

Math 418, Spring 2024 – Homework 7

Due: Friday, March 29th, at 9:00am via Gradescope.

Instructions: Students should complete and submit all problems. Textbook problems are from Dummit and Foote, *Abstract Algebra, 3rd Edition*. All assertions require proof, unless otherwise stated. Typesetting your homework using LaTeX is recommended, and will gain you 2 bonus points per assignment.

1. **Dummit and Foote #13.4.3:** *Prove that an algebraically closed field must be infinite.*
2. **Dummit and Foote #13.3.4:** *Construct the finite field of 16 elements and find a generator for the multiplicative group. How many generators are there?*
3. **Dummit and Foote #13.3.8:** *Determine the splitting field of the polynomial $f(x) = x^p - x - a$ over \mathbb{F}_p where $a \neq 0, a \in \mathbb{F}_p$. Show explicitly that the Galois group is cyclic.*
4. **Dummit and Foote #13.4.2:** *Find a primitive element for $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ over \mathbb{Q} .*
5. **Dummit and Foote #13.4.3:** *Let F be a field contained in the ring of $n \times n$ matrices over \mathbb{Q} . Here, $\mathbb{Q} \in \text{Mat}_n(\mathbb{Q})$ is given by the diagonal matrices. Prove that $[F : \mathbb{Q}] \leq n$. (I do have a hint for this one, if you ask)*