

Announcements

Midterm 1: Wednesday 2/19 7:00-8:30pm Sidney Lu 1043

- Material: everything through Tower Law
 - i.e. through §13.2, except for composite ext's
- One reference sheet allowed (regular size, two sided)
- see policy email for more

Practice problem sol'n sketches posted

Tomorrow's problem session

↳ Wednesday's class: review

Recall: If $K_1, K_2 \subseteq L$, the composite $K_1 K_2$ of K_1 and K_2 is the smallest field containing K_1 and K_2 .

Prop: Let K_1/F , K_2/F be finite ext's w/ $K_1, K_2 \subseteq L$.

a) $[K_1 K_2 : K_2] \leq [K_1 : F]$

b) $[K_1 K_2 : F] \leq [K_1 : F][K_2 : F]$

PF: Let $\{\alpha_1, \dots, \alpha_n\}$ be a basis for K_1 over F .

Let $K = \{f_1 \alpha_1 + \dots + f_n \alpha_n \mid f_i \in K_2\}$

We have $K_1 \subseteq K$, $K_2 \subseteq K$, and $\dim_{K_2} K \leq n$, so if it's a field it is $K_1 K_2$, and a) will hold.

Closed under $+$, $-$: yes, since K is a v.s.

Closed under \cdot :

Since $\alpha_1, \dots, \alpha_k$ is an F -basis for K_1 , write

$$\alpha_i \alpha_j = \sum_k h_k \alpha_k$$

$\text{where } h_k \in F \subseteq K_2$

Then,

$$(f_1 \alpha_1 + \dots + f_n \alpha_n) (g_1 \alpha_1 + \dots + g_n \alpha_n)$$

$$= \sum_{i,j,k} \underbrace{f_i g_j}_{\in K_2} \underbrace{\alpha_i \alpha_j}_{\in K_1} = \sum_{i,j,k} f_i g_j h_k \alpha_k = \sum_k \underbrace{\left(\sum_{i,j} f_i g_j h_k \right)}_{\in K_2} \alpha_k$$

Inverses: Let $\gamma \in K \setminus \{0\}$, and consider the K_2 -linear transformation

$$T_\gamma: K \longrightarrow K$$

$$a \mapsto a\gamma$$

(additive gp. homom.,
but not ring homom.)

Since L is an integral domain,

$\ker(T_Y) = \{0\}$, so by the rank-nullity theorem,

$\dim \operatorname{im} T_Y + \underbrace{\dim \ker T_Y}_0 = n$, so T_Y is onto.

Thus Y has inverse $T_Y^{-1}(1) \in K$.

b) Using the Tower Law,

$$[K_1:F][K_2:F] \geq [K_1, K_2:K_2][K_2:F] = [K_1, K_2:F]$$

□

Alternate pf (see D&F): Finite ext's are iterated simple extensions. Prove a) for simple ext's by considering degrees of min'l polys, and use induction for the general case

Straightedge and Compass Constructions

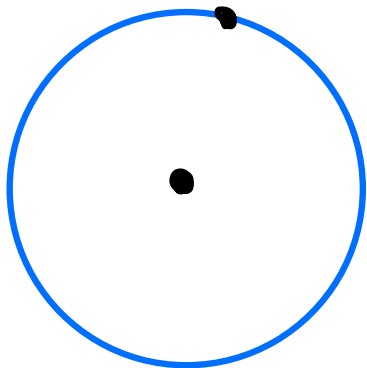
Game (ancient Greeks): Given a straightedge (ruler w/ out markings) and compass, what can we construct?

Operations:

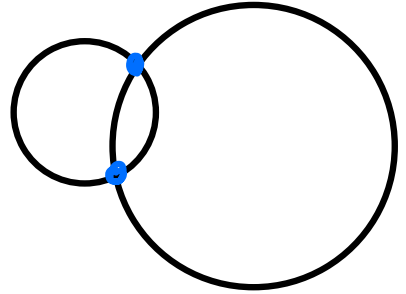
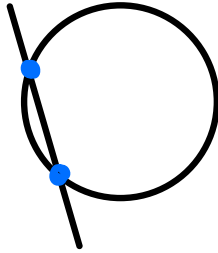
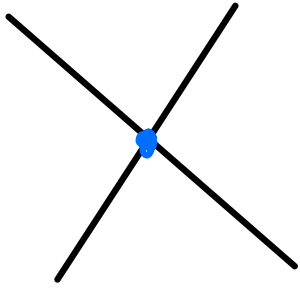
1) Connect two pts. by a line



2) Draw a circle w/ a given center and point



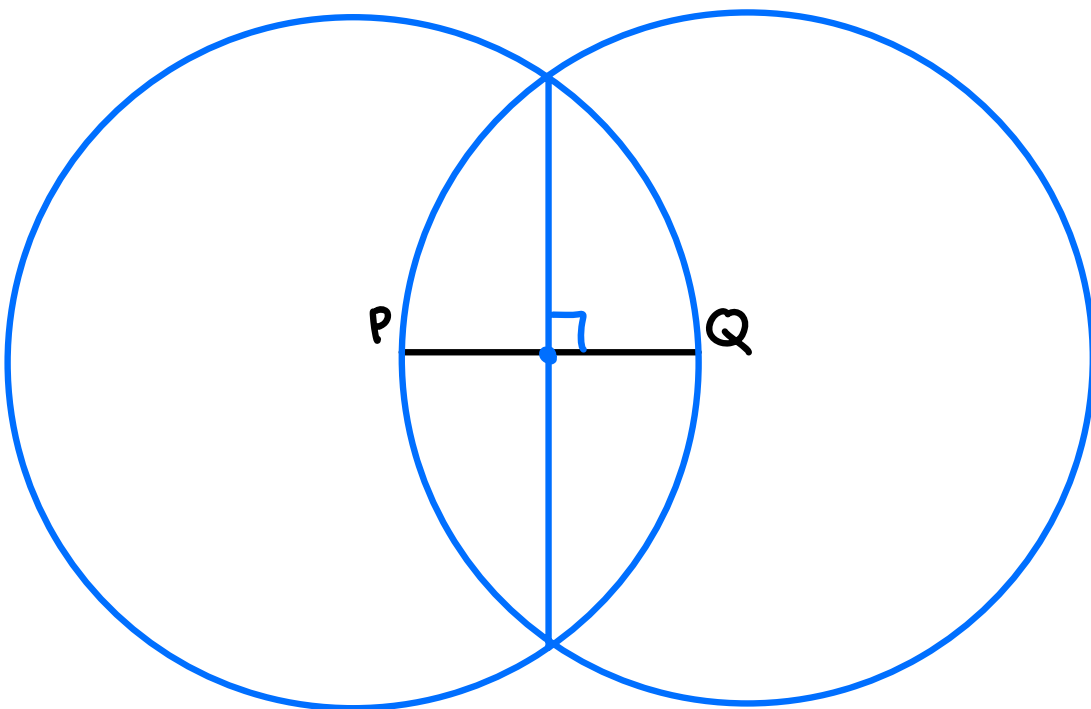
3) Find int. pt. of lines/circles



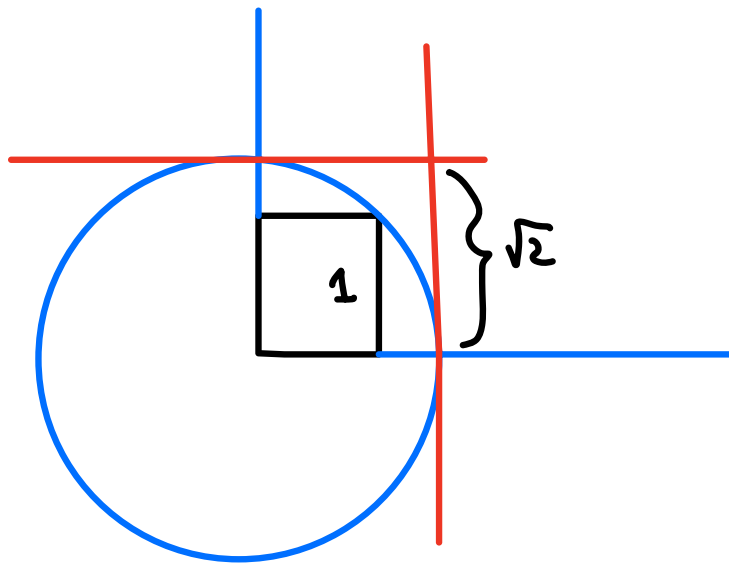
No measuring allowed!

With these operations, can do many things:

a) Perpendicular bisector



b) Double the area of a square



c) Construct the n -gon for certain n
(Gauss: 17-gon)

3 problems that the Greeks couldn't solve:

I) "Double the cube"

II) Trisect an arbitrary angle

III) "Square the circle"

Big idea: constructible numbers

Start w/ two points $\begin{matrix} \bullet & \bullet \\ 0 & 1 \end{matrix}$

Constructible numbers:

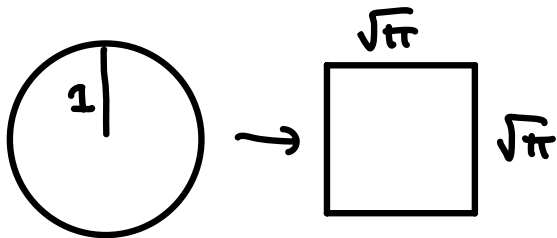
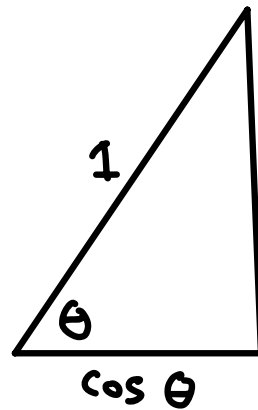
$$\mathcal{C} := \left\{ z \in \mathbb{C} \mid \begin{array}{l} \text{the pt. } z \text{ is constructible} \\ \text{from } 0 \text{ and } 1 \end{array} \right\}$$

Rephrase:

I) Construct $\sqrt[3]{2}$

II) Construct $\cos \frac{\theta}{3}$ given $\cos \theta$

III) Construct $\sqrt{\pi}$



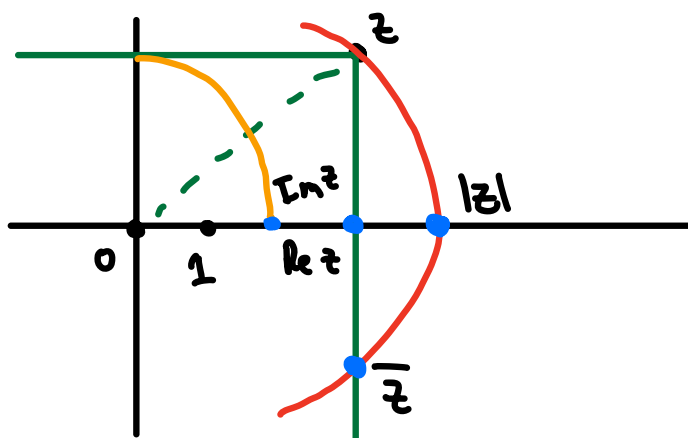
Prop: \mathbb{C} is closed under

a) $z \mapsto |z|$

b) $z \mapsto \bar{z}$

c) $z \mapsto \operatorname{Re}(z)$

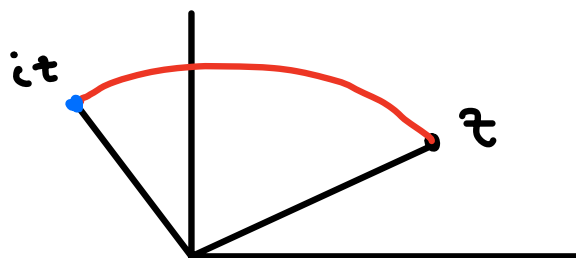
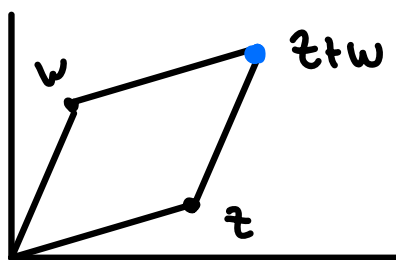
d) $z \mapsto \operatorname{Im}(z)$



e) Addition

f) Subtraction

g) Mult by i



□