Math and Proofs Class 2

September 26th, 2017

Recap of Last Class

- We looked at 2 examples of axiom systems
  - Euclidean Geometry
  - Peano Axioms of Arithmetic
- Left off before proving that addition is commutative

Peano Axioms

1. Zero is a number
2. If a is a number, the successor of a is a number
3. Zero is not the successor of a number
4. Two numbers of which the successors are equal are themselves equal
5. If a set S of numbers contains zero and also the successor of every number in S, then every number is in S.

Set Theory

- System we use today
- Pioneered by Georg Cantor in the 1890s
- Kronecker: *I don't know what predominates in Cantor's Theory – philosophy or theology, but I am sure that there is no mathematics there*
- Feferman: *Simply not relevant to everyday mathematics*
- Hilbert: *No one will drive us from the paradise which Cantor created for us*
- Subsumes both Euclidean geometry and the Peano axioms (and much else)
- (Much of this material is taken from *Doing Mathematics* by Steven Galovich and from *An Outline of Set Theory* by James M. Henle)

Set Theory (cont.)

- Undefined terms: set, element
- Axioms: ZFC
  - Equality: Two sets are equal if and only if they have the same elements
  - Empty Set: There is a set with no elements (called the *empty set*: $\emptyset$)
  - Union: If d is a set of sets, then the union of these sets is a set
  - Power Set: If d is a set, then the collection of all subsets of d is also a set (called the power set: $\mathcal{P}(d)$)
  - Many more, some of which are quite technical
- Notation: $=, \{\}, \in, \subseteq, \emptyset, \cup, \cap, \setminus, \mathcal{P}(d)$

Set Theory Results

1. For any set A, $\emptyset \subseteq A$
2. For any set A, $A \subseteq A$
3. If A, B, and C are sets where $A \subset B$ and $B \subset C$, then $A \subset C$
4. Let A and B be sets. Then $A = B$ if and only if $A \subset B$ and $B \subset A$.

Exercise 1

Let $A = \{x, y, \{x, y\}\}$. True or false:

1. $\{x, y\} \subset A$
2. $\{x, y\} \in A$
3. $\{y\} \subset A$
4. $\{y\} \in A$

Exercise 2

Let $A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{4, 5, 6\}$. Find each of the following sets:

1. $A \cup B$
2. $A \cap B$
3. $B \cap C$
4. $A \setminus B$
5. $A \cap (B \cup C)$
6. $(A \cap B) \cup (A \cap C)$

More Set Theory Results

Let A, B, C be sets

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$ (exercise)

Next Time

- More set theory!