Announcements

- No class this Friday (10/27)
- No H/w this week (HW8 will be due Wed. 11/8)
- Exam 2 graded

  Mean: 63.6

  Median: 63.5

  Std. dev.: 7.45

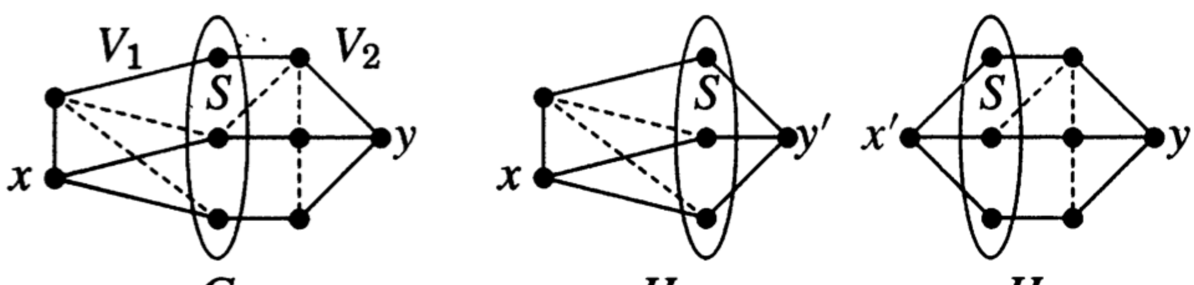
Menger's Theorem: If $x \neq y \in V(G)$ and $xy \notin E(G)$, then $\kappa(x,y) = \lambda(x,y)$

Pf: $\geq$) An x,y-cut must contain an internal vertex from each path in a set of pairwise internally-disjoint x,y-paths, so taking such a set of size $\lambda(x,y)$ gives $\kappa(x,y) \geq \lambda(x,y)$.

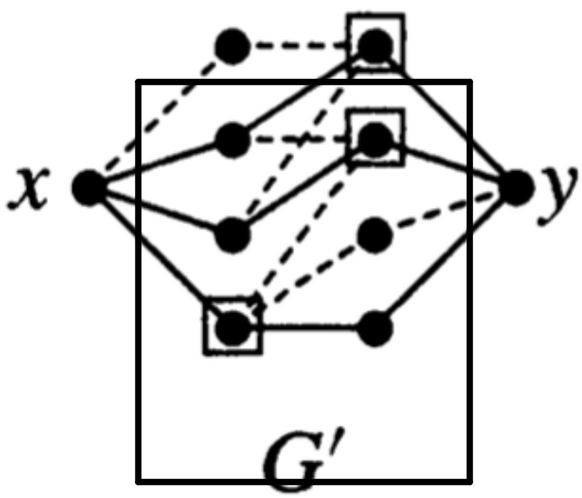
$\leq$) Induction on $n := n(G)$.

Base case: $n=2$. If $xy \notin G$, then there is no x,y-path

$\kappa(x,y) = \lambda(x,y) = 0$.



G H_1 H_2


x
y
G′

Similar results hold for directed graphs and for edge cuts

Def 4.2.11: Let D be a digraph

a) A vertex cut of D is a set $S \subseteq D$ s.t. $D \setminus S$ is not strongly connected

b) If $S, T \subseteq V(D)$, $[S,T]$ denotes the set of edges w/ tail in S and head in T. An edge cut of D is $[S, \bar{S}]$ for some nonempty $S \subseteq V(D)$.

c) (Edge)-connectivity, $\kappa(D)$, $\kappa'(D)$, $\kappa(x,y)$ defined the same w.r.t. vertex/edge cuts.

d) $\lambda(x,y)$ is still the largest # of internally-disjoint x,y-paths

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Def: Let G be a graph or digraph.

a) $\kappa'(x,y)$ = min. size of $F \subseteq E(G)$ s.t. $G \setminus F$ has no x,y-path

b) $\lambda'(x,y)$ = max. size of set of <u>edge-disjoint</u> x,y-paths

Thm: Let G be a graph or digraph.

a) Let $x \neq y \in V(G)$ with no edge from x to y.

  i) If G is a graph, then $\kappa(x,y) = \lambda(x,y)$ (Menger)

  ii) If G is a digraph, then $\kappa(x,y) = \lambda(x,y)$

b) Let $x \neq y \in V(G)$

  i) If G is a graph, then $\kappa'(x,y) = \lambda'(x,y)$

  ii) If G is a digraph, then $\kappa'(x,y) = \lambda'(x,y)$