Math and Proofs Class 7

November 7th, 2017

More about the Axiom of Choice

- *The Axiom of Choice is obviously true, the Well-Ordering Principle is obviously false, and who can tell about Zorn's Lemma?*
- Axiom of Choice: If we have infinitely many buckets, we can form a set with one item from each bucket
- Zorn's Lemma: If S is any nonempty partially ordered set in which every chain has an upper bound, then S has a maximal element. (don't worry about this one too much)
- Well-Ordering Principle: Every set can be "well-ordered"

Ordinal Numbers

- Kind of like counting numbers, but can be infinite
- Two ways of getting ordinals
  1. Successor
  2. Limit

Weak Goodstein's Theorem

- Procedure for the Goodstein sequence with starting point n:
  1. $g_1 = n$
  2. Write this number in base-2 notation
  3. Change all the 2's to 3's
  4. Subtract 1. This is g_2
  5. Continue to find $g_3, g_4, \ldots$.
- Example: starting point 5
  1. $g_1 = 5$
  2. This equals $2^2 + 1$
  3. Change it to $3^2 + 1 (= 10)$
  4. Now subtract 1 from that: $g_2 = 3^2$
- We can prove that this sequence always reaches 0 using ordinal numbers

Goodstein's Theorem

- Now we write numbers in their "hereditary" representation
- Example: starting point 33
  1. $g_1 = 33 = 2^{2^2+1} + 1$
  2. Change 2's to 3's: $3^{3^3+1} + 1$
  3. Then subtract 1: $g_2 = 3^{3^3+1} = 3^10 = 59049$
  4. g_3 starts with 5, and has 155 digits!
- What's different here from the "weak" case (besides being harder)?

Next Time

- The End: Hilbert's Program: Can we build a complete axiom system?