Math 121, Winter 2023, Homework 6 Other Problem

It's suggested to do this before Problem 14.6.22, although you can do that problem without this one.

Homework 6 Extra Problem. *Let* $p_k(x_1, \ldots, x_n) = x_1^k + x_2^k + \cdots + x_n^k$. *Let*

$$E(t) = \sum_{r=0}^{\infty} s_r(x_1, \ldots, x_n) t^r, \qquad P(t) = \sum_{r=1}^{\infty} p_r(x_1, \ldots, x_n) t^{r-1}.$$

Prove that

$$E(t) = \prod_{i=1}^{n} (1 + x_i t), \qquad P(t) = \sum_{i=1}^{\infty} \frac{x_i}{1 - x_i t} = \sum_{i=1}^{\infty} \frac{d}{dt} \ln \frac{1}{1 - x_i t}.$$

Hint for Problem 14.6.22 (optional): compare $P(t)$ with the logarithmic derivative of $E(t)$.