Announcements

Midterm 3: Wed. in class

Covers through Section 10.5

Reference sheet allowed (one A4 sheet w/ writing on both sides)

See policy email (practice problems etc.)

Problem session this week will be Tuesday & review

Midterm 3 Review

(Partial) list of topics:

Everything from first two midterms

(sets, functions, algorithms, induction, counting, probability, etc.)

Relations

Def'n & examples

Properties: reflexive, irref., symmetric, antisym., asymm., transitive

Operations: $R \cup S$, $R \cap S$, $R - S$, $S \circ R$, $\overline{R}$, R^{-1}

Matrices/digraphs for relations

Go btwn ordered pairs, matrices, digraphs

Connections to properties

Operations

Connections to Ch. 10

Equivalence rel'ns

- Definition
- Equiv. classes and set partitions

Graphs/digraphs

- Def'ns: simple/multi./nbhd./deg./bipartite/(induced) subgraph
- Handshake thm.
- Special classes of graphs
- Constructions: deletion/contraction/union
- Adjacency & incidence matrices
- Isomorphism
  - Show that graphs <u>are</u> isomorphic: explicit isom., adj. matrices
  - Show that graphs are <u>not</u> isomorphic: different "label-indep. properties"
- Connectivity (for digraphs, weak vs. strong), cut-edges/cut-vertices
- Paths/circuits
  - Eulerian/Hamiltonian
    + criteria

Other tips:

- Look at HW, quizzes, lecture notes, textbook, other problems
- We have lots of def'ns and constructions — learn them and/or write them on your ref. sheet

Examples:

1) Find the equiv. rel'n corresp. to the following set partition:

$A_i = \{ 5k+i \mid k \in \mathbb{Z} \}$

$A = \mathbb{Z} = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$

Sol'n: $a \sim b$ if and only if $a-b$ is a mult. of 5

$A_i = [i]$ for $i = 1, 2, 3, 4, 5$

2)a) Find a rel'n that is reflexive and transitive, but not symmetric or antisym.

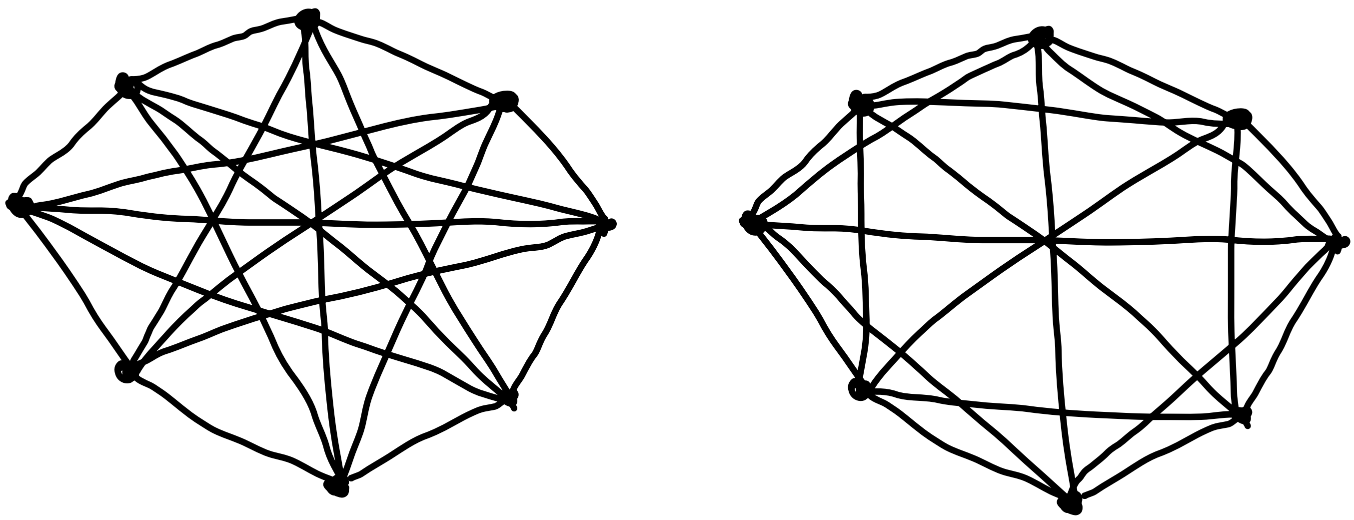
Sol'n: $A = \{1,2,3\}$

$R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,2)\}$

b) Show that any relation which is sym. and antisym. is also transitive.

Pf: Let A be a set and $R \subseteq A \times A$ be a rel'n that is both sym. and antisym. If $x \neq y$ and $(x,y) \in R$, then since R is sym, $(y,x) \in R$, but since R is antisym., $(y,x) \notin R$. Both statements can't be true at the same time, so if $x \neq y$, $(x,y) \notin R$, and every elt. of R is of the form (x,x). Now let $a,b,c \in A$ such that $(a,b) \in R$ and $(b,c) \in R$. By the previous statements, $b=c$, so $(a,c)=(a,b) \in R$, and therefore R is transitive. $\square$

3) (10.3.44) Determine whether or not these graphs are isomorphic.



b) Do they have Eulerian/Hamiltonian paths/circuits?

Sol'n: Neither have Eulerian paths/circuits since both have 8 verts. of deg. 5

Both have Hamiltonian circuits by traversing the outside edges clockwise