MIE 1621 Computational project Junyu Jiang 1002916561

Part 1

(a), (b) & (c)

Text

Description automatically generatedThe algorithm of using gradient-based method with backtracking to solve multivariate function is shown above.

(d)

Text

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Description automatically generated1) Function and variable set-up for f1(x)= x12+x22+x32:

It converges at iteration 1

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Description automatically generated2) Function and variable set-up for f2(x) = x12+2x22-2x1x2-2x2

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Description automatically generated3) Function and variable set-up for f3(x) = 100(x2 - x12)2 + (1-x1)2

It converges at iteration 52

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Description automatically generated4) Function and variable set-up for f4(x) = (x1+x2)4 + x22

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Iteration maximum is reached.

5) Function and variable set-up for f5(x) = (x1-1)2 + (x2-1)2 + c(x12 + x22-0.25)2

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Description automatically generatedWhen c=1 :

It converges at iteration 21.

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Description automatically generatedWhen c = 10:

It converges at iteration 150.

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Description automatically generatedWhen c = 100:

Iteration maximum is reached.

Larger c will make the problem be slower due to more iterations will be made.

e) All the problems above can converge to the optimal solution except for f3. The problem for f3 is that alpha becomes too small after backtracking. Therefore, it can only change the value of x a little after one iteration.

Part 2:

1. Text

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   Description automatically generatedNewton’s method to minimize 100x14 + 0.01x24

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It converges at iteration 17.

Hessians are always positive definite.

1. Text

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   Description automatically generatedGradient-based method with backtracking to minimize 100x14 + 0.01x24 :

Iteration maximum is reached. It is not an efficient method to optimize this function since it needs a great number of runs to get the optimal solution.

1. Text

   Description automatically generatedNewton’s method to minimize

The algorithm starts an infinite loop and cannot converge to an optimal solution.

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Description automatically generatedHere is a screen shot of the output:

1. Text

   Description automatically generatedGradient-based method with backtracking to minimize

Text

Description automatically generated

The algorithm converges at iteration 3.

1. Text

   Description automatically generatedNewton’s method result (x0 = [10, 10]T):

The result shows that it is not converged, but diverged when a larger initial point is typed in.

Text

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Description automatically generatedGradient-based method with backing result (x0 = [10, 10]T):

It converges at iteration 13.

1. Text

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   Description automatically generatedNewton’s method with backtracking result (x0 = [10, 10]T):

Iteration maximum is reached

Here is a summary for (c), (d), (e) and (f):

|  |  |  |
| --- | --- | --- |
| **Method** | **Initial Point** | **# of iteration** |
| Newton’s | (1,1) | Diverged (error) |
| Gradient-based with backtracking | (1,1) | 3 |
| Newton’s | (10,10) | Diverged |
| Gradient-based with backtracking | (10,10) | 13 |
| Newton’s with backtracking | (10,10) | 1000+ |