Announcements

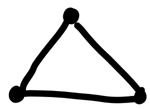
Quiz 7 this Friday

Special (undirected, simple) graphs

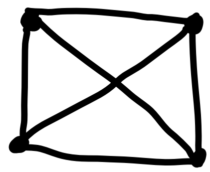
a) Complete graph K_n : all pairs of vertices are adjacent

K_1

K_2

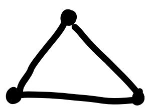


K_3

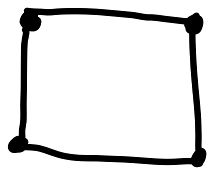


K_4

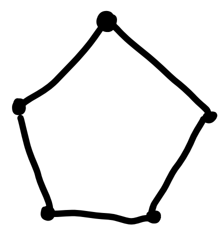
b) Cycle C_n :



C_3

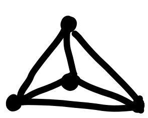


C_4

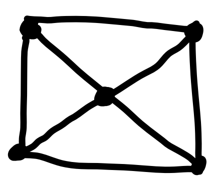


C_5

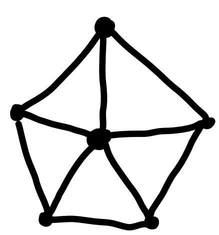
c) Wheel W_n : C_n with a hub



W_3



W_4

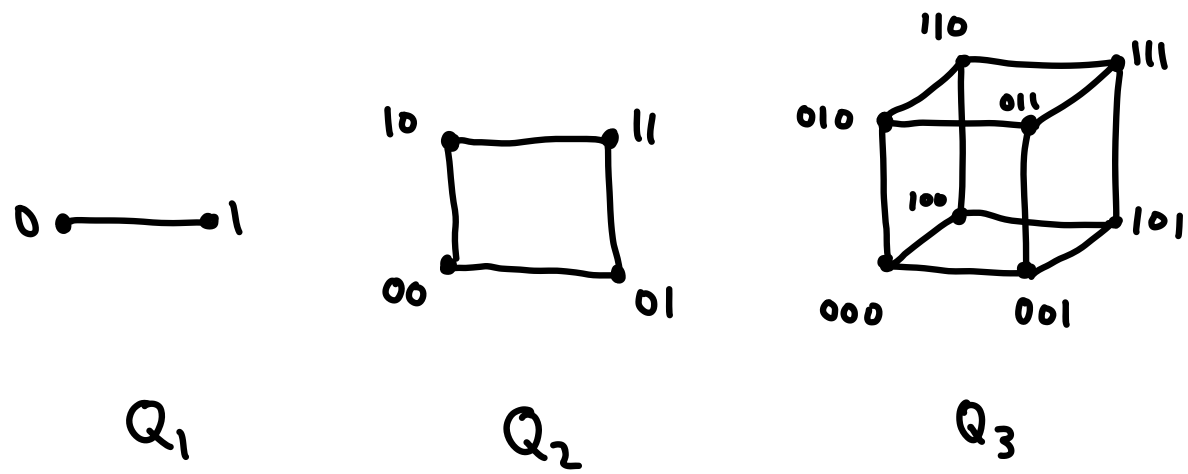


W_5

d) Hypercube Q_n

$V = \{\text{binary strings of length } n\}$

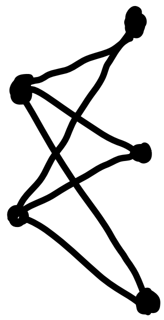
$N(v) = \{\text{all strings off by one digit from } v\}$



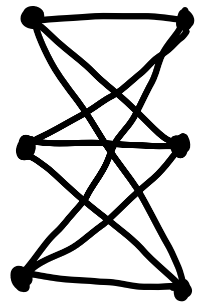
Def: G is <u>bipartite</u> if there is a set partition $V = \underbrace{V_1 \cup V_2}_{\text{disjoint}}$ such that every edge has one endpoint in V_1 and the other in V_2

Class activity: Of the above graphs, which are bipartite?

e) Complete bipartite graphs $K_{m,n}$: all possible edge btwn a set of m vertices and a set of n vertices



$K_{2,3}$



$K_{3,3}$

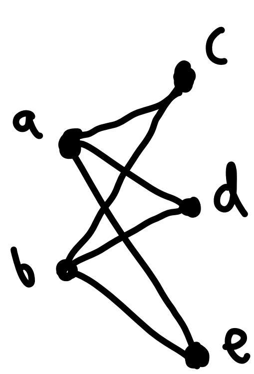
$K_{1,2}$

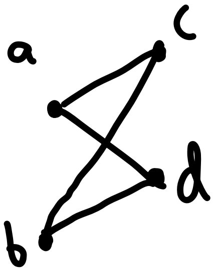
Def: Let $G=(V,E)$ be a graph. The graph $H=(W,F)$ is a *subgraph* of G if $W \subseteq V$ and $F \subseteq E$.

H is an *induced* subgraph of G if F contains every edge of G with both endpoints in W

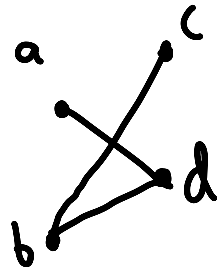
e.g.

G:





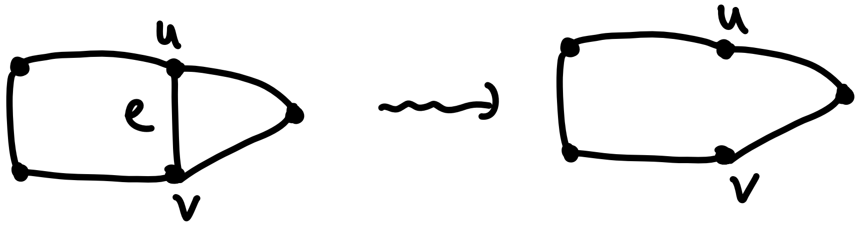
induced subgraph



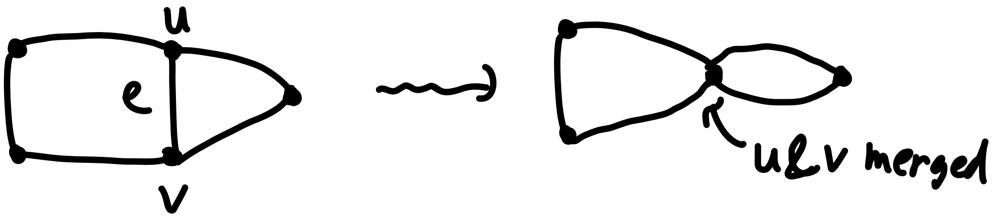
subgraph, but not induced subgraph

Def: Let G be a graph, and let e be an edge of G.

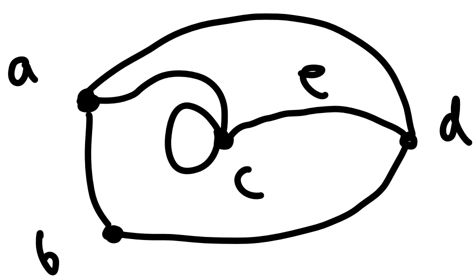
a) Deletion: $G-e$ is the graph formed by deleting e from G



b) Contraction: $G\cdot e$ is the graph formed by deleting e and "merging the endpoints of e.



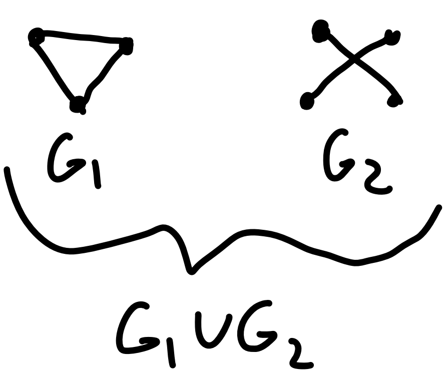
Class activity: Find $G-e$ and $G\cdot e$



