Announcements:

Final exam: Tues. 5/13 8:00am-11:00am,
1047 Sidney Lu Mech. E. Bldg.
Exam will be cumulative
See policy email for full details

- Two reference sheets allowed
- practice problems (solins by the weekend)
- regnade requests will only span a couple days

Review session: Sunday 5/11 10:00 am - 12:00 pm Altaeld 143 (come w/ questions)

Office hours: Friday and Monday 12:00pm - 1:00pm or by email /appointment

ICES questionnaires: go.illinois.edu/ices-online

Final exam review (See previous review topics)

Integral domains, poly rings, irreducibility

Basic tools: irreducibility, field extrs, degrees,

Splitting fields, min'l polys, tower law

Constructibility, III IIII

Constructibility: 4 classical problems, type of extins allowed Separability: derivative criterion, irreds. over char o or finifield Galois theory:

Compute Galois aps. (both up to isom. class and via generators and relins)

Galois correspondence (draw diagrams etc.)
Solvability by radicals

Examples: cyclotomic exths, finite fields, cubics, composite extis

Algebraic geometry:

Ideals, varieties, basic properties
Radical ideals, Nullstellensatz (all forms)

Noetherian rings

Prime wirred., maxil wo pt.

Coordinate ring

Projective space (all defins)

Homogeneous ideals, projective varieties

Projective Nullstellensate

Specific examples

Schemes (only the small amount we covered last time;

also see practice problem)

- · for studying, look at lecture notes, homework/midterm problems, practice problems, textbook
- · midtern length < final exam length < 2. midtern length
- · understand how topics mesh (e.g. ED/PID)UFD w/alg. 9com.)
- · understand theory and examples

Example problems:

i) a) Prove that $V = \{(a, a^2, a^3) | a \in k\}$ is an irreducible affine variety.

Pf: V=V(I) for $I=(k^2-\gamma, x^3-z)$, so V is a variety. We show V is inved. by showing that I is prime. Can show this using the defin of prime: if $f \cdot g \in I$, f or $g \in I$. Alternatively,

$$k[x,y,\overline{z}]/\underline{T} \cong k[x]$$

$$x \longmapsto x^{2}$$

$$z \longmapsto x^{3}$$

and since k[x] is an int. domain, I is prime.

b) Prove that $W = \{ [b^3 : ab^2 : a^2b : a^3] | a,b \in \mathbb{C} \}$ not both $0 \}$ is an irred proj. variety.

Pf: W = V(J) where $J = (xw - yz, xz - y^2, yw - z^2)$ (Fill in the details). J is a homog. ideal, so Wis a proj. variety. Jis prime since

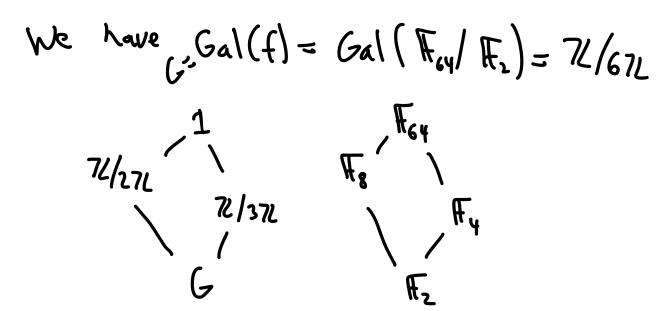
so W is irred.

2) Compute the Galois gp. / Galois corresp. for $f(x) = (x^3 + x + 1)(x^3 + 1)$ over F_3

Soly: $x_2+1=(x+1)(x_2-x+1)$ (over any field) Over $f(x)=(x+1)(x_2-x+1)(x_3+x+1)$ e $f(x)=(x+1)(x_3+x+1)$ e f(x)=(x+1)(x+1) e f(x)=(x+1)(x+1)

By DCF Prop 14.18 (or Prop 14.19)

 $Sp_{ff_2}f = F_{2n}$ where n = lcm(degrees of irred factors)= <math>lcm(1,2,3)= 6



G abelian, hence everythly normal/Galois

3) Prove that a quotient of a PIDR by a prime ideal I is again a PID.

Pf: If I=(0), then R/I=R is a PID

If I = (0), then I is maximal (DEF Prop 8.7),

So R/I is a field, hence a PID.

4) Let k/F be a nontriv. Galois extin of odd order, and let $\alpha \in K \setminus F$. Prove that $|\{\sigma \in Gal(k/F) | \sigma(\alpha) \neq \alpha\}| > |\{\sigma \in Gal(k/F) | \sigma(\alpha) = \alpha\}|$ Pf: Since k/F is Galois, $Gal(k/F(\alpha))$ is a proper subsp. of Gal(k/F). Since [k:F] is odd, so is

|Gal(K/F)|, so every proper subsp. has index > 3. Therefore, the subset of Gal(KF) of automs. that fix x is z = 1 of the total. D b) Give a nontrivial extra of odd order s.t. | { σ ∈ Aut(k/F) | σ (a) ≠ a} | ≤ | { σ ∈ Aut(k/F) | σ (a) = ~ } |

Solh: Let F=Q, k=Q(3/2)

Aut(K/F)=1, so id is the only elt., and this fixer 3/2 e K > F.