

Announcements

HW9 posted (or will be today) — due Sun 11/10

Quiz 7 Wed. in class

§10.1: Graphs

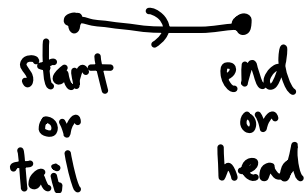
Def: A graph $G=(V,E)$ consists of

V : a nonempty set of vertices, and

E : a set of edges

Each edge has two vertices as endpoints. If they are the same, the edge is called a loop

Def: A digraph D has the same def'n except that edges are directed



Def: A graph or digraph is called simple if

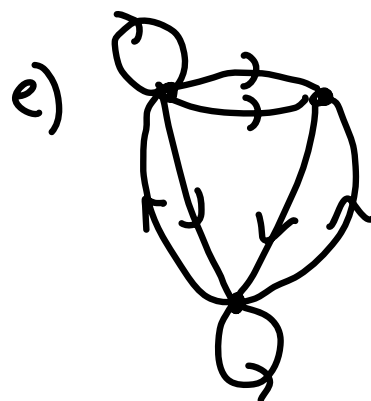
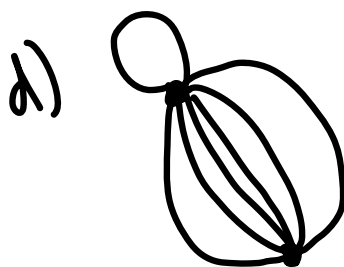
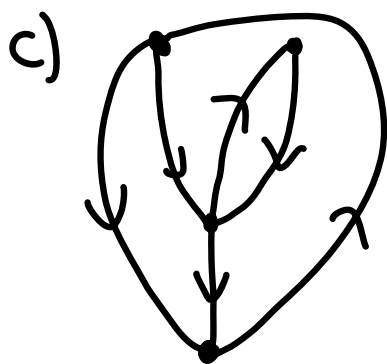
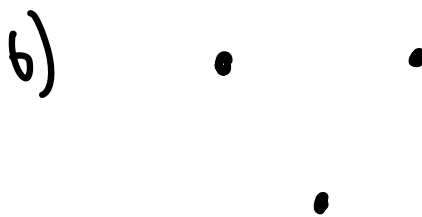
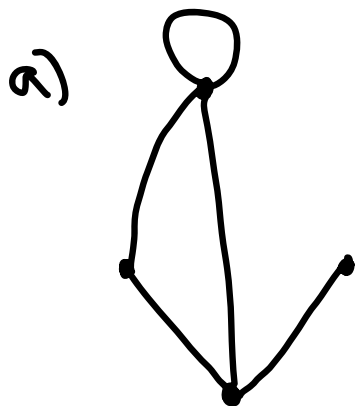
a) it has no loops ("loopless")

b) it has no multiple edges

{ edges w/ same endpoints (graph)
edges w/ same tail/head (digraph)

(Di)graphs w/ multiple edges are called multi(di)graphs

Class activity: Graph or digraph? Simple? Multi-?



Rosen has many examples of how graphs/digraphs can be used to represent real-world data

(Ex 1-13, also on HW)

§10.2: Graph terminology, and special types of graph

Def: Let $G = (V, E)$ be a graph

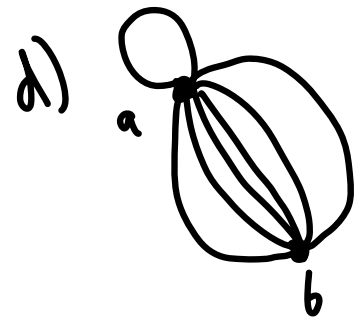
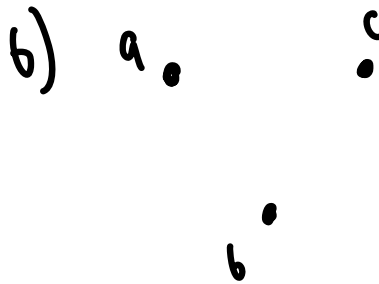
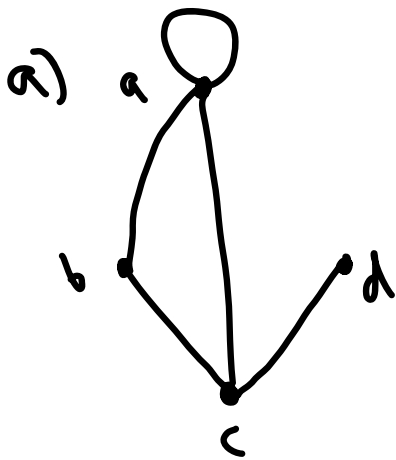
a) $u, v \in V$ ($u \neq v$) are adjacent or neighbors if there is an edge $e \in E$ with endpoints u and v . e is incident to its endpoints.

b) The neighborhood of $v \in V$ is the set $N(v)$ of all neighbors of v . If $A = \{v_1, v_2, \dots, v_k\}$, then

$N(A) = N(v_1) \cup N(v_2) \cup \dots \cup N(v_k)$, the set of all vertices adjacent to any vertex in A

c) The degree of $v \in V$ is the number $d(v)$ of edges incident to v (counting loops twice).

Class activity: Find all neighborhoods and degrees



Handshake theorem: For a graph G with m edges,

$$\sum_{v \in V} \deg(v) = 2m$$

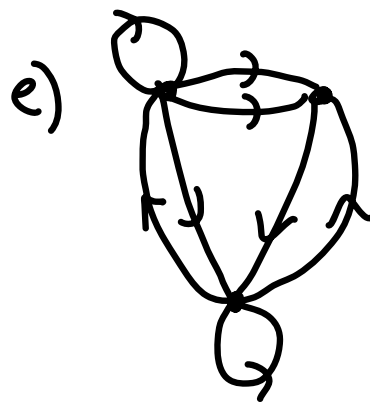
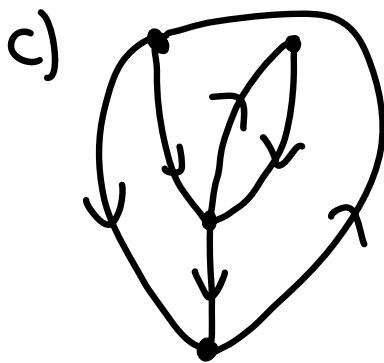
In particular, the number of vertices of odd-degree is always even!

Def: Let $D=(V,E)$ be a digraph, $v \in V$.

The in-degree $\deg^-(v)$ of v is the number of edges w/ end/head v .

The out-degree $\deg^+(v)$ of v is the number of edges w/ start/tail v .

Class activity: Find all in/out-degrees



Handshake theorem: For a digraph D with m edges,

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = m$$

Special (undirected, simple) graphs

a) Complete graph K_n : all pairs of vertices are adjacent



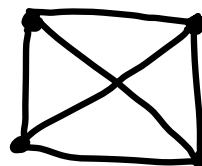
K_1



K_2



K_3

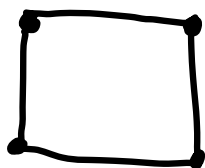


K_4

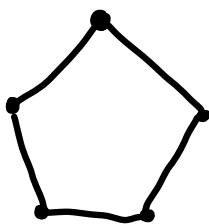
b) Cycle C_n :



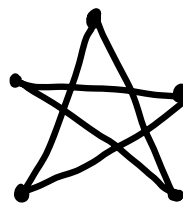
C_3



C_4



C_5

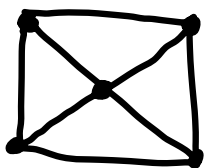


Also C_5
(doesn't matter
how you place the
vertices)

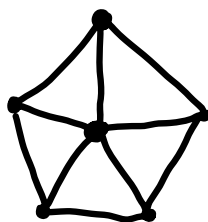
c) Wheel W_n : C_n with a hub



W_3



W_4



W_5

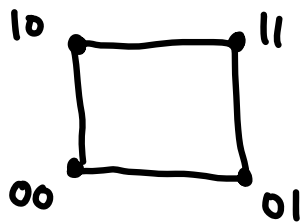
d) Hypercube Q_n

$V = \{\text{binary strings of length } n\}$

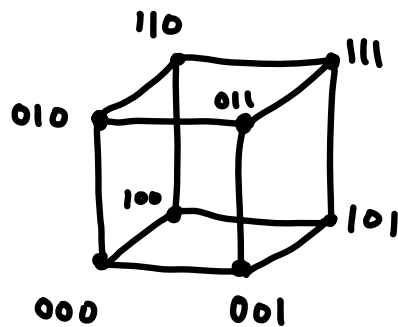
$N(v) = \{\text{all strings off by one digit from } v\}$



Q_1



Q_2



Q_3

Def: G is bipartite if there is a set partition $V = V_1 \cup V_2$ such that every edge has one endpoint in V_1 and the other in V_2 .
 $\underbrace{V_1 \cup V_2}_{\text{disjoint}}$

Class activity: Of the above graphs, which are bipartite?
 (if time)