Solving Quadratic Equations using Δ:

General Form $\boxed{y = ax^2 + bx + c}$

Calculate $\Delta = b^2 - 4ac$

Case 1: If $\Delta < 0$, no real roots

Factorized form: X (no factorized form)

Case 2: If $\Delta > 0$, 2 real roots x_1 and x_2

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} \qquad x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

Factorized form: $y = a(x - x_1)(x - x_2)$

Case 3: If $\Delta = 0$, 1 double root x_0

$$x_0 = \frac{-b}{2a}$$

Factorized form: $y = a(x - x_0)^2$

Example: Factorize $y = 2x^2 - 3x + 1$ $\left(\begin{array}{l} a = 2 \\ b = -3 \\ c = 1 \end{array}\right)$

$\Delta = b^2 - 4ac = (-3)^2 - 4(2)(1) = 9 - 8 = 1 > 0 \Rightarrow$ 2 roots

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{3 - \sqrt{1}}{2(2)} = \frac{2}{4} = \frac{1}{2}$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{3 + \sqrt{1}}{2(2)} = \frac{4}{4} = 1$$

Factorized form: $y = a(x - x_1)(x - x_2)$

$= 2\left(x - \frac{1}{2}\right)(x - 1)$

Special Cases: for $ax^2+bx+c=0$

① $a+b+c=0$

$\Rightarrow \boxed{x_1=1}$ $\quad \boxed{x_2=\frac{c}{a}}$

ex: $x^2+3x-4=0$

$a=1, b=3, c=-4$

$a+b+c=0$

$\Rightarrow \boxed{x_1=1}$ $\quad x_2=\frac{c}{a}=\frac{-4}{1}$

$\boxed{x_2=-4}$

② $a-b+c=0$

$\boxed{x_1=-1}$ $\quad \boxed{x_2=\frac{-c}{a}}$

ex: $x^2-3x-4=0$

$a=1, b=-3, c=-4$

$a-b+c=0$

$\boxed{x_1=-1}$ $\quad x_2=\frac{-c}{a}=\frac{4}{1}$

$\boxed{x_2=4}$