No new announcements today

Prop 1.3.15: If G is simple of order n, and $\delta(G) \geq \frac{n-1}{2}$, then G is connected

Pf:

Recall: If $S \subseteq V(G)$, the induced subgraph $G[S]$ is the graph whose vertex set is S and whose edge set is

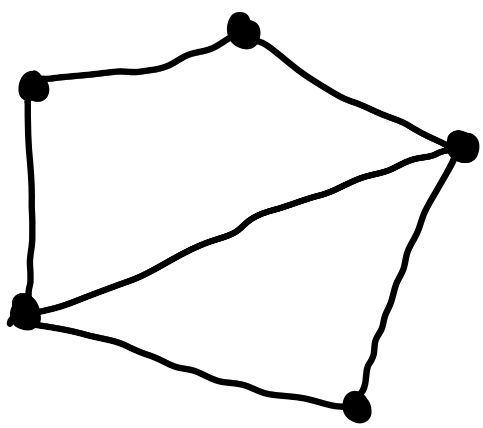
$$\{e \in E(G) \mid \text{both endpoints of } e \text{ are in } S\}$$

Def 1.3.22: G is H-free if G has no induced subgraph isomorphic to H.

Ex: By Konig's Theorem, bipartite graphs have no odd cycles. Therefore, if G is bipartite, G is C_{2k+1}-free for all k

Note: being H-free is <u>not</u> the same as having no subgraph isomorphic to H.

E.g.:



Mantel's Theorem [1907]: The maximu number of edges in an n-vertex triangle-free simple graph is $\lfloor n^2/4 \rfloor$

Pf:

Def: 1.3.27

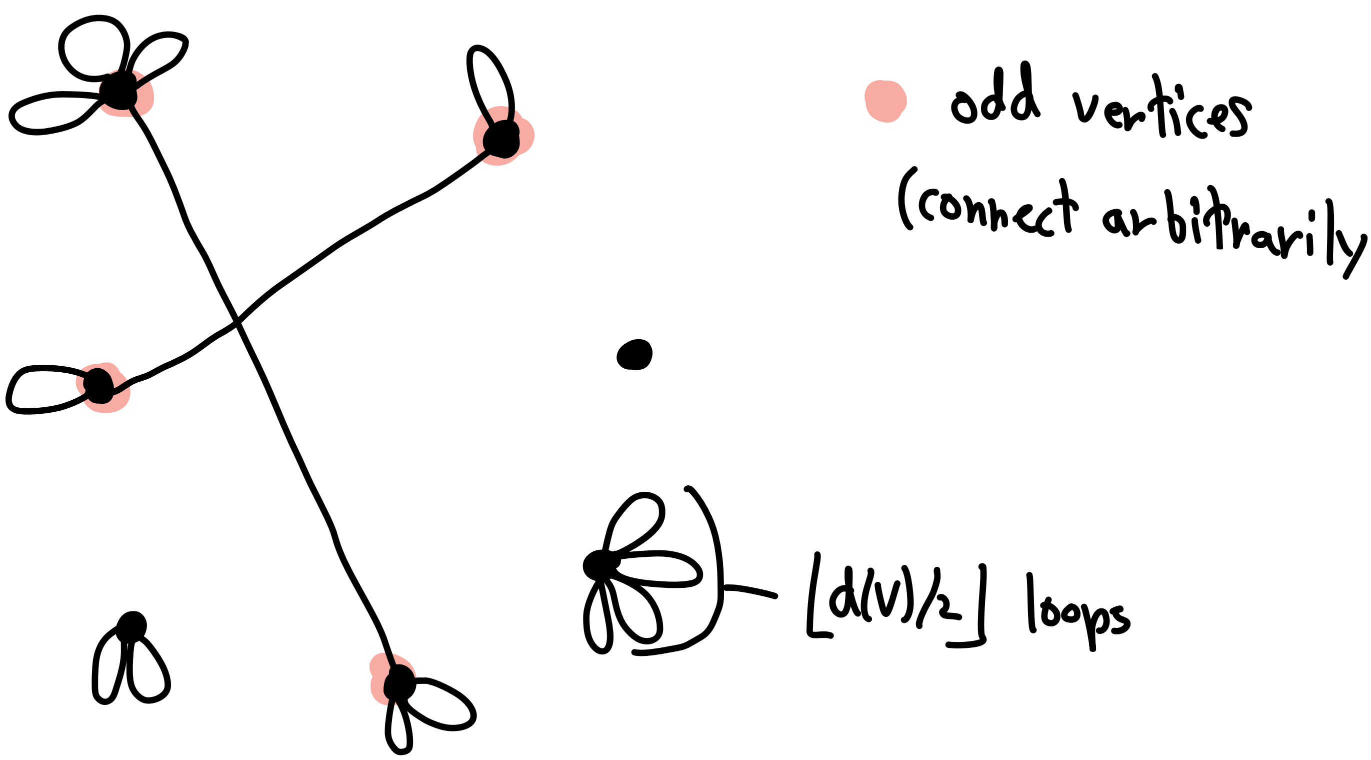
The <u>degree sequence</u> of a graph is a (usually weakly decreasing) list of the vertex degrees: $d_1, d_2, \ldots, d_n$

Question: Which sequences are the degree sequence

of some
- a) graph?
- b) simple graph? "graphic"

Prop 1.3.28: A list $d_1, \dots, d_n$ is the degree sequence of a graph iff $\sum d_i$ is even.

"Proof" by picture:

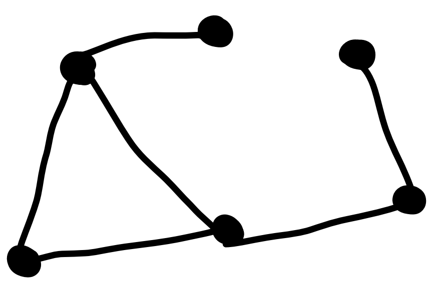


Havel-Hakimi Theorem:

a) For 1 vertex, the only graphic sequence is $d_1 = 0$

b) A list d of $n > 1$ integers is graphic iff d' is graphic, where d' is obtained by deleting the largest element Δ and subtracting 1 from its next Δ largest elements

Ex:



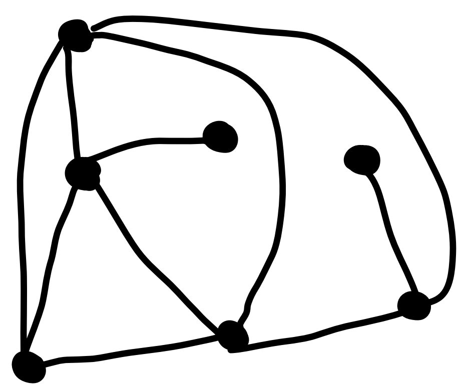
3, 2, 2, 2, 1, 1 is graphic

So 4, 4, 3, 3, 3, 1, 1 is graphic

Since $\cancel{4}, 4, 3, 3, 3, 1, 1$

$\quad -1 \; -1 \; -1 \; -1$

$\quad 3, 2, 2, 2, 1, 1$



Pf: