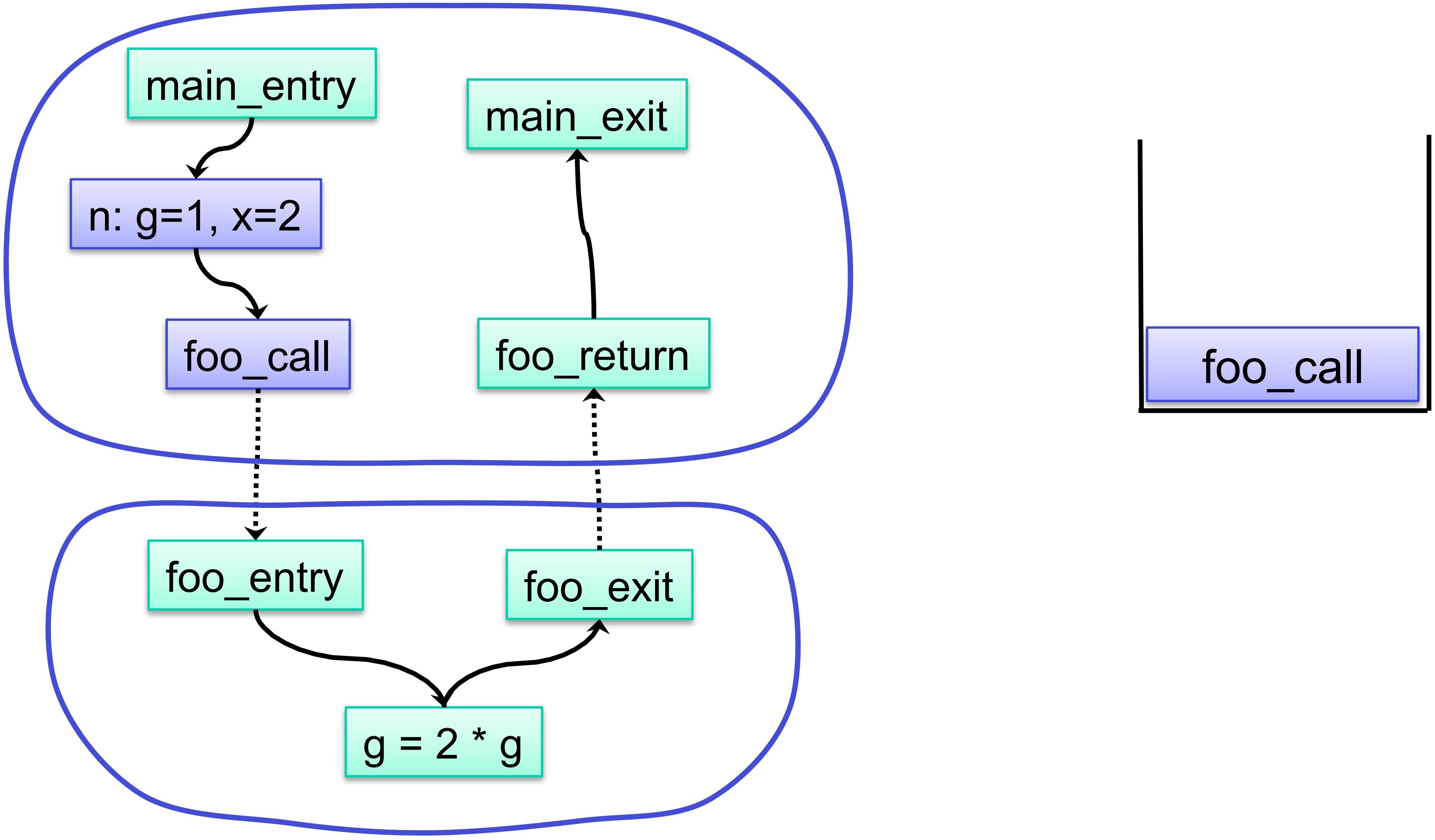

