48. $|A| = 4\begin{vmatrix} 0 & -2 \\ 3 & 0 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 \\ -3 & 0 \end{vmatrix} + 9\begin{vmatrix} -1 & 0 \\ -3 & 3 \end{vmatrix} = 3$

(a) $|A^T| = |A| = 3.$

(b) $|A^2| = |A||A| = |A|^2 = 9.$

(c) $|AA^T| = |A||A^T| = 9$

(d) $\because$ A is a 3×3 matrix

$\therefore |2A| = 2^3|A| = 8 \times 3 = 24.$

(e) $|A^{-1}| = \frac{1}{|A|} = \frac{1}{3}$

76. $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$ the matrix is orthognal

78. $\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ the matrix is orthognal

36 $\left|\begin{vmatrix} -1 & -3 & 1 \\ -4 & 7 & 1 \\ 2 & -13 & 1 \end{vmatrix}\right| = (-1)\cdot 20 + (-3)(-6) + 1\cdot 38 = 0 \Rightarrow$ the points are collinear

8. $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$M_{11} = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$ $\qquad M_{21} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1$

$M_{12} = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1$ $\qquad M_{22} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1$

$M_{13} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1$ $\qquad M_{23} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -2$

$M_{14} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -2$ $\qquad M_{24} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1$

$\begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$

$$M_{31} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1 \qquad M_{41} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -2$$

$$M_{32} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -2 \qquad M_{42} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$$

$$M_{33} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1 \qquad M_{43} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1$$

$$M_{34} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1 \qquad M_{44} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -1$$

$$\operatorname{adj}(A) = \begin{bmatrix} -1 & -1 & -1 & 2 \\ -1 & -1 & 2 & -1 \\ -1 & 2 & -1 & -1 \\ 2 & -1 & -1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 & -1 & 2 \\ -1 & -1 & 2 & -1 \\ -1 & 2 & -1 & -1 \\ 2 & -1 & -1 & -1 \end{bmatrix}$$

$|A| = -3 \Rightarrow$ invertible.

$$\therefore A^{-1} = -\frac{1}{3} \begin{bmatrix} -1 & -1 & -1 & 2 \\ -1 & -1 & 2 & -1 \\ -1 & 2 & -1 & -1 \\ 2 & -1 & -1 & -1 \end{bmatrix} \quad \#$$

18. $$|A| = \begin{vmatrix} 4 & -2 & 3 \\ 2 & 2 & 5 \\ 8 & -5 & -2 \end{vmatrix} = 4\begin{vmatrix} 2 & 5 \\ -5 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 8 & -2 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 8 & -5 \end{vmatrix}$$

$$= -82$$

$$\therefore x = \frac{\begin{vmatrix} -2 & -2 & 3 \\ 16 & 2 & 5 \\ 4 & -5 & -2 \end{vmatrix}}{-82} = \frac{-410}{-82} = 5$$

$$y = \frac{\begin{vmatrix} 4 & -2 & 3 \\ 2 & 16 & 5 \\ 8 & 4 & -2 \end{vmatrix}}{-82} = \frac{-656}{-82} = 8 \quad \Rightarrow \begin{cases} x = 5 \\ y = 8 \\ z = -2 \end{cases}$$

$$z = \frac{\begin{vmatrix} 4 & -2 & -2 \\ 2 & 2 & 16 \\ 8 & -5 & 4 \end{vmatrix}}{82} = \frac{164}{-82} = -2$$