Revision:

Trigonometric Functions:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x}$$

secant

$$\csc x = \frac{1}{\sin x}$$

cosecant

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x} = \sec^2 x = 1+\tan^2 x$$

$$\frac{d}{dx}(\cot x) = \frac{-1}{\sin^2 x} = -\csc^2 x = -(1+\cot^2 x)$$

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{0(\cos x) - (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sec x . \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x . \cot x$$

Trigonometric Identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\sin x = \pm\sqrt{1-\cos^2 x}$$

$$\tan x = \pm\sqrt{\sec^2 x - 1}$$

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b.$$

Cofunction Identities:

$\sin\left(\frac{\pi}{2}-x\right) = \cos x$

$\cos\left(\frac{\pi}{2}-x\right) = \sin x$

$\tan\left(\frac{\pi}{2}-x\right) = \cot x$

$\cot\left(\frac{\pi}{2}-x\right) = \tan x$

$\sec\left(\frac{\pi}{2}-x\right) = \csc x$

$\csc\left(\frac{\pi}{2}-x\right) = \sec x$

Even / Odd Identities:

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

$\csc(-x) = -\csc x$

$\sec(-x) = \sec x$

Inverse Trigonometric Functions:

- $y = \sin^{-1}x = \arcsin x$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin y = x$

  ex: $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

- $y = \cos^{-1}x = \arccos x$ is the angle in $[0, \pi]$ such that $\cos y = x$
- $y = \tan^{-1}x = \arctan x$ is the angle in $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ such that $\tan y = x$.

$$\frac{d}{dx}\left(\sin^{-1}u\right) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\left(\cos^{-1}u\right) = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\left(\tan^{-1}u\right) = \frac{u'}{1+u^2}$$

ex: $y = \tan^{-1}(2x+1)$

$$\frac{dy}{dx} = \frac{2}{1+(2x+1)^2}$$

$$\lim_{x\to\infty} \tan^{-1}x = \frac{\pi}{2}$$

$$\lim_{x\to-\infty} \tan^{-1}x = \frac{-\pi}{2}$$

ex: $\tan^{-1}(1) = \frac{\pi}{4}$

$\tan^{-1}(0) = 0$

$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.