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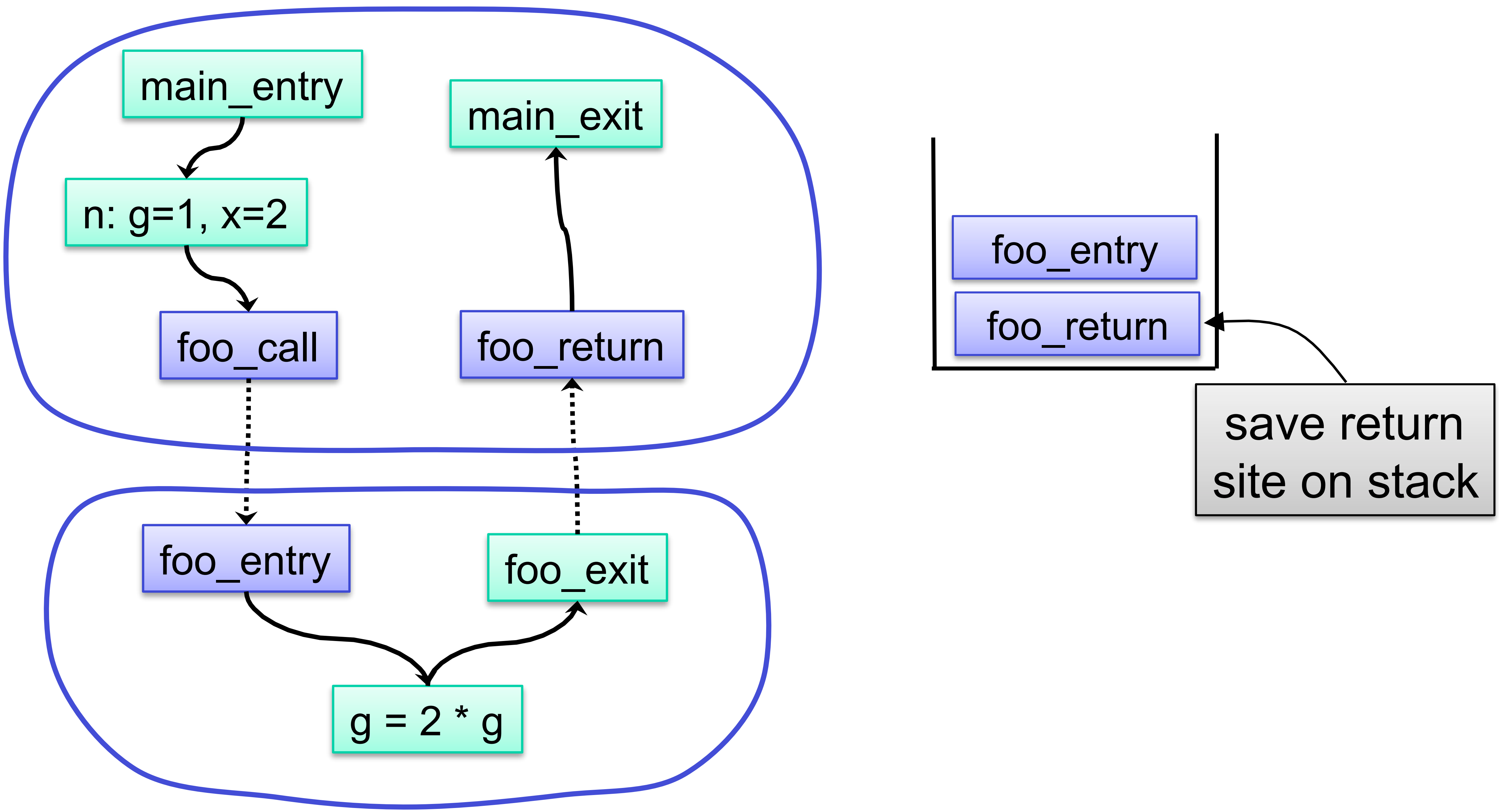
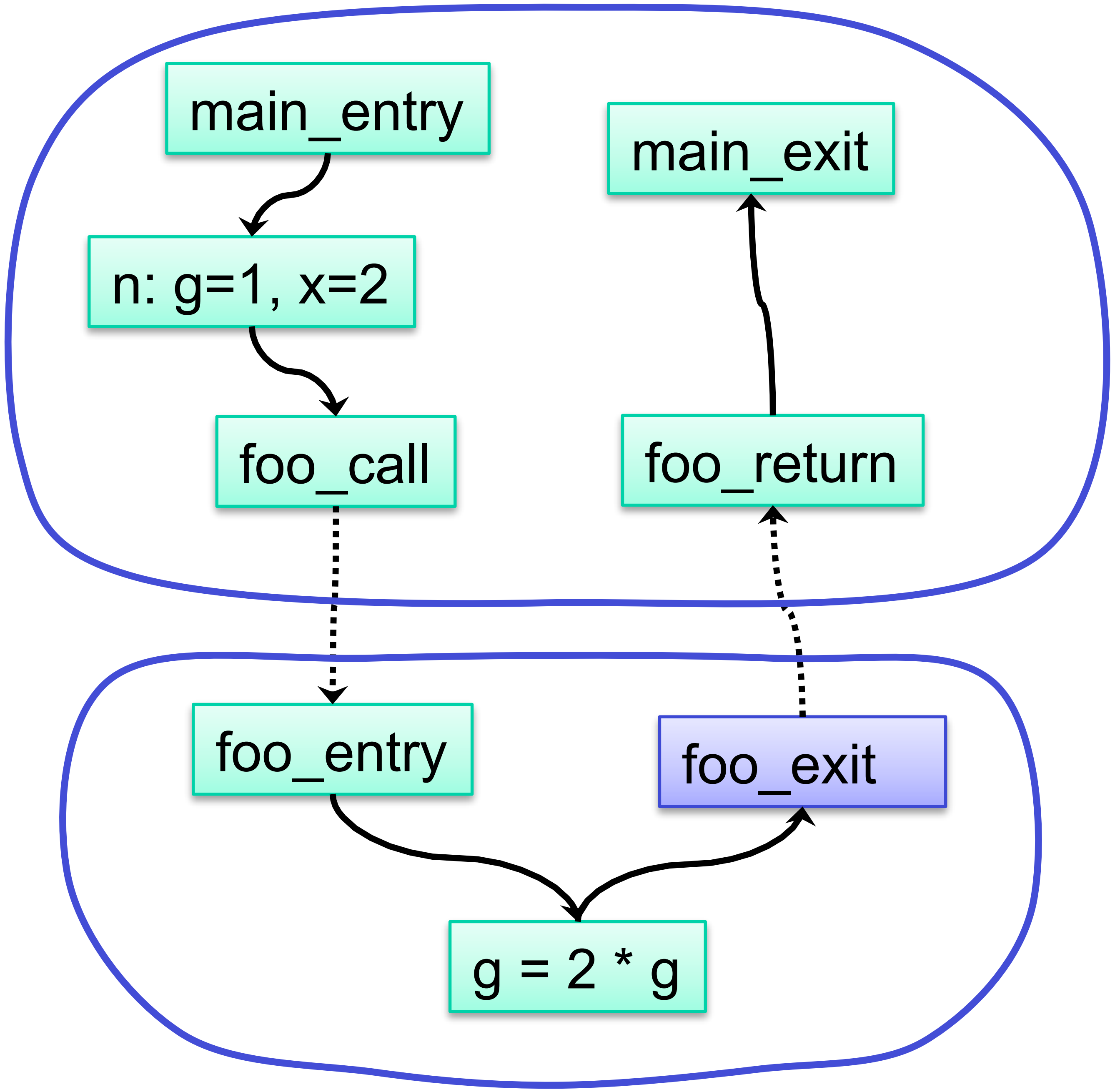
