

# Analog IC Design Homework 3 Report

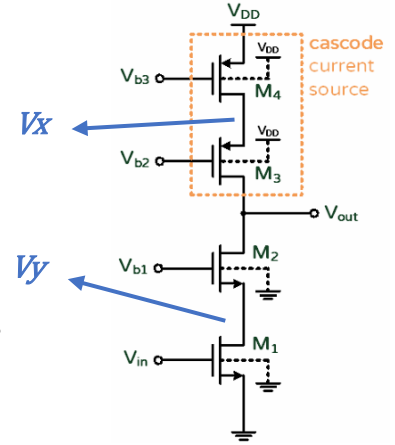
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## Question 1. – Cascode Amplifier

此題所要求的條件如下

- (1)  $I_D < 5\mu A$
- (2)  $|Av| \geq 45dB$
- (3) output swing  $\geq 1.2V$
- (4) M1, M2, M3, M4 operates in saturation region

我們要選定四個電晶體的 W/L、 $V_{in,dc}$ 、 $V_{b1} \sim V_{b3}$  的值來達成目標。



- (a) 首先，在設計之前需要做最初步的電路分析，首先四個電晶體必須操作在飽和狀態，我們可以列出下列式子

$$\begin{cases} V_x - V_{b3} \leq V_{th4} \Rightarrow M4 \text{ saturates} \\ V_{out} - V_{b2} \leq V_{th3} \Rightarrow M3 \text{ saturates} \\ V_{b1} - V_{out} \leq V_{th2} \Rightarrow M2 \text{ saturates} \\ V_{in} - V_y \leq V_{th1} \Rightarrow M1 \text{ saturates} \end{cases}$$

再來，分析電路增益公式（先考慮相對關係，詳細 body effect 公式推導後續說明）

$$\begin{aligned} R_{on} &\approx g_{m2}r_{o2}r_{o1} \\ R_{op} &\approx g_{m3}r_{o3}r_{o4} \\ R_{out} &= R_{op} \parallel R_{on} \\ |Av| &= g_{m1}R_{out} = g_{m1}(g_{m2}r_{o2}r_{o1} \parallel g_{m3}r_{o3}r_{o4}) \end{aligned}$$

假設  $g_{m1} = g_{m2} = g_{m3} = g_{m4} = g_m$ 、 $r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_o$

$$|Av| \cong \frac{1}{2} g_m^2 r_o^2 \dots (1)$$

$$g_m = u_n C_{ox} \left( \frac{W}{L} \right) (V_{ov})^2 \dots (2)$$

最後分析

$$\begin{aligned} \text{output swing} &= V_{DD} - V_{ov4} - V_{ov3} - V_{ov2} - V_{ov1} \geq 1.2V \\ &\Rightarrow V_{ov4} + V_{ov3} + V_{ov2} + V_{ov1} \leq 0.6V \dots (3) \end{aligned}$$

由上述式子可以推論當  $V_{ov} \downarrow \left\{ \begin{aligned} &\Rightarrow g_m \downarrow \Rightarrow I_D \downarrow \Rightarrow |Av| \downarrow, \text{ 因此衡量到 output swing 必須大且 } |Av| \text{ 必須} \\ &\Rightarrow \text{output swing} \uparrow \end{aligned} \right.$

大於 45dB 大約等於 177.827941V/V 的情況下，我們要讓  $V_{ov}$  調整到適合的值來保證所有電晶體操作在 saturation 且達到(1)~(3)之條件。

第一次實驗我隨便設參數觀察電晶體特性

$$\begin{cases} V_{in} = 0.5V \\ V_{b1}, V_{b2}, V_{b3} = 0.6V, 1.1V, 1.2V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{3\mu m}{1\mu m} \end{cases}$$

subckt				
element	0:mm4	0:mm3	0:mm2	0:mm1
model	0:p_18.1	0:p_18.1	0:n_18.1	0:n_18.1
region	Linear	Saturation	Linear	Linear
id	-1.1804u	-1.1804u	1.1804u	1.1804u
ibs	1.434e-22	15.9790a	-3.7407a	-2.327e-22
ibd	15.9787a	311.7440a	-5.7761a	-3.7403a
vgs	-600.0000m	-608.7799m	586.8459m	500.0000m
vds	-91.2201m	-1.6885	7.1593m	13.1541m
vbs	0.	91.2201m	-13.1541m	0.
vth	-498.3871m	-524.6581m	397.0543m	394.1874m
vdsat	-125.5368m	-115.0714m	179.7134m	120.8938m
vod	-101.6129m	-84.1217m	189.7916m	105.8126m
beta	209.3675u	206.2530u	933.9150u	932.2190u
gam eff	557.0847m	556.4343m	507.8133m	507.4459m
gm	13.3257u	16.9666u	6.4398u	10.5094u
gds	5.8945u	39.5736n	161.3395u	84.2451u
gmb	4.0488u	4.8637u	1.3479u	2.2219u
cdtot	7.3687f	3.0818f	28.0051f	23.4239f
cgtot	20.0989f	18.5220f	25.1546f	24.5737f
cstot	22.0212f	21.3145f	26.5183f	25.0319f
cbtot	12.1047f	10.4841f	12.7079f	12.7371f
cgs	16.3732f	15.7900f	14.1385f	15.2742f
cgd	2.4394f	1.0763f	10.3731f	8.5198f

從上述結果可看到仍有三顆電晶體在線性區，觀察 Vds 與 Vov (Vod) 後調整讓 Vin 與 Vb1 更小、Vb2 和 Vb3 更大使電晶體的 |Vgs| 更靠近 |Vth|，這樣可以輕易的讓 Vds 大於 |Vgs| - |Vth|，更容易達到飽和狀態。

第二次實驗選擇

$$\begin{cases} V_{in} = 0.47V \\ V_{b1}, V_{b2}, V_{b3} = 0.5V, 1.2V, 1.25V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{4\mu m}{0.5\mu m}, \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{8\mu m}{0.5\mu m} \end{cases}$$

subckt				
element	0:mm4	0:mm3	0:mm2	0:mm1
model	0:p_18.1	0:p_18.1	0:n_18.1	0:n_18.1
region	Saturation	Saturation	Saturation	Saturation
id	-2.6077u	-2.6077u	2.6077u	2.6077u
ibs	2.671e-22	22.9812a	-12.0415a	-4.812e-22
ibd	22.9807a	678.1721a	-27.8968a	-12.0406a
vgs	-550.0000m	-541.6680m	466.0700m	470.0000m
vds	-58.3320m	-1.6631	44.6805m	33.9300m
vbs	0.	58.3320m	-33.9300m	0.
vth	-504.7800m	-521.6042m	458.7178m	452.0710m
vdsat	-94.6408m	-81.8748m	72.9399m	77.4679m
vod	-45.2200m	-20.0638m	7.3523m	17.9290m
beta	1.1915m	1.1834m	2.5906m	2.5894m
gam eff	557.0847m	556.6653m	508.3882m	507.4460m
gm	37.1745u	48.0768u	46.3676u	42.0895u
gds	25.1769u	200.6062n	34.1073u	55.0842u
gmb	11.0703u	13.8124u	9.0262u	8.4367u
cdtot	14.1111f	8.0644f	7.0888f	8.2018f
cgtot	25.2941f	21.8829f	11.2998f	12.4247f
cstot	29.0428f	25.5811f	12.7818f	13.9666f
cbtot	23.1852f	19.7540f	12.4213f	12.6860f
cgs	18.6630f	15.6823f	7.6512f	8.5876f
cgd	4.1979f	2.8705f	1.8410f	2.2394f

這次實驗得到所有電晶體均於 saturation，但是從公式(1)可以得知 |Av| 與 gm 呈正相關，但由於 gm 均太小，因此增益目前是很小的，而 gm 小是因為 Vov 太小所導致的，因此下一次實驗方向應將 Vov 條大並且讓四顆電晶體還在飽和區運作，因此我們應該讓 Vin 與 Vb1 更大一些，且 Vb2 和

$V_{b3}$  再更小一些；另外，我讓 pmos 與 nmos 的尺寸調整呈 2 比 1 的比例是因為電流公式  $I_d = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_{ov_n})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_{ov_p})^2$ ，因為  $\mu_n \cong 2\mu_p$ ，所以將尺寸比例設成 2 比 1 有助於調整  $V_{ov}$  時更對稱，能更有效的保持在 saturation region。

$$\text{第三次實驗選擇} \begin{cases} V_{in} = 0.5V \\ V_{b1}, V_{b2}, V_{b3} = 0.65V, 1.05V, 1.19V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2\mu m}{0.5\mu m}, \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{4\mu m}{0.5\mu m} \end{cases}$$

```
subckt
element 0:mm4      0:mm3      0:mm2      0:mm1
model   0:p_18.1   0:p_18.1   0:n_18.1   0:n_18.1
region  Saturation Saturation Saturation Saturation
id      -3.5964u    -3.5964u    3.5964u    3.5964u
ibs     4.094e-22   26.2668a   -28.3945a  -7.998e-22
ibd     26.2660a    274.2111a  -117.0725a -28.3929a
vgs     -610.0000m   -630.0227m 517.2198m  500.0000m
vds     -119.9773m   -1.1325    414.7006m  132.7802m
vbs     0.          119.9773m -132.7802m 0.
vth     -510.9441m   -544.5411m 477.0654m  455.4074m
vdsat   -130.6890m   -122.8996m 89.9785m   91.2746m
vod     -99.0559m   -85.4816m  40.1544m   44.5926m
beta    577.0390u    565.4440u   1.2950m    1.2929m
gam_eff 557.0846m     556.2355m   511.0384m   507.4460m
gm       43.8146u    50.9125u    64.8881u    62.1118u
gds      8.9273u     267.3019n   1.1591u     4.0469u
gmb      12.8818u     14.0878u    11.4385u    12.1987u
cdtot    6.2668f       4.3083f     2.8343f     3.2267f
cgtot    13.6489f     13.0826f    6.5057f     6.7011f
cstot    16.4302f     15.8970f    7.7724f     8.0800f
cbtot    11.6432f     10.0930f    5.8243f     6.3533f
cgs      10.9114f     10.6163f    5.0624f     5.2355f
cgd      1.7969f      1.4353f     729.8335a   771.2137a
```

這次實驗可以看到  $V_{ov}$  與  $g_m$  都有明顯的上升且都維持在飽和狀態，且得到的  $|A_v| = 658.0787V/V$ 、 $output\ swing = 1.2653V$ 、 $I_D$  也小於  $5\mu A$ ，已達成所有條件。

(b)

```
subckt
element 0:mm4      0:mm3      0:mm2      0:mm1
model   0:p_18.1   0:p_18.1   0:n_18.1   0:n_18.1
region  Saturation Saturation Saturation Saturation
id      -3.5964u    -3.5964u    3.5964u    3.5964u
ibs     4.094e-22   26.2668a   -28.3945a  -7.998e-22
ibd     26.2660a    274.2111a  -117.0725a -28.3929a
vgs     -610.0000m   -630.0227m 517.2198m  500.0000m
vds     -119.9773m   -1.1325    414.7006m  132.7802m
vbs     0.          119.9773m -132.7802m 0.
vth     -510.9441m   -544.5411m 477.0654m  455.4074m
vdsat   -130.6890m   -122.8996m 89.9785m   91.2746m
vod     -99.0559m   -85.4816m  40.1544m   44.5926m
beta    577.0390u    565.4440u   1.2950m    1.2929m
gam_eff 557.0846m     556.2355m   511.0384m   507.4460m
gm       43.8146u    50.9125u    64.8881u    62.1118u
gds      8.9273u     267.3019n   1.1591u     4.0469u
gmb      12.8818u     14.0878u    11.4385u    12.1987u
cdtot    6.2668f       4.3083f     2.8343f     3.2267f
cgtot    13.6489f     13.0826f    6.5057f     6.7011f
cstot    16.4302f     15.8970f    7.7724f     8.0800f
cbtot    11.6432f     10.0930f    5.8243f     6.3533f
cgs      10.9114f     10.6163f    5.0624f     5.2355f
cgd      1.7969f      1.4353f     729.8335a   771.2137a
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(c) \*\*\*\* small-signal transfer characteristics

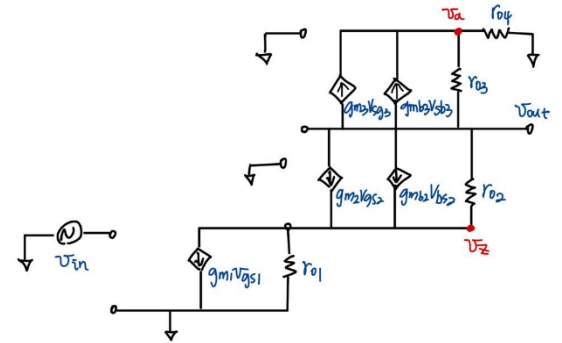
v(vout)/vvin	= -658.0787
input resistance at vvin	= 1.000e+20
output resistance at v(vout)	= 11.1494x

$$|Av| = \left| \frac{vout}{vin} \right| = 658.0787 \left( \frac{V}{V} \right) = 20 \log(658.0787) \text{ dB} = 56.36555668 \text{ dB} > 45 \text{ dB}$$

(d) 因(a)無考慮 body effect，故以下推導詳細考慮其效應

右圖為其小訊號模型，定義  $\begin{cases} g'_{m2} = g_{m2} + g_{mb2} \\ g'_{m3} = g_{m3} + g_{mb3} \end{cases}$

$$\begin{cases} \frac{Va}{r_{o4}} = \frac{Vout - Va}{r_{o3}} + g'_{m3}(-Va) \cdots (1) \\ \frac{Va}{r_{o4}} = \frac{Vz - Vout}{r_{o2}} + g'_{m2}Vz \cdots (2) \\ \frac{Va}{r_{o4}} = -\frac{Vz}{r_{o1}} - g_{m1}Vin \cdots (3) \end{cases}$$



$$(1) \Rightarrow Va \left( \frac{1}{r_{o4}} + \frac{1}{r_{o3}} + g'_{m3} \right) = \frac{Vout}{r_{o3}} \Rightarrow Va = Vout / \left( \frac{r_{o3}}{r_{o4}} + 1 + g'_{m3}r_{o3} \right) \cdots (4)$$

$$(4) \text{ substitute in } (2) \Rightarrow \frac{Vout}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}} = \frac{Vz - Vout}{r_{o2}} + g'_{m2}Vz$$

$$\Rightarrow Vz = Vout \left( \frac{r_{o2}}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}} + 1 \right) / (1 + g'_{m2}r_{o2}) \cdots (5)$$

$$(4) \& (5) \text{ substitute in } (3) \Rightarrow Vout \left\{ \frac{1}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}} \left( \frac{r_{o2}}{r_{o1} + g'_{m2}r_{o1}r_{o2}} + 1 \right) + \frac{1}{r_{o1} + g'_{m2}r_{o1}r_{o2}} \right\}$$

$$\Rightarrow Av = \frac{Vout}{Vin} = - \frac{g_{m1}}{\left\{ \frac{1}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}} \left( \frac{r_{o2}}{r_{o1} + g'_{m2}r_{o1}r_{o2}} + 1 \right) + \frac{1}{r_{o1} + g'_{m2}r_{o1}r_{o2}} \right\}}$$

$$= -g_{m1}Rout$$

利用 python 進行代數運算可得  $Av = -658.108499 \text{ V/V}$ 、 $Rout = 10.595547 \text{ M}\Omega$

$$\begin{cases} Error(|Av|) = \left| \frac{658.0787 - 658.108499}{658.108499} \right| = 0.00453\% \\ Error(Rout) = \left| \frac{11.1494 - 10.595547}{10.595547} \right| = 5.23\% \end{cases}$$

從結果觀察到加入 body effect 後的模型公式誤差較小，但可能還是與模擬使用之模型有些微差異。

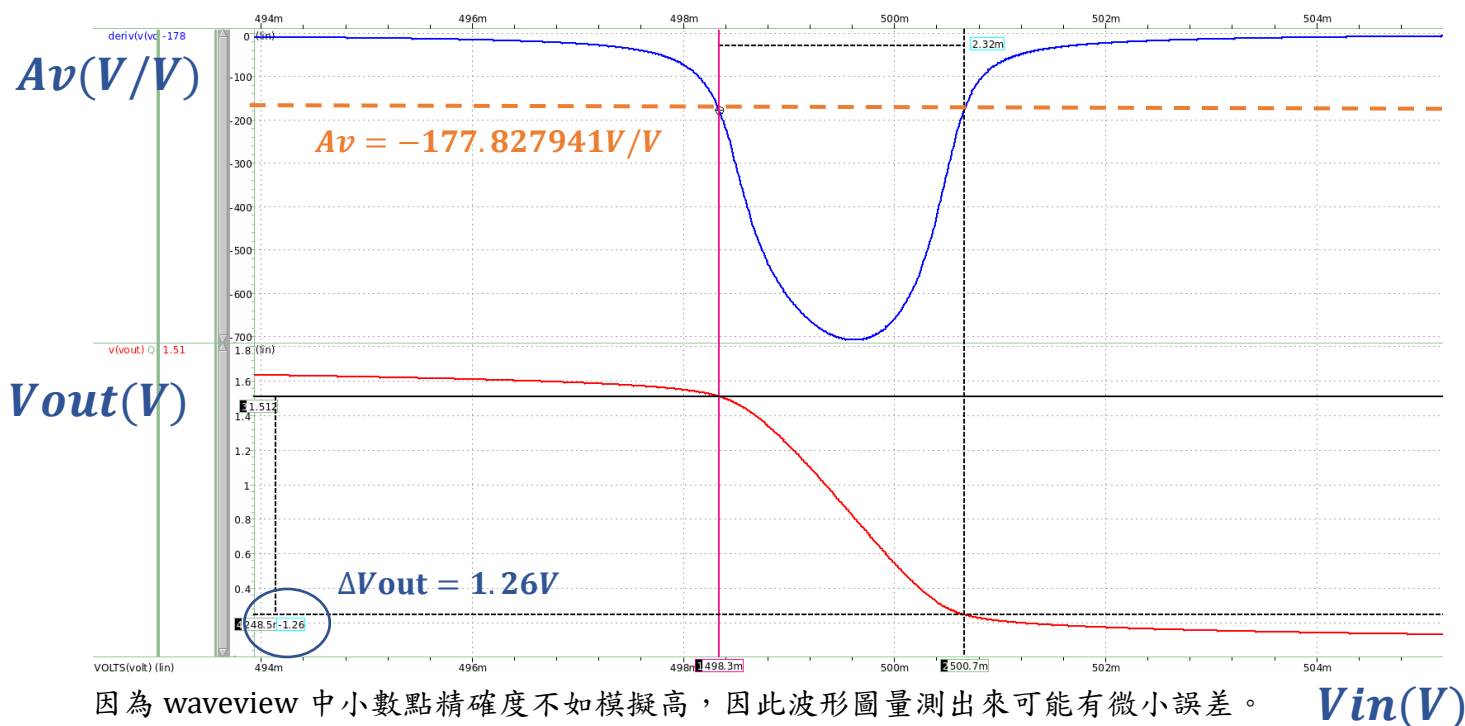
(e) output\_swing\_upper\_bound output\_swing\_lower\_bound output\_swing  
temper alter#

1.5116	0.2464	1.2653	25.0000
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模擬結果顯示  $\begin{cases} \text{output swing upper bound} = 1.5116 \text{ V} \\ \text{output swing lower bound} = 0.2464 \text{ V} \\ \text{output swing} = 1.5116 - 0.2464 = 1.2653 \text{ V} \end{cases}$

Vin(V)	Rin( $\Omega$ )	Rout( $\Omega$ )	Av(V/V)
498.3000m	1.000e+20	2.6618x	-158.1914
498.3100m	1.000e+20	2.7494x	-163.4127
498.3200m	1.000e+20	2.8410x	-168.8652
498.3300m	1.000e+20	2.9365x	-174.5553
498.3400m	1.000e+20	3.0360x	-180.4885
498.3500m	1.000e+20	3.1398x	-186.6696
498.3600m	1.000e+20	3.2477x	-193.1020
498.3700m	1.000e+20	3.3599x	-199.7877
498.3800m	1.000e+20	3.4764x	-206.7273
498.3900m	1.000e+20	3.5971x	-213.9193
498.4000m	1.000e+20	3.7220x	-221.3602
Vin(V)	Rin( $\Omega$ )	Rout( $\Omega$ )	Av(V/V)
500.6000m	1.000e+20	3.7846x	-220.8741
500.6100m	1.000e+20	3.6432x	-212.5771
500.6200m	1.000e+20	3.5072x	-204.5971
500.6300m	1.000e+20	3.3767x	-196.9380
500.6400m	1.000e+20	3.2516x	-189.6007
500.6500m	1.000e+20	3.1319x	-182.5834
500.6600m	1.000e+20	3.0176x	-175.8817
500.6700m	1.000e+20	2.9086x	-169.4896
500.6800m	1.000e+20	2.8047x	-163.3992
500.6900m	1.000e+20	2.7057x	-157.6014
500.7000m	1.000e+20	2.6116x	-152.0861

經過 Vin sweep from 0 to 1.8V, step=10uV 後，可以在 lis 檔中尋找出對應|Av|值在大於 45dB 也就是 177.827941V/V 下的 Vin 值，從表中可以觀察到邊界位於 0.498335V 與 0.500655V 附近，在波形圖中打開 deriv(V(Vout))與 Vout 的波形圖，用 cursor 對準 Av 波形圖縱軸值為 -177.827941V/V 的位置，對下去 Vout 波形圖發現 x 軸值吻合，再利用水平 cursor 量測得到 output swing = 1.26V。



因為 waveview 中小數點精確度不如模擬高，因此波形圖量測出來可能有微小誤差。

(f) 由(a)小題得知

$$\begin{aligned} \text{output swing} &= V_{DD} - V_{ov4} - V_{ov3} - V_{ov2} - V_{ov1} \\ &= 1.8 - 0.0990559 - 0.0854816 - 0.0401544 - 0.0445926 = 1.5307155V \end{aligned}$$

$$\text{Error(output swing)} = \left| \frac{1.2653 - 1.5307155}{1.5307155} \right| = 17.34\%$$

此誤差來源可能是因為 simulation 的 swing 是用 cursor 在達成題目要求的情況下抓出來的，並非固定在同個情況，但手算則是固定在同個 Vin 值算出來的結果。



(g)

	specification	simulation	hand-calculation
$V_{DD}$		1.8V	
$M_1$ (W/L, m)	–	W/L=2um/0.5um, m=1	
$M_2$ (W/L, m)	–	W/L=2um/0.5um, m=1	
$M_3$ (W/L, m)	–	W/L=4um/0.5um, m=1	
$M_4$ (W/L, m)	–	W/L=4um/0.5um, m=1	
$V_{in,DC}$	–	0.50V	
$V_{b1}$	–	0.65V	
$V_{b2}$	–	1.05V	
$V_{b3}$	–	1.19V	
$I_D$	$< 5\mu A$	3.5964 $\mu A$	–
gain $ A_v $	$\geq 45dB$	56.356dB	56.366dB
output impedance	–	11.1494M $\Omega$	10.5955M $\Omega$
output swing	$\geq 1.2V$	1.2653V	1.5307V

## Question 2. – Cascade Amplifier

(a) CS stage

- i. 因為 MOS 電晶體的結構導致若通道長度  $L$  越長，通道等校電阻值  $r_o$  就會愈高，且因為這題的  $R_{out} = r_o$ ，因此對於增益  $|A_v| = g_m \cdot r_o$  來看，設計一個長通道的電晶體可以提高增益；將增益值變大之後，我開始調整  $V_{in}$  想辦法把  $V_{out}$  控制在 0.499V 到 0.501V 之間，因為 common source 的特性是  $V_{in}$  上升， $V_{out}$  下降，因此順著這個特性就能慢慢找到平衡點。因此我最終的  $V_{in,dc} = 0.41932V$ 、 $W/L = 8um/4um$ ，得到的  $V_{out,dc} = 0.5004567V$ 、 $|A_v| = 165.61V/V$ 。

ii.

```

subckt
element 0:mm1
model 0:n_18.1
region Saturation
id 3.0000u
ibs -4.968e-22
ibd -318.7704a
vgs 419.3200m
vds 500.4567m
vbs 0.
vth 330.2636m
vdsat 102.9520m
vod 89.0564m
beta 599.9491u
gam_eff 507.4459m
gm 45.1356u
gds 272.5390n
gmb 9.1562u
cdtot 11.2385f
cgtot 206.6302f
cstot 214.0882f
cbtot 72.5464f
cgs 184.6051f
cgd 2.6654f

```

```

***** operating point status is all simulation time is 0.
node =voltage node =voltage node =voltage
+0:vdd = 1.8000 0:vin = 419.3200m 0:vout1 = 500.4567m
+0:vss = 0.

```

iii. \*\*\*\* small-signal transfer characteristics

v(vout1)/vvin	=	-165.6102
input resistance at vvin	=	1.000e+20
output resistance at v(vout1)	=	3.6692x

iv.  $|A_{v1}| = g_m r_o = \frac{g_m}{g_{ds}} = \frac{45.1356\mu}{272.5390n} = 165.6115272V/V$

手算之增益值與模擬之結果相近，可能稍有小數點取位數或模型公式之誤差存在。

(b) CG stage

- i. 我們可以從許多面向來分析 Common Gate 的特性，首先分析小訊號模型由右圖可以得知  $i_2 = i_1 + i_d = i_3$ ，以下推導  $|A_v|$  之公式。(未包含 body effect)

$$\Rightarrow i_d = -g_m * v_1; i_2 = -\frac{v_o}{R_D}; i_1 = \frac{v_o - v_1}{r_o}; i_3 = \frac{v_1 - v_{in}}{R_S}$$

$$\Rightarrow -g_m * v_1 + \frac{v_o - v_1}{r_o} = -\frac{v_o}{R_D} \Rightarrow v_o \left( \frac{1}{r_o} + \frac{1}{R_D} \right) = v_1 \left( g_m + \frac{1}{r_o} \right)$$

$$\Rightarrow v_o = v_1 \left( g_m + \frac{1}{r_o} \right) (r_o \parallel R_D)$$

$$\therefore i_2 = i_3$$

$$\Rightarrow \frac{v_1 - v_{in}}{R_S} = -\frac{v_o}{R_D}$$

$$\Rightarrow v_{in} = \left( \frac{v_1}{R_S} + \frac{v_o}{R_D} \right) R_S = v_1 + \frac{R_S}{R_D} v_o$$

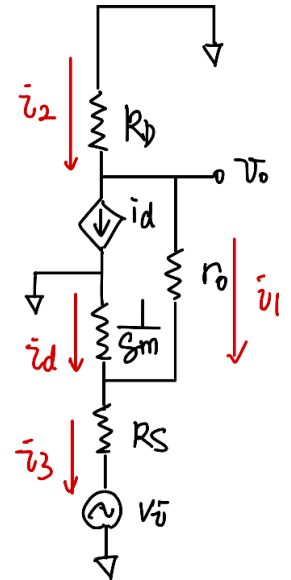
$$= v_1 \left( \left( g_m + \frac{1}{r_o} \right) (r_o \parallel R_D) \frac{R_S}{R_D} + 1 \right)$$

$$\Rightarrow A_v = \frac{v_o}{v_{in}} = \left\{ \left( g_m + \frac{1}{r_o} \right) (r_o \parallel R_D) \right\} / \left\{ \left( g_m + \frac{1}{r_o} \right) (r_o \parallel R_D) \frac{R_S}{R_D} + 1 \right\}$$

$$\therefore \left( g_m + \frac{1}{r_o} \right) (r_o \parallel R_D) = \frac{g_m * r_o * R_D}{r_o + R_D} + \frac{R_D}{r_o + R_D} = \frac{R_D(1 + g_m r_o)}{r_o + R_D}$$

$$\therefore |A_v| = \left| \frac{v_o}{v_{in}} \right| = \left| \frac{\left\{ \frac{R_D(1 + g_m r_o)}{r_o + R_D} \right\}}{\left\{ \frac{R_D(1 + g_m r_o)}{r_o + R_D} * \frac{R_S}{R_D} + 1 \right\}} \right| = \left| \frac{R_D(1 + g_m r_o)}{r_o + R_D + R_S + g_m r_o R_S} \right|$$

由上述式子可以判斷當  $R_D$  上升時， $|A_v|$  也會跟著上升。



Small signal model

在調整參數方面，我選定較小的 L 值但較大的 W/L 值，一方面可以降低  $V_{th}$  使可調控  $V_{ov}$  提升，另一方面也可以增加電流  $I_d$  的值； $R_D$  的部分有兩個面向需要做取捨，分別是  $V_{out,dc}$  的值與  $|A_v|$  的大小，從上述分析來看要增加  $|A_v|$  勢必要增加  $R_D$ ，但是若  $R_D$  值太大會造成 IR drop 太大，進而造成  $V_{out}$  可能太小無法達到 saturation 的狀態，因此我試了幾次調整至  $|A_v|$  值可以達到超過 10V/V 但又可以達到飽和的  $R_D$  值；接下來，因為電流還沒達到要求的 10uA，且已知  $V_b$  與  $I_d$  呈正相關，因此利用微調  $V_b$  的值可以精準的控制  $I_d$  的電流值，達到要求。最終選定的  $V_b = 1.0475V$ 、 $R_D = 100k\Omega$ 、 $W/L = 8\mu m/0.8\mu m$ 。

```

ii.  subckt
      element 0:mm2
      model 0:n_18.1
      region Saturation
      id 10.0439u
      ibs -318.4817a
      ibd -506.7693a
      vgs 547.5000m
      vds 295.6087m
      vbs -500.0000m
      vth 495.9257m
      vdsat 94.0189m
      vod 51.5743m
      beta 3.1682m
      gam_eff 519.9006m
      gm 177.1970u
      gds 3.1199u
      gmb 27.2853u
      cdtot 10.5756f
      cgtot 40.5341f
      cstot 43.6976f
      cbtot 23.5808f
      cgs 33.9086f
      cgd 2.8530f

```

iii. \*\*\*\* small-signal transfer characteristics

```

v(vout)/vvin1 = 15.8228
input resistance at vvin1 = 6.3200k
output resistance at v(vout) = 76.2212k

```

iv. 以下推導會加入 body effect 之影響

(1) 由上頁的小訊號模型可以得知  $R_{in} = \frac{v_1}{-i_3}$

$$\Rightarrow \text{上頁推導 } v_{in} = v_1 \left( \left( g_m + g_{mb} + \frac{1}{r_o} \right) (r_o \parallel R_D) \frac{R_S}{R_D} + 1 \right)$$

$$\Rightarrow i_3 = \frac{v_1 - v_{in}}{R_S} \text{ 將上式 } v_{in} \text{ 帶入 } i_3 = \frac{v_1 \left( 1 - \left( g_m + g_{mb} + \frac{1}{r_o} \right) (r_o \parallel R_D) \frac{R_S}{R_D} - 1 \right)}{R_S}$$

$$\begin{aligned} \Rightarrow R_{in} &= -\frac{v_1}{i_3} = \frac{R_S}{\left( g_m + g_{mb} + \frac{1}{r_o} \right) (r_o \parallel R_D) \left( \frac{R_S}{R_D} \right)} = \frac{R_D}{(g_m + g_{mb} + g_{ds})(r_o \parallel R_D)} \\ &= \frac{100k}{(177.1970u + 3.1199u + 27.2853u) \left( \frac{1}{3.1199u + \frac{1}{100k}} \right)} = 6.3197307k\Omega \end{aligned}$$

(2) 由小訊號模型推得

$$\Rightarrow R_{out} = r_o \parallel R_D = \frac{r_o R_D}{r_o + R_D} = \frac{\frac{R_D}{g_{ds}}}{\frac{1}{g_{ds}} + R_D} = \frac{R_D}{1 + g_{ds} * R_D} = \frac{100k}{1 + 3.1199u * 100k} = 76.2209314k\Omega$$



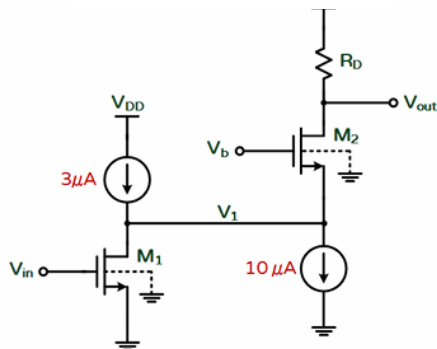
$$(3) |Av| = \left| \frac{R_D(1+(g_m+g_{mb})r_o)}{r_o+R_D+R_S+(g_m+g_{mb})r_oR_S} \right| = \left| \frac{100k \left( 1 + \frac{177.1970u+27.2853u}{3.1199u} \right)}{100k + \frac{1}{3.1199u}} \right| = 15.823459 \left( \frac{V}{V} \right) \text{ if } R_S = 0$$

$$\begin{cases} \text{Error}(R_{in}) = \left| \frac{6.3200 - 6.3197307}{6.3197307} \right| = 0.00426\% \\ \text{Error}(R_{out}) = \left| \frac{76.2212 - 76.2209314}{76.2209314} \right| = 0.00035\% \\ \text{Error}(|Av|) = \left| \frac{15.8228 - 15.823459}{15.823459} \right| = 0.00416\% \end{cases}$$

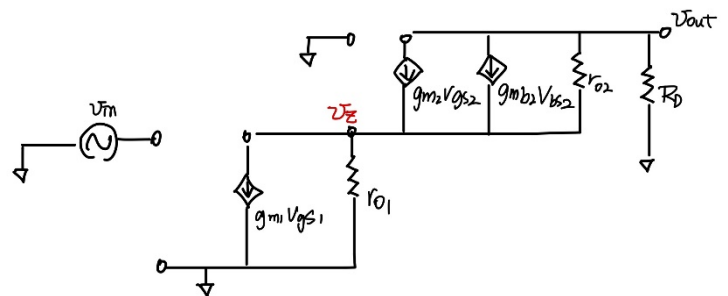
因為此電路存在 body effect，加入考慮之後誤差變得很小。但是，電路中可能含有一些非理想特性，或是現代製成的進步導致手算公式與模擬使用的模型有些差異，因此造成些微誤差。

### (c) Cascade Structure

*Cascade (CS+CG)*



*Small signal model*



i. \*\*\*\*\* operating point status is all simulation time is 0.  
node =voltage node =voltage node =voltage  
+0:v1 = 500.2779m 0:vb = 1.0475 0:vdd = 1.8000  
+0:vin = 419.3200m 0:vout = 800.0047m 0:vss = 0.

可以看到 V1 仍維持在 0.5V 附近。

ii. \*\*\*\*\* small-signal transfer characteristics

v(vout)/vvin = -4.5057  
input resistance at vvin = 1.000e+20  
output resistance at v(vout) = 99.9594k

從模擬結果可以看到增益 $|Av|=4.5057$  比原本 CS 與 CG stage 個別增益相乘小了非常多，原本的 $|Av|$ 在 CS stage 為 165.6102V/V、CG stage 為 15.8228V/V，但接在一起之後的增益為 4.5057V/V，這是因為從 CS 輸出端看進去的 Rout 比單 CS 級看進去的阻抗小了非常多，造成這個現象的原因是因為原本的阻抗 ro1 並聯了右側接的 CG 級的阻抗，導致整體阻抗大幅減少，造成 CS 級的增益值 $|Av1|$ 也跟著大幅下降，詳細的分析如下頁。

首先分析從 V1 看進 CG 級的輸出阻抗，V1 在小訊號模型中令為 Vz 點

令  $g'_{m2} = g_{m2} + g_{mb2}$

$$\begin{aligned} \therefore \begin{cases} i_z = \frac{V_{out}}{R_D} = g'_{m2} V_z + \frac{V_z - V_{out}}{r_{o2}} \\ V_{out} = i_z R_D \end{cases} \\ \therefore i_z = g'_{m2} V_z + \frac{V_z - i_z R_D}{r_{o2}} \\ \Rightarrow i_z r_{o2} = g'_{m2} r_{o2} V_z + V_z - i_z R_D \\ \Rightarrow i_z (r_{o2} + R_D) = V_z (1 + g'_{m2} r_{o2}) \\ \Rightarrow R = \frac{r_{o2} + R_D}{1 + g'_{m2} r_{o2}} \end{aligned}$$

又從原本的 CS stage 看進去的輸出阻抗為 ro1，所以新的 CS stage 輸出阻抗

$$\begin{aligned} R_{out} = R \parallel r_{o1} &= \frac{r_{o2} + R_D}{1 + g'_{m2} r_{o2}} \parallel r_{o1} = \frac{0.324701245M + 100k}{1 + (176.6593u + 3.0848u) * 0.3241701245M} \\ &= 7.165816786 \text{ k}\Omega \ll 3.6692 \text{ M}\Omega \end{aligned}$$

由上述推導可見，cascade 架構的 CS stage 輸出阻抗比原先單一 CS stage 小了非常多，因此整體 CS stage 的增益也下降非常多，造成整個 cascade 架構的增益變非常小。

iii. 由上頁的小訊號模型來分析

$$\begin{cases} -\frac{V_{out}}{R_D} = \frac{V_{out} - V_z}{r_{o2}} + g_{mb2}(-V_z) + g_{m2}(-V_z) \quad \because V_{gs2} = -V_z \\ -\frac{V_{out}}{R_D} = \frac{V_z}{r_{o1}} + g_{m1} V_{in} \Rightarrow r_{o1} \left( -\frac{V_{out}}{R_D} - g_{m1} V_{in} \right) = V_z \end{cases}$$

將上述第二式帶入第一式

$$\begin{aligned} \Rightarrow -\frac{V_{out}}{R_D} &= \frac{V_{out}}{r_{o2}} + V_z \left( -\frac{1}{r_{o2}} - g_{mb2} - g_{m2} \right) \\ &= \frac{V_{out}}{r_{o2}} + r_{o1} \left( \frac{V_{out}}{R_D} + g_{m1} V_{in} \right) \left( \frac{1}{r_{o2}} + g_{mb2} + g_{m2} \right) \\ \Rightarrow V_{out} \left( -\frac{1}{R_D} - \frac{1}{r_{o2}} \right) &= r_{o1} \left( \frac{V_{out}}{R_D r_{o2}} + \frac{g'_{m2} V_{out}}{R_D} + \frac{g_{m1} V_{in}}{r_{o2}} + g_{m1} g'_{m2} V_{in} \right) \\ \Rightarrow -\frac{V_{out}(r_{o2} + R_D + r_{o1} + r_{o1} r_{o2} g'_{m2})}{R_D r_{o2}} &= \frac{V_{in}(g_{m1} r_{o1} + g_{m1} g'_{m2} r_{o2} r_{o1})}{r_{o2}} \\ \Rightarrow A_v = \frac{V_{out}}{V_{in}} &= -\frac{g_{m1} r_{o1} R_D + g_{m1} g'_{m2} r_{o1} r_{o2} R_D}{r_{o1} + r_{o2} + R_D + g'_{m2} r_{o1} r_{o2}} = -\frac{16560312.78 + 964930795.1}{2.7880934.9} \\ &= -4.504713128 \left( \frac{V}{V} \right) \end{aligned}$$

從上式得出  $A_v = -4.504713128V/V$ ，與模擬的  $A_v = -4.5057V/V$  差距非常小，表示此模型加入 body effect 的影響後是很精準的。(以上的參數都在.lis 檔中取得)

```
subckt
element 0:mm1      0:mm2
model   0:n_18.1   0:n_18.1
region  Saturation Saturation
id      3.0000u    10.0000u
ibs     -4.968e-22 -318.6587a
ibd     -318.6565a -509.5694a
vgs     419.3200m  547.2221m
vds     500.2779m  299.7268m
vbs     0.         -500.2779m
vth     330.2642m  495.9243m
vdsat   102.9517m  93.8573m
vod     89.0558m   51.2978m
beta    599.9491u   3.1683m
gam_eff 507.4459m  519.9068m
gm      45.1350u    176.6593u
gds     272.5492n    3.0848u
gmb     9.1562u     27.1968u
```

(d)

	specification	simulation	hand-calculation
<b>Fig. 2.(a) Common-Source stage</b>			
$V_{DD}$	1.8V		
current source load	$3\mu A$		
$M_1$ (W/L, m)	–	W/L=8um/4um, m=1	
$V_{in,DC}$	–	0.41932V	
$V_{out1,DC}$	0.5V±10mV	0.5004567V	–
gain $ A_{v1} $	$\geq 100V/V$	165.6102V/V	165.6115272V/V
output impedance	–	3.6692MΩ	–
<b>Fig. 2.(b) Common-Gate stage</b>			
$V_{DD}$	1.8V		
$V_{in1,DC}$	0.5V		
$M_2$ (W/L, m)	–	W/L=8um/0.8um, m=1	
$V_b$	–	1.0475V	
$R_D$	–	100kΩ	
$I_D$	10μA±1%	10.0439uA	–
gain $ A_{v2} $	$\geq 10V/V$	15.8228V/V	15.823459V/V
input impedance	–	6.3200kΩ	6.3197307kΩ
output impedance	–	76.2212kΩ	76.2209314kΩ
<b>Fig. 3. Cascade CS-CG amplifier</b>			
$V_{DD}$	1.8V		
DC bias ( $V_1$ )	–	0.5002779V	–
overall gain $ A_v $	–	4.5057V/V	4.504713128V/V

### Question 3. – Comparison between “cascode” & “cascade” structure

1. From the perspective of “gain amplitude”, the “cascode” structure is preferred over the “cascade” structure. This preference arises because the output impedance of the cascade structure can be influenced by different stages when connected to each other, potentially leading to degradation in the overall gain. However, in the cascode structure, as derived from the formula above, the gain amplitude can be increased by stacking MOSFETs. In summary, the “cascode” structure outperforms the “cascade” structure in terms of gain amplitude.
2. From the perspective of “output swing”, the “cascade” structure typically offers a larger output swing compared to the “cascode” structure. This is because the output swing of each stage adds up, allowing for a larger overall swing. The cascode structure may have a limited output swing compared to the cascade structure due to the voltage headroom required for the cascode transistor.