109703040 洪晙宸

22. $P^2 = \begin{bmatrix} \frac{2}{5} & \frac{7}{10} \\ \frac{3}{5} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} \frac{2}{5} & \frac{7}{10} \\ \frac{3}{5} & \frac{3}{10} \end{bmatrix} = \begin{bmatrix} \frac{29}{50} & \frac{35}{50} \\ \frac{21}{50} & \frac{51}{100} \end{bmatrix} \Rightarrow$ Regular.

令 $\bar{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad \because P\bar{x} = \bar{x}$

$$\begin{bmatrix} \frac{2}{5} & \frac{7}{10} \\ \frac{3}{5} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} \frac{2}{5}x + \frac{7}{10}y = x \\ \frac{3}{5}x + \frac{3}{10}y = y \\ x + y = 1 \end{cases} \Rightarrow \begin{array}{l} x = \frac{7}{13} \\ y = \frac{6}{13} \end{array} \qquad \therefore \bar{x} = \begin{bmatrix} \frac{7}{13} \\ \frac{6}{13} \end{bmatrix} \#$$

24. $P^2 = \begin{bmatrix} \frac{2}{9} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \\ \frac{4}{9} & \frac{1}{4} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{9} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \\ \frac{4}{9} & \frac{1}{4} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{91}{324} & \frac{19}{72} & \frac{29}{108} \\ \frac{126}{324} & \frac{30}{72} & \frac{42}{108} \\ \frac{107}{324} & \frac{23}{72} & \frac{37}{108} \end{bmatrix} \Rightarrow$ Regular.

令 $\bar{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\quad \because P\bar{x} = \bar{x}$

$$\begin{bmatrix} \frac{2}{9} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \\ \frac{4}{9} & \frac{1}{4} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} \frac{2}{9}x + \frac{1}{4}y + \frac{1}{3}z = x \\ \frac{1}{3}x + \frac{1}{2}y + \frac{1}{3}z = y \\ \frac{4}{9}x + \frac{1}{4}y + \frac{1}{3}z = z \\ x + y + z = 1 \end{cases} \Rightarrow \begin{array}{l} x = \frac{27}{100} \\ y = \frac{2}{5} \\ z = \frac{33}{100} \end{array} \qquad \therefore \bar{x} = \begin{bmatrix} \frac{27}{100} \\ \frac{2}{5} \\ \frac{3}{100} \end{bmatrix}$$

42. $P\bar{x} = \bar{x}$ $\qquad$ 令 $\bar{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 0.1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0.7 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} 0.1x = x \\ 0.2x + y = y \\ 0.7x + z = z \\ x + y + z = 1 \end{cases} \Rightarrow \begin{array}{l} x = 0. \\ y = t \\ z = 1-t \end{array} \;,\; 0 \le t \le 1 \qquad \therefore \bar{x} = \begin{bmatrix} 0 \\ t \\ 1-t \end{bmatrix} ,\; 0 \le t \le 1$$

44. $P\overline{X} = \overline{X}$ 令 $\overline{X} = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 0.7 & 0 & 0.2 & 0.1 \\ 0.1 & 1 & 0.5 & 0.6 \\ 0 & 0 & 0.2 & 0.2 \\ 0.2 & 0 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} 0.7w + 0.2y + 0.1z = w \\ 0.1w + x + 0.5y + 0.6z = x \\ \quad + 0.2y + 0.2z = y \\ 0.2w + 0.1y + 0.1z = z \\ w + x + y + z = 1 \end{cases} \Rightarrow \begin{array}{l} w=0 \\ x=1 \\ y=0 \\ z=0 \end{array} \qquad \overline{X} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \#$$

10. (a) $\begin{bmatrix} 45 & -35 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 10 & 15 \end{bmatrix} \Rightarrow \begin{cases} 45w - 35y = 10 \cdots ① \\ 45x - 35z = 15 \cdots ② \end{cases}$

$\begin{bmatrix} 38 & -30 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 8 & 14 \end{bmatrix} \Rightarrow \begin{cases} 38w - 30y = 8 \cdots ③ \\ 38x - 30z = 14 \cdots ④ \end{cases}$

①③ 解得 $w=1,\ y=1$

②④ 解得 $x=-2 \quad z=-3$

$\therefore A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 45 & -35 \end{bmatrix} A^{-1} = \begin{bmatrix} 10 & 15 \end{bmatrix}$

$\begin{bmatrix} 38 & -30 \end{bmatrix} A^{-1} = \begin{bmatrix} 8 & 14 \end{bmatrix}$

$\begin{bmatrix} 18 & -18 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 18 \end{bmatrix}$

$\begin{bmatrix} 35 & -30 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 20 \end{bmatrix}$

$\begin{bmatrix} 81 & -60 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 21 & 18 \end{bmatrix}$

$\begin{bmatrix} 42 & -28 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 14 & 0 \end{bmatrix}$

$\begin{bmatrix} 75 & -55 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 20 & 15 \end{bmatrix}$

$\begin{bmatrix} 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 22 & -21 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 19 \end{bmatrix}$

$\begin{bmatrix} 15 & -10 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \end{bmatrix}$

$\therefore$ code =

10, 15, 8, 14, 0, 18, 5, 20, 21, 18

14, 0, 20, 15, 0, 2, 1, 19, 5, 0.

