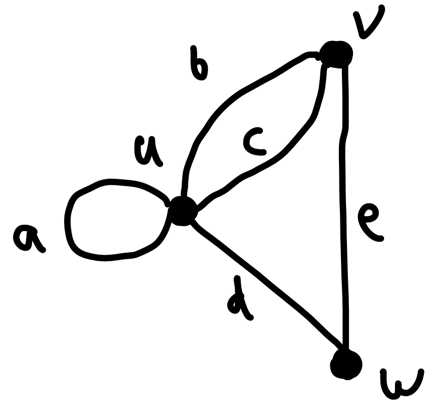
Announcement: H/w 2 will be posted later today

Today: Connectivity, cut-edges, and cycles
Konig's Theorem

Recall: Lemma 1.2.5: Every u,v-walk contains a u,v-path

Key step: If w appears more than once, delete everything btwn first and last accurence (see notes from last time for full proof)

Ex:



$u, a, u, c, v, b, u, d, w$

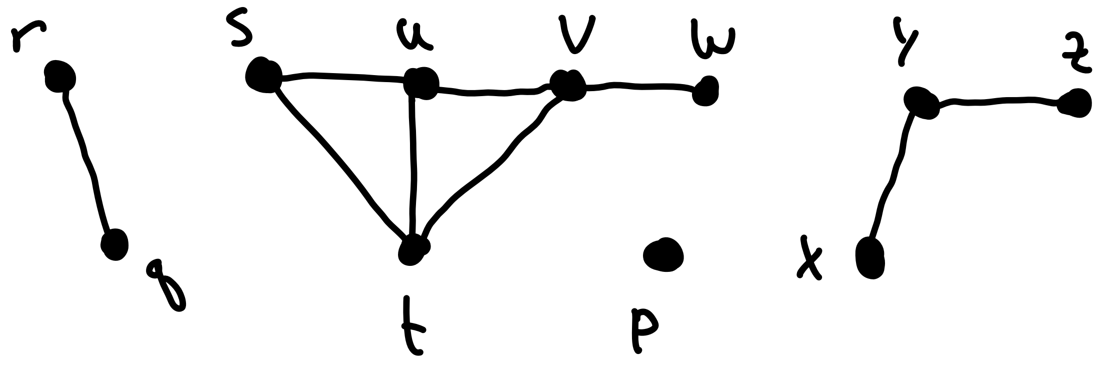
Def 1.2.6 / 1.2.8:

a) G is <u>connected</u> is $\forall u, v \in V(G)$, G contains a u,v-path (or walk or trail)

b) The <u>(connected) components</u> of G are its maximal connected subgraphs

c) An <u>isolated vertex</u> is a vertex of deg 0

Ex 1.2.9:

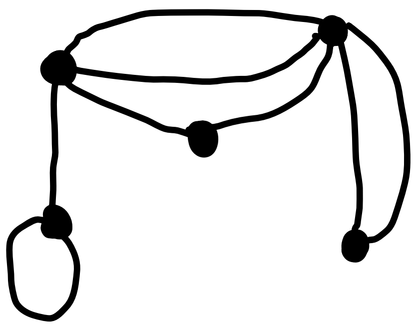


Remark 1.2.7: "u and v are in the same connected component" is an equivalence rel'n

Def 1.2.12:

a) If $T \subseteq V(G)$, the <u>induced subgraph</u> $G[T]$ is the graph w/ vertex set T and edge set

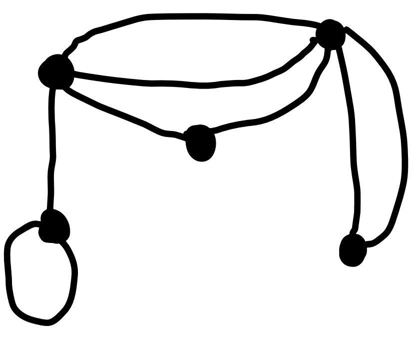
$E(G) \cap \{$edges w/ both endpoints in T



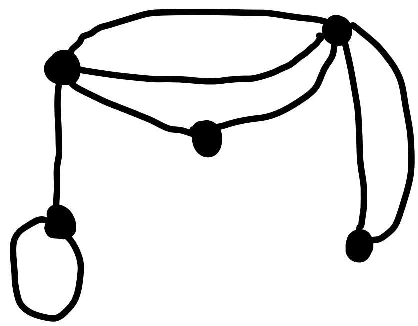
b) An edge $e \in E(G)$ is a <u>cut-edge</u> if the graph $G \setminus e := (V(G), E(G) \setminus e)$ has one more conn. cmpt. than G

vertex set

edge set



c) A vertex $v \in V(G)$ is a cut-vertex if $G[V(G) \setminus v]$ has one more conn. cmpt. than G



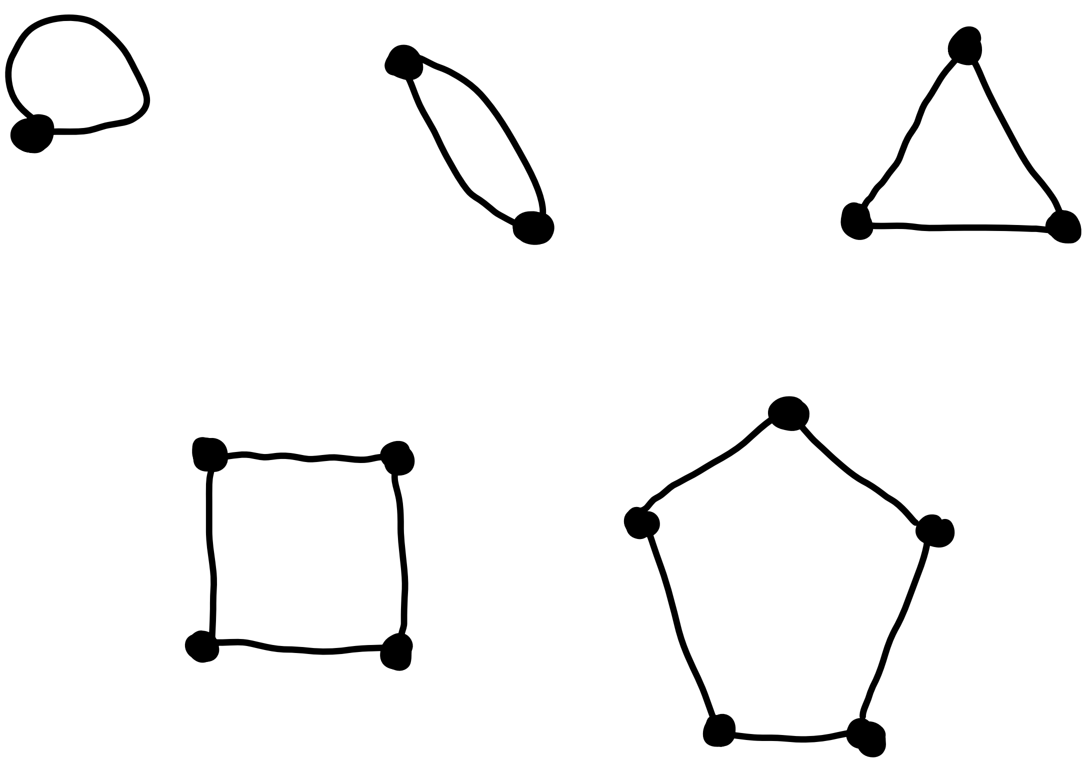
Thm 1.2.14: An edge $e \in E(G)$ is a cut-edge iff it belongs to no cycle

Pf:

Next goal: Characterize bipartite graphs using cycles

Class activity (toy example):

Which cycles C_n are bipartite?



Proposition [Us, 2023]: C_n is bipartite if and only if ________

Konig's Theorem [1936]: G: graph

G is bipartite $\Longleftrightarrow$ G has no odd cycle

Pf: