2) $V = \mathbb{R}^2$; $u = (u_1, u_2)$ $v = (v_1, v_2)$

$u + v = (u_1 + v_1 - 1, u_2 + v_2 - 1)$ $\quad ku = (ku_1, ku_2)$

a) $\left.\begin{array}{l} u = (1, -2) \\ v = (2, 0) \end{array}\right\} \Rightarrow u + v = (2, -3)$; $3u = (3, -6)$

b) show that $(0,0) \neq \vec{0} \overset{?}{=\!=} (1,1)$

c) // $(1,1) = 0$ ✓ Done

Ax 8 is False

d) $-u = (2 - u_1, 2 - u_2)$

e) Ax 7 is False because $k(u+v) \neq ku + kv$ if $k \neq 1$

4) $V = \{(0,y) : y \in \mathbb{R}\}$. ($y$-axis).

<u>Ax1</u>: If u and v are in V, then
$u+v = (0,u_2)+(0,v_2) = (0, u_2+v_2) \in V$

<u>Ax2</u>: $u+v = v+u$ ✓ ?

<u>Ax3</u>: $(u+v)+w = u+(v+w)$ True?

<u>Ax4</u>: $\vec{0} = (0,0)$? because $\vec{0}+\vec{u} = \vec{u}$

<u>Ax5</u>: $-\vec{u} = (0,-u_2)$; $\vec{u}+(-\vec{u}) = \vec{0}$

Ax 6: $k\vec{u} = k(0, u_2) = (0, ku_2) \in V$ ✓

Ax 7: $k(\vec{u}+\vec{v}) = k(0, u_2+v_2) = (0, ku_2+kv_2) = k\vec{u} + k\vec{v}$ ✓

Ax 8: $(k+l)\vec{u} = (0, (k+l)u_2) \cdots = k\vec{u} + l\vec{u}$

Ax 9: $k(m\vec{u}) = k(0, mu_2) = (0, kmu_2) = (km)\vec{u}$.

Ax 10 $1\vec{u} = 1(0, u_2) = (0, u_2) = \vec{u}$.

7) $V = \mathbb{R}^3$ $\vec{u} = (u_1, u_2, u_3)$

$k\vec{u} = (k^2 u_1, k^2 u_2, k^2 u_3)$

Ax1 — Ax5 are True since + is standard.

Ax6: T ?

Ax7: $k(u+v) = ku + kv$ True ?

Ax8: $(k+l)\vec{u} = k\vec{u} + l\vec{u}$ False?

Ax9 $k(m\vec{u}) = (km)\vec{u}$ True?

Ax10: $1\vec{u} = \vec{u}$ True ?

8) $V = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ invertible

$u = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$ $v = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$

Ax 1: $u + v = \begin{bmatrix} u_1 + v_1 & u_2 + v_2 \\ u_3 + v_3 & u_4 + v_4 \end{bmatrix}$

False

let $u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $v = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $u+v = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ not invertible

Ax 2: $u + v = v + u$ (Known from chap 1)

True

Ax 3: $(u+v)+w = u+(v+w)$ True

Ax 4: there is no 0 in V such that $u+0=0+u=u$

False

Ax 5: False (everytime Ax 4 fails, then Ax 5 also fails)

Ax 6: $Ku = \begin{bmatrix} Ku_1 & Ku_2 \\ Ku_3 & Ku_4 \end{bmatrix}$ fails when $K=0$

False

Ax 7: $K(u+v) = Ku + Kv$ True

Ax 8: $(K+m)u = Ku + mu$ True

Ax 9: True

Ax 10: True

9) $V = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$u = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$ $\quad v = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$

<u>Ax 1:</u> $u+v = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix}$ is in V

<u>Ax 2:</u> $u+v = v+u$ true

<u>Ax 3:</u> $(u+v)+w = u+(v+w)$ true

Ax 4: $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is in $V \Rightarrow$ True

Ax 5: $-u = \begin{bmatrix} -a_1 & 0 \\ 0 & -b_1 \end{bmatrix}$ is in V

$(-u) + u = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ True

Ax6: $ku = \begin{pmatrix} ka_1 & 0 \\ 0 & kb_1 \end{pmatrix}$ is in V for $k \in \mathbb{R}$

True

Ax 7: $k(u+v) = ku + kv$ true

Ax 8: $(k+m)u = ku+mu$ true

Ax 9: $k(mu) = (km)u$ True

Ax 10: $1u = u$ true