Announcements:

- Discord server: email me if you want to be added
- H/w 1 graded (1 week for regrade requests)
- H/w 3 will be posted later today
- Midterm 1: Wed. 9/20 7:00-8:30pm
  (Noyes Lab. 217)
- Quiz 1: Fri. 9/15 (in class)

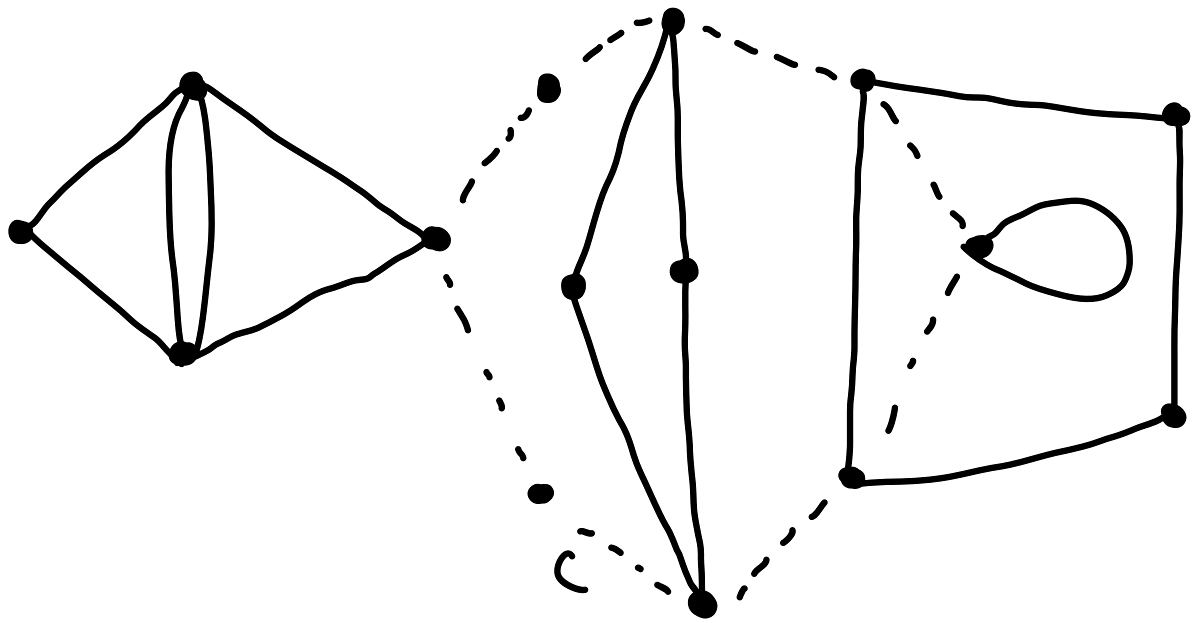
Thm 1.2.26 [Euler]:

G has an Eulerian circuit $\iff$ a) G has ≤ 1 "nontrivial" connected component AND b) G is even

containing edges

Pf: $\Rightarrow$) Done last time

$\Leftarrow$)



Def 1.1.32: A decomposition of G is a list of subgraphs s.t. each edge appears in exactly one subgraph from the list

Corollary (Prop 1.2.27): Every even graph decomposes into cycles.

Pf: In the previous proof, G decomposes into G' and C; use induction on $|E(G)|$. $\square$

§1.3: Vertex Degrees and Counting

Def 1.3.1:

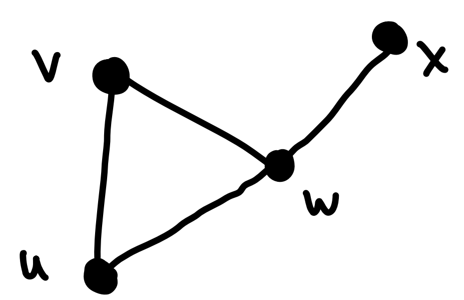
a) Max. degree $= \Delta(G)$

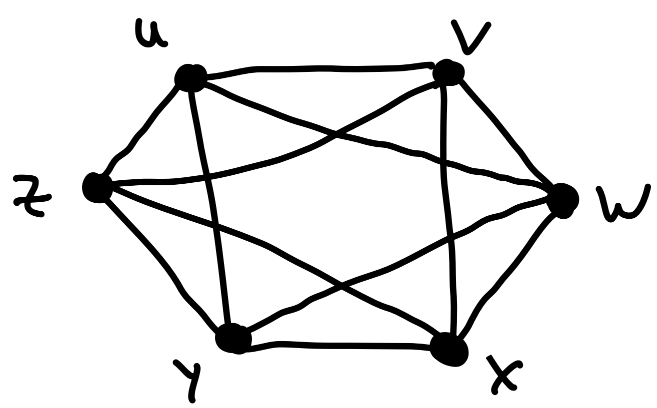
b) Min. degree $= \delta(G)$

c) If $\Delta(G) = \delta(G) = k$, G is <u>k-regular</u>

d) $N_G(v) = N(v) = \{\text{vertices adjacent to } v\}$

Class activity:





$\Delta(G) =$

$\delta(G) =$

regular?

$N(u) =$

$\Delta(G) =$

$\delta(G) =$

regular?

$N(u) =$

Def 1.3.2:

a) $n(G) = |V(G)|$ "order"

b) $e(G) = |E(G)|$ "size"

Important idea:

We can prove a lot about a graph using simple counting arguments

Prop (1.3.3 - 1.3.6):

a) (degree sum formula):

$$\sum_{v \in V(G)} \underbrace{d(v)}_{\text{degree}} = 2e(G)$$

b) $\delta(G) \leq \dfrac{2e(G)}{n(G)} \leq \Delta(G)$

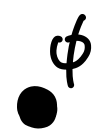
c) G has an even number of vertices of odd degree

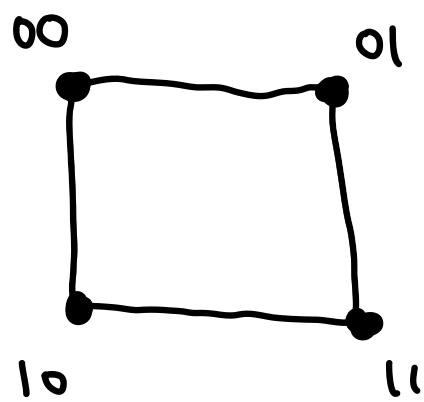
d) A k-regular graph of order n has $nk/2$ edges

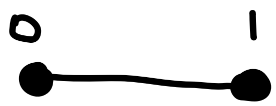
Example 1.3.8: The k-hypercube Q_k

Q_k has vertices labelled by length-k strings of 0's and 1's

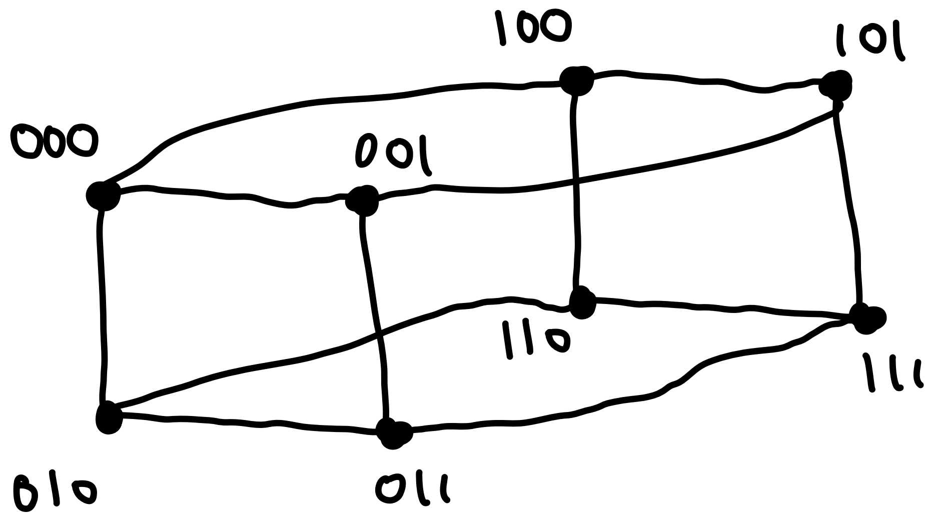
Two vertices are adjacent iff their labels differ in exactly one position

Q_0: 

Q_2: 

Q_1: 

Q_3:



Facts:

(i) Q_k is k-regular

(ii) Q_k is bipartite

(iii) $n(Q_k) = 2^k$

(iv) $e(Q_k) = k 2^{k-1}$

(v) If $j \leq k$, Q_k has $\binom{k}{j} 2^{k-j}$

subgraphs isomorphic to Q_j.

Extremal Problems

Questions involving the word "minimum" or "maximum"

Q: What is the maximum number of edges in a simple graph w/ n vertices

A:

Q: What is the minimum number of edges in a simple graph w/ n vertices

A:

Q: What is the minimum number of edges in a connected simple graph w/ n vertices

A:

Prop 1.3.15: If G is simple of order n, and $\delta(G) \geq \frac{n-1}{2}$, then G is connected

Can rephrase in terms of extremality:

Q: What is the minimum value d such that all n-vertex graphs with $\delta(G) \geq d$ are connected

A:

Pf: