24. let $A = (a, \frac{1}{2}a)$ $B = (b, \frac{1}{2}b)$ $a, b, c \in \mathbb{R}$

$A + B = (a+b, \frac{1}{2}(a+b))$, $\because a+b \in \mathbb{R}$, $\therefore$ $A+B$ is in the set

$cA = (ca, \frac{1}{2}ca)$ $\because$ $ca \in \mathbb{R}$. $\therefore$ cA is in the set

$\Rightarrow$ the set is a vector space

26. Let $M_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & 1 \end{bmatrix} \in V$

$M_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & 1 \end{bmatrix} \in V$

$M_1 + M_2 = \begin{bmatrix} a_1 & b_1 \\ c_1 & 1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & 1 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & 2 \end{bmatrix} \notin V$

$\Rightarrow$ the set is not a vector space

30. Let $A = \begin{bmatrix} 0 & b_1 & c_1 & d_1 \\ a_1 & 0 & c_1 & d_1 \\ a_1 & b_1 & 0 & d_1 \\ a_1 & b_1 & c_1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & b_2 & c_2 & d_2 \\ a_2 & 0 & c_2 & d_2 \\ a_2 & b_2 & 0 & d_2 \\ a_2 & b_2 & c_2 & 0 \end{bmatrix}$

$A+B = \begin{bmatrix} 0 & b_1+b_2 & c_1+c_2 & d_1+d_2 \\ a_1+a_2 & 0 & c_1+c_2 & d_1+d_2 \\ a_1+a_2 & b_1+b_2 & 0 & d_1+d_2 \\ a_1+a_2 & b_1+b_2 & c_1+c_2 & 0 \end{bmatrix} \in V$

$cA = \begin{bmatrix} 0 & cb_1 & cc_1 & cd_1 \\ ca_1 & 0 & cc_1 & cd_1 \\ ca_1 & cb_1 & 0 & cd_1 \\ ca_1 & cb_1 & cc_1 & 0 \end{bmatrix} \in V$

$\Rightarrow$ the set is a vector space.

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2. let $u = (x_1, y_1, 4x_1 - 5y_1)$

$v = (x_2, y_2, 4x_2 - 5y_2)$

$u + v = (x_1 + x_2, y_1 + y_2, 4(x_1 + x_2) - 5(y_1 + y_2)) \in W$

$cu = (cx_1, cy_1, 4cx_1 - 5cy_1) \in W$

$\Rightarrow$ it is a subspace of V.

16. let $A = [1, 0, 3]^T$, $B = [\sqrt{2}, 0, 6]^T$

$A + B = [1+\sqrt{2}, 0, 9]^T$

$\sqrt{C} = 1+\sqrt{2}$

$C = 3 + 2\sqrt{2}$

$3C = 9 + 6\sqrt{2} \neq 9$

$\Rightarrow$ W is not a subspace of a vector space

40. as $s=0$, $t=0$, $(0,0,0) \in W \Rightarrow W$ is nonempty

let $A = (s_1, t_1, s_1+t_1)$ $B = (s_2, t_2, s_2+t_2)$

$A + B = (s_1+s_2, t_1+t_2, (s_1+s_2)+(t_1+t_2)) \in W$

let c be scalar

$cA = (cs_1, ct_1, cs_1+ct_1) = (s_1, t_1, s_1+t_1) \in W$

$\Rightarrow$ W is a subspace of R^3

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6. $aA + bB = \begin{bmatrix} 6 & 2 \\ 9 & 11 \end{bmatrix}$

$a\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 9 & 11 \end{bmatrix}$

$\begin{cases} 2a = 6 & \cdots ① \\ -3a + 5b = 2 & \cdots ② \\ 4a + b = 9 & \cdots ③ \\ a - 2b = 11 & \cdots ④ \end{cases}$ from ①, ② solve $\begin{cases} a = 3 \\ b = -3 \end{cases}$

$\begin{cases} a=3 \\ b=-3 \end{cases}$ put into ④ $\quad 3 + 6 \neq 11$

$\therefore$ The given matrix is not a linear combination of A & B

18. let (u_1, u_2) be any vector R^2, scalar C_1, C_2, C_3

$$(u_1, u_2) = C_1(-1,2) + C_2(2,-1) + C_3(1,1)$$
$$= (-C_1 + 2C_2 + C_3,\ 2C_1 - C_2 + C_3)$$

$$\begin{cases} -C_1 + 2C_2 + C_3 = u_1 \\ 2C_1 - C_2 + C_3 = u_2 \end{cases}$$

let $C_3 = t \in R$, $C_1 = \frac{u_1 + 2u_2}{3} - t$, $C_2 = \frac{2u_1 + u_2}{3} - t$

$\Rightarrow$ a solution exist

$\Rightarrow$ S spans R^2

26. $P^3 = a_0 x^3 + a_1 x^2 + a_2 x + a_3$

$$C_1(-2x + x^2) + C_2(8 + x^3) + C_3(-x^2 + x^3) + C_4(-4 + x^2) = P^3$$

$$\Rightarrow \begin{cases} C_2 + C_3 = a_0 \\ C_1 - C_3 + C_4 = a_1 \\ -2C_1 = a_2 \\ 8C_2 - 4C_4 = a_3 \end{cases}$$

$$\Rightarrow \begin{vmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 1 \\ -2 & 0 & 0 & 0 \\ 0 & 8 & 0 & 4 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \Rightarrow \text{idetity matrix}$$

$\Rightarrow$ S spans P^3

44 take c_1, c_2,

$$c_1 x^2 + c_2(1+x^2) = 0$$

$$(c_1+c_2)x^2 + 0x + c_2 = 0x^2 + 0x + 0$$

$\Rightarrow c_2 = 0,\ c_1 = 0$

$\therefore S$ is linear independent set in P_2