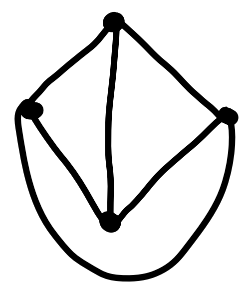
Announcements:

- Quiz 4: Monday in class (covers Ch. 6)
- Exam review: Wed., plus something else (?)
- Final exam: Thurs 12/14, 8:00-11:00am, 132 Berier Hall

Recall: "k-color theorem" means "every planar graph is k-colorable."

No 3-color theorem. Counterexample: K_4



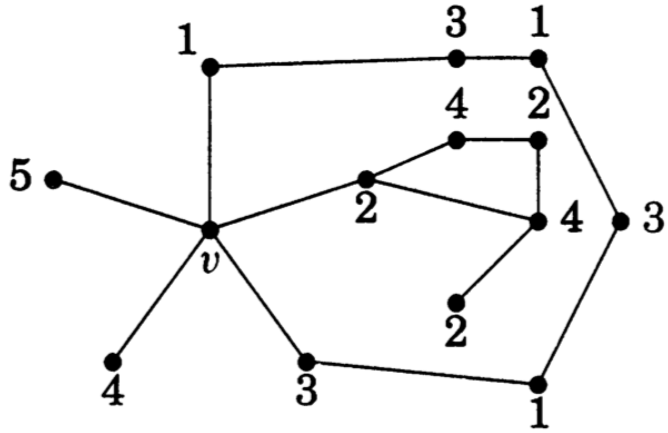
Last time: 6-color theorem ✓

Five-color theorem [Heawood, 1890]: Every planar graph is 5-colorable.

Pf: Induction on $n(G)$.

Base case: $n(G) \le 5$. Can color every vertex a diff. color.

Inductive step: $n(G) > 5$. Let $v \in G$ have degree ≤ 5 (see pf. of 6-color thm).


1
3
1
4
2
5
2
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v
2
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3
1

Let's take these ideas to the 4-color problem

Def 6.3.2:

a) A <u>configuration</u> in a planar triangulation is a cycle C called the <u>ring</u> together with the portion of the graph inside C.

b) For the 4-color problem,

i) a set of configurations is <u>unavoidable</u> if a minimal couterexample must contain a member of it.

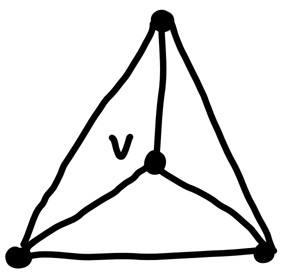
ii) a configuration is <u>reducible</u> if a planar graph containing it cannot be a min'l counterexample

Proof idea:

- Work w/ triangulations; for an arbitrary graph, simply remove some edges
- Find an unavoidable set of configurations
- Prove that each of these configurations is reducible

Four-Color Theorem: Every planar graph is 4-colorable

Pf [Kempe, 1879]: In a planar triangulation, $3 \leq \delta(G) \leq 5$, so the following set of configs. is unavoidable:



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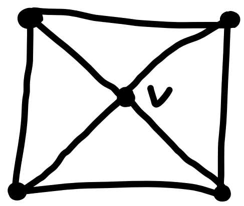

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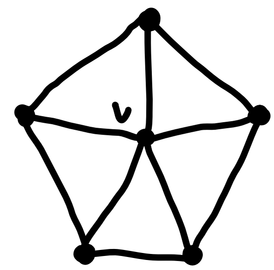

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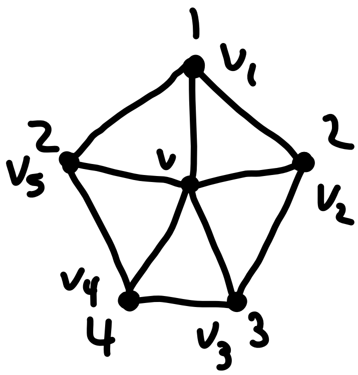
Let G be a minimal counterexample, so that $G \setminus v$ is 4-colorable.


1
v_1
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v_5
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v_4
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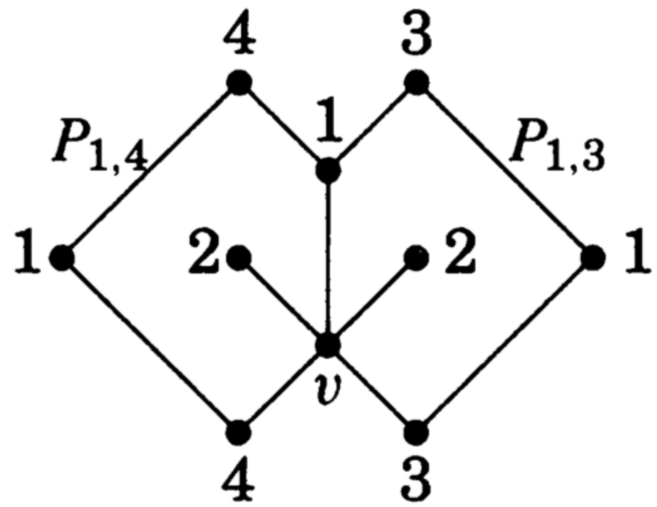
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$P_{1,4}$
$P_{1,3}$
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v
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