Math 418, Spring 2024 – Homework 4

Due: Wednesday, February 14th, at 9:00am via Gradescope.

Instructions: Students should complete and submit all problems. Textbook problems are from Dummit and Foote, *Abstract Algebra, 3rd Edition*. All assertions require proof, unless otherwise stated. Typesetting your homework using LaTeX is recommended, and will gain you 2 bonus points per assignment.

1. **Dummit and Foote #13.2.1:** *Let $\mathbb{F}$ be a finite field of characteristic p. Prove that $|\mathbb{F}| = p^n$ for some positive integer n.*
2. **Dummit and Foote #13.2.4:** *Determine the degree over $\mathbb{Q}$ of $2 + \sqrt{3}$ and of $1 + \sqrt[3]{2} + \sqrt[3]{4}$.*
3. **Dummit and Foote #13.2.5:** *Let $F = \mathbb{Q}(i)$. Prove that $x^3 - 2$ and $x^3 - 3$ are irreducible over F.*
4. **Dummit and Foote #13.2.7:** *Prove that $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Conclude that $[\mathbb{Q}(\sqrt{2} + \sqrt{3}) : \mathbb{Q}] = 4$. Find an irreducible polynomial satisfied by $\sqrt{2} + \sqrt{3}$.*

MORE PROBLEMS TO COME!!!