# Announcements

No class Monday 11/27 (day after Fall Break)

Quiz 3: Fri. 11/10 in class

Midterm 3: Wed. 11/15 7:00-8:30pm Noyes 217

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We already know using greedy coloring that

$$\chi(G) \leq \Delta(G) + 1$$

And equality is possible.

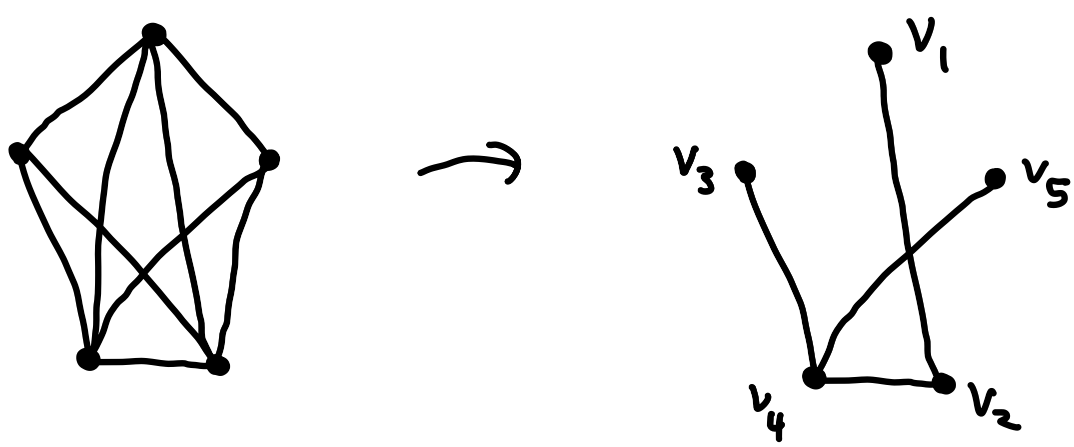
$$\chi(K_n) = n = \Delta(K_n) + 1$$

$$\chi(C_{2k+1}) = 3 = \Delta(C_{2k+1}) + 1$$

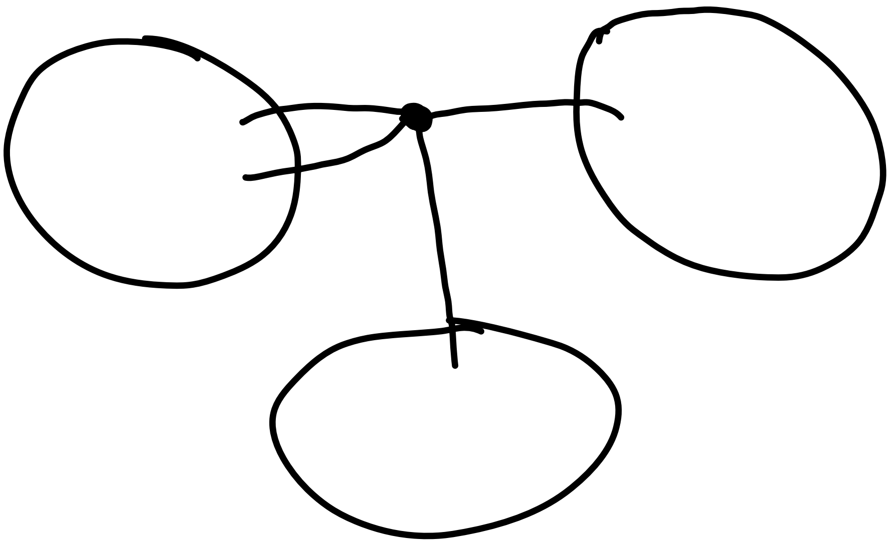
Brooks' Thm (5.1.22): If G is connected and G is <u>not</u> a complete graph or odd cycle, then

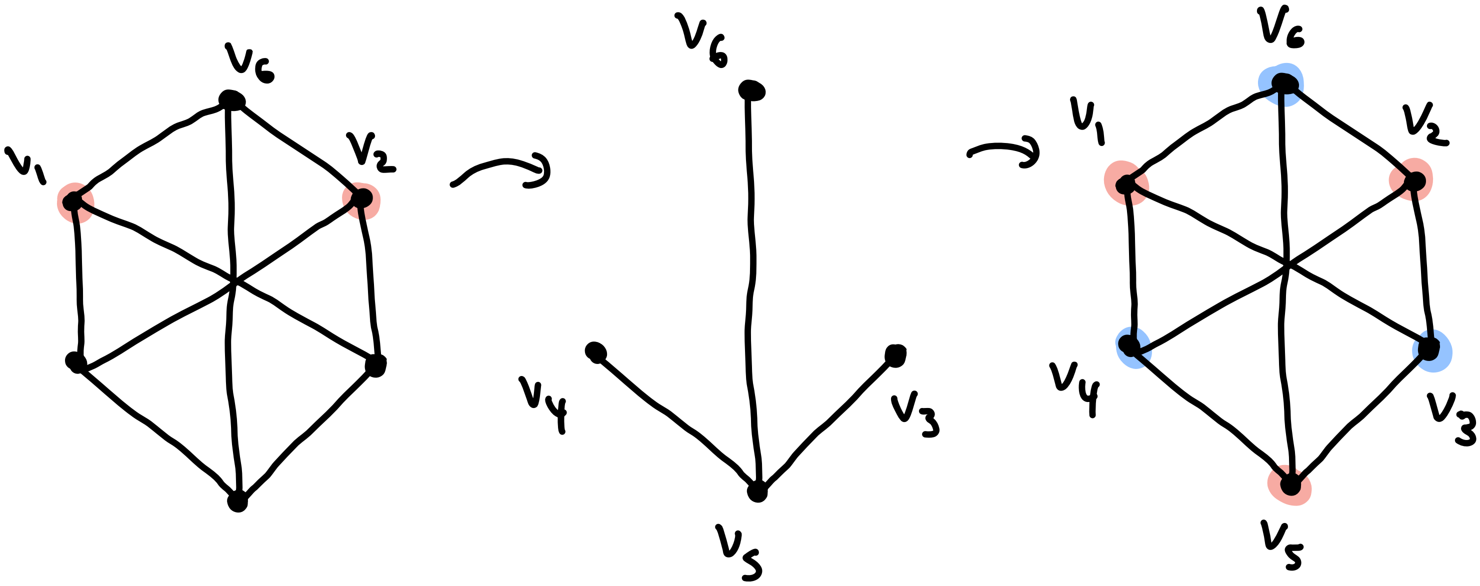
$$\chi(G) \leq \Delta(G).$$

Pf: Let $G$ be a connected graph, and let $k = \Delta(G)$.

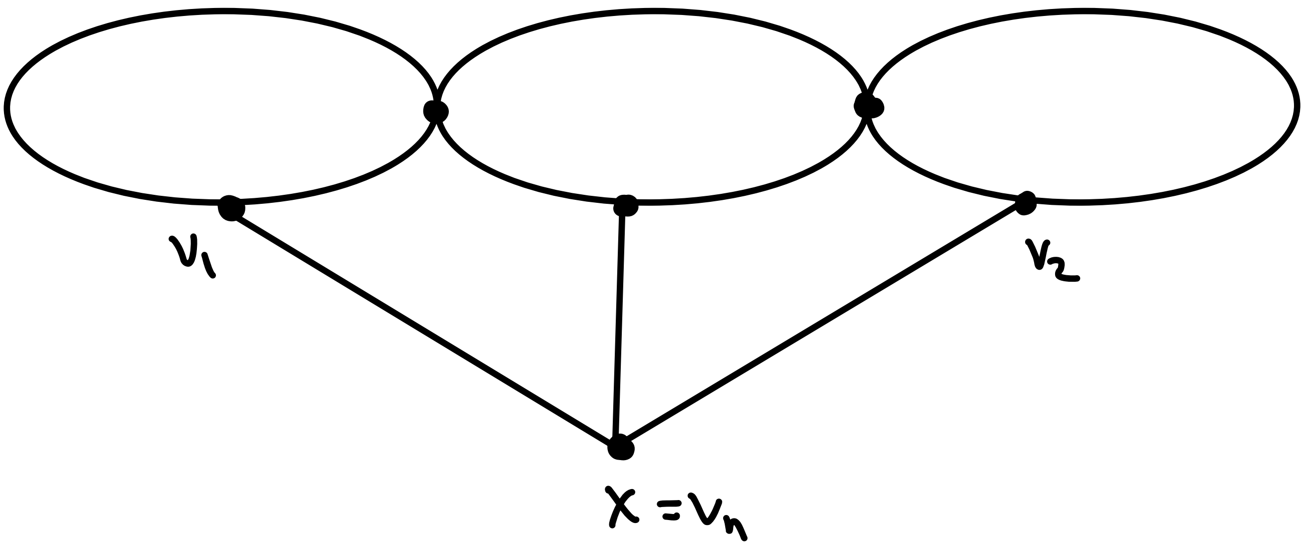



$G$ $\qquad\qquad\qquad$ $T$





v1
v6
v2
v6
v4
v3
v5
v6
v1
v2
v4
v3
v5



$v_1$
$v_2$
$x = v_n$

Last time: showed that for all $G$, $\chi(G) \geq \omega(G)$, and for interval graphs, $\chi(G) = \omega(G)$.

Turns out $\chi(G)$ can be way bigger than $\omega(G)$.

In fact,

Thm 5.2.3: For all $k \geq 1$, there exists a triangle-free graph $G$ with $\chi(G) = k$.

Def 5.2.1: Let $G$ be a simple graph with $V(G) = \{v_1, \dots, v_n\}$. Let $U = \{u_1, \dots, u_n\}$.

<u>Mycielski's construction</u> gives a graph $G' := Myc(G)$ with

$$V(G') = V(G) \sqcup U \sqcup \{w\}$$

$$E(G') = E(G) \sqcup \{u_i v \mid 1 \leq i \leq n, v \in N(v_i)\} \sqcup \{u_i w \mid 1 \leq i \leq n\}$$

Class activity: Find

a) $Myc(K_2)$

b) $Myc(Myc(K_2))$

Pf of Thm 5.2.3: