4-5.

48. $a_0(4t-t^2) + a_1(5+t^3) + a_2(5+3t) + a_3(-3t^2+2t^3)$

$= (a_1+2a_3)t^3 + (-a_0+2a_3)t^2 + (4a_0+3a_2)t + (5a_1+5a_2)$ determine solutions.

$$\det\left|\begin{matrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 2 \\ 4 & 0 & 3 & 0 \\ 0 & 5 & 5 & 0 \end{matrix}\right| = -1 \times \det\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 5 & 5 & 0 \end{vmatrix} + 4\times \det\begin{vmatrix} 1 & 0 & 2 \\ 0 & 0 & 2 \\ 5 & 5 & 0 \end{vmatrix}$$

$$= -1\times(-30) + 4\times 0 = 30 \neq 0.$$

$\Rightarrow$ has solutions. $\Rightarrow$ S spans P_3

$(a_1+2a_3)t^3 + (-a_0+2a_3)t^2 + (4a_0+3a_2)t + (5a_1+5a_2) = 0$

$$\begin{vmatrix} -1 & 0 & 0 & 2 \\ 4 & 0 & 3 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & -2 \\ 0 & 0 & 15 & 40 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 15 & -30 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 3 & 8 \\ 0 & 0 & 0 & -70 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

a_0, a_1, a_2, a_3 has trivial one solution $= 0$.

$\Rightarrow$ linear independent

$\Rightarrow$ S is a basis for P_3

52. $a_0\begin{bmatrix} 1 & 2 \\ -5 & 4 \end{bmatrix} + a_1\begin{bmatrix} 2 & -7 \\ 6 & 2 \end{bmatrix} + a_2\begin{bmatrix} 4 & -9 \\ 11 & 12 \end{bmatrix} + a_3\begin{bmatrix} 12 & -16 \\ 17 & 42 \end{bmatrix}$

$$= \begin{bmatrix} a_0+2a_1+4a_2+12a_3 & 2a_0-7a_1-9a_2-16a_3 \\ -5a_0+2a_1+11a_2+17a_3 & 4a_0+2a_1+12a_2+42a_3 \end{bmatrix}$$

$$\det\left|\begin{matrix} 1 & 2 & 4 & 12 \\ 2 & -7 & -9 & -16 \\ -5 & 6 & 11 & 17 \\ 4 & 2 & 12 & 42 \end{matrix}\right| = 1\times(-7\times(11\times42-17\times12)+6(-9\times42+12\times16)+2(-9\times17+11\times16))$$

$$+2\times(2\times(11\times42-12\times17)+6\times(4\times42-12\times12)+2(4\times17-12\times11))$$

$$-5\times(2\times(-9\times42+12\times16)-7\times(4\times42-12\times12)+2(4\times(-16)+9\times12))$$

$$+4\times(2\times(-9\times17+11\times16)-7\times(4\times17-11\times12)+6(4\times-16+9\times12))$$

$$= 0$$

⇒ a_0, a_1, a_2, a_3 don't have unique solution.

⇒ S doesn't span $M_{2,2}$

⇒ S is not a basis for $M_{2,2}$

68. Subset of S: a: $\{(-4,1,1), (-2,7,-3), (2,1,1)\}$

b: $\{(1,3,-2), (-2,7,-3), (2,1,1)\}$

c: $\{(1,3,-2), (-4,1,1), (2,1,1)\}$

d: $\{(1,3,-2), (-4,1,1), (-2,7,-3)\}$

$$a: \begin{vmatrix} -4 & 1 & 1 \\ -2 & 7 & -3 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 8 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$b: \begin{vmatrix} 1 & 3 & -2 \\ -2 & 7 & -3 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -2 \\ 0 & 13 & -7 \\ 0 & -5 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$c: \begin{vmatrix} 1 & 3 & -2 \\ -4 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -2 \\ 0 & 13 & -7 \\ 0 & -5 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$d: \begin{vmatrix} 1 & 3 & -2 \\ -4 & 1 & 1 \\ -2 & 7 & -3 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -2 \\ 0 & 13 & -7 \\ 0 & 13 & -7 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 13 & -7 \\ 0 & 0 & 0 \end{vmatrix}$$ ⇒ linear dependence

and a, b, c has trivial solution forming R^3

⇒ $\{(-4,1,1), (-2,7,-3), (2,1,1)\}$

$\{(1,3,-2), (-2,7,-3), (2,1,1)\}$

$\{(1,3,-2), (-4,1,1), (2,1,1)\}$

4-6

14. use S form A

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 3 & -9 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -9 \\ 0 & 1 & 5 \\ 0 & -3 & 17 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

basis = $\{(1,0,0), (0,1,0), (0,0,1)\}$

20. use S form A

$$A = \begin{bmatrix} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 2 \\ -1 & -5 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & -1 & 1 & -6 \\ 0 & 3 & -2 & -1 \\ 0 & -2 & 1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & -1 & 1 & -6 \\ 0 & 0 & 1 & -19 \\ 0 & 0 & -1 & 19 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -19 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

basis $= \{(1,0,0,3), (0,1,0,-13), (0,0,1,-19)\}$

42. (a) Because A & B are row-equivalent

rank $(A) = 3$

nullity $(A) = 5-3 = 2$

(b) $\begin{cases} a + c + e = 0 \\ b - 2c + 3e = 0 \\ d - 5e = 0 \end{cases} \Rightarrow a = -s-t,\ b = 2s-3t,\ c = s,\ d = 5t,\ e = t$

$\therefore N(A) = s\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{bmatrix}$, $\because$ two vectors are linear independence

$\therefore \left\{\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{bmatrix}\right\}$ are basis of $N(A)$

(c) $\because$ B is A's row echelon form.

$\therefore$ basis for the row space of A is $\{(1,0,1,0,1), (0,1,-2,0,3), (0,0,0,1,-5)\}$

(d) $\because$ column 1, 2, 4 have the leading ones

$\therefore$ basis for the column space of A is $\{(-2,1,3,1), (-5,3,11,7), (0,1,7,5)\}$

(e) $\because$ B contains a row of 0s, so the rows of A are linearly independent.

(f) $\because a_3 = a_1 - 2a_2 \therefore \{a_1, a_2, a_3\}$ ar linear dependent

$\Rightarrow$ (i), (iii) are linealy independent.