Recall: Dijkstra's Algorithm

Input: A weighted graph G and a vertex $u \in V(G)$

Start: $S = \{u\}$, $t(u) = 0$,

$$t(z) = \min_{u \overset{e}{\bullet\!-\!\bullet} z} wt(e) \quad \text{if } z \neq u$$

While $\exists z \notin S$, $t(z) < \infty$:

  Choose $v \notin S$ s.t. $t(v) = \min\limits_{z \notin S} t(z)$

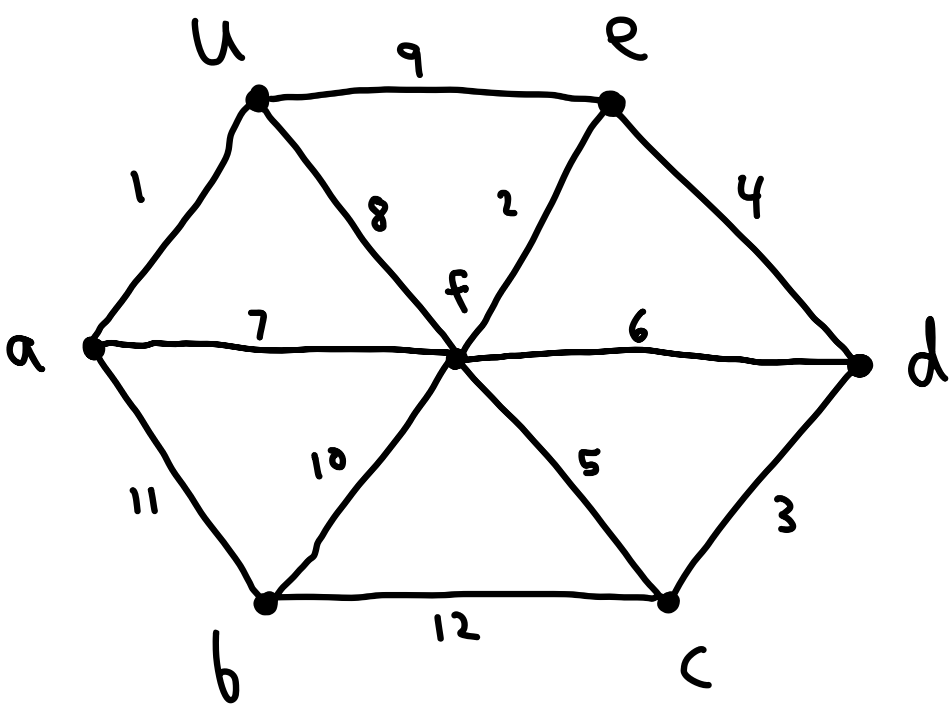
  Add v to S

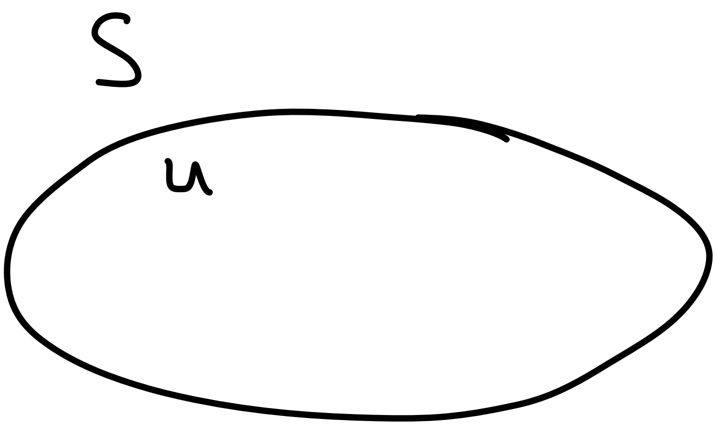
  For all edges $v \overset{e}{\bullet\!-\!\bullet} z$, $z \notin S$:

    Replace $t(z)$ w/ $\min\big(t(z), t(v) + wt(e)\big)$

Output: $t(v) = d(u,v)$ for all $v \in V(G)$

Class activity: Dijkstra!





t(u) =

t(a) =

t(b) =

t(c) =

t(d) =

t(e) =

t(f) =

Thm 2.3.7: The output of Dijkstra's Algorithm is always the distance function $d(u,v)$.

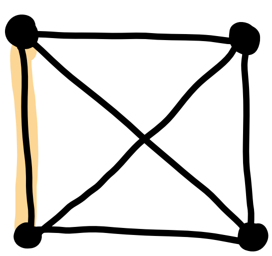
Pf:

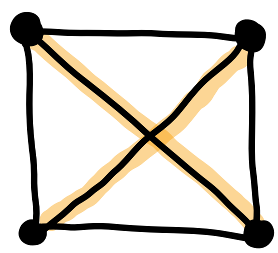
Special case: breadth-first search (all weights are 1)

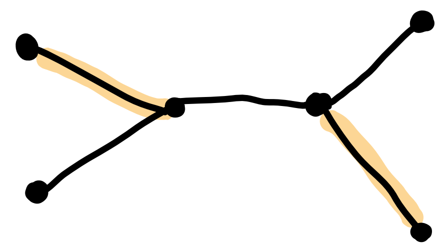
Chapter 3: Matchings and Factors

Def 3.1.1/3.1.4: Let G be a graph

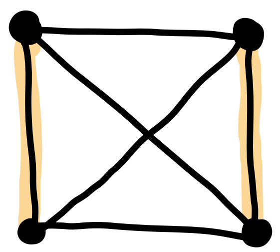
a) A matching in G is a spanning subgraph $M \subseteq G$ such that each vertex has degree ≤ 1 in M

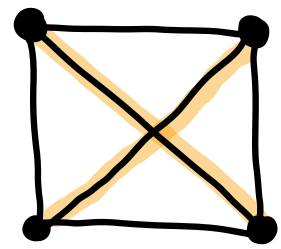


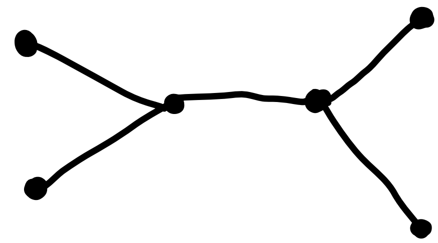




b) A perfect matching is a matching $M \subseteq G$ such that each vertex has degree exactly 1 in M

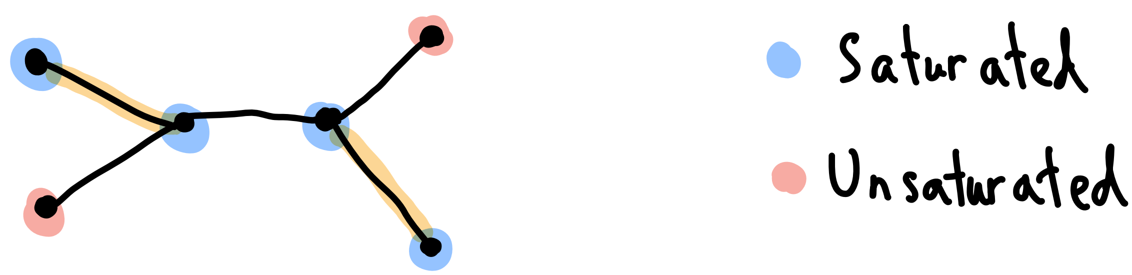






none exists

c) We call a vertex saturated if it has deg. 1 in M
We call a vertex unsaturated if it has deg. 0 in M

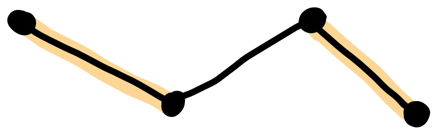


d) M is a maximal matching if there is no matching M' with $M \subsetneq M' \subseteq G$

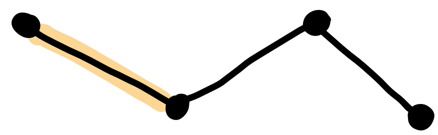
M is a maximum matching if there is no matching M' with $|E(M)| < |E(M')|$

Class activity: Maximal? Maximum? Perfect?

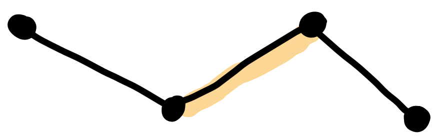
a)



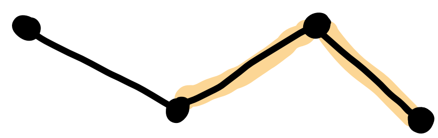
b)



c)



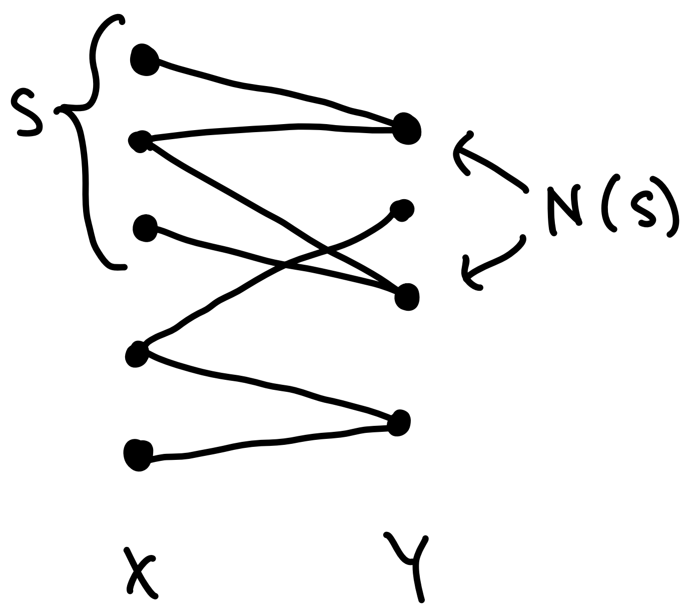
d)



Hall's (Marriage) Thm (3.1.11): Let G be a bipartite graph w/ parts X and Y. Then,

G has a matching that saturates X $\iff$ $|N(S)| \geq |S|$ for all $S \subseteq X$

Pf: $\Rightarrow$



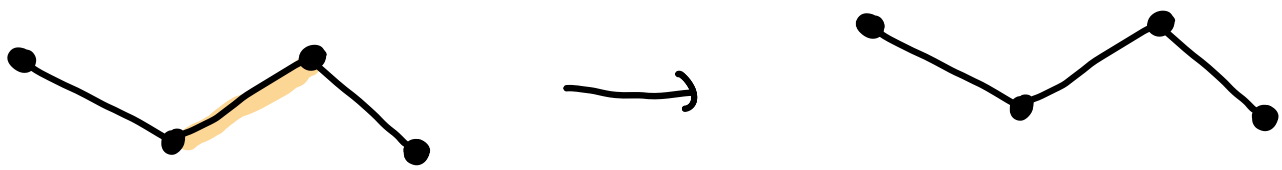
$\Leftarrow$ Need a def'n first

Def 3.1.6: Let $M \subseteq G$ be a matching.

a) An M-alternating path is a path $P \subseteq G$ which alternates btwn. edges in M and edges not in M

b) An M-augmenting path is an M-alternating path whose endpoints are unsaturated

Idea: given an M-augmenting path, swap the edges and non-edges

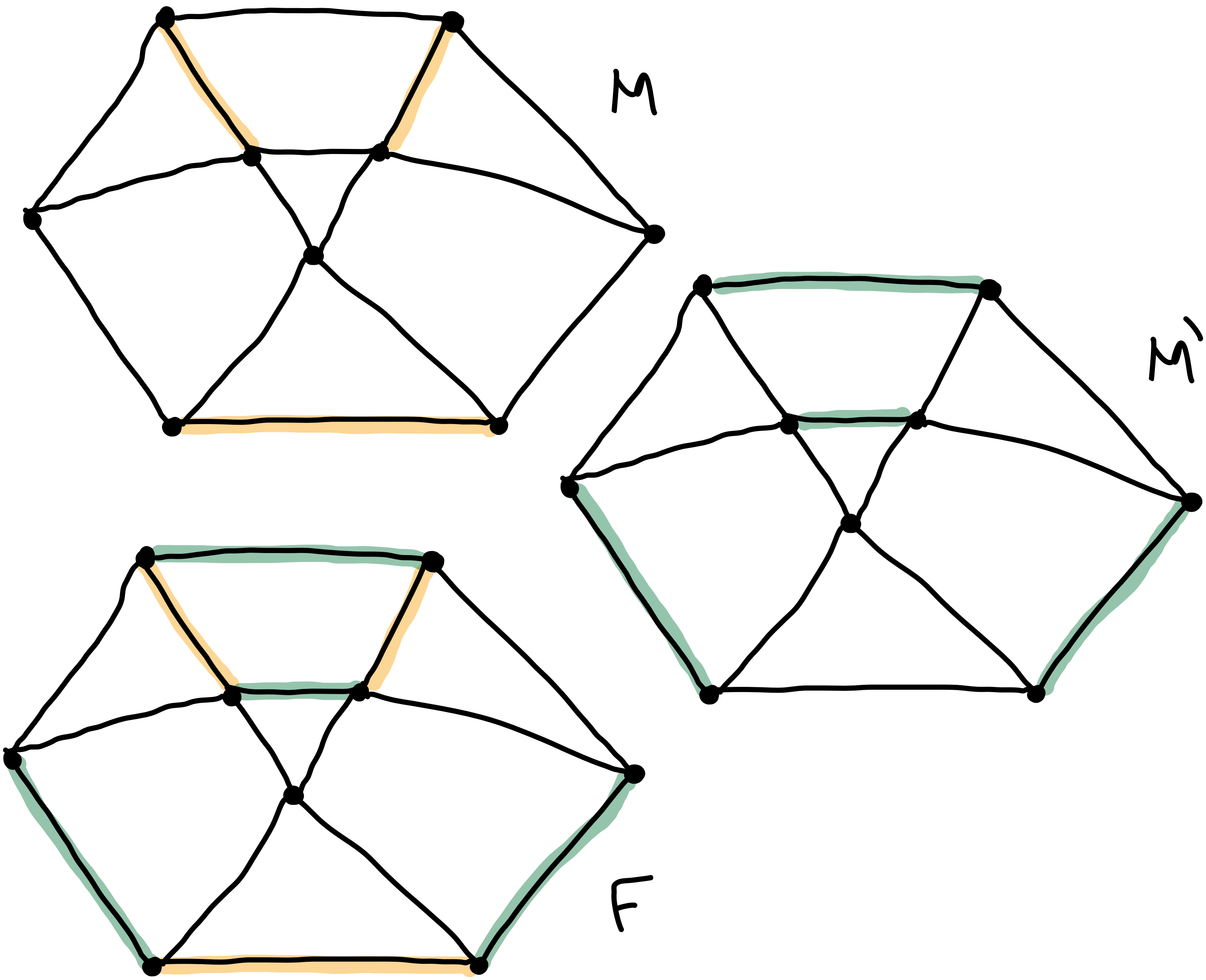


Always gives a larger matching

Thm 3.1.10: Let $M \subseteq G$ be a matching. Then,

M is maximum $\Leftrightarrow$ G has no M-augmenting path

Pf: We prove the contrapositive.


M
M'
F