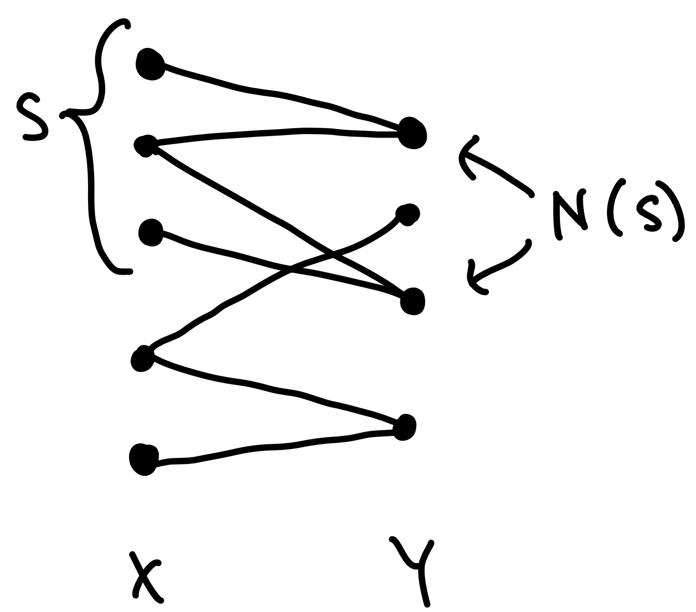
Hall's (Marriage) Thm (3.1.11): Let G be a bipartite graph w/ parts X and Y. Then,

G has a matching that saturates X $\iff$ $|N(S)| \geq |S|$ for all $S \subseteq X$

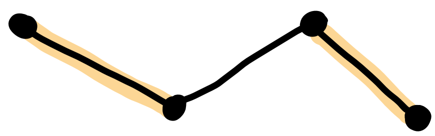
Pf: $\Rightarrow$ If G has such a matching M, the vertices in S are matched to $|S|$ vertices, all of which must be in $|N(S)|$

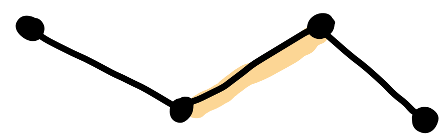


$\Leftarrow$ Need a def'n first

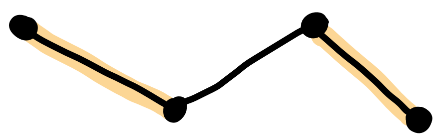
Def 3.1.6: Let $M \subseteq G$ be a matching.

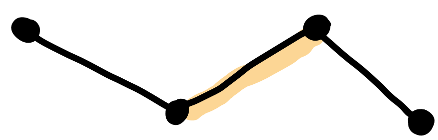
a) An M-alternating path is a path $P \subseteq G$ which alternates btwn. edges in M and edges not in M



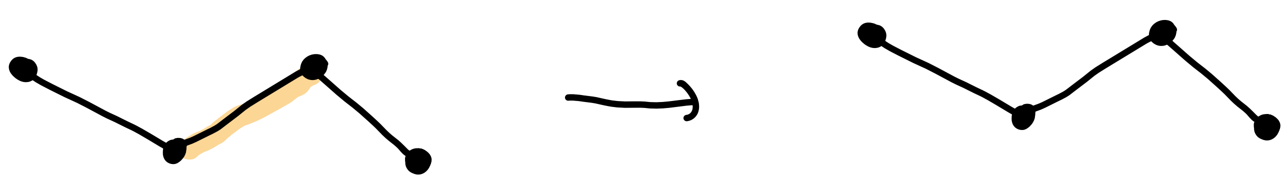


b) An M-augmenting path is an M-alternating path whose endpoints are unsaturated





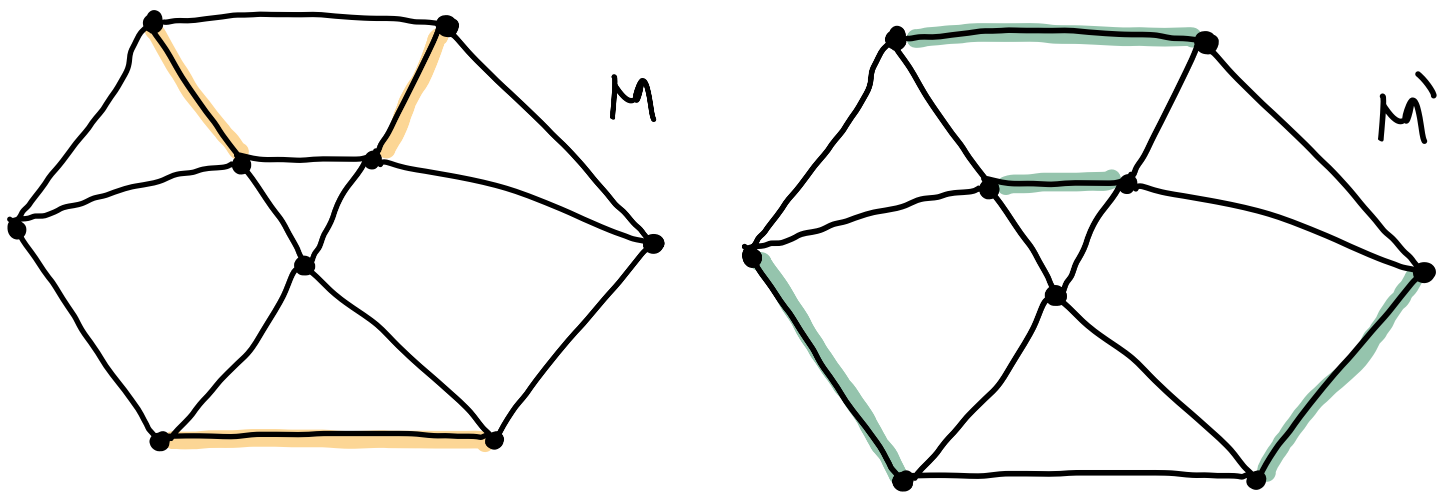
Idea: given an M-augmenting path, swap the edges and non-edges

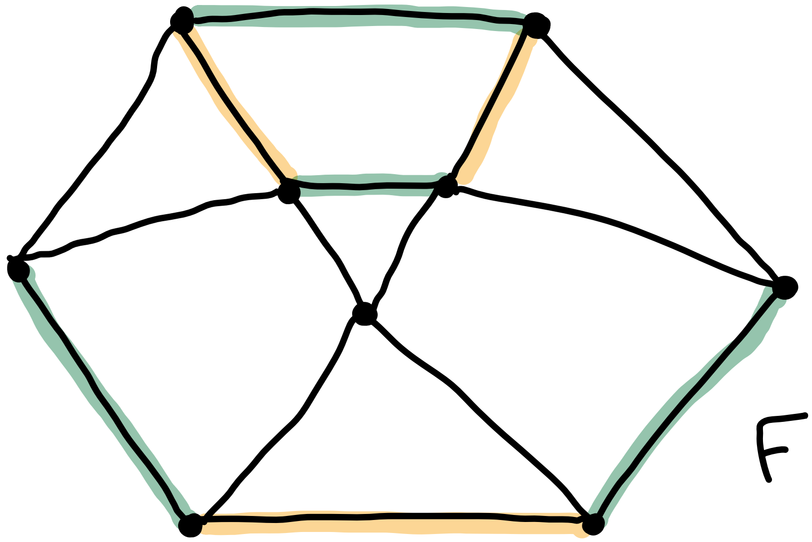


Always gives a larger matching

Thm 3.1.10: Let $M \subseteq G$ be a matching. Then, M is maximum $\Leftrightarrow$ G has no M-augmenting path

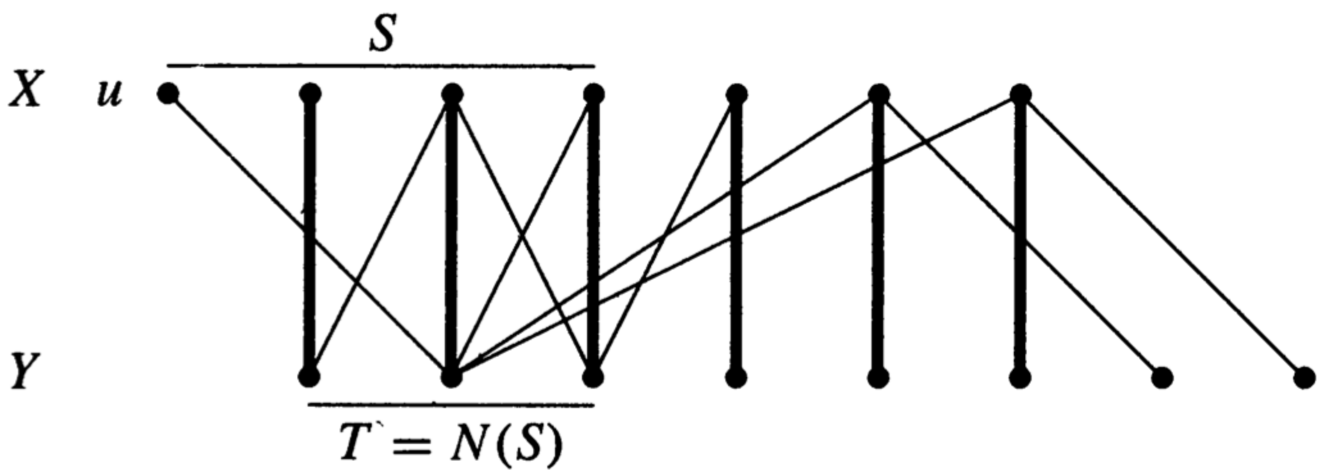
Pf:





F

Back to pf of Hall's Thm:



Def (3.1.14/3.1.19): Let G be a graph

a) $Q \subseteq V(G)$ is a <u>vertex cover</u> of G if every edge in $E(G)$ has ≥ 1 endpoint in Q

b) $L \subseteq E(G)$ is an <u>edge cover</u> of G if every edge in $V(G)$ is incident to ≥ 1 edge in L

c) $\alpha(G) :=$ maximum size of independent set

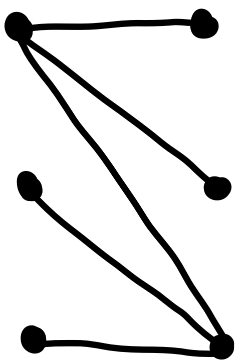
$\alpha'(G) :=$ maximum size of matching

$\beta(G) :=$ minimum size of vertex cover

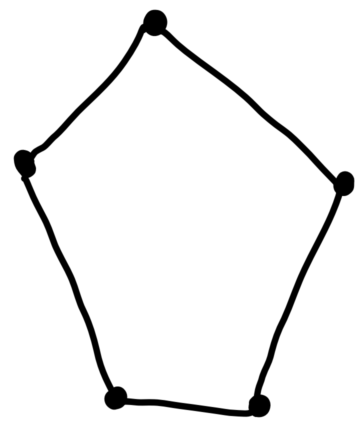
$\beta'(G) :=$ minimum size of edge cover

Class activity: compute $\alpha(G)$, $\alpha'(G)$, $\beta(G)$, $\beta'(G)$ for the graphs below

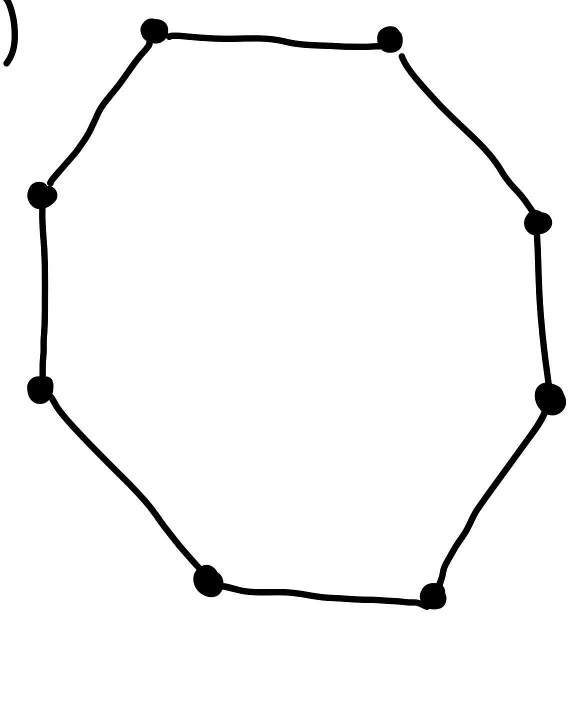
a)



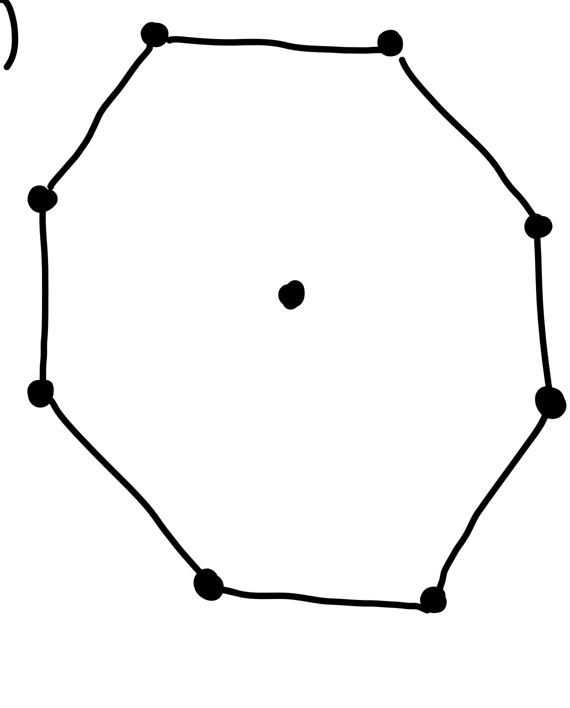
b)



c)



d)



e)