Math 213-A1: Introduction to Discrete Mathematics

Lecture: MWF 1:00-1:50pm
Everett Hall 3117

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Textbook: Discrete Mathematics and its Applications, 7th Edition
By Kenneth H. Rosen

Course website:
andyhardt.github.io/213_S26/course_page.html

Homework 1 has been posted (due Friday 1/30)

"Problem session" after class today (2:00-3:20pm)

See also: LaTeX tutorial (on course website)

Go through syllabus

First two classes: propositional logic, intro to sets

Today: More on set operations, cardinality, membership tables, how to write a proof

Operations on sets

Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Power set (set of all subsets): $P(A) = \{B \mid B \subseteq A\}$

Cartesian product: $A \times B = \{(a,b) \mid a \in A,\ b \in B\}$

Set difference: $\underbrace{A \setminus B}_{\text{or } A-B} = \{x \mid x \in A \text{ and } x \notin B\}$

Complement: $\overline{A} = \{x \mid x \notin A \text{ (and } x \in U)\}$

$= U \setminus A$ fixed "universal" set

e.g. $U = \mathbb{Z}$

$A = \{x \mid x \text{ is an odd integer}\}$

$\overline{A} = U \setminus A = \{x \mid x \text{ is an even integer}\}$

Cardinality:

$|A| =$ the number of elts. in A

e.g. $|\emptyset| = 0$ $|\{a,b,c\}| = 3$ $|\{a,a\}| = 1$ $|\mathbb{Q}| = \infty$

Proof Techniques

A proof is an argument that is

- precise (say exactly what you mean)
- rigorous (justify each step)
- complete (no logical holes)
- clear (easy to read / understand)

Q: Shouldn't all solutions have these properties?

A: Yes, but we'll have particularly high standards on proofs
and you should always show your work!

Problems which say "prove", "show", "demonstrate" require proof

On HW1, this is

2.1.26, 2.2.15, 2.2.24

If in doubt, ask!

Examples of good proofs: §2.2 Examples 10, 11, 12, 13, 14

Example: Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Pf: We show that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

<u>$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$:</u>

Let $x \in A \cup (B \cap C)$. Then $x \in A$ or $x \in B \cap C$ (or both). In the first case, $x \in A \cup B$ since $x \in A$, and $x \in A \cup C$ since $x \in A$. Therefore $x \in (A \cup B) \cap (A \cup C)$. In the second case, $x \in B$ and $x \in C$, so $x \in A \cup B$ and $x \in A \cup C$, so $x \in (A \cup B) \cap (A \cup C)$. Hence, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

<u>$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$</u>

Let $x \in (A \cup B) \cap (A \cup C)$. If $x \in A$, then $x \in A \cup (B \cap C)$. If $x \notin A$, then since $x \in A \cup B$, $x \in B$, and since $x \in A \cup C$, $x \in C$. Thus, $x \in B \cap C$, so $x \in A \cup (B \cap C)$. Hence, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

end of proof $\longrightarrow$ $\square$

Another method: use membership tables.

Every elt. x has 8 possibilities:

$x \in A$, $x \in B$, $x \in C$

$x \in A$, $x \in B$, $x \notin C$,

etc.

A	B	C
1	1	1
1	1	0

means $x \in$ set

means $x \notin$ set

Pf (alt strategy):

We draw the membership table for both sides of the desired equality. Since the columns for $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ are identical, the sets are equal.

A	B	C	B∩C	A∪(B∩C)	A∪B	A∪C	(A∪B)∩(A∪C)
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

$\square$