Announcements:

HW9 posted (due Wed. 11/29 9am)

No class or office hours Mon. 11/27

Remark 6.1.9:

a) G^* is always connected

b) If G is connected, $|V(G)| = |\{\text{faces of } G^*\}|$

and $|V(G^*)| = |\{\text{faces of } G\}|$

c) $(G^*)^* \cong G \iff G$ is connected

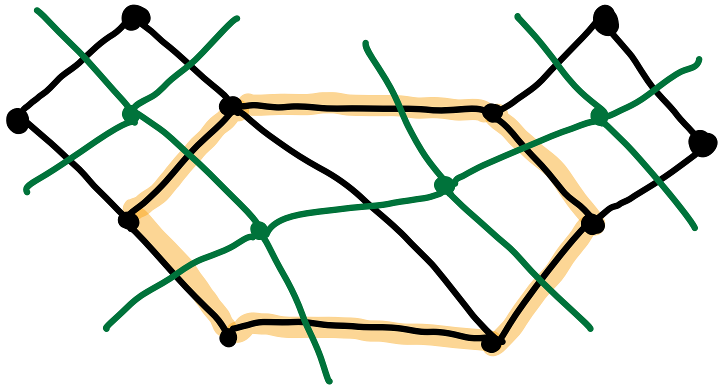
Pf: Homework!

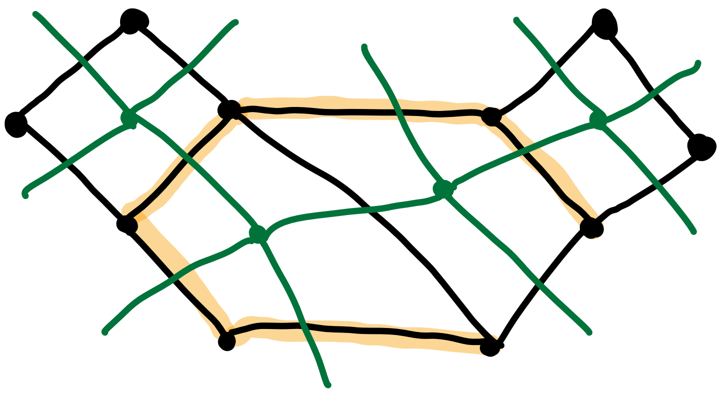
Thm 6.1.14: Let G be a connected graph.

Let $D \subseteq E(G)$, and let $D^* \in E(G^*)$ be the corresponding edges in G^*. Then,

D is the edge set of a cycle $\iff$ D^* is a minimal edge cut.

Pf:





What can we say about the numbers of vertices $n := |V(G)|$, edges $e := |E(G)|$ and faces f of a planar graph?

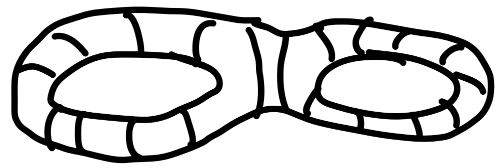
Euler's Formula: For any conn. planar graph G,

$$n - e + f = 2$$

Remark: This is the tip of the iceberg of one of the most important ideas in algebraic topology, called the <u>Euler characteristic</u>. For instance, for a graph drawn on a g-hole torus with no crossings,

$$n - e + f = 2 - g$$

e.g. $g = 2$

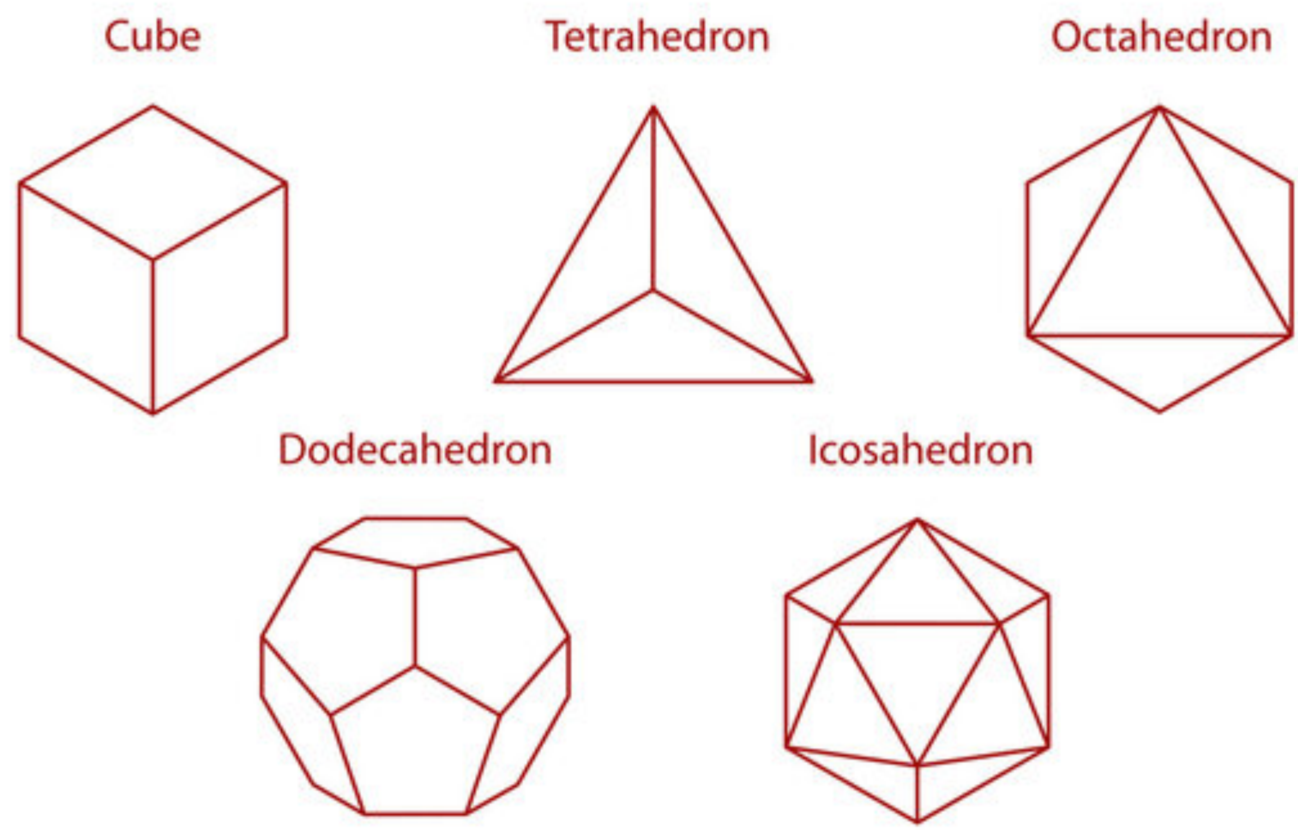


Pf of Euler's formula:

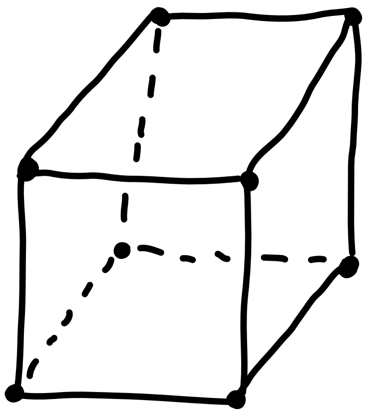
Application: regular polyhedra

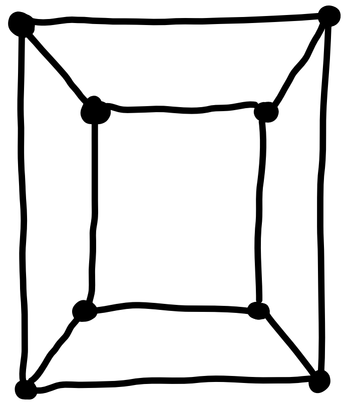
Def: A polyhedron is a 3D solid whose boundary consists of polygons, called faces. The edges/vertices are the edges/vertices of the polygons.

Def: A regular polyhedron is a solid whose boundary consists of identical regular polygons with the same number of faces around each vertex.



View these as a graph on a sphere, and "pull open" the back face to make a plane graph





8 vertices

12 edges

6 faces

$8 - 12 + 6 = 2$

Cor: Every polyhedron satisfies $n - e + f = 2$

Let's determine all the regular polyhedra:

n vertices

e edges

f faces

faces have l edges

vertices have k faces