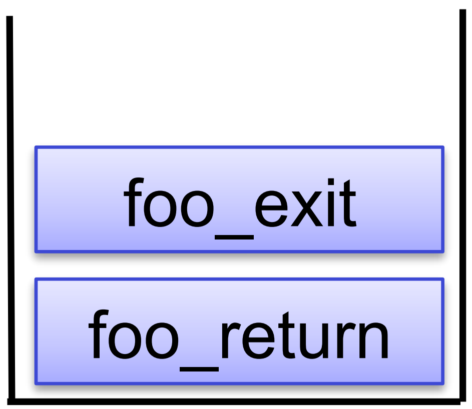
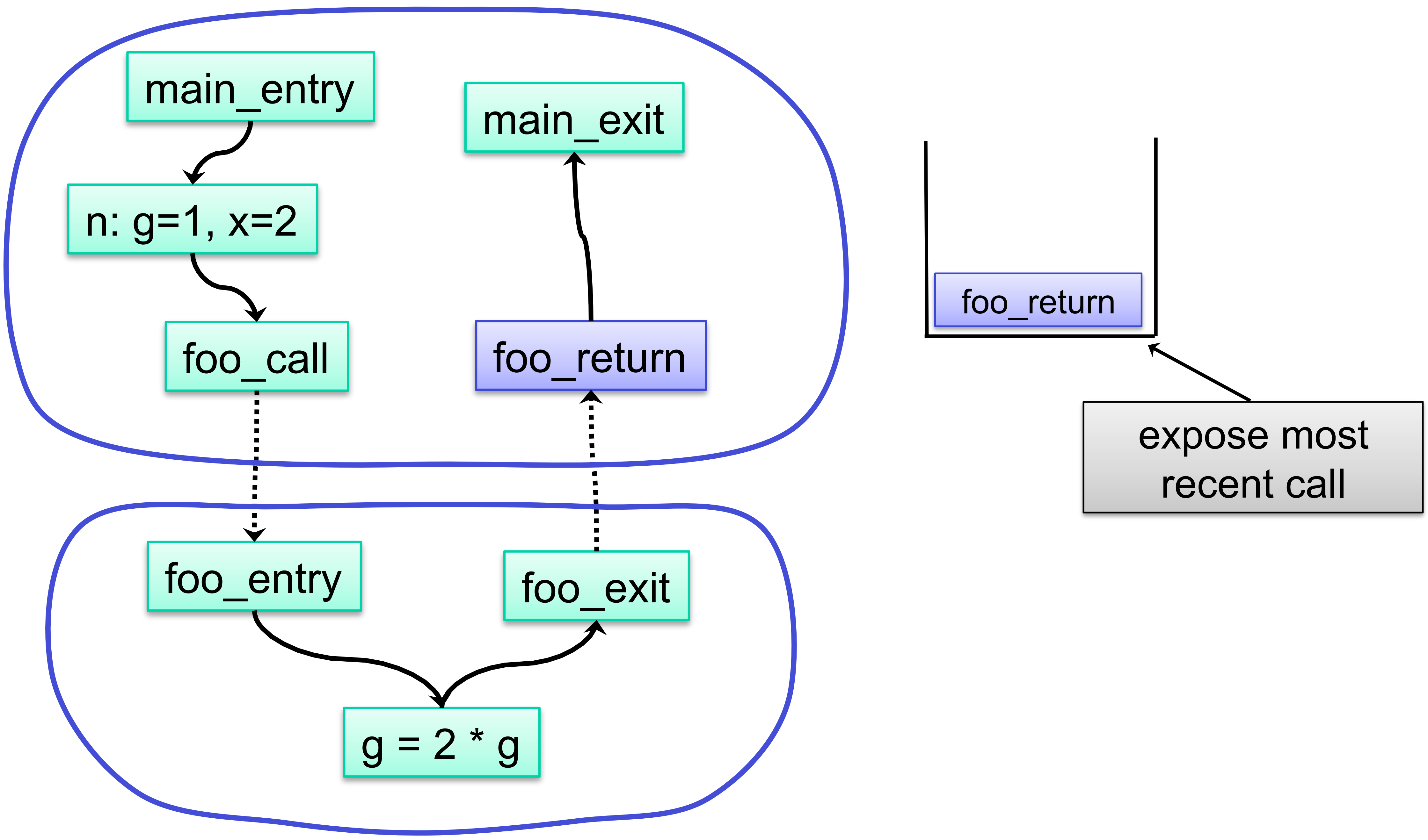


Modeling the Stack

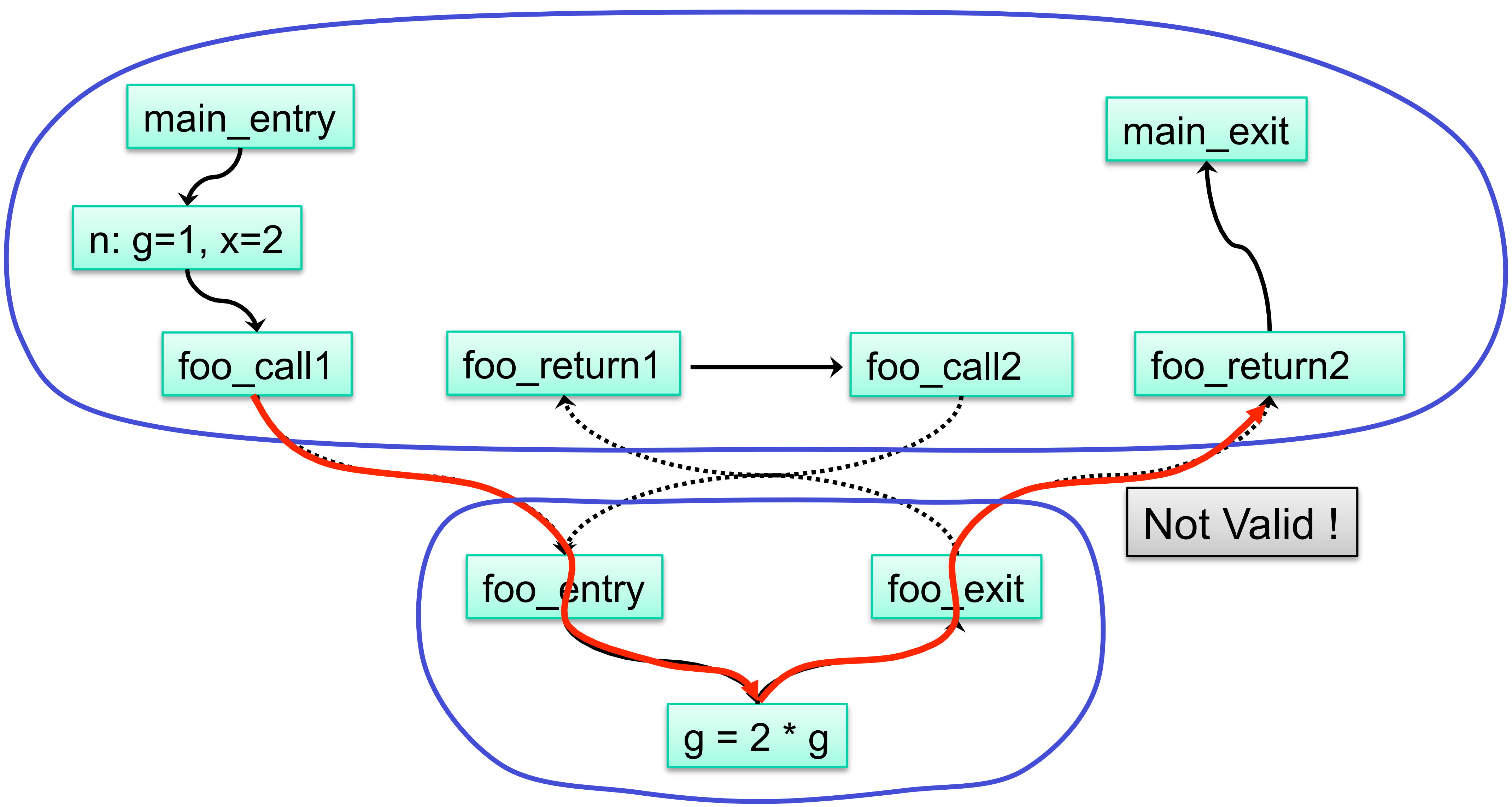




Modeling the Stack



Modeling the Stack



Pushdown Systems

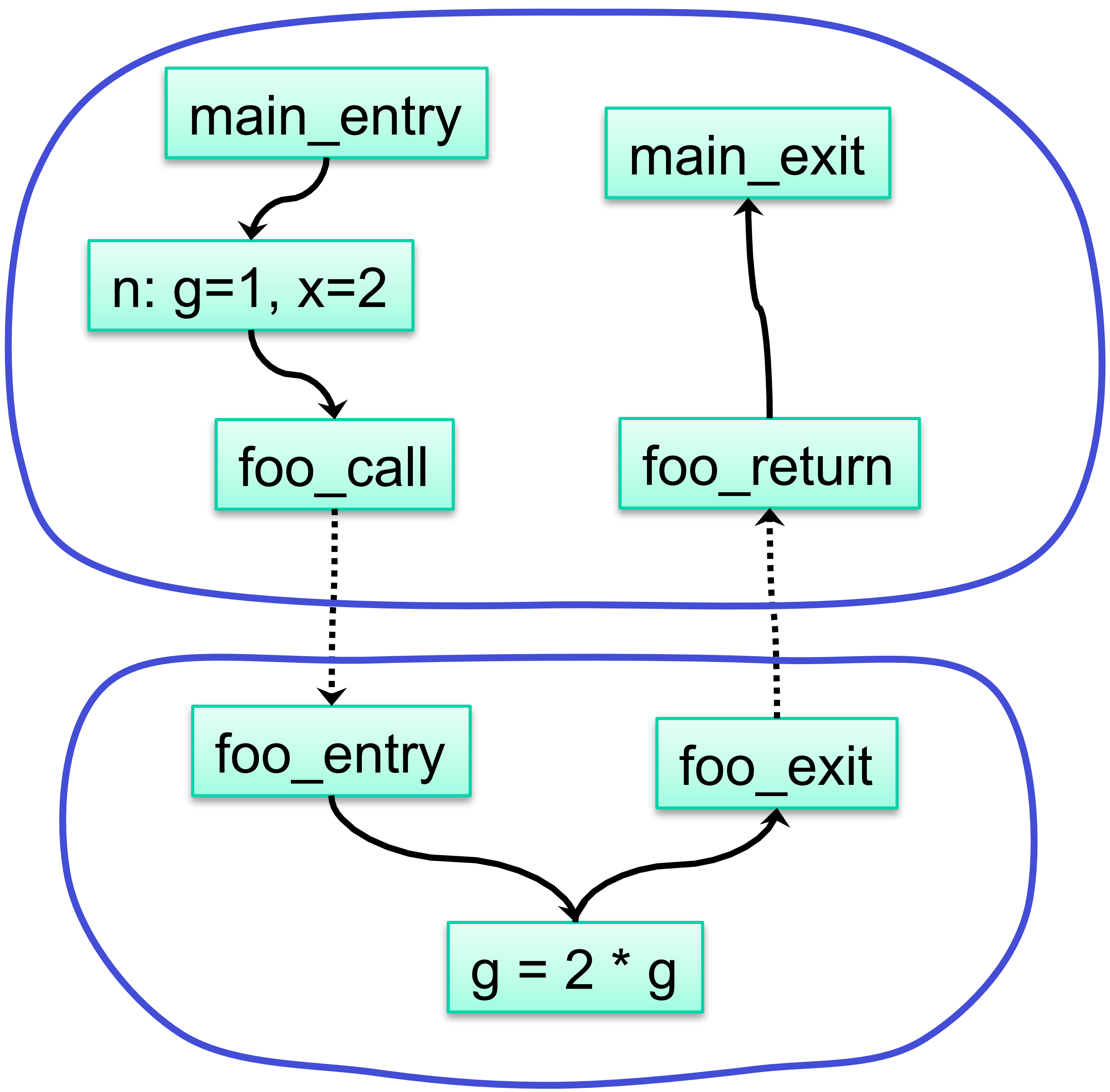
- Finite-state machine with a stack (P, Γ, Δ)
  - $P = \{p,q,\ldots\}$ is a finite set of states
  - $\Gamma = \{y,y',y''\}$ is a finite set of stack symbols
  - Δ: finite set of rules $\subseteq (P \times \Gamma \times P \times \Gamma^*)$
    - if $(p,y,p',u') \in \Delta$,
      then $(p, y \cdot u) \Rightarrow (p', u' \cdot u)$

      for $u \in \Gamma^*$

Pushdown Systems

ICFG	PDS	r∈Δ
Step	Step	(p,y,p',y')
Call	Push	(p,y,p',y',y'')
Return	Pop	(p,y,p')

Pushdown Systems



(p, main_entry, p, n)
(p, foo_call, p, foo_entry, foo_return)
(p, foo_exit, p,)

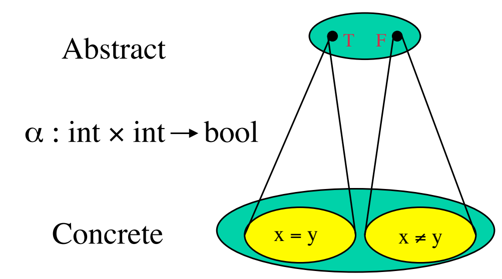
Pushdown Systems

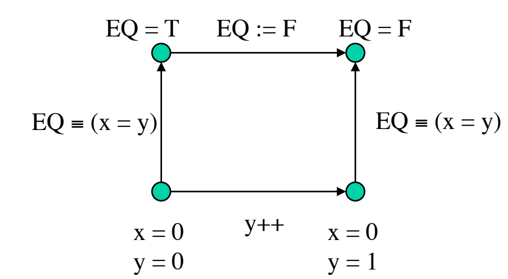
- Finite-state machine with a stack (P, Γ, Δ)
- Pushdown Model Checking algorithms for
  - LTL, CTL, CTL*
  - LTL is polynomial!
- Come to the PL Seminar next semester for more info :-)

SLAM (or now SDV)

[Ball & Rajamani]

- Predicate Abstraction defines a PDS (P,Γ,Δ)
    - Global predicates in P
    - Local predicates and PC in Γ
    - Δ encodes the transition relation
- Check LTL properties on abstract system





SLAM Example

[Ball & Rajamani]

```
      numUnits : int;
      level : int;
      void getUnit() {
[1]     canEnter: bool := F;
[2]     if (numUnits = 0) {
[3]       if (level > 10) {
[4]         NewUnit();
[5]         numUnits := 1;
[6]         canEnter := T;
          }
        } else
[7]         canEnter := T;

[8]     if (canEnter)
[9]       if (numUnits = 0)
[10]        assert(F);
          else
[11]        gotUnit();
      }
```

