Announcement

HW2 due Sunday @ 11:59pm via Gradescope

Def: $f: A \to B$

f is one-to-one / injective if whenever $a \neq b, f(a) \neq f(b)$

f is onto/surjective if $f(A) = B$ range

f is bijective if it is injective and surjective

Ex (cont.):

f is not injective since $f(a) = x = f(c)$, but $a \neq c$

f is not surjective since $y \notin f(A)$

Ex: $g: \mathbb{R} \to \mathbb{R}$

$g(x) = x+1$

g is injective since if $g(x) = g(y)$ then $x+1 = y+1$, so $x = y$

g is surjective since if $z \in \mathbb{R}$, $g(z-1) = z$

See book for increasing/decreasing functions

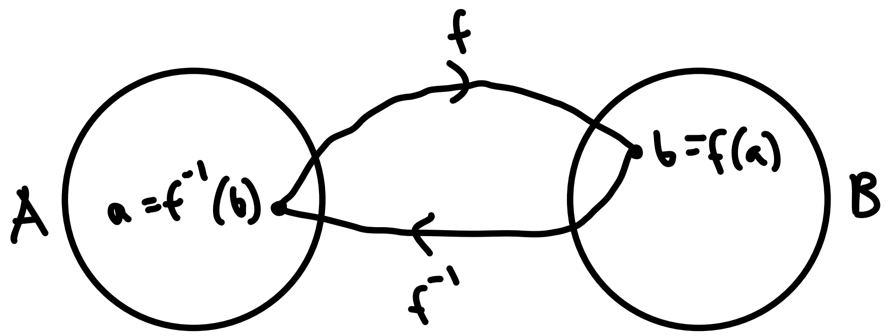
Bijections have inverse functions

$f: A \to B$ bijection

$f^{-1}: B \to A$ (also a bijection)

f^{-1} "undoes" f: if $f(a)=b$, then $f^{-1}(b)=a$

We call a function with an inverse <u>invertible</u>



Ex:

a) $f: \mathbb{R}_+ \to \mathbb{R}_+$, $f(x) = x^2$ ← set of pos. real nums.

is invertible w/ $f^{-1}(x) = \sqrt{x}$ ← pos. sqrt.

b) $A = \{a, b, c\}$ $f: A \to A$

$f(a) = b$ $f(b) = c$ $f(c) = a$

is invertible w/

$f^{-1}(a) = c$ $f^{-1}(b) = a$ $f^{-1}(c) = b$

Composition: apply functions in sequence

Let $f: A \to B$ $g: B \to C$

need these to be the same

Then $g \circ f: A \to C$ is given by

$$g \circ f(a) = g(f(a))$$

Ex: $f: \mathbb{Z} \to \mathbb{Z}$ $g: \mathbb{Z} \to \mathbb{N}$

$f(x) = x+1$ $g(x) = x^2$

$g \circ f: \mathbb{Z} \to \mathbb{N}$

$(g \circ f)(x) = (x+1)^2$

$f \circ g$ is not defined since $\text{dom}(f) \neq \text{codom}(g)$

§3.1: Algorithms

Def: An <u>algorithm</u> is a finite sequence of precise steps

Properties:

- Input
- Output
- Definitensss: Steps are precisely-defined
- Correctness: Always gives the right answer
- Finiteness: Finite # steps for any input
- Effectiveness: You can actually do each step
- Generality: Works for all possible inputs

Ex: Finding max. elt. in a finite sequence

procedure max($a_1, \ldots, a_n$: integers)

$m := a_1$

for $i := 2$ to n

    if $m < a_i$ then $m := a_i$ set m equal to a_i

return m

Class activity (if time): check these properties

Optimization problem: maximize/minimize some parameter
e.g. Give change using the fewest num. coins possible

Greedy algorithm: Try to solve the optimization problem by making the "best" choice at each step

doesn't always give the optimal solution

Greedy Change-Making Algorithm:

procedure change($c_1, c_2, \ldots, c_r$: values of coins, where $c_1 > c_2 > \cdots > c_r$; n: pos. int)

for $i := 1$ to r

  $d_i := 0$ (d_i is the num coins of value c_i)

  while $n \geq c_i$

    $d_i := d_i + 1$ (add a coin of value c_i)

    $n := n - c_i$

return $d_1, d_2, \ldots, d_r$

Class activity (if time): run this algorithm with coins of values: 50, 20, 10, 5, 2, 1 and a starting value of 79

Answer: We have $r=6$, $n=79$, and

$c_1=50$, $c_2=20$, $c_3=10$, $c_4=5$, $c_5=2$, $c_6=1$.

$i=1$: The algorithm first sets d_1 as high as possible, which is $d_1=1$; now, $n=29$.

$i=2$: The algorithm next sets d_2 as high as possible, which is $d_2=1$; now, $n=9$.

$i=3$: The algorithm next sets d_3 as high as possible, which is $d_3=0$; now, $n=9$.

$i=4$: The algorithm next sets d_4 as high as possible, which is $d_4=1$; now, $n=4$.

$i=5$: The algorithm next sets d_5 as high as possible, which is $d_5=2$; now, $n=0$.

$i=6$: The algorithm next sets d_6 as high as possible, which is $d_6=0$; now, $n=0$.

Output: $d_1=1$, $d_2=1$, $d_3=0$, $d_4=1$, $d_5=2$, $d_6=0$