

Announcements

HW2 posted (due Wed. @ 9am via Gradescope)

Quiz 1: score will be higher of $\underbrace{Q1+Q2+Q3}$ or $\frac{3}{2}(Q2+Q3)$
each question worth 4 pts.

Recall: A function $f: A \rightarrow B$ is an assignment of exactly one elt. of B to each elt. of A

- A is the domain of f
- B is the codomain of f
- The range/image of f is the set $\{f(a) \mid a \in A\}$
- If $a \in A$, $f(a)$ is the image of a under f
- If $b \in B$, the preimage of b under f is the set

$$f^{-1}(b) = \{a \in A \mid f(a) = b\}$$

Ex: $A = \{a, b, c\}$ $B = \{x, y, z\}$

$$f: A \rightarrow B$$

$$f(a) = x \quad f(b) = z \quad f(c) = x$$

f has:

- domain A
- codomain B
- range $\{x, z\}$

The image of c is x

The preimage of x is $\{a, c\}$

The preimage of y is \emptyset

Can also do image/preimage of sets

Def: Let $f: A \rightarrow B$. Let $C \subseteq A$ and $D \subseteq B$

The image of C is $f(C) = \{f(c) \mid c \in C\}$

The preimage of D is $f^{-1}(D) = \{a \in A \mid f(a) \in D\}$

Ex (cont): $f(\{a, c\}) = \{x\}$

$$f^{-1}(\{x, z\}) = A$$

Def: $f: A \rightarrow B$

f is one-to-one / injective if whenever $a \neq b, f(a) \neq f(b)$

f is onto / surjective if $f(A) = B$ ← range

f is bijective if it is injective and surjective

Ex (cont.):

f is not injective since $f(a) = x = f(c)$, but $a \neq c$

f is not surjective since $y \notin f(A)$

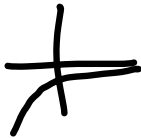
Ex: $g: \mathbb{R} \rightarrow \mathbb{R}$

$$g(x) = x + 1$$

g is injective since if $g(x) = g(y)$ then $x + 1 = y + 1$, so $x = y$

g is surjective since if $z \in \mathbb{R}$, $g(z - 1) = z$

Note: Every function $f: \mathbb{R} \rightarrow \mathbb{R}$

that is strictly increasing 

or strictly decreasing 

is injective

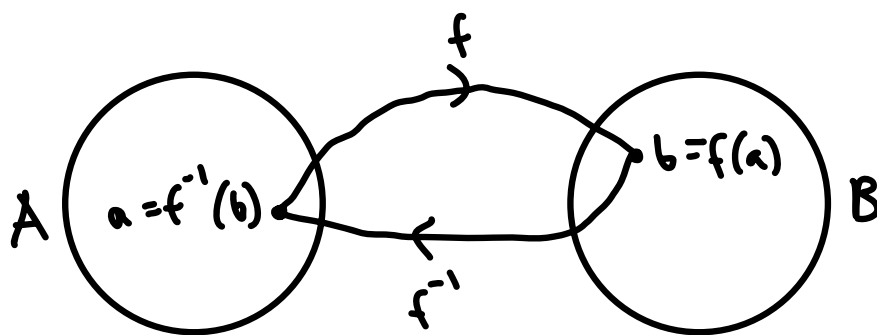
Bijections have inverse functions

$f: A \rightarrow B$ bijection

$f^{-1}: B \rightarrow A$ (also a bijection)

f^{-1} "undoes" f : if $f(a)=b$, then $f^{-1}(b)=a$

We call a function with an inverse invertible



Ex:

↙ set of pos. real nums.

a) $f: \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $f(x) = x^2$

is invertible w/ $f^{-1}(x) = \sqrt{x}$ ↙ pos. sqrt.

$$b) A = \{a, b, c\} \quad f: A \rightarrow A$$

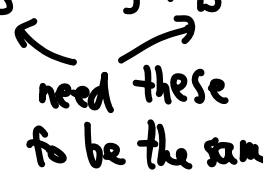
$$f(a) = b \quad f(b) = c \quad f(c) = a$$

is invertible w/

$$f^{-1}(a) = c \quad f^{-1}(b) = a \quad f^{-1}(c) = b$$

Composition: apply functions in sequence

$$\text{Let } f: A \rightarrow B \quad g: B \rightarrow C$$



Then $g \circ f: A \rightarrow C$ is given by

$$g \circ f(a) = g(f(a))$$

$$\text{Ex: } f: \mathbb{Z} \rightarrow \mathbb{Z} \quad g: \mathbb{Z} \rightarrow \mathbb{N}$$

$$f(x) = x+1$$

$$g(x) = x^2$$

$$g \circ f: \mathbb{Z} \rightarrow \mathbb{N}$$

$$(g \circ f)(x) = (x+1)^2$$

$f \circ g$ is not defined since

$$\text{dom}(f) \neq \text{codom}(g)$$

See textbook for more examples