

Quiz 4 today!

§ 7.3: Bayes Theorem

Recall: conditional probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Bayes' Theorem: Assume $P(E), P(F) > 0$. Then,

$$P(F|E) = \frac{P(E|F) P(F)}{P(E)}$$

PF: By definition,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \quad \text{and} \quad P(F|E) = \frac{P(E \cap F)}{P(E)},$$

so $P(E \cap F) = P(E|F)P(F)$, and combining this w/ the 2nd eqn,

$$P(F|E) = \frac{P(E|F) P(F)}{P(E)}.$$

□

Important note: often, $P(E)$ is unknown, so

Rosen gives the alternate version:

$$P(F|E) = \frac{P(E|F) P(F)}{P(E|F) P(F) + P(E|\bar{F}) P(\bar{F})}$$

This is equivalent to our version, since

$$p(E|F)p(F) + p(E|\bar{F})p(\bar{F}) = p(E \cap F) + p(E \cap \bar{F}) = p(E)$$

↑
sum rule, since
 $E \cap F$ and $E \cap \bar{F}$
have no overlap

Ex 2: Suppose one person in 100,000 has a particular rare disease. There is a diagnostic test which is correct

- 99% of the time, when given to a person w/ the disease
- 99.5% of the time, when given to a person w/out the disease

Find the probability that a person who tests positive actually has the disease.

Sol'n: E : tests positive, F : has the disease

Want: $p(F|E)$.

$$p(F) = \frac{1}{100000} = 0.00001 \quad p(\bar{F}) = 0.99999$$

$$p(E|F) = 0.99 \quad p(E|\bar{F}) = 1 - 0.995 = 0.005$$

By Bayes' Theorem,

$$p(F|E) = \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\bar{F})p(\bar{F})} = \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\bar{F})p(\bar{F})}$$

$$= \frac{0.99 \cdot 0.00001}{0.99 \cdot 0.00001 + 0.005 \cdot 0.99999} = 0.002 = 0.2\%$$

Even though the test is very good, almost all of the positive tests are false positives!

Ex 1 (Class activity if time):

Two boxes

Box 1: 2 Green balls, 7 Red balls

Box 2: 4 Green balls, 3 Red balls

We

- Choose a box at random ($p(\text{Box 1}) = 0.5$)
- Choose a ball at random (equal prob for each ball in the box) from that box

If we select a Red ball, what is the probability it came from the first box?