24-b $\quad V = \{ (a,1,0) \} \subset \mathbb{R}^3$

V is not closed under addition because

$(a,1,0)+(b,1,0) = (a+b,2,0) \notin V$

Conc: V is not a subspace of $\mathbb{R}^3$.

24-c $\quad V = \{(a,b,c) : b = a+c\} \subset \mathbb{R}^3$

1) $V \neq \phi$ because $(1,2,1) \in V$

2) let (a_1, b_1, c_1) and (a_2, b_2, c_2) be in V

then $(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1+a_2, b_1+b_2, c_1+c_2) \in V$

because $b_1+b_2 = a_1+c_1+a_2+c_2$; $\boxed{V \text{ is closed under } +}$

3) let (a,b,c) be in V, then $k(a,b,c) = (ka, kb, kc) \in V$

because $kb = k(a+c) = ka + kc$. Hence V is closed under scalar mult.

therefore V is a subspace of $\mathbb{R}^3$.

25-b $V = \{ A \in M_{nn} : |A| = 0 \}$.

V is not closed under addition because

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad ; \quad A+B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$\therefore$ V is not a subspace of M_{nn}

<u>25-C</u> $V = \{ A \in M_{nn} : tr(A) = 0 \}$.

1) See that $0 \in V$, hence $V \neq \phi$

2) If A and B are in V, then
$tr(A+B) = tr(A) + tr(B) = 0 + 0 = 0$
$\therefore A+B \in V$

3) If $A \in V$ and $k \in \mathbb{R}$, then $tr(kA) = k\,tr(A) = 0$
$\therefore kA \in V$

Conc: V is a subspace of M_{nn}

25-e $\quad V = \{ A \in M_{nn} : A^T = -A \}$.

1) $0 \in V$?

2) If A and B are in V, then $A+B \in V$?

$(A+B)^T = A^T + B^T = -A - B = -(A+B)$

3) If $A \in V$ and $k \in \mathbb{R}$, then $kA \in V$?

$(kA)^T = k A^T = k(-A) = -(kA)$.

. . . . . .

§6 - a

Def: P_3 all real polynomials of deg ≤ 3.

i.e: $a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

$V = \{ a_0 + a_1 x + a_2 x^2 + a_3 x^3$ s.t. $a_1 = a_2 \}$

1) $V \neq \phi$ because $1 + 2x + 2x^2 + 1000x^3 \in V$

2) V is closed under addition ?

3) " " -- scalar mult.?

26 - c : V is not a subspace of P_3

because it's not closed under scalar mult.

$V = \{ a_0 + a_1 x + a_2 x^2 + a_3 x^3$ s.t. a_0, a_1, a_2, a_3 are in $\mathbb{Z} \}$

for ex : $P = 1 + x + x^2 + x^3$ and $k = \frac{1}{2}$.

30 Given $\vec{u} = (1, -3, 2)$ $\vec{v} = (1, 0, -4)$

a) $\vec{w} = (0, -3, 6)$

see that $\vec{w} = \vec{u} - \vec{v}$

$\therefore$ $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$.

b) $\vec{w} = (3, -9, -2)$. If $\vec{w} = k_1\vec{u} + k_2\vec{v}$

then

$\longrightarrow$

$$k_1\begin{pmatrix}1\\-3\\2\end{pmatrix} + k_2\begin{pmatrix}1\\0\\-4\end{pmatrix} = \begin{pmatrix}3\\-9\\-2\end{pmatrix}$$

$\therefore$

$$\begin{aligned} k_1 + k_2 &= 3 \\ -3k_1 &= -9 \\ 2k_1 - 4k_2 &= -2 \end{aligned}$$

which is inconsistant ?

Hence $\vec{W}$ is not a linear combination of $\vec{u}$ and $\vec{v}$.

