Reminder: HW4 due Wed. 2/25 @ 9am

Recall:

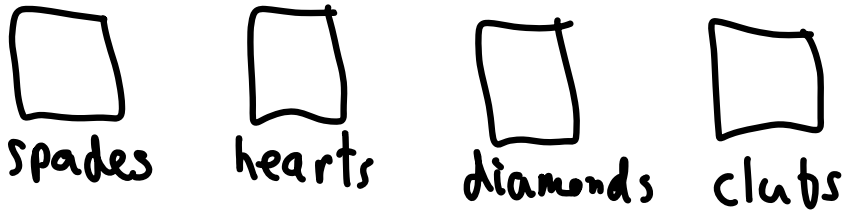
Pigeonhole Principle: Put n pigeons into n boxes. If $m > n$, there must be at least one box w/ multiple pigeons.

Generalized pigeonhole principle: Put m pigeons into n boxes. Then there is at least one box w/ $\lceil m/n \rceil$ pigeons

e.g. $m = 31 \quad n = 10 \rightsquigarrow \lceil m/n \rceil = \lceil 3.1 \rceil = 4$

$m = 40 \quad n = 10 \rightsquigarrow \lceil m/n \rceil = \lceil 4 \rceil = 4$

Ex 7: How many cards must be chosen from a deck to ensure there are ≥ 3 of the same suit



Ans: We want the smallest m s.t.

$\left\lceil \frac{m}{4} \right\rceil \geq 3$ i.e. $m > 2 \cdot 4 \rightsquigarrow m = 9$

Ex 8: Telephone numbers are of the form,

$$\underbrace{NXX}_{\text{area code}} - NXX - XXXX$$

where each N can be a digit from 2 to 9 and each X can be a digit from 0 to 9.
A state has 25,000,000 phones. How many area codes does it need to ensure each phone has a diff. num?

Soln: NXX-XXXX

$8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 8{,}000{,}000$ numbers per area code

$m = 25$ million, $n = 8$ million

§6.3: Permutations and Combinations

Ex I: How many ways are there to order 3 people in a line?

ABC BAC CAB
ACB BCA CBA
} permutations of $\{A, B, C\}$

Ans: $3 \cdot 2 \cdot 1 = 6$ (from prod. rule)

Ex II: How many ways are there to order 2 out of 5 people in a line?

AB BA CA DA EA
AC BC CB DB EB
AD BD CD DC EC
AE BE CE DE ED
} 2-permutations of $\{A, B, C, D, E\}$

Ans: $5 \cdot 4 = 20$

Def: A permutation of a set is an ordered arrangement of its elts. An r-permutation is an ordered arrangement of r of its elts.

The number of r-permutations of a set of card. n is

$$P(n,r) = \underbrace{n(n-1)(n-2)\cdots(n-r+1)}_{r \text{ factors}} = \frac{n!}{(n-r)!} \qquad (P(n,n) = n!)$$

"n permute r"

Ex 7: How many permutations of the letters ABCDEFGH contain the string ABC?

e.g. DGHABCFE

Ans: Just take permutations of {ABC, D, E, F, G, H}

$P(6,6) = 6! = 5040$

Ex III: How many committees of 2 students can be formed out of a total of 5 students?

AB BC CE
AC BD DE
AD BE
AE CD

10 total

Note: here AB and BA represent the same committee

Half as many as in Ex II since there are 2! orders for each committee.

Ex IV: There are $P(10,3) = 10 \cdot 9 \cdot 8 = 720$ ways to order 3 out of 10 people. But,

ABC BCA
ACB CAB
BAC CBA
} same committee

BFG FGB
BGF GBF
FBG GFB
} same committee

So there are only $\frac{720}{3!} = 120$ 3-person committees out of 10 people

Def: A k-combination of a set is an unordered arrangement of k of its elts. i.e. an k-elt. subset!

The number of k-combinations of a set of card. n is

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{P(n,k)}{k!}$$

"n choose k" ← binomial coefficients

Ex 11: How many ways are there to choose 5 (unordered) cards from a 52-card deck?

$$\text{Ans: } \binom{52}{5} = \frac{52!}{5!\ 47!} = \frac{52\cdot 51\cdot 50\cdot 49\cdot 48\cdot 47\cdot 46\cdots}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 47\cdot 46\cdots}$$

$$= \frac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 2598960$$

How many ways to choose 47 cards?

$$\text{Ans: } \binom{52}{47} = \frac{52!}{47!\ 5!} = \binom{52}{5} =$$

$$\binom{n}{k} = \binom{n}{n-k} \text{ always}$$

Class activity:

a) How many 3-elt subsets of $\{A, B, C, D, E, F\}$ are there?

b) How many contain A?

c) How many don't contain A?

d) Can you express the above 3 quantities as binom. coeffs. $C(n,r)$?

e) Does the above say anything about those binom. coeffs.?