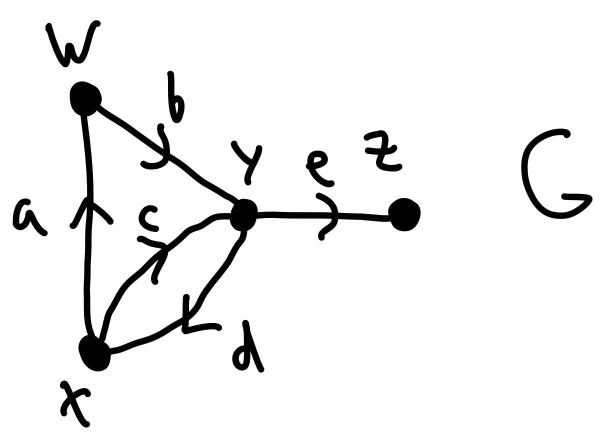
Announcements:

- H/w 2 graded; H/w 4 due 9/27 (2 weeks from today)
- Quiz 1 this Friday in class (20 mins)
  - Content: anything covered thru. today
- Midterm 1: Wed. 9/20 7:00-8:30pm (Noyes Lab. 217)
  - Reference sheet allowed (two-sided) otherwise, no resources allowed
  - See Mondy's email for full policies

Class activity:



$$\begin{array}{c|cccc} & w & x & y & z \\ \hline w & & 0 & & \\ x & 1 & & & \\ y & & & & \\ z & & & & \end{array}$$

$A(G)$

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline w & -1 & & & & \\ x & 1 & & & & \\ y & & & & & \\ z & & & & & \end{array}$$

$M(G)$

g) For a vertex v,

$d^+(v)$: outdegree, # edges w/ tail v

$d^-(v)$: indegree, # edges w/ head v

$\delta^\pm(G)$: min out/indegree, $\Delta^\pm(G)$: max out/indegree

Successor: a vertex w s.t. $\exists$ an edge $v \to w$

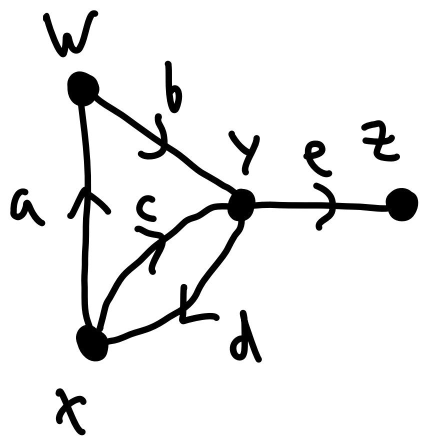
Predecessor: a vertex u s.t. $\exists$ an edge $u \to v$

$N^+(v)$: Out-nbhd/sucessor set, set of successors of v

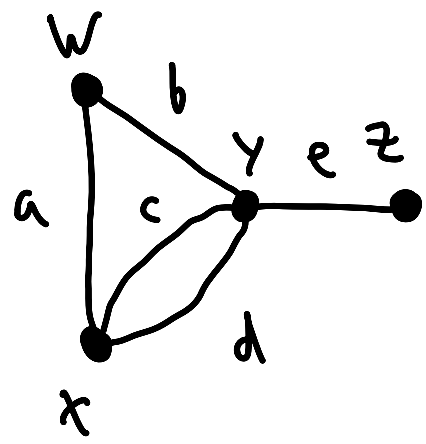
$N(v)$: In-nbhd/predecessor set, set of predecessors of v

Degree-sum formula: $e(G) = \sum_{v \in V(G)} d^+(v) = \sum_{v \in V(G)} d^-(v)$

h) The underlying graph of a digraph D is the graph G obtained by removing directions

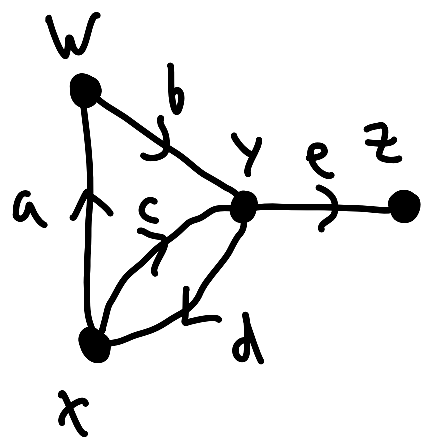


D



G

i) A digraph is weakly connected if the underlying graph is connected, and strongly connected if $\exists$ path from u to v $\forall$ vertices u,v



Thm 1.4.24: D : digraph

D has an Eulerian circuit $\iff$ a) $d^+(v) = d^-(v) \quad \forall v \in V(D)$

b) the underlying graph has ≤ 1 nontrivial component

D has an Eulerian trail $\iff$ a) $\sum_{v \in V(D)} |d^+(v) - d^-(v)| \leq 2$

b) the underlying graph has ≤ 1 nontrivial component

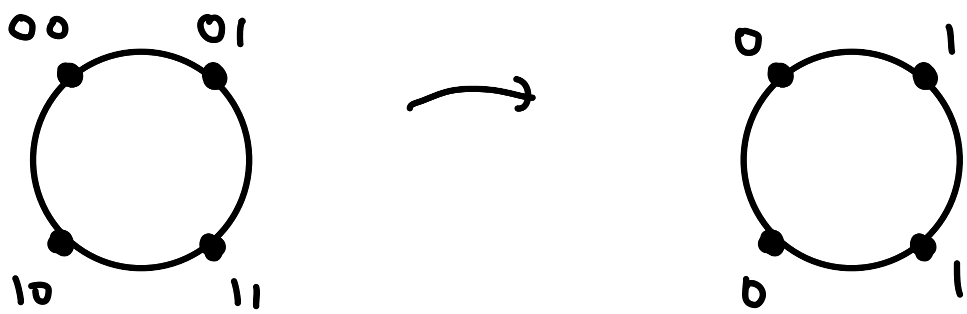
Pf (of first part):

$\Rightarrow$):

$\Leftarrow)$:

Remark: If D has an Eulerian circuit, it's nontrivial component is strongly connected.

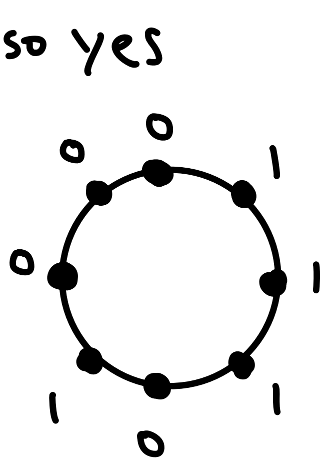
Application 1.4.25: de Bruijn cycles



Is there a cyclic arrangement of 2^n binary digits s.t. the 2^n strings of n consecutive digits are distinct?.

$n=2$. Yes

$n=3$: Also yes

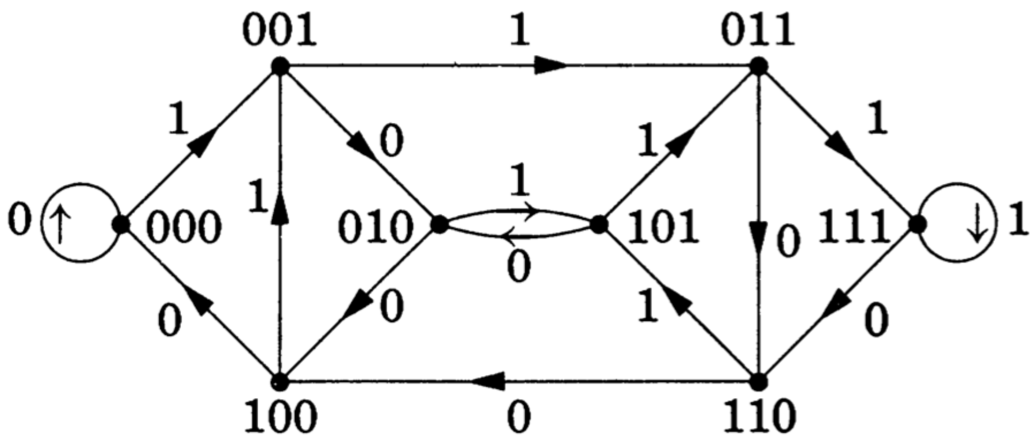


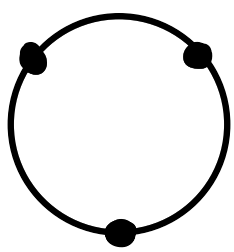
Let D_n be a digraph w/

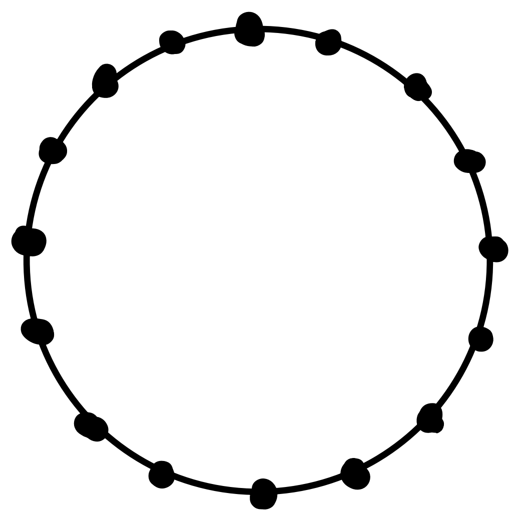
$V(D_n)$ = binary strings of length $n-1$

$a \xrightarrow{x} b$ if $a = a_1 a_2 \cdots a_{n-1}$, $b = a_2 \cdots a_{n-1} x$ i.e. the last $n-2$ entries of a are the first $n-2$ entries of b

$n = 4$:







Eulerian circuit in D_n $\longleftrightarrow$ Cyclic arrangement w/ distinct n-strings

Thm 1.4.26: D_n has an Eulerian circuit

Pf:

Def 1.4.27:

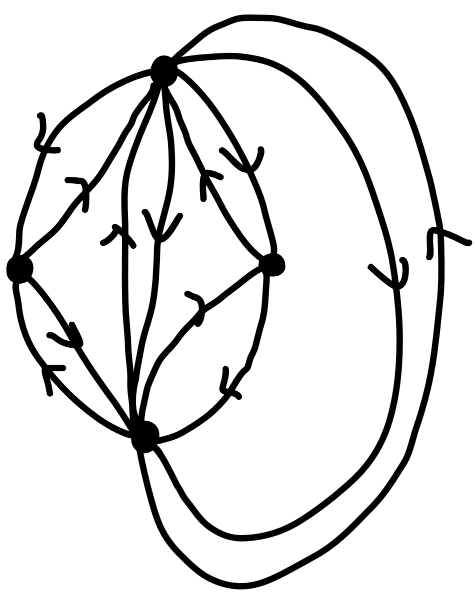
a) A digraph D is an <u>orientation</u> of a graph G if G is the underlying graph of D.

b) An <u>oriented graph</u> is an orientation of a simple graph

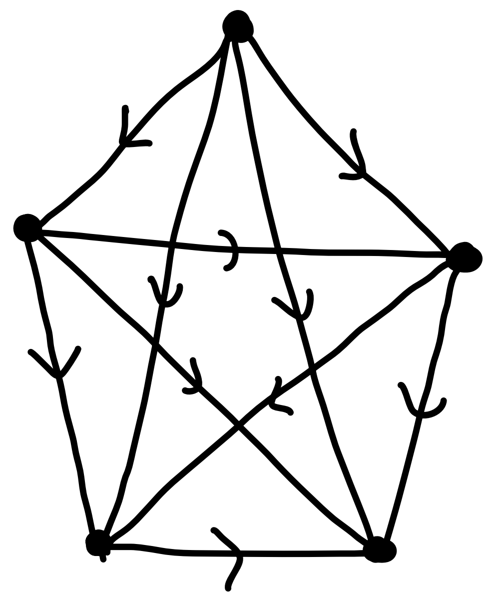
c) A tournament is an orientation of a complete graph

Class activity: Oriented graph? Tournament?

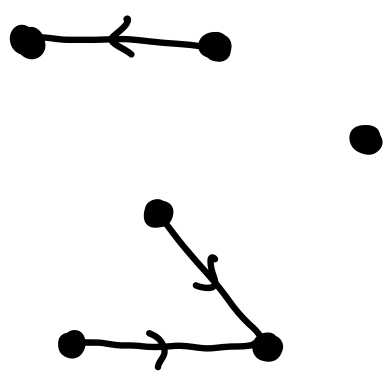
a)



b)



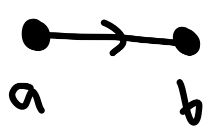
c)

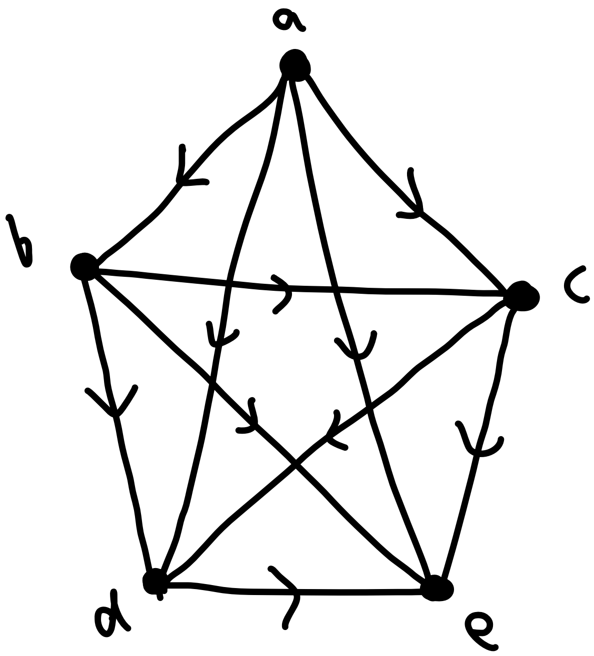


Reason for name "tournament":

Every player plays every other player ("round robin")

If a beats b, orient the edge like this





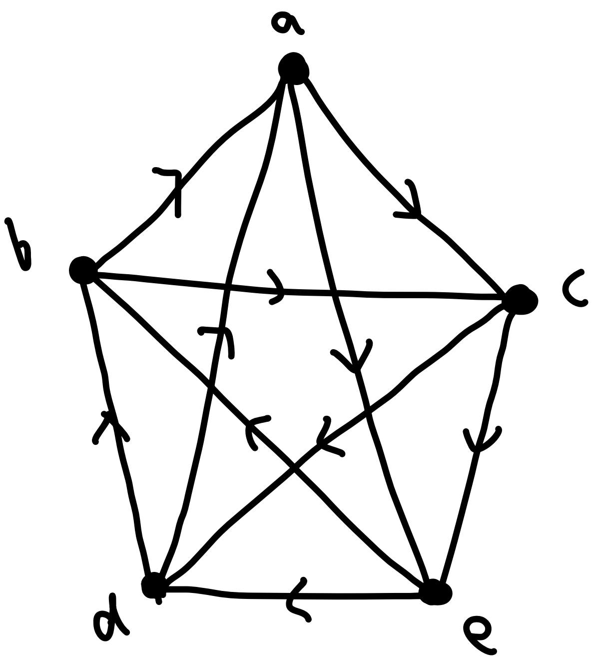
a beats b, c, d, e

b beats c, d, e

c beats d, e

d beats e

a is the champion



a beats c, e

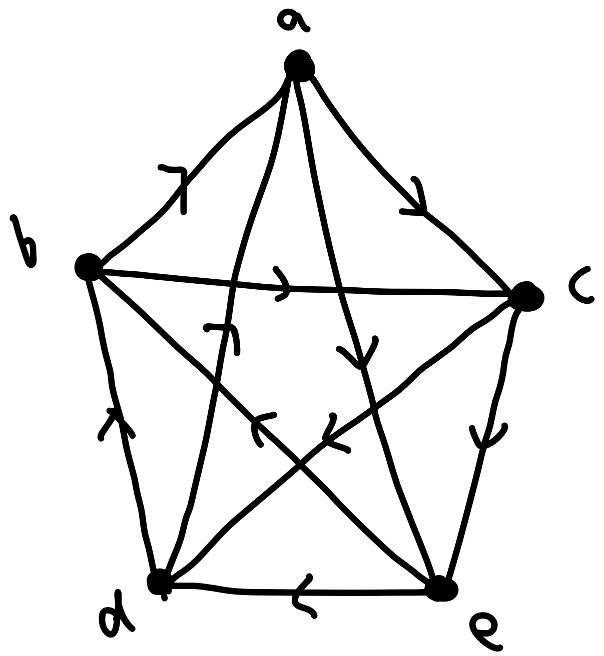
b beats a, c

c beats d, e

d beats a, b

e beats b, d

Def 1.4.29: $v \in V(D)$ is called a <u>king</u> if there is a path of length ≤ 2 from v to every other vertex.



a beats c

a beats e

a beats e beats b

a beats c beats d

So a is a king

Prop 1.4.30: Every tournament T has at least one king

Pf: