Math and Proofs Class 1

September 19, 2017

What do we need to create a mathematical framework?

- Undefined Terms
- Assumptions
- Method of Proof

Euclid's Elements (for geometry)

Undefined Terms

1. Point
2. Line
3. Plane

Euclid's Elements (cont.)

Common Notions

1. That which are equal to the same thing are also equal to one another
2. If equals are added to equals, the wholes are equal
3. If equals are subtracted from equals, the remainders are equal
4. Things which coincide with one another are equal to one another
5. The whole is greater than the part

Euclid's Elements (cont.)

Postulates

1. It is possible to draw a straight line from any point to any other point
2. It is possible to extend a line segment continuously in both directions
3. It is possible to describe a circle with any center and any radius
4. It is true that all right angles are equal to one another
5. It is true that, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, intersect on that side.

Things to Note

- Not very precise
- More assumptions needed
- Common notions and postulates are both just assumptions, or axioms
- Parallel postulate was important historically

A couple of Elements Proofs

Proposition 1.4: If two triangles have two sides equal to two sides, respectively, and have the angles enclosed by the straight lines equal, then they will also have the base equal to the base, and the triangle will be equal to the triangle, and the remaining angles subtended by the equal sides will be equal to the corresponding remaining angles.

A couple of Elements Proofs (cont.)

Proposition 1.6: If a triangle has two angles equal to one another, then the sides subtending the equal angles will also be equal to one another.

Peano Axioms (for arithmetic)

1. Zero is a number
2. If a is a number, the successor of a is a number
3. Zero is not the successor of a number
4. Two numbers of which the successors are equal are themselves equal
5. If a set S of numbers contains zero and also the successor of every number in S, then every number is in S.

Let's use the Peano Axioms to do some (sort of boring, but essential) things

- Define addition
- Show that addition is commutative
- Show that addition is associative
- Define multiplication
- Show that multiplication is commutative, associative, and distributive over addition

Next Time

- Start to look at set theory, which we can use to describe *both* geometry and arithmetic, and also everything else in math!
- Maybe do a couple proofs in groups or on your own (interest?)