$\Rightarrow$ JOHN RETERN TO BASE

16.

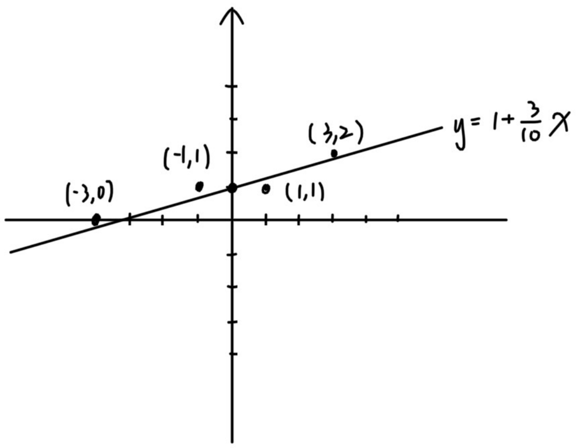
$$X = \begin{bmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 20 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$A = (X^T X)^{-1} X^T Y = \frac{1}{80} \begin{bmatrix} 20 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \frac{1}{80} \begin{bmatrix} 80 \\ 24 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{10} \end{bmatrix}$$

$\therefore$ the least squares regression line is $y = 1 + \frac{3}{10}x$.



$x = -3$, $y = -\frac{1}{10}$

$x = -1$, $y = \frac{7}{10}$

$x = 1$, $y = \frac{13}{10}$

$x = 3$, $y = \frac{19}{10}$

$$\left(0 - \left(-\frac{1}{3}\right)\right)^2 + \left(1 - \frac{7}{10}\right)^2 + \left(\frac{13}{10} - 1\right)^2 + \left(2 - \frac{19}{10}\right)^2 = \frac{1}{5} \ \#$$

18.

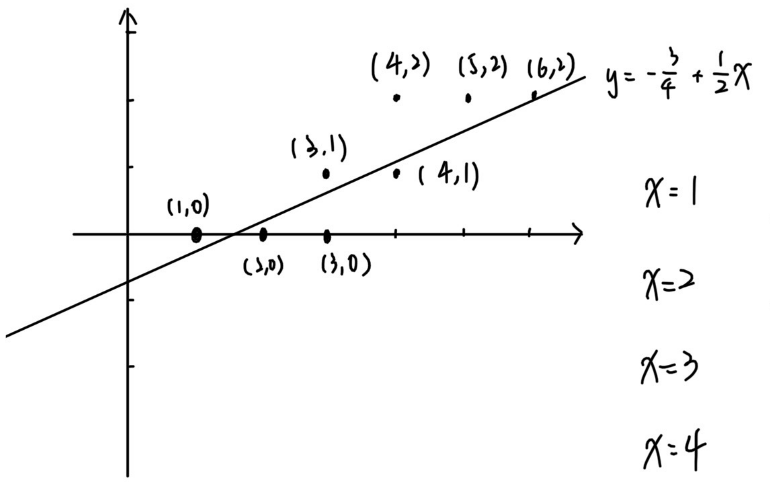
$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 & 4 & 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 28 \\ 28 & 116 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 & 4 & 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 37 \end{bmatrix}$$

$$A = (X^T X)^{-1} X^T Y = \frac{1}{144} \begin{bmatrix} 116 & -28 \\ -28 & 8 \end{bmatrix} \begin{bmatrix} 8 \\ 37 \end{bmatrix} = \frac{1}{144} \begin{bmatrix} -108 \\ 72 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} \\ \frac{1}{2} \end{bmatrix}$$

$\therefore$ the least squares regression line is $y = \frac{-3}{4} + \frac{1}{2}x$.



$x = 1 \quad y = \frac{-1}{4}$

$x = 2 \quad y = \frac{1}{4}$

$x = 3 \quad y = \frac{3}{4}$

$x = 4 \quad y = \frac{5}{4}$

$x = 5 \quad y = \frac{7}{4}$

$x = 6 \quad y = \frac{9}{4}$

$$\left[0-\left(-\tfrac{1}{4}\right)\right)^2 + \left(\tfrac{1}{4}-0\right)^2 + \left(\tfrac{3}{4}-0\right)^2 + \left(\tfrac{3}{4}-1\right)^2 + \left(\tfrac{5}{4}-1\right)^2 + \left(\tfrac{5}{4}-2\right)^2 + \left(\tfrac{7}{4}-2\right)^2 + \left(\tfrac{9}{4}-2\right)^2$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{9}{16} + \frac{1}{16} + \frac{1}{16} + \frac{9}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{2}$$