

Math 418, Spring 2024 – Homework 4

Due: Wednesday, February 21st, at 9:00am via Gradescope.

Instructions: Students should complete and submit all problems. Textbook problems are from Dummit and Foote, *Abstract Algebra, 3rd Edition*. All assertions require proof, unless otherwise stated. Typesetting your homework using LaTeX is recommended, and will gain you 2 bonus points per assignment.

1. **Dummit and Foote #13.2.1:** Let \mathbb{F} be a finite field of characteristic p . Prove that $|\mathbb{F}| = p^n$ for some positive integer n .
2. **Dummit and Foote #13.2.4:** Determine the degree over \mathbb{Q} of $2 + \sqrt{3}$ and of $1 + \sqrt[3]{2} + \sqrt[3]{4}$.
3. **Dummit and Foote #13.2.5:** Let $F = \mathbb{Q}(i)$. Prove that $x^3 - 2$ and $x^3 - 3$ are irreducible over F .
4. **Dummit and Foote #13.2.7:** Prove that $\mathbb{Q}(\sqrt{2} + \sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Conclude that $[\mathbb{Q}(\sqrt{2} + \sqrt{3}) : \mathbb{Q}] = 4$. Find an irreducible polynomial satisfied by $\sqrt{2} + \sqrt{3}$.

MORE PROBLEMS TO COME!!!