Announcements:

- Midterm 2 Wed.

  Wed. 10/18 7:00pm - 8:30pm in 217 Noyes Lab.

  See email for policies

- Tues. problem session will be study session

- Wed. class will be review

Chapter 4: Connectivity & Paths

Idea of connectivity:

How many vertices/edges do we need to delete to form a disconnected graph?

Def 4.1.1: Let G be a graph

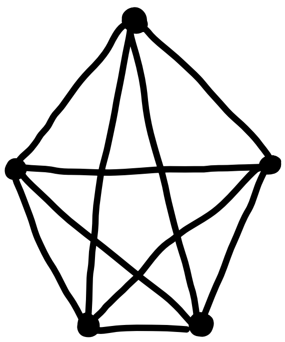
a) A <u>vertex cut</u> is a set $S \subseteq V(G)$ s.t $G \setminus S$ is disconn.

b) The <u>(vertex) connectivity</u> $\overset{\text{"kappa"}}{\kappa(G)}$ is the min. size of a vertex cut (OR $n-1$ if $\nexists$ vertex cut)

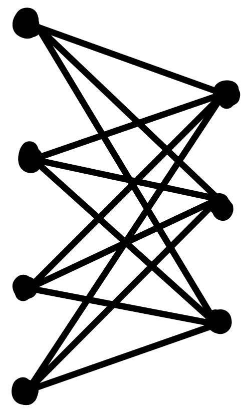
c) G is <u>k-connected</u> if $\kappa(G) \geq k$

Class activity: Find $\kappa(G)$ for the following graphs

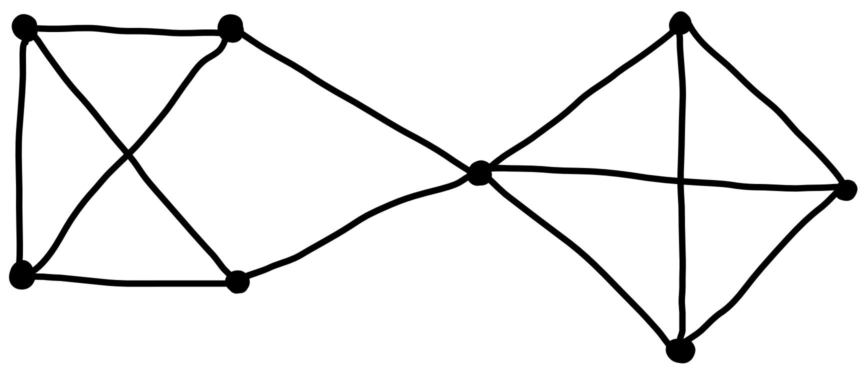
a)



b)



c)



Def 4.1.7:

a) A <u>disconnecting set</u> is a set $F \subseteq E(G)$ s.t. $G \setminus F$ is disconn.

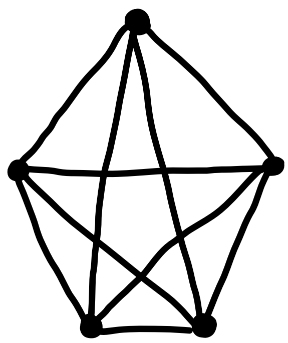
' means "edges"

b') The <u>edge connectivity</u> $\kappa'(G)$ is the min. size of a disconn. set (or $|E(G)|$ if $\nexists$ disconn. set)

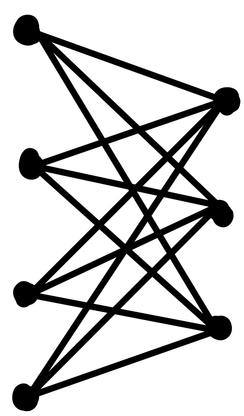
c') G is <u>k-edge-connected</u> if $\kappa'(G) \geq k$

Class activity: Find $\kappa'(G)$ for the following graphs

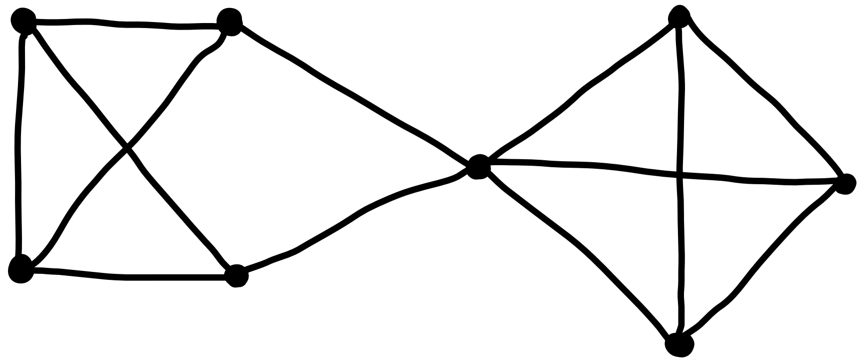
a)



b)



c)



d') An <u>edge cut</u> is a disconn. set F s.t $\exists S \subseteq V(G)$ where each edge in F has exactly one endpoint in S.
(every min'l disconn. set is an edge cut)

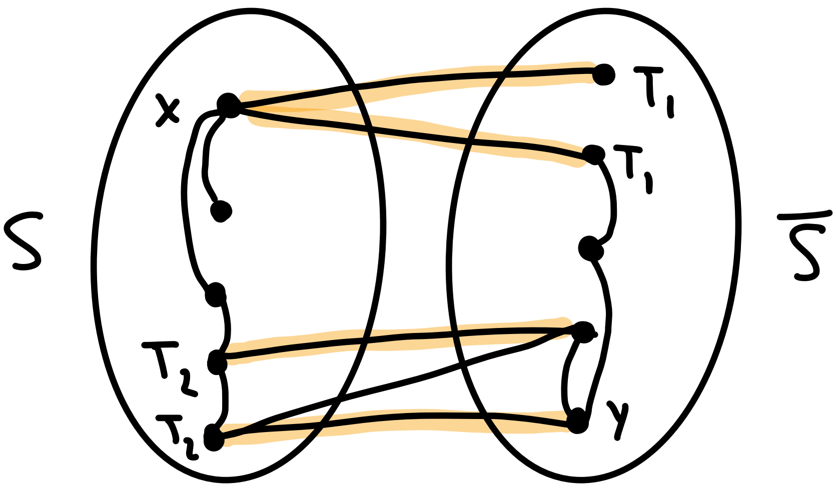
e') Every edge cut has the form

$$[S,T] := \{e \in E(G) \mid e \text{ has one endpoint in } S \text{ and the other in } T\}$$

Thm 4.1.9: Let G be a simple graph. Then,

$$\kappa(G) \leq \kappa'(G) \leq \delta(G)$$

Pf:



S
x
T_1
T_1
T_2
T_2
y
$\bar{S}$

Thm 4.1.11: If G is 3-regular, then $\kappa(G) = \kappa'(G)$

Pf: