Announcements:

- HW9 due tomorrow (Thurs. 10/30) at 9am (office hour today)
- Exam 3 graded

Mean: 62.9 } out of 95
Median: 64.5
Std. dev.: 15.0

Problem Scores:

Q_1: 82% Q_3: 50%
Q_2: 50% Q_4: 75%

- Plan for rest of semester (rough!)

Wed 11/29: §6.1, §6.3 if time
Fri 12/1: §6.3
Mon 12/4: §6.3 (cont.) and Quiz 4
Wed 12/6: Final exam review
(Some sort of review session + office hours)
Thurs 12/14, 8:00-11:00am: Final exam!
132 Berier Hall (<u>not</u> one of our usual rooms!)

Recall:

Euler's Formula: Let G be a connected plane graph w/ n vertices, e edges, and f faces. Then,

$$n - e + f = 2$$

Last time: used this to study regular polyhedra

Today: a bunch of corollaries

Remark 6.1.22:

a) Since n and e don't depend on the planar embedding, neither does f.

b) Recall that the dual graphs of two different planar embeddings of G can be nonisomorphic. However, if the dual graph has n^* vertices, e^* edges, and f^* faces, $n^* = f$, $e^* = e$, $f^* = n$, so these numbers are independent of planar embedding.

c) For a graph w/ k conn. components, we have

$$n - e + f = k + 1$$

Thm 6.1.23:

a) If G is a simple planar graph w/ ≥ 3 vertices, then

$$e(G) \leq 3n(G) - 6$$

b) If G is also $\triangle$-free, then

$$e(G) \leq 2n(G) - 4$$

Pf:

Corollary (6.1.24): K_5 and $K_{3,3}$ are nonplanar

(already proved this)

Pf: Class activity!

Def 6.1.25:

a) A <u>maximal planar graph</u> is a simple planar graph that is not a spanning subgraph of another planar graph (i.e. adding edges makes the graph nonplanar)

b) A <u>triangulation</u> is a simple plane graph where every face boundary is a 3 cycle.

Prop 6.1.26: Let G be a simple n-vertex plane graph. The following are equivalent:

A) $e(G) = 3n - 6$

B) G is a triangulation

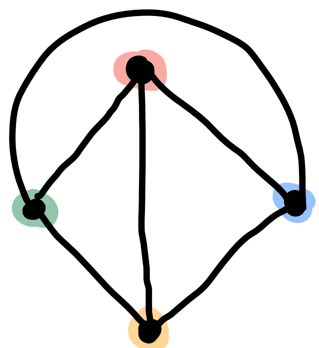
C) G is (an embedding of) a maximal planar graph.

Pf:

Recall our main question for this section: what is the maximum number of colors needed to give any (loopless/simple) planar graph a proper coloring?

k-Color Theorem: Every planar graph is k-colorable.

There is no 3-color theorem since $\chi(K_4) = 4$ and K_4 is planar



Six-Color Theorem (Exercise 6.3.2): Every planar graph is 6-colorable.

Pf: Induction on $n(G)$.

What about 5 colors? 4 colors?

Next time.