32 - 9

$A = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$; $B = \begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix}$; $C = \begin{pmatrix} 1 & 1 \\ -2 & 5 \end{pmatrix}$

$D = \begin{pmatrix} 2 & 5 \\ -2 & 4 \end{pmatrix}$

is $D = k_1 A + k_2 B + k_3 C$?

$= \begin{pmatrix} 3k_1 + k_3 & 2k_1 + 2k_2 + k_3 \\ -2k_2 - 2k_3 & k_1 + 4k_2 + 5k_3 \end{pmatrix}$

$\therefore$

$3k_1 + k_3 = 2$

$2k_1 + 2k_2 + k_3 = 5$

$-2k_2 - 2k_3 = -2$

$k_1 + 4k_2 + 5k_3 = 4$

. . . .

33- a

$P_1 = 2 + x + 4x^2$

$P_2 = 1 - x + 3x^2$

$P_3 = 3 + 2x + 5x^2$

a) $P = 6 + 2x + 12x^2$.

If $P = k_1 P_1 + k_2 P_2 + k_3 P_3$, then

$2k_1 + k_2 + 3k_3 = 6$

$k_1 - k_2 + 2k_3 = 2$

$4k_1 + 3k_2 + 5k_3 = 12$

Recall:

$P(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

$Q(x) = b_0 + b_1 x + b_2 x^2 + \cdots + b_n x^n$

$P(x) = Q(x)$ for all x in $\mathbb{R}$ $\Longleftrightarrow$

$a_i = b_i$ for $0 \le i \le n$

_ _ _ _ _

34 - a $\quad \vec{v}_1 = (1,2,3)$; $\vec{v}_2 = (2,0,0)$; $\vec{v}_3 = (-2,1,0)$ in $\mathbb{R}^3$

Is $\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \mathbb{R}^3$?

let $\vec{b} = (b_1, b_2, b_3)$. If $\vec{b} \in \text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$, then

$\vec{b} = k_1\vec{v}_1 + k_2\vec{v}_2 + k_3\vec{v}_3$ Hence

$(b_1, b_2, b_3) = k_1(1,2,3) + k_2(2,0,0) + k_3(-2,1,0)$ and

$$(*)\quad \begin{aligned} k_1 + 2k_2 - 2k_3 &= b_1 \\ 2k_1 + 0 + k_3 &= b_2 \\ 3k_1 &= b_3 \end{aligned}$$

$A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 0 & 1 \\ 3 & 0 & 0 \end{pmatrix}$; $|A| = 3\begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} = 6$

$\therefore (*)$ is consistant for all $\vec{b} \in \mathbb{R}^3$

$\therefore \mathbb{R}^3 = \text{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$

35 - a

Given

$\vec{v}_1 = (2, 1, 3, 0)$

$\vec{v}_2 = (3, -1, 2, 5)$

$\vec{v}_3 = (-1, 0, 1, 2)$

$\vec{v} = (9, 0, 11, 12)$

Is $\vec{v} = k_1\vec{v}_1 + k_2\vec{v}_2 + k_3\vec{v}_3$?

$\Updownarrow$

$$2k_1 + 3k_2 - k_3 = 9$$

$$k_1 - k_2 = 0$$

$$3k_1 + 2k_2 + k_3 = 11$$

$$5k_2 + 2k_3 = 12$$

. . . . . .

36) Is $P_2 = \text{span}(P_1, P_2, P_3, P_4)$?

$P_1 = 1 + x + 2x^2$

$P_2 = 3 + x$

$P_3 = 5 - x + 4x^2$

$P_4 = -2 - 2x + 2x^2$

Let $b = b_1 + b_2 x + b_3 x^2$

$b = k_1 P_1 + k_2 P_2 + k_3 P_3 + k_4 P_4$

$k_1 + 3k_2 + 5k_3 - 2k_4 = b_1$

$k_1 + k_2 - k_3 - 2k_4 = b_2$

$2k_1 + 0 + 4k_3 + 2k_4 = b_3$

- - —