

OFDM-MIMO Channel with Equalizers

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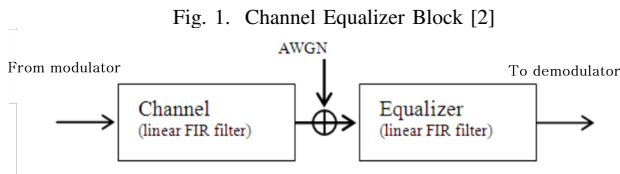
Abstract—In this simulation task, we explore three different equalization schemes in MIMO flat fading channel, frequency-selective OFDM channel, and combined OFDM-MIMO wireless links on the bit error rate (BER) metric. Throughout the simulations, we modulate in 4-QAM (MIMO, OFDM-MIMO) and in 16-QAM (OFDM) and observe BER performances for signal-to-noise ratio (SNR) up to 30. We find that given the specifications for OFDM as defined in IEEE 802.11a, precoding and zero forcing schemes in MIMO yield similar BER performances, while MMSE scheme performs slightly worse at higher SNR's. Based on the equalization scheme, we assume perfect channel state information at the transmitter (CSIT) (for precoding) and at the receiver (CSIR) (for zero-forcing and MMSE).

Index Terms—Equalization, MIMO, OFDM, OFDM-MIMO, CSIT, CSIR

I. INTRODUCTION

In digital communication systems, achieving the target data rate and bit error rate through a transmission channel is crucial for data transfer. In this regard, an equalizer is usually employed at the receiver side in order to reduce the number of incorrect instances of the transmitted signal. In general, we consider the sequence of bit transmission in the order described in Figure 1. The modulated signal is encoded by the channel, an additive white Gaussian noise (AWGN) is added, and an equalizer is applied before demodulating the received signal for signal recovery. This equalization block at the receiver serves to minimize the effect of inter-symbol interference (ISI) and inter-channel interference (ICI).

In this paper, we study the effects of three equalization schemes – precoding, zero-forcing and MMSE in MIMO links, zero-forcing and MMSE in OFDM models, and all combinations in OFDM-MIMO channel models. These linear equalizers minimize the error between the received and transmitted symbols without enhancing the noise [1]. We do not take into account the impairments in transmission in these simulations, such as timing offset, frequency and phase offsets and clock drift, and thus we can work in the complex band instead of the baseband.

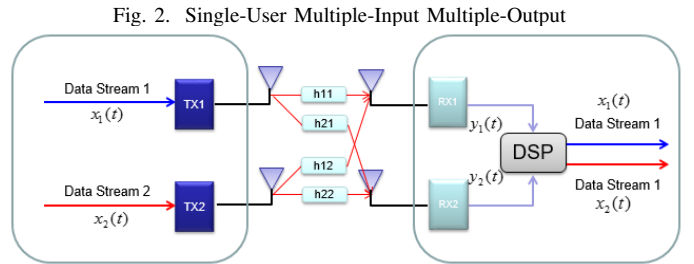


A. MIMO Channel

We model 2×2 MIMO channels with three different flat-fading channels. For $M_{transmit} = 2$ and $M_{receive} = 2$, the channel state matrix is represented in dimensions $2 \times 2 \times \text{symbol size}$ (see Equation 1).

$$H_{MIMO} = \begin{bmatrix} H_{11} & \cdots & H_{1M_T} \\ \vdots & \ddots & \vdots \\ H_{M_R1} & \cdots & H_{M_RM_T} \end{bmatrix} \quad (1)$$

The single-user MIMO channel that we model takes two data streams given two transmit antennas, and encodes (transmits) data through the channel matrix H and multiplexes at the receiver. To the model described in Figure 2, we add an equalizer as seen in Figure 1, in three different schemes described in Section II [3].



B. OFDM Encoding

In OFDM symbol coding, we adopt the specifications as describe in the IEEE 802.11a standards. In each OFDM burst, there are a total of 80 symbols including 64 subcarriers containing 48 data samples to transmit, 12 zeros to reduce inter-channel interference (ICI), and 4 pilot symbols for channel estimation (e.g. frequency and phase offset estimation). The subcarriers are evenly spaced over 20 MHz bandwidth into 312.5 kHz. We also need to prepend 16 sample-long cyclic prefixes, which we take from the data samples in order for linear convolution to become circular convolution and to further mitigate channel distortion. Since the number of subcarriers is 64, we use 64-point IDFT (IFFT) and DFT (FFT) throughout the OFDM symbol generation stage. We transmit the OFDM symbols through a frequency-selective channel with the following parameters¹: [4], [5]

¹<https://www.mathworks.com/help/comm/ref/rayleighchan.html>

- 1) Symbol time per subchannel (sampling period of channel) = $4 \mu s$
- 2) Max Doppler frequency shift (Hz) = 0
- 3) Path delays = $[0, 1e-5, 3.5e-5, 12e-5]$
- 4) Average path power gains in each path = $[0, -1, -1, -3]$

C. OFDM-MIMO

We put together the MIMO link and OFDM symbol transmission models into a OFDM-MIMO model, such that OFDM bursts are generated and then transmitted over MIMO flat-fading channel. The assumptions for CSIT and CSIR (described in Section II) are valid for this model. We note that in this combined link, the frequency selective MIMO channels essentially become a group of independent flat-fading MIMO channels [6].

II. EQUALIZER SCHEME

We employ three different equalizer approaches in each model – precoding, zero-forcing and minimum mean-squared error. All of these schemes are placed at the receiver in order to invert the effect of the channel on the transmitted symbols. Note that we assume perfect CSIT for precoding and perfect CSIR for zero-forcing and MMSE. The added AWGN level at the channel, for which its half-power is accounted, should not be too large to largely distort the transmitted signal.

A. Transmit precoding and Receiver Shaping

For channel matrix H , we perform parallel decomposition by SVD, such that

$$H = U\Sigma V^H$$

where we apply linear transformation as follows:

$$x = V\tilde{x}$$

where \tilde{x} = input vector.

After transmitting over a channel H with additive white Gaussian noise n ,

$$\tilde{y} = U^H(Hx + n) = U^H U \Sigma V^H V \tilde{x} + U^H n = \Sigma \tilde{x} + \tilde{n},$$

where $\tilde{n} = U^H n$.

B. Zero-forcing

Zero-forcing equalizer approximates the inverse of the channel with a linear filter, by applying W_{ZF} to the received symbols (see Equation 3) [4].

$$W_{ZF} = (H^H H)^{-1} H^H \quad (2)$$

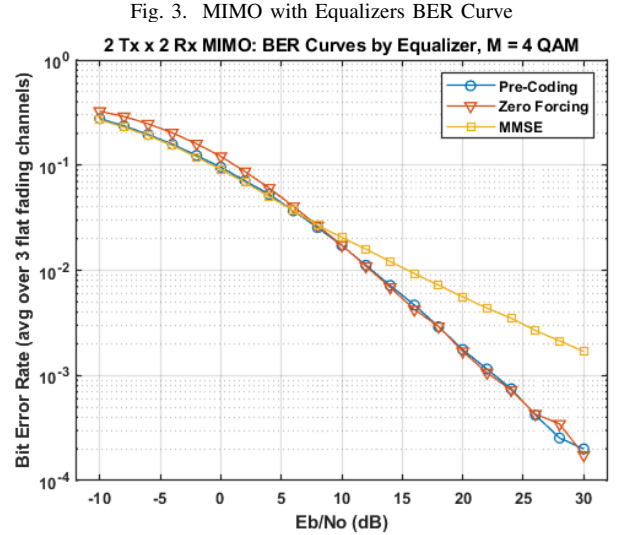
C. Minimum Mean Squared Error (MMSE)

MMSE equalizer designs the filter to minimize the expected value of the squared error signal (filter output minus the transmitted signal). In a similar fashion to zero-forcing, we apply W_{MMSE} , which takes into account the noise component of the channel [4], [7].

$$W_{MMSE} = (H^H H + \sigma_n^2 I)^{-1} H^H \quad (3)$$

III. RESULTS

In the MIMO channels with the three equalizers, we observed that at lower SNR's (SNR < 8) MMSE corrected the most, and precoding then zero-forcing in order, and at higher SNR's precoding and zero-forcing performed similarly in BER, but MMSE yielded worse performance than the other two (see Figure 3). Although MMSE equalizers take into account the interferences from adjacent symbols and adjacent subcarriers, some ICI and ISI still remain, thereby limiting the performance at high SNR's. One way to improve the equalizer performance at high SNR's is to use interference cancellation techniques [8].



For transmission of OFDM symbols through flat-fading Rayleigh channels, zero-forcing and MMSE equalizers gave similar BER performances (see Figure 4). At lower SNR's, the MMSE approach yielded slightly better BER's, yet not significantly, yet at higher SNR's zero-forcing performed better. We note that due to the specifications of the channel, the general BER level remains in the order of 10^{-1} . With different values for the taps (paths) for the channel, the order would be much lower.

In combination of the two approaches, we noticed that OFDM with zero-forcing equalizers resulted in lower BER's in general than OFDM with MMSE equalizers (see Figure 5). Within OFDM zero-forcing scheme, zero-forcing was the best, and precoding then MMSE in order, while within OFDM MMSE scheme, in the order of precoding, zero-forcing, and MMSE, at SNR of 22.

IV. CONCLUSION

We simulated the MIMO, OFDM and OFDM-MIMO models with three different equalization schemes and in combination for 4-QAM in MIMO and OFDM-MIMO and 16-QAM in OFDM modulation technique. We observed that the MMSE performance varied by SNR, while precoding and

Fig. 4. OFDM with Equalizers BER Curve

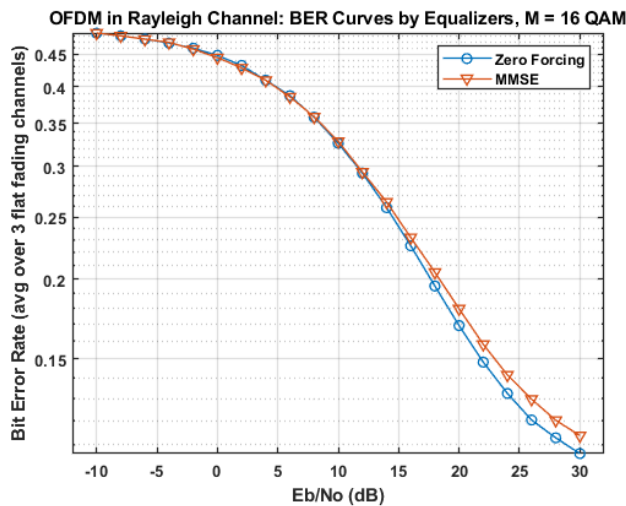
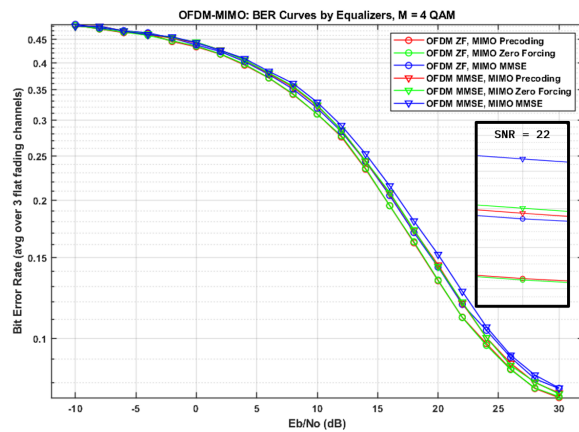


Fig. 5. OFDM-MIMO with Equalizers BER Curve



zero-forcing were very similar in MIMO. For OFDM symbols, zero-forcing and MMSE yielded similar BER performances, but slightly better overall for the latter. In OFDM-MIMO, besides the overall order of BER being lower due to the channel parameters defined in the OFDM (as per IEEE 802.11a standard), the performance in BER resulted as observed in each respective model.

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