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ECE302 Stochastics
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Stochastics – Dungeons and Dragons

- Notation: XdY == sum of all results from rollig a Y-sided die X times
  ex) 3d6: roll a 6-sided die 3 times, and sum the results
  2d10: roll 10-sided die and sum the results
- Assumption: All dice rolls are fair (each outcome is equally likely)

Question 1. Dice Rolls and Character Creation

- Number of ability score for a human can have ranges from 3 to 18 (Average = 9)
- 6 provided ability scores = {strength, dexterity, intelligence, wisdom, constitution, charisma}
- * Fun Method: Generate 3 potential values for each of N ability scores (from 3d6), and take maximum of those three for each ability score. Then the ability scores with those highest potential values become a character. In notation, letting X = outcome of 3d6, outcome of fun method = $\max(X_1, X_2, X_3)$.

1) 3d6 die roll

Let X = ability score (= sum of rolling a 6-sided die 3 times)

Sample space S = {(1,1,1), (1,1,2), (1,1,3), ... , (6,6,5), (6,6,6)}

Sample space of X, Sx = {3,4,5, ... , 16, 17, 18}

Pmf of X:

X	Combinations ((a,b,c) represented as abc)						# Cases	Pr[X] [%]
3	111						1	0.46
4	112						3	1.39
5	113	221					6	2.78
6	114	123	222				10	4.63
7	115	223	331	124			15	6.94
8	116	233	431	125	224		21	9.72
9	621	432	333	225	135	144	25	11.57
10	631	622	532	442	433	145	27	12.50
11	641	155	542	335	434	632	27	12.50
12	651	435	444	525	642	633	25	11.57
13	661	544	346	652	553		21	9.72
14	662	554	446	653			15	6.94
15	663	654	555				10	4.63
16	664	556					6	2.78
17	665						3	1.39
18	666						1	0.46
						Total	**216**	**100 %**

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a) Probability that the sum of rolling three 6-sided dice once is 18?

Let X = sum of die rolls from rolling 3 6-sided dice once

actual sum from the experiment = 18

Assuming equiprobability of each face appearing for each die,

Pr[X = 18] = 1 / 216. The only case for the sum to be 18 is (6,6,6), which has the probabilities of each of the three dice multiplied (= (1/6)*(1/6)*(1/6)) since rolling a 6 in one die bears 1 chance out of 6 in each outcome sample space of {1,2,3,4,5,6}.

b) Probability that the result of the fun-method is 18?

Pr[max(X_1, X_2, X_3) = 18] = $\sum(\exists\ one\ 18), (\exists\ two\ 18's), (\exists\ three\ 18's)$

= $\binom{3}{1}P(X=18)^1\left(1-P(X=18)\right)^{3-1} + \binom{3}{2}P(X=18)^2\left(1-P(X=18)\right)^{3-2} + \binom{3}{3}P(X=18)^3\left(1-P(X=18)\right)^{3-3}$

= 3 * (1/216)^1*(215/216)^2 + 3 * (1/216)^2 *(215/216)^1 + (1/216)^3

= 1.38 %

c) Probability that the character has 18's in all 6 scores, using fun method?

Since rolling for each ability score is independent,

Pr[most perfect form of human] = (Pr[max(X_1, X_2, X_3) = 18])^6

≒ 6.79*10^{-12} = 0

d) Probability that the character has all 9's in all 6 scores, using fun method?

Similar to (b),

(Pr[max(X_1, X_2, X_3) = 9])^6 = ($\sum(\exists\ one\ 18), (\exists\ two\ 18's), (\exists\ three\ 18's)$)^6

= ($\binom{3}{1}P(X=9)^1\left(1-P(X=9)\right)^{3-1} + \binom{3}{2}P(X=9)^2\left(1-P(X=9)\right)^{3-2} + \binom{3}{3}P(X=9)^3\left(1-P(X=9)\right)^{3-3}$)^6

= (3*(25/216)^1*(191/216)^2 + 3*(25/216)^2*(191/216)^1 + (25/216)^3) ^{6}

≒ 6.83*10^{-4} = 0

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Question 2. Wizards, Trolls and Warriors

- Trolls(=A) have hit points of 1d4 = X
- Keene(=B) can damage 2d2 points with FIREBALL spell = Y

1) Sample space for X: S_x = {1,2,3,4} with each probability p_x = ¼
   Sample space for Y: S_y = { 2,3,4 } with probability p_y = { ¼, ½, ¼ }

a) E[X] = $\sum x * p(x) = 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{4}\right) = 2.5\ hit\ points$
   E[Y] = $\sum y * p(y) = 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{2}\right) + 4\left(\frac{1}{4}\right) = 3\ hit\ points$
   Pr[Y > 3] = Pr[Y = 4] = ¼

b) pmf(X ϵ S_x): P[X=1] = ¼, P[X=2] = ¼, P[X=3] = ¼, P[X=4] = ¼
   pmf(Y ϵ S_y): P[Y=2] = ¼, P[Y=3] = ½, P[Y=4] = ¼

e) Sword ⇒ 2d6 to hit
   Hammer(only if successful hit with Sword) ⇒ 1d4 to hit
   Let X' = outcome of 1d20

	Second Die Outcome						
First Die Outcome		**1**	**2**	**3**	**4**	**5**	**6**
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Sword Damage	P[Sword]
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

For Sword:

P[Sword hit] = P[X'≥11] = 10/20 = 1/2

E[Sword] = $\sum(sword) * p[sword]$

= 2*(1/36) + 3*(2/36) + 4*(3/36)+5*(4/36)
+ 6*(5/36) + 7*(6/36) + 8*(5/36) = 9*(4/36)
+ 10*(3/36) + 11*(2/36) 12*(1/36)

= 252/36 = 7

For 1d20:

Sample space = {1,2,3, … , 18, 19, 20}

Sample space for X'≥11 = {11, 12, … , 18, 19, 20}

Thus P[X'≥11] = 10/20 = 1/2, assuming equiprobability of events

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For Hammer:

Hammer Damage	P[Hammer]
1	1/4
2	1/4
3	1/4
4	1/4

P[Hammer hit] = P[Sword hit] * P[X'≥11] = $(P[X' \geq 11])^2$ = 1/4

E[Hammer] = $\sum(hammer) * p[hammer]$

= 1 * (1/4) + 2*(1/4) + 3*(1/4) + 4*(1/4) = 2.5

Let D = total damage on Keene by Shedjam with Sword and Hammer

E[D] = $\sum d * p[d]$ = (sword)*p[sword] + (hammer)*p[hammer]

= E[Sword]P[Sword hit] + E[Hammer]P[Hammer hit]

= 7 * 1/2 + 2.5 * 1/4 = 33/8 = 4.125 points

f) Since Roby Tumblesnatch has a remarkably effective shield, which keeps Keene from incurring damage from attacks, Keene cannot be hurt; he can only attack. Thus, even if damage is greater than Keene's hit points, Keene can only win in this case, with D = 0. $P[Y \geq D]_{D=0} = 1 \because Y = any(\{2,3,4\}) \geq D = 0$

Contents


```
% ECE302: Stochastics
clear all; close all; clc;
% Andy Jeong
% Project 1: Dungeons and Dragons
```

Question 1

(a) rolling once, get 18

```
count = 0; success = 0;
while (count < 100000)
    num1 = randi(6,1);
    num2 = randi(6,1);
    num3 = randi(6,1);
    sumNum = num1 + num2 + num3;
    if (sumNum == 18)
        success = success + 1;
    end
    count = count + 1;
end
AbilityScore18 = success/count;
sprintf('Ability Score prob: %.5f',AbilityScore18)
```

```
ans =

    'Ability Score prob: 0.00459'
```

(b) 18, fun method

```
count = 0; success = 0;
while (count < 100000)
    for i = 1:3
        num1 = randi(6,1); num2 = randi(6,1); num3 = randi(6,1);
        sumNum(i) = num1 + num2 + num3;
    end
```

```
    if (max(sumNum) == 18)
        success = success + 1;
    end
    count = count + 1;
end
AbilityScore18fun = success/count;
sprintf('Ability Score fun method prob: %.5f',AbilityScore18fun)
```

```
ans =

    'Ability Score fun method prob: 0.01418'
```

(c) all 18, fun method

```
count = 0; success = 0;
while (count < 100000)
    for i = 1:6
        num1 = randi(6,1); num2 = randi(6,1); num3 = randi(6,1);
        sumNum(i) = num1 + num2 + num3;
    end
    if all(sumNum == 18)
        success = success + 1;
    end
    count = count + 1;
end
AbilityScoreAll18fun = success/count;
sprintf('Ability Score all 18 fun method prob: %.5f',AbilityScoreAll18fun)
```

```
ans =

    'Ability Score all 18 fun method prob: 0.00000'
```

(d) all 9, fun method

```
count = 0; success = 0;
while (count < 100000)
    for i = 1:6
        num1 = randi(6,1); num2 = randi(6,1); num3 = randi(6,1);
        sumNum(i) = randi(6,1)+randi(6,1)+randi(6,1);
    end
    if all(sumNum == 9)
        success = success + 1;
    end
    count = count + 1;
end
AbilityScoreAll9fun = success/count;
sprintf('Ability Score all 9 fun method prob: %.5f',AbilityScoreAll9fun)
```

```
ans =

    'Ability Score all 9 fun method prob: 0.00001'
```

Question 2

(a) Expected HP, Damage; Damage > 3

```
% Trolls
count = 0;
one = 0; two = 0; three = 0; four = 0;
while (count < 100000)
    num = randi(4,1);
    if (num == 1)
        one = one + 1;
    end
    if (num == 2)
        two = two + 1;
    end
    if (num == 3)
        three = three + 1;
    end
    if (num == 4)
        four = four + 1;
    end
    count = count + 1;
end
ptroll = [one/count two/count three/count four/count]
% Keene
count = 0;
two = 0; three = 0; four = 0;
while (count < 100000)
    num = randi(2,1) + randi(2,1);
    if (num == 2)
        two = two + 1;
    end
    if (num == 3)
        three = three + 1;
    end
    if (num == 4)
        four = four + 1;
    end
    count = count + 1;
end
pkeene = [two/count three/count four/count]
troll = [1,2,3,4];
keene = [2,3,4];
Etroll = sum(troll.*ptroll)
Ekeene = sum(keene.*pkeene)
KeeneBoundMoreThanThree = Ekeene/keene(keene>3);
sprintf('Pr[FIREBALL more than three point damage] bound <= %.3f', KeeneBoundMoreThanThree
)
```

```
ptroll =

    0.2497    0.2495    0.2509    0.2499

pkeene =

    0.2488    0.5021    0.2492
```

```
Etroll =

    2.5011


Ekeene =

    3.0004


ans =

    'Pr[FIREBALL more than three point damage] bound <= 0.750'
```

(b) pmf of HP, Damage

simulated from (a)

```
ptroll % ptroll = [1/4 1/4 1/4 1/4];
pkeene %pkeene = [1/4 1/2 1/4];
```

```
ptroll =

    0.2497    0.2495    0.2509    0.2499


pkeene =

    0.2488    0.5021    0.2492
```

(c) Probability that Keene slays all the trolls

```
count = 0; allSlayed = 0;
while count < 100000
    keenept = randi(2,1) + randi(2,1);
    trollpt = [randi(4,1), randi(4,1), randi(4,1), randi(4,1), randi(4,1), randi(4,1)];
    afterDamage = trollpt - keenept;
    if all(afterDamage <= 0)
        allSlayed = allSlayed + 1;
    end
    count = count + 1;
end
All6Slayed = allSlayed/count;
sprintf('All 6 Slayed prob: %.5f',All6Slayed)
```

```
ans =

    'All 6 Slayed prob: 0.34395'
```

(d) Expected HP of the remaining troll

check 5 are 0, and then check if there is nonzero term

```
count = 0; remainingHP = 0; countRemainingHP = 0;
while count < 100000
    keenept = randi(2,1) + randi(2,1);
    trollpt = [randi(4,1), randi(4,1), randi(4,1), randi(4,1), randi(4,1), randi(4,1)];
    afterDamage = trollpt - keenept;
    indexNegZero = find(afterDamage<=0);
    afterDamage(indexNegZero) = 0;
    if length(find(afterDamage>0)) == 1
        indexNonZero = find(afterDamage>0);
        remainingHP = remainingHP + afterDamage(indexNonZero);
        countRemainingHP = countRemainingHP + 1;
    end
    count = count + 1;
end
expectedHP = sum(remainingHP)/countRemainingHP;
sprintf('Expected HP of the last troll: %.5f',expectedHP)
```

```
ans =

    'Expected HP of the last troll: 1.05882'
```

(e) Expected Damage

```
count = 0; i = 0;
damage = 0;
while count < 100000
    sword = randi(6,1)+randi(6,1);
    hammer = randi(4,1);
    roll1 = randi(20,1,20);
    probSword = length(find(roll1>=11))/length(roll1);
    roll2 = randi(20,1,20);
    probHammer = probSword * length(find(roll2>=11))/length(roll2);
    damage(end+1) = sword*probSword + hammer*probHammer;
    i = i + 1;
    count = count + 1;
end
expectDamage = sum(damage)/(length(damage)-1);
sprintf('Expected Damage: %.5f',expectDamage)
```

```
ans =

    'Expected Damage: 4.12132'
```

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