Analog IC Design Homework 3 Report
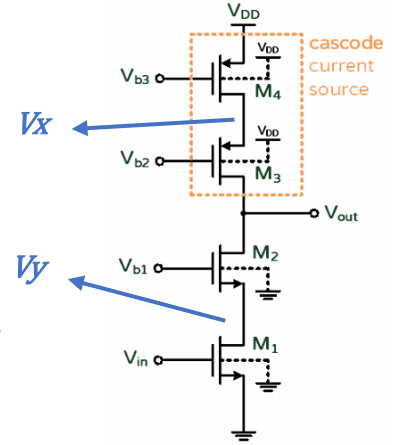
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◆ Department : ESS 工科系 25 級

Question 1. – Cascode Amplifier

此題所要求的條件如下

(1) $I_D < 5\mu A$

(2) $|Av| \geq 45dB$

(3) output swing $\geq$ 1.2V

(4) M1, M2, M3, M4 operates in saturation region

我們要選定四個電晶體的 W/L、Vin,dc、Vb1~Vb3 的值來達成目標。

(a) 首先，在設計之前需要做最初步的電路分析，首先四個電晶體必須操作在飽和狀態，我們可以列出下列式子

$$\begin{cases} V_x - V_{b3} \leq V_{th4} \Rightarrow M4\ saturates \\ Vout - V_{b2} \leq V_{th3} \Rightarrow M3\ saturates \\ V_{b1} - Vout \leq V_{th2} \Rightarrow M2\ saturates \\ Vin - V_y \leq V_{th1} \Rightarrow M1\ saturates \end{cases}$$

再來，分析電路增益公式 (先考慮相對關係，詳細 body effect 公式推導後續說明)

$$R_{on} \approx g_{m2} r_{o2} r_{o1}$$
$$R_{op} \approx g_{m3} r_{o3} r_{o4}$$
$$R_{out} = R_{op} \parallel R_{on}$$
$$|Av| = g_{m1} R_{out} = g_{m1}(g_{m2} r_{o2} r_{o1} \parallel g_{m3} r_{o3} r_{o4})$$

假設 $g_{m1} = g_{m2} = g_{m3} = g_{m4} = gm$、$r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_o$

$$|Av| \cong \frac{1}{2} g_m^2 r_o^2 \cdots (1)$$

$$g_m = u_n C_{ox} \left(\frac{W}{L}\right)(V_{ov})^2 \cdots (2)$$

最後分析

$$output\ swing = V_{DD} - V_{ov4} - V_{ov3} - V_{ov2} - V_{ov1} \geq 1.2V$$
$$\Rightarrow V_{ov4} + V_{ov3} + V_{ov2} + V_{ov1} \leq 0.6V \cdots (3)$$

由上述式子可以推論當 $V_{ov} \downarrow \begin{cases} \Rightarrow g_m \downarrow \Rightarrow I_D \downarrow \Rightarrow |Av| \downarrow \\ \Rightarrow output\ swing \uparrow \end{cases}$，因此衡量到 output swing 必須大且|Av|必須大於 45dB 大約等於 177.827941V/V 的情況下，我們要讓 Vov 調整到適合的值來保證所有電晶體操作在 saturation 且達到(1)~(3)之條件。

第一次實驗我隨便設參數觀察電晶體特性
$$\begin{cases} Vin = 0.5V \\ V_{b1}, V_{b2}, V_{b3} = 0.6V, 1.1V, 1.2V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{3um}{1um} \end{cases}$$

```
subckt
element   0:mm4      0:mm3      0:mm2      0:mm1
model     0:p_18.1   0:p_18.1   0:n_18.1   0:n_18.1
region    Linear     Saturation Linear     Linear
  id      -1.1804u   -1.1804u    1.1804u    1.1804u
  ibs     1.434e-22  15.9790a   -3.7407a  -2.327e-22
  ibd     15.9787a   311.7440a  -5.7761a  -3.7403a
  vgs     -600.0000m -608.7799m  586.8459m  500.0000m
  vds     -91.2201m  -1.6885     7.1593m    13.1541m
  vbs     0.         91.2201m   -13.1541m   0.
  vth     -498.3871m -524.6581m  397.0543m  394.1874m
  vdsat   -125.5368m -115.0714m  179.7134m  120.8938m
  vod     -101.6129m -84.1217m   189.7916m  105.8126m
  beta    209.3675u  206.2530u   933.9150u  932.2190u
  gam eff 557.0847m  556.4343m   507.8133m  507.4459m
  gm      13.3257u   16.9666u    6.4398u    10.5094u
  gds     5.8945u    39.5736n    161.3395u  84.2451u
  gmb     4.0488u    4.8637u     1.3479u    2.2219u
  cdtot   7.3687f    3.0818f     28.0051f   23.4239f
  cgtot   20.0989f   18.5220f    25.1546f   24.5737f
  cstot   22.0212f   21.3145f    26.5183f   25.0319f
  cbtot   12.1047f   10.4841f    12.7079f   12.7371f
  cgs     16.3732f   15.7900f    14.1385f   15.2742f
  cgd     2.4394f    1.0763f     10.3731f   8.5198f
```

