Announcements:

- Wednesday's class will have a sub
- Final exam room set

  Thurs. 12/14 8:00am - 11:00am in 132 Bevier Hall
- Quiz this Friday (all material thru. today)
- Midterm 2 next Wed.

  Wed. 10/18 7:00pm - 8:30pm in 217 Noyes Lab.

  Policies similar to Midterm 1; email coming soon)

Recall: Def (3.1.14/3.1.19): Let G be a graph

a) $Q \subseteq V(G)$ is a <u>vertex cover</u> of G if every edge in $E(G)$ has ≥ 1 endpoint in Q

b) $L \subseteq E(G)$ is an <u>edge cover</u> of G if every vertex in $V(G)$ is incident to ≥ 1 edge in L

c) $\alpha(G) :=$ maximum size of independent set

$\alpha'(G) :=$ maximum size of matching

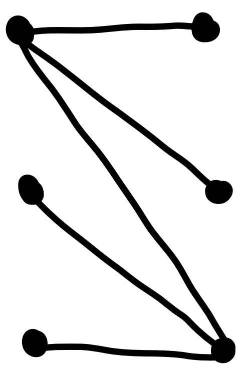
$\beta(G) :=$ minimum size of vertex cover

$\beta'(G) :=$ minimum size of edge cover

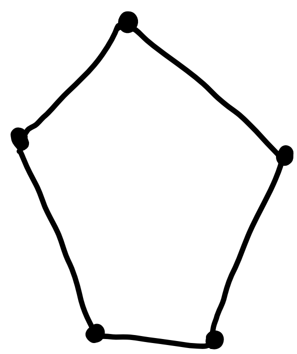
Class activity:

(I) compute $\alpha(G)$, $\alpha'(G)$, $\beta(G)$, $\beta'(G)$ for these graphs

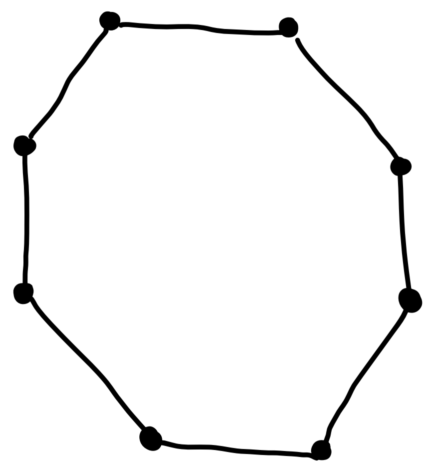
a)



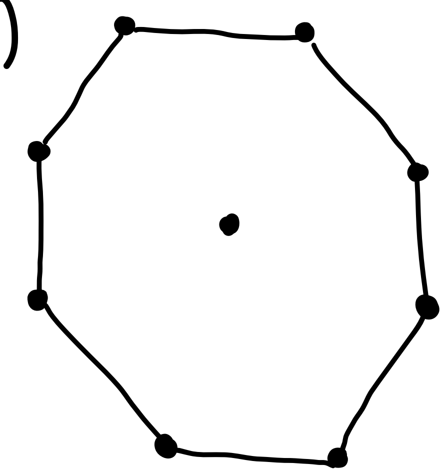
b)



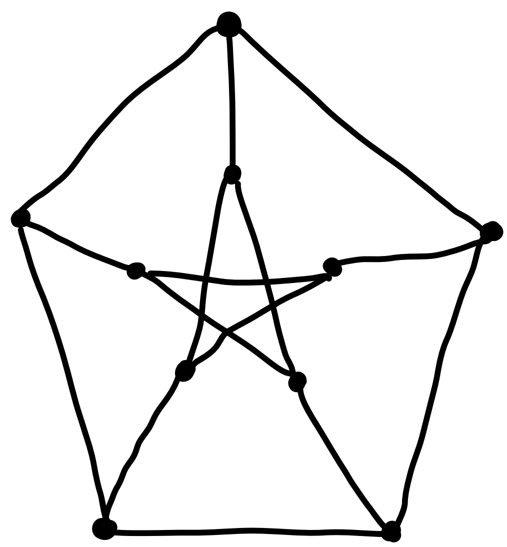
c)



d)



e)



(II) Do your own examples, and make conjectures

Stable Matchings

Let G be a complete X,Y-bigraph w/ $|X| = |Y| = k$

Assign each vertex $x \in X$ a preference list

$$y_{i_1} > \cdots > y_{i_k}$$

i.e. an ordering of the elements of Y.

Similarly, assign each vertex $y \in Y$ a preference list of the vertices in X.

Let M be a perfect matching of G.

An <u>unstable pair</u> is an unmatched pair (x,y), $x \in X$, $y \in Y$ s.t.

y is higher on the preference list of x than x's match

x is higher on the preference list of y than y's match

If M yields no unstable pairs, it is called a stable matching

Ex: Children $X = \{x, y, z, w\}$

Puppies $Y = \{a, b, c, d\}$

Preference lists:

$x: a > b > c > d$ $\qquad$ $a: z > x > y > w$

$y: a > c > b > d$ $\quad$ $b: y > w > x > z$

$z: c > d > a > b$ $\quad$ $c: w > x > y > z$

$w: c > b > a > d$ $\quad$ $d: x > y > z > w$

$\{xb, ya, zd, wc\}$ – unstable: x & a prefer each other to their matches

$\{xa, yb, zd, wc\}$ – stable

Gale-Shapley Algorithm (3.2.18):

Input: Preference rankings by all children and puppies

Iteration: Each puppy bounds up to the highest child on its preference list who hasn't already rejected it.

If Each puppy chooses a different child:

Stop, and use the resulting matching

Otherwise:

Each child rejects every puppy that bounds up to it, except the child says "maybe" to his/her favorite of the puppies bounding up to him/her.

Repeat iteration

Thm 3.2.18: The Gale-Shapley Algorithm always produces a stable matching

Pf next time.

Question: Who is happier?

i.e. more likely to get a higher choice

Answer: The puppies!