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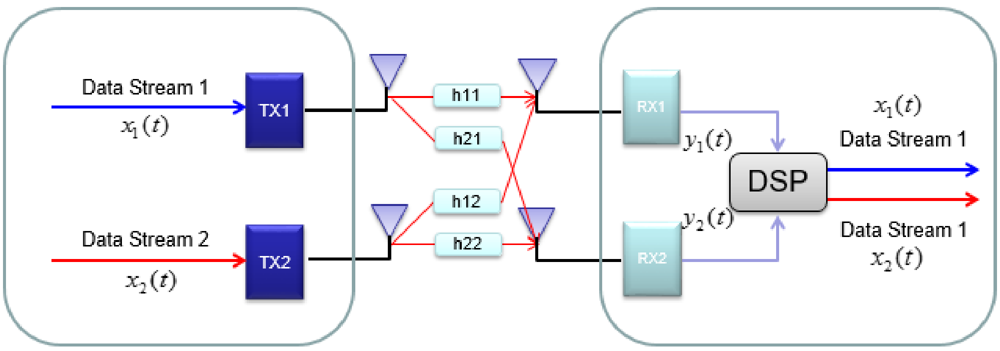
```
% ECE408 - Wireless Communications
% Jongoh (Andy) Jeong
% Project: MIMO, OFDM, MIMO-OFDM
% Date: April 29, 2020
clear all; close all; clc;
warning ('off','all');

% Reference:
%
% [1] Stuber et al. $Broadband MIMO-OFDM Wireless Communications$
```

Part 1: MIMO

model a 2x2 MIMO link with flat fading gains and 3 equalizer schemes: Pre-coding, Zero-forcing and MMSE Pre-coding has CSIT, where Zero-forcing and MMSE has CSIR

Background

Single-User MIMO system with 2 Tx and 2 Rx antennas are described below:



Reference: [2] https://www.sharetechnote.com/html/BasicProcedure_LTE_MIMO.html

Parameter Setup

```
M = 16;           % modulation order
k = log2(M);      % coded bits per symbol
nSyms = 1e3/2;      % number of symbols to send
nBits = nSyms * k;

nChan = 3;       % number of flat fading MIMO channels
EbNo = -10:2:30; % Eb/No
snrVector = EbNo + 10*log10(k); % Es/No before adding noise

% 2 x 2 MIMO channel
Mt = 2;
Mr = 2;
```

```
% initialize
berPreCoding = zeros(nChan, length(snrVector));
berZeroForcing = zeros(nChan, length(snrVector));
berMMSE = zeros(nChan, length(snrVector));
```

Transmit Precoding and Receiver Shaping Scheme

Transmit precoding and Receiver Shaping

For channel matrix H, we perform parallel decomposition by SVD, such that

$$H = U\Sigma V^H$$

where we apply linear transformation as follows:

$$x = V\bar{x},$$

where $\bar{x}$ =input vector

After transmitting over a channel H and adding white noise n

$$\bar{y} = U^H(Hx + n)$$
$$= U^H U\Sigma V^H V\bar{x} + U^H n$$
$$= \Sigma\bar{x} + \bar{n},$$

where $\bar{n} = U^H n$

$\therefore$ The received data $(\hat{x})$ can be recovered by taking inverse operation:

$$\hat{x} = (\Sigma)^{-1}\bar{y} = (\Sigma)^{-1}(\Sigma\bar{x} + \bar{n}) = \bar{x} + (\Sigma)^{-1}\bar{n}$$

Reference: Goldsmith, $Wireless Communications$ [pp. 323-324]

```
% Transmit precoding: x = V*(x_hat)
% Receiver shaping: (y_hat) = (U_hermitian_transposed)*y
U = zeros(Mr, Mt, nBits);
S = zeros(Mr, Mt, nBits);
V = zeros(Mr, Mt, nBits);
prefiltered = zeros(Mt, 1, nBits);
txData = zeros(Mt, 1, nBits);
postfiltered = zeros(Mr, 1, nBits);
rxData = zeros(Mr, 1, nBits);

disp('MIMO precoding');
for i = 1:nChan
    fprintf('Channel: %d\n',i);
    % unique MIMO channel for 'Mr' receive and 'Mt' transmit antennas
    H = ( randn(Mr, Mt, nBits) + 1j*randn(Mr, Mt, nBits) ) / sqrt(2);

    % generate a sequence of random message bits and QAM modulate
    data = randi([0 M-1], Mt, 1, nBits);
    dataMod = qammod(data, M);

    % precode
    for bit = 1:nBits
        % decompose channel matrix H by SVD
        [U(:,:,bit), S(:,:,bit), V(:,:,bit)] = svd(H(:,:,bit));
        % pre-code data for each bit: inverting fading at transmitter (x = V * x_hat)
        prefiltered(:,:,bit) = V(:,:,bit) * dataMod(:,:,bit);
        % send over the fading channel
        txData(:,:,bit) = H(:,:,bit)*prefiltered(:,:,bit);
    end

    fprintf('SNR:\t');
    for j = 1:length(snrVector)
       fprintf('%d\t',j);
       % add white Gaussian noise (x_noisy <-- x + noise)
       % for double-sided white noise, account for 1/2 in power (y_hat = U^(H) * y)
       noise = randn(Mr, 1, nBits) + 1j*randn(Mr, 1, nBits) / sqrt(2);
       txNoisy = txData +  noise * 10^(-snrVector(j)/10/2);
```

```
        for bit = 1:nBits
            % post-code data for each bit: remove fading channel components
            postfiltered(:,:,bit) = U(:,:,bit)' * txNoisy(:,:,bit);
            % recover data
            rxData(:,:,bit) = S(:,:,bit)^-1 * postfiltered(:,:,bit);
        end

        % QAM demodulate and compute bit error rate
        rxData = qamdemod(rxData,M);
        [~,berPreCoding(i,j)] = biterr(data, rxData);
    end
    fprintf('\n');
end
% take average of all 3 fading channels
berPreCoding = mean(berPreCoding);
```

```
MIMO precoding
Channel: 1
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
Channel: 2
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
Channel: 3
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
```

Zero Forcing Scheme

```
txData = zeros(Mt, 1, nBits);
rxData = zeros(Mr, 1, nBits);
W = zeros(Mr, Mt, nBits);

disp('MIMO zero forcing');
for i = 1:nChan
    fprintf('Channel: %d\n',i);
    % unique MIMO channel for 'Mr' receive and 'Mt' transmit antennas
    H = ( randn(Mr, Mt, nBits) + 1j*randn(Mr, Mt, nBits) ) / sqrt(2);

    % generate a sequence of random message bits and QAM modulate
    data = randi([0 M-1], Mt, 1, nBits);
    dataMod = qammod(data, M);

    for bit = 1:nBits
        % send over the fading channel
        txData(:,:,bit) = H(:,:,bit) * dataMod(:,:,bit);
    end

    fprintf('SNR:\t');
    for j = 1:length(snrVector)
       fprintf('%d\t',j);
       % add white Gaussian noise (x_noisy <-- x + noise)
       % for double-sided white noise, account for 1/2 in power (y_hat = U^(H) * y)
       noise = randn(Mr, 1, nBits) + 1j*randn(Mr, 1, nBits) / sqrt(2);
       txNoisy = txData +  noise * 10^(-snrVector(j)/10/2);

       for bit = 1:nBits
           % (1) W_{zf} = H_{Pseudoinverse} = (H^{H} * H)^{-1} * H^{H}
           W(:,:,bit) = (H(:,:,bit)' * H(:,:,bit))^-1 * H(:,:,bit)';
           rxData(:,:,bit) = W(:,:,bit) * txNoisy(:,:,bit);
           % (2) or simply solve linear system H*x = y for x, if full rank
           % rxData(:,:,bit) = H(:,:,bit) \ txNoisy(:,:,bit);
       end

       % QAM demodulate and compute bit error rate
       rxData = qamdemod(rxData,M);
       [~,berZeroForcing(i,j)] = biterr(data, rxData);
    end
    fprintf('\n');
end
% take average of all 3 fading channels
berZeroForcing = mean(berZeroForcing);
```

```
MIMO zero forcing
Channel: 1
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
Channel: 2
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
```

```
        16      17      18      19      20      21
Channel: 3
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
```

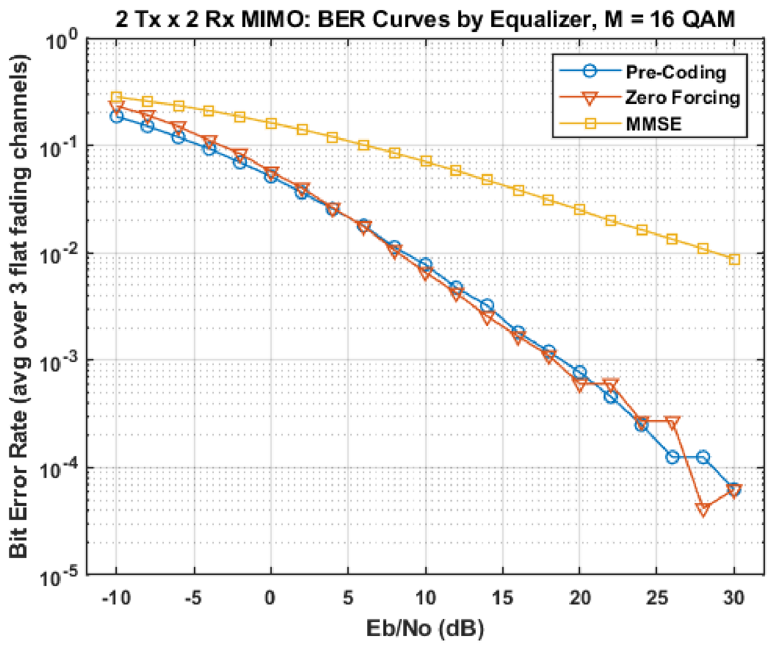
MMSE Scheme

```
txData = zeros(Mt, 1, nBits);
rxData = zeros(Mr, 1, nBits);
W = zeros(Mr, Mt, nBits);

disp('MIMO MMSE');
for i = 1:nChan
    fprintf('Channel: %d\n',i);
    % unique MIMO channel for 'Mr' receive and 'Mt' transmit antennas
    H = ( randn(Mr, Mt, nBits) + 1j*randn(Mr, Mt, nBits) ) / sqrt(2);

    % generate a sequence of random message bits and QAM modulate
    data = randi([0 M-1], Mt, 1, nBits);
    dataMod = qammod(data, M);

    for bit = 1:nBits
        % send over the fading channel
        txData(:,:,bit) = H(:,:,bit) * dataMod(:,:,bit);
    end

    fprintf('SNR:\t');
    for j = 1:length(snrVector)
       fprintf('%d\t',j);
       % add white Gaussian noise (x_noisy <-- x + noise)
       % for double-sided white noise, account for 1/2 in power (y_hat = U^(H) * y)
       noise = randn(Mr, 1, nBits) + 1j*randn(Mr, 1, nBits) / sqrt(2);
       txNoisy = txData +  noise * 10^(-snrVector(j)/10/2);

       for bit = 1:nBits
           % add noise variations before taking inverse of the first parenthesis
           W(:,:,bit) = (H(:,:,bit)' * H(:,:,bit) + ...
                         + eye(Mt)*10^(-snrVector(j)/10/2) ...
                         )^-1 * H(:,:,bit)';
           rxData(:,:,bit) = W(:,:,bit) * txNoisy(:,:,bit);
       end

       % QAM demodulate and compute bit error rate
       rxData = qamdemod(rxData,M);
       [~,berMMSE(i,j)] = biterr(data, rxData);
    end
    fprintf('\n');
end
% take average of all 3 fading channels
berMMSE = mean(berMMSE);
```

```
MIMO MMSE
Channel: 1
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
Channel: 2
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
Channel: 3
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
```

MIMO BER Curves

```
figure;
semilogy(EbNo, berPreCoding,'-o', ...
         EbNo, berZeroForcing ,'-v', ...
         EbNo, berMMSE,'-s','LineWidth',1);
grid on;
xlim([EbNo(1)-2 EbNo(end)+2]);
title(sprintf('%d Tx x %d Rx MIMO: BER Curves by Equalizer, M = %d QAM', Mt, Mr, M));
set(gca, 'FontWeight','bold','LineWidth',1);
xlabel('Eb/No (dB)');
ylabel('Bit Error Rate (avg over 3 flat fading channels)');
legend('Pre-Coding','Zero Forcing','MMSE');
snapnow;
```



Part 2: OFDM

Reference:

- mmse equalizer: [3] https://www.researchgate.net/publication/313955547_Performance_of_MMSE_channel_equalization_for_MIMO_OFDM_system

- 801.11a: [4] http://rfmw.em.keysight.com/wireless/helpfi0les/89600b/webhelp/subsystems/wlan-ofdm/Content/ofdm_80211-overview.htm

[5] Goldsmith, $Wireless Communications$, [p.397]

```
% assume perfect CSIR
```

Parameter Setup

```
% close all; clear all; clc;
% warning ('off','all');

% 802.11a
% >> each OFDM symbol has a total of 80 for data samples and cyclic prefix
%   - N = 64 subcarriers are generated,
%       of which 48 are used for data tx, 12 are zeroed to reduce ISI,
%       4 used for pilot symbols for channel estimation;
%   - subcarriers are evenly spaced over 20 MHz bandwidth into 312.5 kHz
%   - CP: 16 samples


nSubcarrier = 64;               % number of subcarriers
nCP = 16;                       % length of cyclic prefix
nData = nSubcarrier - nCP;      % number of subcarriers used for data
nPilot = 4;                     % number of pilots

M = 16;                         % modulation order: 16 (QAM)
nChan = 3;                      % number of unique (random) Rayleigh channels
nSyms = 1e3/2;                    % number of OFDM symbols

% single Rayleigh frequency selective channel with 4 taps by 802.11a PHY standard
Ts = 4e-6;                      % symbol time per subchannel (sampling period of channel)
Fd = 0;                         % maximum Doppler frequency shift (Hz)
tau = [0 1e-5 3.5e-5 12e-5];    % path delays for 4 paths
pdb = [0 -1 -1 -3];             % average path power gains in each path
h = rayleighchan(Ts, Fd, tau, pdb);
h.StoreHistory = 0;
h.StorePathGains = 1;
h.ResetBeforeFiltering = 1;

EbNo = -10:2:30;
snrVector = EbNo + 10*log10(nSubcarrier/(nSubcarrier+nCP));

% initialize ber vectors
berZeroForcing = zeros(nChan, length(snrVector));
berMMSE = zeros(nChan, length(snrVector));
```

