Announcements

§10.1: Graphs

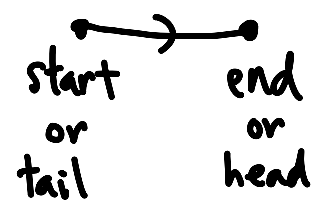
Def: A graph $G=(V,E)$ consists of

V: a nonempty set of vertices, and

E: a set of edges

Each edge has two vertices as endpoints. If they are the same, the edge is called a loop

Def: A digraph D has the same def'n except that edges are directed



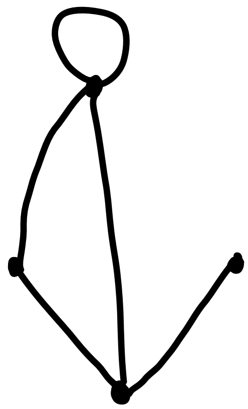
Def: A graph or digraph is called simple if

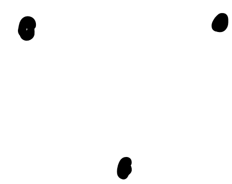
a) it has no loops ("loopless")

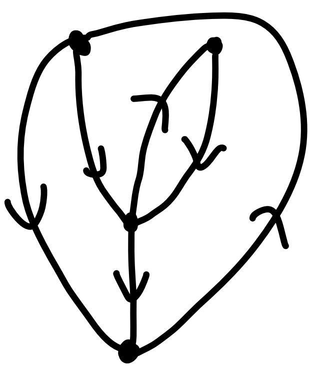
b) it has no multiple edges
- edges w/ same endpoints (graph)
- edges w/ same tail/head (digraph)

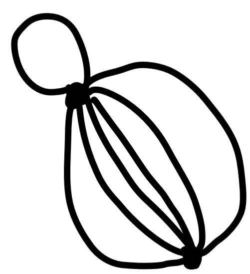
(Di)graphs w/ multiple edges are called multi(di)graphs

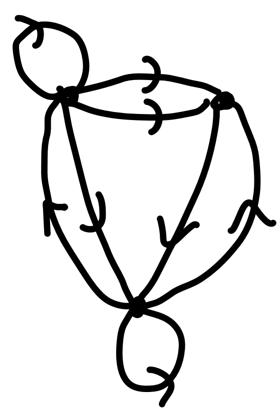
Class activity: Graph or digraph? Simple? Multi-?

a) 

b) 

c) 

d) 

e) 

Rosen has many examples of how graphs/digraphs can be used to represent real-world data (Ex 1 - 13, also on HW)

§10.2: Graph terminology, and special types of graph

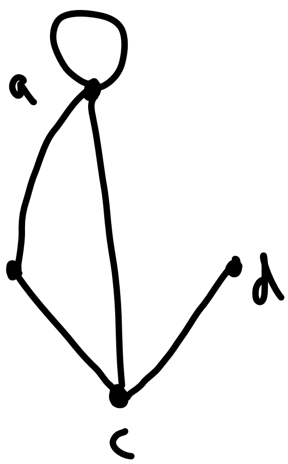
Def: Let $G = (V, E)$ be a graph

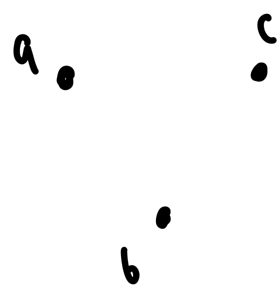
a) $u, v \in V$ $(u \neq v)$ are <u>adjacent</u> or <u>neighbors</u> if there is an edge $e \in E$ with endpoints u and v. e is <u>incident</u> to its endpoints.

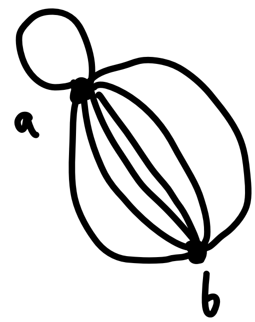
b) The <u>neighborhood</u> of $v \in V$ is the set $N(v)$ of all neighbors of v. If $A = \{v_1, v_2, .., v_k\}$, then $N(A) = N(v_1) \cup N(v_2) \cup \cdots \cup N(v_k)$, the set of all vertices adjacent to any vertex in A

c) The <u>degree</u> of $v \in V$ is the number $d(v)$ of edges incident to v (counting loops twice).

Class activity: Find all neighborhoods and degrees

a) 

b) 

d) 

Handshake theorem: For a graph G with m edges,

$$\sum_{v \in V} \deg(v) = 2m$$

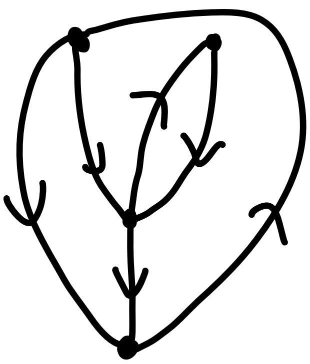
In particular, the number of vertices of odd-degree is always even!

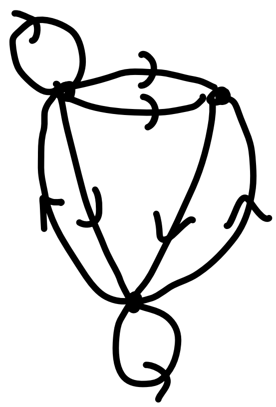
Def: Let $D=(V,E)$ be a digraph, $v \in V$.

The in-degree $\deg^-(v)$ of v is the number of edges w/ end/head v.

The out-degree $\deg^+(v)$ of v is the number of edges w/ start/tail v.

Class activity: Find all in/out-degrees

c) 

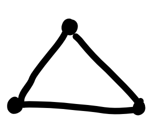
e) 

Special (undirected, simple) graphs

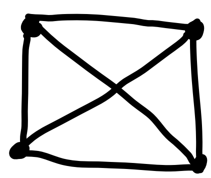
a) Complete graph K_n: all pairs of vertices are adjacent

K_1

K_2

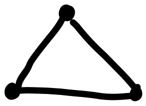


K_3

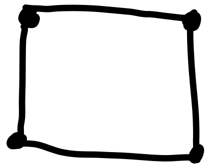


K_4

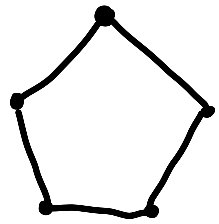
b) Cycle C_n :



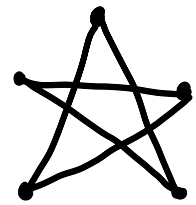
C_3



C_4

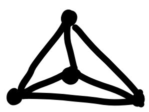


C_5

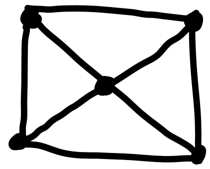


Also C_5
(doesn't matter how you place the vertices)

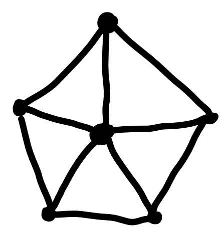
c) Wheel W_n : C_n with a hub



W_3



W_4

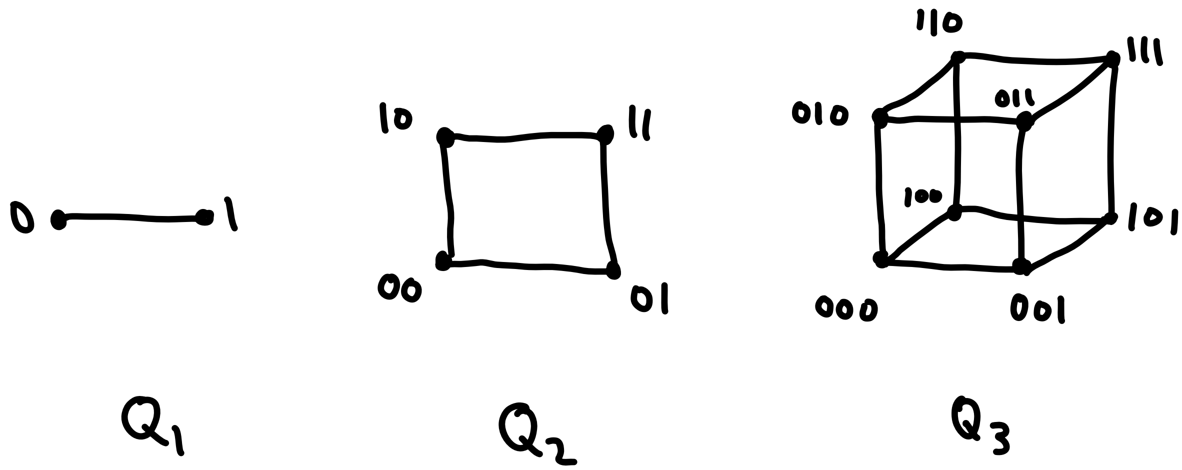


W_5

d) Hypercube Q_n

$V = \{$ binary strings of length $n \}$

$N(v) = \{$ all strings off by one digit from $v \}$



Def: G is <u>bipartite</u> if there is a set partition $V = \underbrace{V_1 \cup V_2}_{\text{disjoint}}$ such that every edge has one endpoint in V_1 and the other in V_2

Class activity: Of the above graphs, which are bipartite?
(if time)