Announcements:

HW8 posted (due Wed. 11/8)

Quiz 3: Fri. 11/10 in class

Midterm 3: Wed. 11/15 7:00-8:30pm Noyes 217

Recall Cor 4.3.8: Let N be a network. If f is a feasible flow and $[S,T]$ is a source-sink cut, then

$$val(f) \leq cap(S,T)$$

Implication: $\max_f val(f) \leq \min_{[S,T]} cap[S,T]$

Max-flow, min-cut theorem (4.3.11):

$$\max_f val(f) = \min_{[S,T]} cap[S,T]$$

Remark: This result has connections to Menger's Thm., Halls Thm.*, etc. *see homework

Pf. idea: If $val(f) < \min_{[S,T]} cap[S,T]$, find an f-augmenting path.

Ford-Fulkerson algorithm:

Input: A feasible flow f in a network N

Start: $R = \{s\}$, $S = \emptyset$, $\Pi = \{\pi_s := s\}$

"reached" "searched" (Π: paths in underlying graph)

While $R \neq S$ and $t \notin R$:

  Let $v \in R \setminus S$

  For all $vw \in E(N)$:

    If $f(vw) < c(vw)$ and $w \notin R$:

      Add w to R

      Add $\pi_w := \pi_v, w$ to Π

  For all $uv \in E(N)$:

    If $f(uv) > 0$ and $u \notin R$:

      Add u to R

      Add $\pi_u := \pi_v, u$ to Π

  Add v to S

If $t \in R$:

  Output π_t (f-augmenting path)

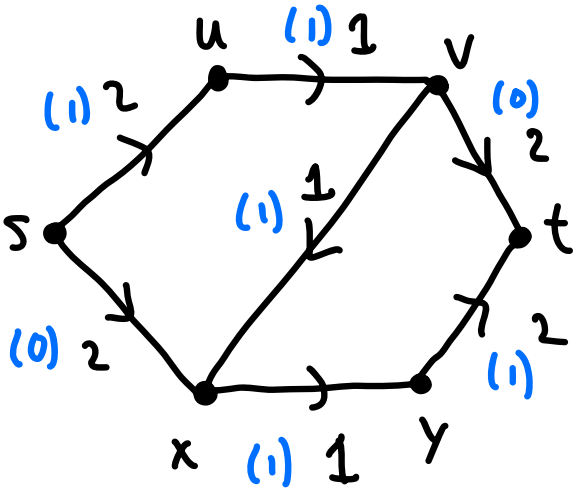
Otherwise (i.e. $R = S$):

  Output $[S, \bar{S}]$ (cut w/ capacity $\text{val}(f)$)

If Ford-Fulkerson returns an f-augmenting path, can augment along the path, and rerun.

Class activity: Run FF on the following graph repeatedly, and obtain a max. flow and min. cut

f:



R: s

S:

Π: $\pi_s = s$

Pf of max-flow, min-cut theorem when $c(e) \in \mathbb{Q}_{\geq 0}$: