TMA 4130 Øving 1

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Oppgave 1

$P = 2\pi$

$$\boxed{a_0 = \frac{1}{P}\int_{-\frac{P}{2}}^{\frac{P}{2}} f(x)\,dx}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 5 - 4\cos(2x) - 2\sin(5x) + 5\cos(8x)\,dx$$

$$= \frac{1}{2\pi}\Big[5x - 2\sin(2x) + \frac{2}{5}\cos(5x) + \frac{5}{8}\sin(8x)\Big]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\Big[5\pi - \cancel{\frac{2}{5}} + 5\pi + \cancel{\frac{2}{5}}\Big]$$

$$\underline{\underline{a_0 = 5}}$$

$$a_n = \frac{1}{\frac{1}{2}P}\int_{-\frac{1}{2}P}^{\frac{1}{2}P} f(x)\cos\left(\frac{n\pi x}{\frac{1}{2}P}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\big(5 - 4\cos(2x) - 2\sin(5x) + 5\cos(8x)\big)\cos\frac{n\pi x}{\pi}\,dx$$

= vanskelig ser heller på cos funksjonen

$\underline{a_n = -4}$ når $n = 2$ $(-2\cos(2x))$

$\underline{a_n = 5}$ når $n = 8$ $(5\cos(8x))$

$\underline{a_n = 0}$ når $n \neq 2$ eller 8

gjør det same for b_n med sin

$b_n = -2$ når $n = 5$ $(-2\sin(5x))$

$b_n = 0$ når $n \neq 5$

oppgave 2

a) p, $k > 0$

$f(kx)$: tester for $\frac{p}{k}$

$f(x+p) = f(x) \Rightarrow g(x) = f(kx)$

$\Rightarrow g(x + \frac{p}{k}) = f(k(x + \frac{p}{k}))$

$= f(kx + p) = f(kx) = g(x)$ ok

$f(\frac{x}{k})$: tester for kp

$\Rightarrow g(x + pk) = f(\frac{x + pk}{k}) =$

$= f(\frac{x}{k} + p) = g(x)$ ok

b) $k = [0, 1, \ldots]$

$f(x + kp) = f(x)$

$g(x) = f(kx) \Rightarrow g(x+p) = f(k(x+p))$

$= f(kx + kp) = f(kx + p) = f(kx)$ ok

c) $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

$P = 2\pi$ $g(x) = f(3x)$

Skal finne sammenhengen mellom

a_0, a_n, b_n til $f(x)$ og a_0', a_n', b_n' til $g(x)$

$g(x) = a_0' + \sum_{n=1}^{\infty} (a_n' \cos 3nx + b_n' \sin 3nx)$

det gir

$a_0' = a_0$

$a_n' = a_{n/3}$ når n er delig på 3 ellers 0

$a_n' = b_{n/3}$ når n er delig på 3 ellers 0

Oppgave 3

$P = 2\pi \quad [-\pi, \pi]$

$$f(x) = \begin{cases} -\sin(x), & -\pi < x \le 0 \\ \sin(x), & 0 < x < \pi \end{cases}$$

a)

$$a_0 = \frac{1}{2\pi}\left(\int_{-\pi}^{0} -\sin(x)\,dx + \int_{0}^{\pi}\sin(x)\,dx\right)$$

$$= \frac{1}{2\pi}\left(\left[\cos(x)\right]_{-\pi}^{0} + \left[-\cos(x)\right]_{0}^{\pi}\right)$$

$$= \frac{1}{\pi}$$

$b_n = 0$ pga $f(x)$ er en like funksjon

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -\sin(x)\cos(x) + \frac{1}{\pi}\int_{0}^{\pi}\sin x \cos x$$

$$= \frac{2}{\pi}\int_{0}^{\pi}\sin(x)\cos(x)\,dx$$

$$\underline{2\sin x \cos x = \sin((n+1)x) - \sin((n-1)x)}$$

$$= \frac{1}{\pi}\int_{0}^{\pi}\sin((n+1)x) - \sin((n-1)x)\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos((n+1)x)}{n+1} + \frac{\cos((n-1)x)}{n-1}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\cos((n+1)\pi)}{n+1} + \frac{\cos((n-1)\pi)}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$\cos n\pi = (-1)^n$

$$= \frac{1}{\pi}\left(\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right)$$

$$= \frac{1}{\pi}\left(\frac{(-1)^n+1}{n+1} - \frac{(-1)^n+1}{n-1}\right) = \frac{((-1)^n+1)(n-1) - ((-1)^n+1)(n+1)}{n^2+1}$$

$$= -\frac{2((-1)^n+1)}{\pi(n^2-1)}$$

oppgave 3

a) $a_n = \begin{cases} -\frac{4}{\pi(4n^2-1)} & \text{når } n \text{ er delig på } 2 \\ 0 & \text{når } n \text{ ikke er delig på } 2 \end{cases}$

$$f(x) = \frac{1}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{4n^2-1}\cos 2nx$$

b)

$$f(0) = -\sin 0 = 0$$

$$f(0) = \frac{1}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{4n^2-1} \cdot 1 = 0$$

$$\sum \frac{1}{(4n^2-1)} = \frac{1}{\pi}\frac{\pi}{4} = \underline{\underline{\frac{1}{4}}}$$

oppgave 4

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x(\pi - x), & 0 \leq x < \pi \end{cases} \qquad P = (-\pi, \pi)$$

a)

$$a_0 = \frac{1}{2\pi} \int_0^{\pi} x\pi - x^2 \, dx$$

$$= \frac{1}{2\pi} \left[\frac{x^2 \pi}{2} - \frac{x^3}{3} \right]_0^{\pi} = \frac{1}{2\pi}\left(\frac{\pi^3}{2} - \frac{\pi^3}{3}\right) = \underline{\frac{\pi^2}{12}}$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} x(\pi - x) \cos nx \, dx$$

$$= \frac{1}{\pi} \Big[\underbrace{\int_0^{\pi} x\pi \cos nx \, dx}_{\mathrm{I}} - \underbrace{\int_0^{\pi} x^2 \cos nx \, dx}_{\mathrm{II}}$$

$$\mathrm{I} = \left[\frac{1}{n} x \sin nx\right]_0^{\pi} - \int_0^{\pi} \frac{\sin nx}{n} = 0 + \left[\frac{1}{n^2} \cos nx\right]_0^{\pi}$$

$$= \frac{\pi(-1)^n - 1}{n^2}$$

$$\mathrm{II} = \left[\frac{1}{n} x^2 \sin nx\right]_0^{\pi} - \int_0^{\pi} \frac{2x \sin nx}{n}$$

$$= 0 + \frac{2}{n}\left[\boxed{\mathrm{I}}\right] = \underline{\frac{2\pi(-1)^n}{n^2}}$$

$$a_n = \underline{\frac{(-1)^n - 1 - 2(-1)^n}{n^2}}$$

oppgave 3

a) $$b_n = \underbrace{\frac{1}{\pi}\int_0^{\pi} x\pi \sin nx}_{\mathrm{I}} + \underbrace{\frac{1}{\pi}\int_0^{\pi} x^2 \sin nx}_{\mathrm{II}}$$

$$\mathrm{I} = \frac{1}{\pi}\left[-\frac{1}{n}x\cos nx\right]_0^{\pi} + \int_0^{\pi}\frac{\cos nx}{n}$$

$$= \underline{-\frac{1}{n}(-1)^n + 0}$$

$a_n \mathrm{I}$

$$\mathrm{II} = \frac{1}{\pi}\left[-\frac{1}{n}x^2\cos nx\right]_0^{\pi} + \frac{1}{\pi}\int_0^{\pi}\frac{2x\cos nx}{n}$$

$$= -\frac{1}{n}\pi(-1)^n + \frac{2}{\pi n^3}((-1)^n - 1)$$

$$b_n = -\frac{\pi}{n}(-1)^n + \frac{\pi(-1)^n}{n} - \frac{2((-1)^n-1)}{\pi n^3}$$

$$= \frac{-(-1)^n n^2 + (-1)^n n^2 - 2((-1)^n-1)}{\pi n^3}$$

$$= \begin{cases} \frac{4}{n^3\pi} & \text{når } n \text{ er delbar på } 2 \\ 0 & \text{når } n \text{ ikke er delbar på } 2 \end{cases}$$

b) $$f(x) = \frac{\pi^2}{12} - \sum \frac{1}{2n^2}\cos 2nx + \frac{4}{\pi}\sum \frac{1}{(2n-1)^3}\sin((2n-1)x)$$

b) $$f(0) = 0 = \frac{\pi^2}{12} - \sum \frac{1}{2n^2}$$

$$\Rightarrow \sum \frac{1}{n^2} = \frac{\pi^2}{12}\cdot 2 = \underline{\underline{\frac{\pi^2}{6}}}$$