Def: If $G_1=(V_1,E_1)$, $G_2=(V_2,E_2)$ are graphs, their <u>union</u> is

$$G_1\cup G_2=(V_1\cup V_2,\ E_1\cup E_2)$$

(just draw them side by side)



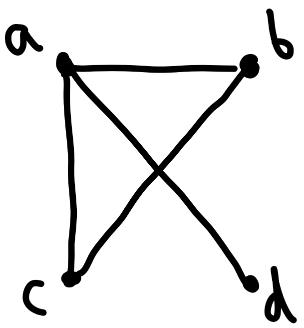
§ 10.3: Representing graphs & graph isomorphism

Def: Let G be a graph w/ vertices $v_1, \dots, v_n$.

The <u>adjacency matrix</u> of G is the matrix $\mathrm{Adj}_G = [a_{ij}]$

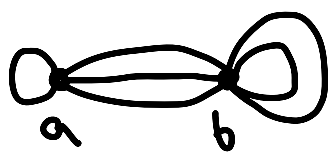
where $a_{ij} = \#$ edges with endpoints v_i & v_j

Ex 3:

G: 

$$\mathrm{Adj}_G = \begin{array}{c c} \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\ & \begin{array}{cccc} a & b & c & d \end{array} \end{array}$$

Ex:

G: 

$$\mathrm{Adj}_G = \begin{array}{c c} \begin{array}{c} a \\ b \end{array} & \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \\ & \begin{array}{cc} a & b \end{array} \end{array}$$

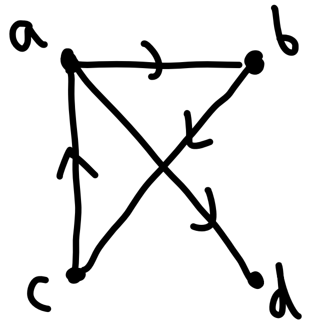
Def: Let D be a digraph w/ vertices $v_1, \ldots, v_n$.

The <u>adjacency matrix</u> of D is the matrix $Adj_D = [a_{ij}]$

where a_{ij} = # edges from v_i to v_j

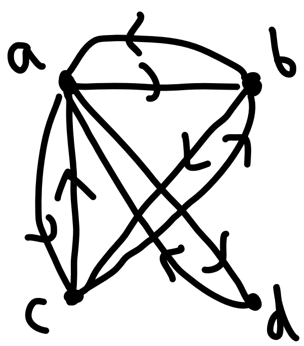
Ex:

D :



$$Adj_D = \begin{array}{c c} \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ & \begin{array}{cccc} a & b & c & d \end{array} \end{array}$$

Ex:



$$Adj_D = \begin{array}{c c} \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\ & \begin{array}{cccc} a & b & c & d \end{array} \end{array}$$

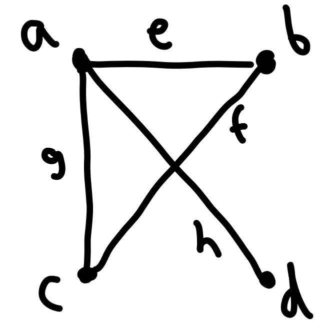
Def: Let G be a graph w/ vertices $v_1, \dots, v_n$.
and edges $e_1, \dots, e_m$

The incidence matrix of G is the matrix $\mathrm{Inc}_G = [m_{ij}]$

or both endpoints!

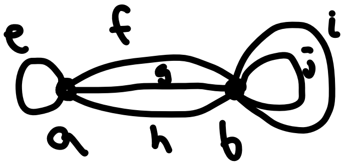
where $m_{ij} = \begin{cases} 1, & \text{if } v_i \text{ is an endpoint of } e_j \\ 0, & \text{otherwise} \end{cases}$

Ex:

G: 

$$\mathrm{Inc}_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ex:

G: 

$$\mathrm{Inc}_G = \begin{array}{c} \\ a \\ b \end{array} \begin{array}{c} \begin{array}{cccccc} e & f & g & h & i & j \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$