Announcements:

Quiz today!

Midterm 3: Next Wed. 11/15 7:00-8:30pm Noyes 217

Recall: The chromatic polynomial of G is

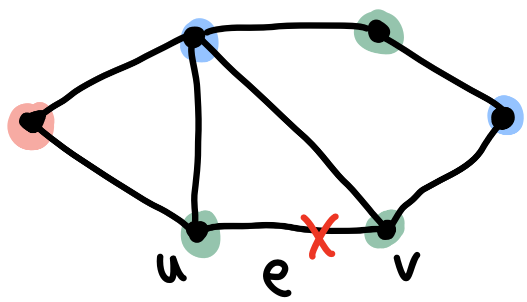
$\chi(G;k) :=$ number of proper k-colorings of G

There is a method to compute $\chi(G;k)$ recursively using deletion-contraction, allowing for a computation of $\chi(G;k)$, and thus $\chi(G)$, for any (individual) graph G.

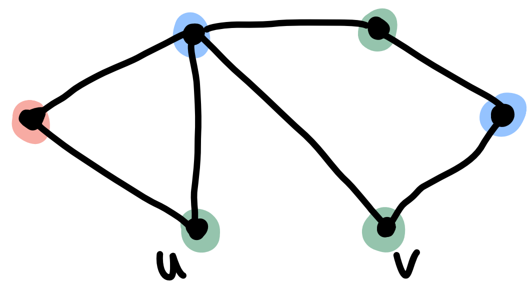
Thm 5.3.6: Let G be a simple graph and $e \in E(G)$. Then,

$$\chi(G;k) = \chi(G \setminus e; e) - \chi(G \cdot e; k)$$

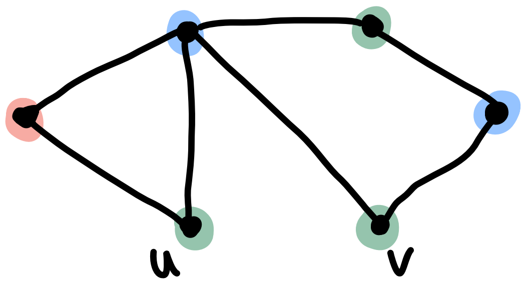
Pf:



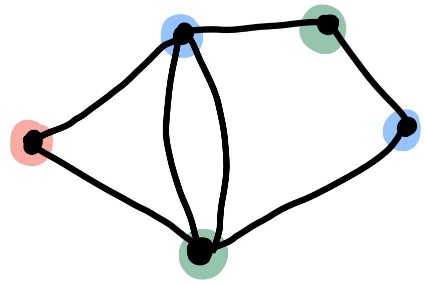
G



G \ e



$G \setminus e$



$G \cdot e$

$\square$

Example 5.3.7:

a) Let $G = C_4$, $e \in E(G)$ any edge

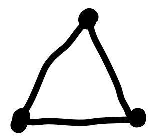
Chapter 6: Planar Graphs

Goal: Find possible values of $\chi(G)$ for planar graphs G

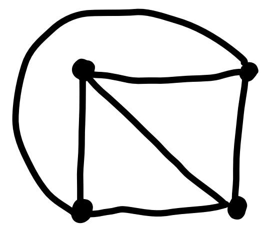
planar: can be drawn on a piece of paper w/out crossings

$\chi(K_1)=1$

$\chi(K_2)=2$

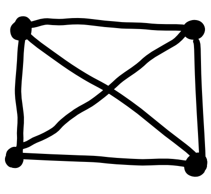


$\chi(K_3)=3$

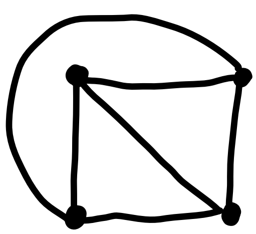


$\chi(K_4)=4$

Def 6.1.4: A graph G is planar if it has a drawing w/out crossings, called a <u>planar embedding</u> or a <u>plane graph</u>



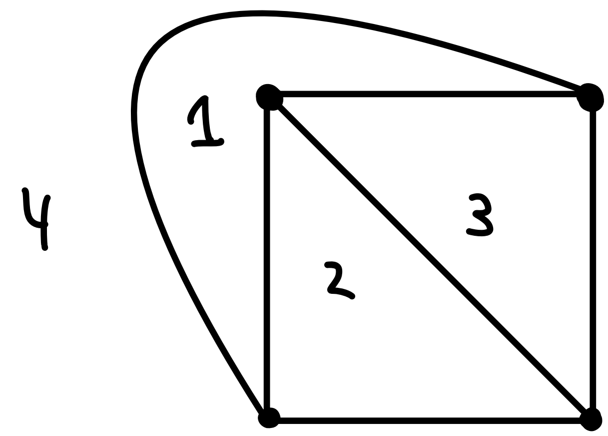
not a planar embedding



planar embedding

[so K_4 is planar]

Def: The faces of planar embedding are the maximal regions of the plane not intersecting vertices and edges



Remark: It is surprisingly difficult to make some of these ideas rigorous. Need topology and the "Jordan Curve Theorem"

Prop 6.1.2: K_5 and $K_{3,3}$ are not planar