Analog IC Design Homework 3 Report

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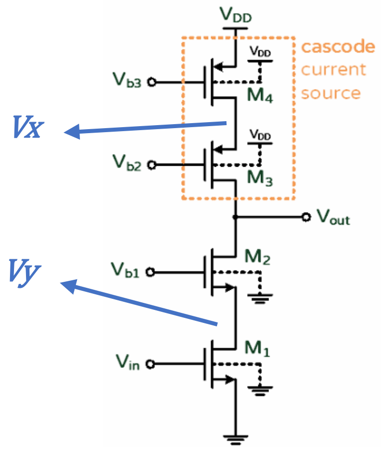
Question 1. – Cascode Amplifier



此題所要求的條件如下

(1) $I_D < 5\mu A$

(2) $|Av| \geq 45dB$

(3) output swing $\geq 1.2V$

(4) M1, M2, M3, M4 operates in saturation region

我們要選定四個電晶體的 W/L、Vin,dc、Vb1~Vb3 的值來達成目標。

(a) 首先，在設計之前需要做最初步的電路分析，首先四個電晶體必須操作在飽和狀態，我們可以列出下列式子

$$\begin{cases} V_x - V_{b3} \leq V_{th4} \Longrightarrow M4\ saturates \\ Vout - V_{b2} \leq V_{th3} \Longrightarrow M3\ saturates \\ V_{b1} - Vout \leq V_{th2} \Longrightarrow M2\ saturates \\ Vin - V_y \leq V_{th1} \Longrightarrow M1\ saturates \end{cases}$$

再來，分析電路增益公式

$$R_{on} \approx (g_{m2} + g_{mb2}) r_{o2} r_{o1}$$
$$R_{op} \approx (g_{m3} + g_{mb3}) r_{o3} r_{o4}$$
$$R_{out} = R_{op} \parallel R_{on}$$
$$|Av| = g_{m1} R_{out} = g_{m1}((g_{m2} + g_{mb2}) r_{o2} r_{o1} \parallel (g_{m3} + g_{mb3}) r_{o3} r_{o4})$$

假設 $g_{m1} = g_{m2} = g_{m3} = g_{m4} = gm$、$r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_o$

$$|Av| \cong \frac{1}{2} g_m^2 r_o^2 \cdots (1)$$

$$g_m = u_n C_{ox} \left(\frac{W}{L}\right) (V_{ov})^2 \cdots (2)$$

最後分析

$$\text{output swing} = V_{DD} - V_{ov4} - V_{ov3} - V_{ov2} - V_{ov1} \geq 1.2V$$
$$\Rightarrow V_{ov4} + V_{ov3} + V_{ov2} + V_{ov1} \leq 0.6V \cdots (3)$$

由上述式子可以推論當 $V_{ov} \downarrow \begin{cases} \Rightarrow g_m \downarrow \Rightarrow I_D \downarrow \Rightarrow |Av| \downarrow \\ \Rightarrow output\ swing \uparrow \end{cases}$，因此衡量到 output swing 必須大且|Av|必須大於 45dB 大約等於 177.827941V/V 的情況下，我們要讓 Vov 調整到適合的值來保證所有電晶體操作在 saturation 且達到(1)~(3)之條件。

第一次實驗我隨便設參數觀察電晶體特性 $\begin{cases} Vin = 0.5V \\ V_{b1}, V_{b2}, V_{b3} = 0.6V, 1.1V, 1.2V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{3um}{1um} \end{cases}$

subckt

element	0:mm4	0:mm3	0:mm2	0:mm1
model	0:p_18.1	0:p_18.1	0:n_18.1	0:n_18.1
region	Linear	Saturation	Linear	Linear
id	-1.1804u	-1.1804u	1.1804u	1.1804u
ibs	1.434e-22	15.9790a	-3.7407a	-2.327e-22
ibd	15.9787a	311.7440a	-5.7761a	-3.7403a
vgs	-600.0000m	-608.7799m	586.8459m	500.0000m
vds	-91.2201m	-1.6885	7.1593m	13.1541m
vbs	0.	91.2201m	-13.1541m	0.
vth	-498.3871m	-524.6581m	397.0543m	394.1874m
vdsat	-125.5368m	-115.0714m	179.7134m	120.8938m
vod	-101.6129m	-84.1217m	189.7916m	105.8126m
beta	209.3675u	206.2530u	933.9150u	932.2190u
gam eff	557.0847m	556.4343m	507.8133m	507.4459m
gm	13.3257u	16.9666u	6.4398u	10.5094u
gds	5.8945u	39.5736n	161.3395u	84.2451u
gmb	4.0488u	4.8637u	1.3479u	2.2219u
cdtot	7.3687f	3.0818f	28.0051f	23.4239f
cgtot	20.0989f	18.5220f	25.1546f	24.5737f
cstot	22.0212f	21.3145f	26.5183f	25.0319f
cbtot	12.1047f	10.4841f	12.7079f	12.7371f
cgs	16.3732f	15.7900f	14.1385f	15.2742f
cgd	2.4394f	1.0763f	10.3731f	8.5198f

從上述結果可看到仍有三顆電晶體在線性區，觀察 Vds 與 Vov (Vod)後調整讓 Vin 與 V_{b1} 更小、V_{b2} 和 V_{b3} 更大使電晶體的|Vgs|更靠近|Vth|，這樣可以輕易的讓 Vds 大於|Vgs|-|Vth|，更容易達到飽和狀態。

第二次實驗選擇 $\begin{cases} Vin = 0.47V \\ V_{b1}, V_{b2}, V_{b3} = 0.5V, 1.2V, 1.25V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{4um}{0.5um}, \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{8um}{0.5um} \end{cases}$

subckt

element	0:mm4	0:mm3	0:mm2	0:mm1
model	0:p_18.1	0:p_18.1	0:n_18.1	0:n_18.1
region	Saturation	Saturation	Saturation	Saturation
id	-2.6077u	-2.6077u	2.6077u	2.6077u
ibs	2.671e-22	22.9812a	-12.0415a	-4.812e-22
ibd	22.9807a	678.1721a	-27.8968a	-12.0406a
vgs	-550.0000m	-541.6680m	466.0700m	470.0000m
vds	-58.3320m	-1.6631	44.6805m	33.9300m
vbs	0.	58.3320m	-33.9300m	0.
vth	-504.7800m	-521.6042m	458.7178m	452.0710m
vdsat	-94.6408m	-81.8748m	72.9399m	77.4679m
vod	-45.2200m	-20.0638m	7.3523m	17.9290m
beta	1.1915m	1.1834m	2.5906m	2.5894m
gam eff	557.0847m	556.6653m	508.3882m	507.4460m
gm	37.1745u	48.0768u	46.3676u	42.0895u
gds	25.1769u	200.6062n	34.1073u	55.0842u
gmb	11.0703u	13.8124u	9.0262u	8.4367u
cdtot	14.1111f	8.0644f	7.0888f	8.2018f
cgtot	25.2941f	21.8829f	11.2998f	12.4247f
cstot	29.0428f	25.5811f	12.7818f	13.9666f
cbtot	23.1852f	19.7540f	12.4213f	12.6860f
cgs	18.6630f	15.6823f	7.6512f	8.5876f
cgd	4.1979f	2.8705f	1.8410f	2.2394f

這次實驗得到所有電晶體均於 saturation，但是從公式(1)可以得知|Av|與 gm 呈正相關，但由於 gm 均太小，因此增益目前是很小的，而 gm 小是因為 Vov 太小所導致的，因此下一次實驗方向應將 Vov 條大並且讓四顆電晶體還在飽和區運作，因此我們應該讓 Vin 與 V_{b1} 更大一些，且 V_{b2} 和

V_{b3} 再更小一些；另外，我讓 pmos 與 nmos 的尺寸調整呈 2 比 1 的比例是因為電流公式 $Id = \frac{1}{2}\mu nCox\left(\frac{W}{L}\right)_n (Vov_n)^2 = \frac{1}{2}\mu pCox\left(\frac{W}{L}\right)_p (Vov_p)^2$，因為 $\mu_n \cong 2\mu_p$，所以將尺寸比例設成 2 比 1 有助於調整 Vov 時更對稱，能更有效的保持在 saturation region。

第三次實驗選擇 $\begin{cases} Vin = 0.5V \\ V_{b1}, V_{b2}, V_{b3} = 0.65V, 1.05V, 1.19V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2um}{0.5um}, \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{4um}{0.5um} \end{cases}$

subckt				
element	0:mm4	0:mm3	0:mm2	0:mm1
model	0:p_18.1	0:p_18.1	0:n_18.1	0:n_18.1
region	Saturation	Saturation	Saturation	Saturation
id	-3.5964u	-3.5964u	3.5964u	3.5964u
ibs	4.094e-22	26.2668a	-28.3945a	-7.998e-22
ibd	26.2660a	274.2111a	-117.0725a	-28.3929a
vgs	-610.0000m	-630.0227m	517.2198m	500.0000m
vds	-119.9773m	-1.1325	414.7006m	132.7802m
vbs	0.	119.9773m	-132.7802m	0.
vth	-510.9441m	-544.5411m	477.0654m	455.4074m
vdsat	-130.6890m	-122.8996m	89.9785m	91.2746m
vod	-99.0559m	-85.4816m	40.1544m	44.5926m
beta	577.0390u	565.4440u	1.2950m	1.2929m
gam eff	557.0846m	556.2355m	511.0384m	507.4460m
gm	43.8146u	50.9125u	64.8881u	62.1118u
gds	8.9273u	267.3019n	1.1591u	4.0469u
gmb	12.8818u	14.0878u	11.4385u	12.1987u
cdtot	6.2668f	4.3083f	2.8343f	3.2267f
cgtot	13.6489f	13.0826f	6.5057f	6.7011f
cstot	16.4302f	15.8970f	7.7724f	8.0800f
cbtot	11.6432f	10.0930f	5.8243f	6.3533f
cgs	10.9114f	10.6163f	5.0624f	5.2355f
cgd	1.7969f	1.4353f	729.8335a	771.2137a

這次實驗可以看到 Vov 與 gm 都有明顯的上升且都維持在飽和狀態，且得到的|Av|=658.0787V/V、output swing=1.2653V、I_D 也小於 5uA，已達成所有條件。

(b)

subckt				
element	0:mm4	0:mm3	0:mm2	0:mm1
model	0:p_18.1	0:p_18.1	0:n_18.1	0:n_18.1
region	Saturation	Saturation	Saturation	Saturation
id	-3.5964u	-3.5964u	3.5964u	3.5964u
ibs	4.094e-22	26.2668a	-28.3945a	-7.998e-22
ibd	26.2660a	274.2111a	-117.0725a	-28.3929a
vgs	-610.0000m	-630.0227m	517.2198m	500.0000m
vds	-119.9773m	-1.1325	414.7006m	132.7802m
vbs	0.	119.9773m	-132.7802m	0.
vth	-510.9441m	-544.5411m	477.0654m	455.4074m
vdsat	-130.6890m	-122.8996m	89.9785m	91.2746m
vod	-99.0559m	-85.4816m	40.1544m	44.5926m
beta	577.0390u	565.4440u	1.2950m	1.2929m
gam eff	557.0846m	556.2355m	511.0384m	507.4460m
gm	43.8146u	50.9125u	64.8881u	62.1118u
gds	8.9273u	267.3019n	1.1591u	4.0469u
gmb	12.8818u	14.0878u	11.4385u	12.1987u
cdtot	6.2668f	4.3083f	2.8343f	3.2267f
cgtot	13.6489f	13.0826f	6.5057f	6.7011f
cstot	16.4302f	15.8970f	7.7724f	8.0800f
cbtot	11.6432f	10.0930f	5.8243f	6.3533f
cgs	10.9114f	10.6163f	5.0624f	5.2355f
cgd	1.7969f	1.4353f	729.8335a	771.2137a

(c)

```
****     small-signal transfer characteristics

 v(vout)/vvin                                    = -658.0787
 input resistance at                vvin         =   1.000e+20
 output resistance at v(vout)                    =    11.1494x
```

$$|\mathrm{Av}| = \left|\frac{\mathrm{vout}}{\mathrm{vin}}\right| = 658.0787\left(\frac{\mathrm{V}}{\mathrm{V}}\right) = 20\log(658.0787)\ \mathrm{dB} = 56.36555668\mathrm{dB} > 45\mathrm{dB}$$

(d) 由(a)部分的推導可得知

$$R_{out} = (\mathrm{g}_{\mathrm{m2}} + g_{mb2})r_{o2}r_{o1} \parallel (\mathrm{g}_{\mathrm{m3}} + g_{mb3})r_{o3}r_{o4}$$
$$= 16271685.37 \parallel 27239127.09 = 10.186583M\Omega$$

$$|\mathrm{Av}| = \mathrm{g}_{\mathrm{m1}}R_{out} = g_{m1}((\mathrm{g}_{\mathrm{m2}} + g_{mb2})r_{o2}r_{o1} \parallel (\mathrm{g}_{\mathrm{m3}} + g_{mb3})r_{o3}r_{o4})$$
$$= 62.1118\mathrm{u} * 10.186583M = 632.707\left(\frac{V}{V}\right)$$

$$\begin{cases} Error(Rout) = \left|\dfrac{11.1494 - 10.186583}{10.186583}\right| = 9.45\% \\ Error(|Av|) = \left|\dfrac{658.0787 - 632.707}{632.707}\right| = 4.01\% \end{cases}$$

雖然這次的模型中已經加入了 body effect 來計算，但可能有些現代製成或模型上的差異導致手算出來的結果還是有些誤差。

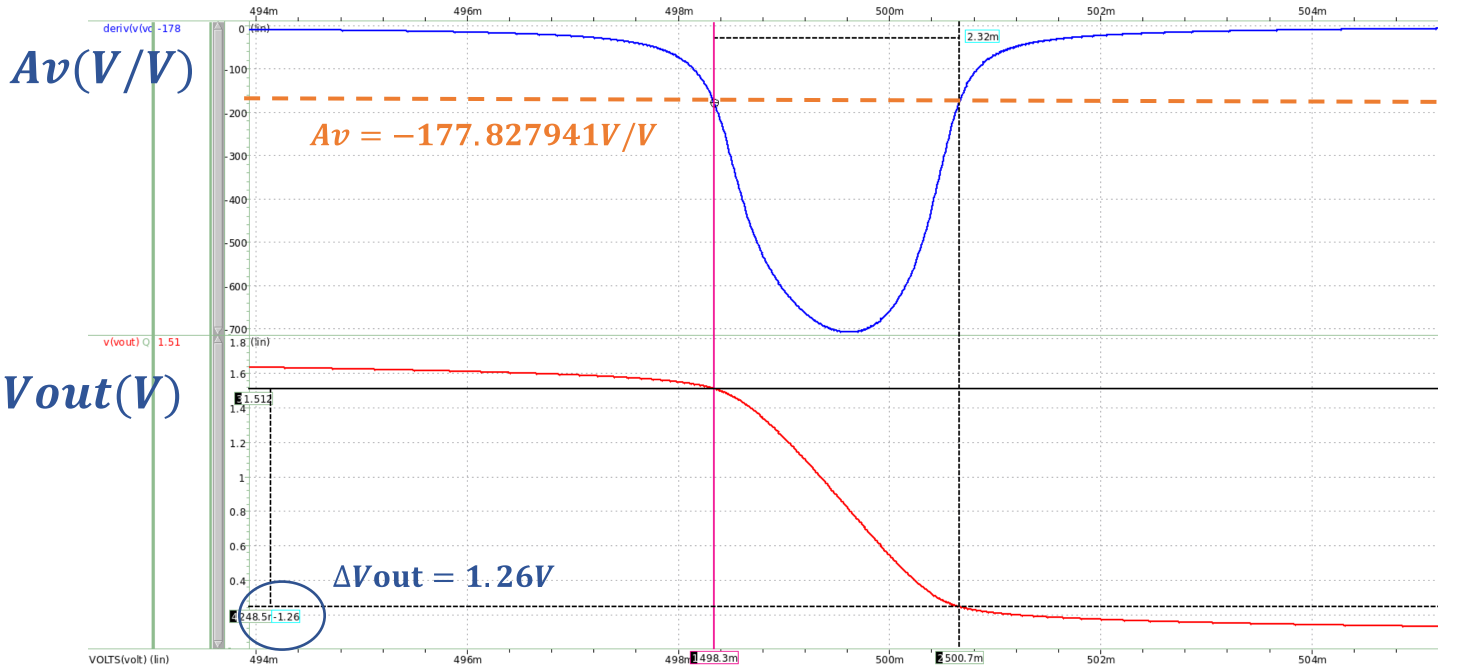
(e)

```
output_swing_upper_bound output_swing_lower_bound output_swing
temper             alter#
  1.5116               0.2464               1.2653           25.0000
```

模擬結果顯示 $\begin{cases} output\ swing\ upper\ bound = 1.5116V \\ output\ swing\ lower\ bound = 0.2464V \\ output\ swing = 1.5116 - 0.2464 = 1.2653V \end{cases}$

Vin(V)	Rin(Ω)	Rout(Ω)	Av(V/V)
498.3000m	1.000e+20	2.6618x	-158.1914
498.3100m	1.000e+20	2.7494x	-163.4127
498.3200m	1.000e+20	2.8410x	-168.8652
498.3300m	1.000e+20	2.9365x	-174.5553
498.3400m	1.000e+20	3.0360x	-180.4885
498.3500m	1.000e+20	3.1398x	-186.6696
498.3600m	1.000e+20	3.2477x	-193.1020
498.3700m	1.000e+20	3.3599x	-199.7877
498.3800m	1.000e+20	3.4764x	-206.7273
498.3900m	1.000e+20	3.5971x	-213.9193
498.4000m	1.000e+20	3.7220x	-221.3602
500.6000m	1.000e+20	3.7846x	-220.8741
500.6100m	1.000e+20	3.6432x	-212.5771
500.6200m	1.000e+20	3.5072x	-204.5971
500.6300m	1.000e+20	3.3767x	-196.9380
500.6400m	1.000e+20	3.2516x	-189.6007
500.6500m	1.000e+20	3.1319x	-182.5834
500.6600m	1.000e+20	3.0176x	-175.8817
500.6700m	1.000e+20	2.9086x	-169.4896
500.6800m	1.000e+20	2.8047x	-163.3992
500.6900m	1.000e+20	2.7057x	-157.6014
500.7000m	1.000e+20	2.6116x	-152.0861

經過 Vin sweep from 0 to 1.8V, step=10uV 後，可以在 lis 檔中尋找出對應|Av|值在大於 45dB 也就是 177.827941V/V 下的 Vin 值，從表中可以觀察到邊界位於 0.498335V 與 0.500655V 附近，在波形圖中打開 deriv(V(Vout))與 Vout 的波形圖，用 cursor 對準 Av 波形圖縱軸值為-177.827941V/V 的位置，對下去 Vout 波形圖發現 x 軸值吻合，再利用水平 cursor 量測得到 output swing = 1.26V。



因為 waveview 中小數點精確度不如模擬高，因此波形圖量測出來可能有微小誤差。

(f) 由(a)小題得知

$$\text{output swing} = V_{DD} - V_{ov4} - V_{ov3} - V_{ov2} - V_{ov1}$$
$$= 1.8 - 0.0990559 - 0.0854816 - 0.0401544 - 0.0445926 = 1.5307155V$$

$$\text{Error(output swing)} = \left|\frac{1.2653 - 1.5307155}{1.5307155}\right| = 17.34\%$$

此誤差來源可能是因為 simulation 的 swing 是用 cursor 在達成題目要求的情況下抓出來的，並非固定在同個情況，但手算則是固定在同個 Vin 值算出來的結果。

(g)

	specification	simulation	hand-calculation
V_{DD}	1.8V		
M_1 (W/L, m)	–	W/L=2um/0.5um, m=1	
M_2 (W/L, m)	–	W/L=2um/0.5um, m=1	
M_3 (W/L, m)	–	W/L=4um/0.5um, m=1	
M_4 (W/L, m)	–	W/L=4um/0.5um, m=1	
$V_{in,DC}$	–	0.50V	
V_{b1}	–	0.65V	
V_{b2}	–	1.05V	
V_{b3}	–	1.19V	
I_D	< 5μA	3.5964μA	–
gain $\lvert A_v \rvert$	≥ 45dB	56.356dB	56.024dB
output impedence	–	11.1494MΩ	10.1866MΩ
output swing	≥ 1.2V	1.2653V	1.5307V

Question 2. – Cascade Amplifier

(a) CS stage

i. 因為 MOS 電晶體的結構導致若通道長度 L 越長，通道等校電阻值 ro 就會愈高，且因為這題的 Rout = ro，因此對於增益|Av|=gm*ro 來看，設計一個長通道的電晶體可以提高增益；將增益值變大之後，我開始調整 Vin 想辦法把 Vout 控制在 0.499V 到 0.501V 之間，因為 common source 的特性是 Vin 上升，Vout 下降，因此順著這個特性就能慢慢找到平衡點。因此我最終的 Vin,dc＝0.41932V、W/L＝8um/4um，得到的 Vout,dc＝0.5004567V、|Av1|=165.61V/V。

ii.

```
subckt
element  0:mm1
model    0:n_18.1
region   Saturation
 id         3.0000u
 ibs      -4.968e-22
 ibd      -318.7704a
 vgs       419.3200m
 vds       500.4567m
 vbs         0.
 vth       330.2636m
 vdsat     102.9520m
 vod        89.0564m
 beta      599.9491u
 gam eff   507.4459m
 gm         45.1356u
 gds       272.5390n
 gmb         9.1562u
 cdtot      11.2385f
 cgtot     206.6302f
 cstot     214.0882f
 cbtot      72.5464f
 cgs       184.6051f
 cgd         2.6654f

****** operating point status is all       simulation time is     0.
   node        =voltage      node     =voltage      node     =voltage

+0:vdd      =   1.8000  0:vin      = 419.3200m 0 vout1    = 500.4567m
+0:vss      =   0.
```

iii.

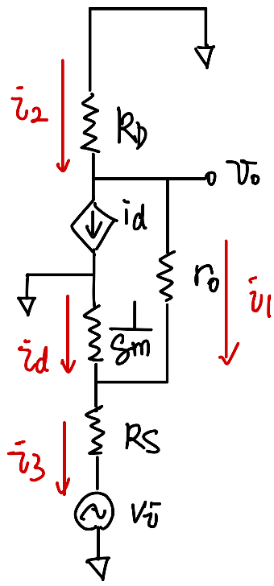
```
****     small-signal transfer characteristics

      v(vout1)/vvin                                  = -165.6102
      input resistance at               vvin         =   1.000e+20
      output resistance at v(vout1)                  =     3.6692x
```

iv. $|A_{v1}| = g_m r_o = \frac{g_m}{g_{ds}} = \frac{45.1356u}{272.5390n} = 165.6115272V/V$

手算之增益值與模擬之結果相近，可能稍有小數點取位數或模型公式之誤差存在。

(b) CG stage

i. 我們可以從許多面向來分析 Common Gate 的特性，首先分析小訊號模型由右圖可以得知 i2=i1+id=i3，以下推導|Av|之公式。(未包含 body effect)



Small signal model

$$\Rightarrow \mathrm{id} = -\mathrm{gm} * \mathrm{v1};\ \mathrm{i2} = -\frac{\mathrm{vo}}{R_D}; i1 = \frac{vo - v1}{ro}; i3 = \frac{v1 - vin}{R_S}$$

$$\Rightarrow -\mathrm{gm} * \mathrm{v1} + \frac{vo - v1}{ro} = -\frac{\mathrm{vo}}{R_D} \Rightarrow vo\left(\frac{1}{ro} + \frac{1}{R_D}\right) = v1\left(gm + \frac{1}{ro}\right)$$

$$\Rightarrow \mathrm{vo} = \mathrm{v1}\left(\mathrm{gm} + \frac{1}{\mathrm{ro}}\right)(\mathrm{ro}\ \parallel R_D)$$

$$\because \mathrm{i2} = \mathrm{i3}$$

$$\Rightarrow \frac{\mathrm{v1} - \mathrm{vin}}{R_S} = -\frac{\mathrm{vo}}{R_D}$$

$$\Rightarrow vin = \left(\frac{v1}{R_S} + \frac{\mathrm{vo}}{R_D}\right)R_S = v1 + \frac{R_S}{R_D}vo$$

$$= \mathrm{v1}(\left(\mathrm{gm} + \frac{1}{\mathrm{ro}}\right)(\mathrm{ro} \parallel R_D)\frac{R_S}{R_D} + 1)$$

$$\Rightarrow \mathrm{Av} = \frac{\mathrm{vo}}{\mathrm{vin}} = \{\left(\mathrm{gm} + \frac{1}{\mathrm{ro}}\right)(\mathrm{ro}\ \parallel R_D)\}/\{\left(\mathrm{gm} + \frac{1}{\mathrm{ro}}\right)(\mathrm{ro} \parallel R_D)\frac{R_S}{R_D} + 1)\}$$

$$\because \left(\mathrm{gm} + \frac{1}{\mathrm{ro}}\right)(\mathrm{ro}\ \parallel R_D) = \frac{gm * ro * R_D}{ro + R_D} + \frac{R_D}{ro + R_D} = \frac{R_D(1 + gmro)}{ro + R_D}$$

$$\therefore |\mathrm{Av}| = |\frac{\mathrm{vo}}{\mathrm{vin}}| = |\frac{\left\{\frac{R_D(1 + gmro)}{ro + R_D}\right\}}{\left\{\frac{R_D(1 + gmro)}{ro + R_D} * \frac{R_S}{R_D} + 1\right\}}| = |\frac{R_D(1 + gmro)}{ro + R_D + R_s + gmroR_s}|$$

由上述式子可以判斷當 R_D 上升時，|Av|也會跟著上升。

在調整參數方面，我選定較小的 L 值但較大的 W/L 值，一方面可以降低 Vth 使可調控 Vov 提升，另一方面也可以增加電流 Id 的值；R_D 的部分有兩個面向需要做取捨，分別是 Vout,dc 的值與|Av|的大小，從上述分析來看要增加|Av|勢必要增加 R_D，但是若 R_D 值太大會造成 IR drop 太大，進而造成 Vout 可能太小無法達到 saturation 的狀態，因此我試了幾次調整至|Av|值可以達到超過 10V/V 但又可以達到飽和的 R_D 值；接下來，因為電流還沒達到要求的 10uA，且已知 Vb 與 Id 呈正相關，因此利用微調 Vb 的值可以精準的控制 Id 的電流值，達到要求。最終選定的 Vb = 1.0475V、R_D = 100kΩ、W/L = 8um/0.8um。

ii.

```
subckt
element  0:mm2
model    0:n_18.1
region   Saturation
 id         10.0439u
 ibs       -318.4817a
 ibd       -506.7693a
 vgs        547.5000m
 vds        295.6087m
 vbs       -500.0000m
 vth        495.9257m
 vdsat       94.0189m
 vod         51.5743m
 beta         3.1682m
 gam eff    519.9006m
 gm         177.1970u
 gds          3.1199u
 gmb         27.2853u
 cdtot       10.5756f
 cgtot       40.5341f
 cstot       43.6976f
 cbtot       23.5808f
 cgs         33.9086f
 cgd          2.8530f
```

iii.

```
 ****     small-signal transfer characteristics

  v(vout)/vvin1                            =    15.8228
  input resistance at               vvin1  =     6.3200k
  output resistance at v(vout)             =    76.2212k
```

iv. 以下推導會加入 body effect 之影響

(1) 由上頁的小訊號模型可以得知 $\mathrm{Rin}=\frac{\mathrm{v1}}{-\mathrm{i3}}$

$$\Rightarrow \text{上頁推導 vin} = \mathrm{v1}\left(\left(\mathrm{g_m}+\mathrm{g_{mb}}+\frac{1}{\mathrm{ro}}\right)(\mathrm{ro}\parallel R_D)\frac{R_S}{R_D}+1\right)$$

$$\Rightarrow \mathrm{i3}=\frac{\mathrm{v1}-\mathrm{vin}}{\mathrm{R_S}} \text{將上式 vin 帶入 i3} = \frac{\mathrm{v1}\left(1-\left(\mathrm{g_m}+\mathrm{g_{mb}}+\frac{1}{\mathrm{ro}}\right)(\mathrm{ro}\parallel R_D)\frac{R_S}{R_D}-1\right)}{\mathrm{R_S}}$$

$$\Rightarrow \mathrm{Rin}=-\frac{\mathrm{v1}}{\mathrm{i3}}=\frac{\mathrm{R_S}}{\left(\mathrm{g_m}+\mathrm{g_{mb}}+\frac{1}{\mathrm{ro}}\right)(\mathrm{ro}\parallel \mathrm{R_D})\left(\frac{R_S}{R_D}\right)}=\frac{R_D}{(\mathrm{g_m}+\mathrm{g_{mb}}+g_{ds})(ro\parallel R_D)}$$

$$=\frac{100k}{(177.1970u+3.1199u+27.2853u)\left(\frac{1}{3.1199u+\frac{1}{100k}}\right)}=6.3197307k\Omega$$

(2) 由小訊號模型推得

$$\Rightarrow \mathrm{Rout}=\mathrm{ro}\parallel \mathrm{R_D}=\frac{roR_D}{ro+R_D}=\frac{\frac{R_D}{gds}}{\frac{1}{gds}+R_D}=\frac{R_D}{1+gds*R_D}=\frac{100k}{1+3.1199u*100k}=76.2209314k\Omega$$

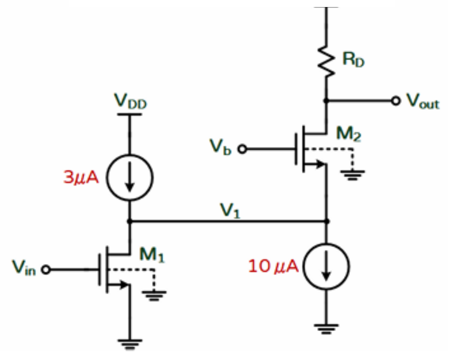
$$(3)\ |Av| = \left|\frac{R_D(1+(gm+gmb)*ro)}{ro+R_D+R_S+(gm+gmb)*ro*Rs}\right| = \left|\frac{100k\left(1+\frac{177.1970u+27.2853u}{3.1199u}\right)}{100k+\frac{1}{3.1199u}}\right| = 15.823459\left(\frac{V}{V}\right)\ if\ Rs = 0$$

$$\begin{cases} Error(Rin) = \left|\dfrac{6.3200 - 6.3197307}{6.3197307}\right| = 0.00426\% \\ Error(Rout) = \left|\dfrac{76.2212 - 76.2209314}{76.2209314}\right| = 0.00035\% \\ Error(|Av|) = \left|\dfrac{15.8228 - 15.823459}{15.823459}\right| = 0.00416\% \end{cases}$$

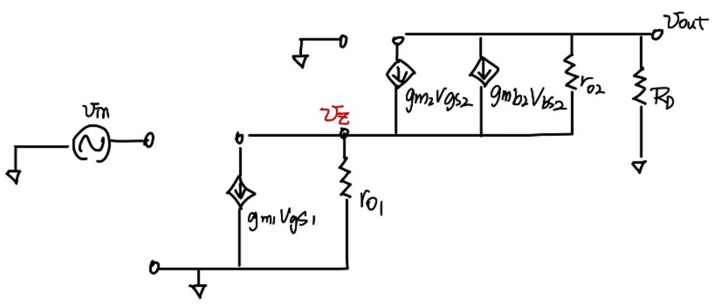
因為此電路存在 body effect，加入考慮之後誤差變得很小。但是，電路中可能含有一些非理想特性，或是現代製成的進步導致手算公式與模擬使用的模型有些差異，因此造成些微誤差。

(c) Cascade Structure

Cascade (CS+CG)



Small signal model



i.

```
****** operating point status is all       simulation time is     0.
    node        =voltage        node      =voltage        node      =voltage

+0:v1          =  500.2779m 0:vb          =    1.0475   0:vdd          =    1.8000
+0:vin         =  419.3200m 0:vout        =  800.0047m 0:vss          =    0.
```

可以看到 V1 仍維持在 0.5V 附近。

ii.

```
 ****      small-signal transfer characteristics

      v(vout)/vvin                              =    -4.5057
      input resistance at              vvin     =   1.000e+20
      output resistance at v(vout)              =    99.9594k
```

從模擬結果可以看到增益|Av|=4.5057 比原本 CS 與 CG stage 個別增益相乘小了非常多，原本的|Av|在 CS stage 為 165.6102V/V、CG stage 為 15.8228V/V，但接在一起之後的增益為 4.5057V/V，這是因為從 CS 輸出端看進去的 Rout 比單 CS 級看進去的阻抗小了非常多，造成這個現象的原因是因為原本的阻抗 ro1 並聯了右側接的 CG 級的阻抗，導致整體阻抗大幅減少，造成 CS 級的增益值|Av1|也跟著大幅下降，詳細的分析如下頁。

首先分析從 V1 看進 CG 級的輸出阻抗，V1 在小訊號模型中令為 Vz 點
令 $g'_{m2} = g_{m2} + g_{mb2}$

$$\because \begin{cases} \mathrm{i_Z} = \dfrac{\mathrm{V_{out}}}{\mathrm{R_D}} = \mathrm{g'_{m2}V_Z} + \dfrac{\mathrm{V_Z - V_{out}}}{\mathrm{r_{o2}}} \\ Vout = i_Z R_D \end{cases}$$

$$\therefore \mathrm{i_z} = g'_{m2}Vz + \frac{Vz - i_z R_D}{r_{o2}}$$

$$\Rightarrow \mathrm{i_z} r_{o2} = g'_{m2} r_{o2} Vz + Vz - i_z R_D$$

$$\Rightarrow \mathrm{i_z}(r_{o2} + R_D) = Vz(1 + g'_{m2} r_{o2})$$

$$\Rightarrow \mathrm{R} = \frac{\mathrm{r_{o2} + R_D}}{1 + \mathrm{g'_{m2}} r_{o2}}$$

又從原本的 CS stage 看進去的輸出阻抗為 ro1，所以新的 CS stage 輸出阻抗

$$\mathrm{Rout} = \mathrm{R} \parallel \mathrm{r_{o1}} = \frac{\mathrm{r_{o2} + R_D}}{1 + \mathrm{g'_{m2}} r_{02}} \parallel \mathrm{r_{o1}} = \frac{0.324701245\mathrm{M} + 100\mathrm{k}}{1 + (176.6593\mathrm{u} + 3.0848\mathrm{u}) * 0.3241701245\mathrm{M}}$$
$$= 7.165816786\ \mathrm{k\Omega} \ll 3.6692\ \mathrm{M\Omega}$$

由上述推導可見，cascade 架構的 CS stage 輸出阻抗比原先單一 CS stage 小了非常多，因此整體 CS stage 的增益也下降非常多，造成整個 cascade 架構的增益變非常小。

iii. 由上頁的小訊號模型來分析

$$\begin{cases} -\dfrac{Vout}{R_D} = \dfrac{Vout - Vz}{r_{o2}} + g_{mb2}(-Vz) + g_{m2}(-Vz) \quad \because Vgs_2 = -Vz \\ -\dfrac{Vout}{R_D} = \dfrac{Vz}{r_{o1}} + g_{m1}Vin \Rightarrow r_{o1}\left(-\dfrac{Vout}{R_D} - g_{m1}Vin\right) = Vz \end{cases}$$

將上述第二式帶入第一式

$$\Rightarrow -\frac{Vout}{R_D} = \frac{Vout}{r_{o2}} + Vz\left(-\frac{1}{r_{o2}} - g_{mb2} - g_{m2}\right)$$
$$= \frac{Vout}{r_{o2}} + r_{o1}\left(\frac{Vout}{R_D} + g_{m1}Vin\right)\left(\frac{1}{r_{o2}} + g_{mb2} + g_{m2}\right)$$

$$\Rightarrow Vout\left(-\frac{1}{R_D} - \frac{1}{r_{o2}}\right) = r_{o1}\left(\frac{Vout}{R_D r_{o2}} + \frac{g'_{m2}Vout}{R_D} + \frac{g_{m1}Vin}{r_{o2}} + g_{m1}g'_{m2}Vin\right)$$

$$\Rightarrow -\frac{Vout(r_{o2} + R_D + r_{o1} + r_{o1}r_{o2}g'_{m2})}{R_D r_{o2}} = \frac{Vin(g_{m1}r_{o1} + g_{m1}g'_{m2}r_{o2}r_{o1})}{r_{o2}}$$

$$\Rightarrow Av = \frac{Vout}{Vin} = -\frac{g_{m1}r_{o1}R_D + g_{m1}g'_{m2}r_{o1}r_{o2}R_D}{r_{o1} + r_{o2} + R_D + g'_{m2}r_{o1}r_{o2}} = -\frac{16560312.78 + 964930795.1}{2.7880934.9}$$
$$= -4.504713128\left(\frac{V}{V}\right)$$

從上式得出 Av＝-4.504713128V/V，與模擬的 Av=-4.5057V/V 差距非常小，表示此模型加入 body effect 的影響後是很精準的。(以上的參數都在.lis 檔中取得)

```
subckt
element  0:mm1      0:mm2
model    0:n_18.1   0:n_18.1
region   Saturation Saturation
 id         3.0000u   10.0000u
 ibs      -4.968e-22 -318.6587a
 ibd      -318.6565a -509.5694a
 vgs       419.3200m  547.2221m
 vds       500.2779m  299.7268m
 vbs         0.      -500.2779m
 vth       330.2642m  495.9243m
 vdsat     102.9517m   93.8573m
 vod        89.0558m   51.2978m
 beta      599.9491u    3.1683m
 gam eff   507.4459m  519.9068m
 gm         45.1350u  176.6593u
 gds       272.5492n    3.0848u
 gmb         9.1562u   27.1968u
```

(d)

	specification	simulation	hand-calculation
Fig. 2.(a) Common-Source stage			
V_{DD}	1.8V		
current source load	3μA		
M_1 (W/L, m)	–	W/L=8um/4um, m=1	
$V_{in,DC}$	–	0.41932V	
$V_{out1,DC}$	0.5V±10mV	0.5004567V	–
gain $\|A_{v1}\|$	≥ 100V/V	165.6102V/V	165.6115272V/V
output impedence	–	3.6692MΩ	–
Fig. 2.(b) Common-Gate stage			
V_{DD}	1.8V		
$V_{in1,DC}$	0.5V		
M_2 (W/L, m)	–	W/L=8um/0.8um, m=1	
V_b	–	1.0475V	
R_D	–	100kΩ	
I_D	10μA±1%	10.0439uA	–
gain $\|A_{v2}\|$	≥ 10V/V	15.8228V/V	15.823459V/V
input impedence	–	6.3200kΩ	6.3197307kΩ
output impedence	–	76.2212kΩ	76.2209314kΩ
Fig. 3. Cascade CS-CG amplifier			
V_{DD}	1.8V		
DC bias (V_1)	–	0.5002779V	–
overall gain $\|A_v\|$	–	4.5057V/V	4.504713128V/V

Question 3. – Comparison between "cascode" & "cascade" structure

1. From the perspective of "gain amplitude", the "cascode" structure is preferred over the "cascade" structure. This preference arises because the output impedance of the cascade structure can be influenced by different stages when connected to each other, potentially leading to degradation in the overall gain. However, in the cascode structure, as derived from the formula above, the gain amplitude can be increased by stacking MOSFETs. In summary, the "cascode" structure outperforms the "cascade" structure in terms of gain amplitude.

2. From the perspective of "output swing", the "cascade" structure typically offers a larger output swing compared to the "cascode" structure. This is because the output swing of each stage adds up, allowing for a larger overall swing. The cascode structure may have a limited output swing compared to the cascade structure due to the voltage headroom required for the cascode transistor.