Zero Forcing Scheme

```
% initialize
chan = zeros(nSubcarrier+nCP, nSyms);
txData = zeros(nSubcarrier+nCP, nSyms);

disp('OFDM zero forcing');
for i = 1:nChan
    fprintf('Channel: %d\t',i);
    % generate a sequence of random message bits and QAM modulate
    msg = randi([0 M-1], 1, nData*nSyms);
    dataMod = qammod(msg, M);
    dataMod = reshape(dataMod, nData,[]);

    % 48 = data, 4 = pilot, rest (12) = zeros to reduce ISI
    frame = zeros(nSubcarrier, nSyms);
    randomIdx = randperm(nSubcarrier);
    dataIdx = randomIdx(1:nData);
    frame(dataIdx,:) = dataMod;             % data
    frame(nData+1:nData+nPilot,:) = 1;      % pilot
    dataIFFT = ifft(frame,64);              % 64-point IFFT

    % insert cyclic prefix
    dataIFFTWithCP = [dataIFFT(nSubcarrier-nCP+1:nSubcarrier,:); dataIFFT];

    % OFDM encode (flat fading Rayleigh channel)
    for k=1:nSyms
        chan(:,k) = filter(h,ones((nSubcarrier+nCP),1));
        txData(:,k) = chan(:,k) .* dataIFFTWithCP(:,k);
    end

    fprintf('SNR:\t');
    for j = 1:length(snrVector)
        fprintf('%d\t',j);
        noise = (randn(nSubcarrier+nCP,nSyms)+1j*randn(nSubcarrier+nCP,nSyms)) / sqrt(2);
        txNoisy = txData + 10^(-snrVector(j)/10/2) * noise;

        % take 64-point FFT back on data + zero + pilot portion (w/o CP)
        rxWithoutCP = txNoisy(nCP+1:(nSubcarrier+nCP),:);
        rxFFT = fft(rxWithoutCP, nSubcarrier);

        W = ((conj(chan(nCP+1:end,:)) .* chan(nCP+1:end,:)) ./ conj(chan(nCP+1:end,:)));
        ofdmZF = rxFFT ./ W;

        ofdmRxData = ofdmZF(dataIdx,:);
        ofdmRxDataToDemod = reshape(ofdmRxData,1,[]);
        rxData = qamdemod(ofdmRxDataToDemod, M);
        [~, berZeroForcing(i,j)] = biterr(msg, rxData);

    end
    fprintf('\n');
end
```

```
OFDM zero forcing
Channel: 1      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
Channel: 2      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
Channel: 3      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
```

MMSE Scheme

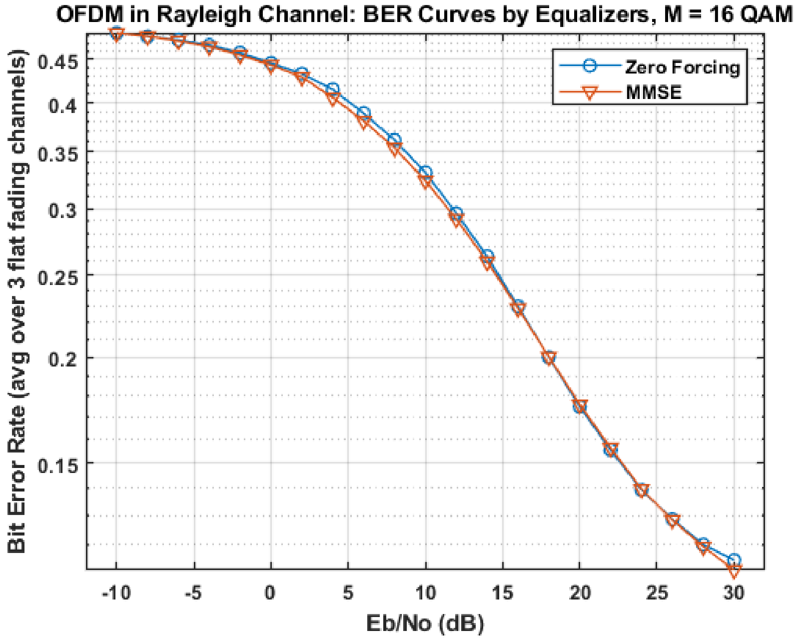
```
% initialize
chan = zeros(nSubcarrier+nCP, nSyms);
txData = zeros(nSubcarrier+nCP, nSyms);
rxData = zeros(1, nData * nSyms);

disp('OFDM MMSE');
for i = 1:nChan
    fprintf('Channel: %d\n',i);
    % generate a sequence of random message bits and QAM modulate
    msg = randi([0 M-1], 1, nData*nSyms);
    dataMod = qammod(msg, M);
    dataMod = reshape(dataMod, nData,[]);

    % 48 = data, 4 = pilot, rest (12) = zeros to reduce ISI
    frame = zeros(nSubcarrier, nSyms);
    randomIdx = randperm(nSubcarrier);
    dataIdx = randomIdx(1:nData);
```

```
    frame(dataIdx,:) = dataMod;              % data
    frame(nData+1:nData+nPilot,:) = 1;       % pilot
    dataIFFT = ifft(frame,64);               % 64-point IFFT

    % insert cyclic prefix
    dataIFFTWithCP = [dataIFFT(nSubcarrier-nCP+1:nSubcarrier,:); dataIFFT];

    % OFDM encode (flat fading Rayleigh channel)
    for k=1:nSyms
        chan(:,k) = filter(h,ones((nSubcarrier+nCP),1));
        txData(:,k) = chan(:,k).* dataIFFTWithCP(:,k);
    end

    fprintf('SNR:\t');
    for j = 1:length(snrVector)
        fprintf('%d\t',j);
        noise = (randn(nSubcarrier+nCP,nSyms)+1j*randn(nSubcarrier+nCP,nSyms)) / sqrt(2);
        noiseSNR = 10^(-snrVector(j)/10/2);
        txNoisy = txData + noise * noiseSNR;

        % take 64-point FFT back on data + zero + pilot portion (w/o CP)
        rxWithoutCP = txNoisy(nCP+1:(nSubcarrier+nCP),:);
        rxFFT = fft(rxWithoutCP, nSubcarrier);

        W = ((conj(chan(nCP+1:end,:)) .* chan(nCP+1:end,:) + ...
            ones(nSubcarrier, nSyms).*noiseSNR) ./ conj(chan(nCP+1:end,:)));
        ofdmMMSE = rxFFT ./ W;

        ofdmRxData = ofdmMMSE(dataIdx,:);
        ofdmRxDataToDemod = reshape(ofdmRxData,1,[]);
        rxData = qamdemod(ofdmRxDataToDemod, M);
        [~, berMMSE(i,j)] = biterr(msg, rxData);

    end
    fprintf('\n');
end
```

```
OFDM MMSE
Channel: 1
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
Channel: 2
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
Channel: 3
SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13      14      15
        16      17      18      19      20      21
```

Plot BER Curves

```
figure;
semilogy(EbNo, mean(berZeroForcing),'-o', ...
         EbNo, mean(berMMSE),'-v','LineWidth',1);
grid on;
xlim([EbNo(1)-2 EbNo(end)+2]);
title(sprintf('OFDM in Rayleigh Channel: BER Curves by Equalizers, M = %d QAM', M));
set(gca, 'FontWeight','bold','LineWidth',1);
xlabel('Eb/No (dB)');
ylabel('Bit Error Rate (avg over 3 flat fading channels)');
legend('Zero Forcing','MMSE');
snapnow;
```



Part 3: MIMO-OFDM

```
% close all; clear all; clc;
% warning ('off','all');

% Take the OFDM symbol and put it through a 2x2 frequency selective MIMO link.
% You can assume perfect CSIT or CSIR, whichever you want.
% The magic of OFDM and MIMO is the frequency selective MIMO channel
% just becomes a bunch of independent flat fading MIMO channels.

% assume perfect CSIR
```

Parmeter Setup

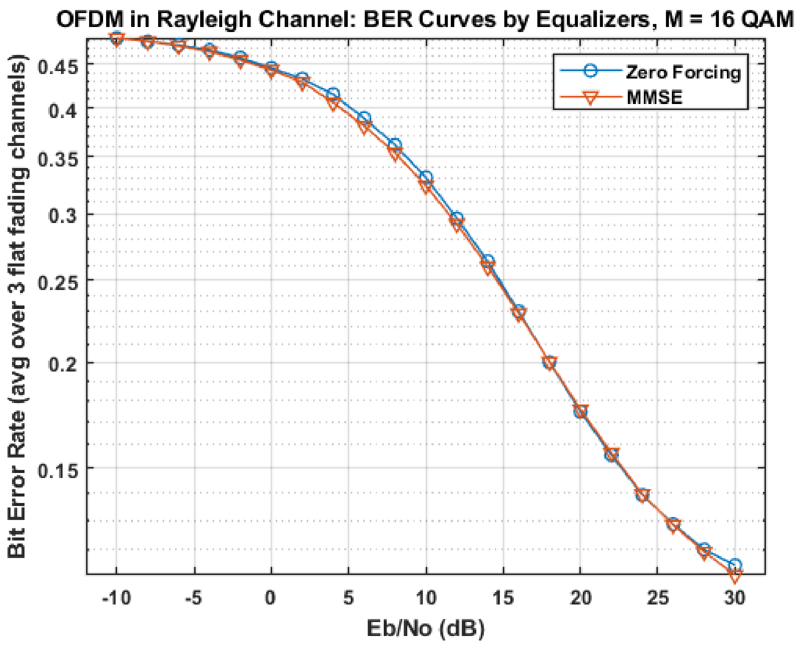
```
nChan = 3;                      % number of unique (random) Rayleigh channels
nSyms = 1e2/2;                     % number of symbols to transmit
M = 16;                         % modulation order: 16 (QAM)
k = log2(M);
nBits = nSyms * k;

% MIMO
Mt = 2;
Mr = 2;

% OFDM
nSubcarrier = 64;               % number of subcarriers
nCP = 16;                       % length of cyclic prefix
nData = nSubcarrier - nCP;      % number of subcarriers used for data
nPilot = 4;                     % number of pilots

% single Rayleigh frequency selective channel with 4 taps by 802.11a PHY standard
Ts = 4e-6;                      % symbol time per subchannel (sampling period of channel)
Fd = 0;                         % maximum Doppler frequency shift (Hz)
tau = [0 1e-5 3.5e-5 12e-5];    % path delays for 4 paths
pdb = [0 -1 -1 -3];             % average path power gains in each path
h = rayleighchan(Ts, Fd, tau, pdb);
h.StoreHistory = 0;
h.StorePathGains = 1;
h.ResetBeforeFiltering = 1;

% SNR
EbNo = -10:2:30;
snrVector = EbNo + 10*log10(k) + 10*log10(nSubcarrier/(nSubcarrier+nCP));

% initialize
nMIMOequalizers = 3;
berOFDMZeroForcing = zeros(nChan, length(snrVector), nMIMOequalizers);
berOFDMmmse = zeros(nChan, length(snrVector), nMIMOequalizers);
H = zeros(Mr, Mt, nData*nSyms*k);

% order:
% data src - MIMO (3) - OFDM (2) - AWGN - OFDM (2) - MIMO (3) - data sink
% assume perfect CSIT; estimate CSIR
```

OFDM Zero-Forcing, MIMO Precoding/Zero-Forcing/MMSE

```
% initialize
U = zeros(Mr, Mt, nData*nSyms*k);
S = zeros(Mr, Mt, nData*nSyms*k);
V = zeros(Mr, Mt, nData*nSyms*k);
chan = zeros(nSubcarrier+nCP, nSyms);
txOFDM = zeros(nSubcarrier+nCP, nSyms*k*Mt);
prefiltered = zeros(Mr,1,nData*nSyms*k);
txMIMO = zeros(Mr,1,nData*nSyms*k);
postfiltered = zeros(Mr,1,nData*nSyms*k);
rxMIMO = zeros(Mr,1,nData*nSyms*k);

for i = 1:nChan
    fprintf('Channel: %d\t',i);

    % unique MIMO channel for 'Mr' receive and 'Mt' transmit antennas
    H = ( randn(Mr, Mt, nData*nSyms*k) + 1j*randn(Mr, Mt, nData*nSyms*k) ) / sqrt(2);

    % generate a sequence of random message bits and QAM modulate
    msg = randi([0 M-1], 1, nData*nSyms*k*Mt);
    bits = reshape(msg, 2, []);
    dataMod = qammod(bits, M);

    % reshape for OFDM framing
    txMsg = reshape(dataMod, nData,[]);

    % OFDM: 48 = data, 4 = pilot, rest (12) = zeros to reduce ISI
    randomIdx = randperm(nSubcarrier);
    dataIdx = randomIdx(1:nData);
    frame = zeros(nSubcarrier, nSyms*k*Mt);
    frame(dataIdx,:) = txMsg;               % data
    frame(nData+1:nData+nPilot,:) = 1;      % pilot
    dataIFFT = ifft(frame,64);              % 64-point IFFT

    % OFDM: insert cyclic prefix (CP)
    dataIFFTWithCP = [dataIFFT(nSubcarrier-nCP+1:nSubcarrier,:); dataIFFT];

    % OFDM encode (flat fading Rayleigh channel)
    for kk = 1:nSyms*k*Mt
        chan(:,kk) = filter(h,ones((nSubcarrier+nCP),1));
        txOFDM(:,kk) = chan(:,kk) .* dataIFFTWithCP(:,kk);
    end

    fprintf('SNR:\t');
    for j = 1:length(snrVector)
        fprintf('%d\t',j);
        % add noise from OFDM modulation
        noise = (randn(nSubcarrier+nCP,nSyms*k*Mt) + 1j*randn(nSubcarrier+nCP,nSyms*k*Mt)) / sqrt(2);
        txNoisy = txOFDM + 10^(-snrVector(j)/10/2) * noise;

        % OFDM: take 64-point FFT back on data + zero + pilot portion (w/o CP)
        rxWithoutCP = txNoisy(nCP+1:(nSubcarrier+nCP),:);
        rxFFT = fft(rxWithoutCP, 64); % 64-point FFT

        % OFDM: Equalize
        W = ((conj(chan(nCP+1:end,:)) .* chan(nCP+1:end,:)) ./ conj(chan(nCP+1:end,:)));
        ofdmZF = rxFFT ./ W;
        ofdmRxData = ofdmZF(dataIdx,:);

        % OFDM frame
        OFDMsym = reshape(ofdmRxData, 1, []);

        % prepare to send over MIMO channel
        txMIMOsym = reshape(OFDMsym, Mt, 1, []);

        % ---- MIMO precoding
        % precode and send over channel
        for bit = 1:nData*nSyms*k
            % decompose channel matrix H by SVD
            [U(:,:,bit), S(:,:,bit), V(:,:,bit)] = svd(H(:,:,bit));
            % pre-code data for each bit: inverting fading at transmitter (x = V * x_hat)
            prefiltered(:,:,bit) = V(:,:,bit) * txMIMOsym(:,:,bit);
            % send over the fading channel
            txMIMO(:,:,bit) = H(:,:,bit)*prefiltered(:,:,bit);
        end

        % add noise from MIMO channel
        noiseChannel = (randn(Mr,1,nData*nSyms*k) + 1j*randn(Mr,1,nData*nSyms*k)) / sqrt(2);
        txMIMONoisy = txMIMO + 10^(-snrVector(j)/10/2) * noiseChannel;

        for bit = 1:nData*nSyms*k
           % post-code data for each bit: remove fading channel components
```

```
            postfiltered(:,:,bit) = U(:,:,bit)' * txMIMONoisy(:,:,bit);
            % recover data
            rxMIMO(:,:,bit) = S(:,:,bit)^-1 * postfiltered(:,:,bit);
        end
        rxMIMOPrecoded = rxMIMO;
        clear rxMIMO;

        % ---- MIMO zero forcing
        % pass through channel
        for bit = 1:nData*nSyms*k
            txMIMO(:,:,bit) = H(:,:,bit) * txMIMOsym(:,:,bit);
        end
        % add noise from MIMO channel
        noiseChannel = (randn(Mr,1,nData*nSyms*k) + 1j*randn(Mr,1,nData*nSyms*k)) / sqrt(2);
        txMIMONoisy = txMIMO + 10^(-snrVector(j)/10/2) * noiseChannel;
        clear W;
        for bit = 1:nData*nSyms*k
           W(:,:,bit) = (H(:,:,bit)' * H(:,:,bit))^-1 * H(:,:,bit)';
           rxMIMO(:,:,bit) = W(:,:,bit) * txMIMONoisy(:,:,bit);
        end
        rxMIMOzf = rxMIMO;
        clear rxMIMO;

        % ---- MIMO MMSE
                % pass through channel
        for bit = 1:nData*nSyms*k
            txMIMO(:,:,bit) = H(:,:,bit) * txMIMOsym(:,:,bit);
        end
        % add noise from MIMO channel
        noiseChannel = (randn(Mr,1,nData*nSyms*k) + 1j*randn(Mr,1,nData*nSyms*k)) / sqrt(2);
        txMIMONoisy = txMIMO + 10^(-snrVector(j)/10/2) * noiseChannel;

        clear W;
        for bit = 1:nData*nSyms*k
            W(:,:,bit) = (H(:,:,bit)' * H(:,:,bit) + ...
                          + eye(Mt)*10^(-snrVector(j)/10/2) ...
                          )^-1 * H(:,:,bit)';
            rxMIMO(:,:,bit) = W(:,:,bit) * txMIMONoisy(:,:,bit);
        end
        rxMIMOmmse = rxMIMO;
        clear rxMIMO;

        % QAM demodulate and compute bit error rate
        rxDataPC = qamdemod(rxMIMOPrecoded, M);
        rxDataZF = qamdemod(rxMIMOzf, M);
        rxDataMMSE = qamdemod(rxMIMOmmse, M);

        rxDataMsgPC = reshape(rxDataPC, 1, []);
        rxDataMsgZF = reshape(rxDataZF, 1, []);
        rxDataMsgMMSE = reshape(rxDataMMSE, 1, []);

        [~,berOFDMZeroForcing(i,j,1)] = biterr(msg, rxDataMsgPC);
        [~,berOFDMZeroForcing(i,j,2)] = biterr(msg, rxDataMsgZF);
        [~,berOFDMZeroForcing(i,j,3)] = biterr(msg, rxDataMsgMMSE);

    end
    fprintf('\n');
end

berOFDMZeroForcing = mean(berOFDMZeroForcing,1);
berOFDMZeroForcing = permute(berOFDMZeroForcing, [3 2 1]);
```

```
Channel: 1      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
Channel: 2      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
Channel: 3      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
```



OFDM MMSE, MIMO Precoding/Zero-Forcing/MMSE

```
% initialize
U = zeros(Mr, Mt, nData*nSyms*k);
S = zeros(Mr, Mt, nData*nSyms*k);
V = zeros(Mr, Mt, nData*nSyms*k);
chan = zeros(nSubcarrier+nCP, nSyms);
txOFDM = zeros(nSubcarrier+nCP, nSyms*k*Mt);
prefiltered = zeros(Mr,1,nData*nSyms*k);
txMIMO = zeros(Mr,1,nData*nSyms*k);
postfiltered = zeros(Mr,1,nData*nSyms*k);
rxMIMO = zeros(Mr,1,nData*nSyms*k);

for i = 1:nChan
    fprintf('Channel: %d\t',i);

    % unique MIMO channel for 'Mr' receive and 'Mt' transmit antennas
    H = ( randn(Mr, Mt, nData*nSyms*k) + 1j*randn(Mr, Mt, nData*nSyms*k) ) / sqrt(2);

    % generate a sequence of random message bits and QAM modulate
    msg = randi([0 M-1], 1, nData*nSyms*k*Mt);
    bits = reshape(msg, 2, []);
    dataMod = qammod(bits, M);

    % reshape for OFDM framing
    txMsg = reshape(dataMod, nData,[]);

    % OFDM: 48 = data, 4 = pilot, rest (12) = zeros to reduce ISI
    randomIdx = randperm(nSubcarrier);
    dataIdx = randomIdx(1:nData);
    frame = zeros(nSubcarrier, nSyms*k*Mt);
    frame(dataIdx,:) = txMsg;               % data
    frame(nData+1:nData+nPilot,:) = 1;       % pilot
    dataIFFT = ifft(frame,64);               % 64-point IFFT

    % OFDM: insert cyclic prefix (CP)
    dataIFFTWithCP = [dataIFFT(nSubcarrier-nCP+1:nSubcarrier,:); dataIFFT];

    % OFDM encode (flat fading Rayleigh channel)
    for kk = 1:nSyms*k*Mt
        chan(:,kk) = filter(h,ones((nSubcarrier+nCP),1));
        txOFDM(:,kk) = chan(:,kk) .* dataIFFTWithCP(:,kk);
    end

    fprintf('SNR:\t');
    for j = 1:length(snrVector)
        fprintf('%d\t',j);
        % add noise from OFDM modulation
        noise = (randn(nSubcarrier+nCP,nSyms*k*Mt) + 1j*randn(nSubcarrier+nCP,nSyms*k*Mt)) / sqrt(2);
        noiseSNR = 10^(-snrVector(j)/10/2);
        txNoisy = txOFDM + noiseSNR * noise;

        % OFDM: take 64-point FFT back on data + zero + pilot portion (w/o CP)
        rxWithoutCP = txNoisy(nCP+1:(nSubcarrier+nCP),:);
        rxFFT = fft(rxWithoutCP, 64); % 64-point FFT
```

```
% OFDM: Equalize
W = ((conj(chan(nCP+1:end,:)) .* chan(nCP+1:end,:) + ...
    ones(nSubcarrier, nSyms*k*Mt).*noiseSNR) ./ conj(chan(nCP+1:end,:)));
ofdmMMSE = rxFFT ./ W;
ofdmRxData = ofdmMMSE(dataIdx,:);

% OFDM frame
OFDMsym = reshape(ofdmRxData, 1, []);

% prepare to send over MIMO channel
txMIMOsym = reshape(OFDMsym, Mt, 1, []);

% ---- MIMO precoding
% precode and send over channel
for bit = 1:nData*nSyms*k
    % decompose channel matrix H by SVD
    [U(:,:,bit), S(:,:,bit), V(:,:,bit)] = svd(H(:,:,bit));
    % pre-code data for each bit: inverting fading at transmitter (x = V * x_hat)
    prefiltered(:,:,bit) = V(:,:,bit) * txMIMOsym(:,:,bit);
    % send over the fading channel
    txMIMO(:,:,bit) = H(:,:,bit)*prefiltered(:,:,bit);
end

% add noise from MIMO channel
noiseChannel = (randn(Mr,1,nData*nSyms*k) + 1j*randn(Mr,1,nData*nSyms*k)) / sqrt(2);
txMIMONoisy = txMIMO + 10^(-snrVector(j)/10/2) * noiseChannel;

for bit = 1:nData*nSyms*k
   % post-code data for each bit: remove fading channel components
   postfiltered(:,:,bit) = U(:,:,bit)' * txMIMONoisy(:,:,bit);
   % recover data
   rxMIMO(:,:,bit) = S(:,:,bit)^-1 * postfiltered(:,:,bit);
end
rxMIMOPrecoded = rxMIMO;
clear rxMIMO;

% ---- MIMO zero forcing
% pass through channel
for bit = 1:nData*nSyms*k
    txMIMO(:,:,bit) = H(:,:,bit) * txMIMOsym(:,:,bit);
end
% add noise from MIMO channel
noiseChannel = (randn(Mr,1,nData*nSyms*k) + 1j*randn(Mr,1,nData*nSyms*k)) / sqrt(2);
txMIMONoisy = txMIMO + 10^(-snrVector(j)/10/2) * noiseChannel;
clear W;
for bit = 1:nData*nSyms*k
   W(:,:,bit) = (H(:,:,bit)' * H(:,:,bit))^-1 * H(:,:,bit)';
   rxMIMO(:,:,bit) = W(:,:,bit) * txMIMONoisy(:,:,bit);
end
rxMIMOzf = rxMIMO;
clear rxMIMO;

% ---- MIMO MMSE
        % pass through channel
for bit = 1:nData*nSyms*k
    txMIMO(:,:,bit) = H(:,:,bit) * txMIMOsym(:,:,bit);
end
% add noise from MIMO channel
noiseChannel = (randn(Mr,1,nData*nSyms*k) + 1j*randn(Mr,1,nData*nSyms*k)) / sqrt(2);
txMIMONoisy = txMIMO + 10^(-snrVector(j)/10/2) * noiseChannel;

clear W;
for bit = 1:nData*nSyms*k
    W(:,:,bit) = (H(:,:,bit)' * H(:,:,bit) + ...
                  + eye(Mt)*10^(-snrVector(j)/10/2) ...
                  )^-1 * H(:,:,bit)';
    rxMIMO(:,:,bit) = W(:,:,bit) * txMIMONoisy(:,:,bit);
end
rxMIMOmmse = rxMIMO;
clear rxMIMO;

% QAM demodulate and compute bit error rate
rxDataPC = qamdemod(rxMIMOPrecoded, M);
rxDataZF = qamdemod(rxMIMOzf, M);
rxDataMMSE = qamdemod(rxMIMOmmse, M);

rxDataMsgPC = reshape(rxDataPC, 1, []);
rxDataMsgZF = reshape(rxDataZF, 1, []);
rxDataMsgMMSE = reshape(rxDataMMSE, 1, []);

[~,berOFDMmmse(i,j,1)] = biterr(msg, rxDataMsgPC);
[~,berOFDMmmse(i,j,2)] = biterr(msg, rxDataMsgZF);
```

```
        [~,berOFDMmmse(i,j,3)] = biterr(msg, rxDataMsgMMSE);

    end
    fprintf('\n');
end

berOFDMmmse = mean(berOFDMmmse,1);
berOFDMmmse = permute(berOFDMmmse, [3 2 1]);
```

```
Channel: 1      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
Channel: 2      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
Channel: 3      SNR:    1       2       3       4       5       6       7       8       9       10      11      12      13
        14      15      16      17      18      19      20      21
```

Plot BER Curves

```
figure('Renderer', 'painters', 'Position', [300 300 900 600])
semilogy(EbNo, berOFDMZeroForcing(1,:), '-bo', ...
         EbNo, berOFDMZeroForcing(2,:), '-b^', ...
         EbNo, berOFDMZeroForcing(3,:), '-bv', 'LineWidth',1);
hold on;
semilogy(EbNo, berOFDMmmse(1,:), '-ko', ...
         EbNo, berOFDMmmse(2,:), '-k^', ...
         EbNo, berOFDMmmse(3,:), '-kv', 'LineWidth',1);
grid on;
xlim([EbNo(1)-2 EbNo(end)+2]);
legend('OFDM ZF, MIMO Precoding','OFDM ZF, MIMO Zero Forcing','OFDM ZF, MIMO MMSE', ...
       'OFDM MMSE, MIMO Precoding','OFDM MMSE, MIMO Zero Forcing','OFDM MMSE, MIMO MMSE');
title(sprintf('MIMO-OFDM: BER Curves by Equalizers, M = %d QAM', M));
set(gca, 'FontWeight','bold','LineWidth',1);
xlabel('Eb/No (dB)');
ylabel('Bit Error Rate (avg over 3 flat fading channels)');
snapnow;
```