Math 213 Week 1 Notes

~~MWF 12-12:50~~

~~Wymer 1010~~

Lecture 1:

~~Syllabus, HW policies, exam dates/ Quizzes.~~

Discrete Mathematics: Study of

- Logic
- Sets
- Algorithms
- functions
- Induction
- Probability
- Counting

Discrete: individually separate, distinct. "Opposite of continuous" (studied in calculus)

Fundamental building block:

<u>Def.</u> A proposition is a declarative sentence. If is either True or False (discrete outcome)

<u>E.g.</u>

- Urbana is the capital of Illinois
- Springfield is the capital of illinois
- $1+1=2$

<u>Non-Eg.</u>
- What time is it?
- Read chapter 1 of Discrete math textbook.

<u>Def</u>^{n}: A propositional (or sentential) variable is a letter ($p, q, r, s, \ldots$ etc) used to denote a proposition.

<u>Looking Forward</u>: <u>Sets</u> are defined in terms of propositions. Sets underly all discrete math (+ more) so we need to study operations on propositions and their effects on truth values.

Defⁿ: Let p be a proposition. The negation of p, $\neg p$, is the statement

"it is not the case that p".

The proposition $\neg p$ has the opposite truth value as p.

E.g. Let p be "It is raining outside"

$\neg p$ is "it is not the case that it is raining outside"

or equivalently: "it is not raining outside"

Just like negation, we can form new propositions (5)
from old ones using the logical connectives
"and" and "or".

Def: Let p, q be propositions. The conjunction of p and q, denoted $p \wedge q$, is the proposition "p and q". $p \wedge q$ is true when both p and q are true and false otherwise.

E.g. p: "it is raining outside"

q: "There's a snake on the grass"

$p \wedge q$: "it is raining outside and there is a snake in the grass"

is true when and only when p and q are both true.

Def. The disjunction of p, q, denoted $p \vee q$, is the proposition "p or q". The proposition $p \vee q$ is false when both p and q are false and true otherwise.

Eg. $p \vee q$ "it is raining outside or there is a snake in the grass"

is ~~false~~ false when and only when it is not raining and there is not a snake in the grass.

Truth tables for conjunction, disjunction

- List all possible truth value combinations of the component propositions.
- Combine using the above rules.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

End of Lecture 1

Lecture 2:

Def^n: The conditional statement, $p \Rightarrow q$, is the proposition "if p then q". This statement is false when p is true and q is false and true otherwise.

p is called antecedent or hypothesis

q is called conclusion or consequent.

- If it is raining outside, then there is a snake in the grass.

Note: this is true if it is not raining outside, ~~and~~ We say the implication is vacuously true.

E.g. $P(x)$ "x is greater than 5"

$P(4)$ is False

$P(5)$ is False

$P(6)$ is true.

Once we substitute a value for the variable, a prop. function becomes a proposition with a truth value.

Want: A way to say that $P(x)$ is true for all values of x (of a certain type) or for at least one value of x.

These are accomplished by using Quantifiers

Ex: "For every real number x, $x^2 > 0$."

Let $P(x)$ denote the prop. function "$x^2>0$".

So the above is

"For every real number x, $P(x)$".

This is true when $P(x)$ is true for every real number.

Def$^{\underline{n}}$: The <u>universal quantification</u> of $P(x)$ is the statement "$P(x)$ for all values of x in the domain"

denoted $\forall x, P(x)$.

$\forall x\, P(x)$ is true when $P(x)$ is true for every x.

is false when there is <u>an</u> x for which $P(x)$ is false.

So to show a universally quantified statement is false, it is enough to find one counterexample.

Existential quantifier:

Let $P(x)$ be a propositional function.

The existential quantification of $P(x)$ is the statement " There exists an element x in a specified domain so that $P(x)$".

This statement is true if there is at least one x in the domain so that $P(x)$ is true.

This statement is false if $P(x)$ is false for every x in the domain. Denoted $\exists x\, P(x)$.

e.g. " There is a real number x so that $x > 4$" T

" There is a real number x so that $x = x + 1$" F.

Recall:

- Propositions
- Truth values
- logical connectives:
  - not
  - and
  - or

Truth tables: Recipies for determining truth values of compound propositions made out of logical connectives.

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$	$p \vee q$	$\neg(p \wedge q)$	$\neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	F	T	T

[redacted]: Sets

Def$^{\underline{n}}$: A set is an unordered collection of objects, called elements, or members of the set. A set is said to contain its elements. We write

$a \in A$ to indicate that a is an element of the set A.

E.g.: $A =$ The set of all pos. whole numbers less than 6 divisible by 2.

$A = \{0, 2, 4\}$

$A =$ positive even whole numbers less than 6.

Lecture 3

Recall: A set is an <u>unordered</u> collection of <u>distinct</u> objects, called elements or members.

If A is a set, we write $a \in A$ to indicate a is an element of the set A.

<u>Multiple ways to describe sets</u>:

(<u>Roster</u> Notation) 1. List all elements btwn curly braces

e.g. $A = \{a, b, c, d\} = \{d, a, b, c\} = \{d, d, a, a, b, c\}$

unordered distinct.

$$V = \{a, e, i, o, u\}$$

2. Set builder notation: Characterize all elements of a set by stating a property or properties of the members.

$\{x \mid x \text{ has property } P\}$.

e.g. $A = \{x \mid x \text{ is an odd positive integer less than } 10\}$

$E = \{x \mid x \text{ is a vowel}\}$.

$B = \{x \mid x \text{ is a real number and } 1 \leq x < 2\}$.

Special Sets:

$\mathbb{N} = \{0, 1, 2, \ldots\}$ set of natural numbers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ set of integers

$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ set of all positive integers

listing their elements.

$\mathbb{Q} = \{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z} \text{ and } q \neq 0\}$.

$E = \{x \in \mathbb{N} \mid x = 2k \text{ for some } k \in \mathbb{Z}\} = \{x \in \mathbb{N} \mid \exists k : P(x)\}$

$O = \{x \in \mathbb{N} \mid x = 2k+1 \text{ for some } k \in \mathbb{Z}\}$.

$\mathbb{R} =$ the set of real numbers (continuum)

Intervals (subsets of $\mathbb{R}$)

Let $a, b \in \mathbb{R}$ with $a<b$.

Then define $[a,b]$, $(a,b]$, $[a,b)$, (a,b).

Similar to logical equivalence of propositions, we have

Def^n: Two sets A, B are equal if they have the same elements.

$$\forall x \left((x \in A \Rightarrow x \in B) \text{ and } (x \in B \Rightarrow x \in A) \right)$$

We write $A=B$ to mean two sets are equal.

Note: Order of elements in a set doesn't matter

$$\{1,3,7\} = \{7,1,3\}.$$

Further, repetitions are ignored: $\{1,1,1,1,3\} = \{1,3\}$.

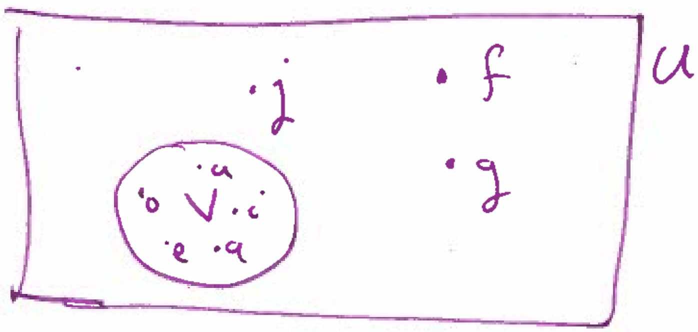
Venn Diagrams + representing sets

A venn diagram is a picture of a set.

E.g. Let $\mathcal{U}$ = 26 letters of English alphabet

V = Vowels = $\{a, e, i, o, u\}$

$V \subseteq \mathcal{U}$,



Note: the bubble for V is completely inside the bubble for $\mathcal{U}$.

Def^n: A set A is a subset of a set B if

$\forall x \in A,\ x \in B$ aka if $x \in A$ then $x \in B$.

(in words every element of A is an element of B)

We write $A \subseteq B$.

Note: To show that A is a subset of B we need to show that the proposition

$$\forall x\,(x \in A \Rightarrow x \in B)$$ is true.

To show A is not a subset of B we need to only find one element of A that is not element of B.

Facts:

1. $\emptyset \subseteq S$ for any set S.
2. $S \subseteq S$ for any set S.

Try to verify 1. (recall truth table for a conditional)

The Empty set:

Def^n The empty set is a set w/ no elements.

Denoted $\emptyset = \{\ \}$.

Note: $\emptyset$ and $\{\emptyset\}$ are <u>different</u> <u>sets</u>

one has no elements, the other has one element (the empty set itself).

Operations on Sets:

$$P(A) = \{B \mid B \subseteq A\}$$

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}.$$

Examples: $A = \{a, b, c\}$ $B = \{b, c, d\}$. $C = \{e, f, g\}$

1. $A \cap B = \{b, c\}$

2. $A \cup B = \{a, b, c, d\}$

3. $A \cap C = \emptyset$

4. $A \cup C = \{a, b, c, e, f, g\}$

5. $$A \times B = \{(a, b), (a, c), (a, d), (b, b), (b, c), (b, d), (c, b), (c, c), (c, d)\}.$$

6. $$P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$