Math and Proofs Class 3

September 26th, 2017

Fun aside: Euclid's Proof that there are infinitely many prime numbers

- A prime number is a number with only two factors: 1 and itself
- Every number is made up of prime numbers multiplied together
- So maybe there are finitely many of them? We can multiply them together in so many different ways. Maybe we can create all the numbers in that way?
- Nope! Multiply them all together and add 1. What does this do?

Recap of Last Class

- We started to learn about set theory
- We looked at some set theory operations and did some simple proofs

Set Theory Reminders

- Empty Set: $\emptyset$
- Union: $A \cup B$
- Intersection: $A \cap B$
- Set minus: $A \setminus B$ (the elements in A but not B)
- Subset: $A \subseteq B$
- Element: $x \in A$

Set Theory Results

1. For any set A, $\emptyset \subseteq A$
2. For any set A, $A \subseteq A$
3. If A, B, and C are sets where $A \subset B$ and $B \subset C$, then $A \subset C$
4. Let A and B be sets. Then $A = B$ if and only if $A \subset B$ and $B \subset A$.

Exercise 1

Let $A = \{x, y, \{x, y\}\}$. True or false:

1. $\{x, y\} \subseteq A$
2. $\{x, y\} \in A$
3. $\{y\} \subseteq A$
4. $\{y\} \in A$

Exercise 2

Let $A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{4, 5, 6\}$. Find each of the following sets:

1. $A \cup B$
2. $A \cap B$
3. $B \cap C$
4. $A \setminus B$
5. $A \cap (B \cup C)$
6. $(A \cap B) \cup (A \cap C)$

More Set Theory Results

Let A, B, C be sets

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$ (exercise)

Cartesian Product

The Cartesian Product of two sets A and B is the set of ordered pairs with first entry in A and second entry in B.

Example: $A = \{1, 2\}, B = \{dog, cat, child\}$. Then
$A \times B = \{(1, dog), (1, cat), (1, child), (2, dog), (2, cat), (2, child)\}$.

Questions:

1. If $A = \{a, b\}, B = \{34\}$, what are $A \times B$ and $B \times A$?
2. If A has 4 elements and B has 3 elements, how many elements does $A \times B$ have?
3. If $\mathbb{R}$ is the set of "real" numbers, you could say that $\mathbb{R}$ represents the number line. In this way of thinking, what is $\mathbb{R} \times \mathbb{R}$?

Power Sets

If A is a set, then the power set $P(A)$ is the set of all subsets of A.

Example: $A = \{a, b, c\}$.
$P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

Exercises:

1. If $A = \{1, 2\}$, what is $P(A)$?
2. Find the power set of the empty set
3. Prove:
    - If $A \subset B$, then $P(A) \subset P(B)$
    - If $A \cap B = \emptyset$, then $P(A) \cap P(B) = \emptyset$

Next Time

- We'll look at functions, bijections, and equivalence relations
- In two weeks, we'll start looking at cardinality and infinity