Math 418, Spring 2025 – Homework 5

Due: Wednesday, March 5th, at 9:00am via Gradescope.

Instructions: Students should complete and submit all problems. Textbook problems are from Dummit and Foote, *Abstract Algebra, 3rd Edition*. All assertions require proof, unless otherwise stated. Typesetting your homework using LaTeX is recommended, and will gain you 1 bonus point per assignment.

1. **Dummit and Foote #13.4.1:** *Determine the splitting field and its degree over* $\mathbb{Q}$ *for* $f(x) = x^4 - 2$.
2. **Dummit and Foote #13.4.2:** *Determine the splitting field and its degree over* $\mathbb{Q}$ *for* $f(x) = x^4 + 2$.
3. **Dummit and Foote #13.4.3:** *Determine the splitting field and its degree over* $\mathbb{Q}$ *for* $f(x) = x^4 + x^2 + 1$.
4. **Dummit and Foote #13.4.6:** *Let* K_1 *and* K_2 *be finite extensions of* F *contained in the field* K*, and assume both are splitting fields over* F*.*
    a. *Prove that their composite* K_1K_2 *is a splitting field over* F*.*
    b. *Prove that* $K_1 \cap K_2$ *is a splitting field over* F*.*