

## Announcements

HW9 posted (or will be today) — due Sun 11/10

Quiz 7 Wed. in class

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## §10.1: Graphs

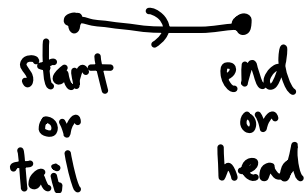
Def: A graph  $G=(V,E)$  consists of

$V$ : a nonempty set of vertices, and

$E$ : a set of edges

Each edge has two vertices as endpoints. If they are the same, the edge is called a loop

Def: A digraph  $D$  has the same def'n except that edges are directed



Def: A graph or digraph is called simple if

a) it has no loops ("loopless")

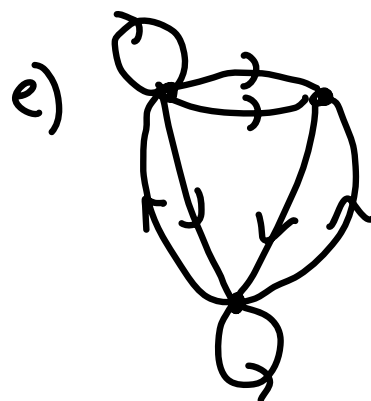
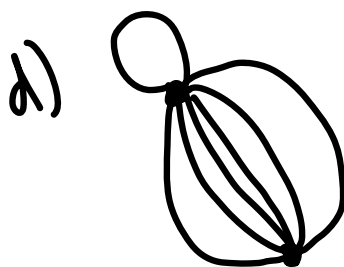
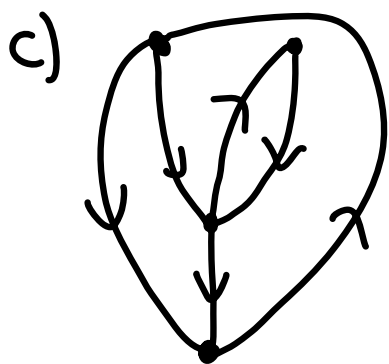
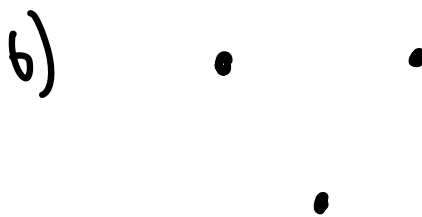
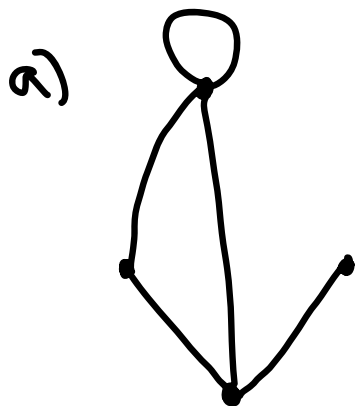
b) it has no multiple edges

{ edges w/ same endpoints (graph)

{ edges w/ same tail/head (digraph)

(Di)graphs w/ multiple edges are called multi(di)graphs

Class activity: Graph or digraph? Simple? Multi-?



Rosen has many examples of how graphs/digraphs can be used to represent real-world data  
(Ex 1-13, also on HW)

### §10.2: Graph terminology, and special types of graph

Def: Let  $G = (V, E)$  be a graph

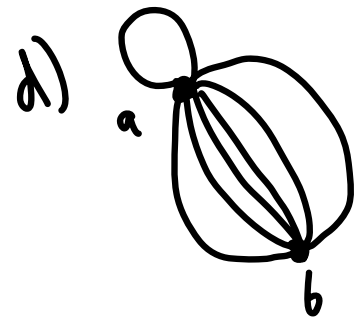
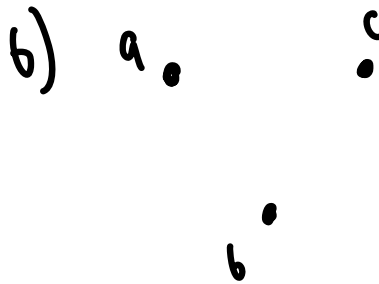
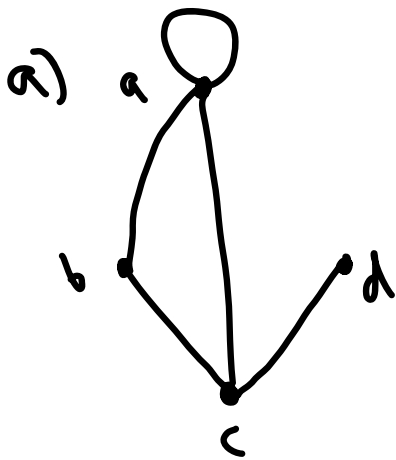
a)  $u, v \in V$  ( $u \neq v$ ) are adjacent or neighbors if there is an edge  $e \in E$  with endpoints  $u$  and  $v$ .  $e$  is incident to its endpoints.

b) The neighborhood of  $v \in V$  is the set  $N(v)$  of all neighbors of  $v$ . If  $A = \{v_1, v_2, \dots, v_k\}$ , then

$N(A) = N(v_1) \cup N(v_2) \cup \dots \cup N(v_k)$ , the set of all vertices adjacent to any vertex in  $A$

c) The degree of  $v \in V$  is the number  $d(v)$  of edges incident to  $v$  (counting loops twice).

Class activity: Find all neighborhoods and degrees



Handshake theorem: For a graph  $G$  with  $m$  edges,

$$\sum_{v \in V} \deg(v) = 2m$$

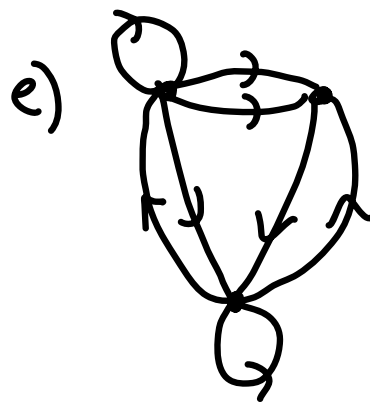
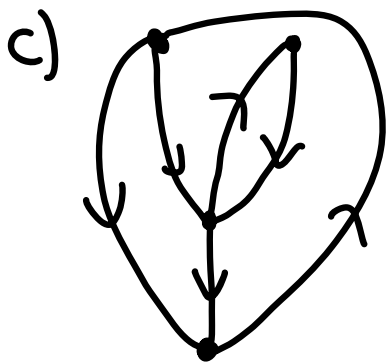
In particular, the number of vertices of odd-degree is always even!

Def: Let  $D=(V,E)$  be a digraph,  $v \in V$ .

The in-degree  $\deg^-(v)$  of  $v$  is the number of edges w/ end/head  $v$ .

The out-degree  $\deg^+(v)$  of  $v$  is the number of edges w/ start/tail  $v$ .

Class activity: Find all in/out-degrees



Handshake theorem: For a digraph  $D$  with  $m$  edges,

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = m$$

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Special (undirected, simple) graphs

a) Complete graph  $K_n$ : all pairs of vertices are adjacent



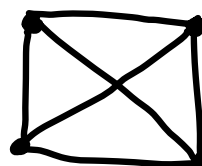
$K_1$



$K_2$



$K_3$

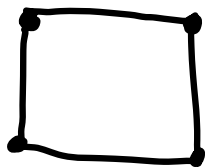


$K_4$

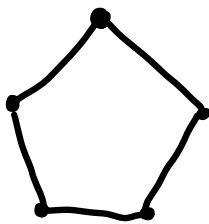
b) Cycle  $C_n$  :



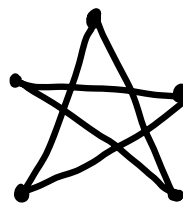
$C_3$



$C_4$



$C_5$

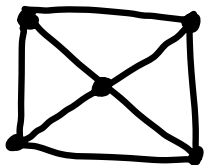


Also  $C_5$   
(doesn't matter  
how you place the  
vertices)

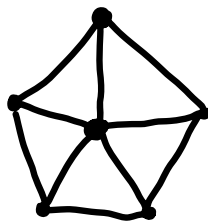
c) Wheel  $W_n$  :  $C_n$  with a hub



$W_3$



$W_4$



$W_5$

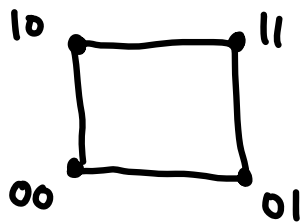
d) Hypercube  $Q_n$

$V = \{ \text{binary strings of length } n \}$

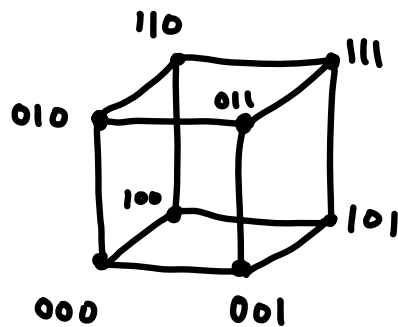
$N(v) = \{ \text{all strings off by one digit from } v \}$



$Q_1$



$Q_2$

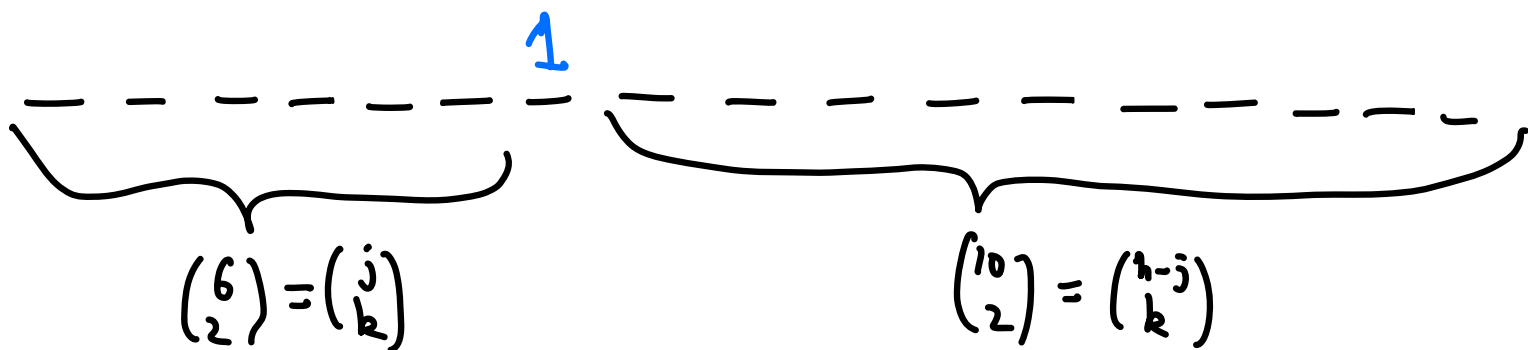


$Q_3$

Def:  $G$  is bipartite if there is a set partition  $V = V_1 \cup V_2$  such that every edge has one endpoint in  $V_1$  and the other in  $V_2$ .  
disjoint

Class activity: Of the above graphs, which are bipartite?  
 (if time)

$$j+1=7 \quad j=6$$



$$3 \leq j+1 \leq 15$$

  
 middle  
 1



$$n+1=17 \quad n=16$$

$$2k+1=5$$

$$k=2$$

$$n+1 - (j+1) = n - j$$

$$2 \leq j \leq 14$$


  
 $k$                    $n-k$