MIMO-OFDM Channel with Equalizers

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***Abstract*—In this simulation task, we explore three equalization schemes in MIMO flat fading channel and OFDM in three and two different ways, respectively. In addition, we combine the two techniques to design a MIMO-OFDM channel and explore the equalizer performances in this scheme. Throughout the simulations, we modulate in 16-QAM and observe bit error rates (BER) performances for signal-to-noise ratio (SNR) up to 30. We find that given the specifications for OFDM as defined in IEEE 802.11a, precoding and zero forcing schemes for MIMO yield similar BER performances, while MMSE scheme performs slightly worse for this implementation. Based on the equalization scheme, we assume perfect channel state information at the transmitter (CSIT) or at the receiver (CSIR).**

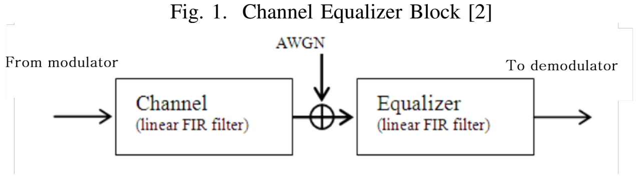
***Index Terms*—Equalization, MIMO-OFDM, CSIT, CSIR**

I. Introduction

In digital communication systems, achieving the target data rate and bit error rate through a transmission channel is crucial for data transfer. In this regard, an equalizer is usually employed at the receiver side in order to reduce the incorrect instances of the transmitted signal. In general, we consider the sequence of bit transmission in the order described in Figure 1. The modulated signal is encoded by the channel, an additive white Gaussian noise (AWGN) is added, and an equalization is applied before demodulating the received signal for recovery. This equalization block serves to minimize the effect of inter-symbol interference (ISI).

In this paper, we study the effects of three equalization schemes – precoding, zero-forcing and MMSE in MIMO links, zero-forcing and MMSE in OFDM models, and all six combinations in MIMO-OFDM channel models. These linear equalizers minimize the error between the received and transmitted symbols without enhancing the noise [1].

Fig. 1. Channel Equalizer Block [2]



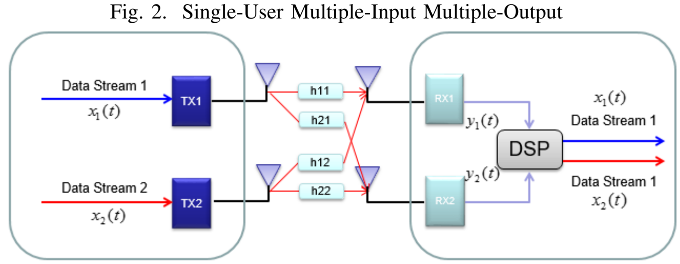
A. MIMO Channel

We model 2×2 MIMO channels with three different flat-fading channels. For $M_{transmit}$ = 2 and $M_{receive}$ = 2, the channel state matrix is represented in dimensions 2×2×symbol size (see Equation 1).

$$H_{MIMO} = \begin{bmatrix} H_{11} & \cdots & H_{1M_T} \\ \vdots & \ddots & \vdots \\ H_{M_R1} & \cdots & H_{M_RM_T} \end{bmatrix} \tag{1}$$

The single-user MIMO channel that we model takes two data streams given two transmit antennas, and encodes (transmits) data through the channel matrix H and multiplexes at the receiver. To the model described in Figure 2, we add an equalizer as seen in Figure 1, in three different schemes described in Section II [3].

Fig. 2. Single-User Multiple-Input Multiple-Output



B. OFDM Encoding

In implementing OFDM symbol coding, we adopt the specifications as describe in the IEEE 802.11a standards. In each OFDM symbol, there are a total of 80 symbols including 64 subcarriers containing 48 data samples to transmit and 12 zeros to reduce inter-channel interference (ICI), and 4 pilot symbols for channel estimation (e.g. frequency and phase offset estimation). We disregard the frequency and phase offset estimation in this simulation (i.e. assume they are perfectly known). The subcarriers are evenly spaced over 20 MHz bandwidth into 312.5 kHz. We also need to prepend 16 sample-long cyclic prefixes, which we take from the data samples in order for linear convolution to become circular convolution. Since the number of subcarriers is 64, we use 64-point IDFT (IFFT) and DFT (FFT) throughout the OFDM symbol generation stage. We transmit the OFDM symbols through a single frequency-selective Rayleigh channel with the following parameters: [4], [5].

1) Symbol time per subchannel (sampling period of channel) = 4 μs
2) Max Doppler frequency shift (Hz) = 0
3) Path delays in each of 4 paths = [0, 1e-5, 3.5e-5, 12e-5]
4) Average path power gains in each path = [0, -1, -1, -3]

C. MIMO-OFDM

We put together the MIMO link and OFDM symbol transmission models into a MIMO-OFDM model. The assumptions for CSIT and CSIR (described in Section II) are valid for this model. We note that in this combined link, the frequency selective MIMO channels become a group of independent flat-fading MIMO channels [6].

II. Equalizer Scheme

We employ three different equalizer approaches in each model we design – precoding, zero-forcing and minimum mean-squared error. All of these schemes are placed at the receiver in order to invert the effect of the channel on the transmitted symbols.

A. Transmit precoding and Receiver Shaping

For channel matrix H, we perform parallel decomposition by SVD, such that

$$H = U\Sigma V^H$$

where we apply linear transformation as follows:

$$x = V\tilde{x}$$

where $\tilde{x}$ = input vector.

After transmitting over a channel H with additive white Gaussian noise n,

$\tilde{y} = U^H(Hx + n) = U^H U\Sigma V^H V\tilde{x} + U^H n = \Sigma\tilde{x} + \tilde{n}$,

where $\tilde{n} = U^H n$

the received data ($\hat{x}$) can be recovered by taking inverse operation:

$$\hat{x} = (\Sigma)^{-1}\tilde{y} = (\Sigma)^{-1}(\Sigma\tilde{x} + \tilde{n}) = \tilde{x} + (\Sigma)^{-1}\tilde{n}$$

B. Zero-forcing

Zero-forcing equalizer approximates the inverse of the channel with a linear filter, by applying W_{ZF} to the received symbols (see Equation 3) [4].

$$W_{ZF} = (H^H H)^{-1} H^H \tag{2}$$

C. Minimum Mean Squared Error (MMSE)

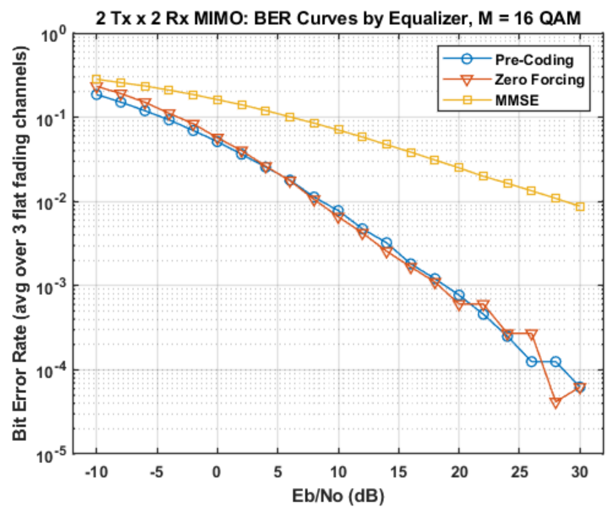
MMSE equalizer designs the filter to minimize the expected value of the squared error signal (filter output minus the transmitted signal). In a similar fashion to zero-forcing, we apply W_{MMSE}, which takes into account the noise component of the channel [4], [7].

$$W_{MMSE} = (H^H H + \sigma_n^2 I)^{-1} H^H \tag{3}$$

III. Results

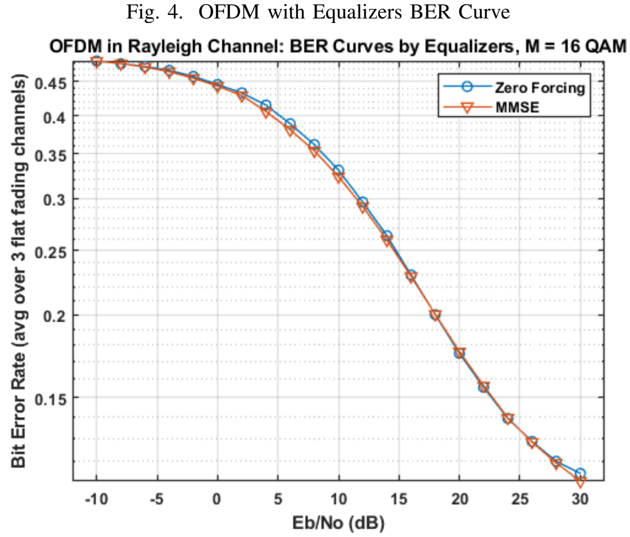
In the MIMO channels with the three equalizers, we observed that the precoding and zero-forcing performed similarly, reaching 10^{-4} at SNR=25, with the latter scheme slightly better. MMSE equalizer, however, did not perform well, with the noise taken into consideration. This could insinuate that distortion due to noise may be severe, or noise compensation in the equalization stage may not have been effective (see Figure 3).

Fig. 3. MIMO with Equalizers BER Curve



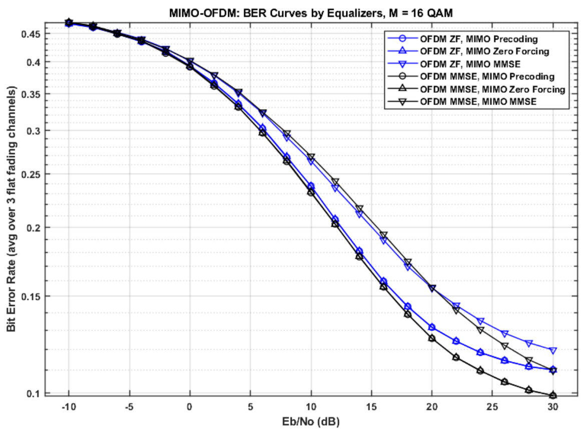
For transmission of OFDM symbols through flat-fading Rayleigh channels, zero-forcing and MMSE equalizers gave similar BER performances. At lower SNR's, the MMSE approach yielded slightly better, but not significantly. We note that due to the specifications of the channel, the general BER level remains in the order of 10^{-1}. With different values for the taps (paths), the order would be much lower.

Fig. 4. OFDM with Equalizers BER Curve



In combination of the two approaches, we noticed that, in general, OFDM with MMSE equalizer performed better than that with zero-forcing. Within each scheme, MIMO precoding and zero forcing were similar, while MMSE slightly worse as seen in the MIMO-only model in Figure 3.

Fig. 5. MIMO-OFDM with Equalizers BER Curve



IV. Conclusion

We simulated the MIMO, OFDM and MIMO-OFDM models with three different equalization schemes and in combination for 16-QAM modulation technique. We observed that the MMSE performed worse than precoding or zero-forcing for MIMO equalization. This may be due to insufficient compensation of the noise in the equalization stage or severe channel noise. For OFDM symbols, zero-forcing and MMSE yielded similar BER performances, but a little better for the latter. In combination, besides the overall order of BER being lower due to the frequency-selective channel parameters defined in the OFDM (as per IEEE 802.11a standard), the performance in BER resulted as seen in each respective models.

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