

MATH 213 – DISCRETE MATH – Spring 2026 – Quiz 2 – Wednesday, Feb. 11

This quiz contains 3 questions – You have 15 minutes

Name: _____

Problem 1. For each of the following functions $f : A \rightarrow B$, is f injective, surjective, both, or neither? (*no work needed*)

a. $A = \{1, 2, 3\}, B = \{1, 2\}; f(1) = 1, f(2) = 2, f(3) = 2$

Solution: Surjective, but not injective

b. $A = \{1, 2, 3\}, B = \{1, 2, 3\}; f(1) = 1, f(2) = 2, f(3) = 2$

Solution: Neither

c. $A = B = \mathbb{N}, f(x) = x^2 + 1$

Solution: Injective, but not surjective

Problem 2. Define what it means for $f(x)$ to be

a. $O(g(x))$

Solution: f is $O(g)$ if and only if there exist C, k such that if $x > k$, $|f(x)| \leq C|g(x)|$.

b. $\Omega(g(x))$

Solution: f is $\Omega(g)$ if and only if there exist C, k such that if $x > k$, $|f(x)| \geq C|g(x)|$.

c. $\Theta(g(x))$

Solution: $f(x)$ is $\Theta(g(x))$ if and only if f is $O(g)$ and $\Omega(g)$.

Problem 3. Recall the binary-search algorithm:

Input: an integer x ; a list a_1, \dots, a_n of distinct integers with $a_1 < a_2 < \dots < a_n$.

Output: the location i of x in the list (or 0 if it's not in the list).

Algorithm:

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Let  $i := 1$ 
Let  $j := n$ 
While  $i < j$ 
    Let  $m := \lfloor \frac{i+j}{2} \rfloor$ 
    If  $x > a_m$ , set  $i := m + 1$ 
    Otherwise, set  $j := m$ 
If  $x = a_i$ , location  $:= i$ 
Otherwise, location  $:= 0$ 
return location
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Implement this algorithm with $x = 8$ and the list $0, 1, 2, 4, 5, 6, 8, 9$ ($n = 8$). Show the result of each while loop.

Solution:

Start: $i = 1, j = 8$

Round 1: $i = 1, j = 8, m = 4, a_m = 4$. Since $x = 8 > 4$, we set $i := m + 1 = 5$

Round 2: $i = 5, j = 8, m = 6, a_m = 6$. Since $x = 8 > 6$, we set $i := m + 1 = 7$

Round 3: $i = 7, j = 8, m = 7, a_m = 8$. Since $x = 8 \leq 8$, we set $j := m = 7$

Now, we have $i = j = 7$, so we exit the while loop. Since $x = 8 = a_7$, we return 7.