Announcements:

- Midterm 1: Wed. 9/20 7:00-8:30pm (Noyes 217)
  - Topics: All of chapter 1
  - Reference sheet allowed (two-sided)
  - See Monday's email for full policies
- Tuesday problem session (4:00-5:30pm) will be a review session (bring your own Q's)
- Wednesday class will be review (I'll bring the Q's)

Chapter 2: Trees & Distance

Thm 2.1.4: Let G be a graph w/ $|V(G)| = n$
The following are equivalent:

a) G is connected and has no cycles

b) G is connected and has $n-1$ edges

c) G has $n-1$ edges and no cycles

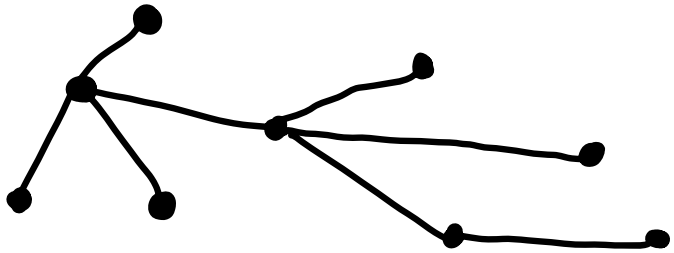
d) $\forall u,v \in V(G)$ $\exists!$ u,v-path in G

e) G is connected and every edge is a cut edge

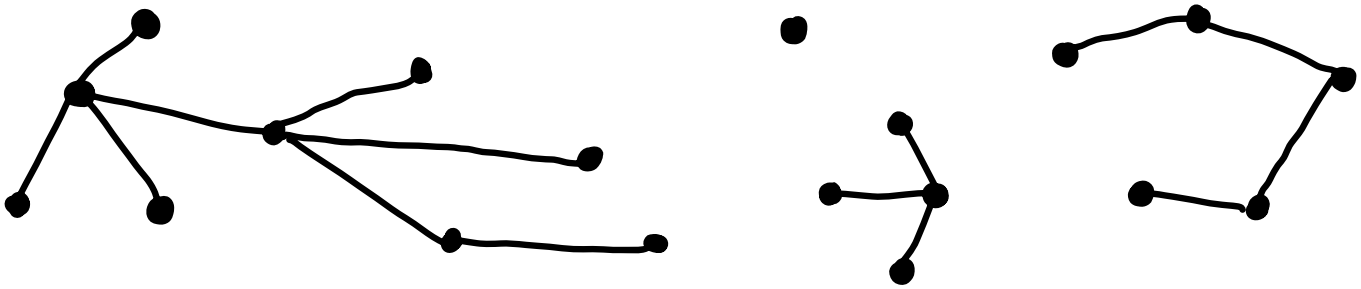
Pf:

Def:

a) A tree is a graph satisfying any/all of the equivalent conditions a) - e) above.



b) A forest is a graph whose conn. components are trees.



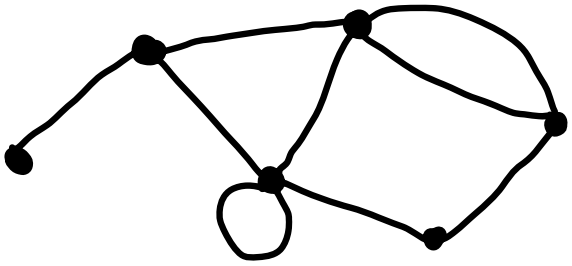
c) A leaf is a degree 1 vertex of a tree

d) $H \subseteq G$ is a spanning subgraph of G if $V(H) = V(G)$

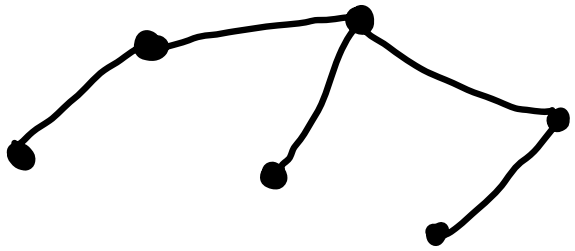
e) Spanning tree $\iff$ spanning subgraph & tree

Class activity: Spanning Subgraph? Spanning Tree?

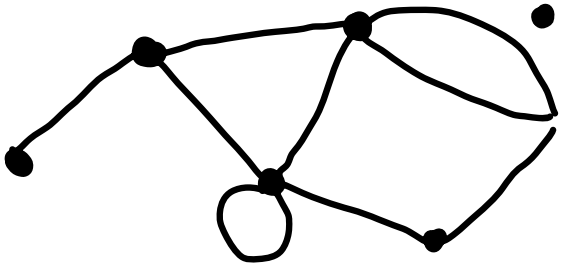
G



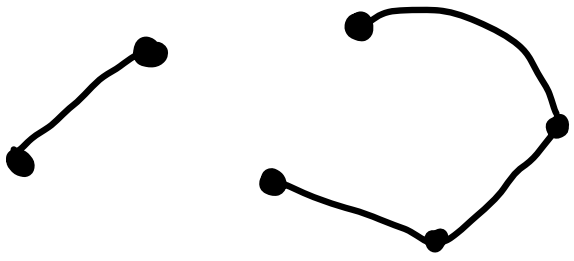
a)



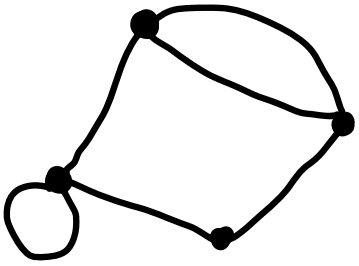
b)



c)



d)



Cor 2.1.5: Every connected graph G contains a spanning tree.

Pf:

Prop (2.1.6/2.1.7): Let G be a graph w/ spanning trees T, T'.

a) For all $e \in E(T)$, $\exists\, e' \in E(T')$ s.t. $(T \cup e') \setminus e$ is a spanning tree of G.

b) For all $e' \in E(T')$, $\exists\, e \in E(T)$ s.t. $(T \cup e') \setminus e$ is a spanning tree of G.

[If you tell me which edge to remove, I'll tell you which edge to add
If you tell me which edge to add, I'll tell you which edge to remove]