Chapter_8 - Techniques of Integration:

Section 8.1 - Using Basic Integration Formulas:

Basic Integration Formulas:

1) $\int k\,dx = kx + C$

ex: $\int 2\,dx = 2x + C$

2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

ex: $\int \frac{dx}{\sqrt{x}} = \int x^{-1/2}\,dx = \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$

3) $\int \frac{dx}{x} = \ln|x| + C$

$\int \frac{du}{u} = \ln|u| + C$

4) $\int e^x\,dx = e^x + C$

$\int e^u\,du = e^u + C$

5) $\int a^x\,dx = \frac{a^x}{\ln a} + C \quad (a > 0)$

ex: $\int 3^x\,dx = \frac{3^x}{\ln 3} + C$

6) $\int \sin x\,dx = -\cos x + C$

7) $\int \cos x\,dx = \sin x + C$

8) $\int \sec^2 x\,dx = \tan x + C$

9) $\int \csc^2 x\,dx = -\cot x + C$

10) $\int \sec x \tan x\,dx = \sec x + C$

Scanned with CamScanner

11) $\int \csc x \cot x \, dx = -\csc x + c$

12) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + c$

$= -\ln\left|\frac{1}{\sec x}\right| + c = -\ln(1) + \ln|\sec x| + c = \ln|\sec x| + c$

13) $\int \cot x \, dx = \ln|\sin x| + c = -\ln|\csc x| + c$

14) $\int \sec x \, dx = \ln|\sec x + \tan x| + c$

Proof: $\int \sec x \, dx = \int \sec x . \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx = \ln|\sec x + \tan x| + c$

15) $\int \csc x \, dx = -\ln|\csc x + \cot x| + c$

16) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

17) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

Exercises:

2) $\int \frac{x^2}{x^2+1} dx = \int \frac{x^2+1-1}{x^2+1} dx = \int \left(1 - \frac{1}{x^2+1}\right) dx$

$= \int 1 \, dx - \int \frac{1}{x^2+1} dx = x - \tan^{-1} x + c$

Scanned with CamScanner

10) $\displaystyle\int_1^2 \frac{8\,dx}{x^2-2x+2} = \int_1^2 \frac{8\,dx}{x^2-2x+1+1} = \int_1^2 \frac{8\,dx}{(x-1)^2+1}$

$\boxed{\begin{array}{l} u = x-1 \\ du = 1\,dx \end{array}}$

$\boxed{\begin{array}{l} x=1 \Rightarrow u=0 \\ x=2 \Rightarrow u=1 \end{array}}$

$\displaystyle = \int_0^1 \frac{8\,du}{u^2+1} = \left[8\tan^{-1}u\right]_0^1 = 8\left(\tan^{-1}1 - \tan^{-1}0\right)$

$\displaystyle = 8\left(\frac{\pi}{4} - 0\right) = 2\pi$

19) $\displaystyle\int \frac{d\theta}{\sec\theta + \tan\theta} = \int \frac{d\theta}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} = \int \frac{d\theta}{\frac{1+\sin\theta}{\cos\theta}}$

$\displaystyle = \int \frac{\cos\theta}{1+\sin\theta}\,d\theta = \int \frac{du}{u} = \ln|u| + c$

$= \ln|1+\sin\theta| + c$

$= \ln(1+\sin\theta) + c \qquad (\sin\theta \geq -1)$

$\boxed{\begin{array}{l} u = 1+\sin\theta \\ du = \cos\theta\,d\theta \end{array}}$

22) $\displaystyle\int \frac{x + 2\sqrt{x-1}}{2x\sqrt{x-1}}\,dx = \int \frac{x}{2x\sqrt{x-1}}\,dx + \int \frac{2\sqrt{x-1}}{2x\sqrt{x-1}}\,dx$

$\displaystyle = \int \frac{1}{2\sqrt{x-1}}\,dx + \int \frac{1}{x}\,dx$

$\boxed{\begin{array}{l} u = x-1 \\ du = 1\,dx \end{array}}$

$\displaystyle = \int \frac{1}{2\sqrt{u}}\,du + \ln|x| + c$

$= \sqrt{u} + \ln|x| + c = \sqrt{x-1} + \ln|x| + c$

$= \sqrt{x-1} + \ln x + c \qquad (x \geq 1)$

Scanned with CamScanner

23) $\displaystyle\int_0^{\pi/2} \sqrt{1-\cos\theta}\, d\theta = \int_0^{\pi/2} \sqrt{2\sin^2\frac{\theta}{2}}\, d\theta$

$$\sin^2\frac{\theta}{2} = \frac{1-\cos 2\left(\frac{\theta}{2}\right)}{2}$$

$$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$$

$$\Longrightarrow 1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

$$0 \leq \frac{\theta}{2} \leq \frac{\pi}{4}$$

$$\Rightarrow \sin\frac{\theta}{2} \geqslant 0$$

$$= \int_0^{\pi/2} \sqrt{2}\,.\left|\sin\frac{\theta}{2}\right| d\theta$$

$$= \int_0^{\pi/2} \sqrt{2}\, \sin\frac{\theta}{2}\, d\theta$$

$$= \sqrt{2}\left[-2\cos\frac{\theta}{2}\right]_0^{\pi/2}$$

$$= -2\sqrt{2}\left[\cos\frac{\pi}{4} - \cos 0\right]$$

$$= -2\sqrt{2}\left(\frac{\sqrt{2}}{2} - 1\right) = -2 + 2\sqrt{2}$$

Scanned with CamScanner