## <u>Announ cements</u>

Middern 2: Wed 3/26 7:00-8:30pm, Sidney Lu 1043
See policy email (reference sheet allowed)

Topics: Everything through today (i.e. thru D&F \$14.1) but focus is on post-Midterm 1 material (\$13.2-onwards)

Practice problems: see email or website

Tues., Wed. after break: review

Conflicts: email me ASAP

HW7 (due hed 4/2): will be posted over break but all problems are from post-midterm 2 material

Recall: K/F !field extín.

Aut(K/F) = {automs. of k which fix F} < Aut(k)

H = Aut(K)

Fix H = subfield of K fixed by every elt. of H

Thm: Let f(x) = F[x], K = Sp.f. Then,

| Aut(x/F)| < [x:F],

w/ equality if f is separable.

Pf by example: (see DLF for full argument)

$$f(x) = x^3 - 2 \in \mathbb{Q}[x]$$

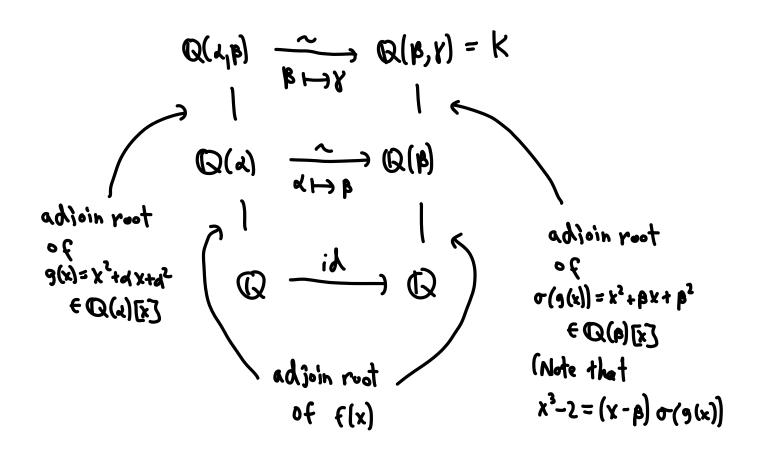
Splits as  $(x - 3/2)(x - 3/3)(x - 3/3)(x - 3/3)(x - 3/3)$  over  $\mathbb{Q}(x, \beta)$ 

$$K = Q(a, \beta) \qquad (x - a)(x - \beta)(x - \gamma)$$

$$L = Q(a) \qquad (x - a)(x^2 + ax + a^2)$$

$$Q \qquad \qquad \chi^3 - 2$$

Build JE Aut (K/Q) in two steps using DEFThm. 13.27



How many such or can we construct?

= 3.5 = (# roots of f)(# roots of g)

= (deg f)(deg g) = [Q(a):Q][k:Q(a)]

= Tk:Q]

f sep. = [K:Q]

Remark: If  $f(x) \in F[x]$  has roots  $d_1, \dots, d_n$  and  $k = Sp_{\overline{x}}f$ ,  $\sigma \in Aut(k/F)$  then the restriction  $\sigma \in Aut(k/F)$  then the restriction  $\sigma \in Aut(k/F)$  then the restriction  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$  and  $\sigma \in Aut(k/F)$  then  $\sigma \in Aut(k/F)$ 

The homom. Aut  $(K/F) \longrightarrow S_n$  (symmetric gp. on n fetters)  $\sigma \longmapsto \overline{\sigma}$ 

is inj. (every autom. gives a different perm.) but not necessarily surj.

Def: A finite extension K/F is <u>Galois</u> if |Aut(K/F)| = [K:F]. In this case, we set |Aut(K/F)| = |Aut(K/F)| and call it the <u>Galois group</u> of |K/F|.

Cor: If fe F[x] is sep., k = Spf, then k/f is Galois (Turns out all Galois extrés are of this form)

## Examples:

$$k = Sp_{\mathcal{Q}} \in \text{ where } \{(x) = (x^2 - 2)(x^2 + 1)\}$$

Note: this is a proper subgp. of S.

$$|Aut(K/Q)| = 6 = [K:Q] \quad |E=Q(\Im z, S_3\Im z)|$$

$$|Aut(K/Q)| = 6 = [K:Q] \quad |E=Q(\Im z, S_3)|$$

$$|E=Q(\Im z, S_3)|$$

Thm: Let 
$$H \leq Aut(K)$$
,  $F = Fix H$ 

finite any
sp. field

Then K/F is Galois!

More precisely,

Enjoy the break!