SLAM Example

```
      numUnits : int;
      level : int;
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[1]     canEnter: bool := F;
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[8]     if (canEnter)
[9]       if (numUnits = 0)
[10]        assert(F);
          else
[11]        gotUnit();
      }
```

```
      void getUnit() {
[1]     ...
[2]     if (?) {
[3]       if (?) {
[4]         ...
[5]         ...
[6]         ...
          }
        } else
[7]         ...

[8]     if (?)
[9]       if (?)
[10]        ...
          else
[11]        ...
      }
```

SLAM Example

```
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       level : int;
       void getUnit() {
[1]      canEnter: bool := F;
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[7]          canEnter := T;

[8]      if (canEnter)
[9]        if (numUnits = 0)
[10]         assert(F);
           else
[11]         gotUnit();
       }
```

```
       nU0: bool;
//     nU0 = T iff (numUnits = 0)
       void getUnit() {
[1]      ...
[2]      if (nU0) {
[3]        if (?) {
[4]          ...
[5]          nU0 := F;
[6]          ...
           }
         } else
[7]          ...

[8]      if (?)
[9]        if (nU0)
[10]         ...
           else
[11]         ...
       }
```

SLAM Example

```
     numUnits : int;
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     void getUnit() {
[1]    canEnter: bool := F;
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[10]       assert(F);
         else
[11]       gotUnit();
     }
```

```
     nU0: bool;

     void getUnit() {
[1]    cE: bool := F;
[2]    if (nU0) {
[3]      if (?) {
[4]        ...
[5]        nU0 := F;
[6]        cE := T;
         }
       } else
[7]        cE := T;

[8]    if (cE)
[9]      if (nU0)
[10]       ...
         else
[11]       ...
     }
```

SLAM Example

```
     numUnits : int;
     level : int;
     void getUnit() {
[1]    canEnter: bool := F;
[2]    if (numUnits = 0) {
[3]      if (level > 10) {
[4]        NewUnit();
[5]        numUnits := 1;
[6]        canEnter := T;
       }
     } else
[7]      canEnter := T;

[8]    if (canEnter)
[9]      if (numUnits = 0)
[10]       assert(F);
       else
[11]       gotUnit();
     }
```

```
     void getUnit() {
[1]    ...
[2]    if (?) {
[3]      if (?) {
[4]        ...
[5]        ...
[6]        ...
         }
     } else
[7]        ...

[8]    if (?)
[9]      if (?)
[10]       ...
         else
[11]       ...
     }
```

```
     nU0: bool;
//   nU0 = T iff (numUnits = 0)
     void getUnit() {
[1]    ...
[2]    if (nU0) {
[3]      if (?) {
[4]        ...
[5]        nU0 := F;
[6]        ...
         }
     } else
[7]        ...

[8]    if (?)
[9]      if (nU0)
[10]       ...
         else
[11]       ...
     }
```

```
     nU0: bool;

     void getUnit() {
[1]    cE: bool := F;
[2]    if (nU0) {
[3]      if (?) {
[4]        ...
[5]        nU0 := F;
[6]        cE := T;
         }
     } else
[7]        cE := T;

[8]    if (cE)
[9]      if (nU0)
[10]       ...
         else
[11]       ...
     }
```

What about Concurrency?

- Easy to combine finite-state systems (cross product)
- Software abstractions can still be infinite-state systems
  - PDSs are finite-state machine *plus a stack!*
  - Two-stacks can simulate a turing machine ...

What about Concurrency?

- Easy to combine finite-state systems (cross product)
- Software abstractions are still infinite-state systems
  - Finite-state machine *plus a stack!*
  - Two-stacks can simulate a turing machine ...
  - Reachability is *undecidable* in general!

What about Concurrency?

- Must bound *global* communication
  - Transfer of global state (think volatile in Java)
  - Happens at context switch (interleaved semantics)
  - Happens at message pass (MPI)