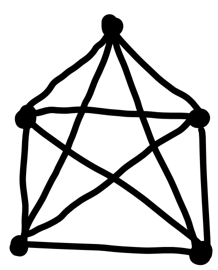
Announcements:

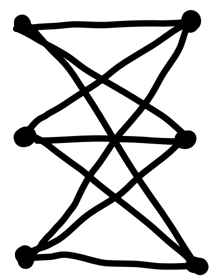
Midterm 3: Wed. 7:00-8:30pm Noyes 217

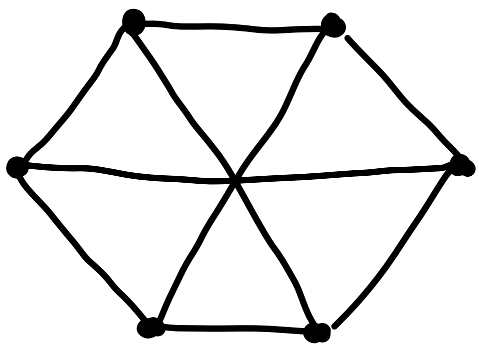
Covers through Chapter 5

Final homework (HW9) will be due Wed. 11/29

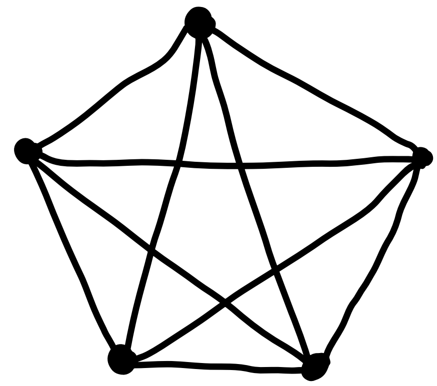
Prop 6.1.2: K_5 and $K_{3,3}$ are not planar





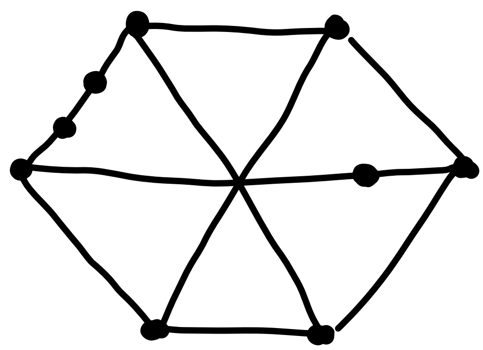


$K_{3,3}$



K_5

Def: A <u>subdivision</u> of a graph G is a graph G' obtained by repeated subdivisions of edges

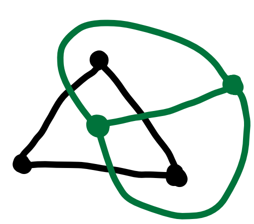


Kuratowski's Theorem (6.2.2): Let G be a graph.

G is planar $\iff$ G does not have a subgraph isomorphic to a subdivision of K_5 or $K_{3,3}$.

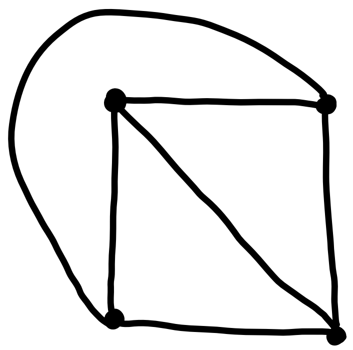
For any plane graph G (loops, mult. edges ok!), there is a nice relationship between vertices, edges, and faces.

Def 6.1.7: Let G be a plane graph. The <u>dual graph</u> G^* of G is a plane graph whose vertices corresp. to the faces of G. For each edge e in G, we create an edge in G^* crossing e, with endpoints at the vertices of G^* corresponding to the faces of G bounding e.

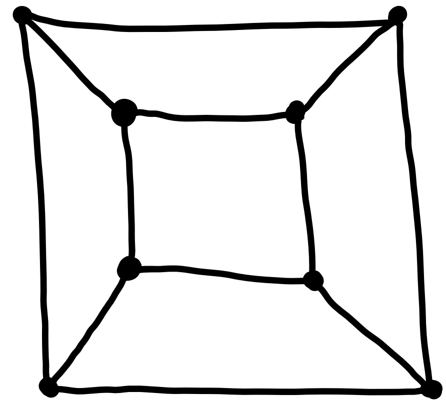


Class activity: Find the dual graphs, and count the vertices, edges, and faces of G and G^*.

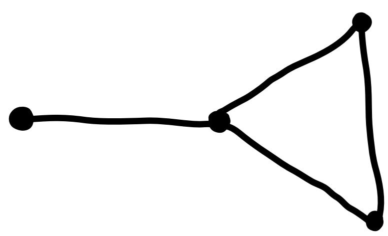
a)



b)

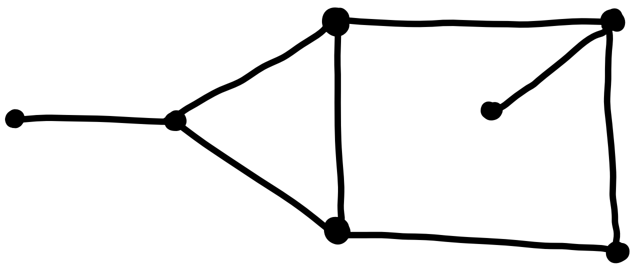


c)

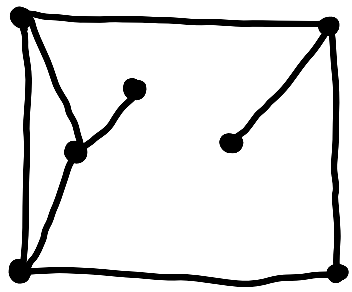


Class activity! Same thing!

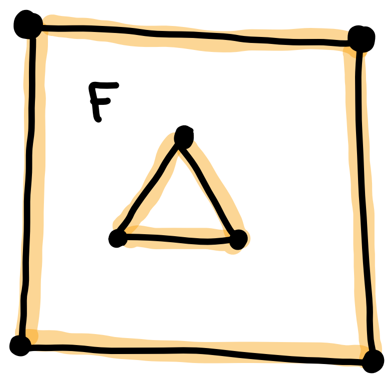
a)



b)



Def 6.1.11: The <u>length</u> $\ell(F)$ of a face F in a plane graph G is the total length of the closed walk(s) in G bounding F.



$\ell(F) = 7$

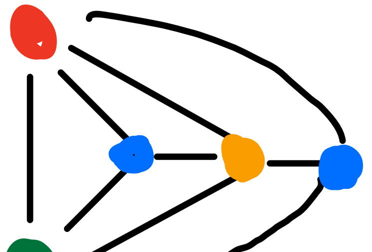
Prop 6.1.13: Let G be a plane graph.

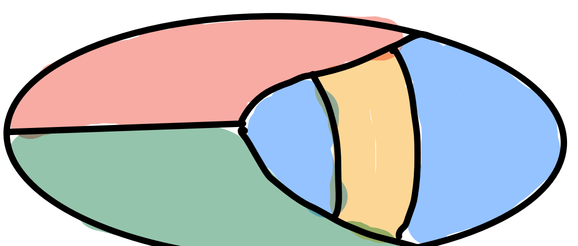
a) Let F be a face of G, and let $v \in V(G^*)$ be the corresponding vertex in G^*. Then, $\ell(F) = d(v)$.

b) If $F_1, \dots, F_k$ are the faces of G, then

$$2e(G) = \sum_{i=1}^{k} \ell(F_i).$$

c) The chromatic number $\chi(G)$ is the smallest number of ways to color the faces of G^* such that no faces which share a boundary edge have the same color.





Thm 6.1.14: Let G be a connected graph.

Let $D \subseteq E(G)$, and let $D^* \in E(G^*)$ be the corresponding edges in G^*. Then,

D is the edge set of a cycle $\iff$ D^* is a minimal edge cut.