從上述結果可看到仍有三顆電晶體在線性區，觀察 Vds 與 Vov (Vod)後調整讓 Vin 與 V_b1 更小、V_b2 和 V_b3 更大使電晶體的|Vgs|更靠近|Vth|，這樣可以輕易的讓 Vds 大於|Vgs|-|Vth|，更容易達到飽和狀態。

第二次實驗選擇
$$\begin{cases} Vin = 0.47V \\ V_{b1}, V_{b2}, V_{b3} = 0.5V, 1.2V, 1.25V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{4um}{0.5um}, \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{8um}{0.5um} \end{cases}$$

```
subckt
element   0:mm4      0:mm3      0:mm2      0:mm1
model     0:p_18.1   0:p_18.1   0:n_18.1   0:n_18.1
region    Saturation Saturation Saturation Saturation
  id      -2.6077u   -2.6077u    2.6077u    2.6077u
  ibs     2.671e-22  22.9812a   -12.0415a -4.812e-22
  ibd     22.9807a   678.1721a  -27.8968a  -12.0406a
  vgs     -550.0000m -541.6680m  466.0700m  470.0000m
  vds     -58.3320m  -1.6631     44.6805m   33.9300m
  vbs     0.         58.3320m   -33.9300m   0.
  vth     -504.7800m -521.6042m  458.7178m  452.0710m
  vdsat   -94.6408m  -81.8748m   72.9399m   77.4679m
  vod     -45.2200m  -20.0638m   7.3523m    17.9290m
  beta    1.1915m    1.1834m     2.5906m    2.5894m
  gam eff 557.0847m  556.6653m   508.3882m  507.4460m
  gm      37.1745u   48.0768u    46.3676u   42.0895u
  gds     25.1769u   200.6062n   34.1073u   55.0842u
  gmb     11.0703u   13.8124u    9.0262u    8.4367u
  cdtot   14.1111f   8.0644f     7.0888f    8.2018f
  cgtot   25.2941f   21.8829f    11.2998f   12.4247f
  cstot   29.0428f   25.5811f    12.7818f   13.9666f
  cbtot   23.1852f   19.7540f    12.4213f   12.6860f
  cgs     18.6630f   15.6823f    7.6512f    8.5876f
  cgd     4.1979f    2.8705f     1.8410f    2.2394f
```

這次實驗得到所有電晶體均於 saturation，但是從公式(1)可以得知|Av|與 gm 呈正相關，但由於 gm 均太小，因此增益目前是很小的，而 gm 小是因為 Vov 太小所導致的，因此下一次實驗方向應將 Vov 條大並且讓四顆電晶體還在飽和區運作，因此我們應該讓 Vin 與 V_b1 更大一些，且 V_b2 和

V_{b3} 再更小一些；另外，我讓 pmos 與 nmos 的尺寸調整呈 2 比 1 的比例是因為電流公式 $Id = \frac{1}{2}\mu n Cox \left(\frac{W}{L}\right)_n (Vov_n)^2 = \frac{1}{2}\mu p Cox \left(\frac{W}{L}\right)_p (Vov_p)^2$，因為 $\mu_n \cong 2\mu_p$，所以將尺寸比例設成 2 比 1 有助於調整 Vov 時更對稱，能更有效的保持在 saturation region。

第三次實驗選擇 $\begin{cases} Vin = 0.5V \\ V_{b1}, V_{b2}, V_{b3} = 0.65V, 1.05V, 1.19V \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2um}{0.5um}, \ \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{4um}{0.5um} \end{cases}$

```
subckt
element   0:mm4       0:mm3       0:mm2       0:mm1
model     0:p_18.1    0:p_18.1    0:n_18.1    0:n_18.1
region    Saturation  Saturation  Saturation  Saturation
id         -3.5964u    -3.5964u    3.5964u     3.5964u
ibs         4.094e-22  26.2668a   -28.3945a  -7.998e-22
ibd        26.2660a    274.2111a  -117.0725a -28.3929a
vgs      -610.0000m  -630.0227m   517.2198m   500.0000m
vds      -119.9773m    -1.1325    414.7006m   132.7802m
vbs         0.         119.9773m -132.7802m    0.
vth      -510.9441m  -544.5411m   477.0654m   455.4074m
vdsat    -130.6890m  -122.8996m    89.9785m    91.2746m
vod       -99.0559m   -85.4816m    40.1544m    44.5926m
beta      577.0390u   565.4440u    1.2950m     1.2929m
gam eff   557.0846m   556.2355m   511.0384m   507.4460m
gm         43.8146u    50.9125u    64.8881u    62.1118u
gds         8.9273u    267.3019n    1.1591u     4.0469u
gmb        12.8818u    14.0878u    11.4385u    12.1987u
cdtot       6.2668f     4.3083f     2.8343f     3.2267f
cgtot      13.6489f    13.0826f     6.5057f     6.7011f
cstot      16.4302f    15.8970f     7.7724f     8.0800f
cbtot      11.6432f    10.0930f     5.8243f     6.3533f
cgs        10.9114f    10.6163f     5.0624f     5.2355f
cgd         1.7969f     1.4353f   729.8335a   771.2137a
```

這次實驗可以看到 Vov 與 gm 都有明顯的上升且都維持在飽和狀態，且得到的|Av|=658.0787V/V、output swing=1.2653V、I_D 也小於 5uA，已達成所有條件。

(b)
```
subckt
element   0:mm4       0:mm3       0:mm2       0:mm1
model     0:p_18.1    0:p_18.1    0:n_18.1    0:n_18.1
region    Saturation  Saturation  Saturation  Saturation
id         -3.5964u    -3.5964u    3.5964u     3.5964u
ibs         4.094e-22  26.2668a   -28.3945a  -7.998e-22
ibd        26.2660a    274.2111a  -117.0725a -28.3929a
vgs      -610.0000m  -630.0227m   517.2198m   500.0000m
vds      -119.9773m    -1.1325    414.7006m   132.7802m
vbs         0.         119.9773m -132.7802m    0.
vth      -510.9441m  -544.5411m   477.0654m   455.4074m
vdsat    -130.6890m  -122.8996m    89.9785m    91.2746m
vod       -99.0559m   -85.4816m    40.1544m    44.5926m
beta      577.0390u   565.4440u    1.2950m     1.2929m
gam eff   557.0846m   556.2355m   511.0384m   507.4460m
gm         43.8146u    50.9125u    64.8881u    62.1118u
gds         8.9273u    267.3019n    1.1591u     4.0469u
gmb        12.8818u    14.0878u    11.4385u    12.1987u
cdtot       6.2668f     4.3083f     2.8343f     3.2267f
cgtot      13.6489f    13.0826f     6.5057f     6.7011f
cstot      16.4302f    15.8970f     7.7724f     8.0800f
cbtot      11.6432f    10.0930f     5.8243f     6.3533f
cgs        10.9114f    10.6163f     5.0624f     5.2355f
cgd         1.7969f     1.4353f   729.8335a   771.2137a
```

(c) **** small-signal transfer characteristics

```
v(vout)/vvin                        = -658.0787
input resistance at         vvin    = 1.000e+20
output resistance at v(vout)        = 11.1494x
```
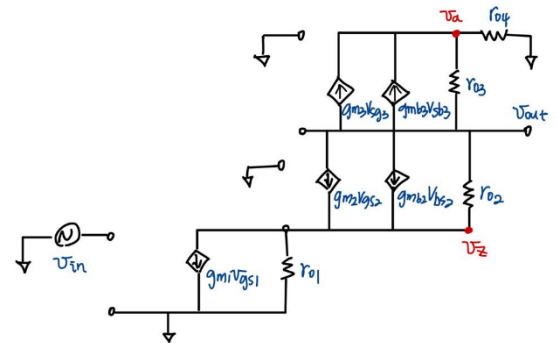
$$|Av| = \left|\frac{vout}{vin}\right| = 658.0787 \left(\frac{V}{V}\right) = 20\log(658.0787)\,dB = 56.36555668dB > 45dB$$

(d) 因(a)無考慮 body effect，故以下推導詳細考慮其效應

右圖為其小訊號模型，定義 $\begin{cases} g'_{m2} = g_{m2} + g_{mb2} \\ g'_{m3} = g_{m3} + g_{mb3} \end{cases}$

$$\begin{cases} \dfrac{Va}{r_{o4}} = \dfrac{Vout - Va}{r_{o3}} + g'_{m3}(-Va) \cdots (1) \\[2mm] \dfrac{Va}{r_{o4}} = \dfrac{Vz - Vout}{r_{o2}} + g'_{m2}Vz \cdots (2) \\[2mm] \dfrac{Va}{r_{o4}} = -\dfrac{Vz}{r_{o1}} - g_{m1}Vin \cdots (3) \end{cases}$$

$$(1) \Rightarrow Va\left(\frac{1}{r_{o4}} + \frac{1}{r_{o3}} + g'_{m3}\right) = \frac{Vout}{r_{o3}} \Rightarrow Va = Vout/(\frac{r_{o3}}{r_{o4}} + 1 + g'_{m3}r_{o3}) \cdots (4)$$

$$(4)\ substitute\ in\ (2) \Rightarrow \frac{Vout}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}} = \frac{Vz - Vout}{r_{o2}} + g'_{m2}Vz$$

$$\Rightarrow Vz = Vout(\frac{r_{o2}}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}} + 1)/(1 + g'_{m2}r_{o2}) \cdots (5)$$

$$(4)\ \&\ (5)\ substitute\ in\ (3) \Rightarrow Vout\left\{\frac{1}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}}\left(\frac{r_{o2}}{r_{o1} + g'_{m2}r_{o1}r_{o2}} + 1\right) + \frac{1}{r_{o1} + g'_{m2}r_{o1}r_{o2}}\right\}$$

$$\Rightarrow Av = \frac{Vout}{Vin} = -\frac{g_{m1}}{\left\{\dfrac{1}{r_{o3} + r_{o4} + g'_{m3}r_{o3}r_{o4}}\left(\dfrac{r_{o2}}{r_{o1} + g'_{m2}r_{o1}r_{o2}} + 1\right) + \dfrac{1}{r_{o1} + g'_{m2}r_{o1}r_{o2}}\right\}}$$

$$= -g_{m1}Rout$$

利用 python 進行代數運算可得 $Av = -658.108499V/V$、$Rout = 10.595547M\Omega$

$$\begin{cases} Error(|Av|) = \left|\dfrac{658.0787 - 658.108499}{658.108499}\right| = 0.00453\% \\[3mm] Error(Rout) = \left|\dfrac{11.1494 - 10.595547}{10.595547}\right| = 5.23\% \end{cases}$$

從結果觀察到加入 body effect 後的模型公式誤差較小，但可能還是與模擬使用之模型有些微差異。

(e)

```
    output_swing_upper_bound output_swing_lower_bound output_swing
 temper              alter#
  1.5116             0.2464                  1.2653              25.0000
```

模擬結果顯示 $\begin{cases} output\ swing\ upper\ bound = 1.5116V \\ output\ swing\ lower\ bound = 0.2464V \\ output\ swing = 1.5116 - 0.2464 = 1.2653V \end{cases}$

Vin(V)	Rin(Ω)	Rout(Ω)	Av(V/V)
498.3000m	1.000e+20	2.6618x	−158.1914
498.3100m	1.000e+20	2.7494x	−163.4127
498.3200m	1.000e+20	2.8410x	−168.8652
498.3300m	1.000e+20	2.9365x	−174.5553
498.3400m	1.000e+20	3.0360x	−180.4885
498.3500m	1.000e+20	3.1398x	−186.6696
498.3600m	1.000e+20	3.2477x	−193.1020
498.3700m	1.000e+20	3.3599x	−199.7877
498.3800m	1.000e+20	3.4764x	−206.7273
498.3900m	1.000e+20	3.5971x	−213.9193
498.4000m	1.000e+20	3.7220x	−221.3602

Vin(V)	Rin(Ω)	Rout(Ω)	Av(V/V)
500.6000m	1.000e+20	3.7846x	−220.8741
500.6100m	1.000e+20	3.6432x	−212.5771
500.6200m	1.000e+20	3.5072x	−204.5971
500.6300m	1.000e+20	3.3767x	−196.9380
500.6400m	1.000e+20	3.2516x	−189.6007
500.6500m	1.000e+20	3.1319x	−182.5834
500.6600m	1.000e+20	3.0176x	−175.8817
500.6700m	1.000e+20	2.9086x	−169.4896
500.6800m	1.000e+20	2.8047x	−163.3992
500.6900m	1.000e+20	2.7057x	−157.6014
500.7000m	1.000e+20	2.6116x	−152.0861

經過 Vin sweep from 0 to 1.8V, step=10uV 後，可以在 lis 檔中尋找出對應|Av|值在大於 45dB 也就是 177.827941V/V 下的 Vin 值，從表中可以觀察到邊界位於 0.498335V 與 0.500655V 附近，在波形圖中打開 deriv(V(Vout))與 Vout 的波形圖，用 cursor 對準 Av 波形圖縱軸值為-177.827941V/V 的位置，對下去 Vout 波形圖發現 x 軸值吻合，再利用水平 cursor 量測得到 output swing＝1.26V。
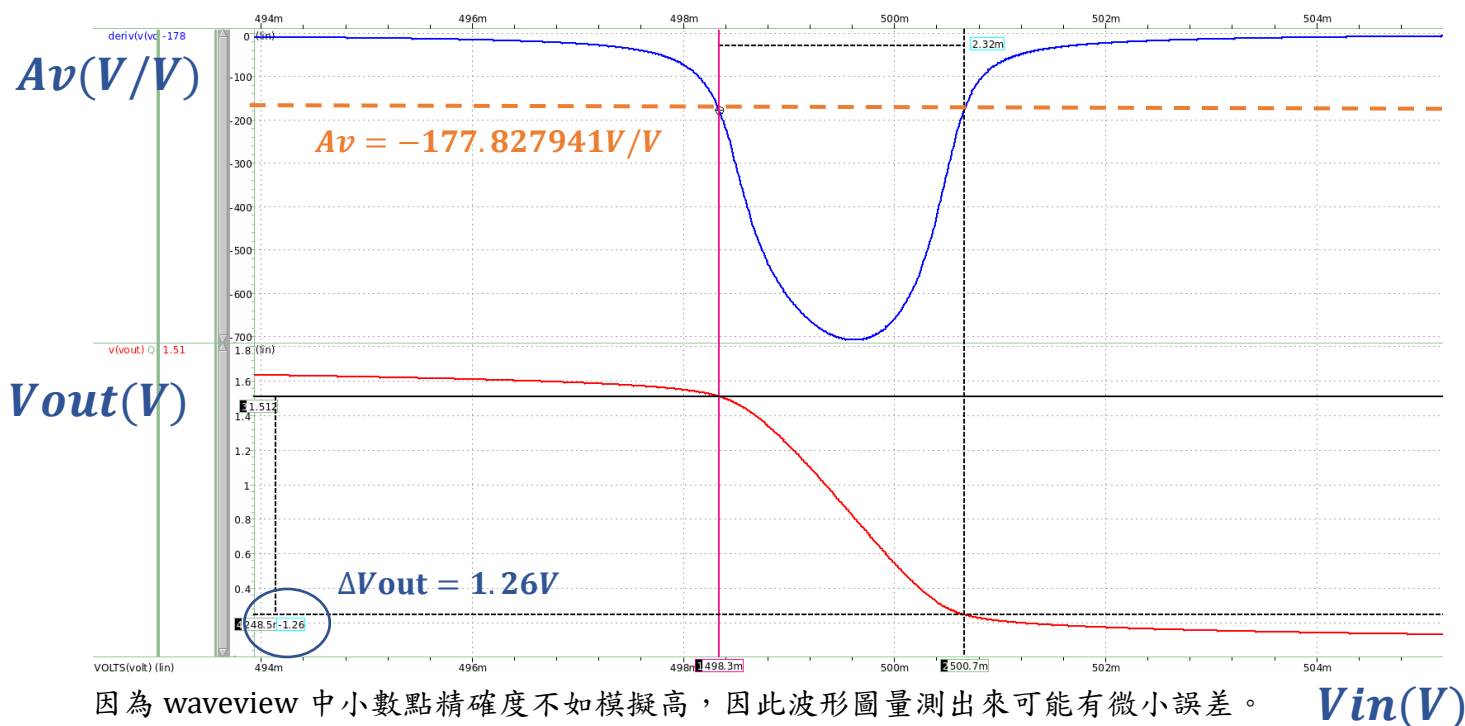
因為 waveview 中小數點精確度不如模擬高，因此波形圖量測出來可能有微小誤差。

(f) 由(a)小題得知

$$\text{output swing} = V_{DD} - V_{ov4} - V_{ov3} - V_{ov2} - V_{ov1}$$
$$= 1.8 - 0.0990559 - 0.0854816 - 0.0401544 - 0.0445926 = 1.5307155V$$

$$\text{Error(output swing)} = \left| \frac{1.2653 - 1.5307155}{1.5307155} \right| = 17.34\%$$

此誤差來源可能是因為 simulation 的 swing 是用 cursor 在達成題目要求的情況下抓出來的，並非固定在同個情況，但手算則是固定在同個 Vin 值算出來的結果。

	specification	simulation	hand-calculation
V_{DD}		1.8V	
M_1 (W/L, m)	–	W/L=2um/0.5um, m=1	
M_2 (W/L, m)	–	W/L=2um/0.5um, m=1	
M_3 (W/L, m)	–	W/L=4um/0.5um, m=1	
M_4 (W/L, m)	–	W/L=4um/0.5um, m=1	
$V_{in,DC}$	–	0.50V	
V_{b1}	–	0.65V	
V_{b2}	–	1.05V	
V_{b3}	–	1.19V	
I_D	< 5μA	3.5964μA	–
gain $|A_v|$	≥ 45dB	56.356dB	56.366dB
output impedence	–	11.1494MΩ	10.5955MΩ
output swing	≥ 1.2V	1.2653V	1.5307V

Question 2. – Cascade Amplifier

(a) CS stage

i.　因為 MOS 電晶體的結構導致若通道長度 L 越長，通道等校電阻值 ro 就會愈高，且因為這題的 Rout = ro，因此對於增益|Av|=gm*ro 來看，設計一個長通道的電晶體可以提高增益；將增益值變大之後，我開始調整 Vin 想辦法把 Vout 控制在 0.499V 到 0.501V 之間，因為 common source 的特性是 Vin 上升，Vout 下降，因此順著這個特性就能慢慢找到平衡點。因此我最終的 Vin,dc = 0.41932V、W/L = 8um/4um，得到的 Vout,dc = 0.5004567V、|Av1|=165.61V/V。

ii.
```
subckt
element   0:mm1
model     0:n_18.1
region    Saturation
 id        3.0000u
 ibs      -4.968e-22
 ibd      -318.7704a
 vgs       419.3200m
 vds       500.4567m
 vbs       0.
 vth       330.2636m
 vdsat     102.9520m
 vod        89.0564m
 beta      599.9491u
 gam eff   507.4459m
 gm         45.1356u
 gds       272.5390n
 gmb         9.1562u
 cdtot      11.2385f
 cgtot     206.6302f
 cstot     214.0882f
 cbtot      72.5464f
 cgs       184.6051f
 cgd         2.6654f


 ****** operating point status is all       simulation time is     0.
    node   =voltage       node   =voltage      node    =voltage

 +0:vdd    =   1.8000  0:vin    = 419.3200m 0:vout1    = 500.4567m
 +0:vss    =   0.
```

iii.   ****   small-signal transfer characteristics

v(vout1)/vvin		= -165.6102
input resistance at	vvin	= 1.000e+20
output resistance at v(vout1)		= 3.6692x

iv. $|A_{v1}| = g_m r_o = \frac{g_m}{g_{ds}} = \frac{45.1356u}{272.5390n} = 165.6115272 V/V$

手算之增益值與模擬之結果相近，可能稍有小數點取位數或模型公式之誤差存在。

(b) CG stage

i.   我們可以從許多面向來分析 Common Gate 的特性，首先分析小訊號模型
由右圖可以得知 i2=i1+id=i3，以下推導|Av|之公式。(未包含 body effect)

$$\Rightarrow id = -gm * v1; \; i2 = -\frac{vo}{R_D}; i1 = \frac{vo - v1}{ro}; i3 = \frac{v1 - vin}{R_S}$$

$$\Rightarrow -gm * v1 + \frac{vo - v1}{ro} = -\frac{vo}{R_D} \Rightarrow vo\left(\frac{1}{ro} + \frac{1}{R_D}\right) = v1\left(gm + \frac{1}{ro}\right)$$

$$\Rightarrow vo = v1\left(gm + \frac{1}{ro}\right)(ro \parallel R_D)$$

$$\because i2 = i3$$

$$\Rightarrow \frac{v1 - vin}{R_S} = -\frac{vo}{R_D}$$

$$\Rightarrow vin = \left(\frac{v1}{R_S} + \frac{vo}{R_D}\right)R_S = v1 + \frac{R_S}{R_D}vo$$
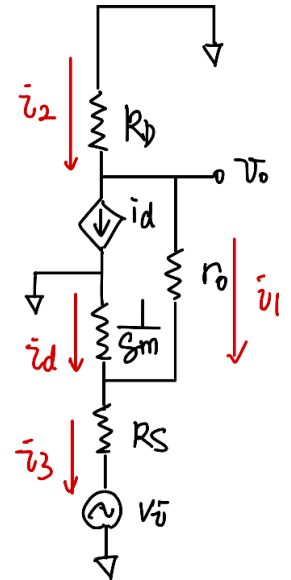
$$= v1\left(\left(gm + \frac{1}{ro}\right)(ro \parallel R_D)\frac{R_S}{R_D} + 1\right)$$

$$\Rightarrow Av = \frac{vo}{vin} = \left\{\left(gm + \frac{1}{ro}\right)(ro \parallel R_D)\right\}/\left\{\left(gm + \frac{1}{ro}\right)(ro \parallel R_D)\frac{R_S}{R_D} + 1\right\}$$

$$\because \left(gm + \frac{1}{ro}\right)(ro \parallel R_D) = \frac{gm * ro * R_D}{ro + R_D} + \frac{R_D}{ro + R_D} = \frac{R_D(1 + gmro)}{ro + R_D}$$

$$\therefore |Av| = |\frac{vo}{vin}| = |\frac{\left\{\frac{R_D(1 + gmro)}{ro + R_D}\right\}}{\left\{\frac{R_D(1 + gmro)}{ro + R_D} * \frac{R_S}{R_D} + 1\right\}}| = |\frac{R_D(1 + gmro)}{ro + R_D + R_S + gmroR_s}|$$

Small signal model

由上述式子可以判斷當 R_D 上升時，|Av|也會跟著上升。

在調整參數方面，我選定較小的 L 值但較大的 W/L 值，一方面可以降低 Vth 使可調控 Vov
提升，另一方面也可以增加電流 Id 的值；R_D 的部分有兩個面向需要做取捨，分別是 Vout,dc
的值與|Av|的大小，從上述分析來看要增加|Av|勢必要增加 R_D，但是若 R_D 值太大會造成 IR
drop 太大，進而造成 Vout 可能太小無法達到 saturation 的狀態，因此我試了幾次調整至
|Av|值可以達到超過 10V/V 但又可以達到飽和的 R_D 值；接下來，因為電流還沒達到要求的
10uA，且已知 Vb 與 Id 呈正相關，因此利用微調 Vb 的值可以精準的控制 Id 的電流值，達
到要求。最終選定的 Vb = 1.0475V、R_D = 100kΩ、W/L = 8um/0.8um。

```
subckt
element   0:mm2
model     0:n_18.1
region    Saturation
 id          10.0439u
 ibs       -318.4817a
 ibd       -506.7693a
 vgs        547.5000m
 vds        295.6087m
 vbs       -500.0000m
 vth        495.9257m
 vdsat       94.0189m
 vod         51.5743m
 beta         3.1682m
 gam eff    519.9006m
 gm         177.1970u
 gds          3.1199u
 gmb         27.2853u
 cdtot       10.5756f
 cgtot       40.5341f
 cstot       43.6976f
 cbtot       23.5808f
 cgs         33.9086f
 cgd          2.8530f
```

iii.   ****   small-signal transfer characteristics

```
v(vout)/vvin1                      =    15.8228
input resistance at       vvin1    =     6.3200k
output resistance at v(vout)       =    76.2212k
```

iv.   以下推導會加入 body effect 之影響

(1) 由上頁的小訊號模型可以得知 $\text{Rin} = \dfrac{v1}{-i3}$

$$\Rightarrow 上頁推導\ \text{vin} = v1\left(\left(g_m + g_{mb} + \frac{1}{ro}\right)(ro \parallel R_D)\frac{R_S}{R_D} + 1\right)$$

$$\Rightarrow i3 = \frac{v1 - vin}{R_S}\ 將上式\ vin\ 帶入\ i3 = \frac{v1\left(1 - \left(g_m + g_{mb} + \frac{1}{ro}\right)(ro \parallel R_D)\frac{R_S}{R_D} - 1\right)}{R_S}$$

$$\Rightarrow \text{Rin} = -\frac{v1}{i3} = \frac{R_S}{\left(g_m + g_{mb} + \frac{1}{ro}\right)(ro \parallel R_D)\left(\frac{R_S}{R_D}\right)} = \frac{R_D}{(g_m + g_{mb} + g_{ds})(ro \parallel R_D)}$$

$$= \frac{100k}{(177.1970u + 3.1199u + 27.2853u)\left(\dfrac{1}{3.1199u + \dfrac{1}{100k}}\right)} = 6.3197307k\Omega$$

(2) 由小訊號模型推得

$$\Rightarrow \text{Rout} = ro \parallel R_D = \frac{ro R_D}{ro + R_D} = \frac{\frac{R_D}{gds}}{\frac{1}{gds} + R_D} = \frac{R_D}{1 + gds * R_D} = \frac{100k}{1 + 3.1199u * 100k} = 76.2209314k\Omega$$

$$(3)\ |Av| = \left|\frac{R_D(1+(gm+gmb)*ro)}{ro+R_D+R_S+(gm+gmb)*ro*Rs}\right| = \left|\frac{100k\left(1+\frac{177.1970u+27.2853u}{3.1199u}\right)}{100k+\frac{1}{3.1199u}}\right| = 15.823459\left(\frac{V}{V}\right)\ if\ Rs = 0$$

$$\begin{cases} \text{Error(Rin)} = \left|\dfrac{6.3200 - 6.3197307}{6.3197307}\right| = 0.00426\% \\[3mm] \text{Error(Rout)} = \left|\dfrac{76.2212 - 76.2209314}{76.2209314}\right| = 0.00035\% \\[3mm] \text{Error(|Av|)} = \left|\dfrac{15.8228 - 15.823459}{15.823459}\right| = 0.00416\% \end{cases}$$
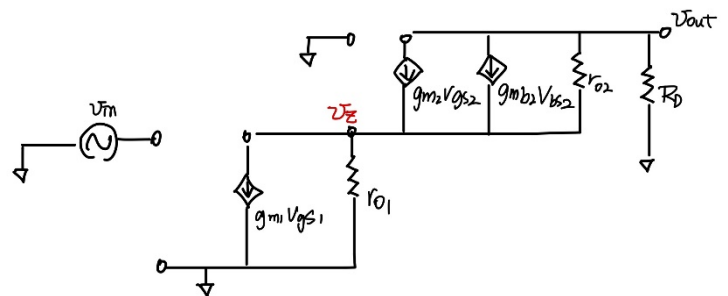
因為此電路存在 body effect，加入考慮之後誤差變得很小。但是，電路中可能含有一些非理想特性，或是現代製成的進步導致手算公式與模擬使用的模型有些差異，因此造成些微誤差。

(c) Cascade Structure
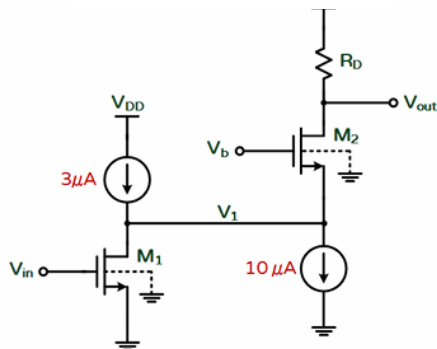
Cascade (CS+CG)　　　　　　　　　　　　　　**Small signal model**

i.
```
******  operating point status is all       simulation time is     0.
   node    =voltage        node    =voltage        node    =voltage

+0:v1     = 500.2779m 0:vb    =   1.0475  0:vdd    =   1.8000
+0:vin    = 419.3200m 0:vout  = 800.0047m 0:vss    =   0.
```

可以看到 V1 仍維持在 0.5V 附近。

ii.
```
****     small-signal transfer characteristics

   v(vout)/vvin                               =    -4.5057
   input resistance at           vvin  =  1.000e+20
   output resistance at v(vout)         =   99.9594k
```

從模擬結果可以看到增益|Av|=4.5057 比原本 CS 與 CG stage 個別增益相乘小了非常多，原本的|Av|在 CS stage 為 165.6102V/V、CG stage 為 15.8228V/V，但接在一起之後的增益為 4.5057V/V，這是因為從 CS 輸出端看進去的 Rout 比單 CS 級看進去的阻抗小了非常多，造成這個現象的原因是因為原本的阻抗 ro1 並聯了右側接的 CG 級的阻抗，導致整體阻抗大幅減少，造成 CS 級的增益值|Av1|也跟著大幅下降，詳細的分析如下頁。

首先分析從 V1 看進 CG 級的輸出阻抗，V1 在小訊號模型中令為 Vz 點

令 $g'_{m2} = g_{m2} + g_{mb2}$

$$\because \begin{cases} i_z = \dfrac{V_{out}}{R_D} = g'_{m2}V_Z + \dfrac{V_Z - V_{out}}{r_{o2}} \\ Vout = i_Z R_D \end{cases}$$

$$\therefore i_z = g'_{m2}Vz + \frac{Vz - i_z R_D}{r_{o2}}$$

$$\Rightarrow i_z r_{o2} = g'_{m2}r_{o2}Vz + Vz - i_z R_D$$

$$\Rightarrow i_z(r_{o2} + R_D) = Vz(1 + g'_{m2}r_{o2})$$

$$\Rightarrow R = \frac{r_{o2} + R_D}{1 + g'_{m2}r_{o2}}$$

又從原本的 CS stage 看進去的輸出阻抗為 ro1，所以新的 CS stage 輸出阻抗

$$Rout = R \parallel r_{o1} = \frac{r_{o2} + R_D}{1 + g'_{m2}r_{o2}} \parallel r_{o1} = \frac{0.324701245M + 100k}{1 + (176.6593u + 3.0848u) * 0.3241701245M}$$

$$= 7.165816786 \, k\Omega \ll 3.6692 \, M\Omega$$

由上述推導可見，cascade 架構的 CS stage 輸出阻抗比原先單一 CS stage 小了非常多，因此整體 CS stage 的增益也下降非常多，造成整個 cascade 架構的增益變非常小。

iii. 由上頁的小訊號模型來分析

$$\begin{cases} -\dfrac{Vout}{R_D} = \dfrac{Vout - Vz}{r_{o2}} + g_{mb2}(-Vz) + g_{m2}(-Vz) \quad \because Vgs_2 = -Vz \\ -\dfrac{Vout}{R_D} = \dfrac{Vz}{r_{o1}} + g_{m1}Vin \Rightarrow r_{o1}\left(-\dfrac{Vout}{R_D} - g_{m1}Vin\right) = Vz \end{cases}$$

將上述第二式帶入第一式

$$\Rightarrow -\frac{Vout}{R_D} = \frac{Vout}{r_{o2}} + Vz\left(-\frac{1}{r_{o2}} - g_{mb2} - g_{m2}\right)$$

$$= \frac{Vout}{r_{o2}} + r_{o1}\left(\frac{Vout}{R_D} + g_{m1}Vin\right)\left(\frac{1}{r_{o2}} + g_{mb2} + g_{m2}\right)$$

$$\Rightarrow Vout\left(-\frac{1}{R_D} - \frac{1}{r_{o2}}\right) = r_{o1}\left(\frac{Vout}{R_D r_{o2}} + \frac{g'_{m2}Vout}{R_D} + \frac{g_{m1}Vin}{r_{o2}} + g_{m1}g'_{m2}Vin\right)$$

$$\Rightarrow -\frac{Vout(r_{o2} + R_D + r_{o1} + r_{o1}r_{o2}g'_{m2})}{R_D r_{o2}} = \frac{Vin(g_{m1}r_{o1} + g_{m1}g'_{m2}r_{o2}r_{o1})}{r_{o2}}$$

$$\Rightarrow Av = \frac{Vout}{Vin} = -\frac{g_{m1}r_{o1}R_D + g_{m1}g'_{m2}r_{o1}r_{o2}R_D}{r_{o1} + r_{o2} + R_D + g'_{m2}r_{o1}r_{o2}} = -\frac{16560312.78 + 964930795.1}{2.7880934.9}$$

$$= -4.504713128 \left(\frac{V}{V}\right)$$

從上式得出 Av＝-4.504713128V/V，與模擬的 Av=-4.5057V/V 差距非常小，表示此模型加入 body effect 的影響後是很精準的。(以上的參數都在.lis 檔中取得)

```
subckt
element   0:mm1        0:mm2
model     0:n_18.1      0:n_18.1
region    Saturation   Saturation
 id          3.0000u      10.0000u
 ibs        -4.968e-22  -318.6587a
 ibd       -318.6565a  -509.5694a
 vgs        419.3200m   547.2221m
 vds        500.2779m   299.7268m
 vbs          0.        -500.2779m
 vth        330.2642m   495.9243m
 vdsat      102.9517m    93.8573m
 vod         89.0558m    51.2978m
 beta       599.9491u     3.1683m
 gam_eff    507.4459m   519.9068m
 gm          45.1350u   176.6593u
 gds        272.5492n     3.0848u
 gmb          9.1562u    27.1968u
```

(d)

	specification	simulation	hand-calculation
Fig. 2.(a) Common-Source stage			
V_{DD}	1.8V		
current source load	3μA		
M_1 **(W/L, m)**	–	W/L=8um/4um, m=1	
$V_{in,DC}$	–	0.41932V	
$V_{out1,DC}$	0.5V±10mV	0.5004567V	–
gain $|A_{v1}|$	≥ 100V/V	165.6102V/V	165.6115272V/V
output impedence	–	3.6692MΩ	–
Fig. 2.(b) Common-Gate stage			
V_{DD}	1.8V		
$V_{in1,DC}$	0.5V		
M_2 **(W/L, m)**	–	W/L=8um/0.8um, m=1	
V_b	–	1.0475V	
R_D	–	100kΩ	
I_D	10μA±1%	10.0439uA	–
gain $|A_{v2}|$	≥ 10V/V	15.8228V/V	15.823459V/V
input impedance	–	6.3200kΩ	6.3197307kΩ
output impedance	–	76.2212kΩ	76.2209314kΩ
Fig. 3. Cascade CS-CG amplifier			
V_{DD}	1.8V		
DC bias (V₁)	–	0.5002779V	–
overall gain $|A_v|$	–	4.5057V/V	4.504713128V/V

Question 3. – Comparison between "cascode" & "cascade" structure

1. From the perspective of "gain amplitude", the "cascode" structure is preferred over the "cascade" structure. This preference arises because the output impedance of the cascade structure can be influenced by different stages when connected to each other, potentially leading to degradation in the overall gain. However, in the cascode structure, as derived from the formula above, the gain amplitude can be increased by stacking MOSFETs. In summary, the "cascode" structure outperforms the "cascade" structure in terms of gain amplitude.

2. From the perspective of "output swing", the "cascade" structure typically offers a larger output swing compared to the "cascode" structure. This is because the output swing of each stage adds up, allowing for a larger overall swing. The cascode structure may have a limited output swing compared to the cascade structure due to the voltage headroom required for the cascode transistor.