Final Project Proposal

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Partners:	Andy Jeong
Course:	Ph235 Simulations
Instructor:	Professor Raja

1 Topic

Purpose

This project aims to demonstrate a nonlinear dynamic system by an application of Duffing Osillator (Equation) through a double-well potential beam setup ("Moon-beam"). This chaotic structural model poses a chaotic behavior for a set of defined parameters and initial conditions, and to approximate the dynamics of the system, a fourth-order Runge-Kutta method will be applied to solve the problem.

Introduction

In order to track and analyze the behavior of a chaotic system such as a randomly stirred fluid, partial differential equations could be used in practice; however, as it can be observed from systems such as the logistic map that the dynamics of a chaotic behavior could be modeled and treated analytically and numerically in a simpler manner, an approach using a single ordinary differential equation - the Duffing Equation - could be employed to route to chaos.

A mechanical example of continuous, nonperiodic, bounded motions in deterministic systems is found in the vibrations of buckled curved plates and beams. ("Moon Beam"). In such nonlinear vibrations of a buckled beam with fixed ends, one can observe both periodic and nonperiodic motions (nonperiodic is also called continuous "intermittent" snap-through under harmonic excitations). The harmonic excitations of this structure exhibits a response similar to strange attractor motions found in a chaotic system.

To characterize the nonperiodic(chaotic) motion of the beam over time, responses over a range of time, amplitude and frequency, including the Poincare map over iterations of periods, will be examined for a number of set of parameters, including the forcing amplitude, driving frequency, and damping.

$$\dot{x} + \delta\dot{x} + \alpha x + \beta x^3 = \gamma cos(\omega t)$$

Equation 1. Duffing Equation

The Duffing equation is described by the above equation, where δ controls damping, α controls linear stiffness, β is the nonlinearity in the restoring force, γ is the amplitude of the driving force, and ω is the angular frequency of the driving force.

Team Roles

(1) *Andy Jeong*

Design a setup for the experiment

Numerically solve the equation using RK4 method

(+ maybe other methods like RKF, ode45) and determine the error (bounds)

Analytically understand and interpret the behavior

Obtain frequency responses and Poincaré sections both analytically and graphically

Ph235 Related Topics

Numerical analysis using fourth-order Runge-Kutta method to solve nonlinear equations (ODE)
Visual representation of time/positional series data and Poincaré sections
Visualization of chaotic behavior with various parameters / animation video

2 Final Demo

Demonstrate linear and nonlinear behaviors, and resulting chaotic behaviors of the system given a set of parameters and initial conditions. Then interpret the the behaviors from the visual representations.

Show and explain different Poincaré section responses aroused from period manipulation (i.e. period doubling) and how chaotic behavior is altered or maintained. Observe bifurcation diagram of the system. Animate each motion through visual packages.

3 Timeline (tentative)

2/6	Initial project proposal
2/13	Feedback on proposal received
3/4	In-class presentation of project proposal
4/10	Working code & animation
5/1	Working draft with documentation
5/8	Final project Demo