Announcement:

HW7 posted (due Wed. 10/25)

Recall:

$\kappa(G)$ = min. size of $S \subseteq V(G)$ s.t. $G \setminus S$ is disconn.

$\kappa'(G)$ = min. size of $F \subseteq E(G)$ s.t. $G \setminus F$ is disconn.

Whitney's Thm: $\kappa(G) \leq \kappa'(G) \leq \delta(G)$ if G: simple

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Today's goal: give several characterizations of 2-connected graphs.

Def 4.2.1: Two u,v-paths are <u>internally disjoint</u> if their intersection is $\{u,v\}$.

Thm 4.2.2: Let G be a graph w/ ≥ 3 vertices. Then,

G is 2-connected $\iff$ $\forall u,v \in V(G)$, $\exists$ two internally disjoint u,v-paths

Pf :

Thm 4.2.2: Let G be a graph w/ ≥ 3 vertices. TFAE:

A) G is conn. and has no cut-vertex

B) $\forall x, y \in V(G)$, $\exists$ internally-disjoint x,y-paths

C) $\forall x, y \in V(G)$, $\exists$ cycle containing x and y

D) $\delta(G) \geq 1$, and $\forall e, f \in E(G)$, $\exists$ cycle containing e and f

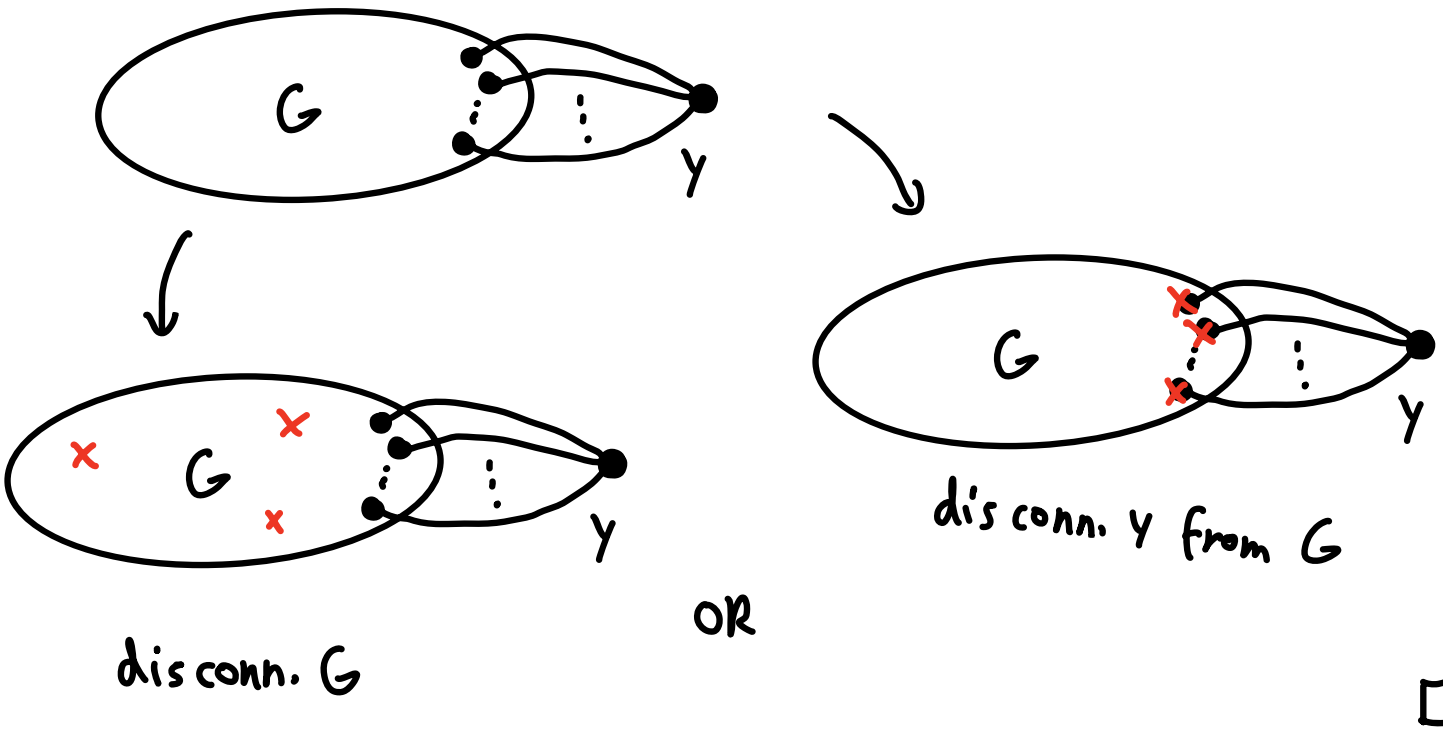
E) G is 2-conn.

Pf:

Interlude:

Expansion Lemma (4.2.3): If G is k-conn. and G' is obtained from G by adding a new vertex y w/ $\geq k$ neighbors in G, then G' is k-conn.

Pf by picture:



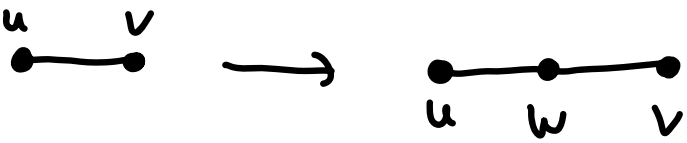
□

Finish pf of Thm 4.2.4:

Def : G : graph

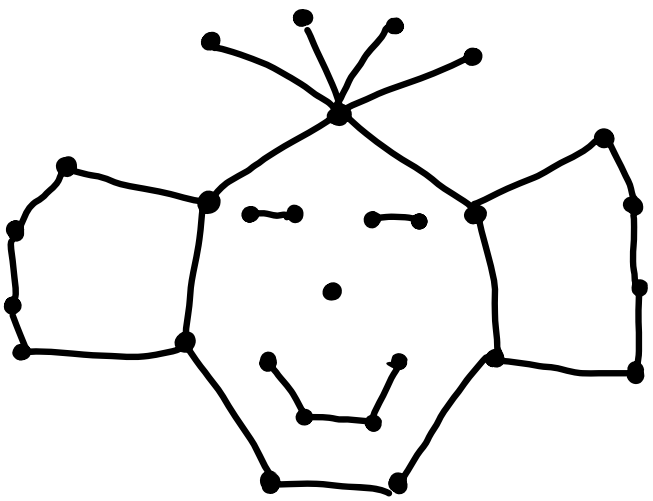
a) A subdivision of an edge 

is



b) An ear of G is a max'l path whose internal vertices have degree 2 in G.

Class activity: Find the ears!



c) An ear decomposition of G is a decomposition $P_0, \ldots, P_k$ s.t. P_0 is a cycle and For $i \geq 1$, P_i is an ear of $P_0 \cup \ldots \cup P_i$

Class activity: find an ear decomposition: