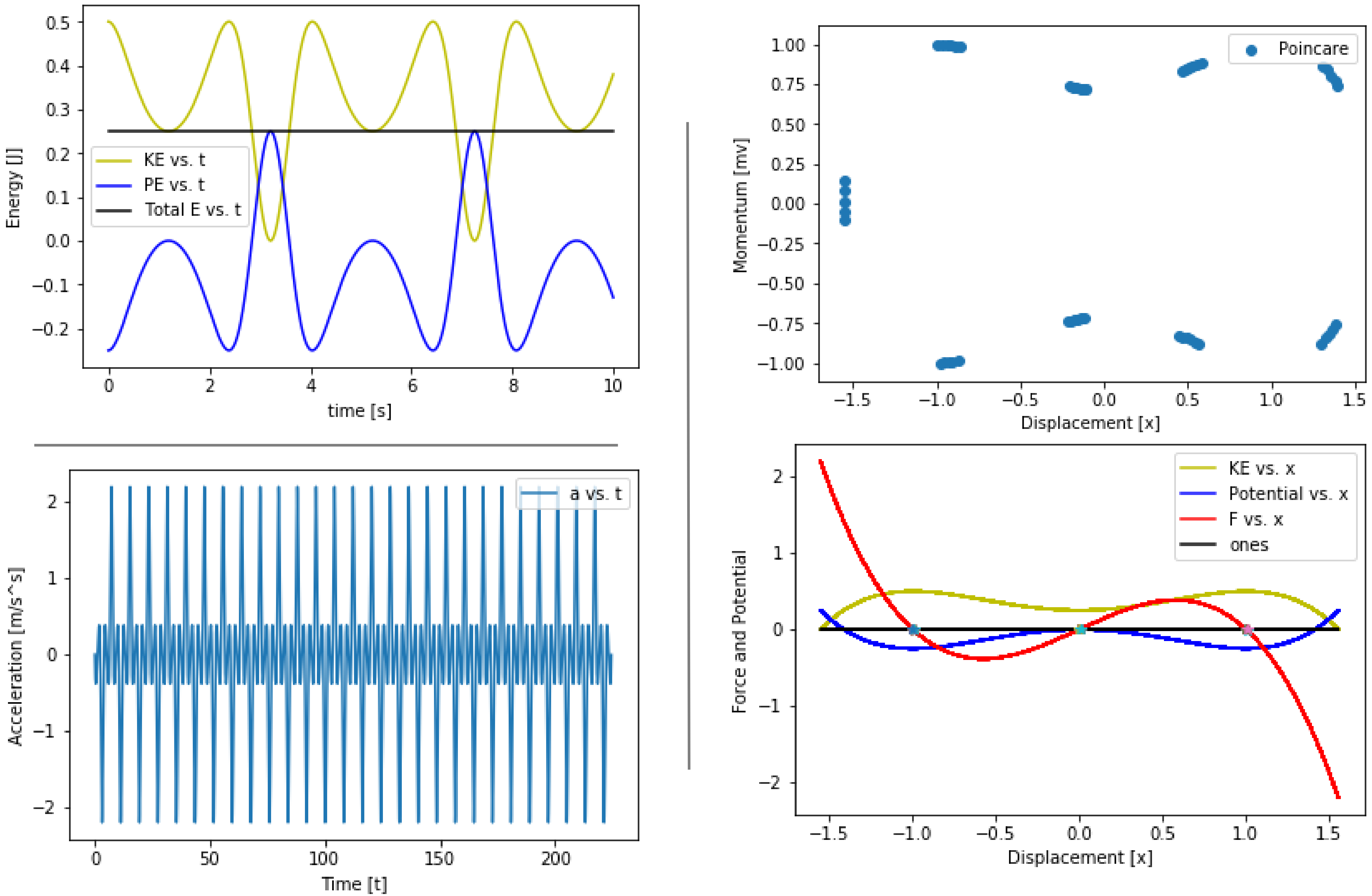
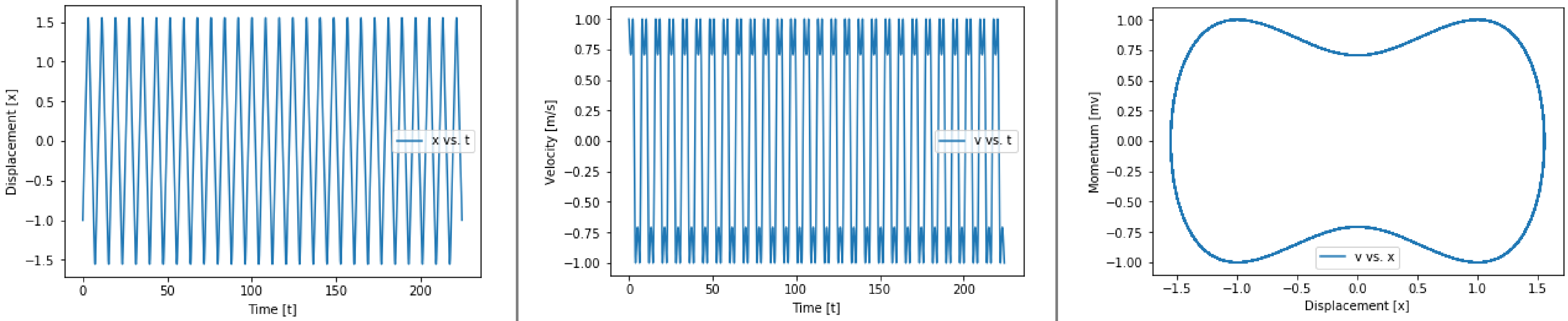
Duffing Oscillator(Equation)

$$\mathrm{m}\frac{d^2x}{dt^2} + \delta\frac{\dot{d}x}{dt} + \alpha x + \beta x^3 = \gamma\cos(\omega t)$$

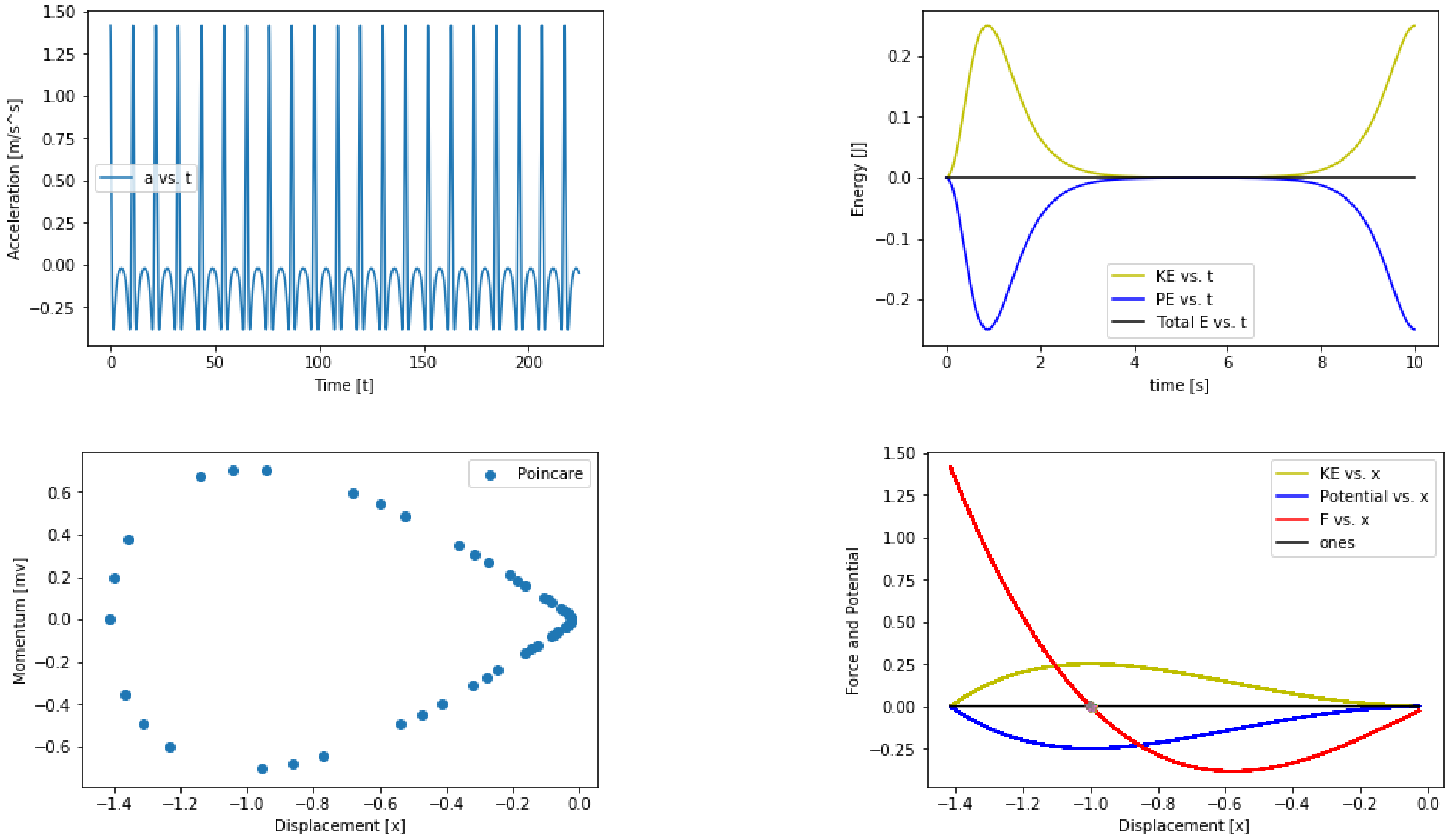
Condition	$\boldsymbol{\delta}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\omega}$	**initial**	
						x	**v**
Case 1. Free Motion (stable equilibrium)	0	-1	1	0	0 (any)	-1	1
Case 2. Free motion (unstable equilibrium)	0	-1	1	0	0 (any)	-1.414	0
Case 3. Damped motion	-0.2	-1	1	0	0 (any)	-1.5	-1.0
Case 4. Damped driven motion	0.1	-1	1	0.1	1.4	0	0
Case 5. Damped driven motion	0.1	-1	1	0.32	1.4	0	0
Case 6. Damped driven motion: (increased driving amplitude)	0.1	-1	1	0.34	1.4	0	0
Case 7. Chaotic Motion *Case 7b.* Poincare Section Chaotic Motion	0.1	-1	1	0.38	1.4	0	0
Case 8. Ratio of gamma to delta causes period doubling, bifurcation	0.05	-1	1	0.34	1.4	0	0
	0.2	-1	1	0.34	1.4	0	0
Case 9. Frequency response curves	-	-	-	-	-	-	-



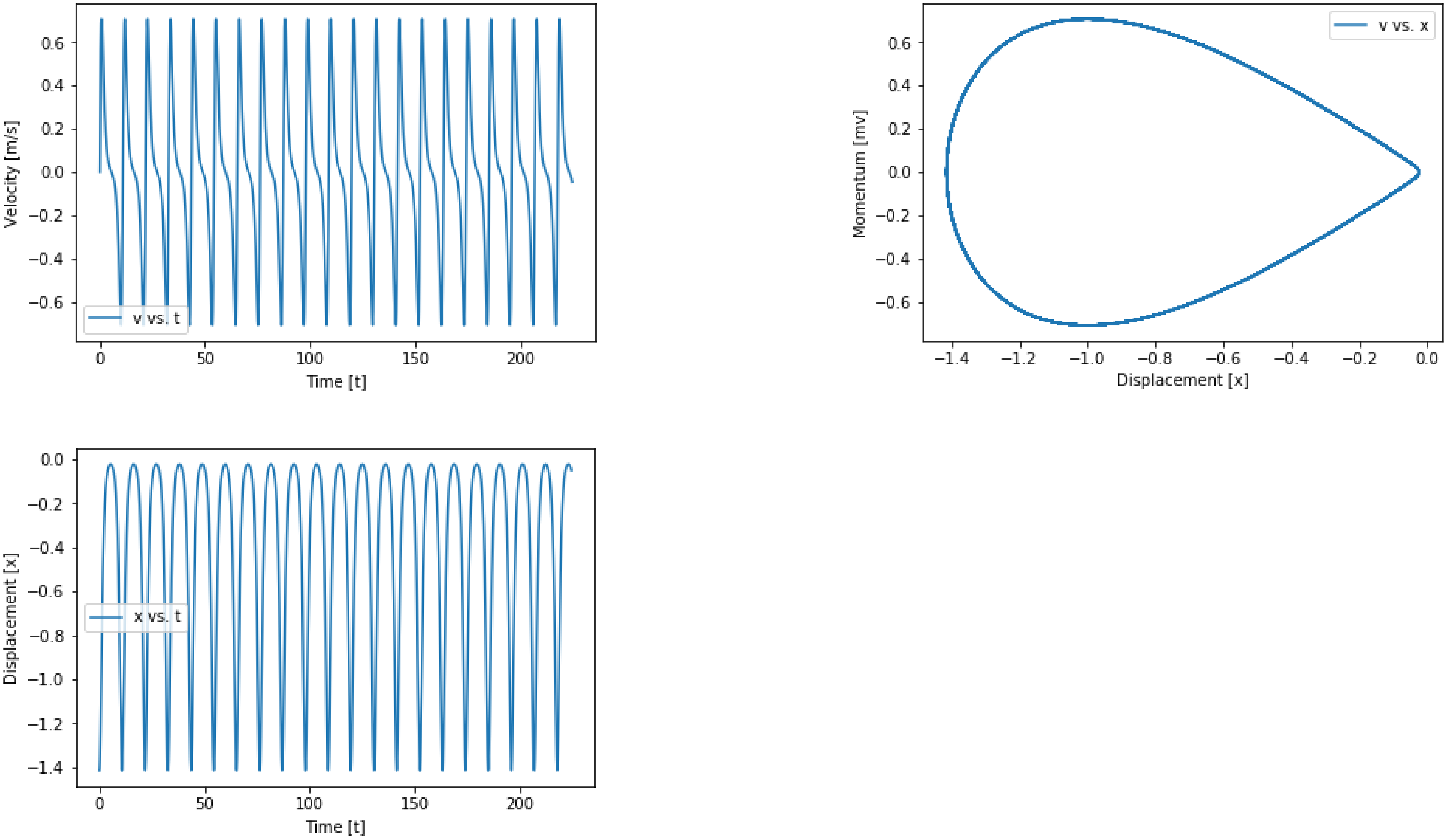
Case 1. Free Motion (stable equilibrium)



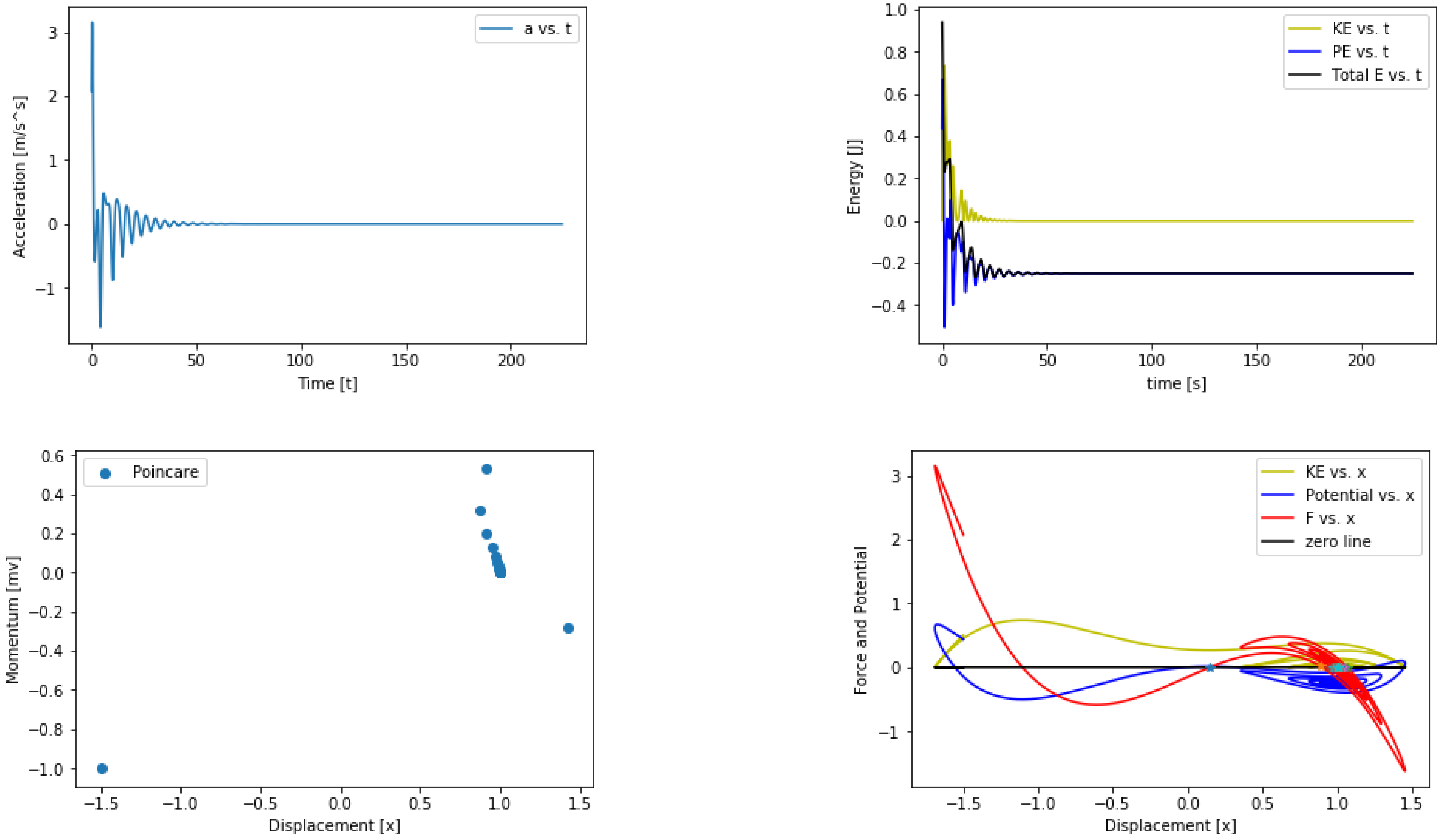
Case 1. Free Motion (stable equilibrium)



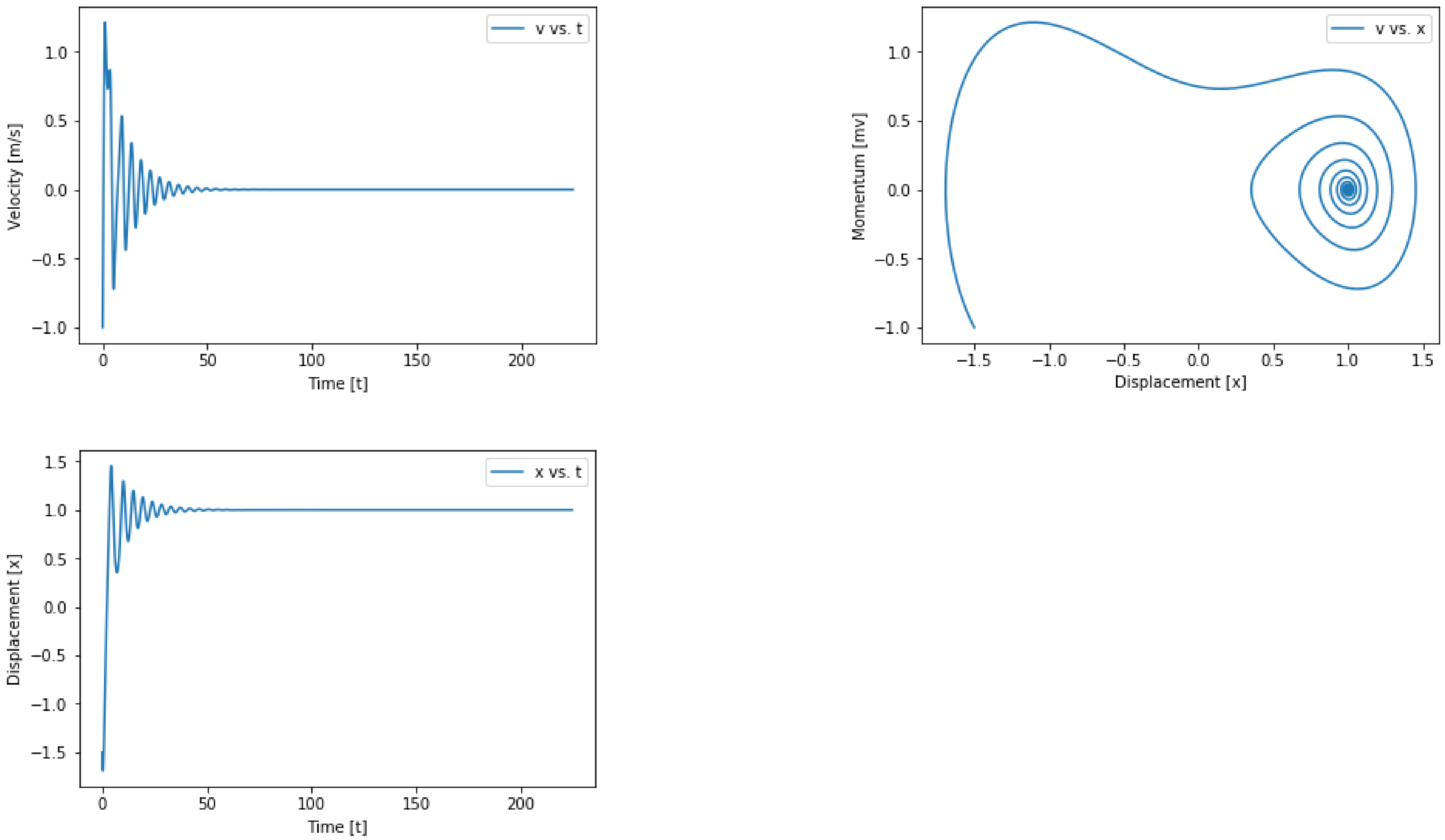
Case 2. Free motion (unstable equilibrium)



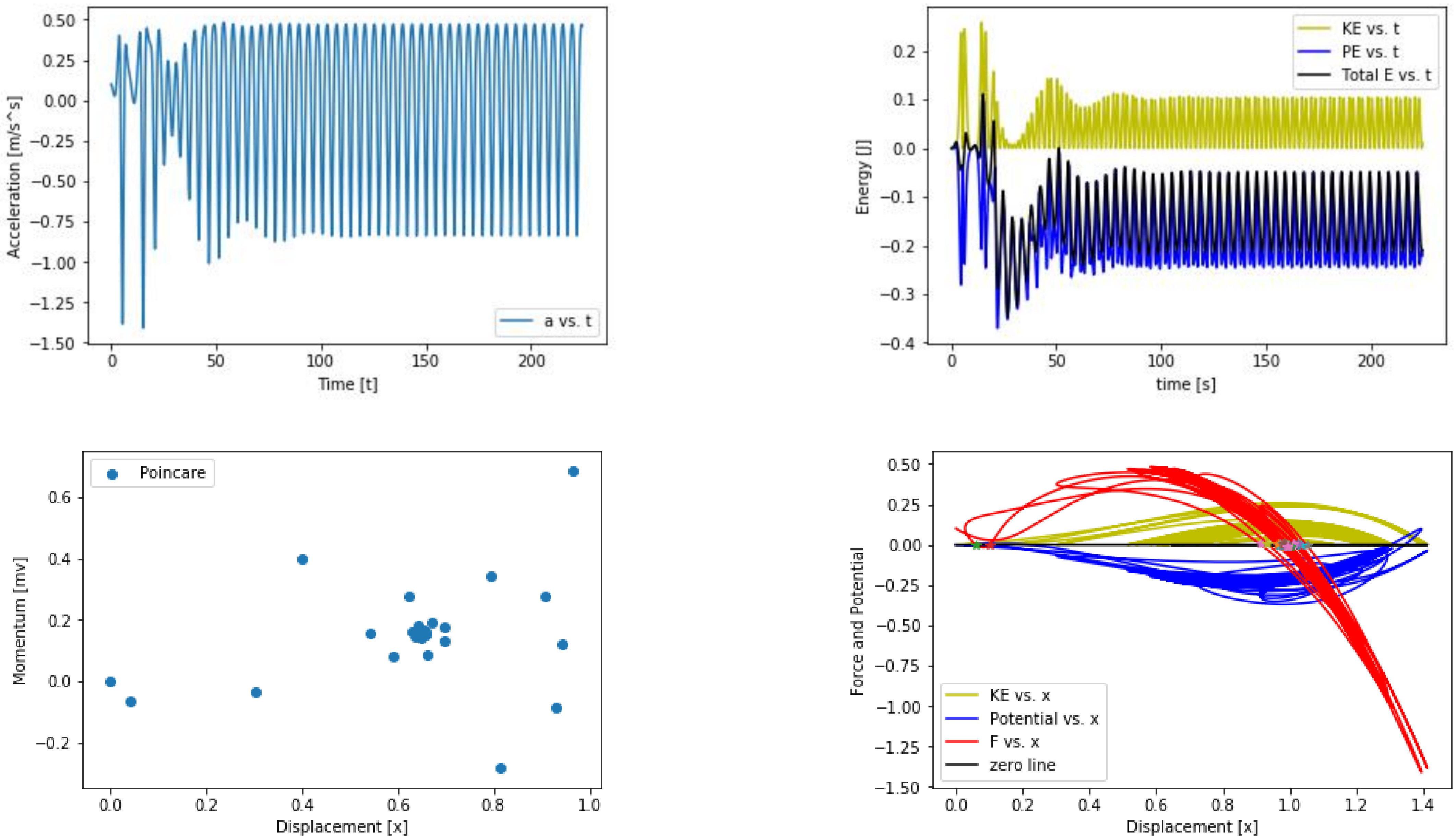
Case 2. Free motion (unstable equilibrium)



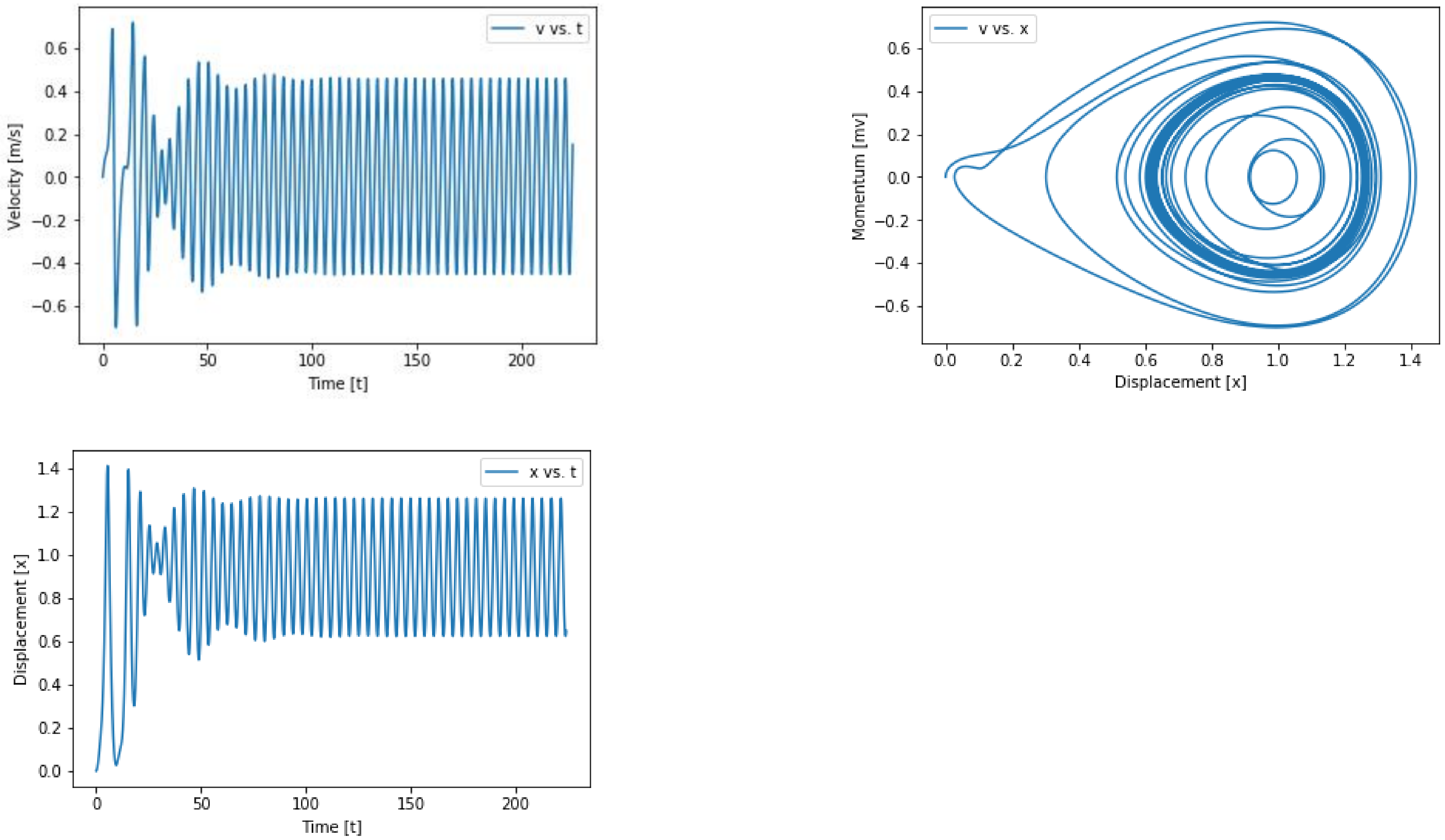
Case 3. Damped motion



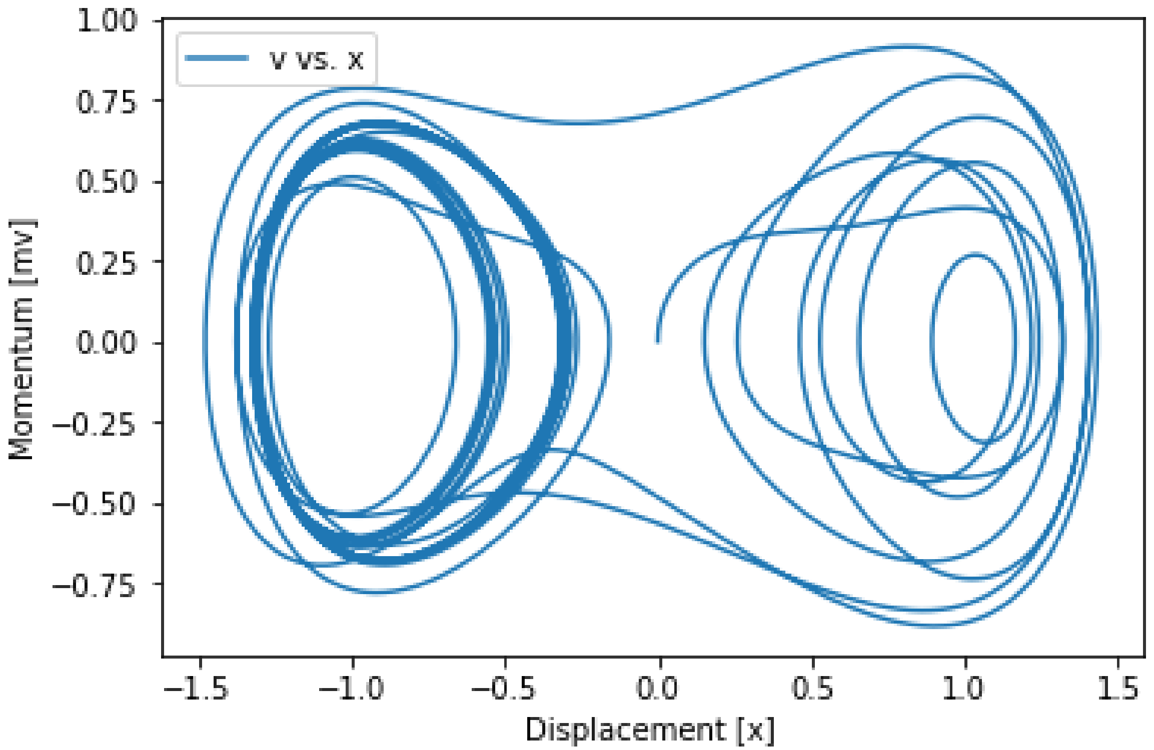
Case 3. Damped motion



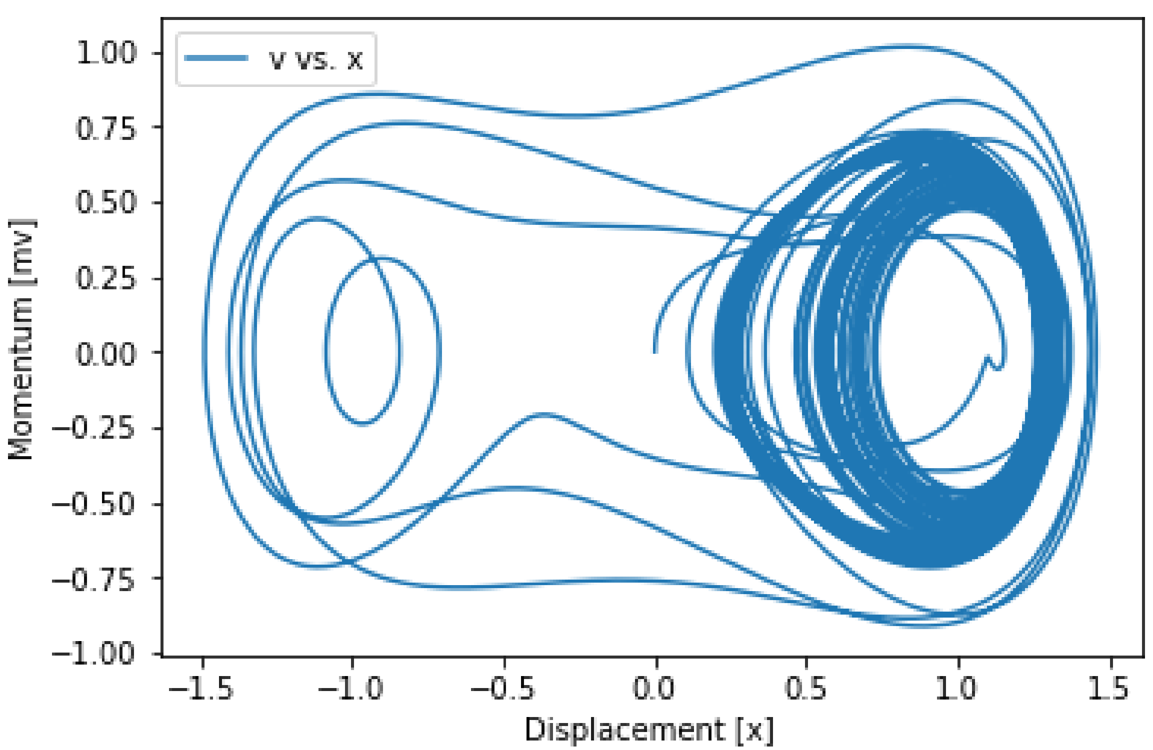
Case 4. Damped driven motion



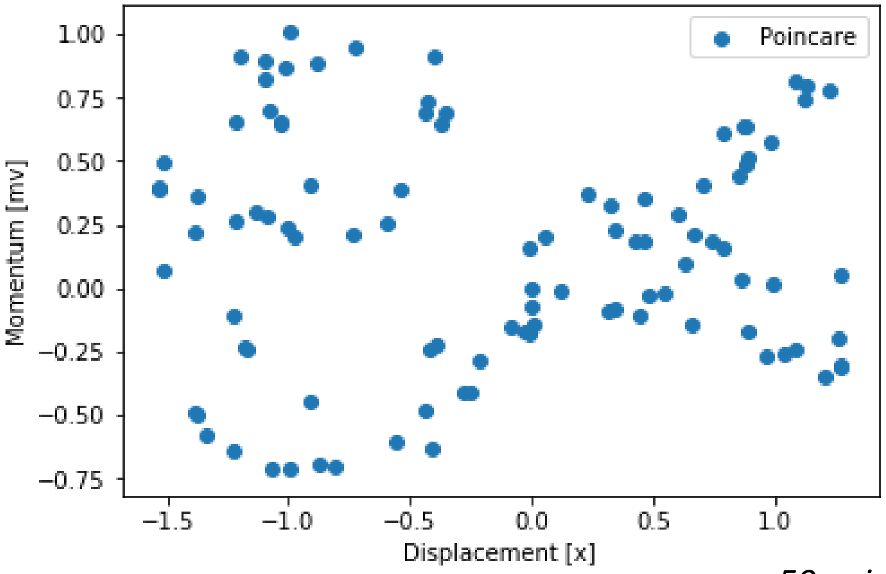
Case 4. Damped driven motion



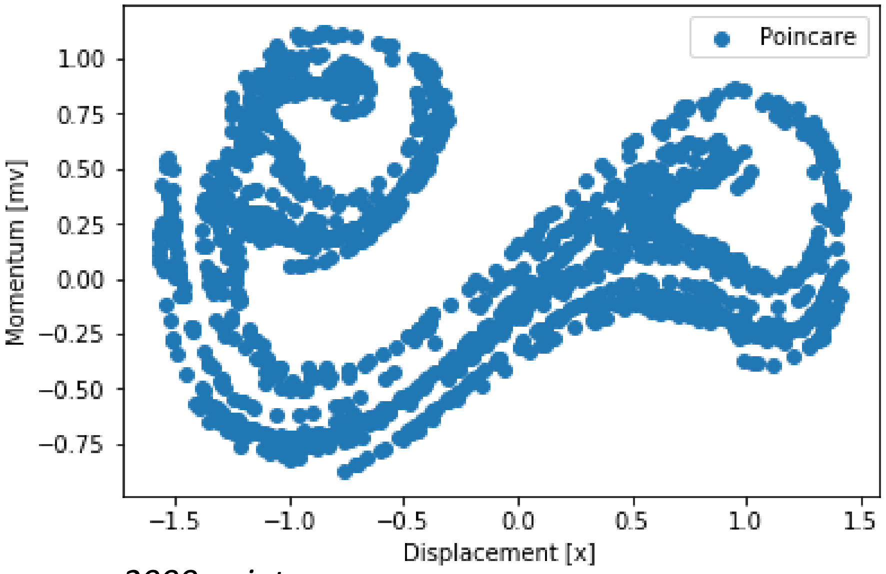
Case 5. Damped driven motion



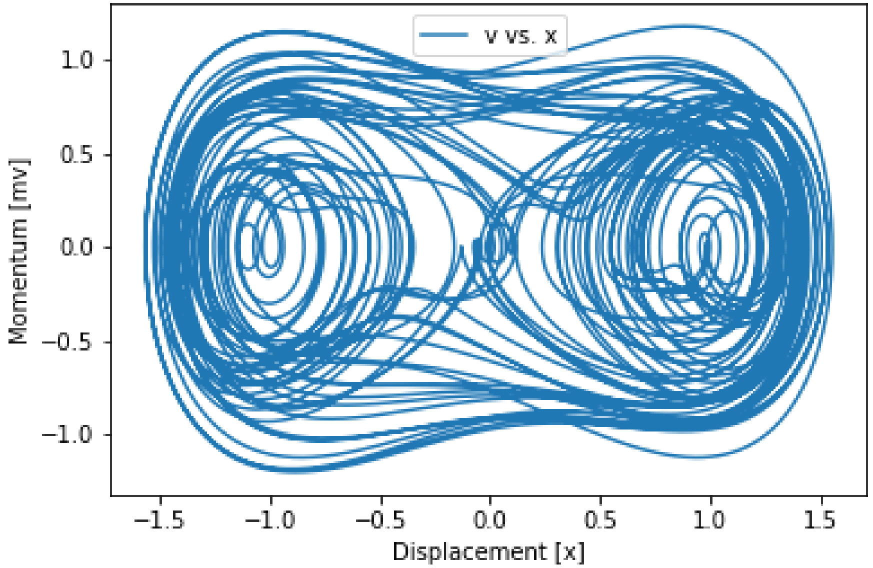
Case 6. Damped driven motion: (increased driving amplitude)



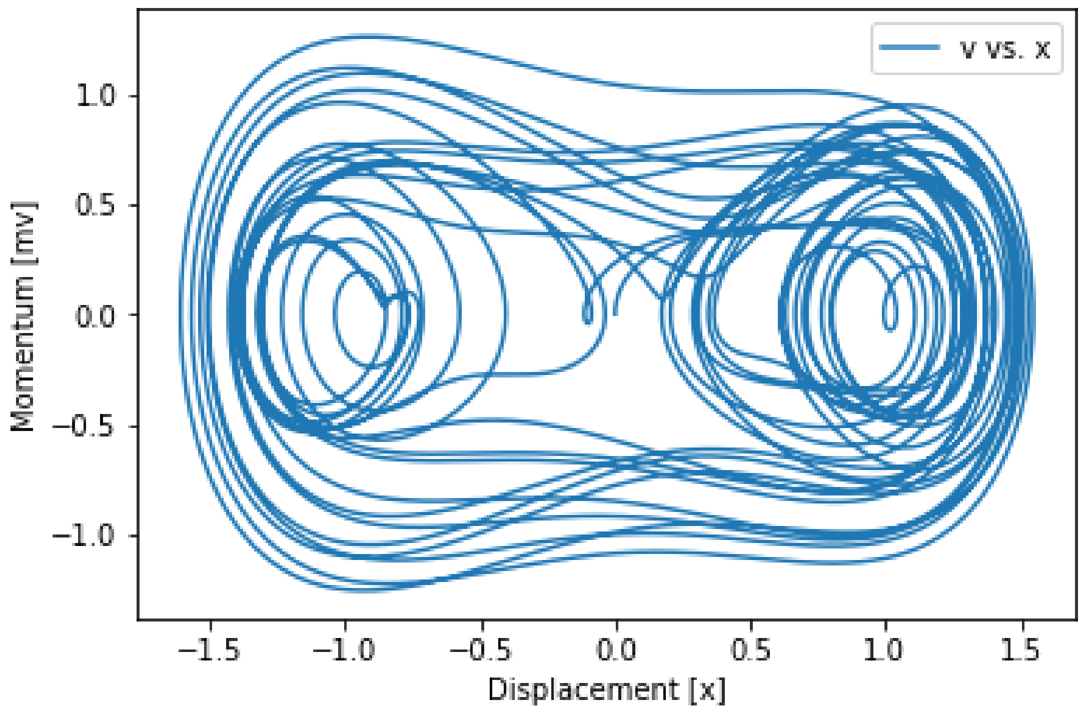
50 points



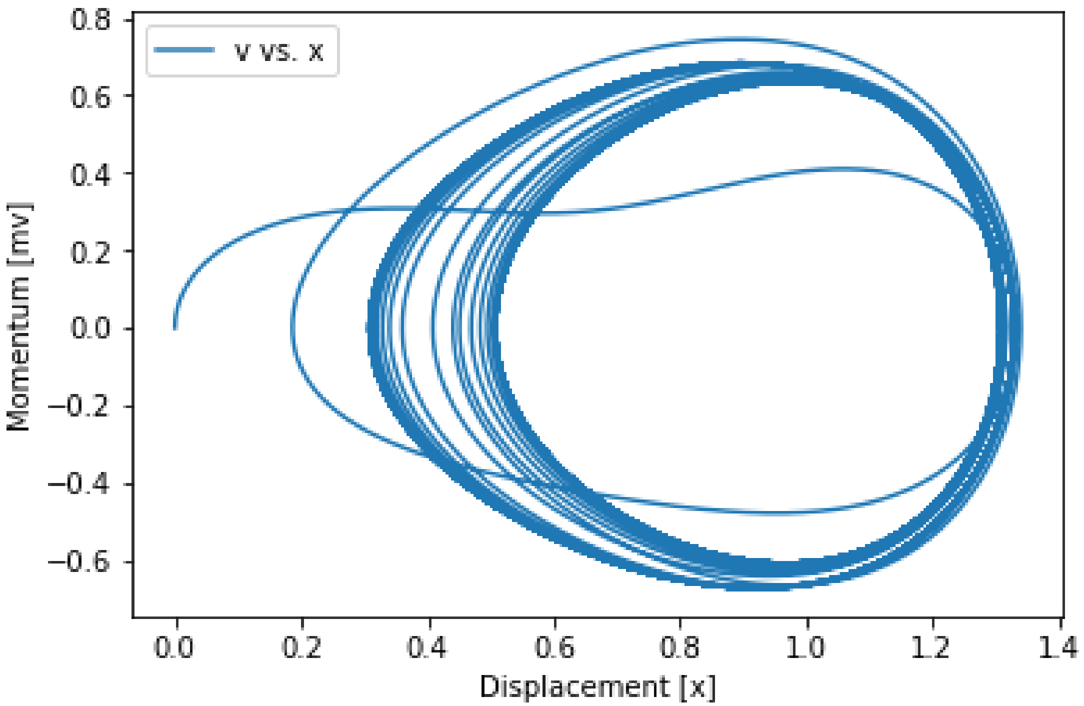
2000 points



Case 7. Chaotic Motion

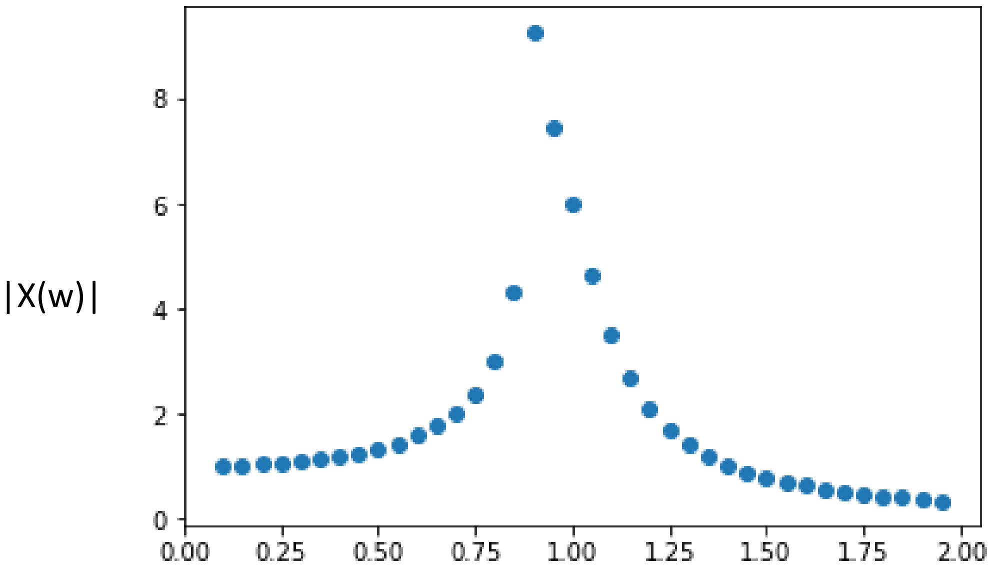


Forcing: 0.34, damping: 0.2

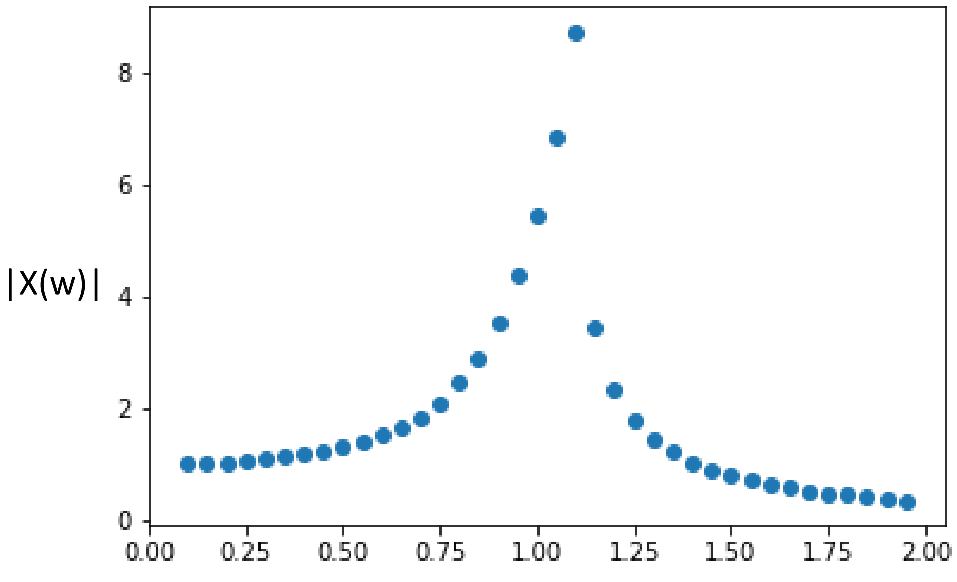


Forcing: 0.34, damping: 0.05

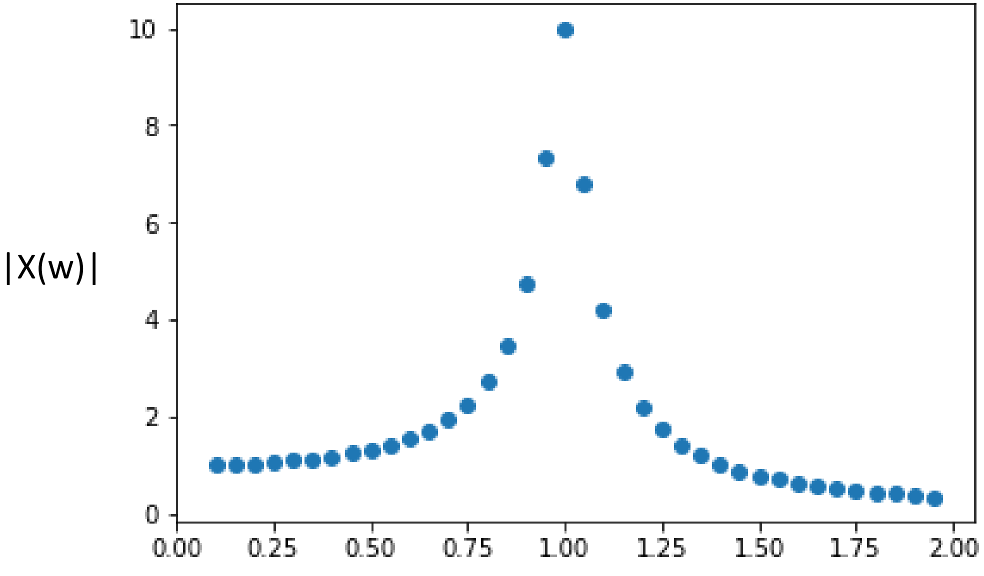
Case 8. Period Doubling (Bifurcation)



Nonlinearity: -0.003

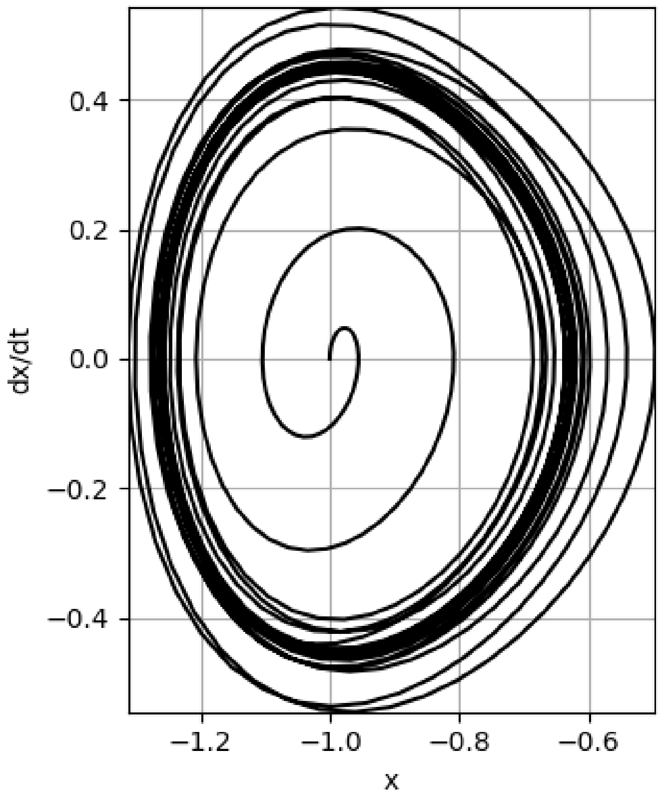


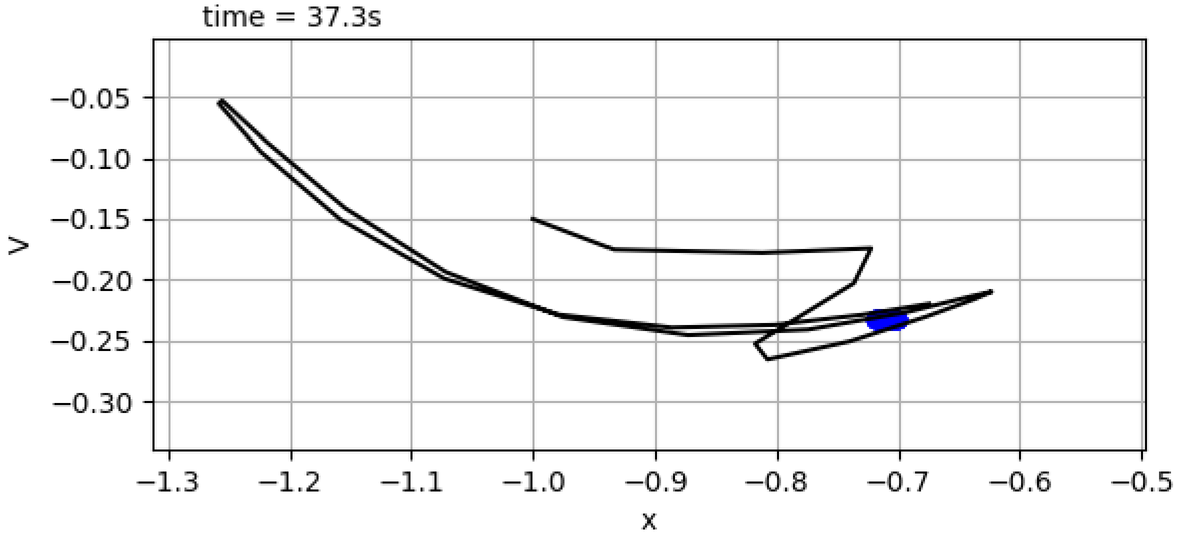
Nonlinearity: 0.004



Nonlinearity: 0

Case 9. Frequency Response





ex) damping: 0.1, forcing: 0.1 / initial x: -1, dx/dt = 0

Error Verification

Step size (h)	Approximation of x at t = 0.1, x_{approx}	Final Global Error(F.G.E) at t = 0.1 (x_{approx}- x_{true})	Approximate Error Ratio
0.1	-0.0018921715207243900	-0.0000000076565361900	
0.05	-0.0018921642988212700	-0.0000000004346330700	17.6160921
0.025	-0.0018921638899839600	-0.0000000000257957601	16.84901197
0.0125	-0.0018921638657577500	-0.0000000000015695500	16.43513065
0.0001 (assume true value)	-0.0018921638641882000 (=x_{true})	-	Expected order: 16 (∵ $O(h^{16})$)

- **As the step sizes halves, the error ratio converges to the order of 16, which is expected for a 4th order Runge-Kutta numerical approximation**