#43 a) $\vec{u}_1 = (3,-1)$ $\vec{u}_2 = (6,-2)$ in $\mathbb{R}^2$

$\vec{u}_1$ and $\vec{u}_2$ are lin. dep. because $\vec{u}_2 = 2\vec{u}_1$.

b) $\vec{u}_1 = (-2,0,1)$, $\vec{u}_2 = (4,-2,0)$, $\vec{u}_3 = (6,-6,3)$ in $\mathbb{R}^3$

$\vec{u}_1 + \vec{u}_2 = (2,-2,1)$

$\vec{u}_3 = 3\vec{u}_1 + 3\vec{u}_2$

$\therefore S = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is lin. dep.

c) $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ in M_{22}

$B = \begin{bmatrix} 0 & -1 \\ -2 & -3 \end{bmatrix}$

A & B are lin. dep because $B = -A$.

d) $P_1 = 2 + x - 3x^2$ in P_2.

$P_2 = -4 - 2x + 6x^2$

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46 - b

$P_1 = 3 + x + x^2$

$P_2 = 2 - x + 5x^2$ in P_2

$P_3 = 4 - 3x^2$

conc: $S = \{P_1, P_2, P_3\}$ is lin. ind.

If $k_1 P_1 + k_2 P_2 + k_3 P_3 = \vec{0}$ then

$k_1(3 + x + x^2) + k_2(2 - x + 5x^2) + k_3(4 - 3x^2) = \vec{0} = 0 + 0x + 0x^2$

$3k_1 + 2k_2 + 4k_3 = 0$

$k_1 - k_2 = 0$

$k_1 + 5k_2 - 3k_3 = 0$

$$|A| = \begin{vmatrix} 3 & 2 & 4 \\ 1 & -1 & 0 \\ 1 & 5 & -3 \end{vmatrix} = - \begin{vmatrix} 2 & 4 \\ 5 & -3 \end{vmatrix} - \begin{vmatrix} 3 & 4 \\ 1 & -3 \end{vmatrix}$$

$$= -[-26 - 13] = 39 \neq 0$$

50) $\vec{v}_1 = (1,2,3,4)$

a) $\vec{v}_2 = (2,2,2,4)$ in $\mathbb{R}^4$

$\vec{v}_3 = (1,0,-1,0)$

by inspection, we see that $\boxed{\vec{v}_3 = \vec{v}_2 - \vec{v}_1}$ (*)

Hence $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is lin. dep.

b) (*) $\Rightarrow \vec{v}_2 = \vec{v}_1 + \vec{v}_3$

and $\vec{v}_1 = \vec{v}_2 - \vec{v}_3$

55) Given $S = \{\vec{v}_1, \vec{v}_2\}$ lin. ind.

and $\vec{v}_3 \notin \text{span}(S)$.

then: $\vec{v}_1 \neq \vec{0}$; $\vec{v}_2 \neq \vec{0}$; $\vec{v}_3 \neq \vec{0}$ and $\vec{v}_3$ is not a linear combination of $\vec{v}_1$ and $\vec{v}_2$

Show that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is lin. ind.

pf: suppose that $k_1\vec{v}_1 + k_2\vec{v}_2 + k_3\vec{v}_3 = \vec{0}$

case 1: If $k_3 = 0$, then $k_1\vec{v}_1 + k_2\vec{v}_2 = \vec{0}$, which implies $k_1 = k_2 = 0$ because $\vec{v}_1$ and $\vec{v}_2$ are lin. ind.

Case 2: If $k_3 \neq 0$, then $\vec{v}_3 = -\frac{k_1}{k_3}\vec{v}_1 - \frac{k_2}{k_3}\vec{v}_2$

i.e. $\vec{v}_3 \in \text{Span}(S)$. contradiction