Announcements:

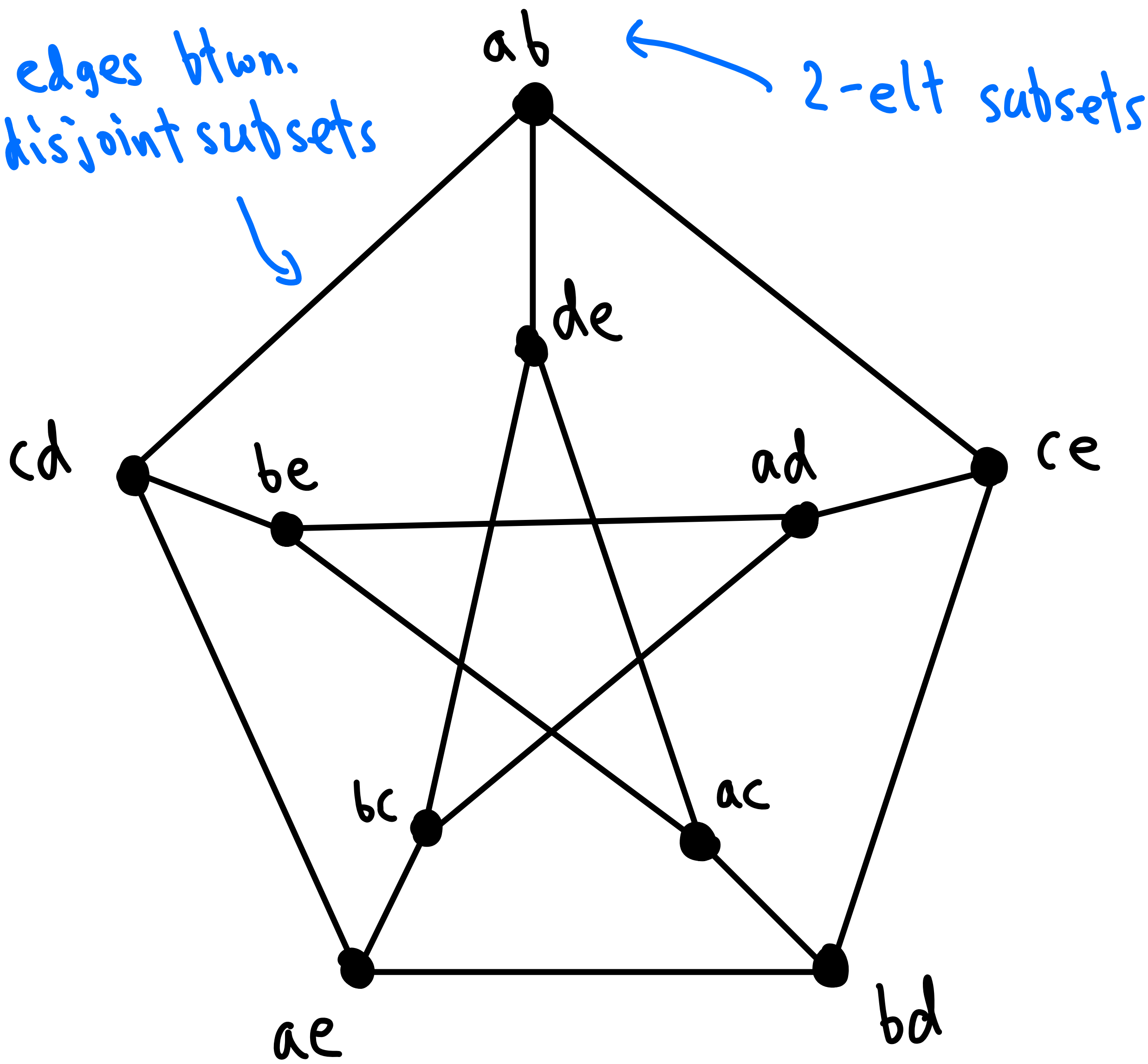
- HW 1 due Wed. 9am via Gradescope
  Course code: 57YPR7

  don't be late!

- Problem session tomorrow 4pm-5:30pm Henry Admin 156

Petersen graph:

$S = \{a, b, c, d, e\}$



Def 1.1.39: The <u>girth</u> of a graph is the length of its shortest cycle (no cyles: girth $= \infty$)

Cor 1.1.40: The Petersen graph G has girth 5.

Def 1.1.41:

a) An <u>automorphism</u> is an isomorphism from a graph to itself [These form a group]

b) A graph G is vertex-transitive if for every pair of vertices $u, v \in V(G)$, there is an automorphism of G mapping u to v

Remark: The Petersen graph is vertex transitive

§1.2: Paths, Cycles, & Trails

Def 1.2.2:

a) A walk is a list

$$v_0, e_1, v_1, e_2, \ldots, e_k, v_k, \quad e_i \in E(G), v_i \in V(G)$$

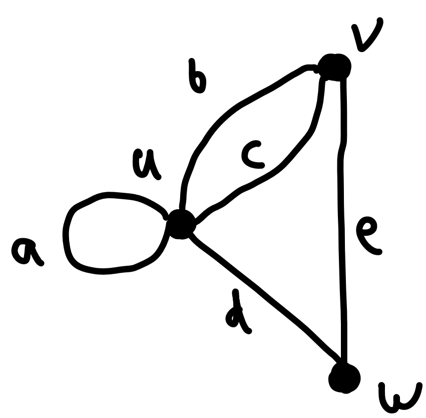
such that e_i has endpoints v_{i-1} and v_i

b) A <u>trail</u> is a walk w/ no repeated edges

c) (Recall) A <u>path</u> is a walk w/ no repeated vertices (or edges)

A walk is <u>closed</u> if $v_0 = v_k$

Class activity: Walk, trail, path, or none? Closed?
(W) (T) (P) (N) (C)



a) u, a, u, c, v, b, u, d, w

b) w, d, v, a, u, c, b

c) u, b, v, e, w

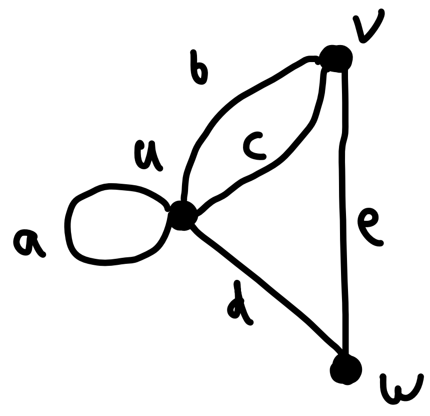
d) u, b, v, e, w, d, u

e) u, c, v, e, w, e, v, b, u

Note: If the graph is simple, we just list vertices

Lemma 1.2.5: Every $\overset{\text{start}}{u},\overset{\text{end}}{v}$-walk contains a u,v-path

Ex:



u, a, u, c, v, b, u, d, w

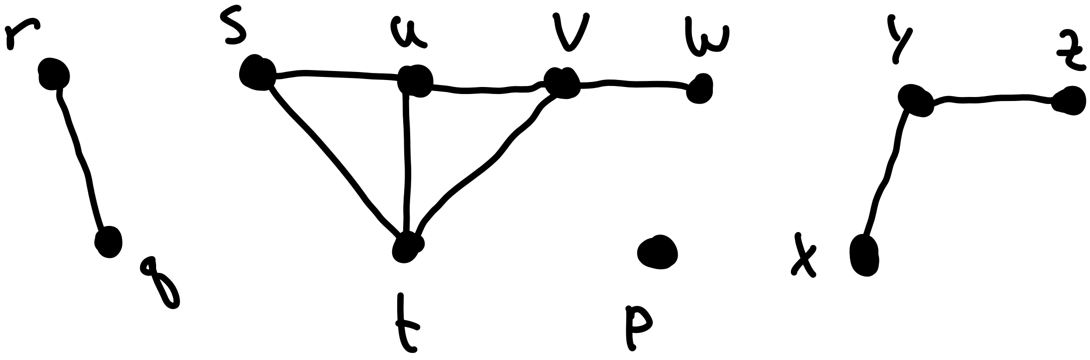
Def 1.2.6 / 1.2.8:

a) G is <u>connected</u> is $\forall u, v \in V(G)$, G contains a u,v-path (or walk or trail)

b) The <u>(connected) components</u> of G are its maximal connected subgraphs

c) An <u>isolated vertex</u> is a vertex of deg 0

Ex 1.2.9:

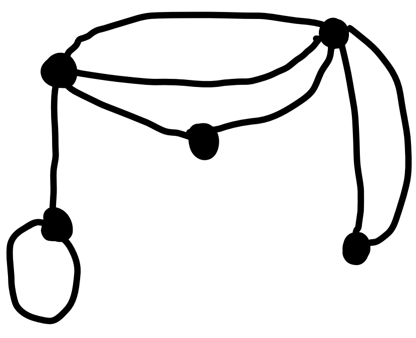


Remark 1.2.7: "u and v are in the same connected component" is an equivalence rel'n

Def 1.2.12:

a) If $T \subseteq V(G)$, the <u>induced subgraph</u> $G[T]$ is the graph w/ vertex set T and edge set

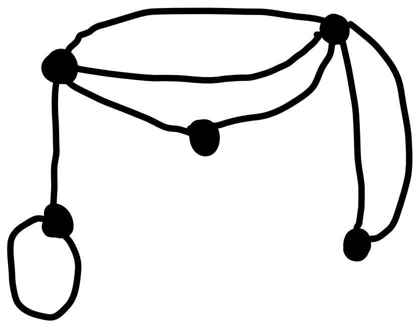
$E(G) \cap \{$edges w/ both endpoints in T



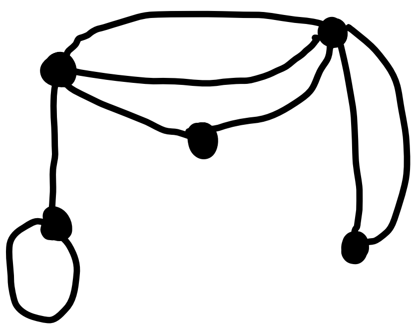
b) An edge $e \in E(G)$ is a <u>cut-edge</u> if the graph $G \setminus e := (V(G), E(G) \setminus e)$ has one more conn. cmpt. than G

vertex set

edge set



c) A vertex $v \in V(G)$ is a cut-vertex if $G[V(G) \setminus v]$ has one more conn. cmpt. than G



Thm 1.2.14: An edge $e \in E(G)$ is a cut-edge iff it belongs to no cycle