Announ cements

Friday class cancelled (start Spring break early)

Email me if you want office hours

Middern 2: Thurs. 3/21 7:00-8:30 pm, Loomis Lab. 144

See policy email (reference sheet allowed)

Topics: Everything through today (i.e. thru DRF \$14.1) but focus is on post-Midtern 1 material (\$13.2-onwards)

Practice problems: see email

Tues., Wed. after break: review

Conflicts: email me ASAP

HW7 (due Wed 3/27): will be posted after break

Recall: K/F !field extín.

Aut(K/F) = {automs. of k which fix F} \le Aut(k)

H = Aut(K)

Fix H = subfield of K fixed by every elt. of H

Thm: Let f(x) = F[x], K = Sp.f. Then,

| Aut(x/F)| < [x:F],

w/ equality if f is separable.

Pf by example: (see DLF for full argument)

$$f(x) = x^3 - 2 \in \mathbb{Q}[x]$$

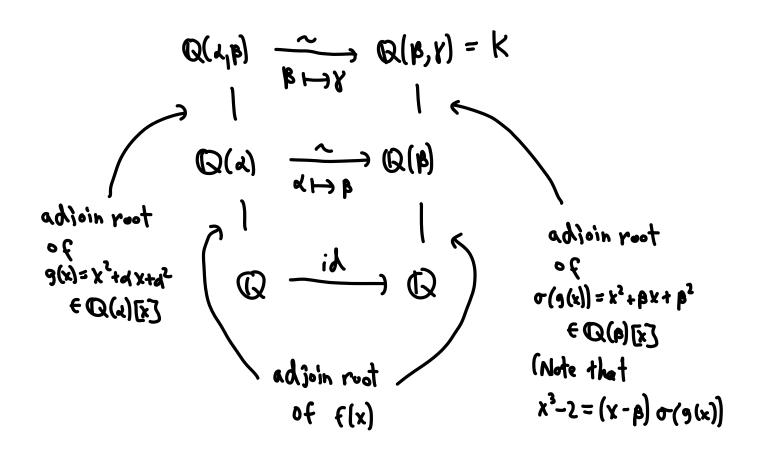
Splits as $(x - 3/2)(x - 3/3)(x - 3/3)(x - 3/3)(x - 3/3)$ over $\mathbb{Q}(x, \beta)$

$$K = Q(a, B) \qquad (x - a)(x - B)(x - Y)$$

$$L = Q(a) \qquad (x - a)(x^2 + ax + a^2)$$

$$Q \qquad \qquad \chi^3 - 2$$

Build JE Aut (K/Q) in two steps



How many such or can we construct?

= 3.5 = (# roots of f)(# roots of g)

= (deg f)(deg g) = [Q(a):Q][k:Q(a)]

= Tk:Q]

f sep. = [K:Q]

Remark: If $f(x) \in F[x]$ has roots d_1, \dots, d_n and $k = Sp_{\overline{x}}f$, $\sigma \in Aut(k/F)$ then the restriction $\sigma \in Aut(k/F)$ then the restriction $\sigma \in Aut(k/F)$ then the restriction $\sigma \in Aut(k/F)$ then $\sigma \in Aut(k/F)$ then $\sigma \in Aut(k/F)$ then $\sigma \in Aut(k/F)$ and $\sigma \in Aut(k/F)$ then $\sigma \in Au$

The homom. Aut $(K/F) \longrightarrow S_n$ (symmetric gp. on n fetters) $\sigma \longmapsto \overline{\sigma}$

is inj. (every autom. gives a different perm.) but not necessarily surj.

Def: A finite extension K/F is <u>Galois</u> if |Aut(K/F)| = [K:F]. In this case, we set |Aut(K/F)| = |Aut(K/F)| and call it the <u>Galois group</u> of |K/F|.

Cor: If fe F[x] is sep., k = Spf, then k/f is Galois (Turns out all Galois extrés are of this form)

Examples:

$$k = Sp_{\mathcal{Q}} \in \text{ where } \{(x) = (x^2 - 2)(x^2 + 1)\}$$

Note: this is a proper subgp. of S.

$$|Aut(K/Q)| = 6 = [K:Q]$$
 Galois!
 $Gal(K/Q) \cong S_3$

Thm: Let
$$H \leq Aut(K)$$
, $F = Fix H$

finite any

sp. field

Then K/F is Galois!

More precisely,

Enjoy the break!