Announcements:

Quiz 1 this Friday in class (20 mins; start in middle)

Content: anything covered thru. Wednesday

Focus on definitions, thm. statements, examples

No outside resources allowed

E.g. "State the Havel-Hakimi Theorem, and give d and d' for the following graph: ..."

Midterm 1: Wed. 9/20 7:00-8:30pm (Noyes Lab. 217)

Will (roughly) cover through this week

Harder than quiz, more like homework

Accomodations/conflicts: contact me ASAP!

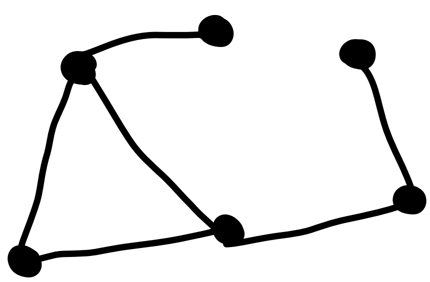
I will send a full email with policies soon

Havel-Hakimi Theorem:

a) For 1 vertex, the only graphic sequence is $d_1 = 0$

b) A list d of $n > 1$ integers is graphic iff d' is graphic, where d' is obtained by deleting the largest element Δ and subtracting 1 from its next Δ largest elements

Ex:



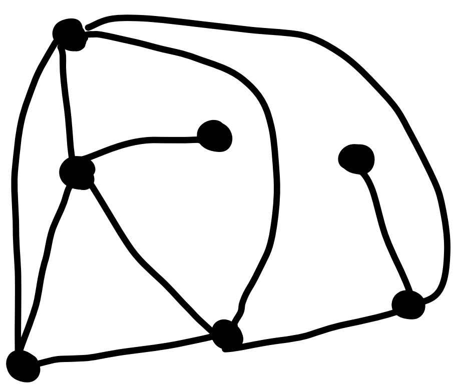
3,2,2,2,1,1 is graphic

So 4,4,3,3,3,1,1 is graphic

Since $\cancel{4}$, 4, 3, 3, 3, 1, 1

-1 -1 -1 -1

3, 2, 2, 2, 1, 1

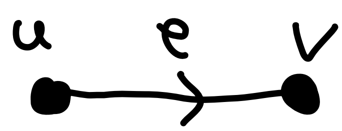


Pf: $n=1$: simple graph can't have edges

$n>1$: Sufficiency:

§1.4: Directed Graphs

Def 1.4.2: A <u>directed graph</u> or <u>digraph</u> is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$, and a function assigning each edge an ordered pair of vertices



"e has endpoints u and v"

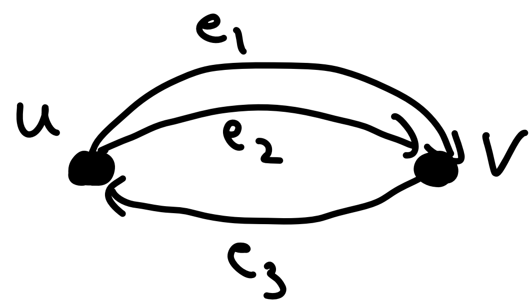
"e goes from u to v"

"e has tail u and head v"

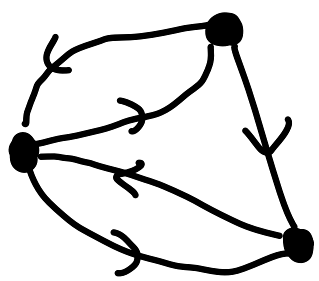
"$u \to v$" or "$u \xrightarrow{e} v$"

Most basic def'ns are similar as for graphs.

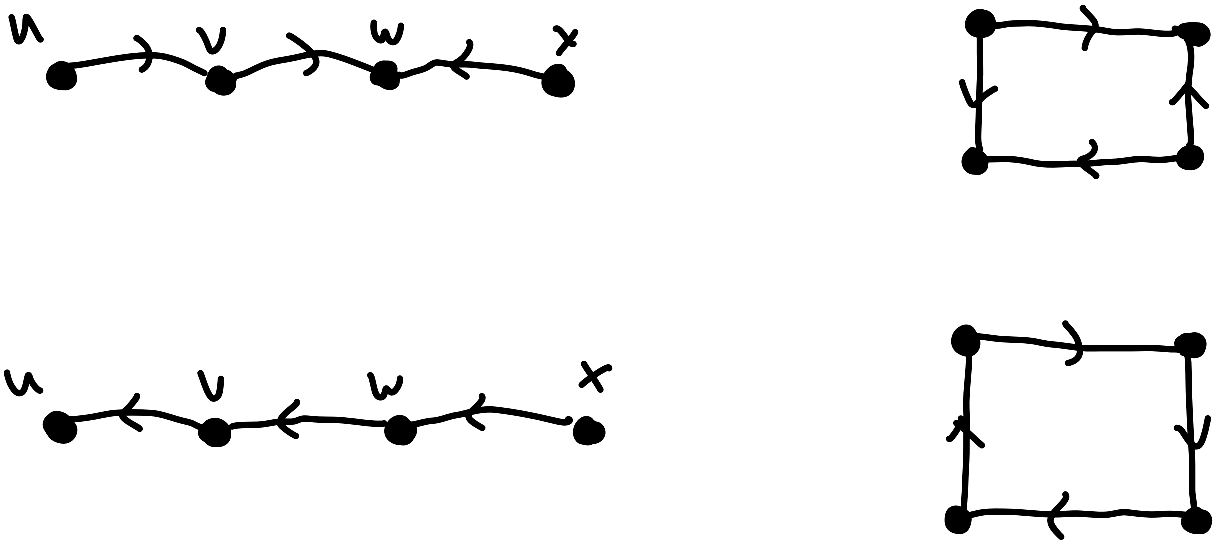
a) <u>Multiple edges</u> are edges w/ the same tail and head



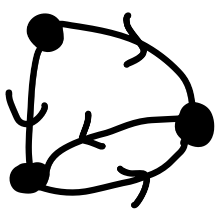
b) A graph is simple if it has no $\underbrace{\text{loops}}_{\text{same as for graphs}}$ or multiple edges



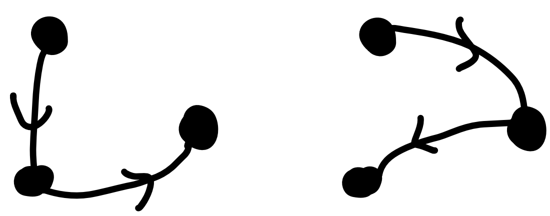
c) To be a path, cycle, walk, trail, circuit, you have to follow the edges tail to head



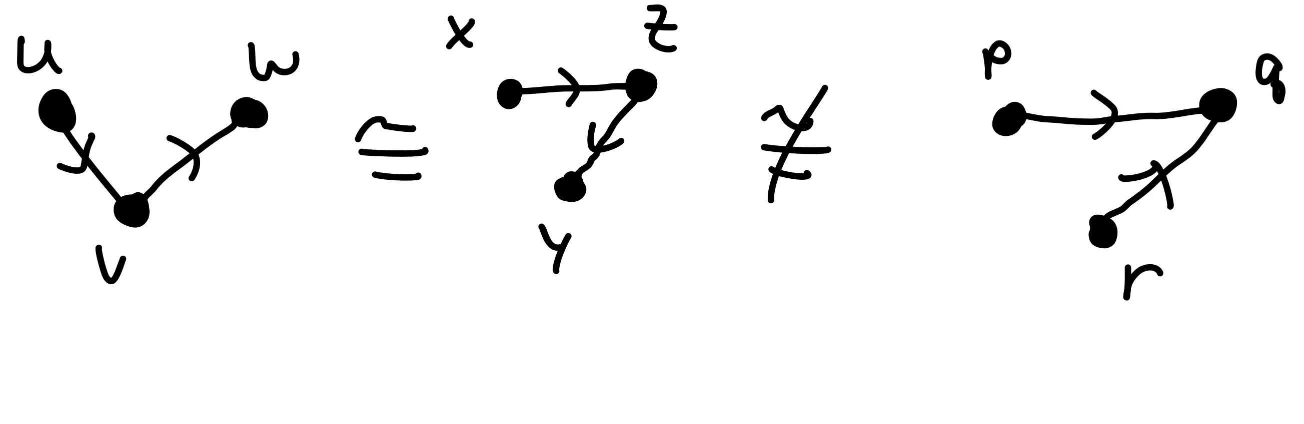
d) Subgraph, decomposition, union the same.



decomposes into



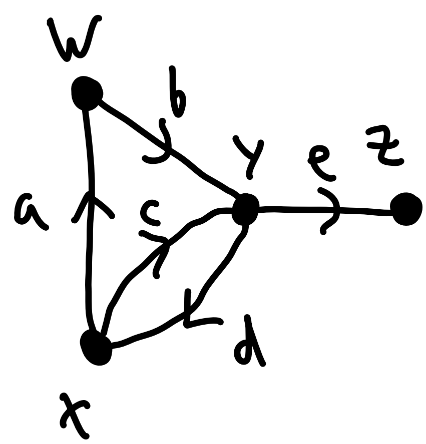
e) Isomorphism same, except edges have to point same direction



f) The (i,j) entry of the adjacency matrix is the number of edges from v_i to v_j

The (i,j) entry of the incidence matrix of a loopless graph is $+1$ if v_i is the tail of e_j and -1 if v_i is the head of e_j

Class activity:



G

$$\begin{array}{c} \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \left[\begin{array}{cccc} \; & \; & \; & \; \\ & & & \\ & & & \\ & & & \end{array}\right] \end{array}$$

$A(G)$

$$\begin{array}{c} \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc} \; & \; & \; & \; & \; \\ & & & & \\ & & & & \\ & & & & \end{array}\right] \end{array}$$

$M(G)$

g) For a vertex v,

$d^+(v)$: outdegree, # edges w/ tail v

$d^-(v)$: indegree, # edges w/ head v

$\delta^{\pm}(G)$: min out/indegree, $\Delta^{\pm}(G)$: max out/indegree

Successor: a vertex w s.t. $\exists$ an edge $v \to w$

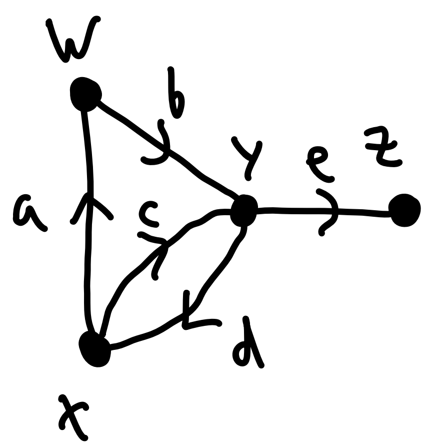
Predecessor: a vertex u s.t. $\exists$ an edge $u \to v$

$N^+(v)$: Out-nbhd/sucessor set, set of successors of v

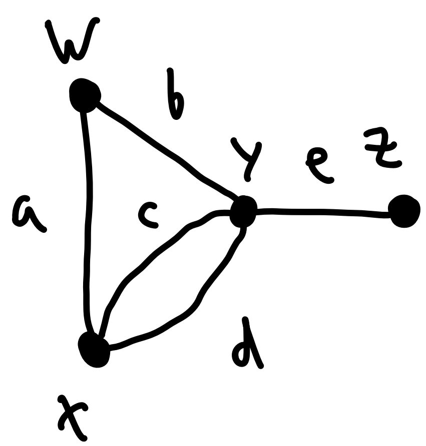
$N(v)$: In-nbhd/predecessor set, set of predecessors of v

Degree-sum formula: $e(G) = \sum_{v \in V(G)} d^+(v) = \sum_{v \in V(G)} d^-(v)$

h) The <u>underlying graph</u> of a digraph D is the graph G obtained by removing directions

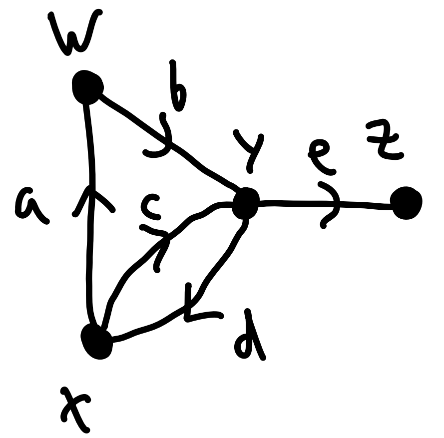


D



G

i) A digraph is weakly connected if the underlying graph is connected, and strongly connected if $\exists$ path from u to v $\forall$ vertices u, v



Thm 1.4.24: D: digraph

D has an Eulerian circuit $\iff$ a) $d^+(v) = d^-(v)$ $\forall v \in V(D)$

b) the underlying graph has ≤ 1 nontrivial component

D has an Eulerian trail $\iff$ a) $\sum_{v \in V(D)} |d^+(v) - d^-(v)| \leq 2$

b) the underlying graph has ≤ 1 nontrivial component