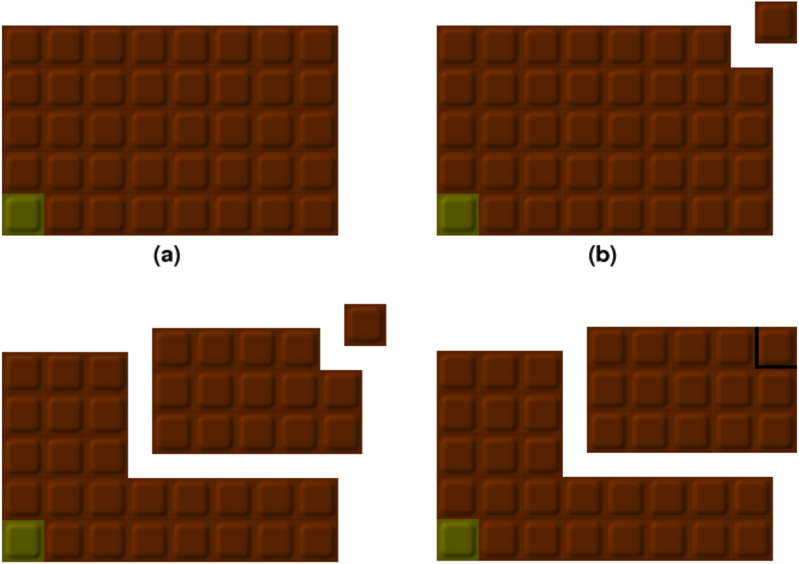
Math and Proofs Class 4

October 17th, 2017

Fun aside: Chomp Game



(a) (b) (c) (d)

Recap of Last Class

- We did some more set theory!
- Now we'll use set theory and look at equivalence relations, functions, and bijections

Equivalence Relations

- An *equivalence relation* on a set A is a set of ordered pairs where each half of each pair is an element of A
- It also has to be these three things:
  1. Reflexive: (a, a) is in the equivalence relation for every $a \in A$.
  2. Symmetric: If (a, b) is in the equivalence relation, then so is (b, a).
  3. Transitive: If (a, b) and (b, c) are in the equivalence relation, then so is (a, c).
- Let $A = \{1, 2, 3\}$. We're going to see what is and isn't an equivalence relation on A.

Functions

- A *function* $f : A \to B$ is a set of ordered pairs in $A \times B$ so that each element of A appears in exactly one ordered pair.
- Exercise: Let $A = \{1, 2, 3\}, B = \{a, b, c\}$. Which of the following sets are functions?
  1. $\{(1, a), (2, b), (3, c)\}$
  2. $\{(1, b), (2, b), (3, b)\}$
  3. $\{(1, a), (1, b), (3, c)\}$
  4. $\{(2, c), (1, b), (3, a)\}$
- What you might think of as a "normal function" is also a function by this definition
  - The function $y = x^2$ corresponds to the set $\{(x, x^2) : x \in \mathbb{R}\}$, which is the set of all pairs of numbers where the first number can be anything but the second number is the first one squared.

Bijections

- A *bijection* is a function where each output appears exactly once too.
- Exercise: Let $A = \{1, 2, 3\}, B = \{a, b, c\}$. Which of the following sets are functions?
    1. $\{(1, a), (2, b), (3, c)\}$
    2. $\{(1, b), (2, b), (3, b)\}$
    3. $\{(1, a), (1, b), (3, c)\}$
    4. $\{(2, c), (1, b), (3, a)\}$
- Since each input has to appear exactly once and each output has to appear exactly once, a bijection is a "pairing off" of the first set and the second set
- So what happens if there are different numbers of elements – you can't have a bijection!

Cardinality

- Let's make an equivalence relation between different sets
- Two sets A and B are equivalent if there's a bijection between them.
- Remember what this means: A and B are equivalent if they have the same **number** of elements
- Let's check that this actually is an equivalence relation: reflexive, symmetric, transitive
- We call this concept cardinality

Next Time

- We'll look at cardinality and infinity