

Math 412, Fall 2023 – Homework 7

Due: Wednesday, October 25th, at 9:00AM via Gradescope

Instructions: Students taking the course for three credit hours (undergraduates, most graduate students) should choose four of the following five problems to solve and turn in—if you do all five, only the first four will be graded. Graduate students taking the course for four credits should solve all five. Problems that use the word “describe”, “determine”, “show”, or “prove” require proof for all claims.

1. Prove that every graph G with no isolated vertices has a matching of size at least $\frac{n(G)}{1+\Delta(G)}$.
2. Let G be a simple graph with $\Delta(G) \leq 3$. Prove that $\kappa(G) = \kappa'(G)$.
3. Let G be k -connected and let $U_1, U_2 \subseteq V(G)$ be disjoint sets with $|U_1| = |U_2| = k$. Prove that there are k pairwise disjoint paths, each of which have one endpoint in U_1 and the other endpoint in U_2 .
4. Prove that the hypercube Q_k is k -connected.
5. Draw a graph G with n vertices and $\kappa(G) = k$ for the following values of n and k .
 - (a) $n = 9, k = 3$
 - (b) $n = 9, k = 4$
 - (c) $n = 10, k = 3$
 - (d) $n = 10, k = 4$

[Hint: see Example 4.1.4]