Math and Proofs Class 6

October 24th, 2017

Fun aside: Monty Hall Problem

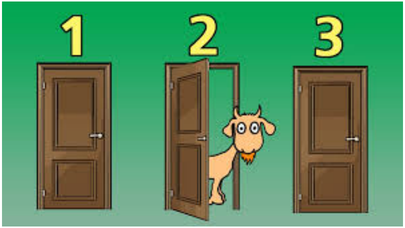

Figure: Monty Hall Problem

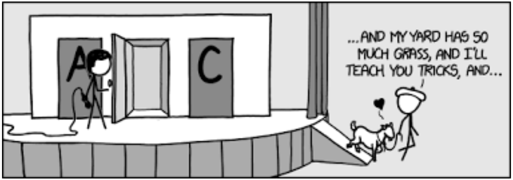

Figure: Or there's another solution...

Recap of Last Class

- Talked about cardinality
- Did some examples of the pigeonhole principle
- Showed that the integers, even integers, odd integers, and rational numbers all have the same cardinality
- But we showed that the real numbers have a LARGER cardinality than the integers

More Cardinality

- Now: we'll show that the power set of A always has a larger cardinality than A.
- $|A| < |P(A)|$
- Examples:
  1. $A = \{1, 2\}, P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$, $|A| = 2, P(A) = 4$
  2. $P(\emptyset) = \{\emptyset\}$, $|\emptyset| = 0, |\{\emptyset\}| = 1$

Axiom of Choice

- Given any set of mutually disjoint nonempty sets, there exists at least one set that contains exactly one element in common with each of the nonempty sets.
- *The Axiom of Choice is necessary to select a set from an infinite number of pairs of socks, but not an infinite number of pairs of shoes*
- Implication: Banach-Tarski

Zorn's Lemma

- If S is any nonempty partially ordered set in which every chain has an upper bound, then S has a maximal element.

The Well-Ordering Principle

- *The Axiom of Choice is obviously true, the well-ordering principle is obviously false, and who can tell about Zorn's Lemma?*
- Well-ordering principle: Every set can be well-ordered
- What is a well-ordering? It means that we order the elements in such a way such that every subset has a least element

Induction

- If a fact is true about 0, and if whenever it's true about n, then it's also true about $n + 1$, then it's true about every integer.
- $\sum_{i=1}^{n} n = \frac{n(n+1)}{2}$
- If n lines are drawn in the plane and no two lines are parallel, how many regions do they separate the plane into?

Next Time

- Transfinite Induction
- Goodstein's Theorem