

**Problem §1.1: 8(a,d,f,h):** Let  $p$  and  $q$  be the propositions

$p$ : I bought a lottery ticket this week.

$q$ : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

- (a)  $\neg p$
- (d)  $p \wedge q$
- (f)  $(\neg p) \implies (\neg q)$
- (h)  $(\neg p) \vee (p \wedge q)$

**Problem §1.2: 6:** Use a truth table to verify the first De Morgan law  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

**Problem §1.4: 14(a,d):** Express each of these quantifications in English, if the domain consists of all real numbers. Then, determine the truth value of the statement

- (a)  $\exists x(x^3 = -1)$
- (d)  $\forall x(2x > x)$

**Problem §2.1: 10(a,c,e,g):** Determine whether the following statements are true or false.

- (a)  $\emptyset \in \{\emptyset\}$
- (c)  $\{\emptyset\} \in \{\emptyset\}$
- (e)  $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (g)  $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

**Problem §2.1: 16:** Use a Venn diagram to illustrate the relationships  $A \subset B$  and  $A \subset C$ .

**Problem §2.1: 20:** What is the cardinality of each of the following sets?

- (a)  $\emptyset$
- (b)  $\{\emptyset\}$
- (c)  $\{\emptyset, \{\emptyset\}\}$
- (d)  $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

**Problem §2.1: 26:** Show that if  $A \subseteq C$  and  $B \subseteq D$ , then  $A \times B \subseteq C \times D$ .

**Problem §2.1: 32(a,c):** Let  $A = \{a, b, c\}$ ,  $B = \{x, y\}$ , and  $C = \{0, 1\}$ . Find the following Cartesian products.

- (a)  $A \times B \times C$

(c) $C \times A \times B$
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**Problem §2.2: 4:** Let  $A = \{a, b, c, d, e\}$  and  $B = \{a, b, c, d, e, f, g, h\}$ . Find:

- (a)  $A \cup B$ .
- (b)  $A \cap B$ .
- (c)  $A - B$ .
- (d)  $B - A$ .

**Problem §2.2: 14:** Find the sets  $A$  and  $B$  if  $A - B = \{1, 5, 7, 8\}$ ,  $B - A = \{2, 10\}$ , and  $A \cap B = \{3, 6, 9\}$ .

**Problem §2.2: 15:** Prove the second De Morgan law in Table 1 by showing that if  $A$  and  $B$  are sets, then  $\overline{A \cup B} = \overline{A} \cap \overline{B}$  (a) showing each side is a subset of the other side and (b) by using a membership table.

**Problem §2.2: 24:** Let  $A, B$ , and  $C$  be sets. Show that  $(A - B) - C = (A - C) - (B - C)$ .

**Problem §2.2: 26:** Draw the Venn diagrams for each of the following combinations of the sets  $A, B$ , and  $C$ .

- (a)  $A \cap (B \cup C)$
- (b)  $\overline{A} \cap \overline{B} \cap \overline{C}$
- (c)  $(A - B) \cup (A - C) \cup (B - C)$

**Note:** here is sample LaTeX code for drawing truth tables and Venn diagrams. It is also acceptable to hand-draw figures like these and attach them at the end of your submission (although please make sure to tag them correctly).

Sample truth table:

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Sample Venn diagrams:

