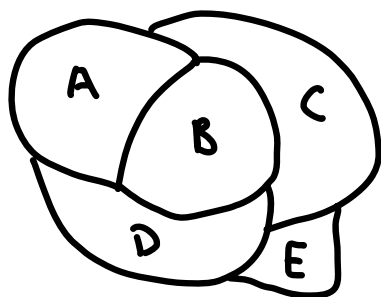


# Quiz today!

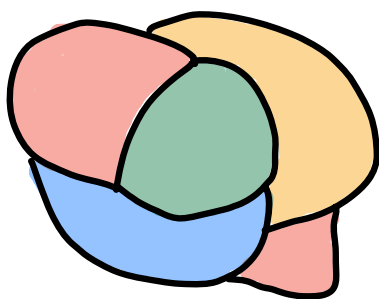
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## §10.8: Graph coloring

Map: Separation of (part of) the plane into contiguous regions



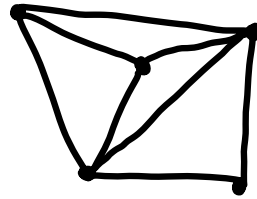
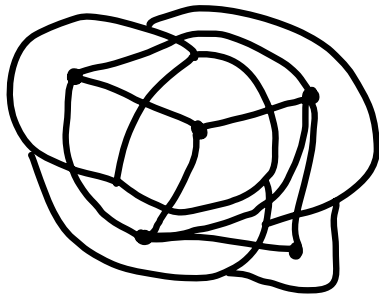
Map coloring: assign each region a color s.t.  
all adjacent regions have different colors



Question: what is the smallest number of colors we need for a given map?

This is secretly a graph theory problem

Def: For a map  $M$ , the dual graph of  $M$  is the graph formed by putting a vertex in the middle of each region of  $M$ , and connecting vertices for adjacent regions



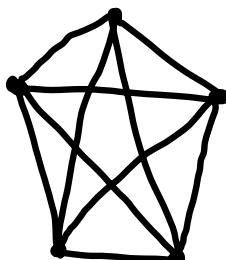
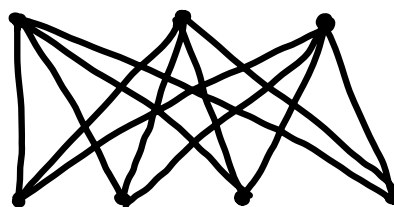
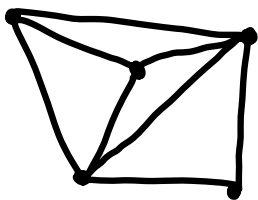
Graph coloring: assign each vertex a color s.t.  
all adjacent vertices have different colors

(coloring the dual graph of a map is equiv. to coloring  
the map itself)

Def: The chromatic number  $\overset{?chi}{\chi}(G)$  is the smallest number  
of colors needed to color  $G$ .

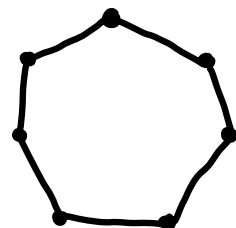
We can do this for any graph; the planar ones are  
the graphs corresponding to maps.

Class activity: Find  $\chi(G)$



$K_5$

$K_{3,4}$



$C_7$

Four-color theorem: For any simple planar graph  $G$ ,  $\chi(G) \leq 4$

1852: Conjectured by Guthrie

1879: ~~Proof given by Kempe~~

1890: Heawood showed that Kempe's proof was flawed (!)  
and also proved the five-color thm.

⋮

many years passed

⋮

1976: Appel & Haken proved the theorem!

@UIUC

Proof technique: break up into 1,834 cases,  
and check them all by computer

To this day, no non-computer exists!