Chapter 2: Force Vectors

Instructor: Christine SAAB

Textbook: R.C. Hibbeler, *Engineering Mechanics: Statics*, 14th Edition, Pearson Prentice Hall, ISBN 978-981-06-8134-0, 2010

Plan

- Scalars and vectors
- Vector operations
- Vector addition of forces
- Addition of a system of coplanar forces
- Cartesian vectors
- Addition of Cartesian vectors
- Position vectors
- Force vector directed along a line
- Dot product

Scalars and Vectors

- **Scalar**: a quantity that has only a magnitude.

Example: mass, length, time, temperature, volume, density

- **Vector**: a quantity that has both magnitude and direction.

Example: position, displacement, velocity, acceleration, momentum, force

Scalars and Vectors

Vector:

- Represented by a letter with an arrow over it such as $\vec{A}$ or by a bold face letter such as **A**
- Represented graphically as an arrow
- Magnitude is represented by $\left|\vec{A}\right|$ or simply A

Scalars and Vectors

Characteristics of a vector:

- **Magnitude** of vector: length of arrow.

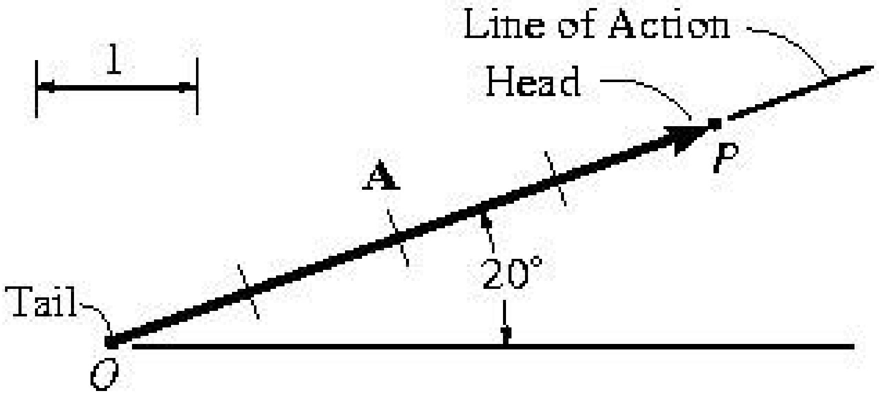
- **Direction** of vector: angle between the reference axis and arrow’s line of action.

- **Sense** of vector: arrowhead.

Scalars and Vectors

Example of vector:

- Magnitude of vector = 4 units
- Direction of vector=20 ° measured counterclockwise from the horizontal axis
- Sense of vector= upward and to the right
- Point O is called **tail** of the vector
- Point P is called the **tip** or **head**



Vector Operations

Multiplication and Division of a Vector by a Scalar

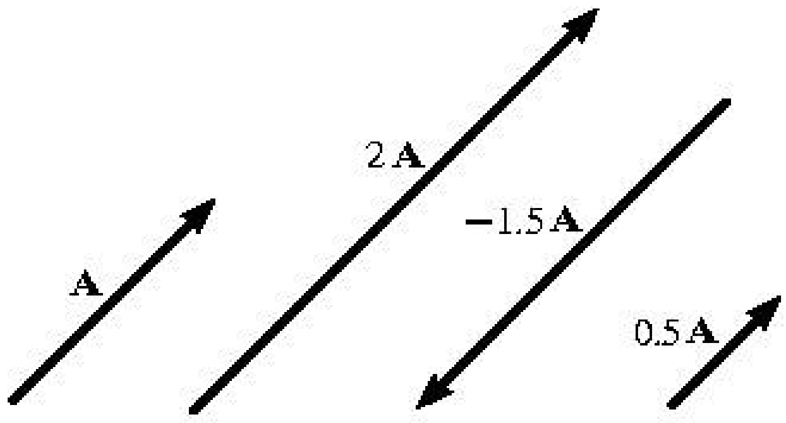
- Product of vector **A** and scalar a = a**A**
- Magnitude = $|aA|$
- If a is positive, sense of a**A** is the same as sense of **A**
- If a is negative sense of a**A**, it is opposite to the sense of **A**

Vector Operations

Multiplication and Division of a Vector by a Scalar

- Negative of a vector is found by multiplying the vector by (-1)
- Law of multiplication applies

e.g: **A**/a = (1/a) **A**, a≠0



Scalar Multiplication and Division

Vector Operations

Vector Addition

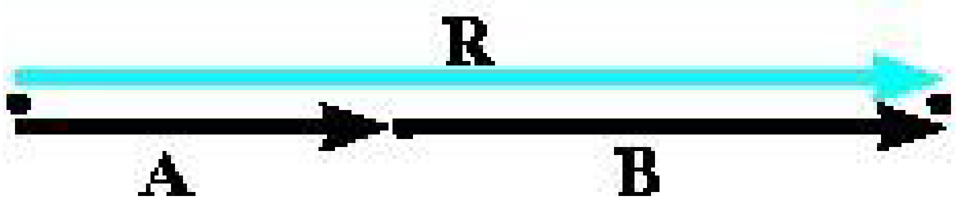
- Addition of two vectors **A** and **B** gives a resultant vector **R** by the parallelogram law
- Result **R** can be found by triangle construction
- Commutative e.g. **R** = **A** + **B** = **B** + **A**

(a) Parallelogram Law (b) Triangle construction (c) Triangle construction (d)

Vector Operations

Vector Addition

- Special case: Vectors **A** and **B** are collinear (both have the same line of action)



$R = A + B$

Addition of collinear vectors

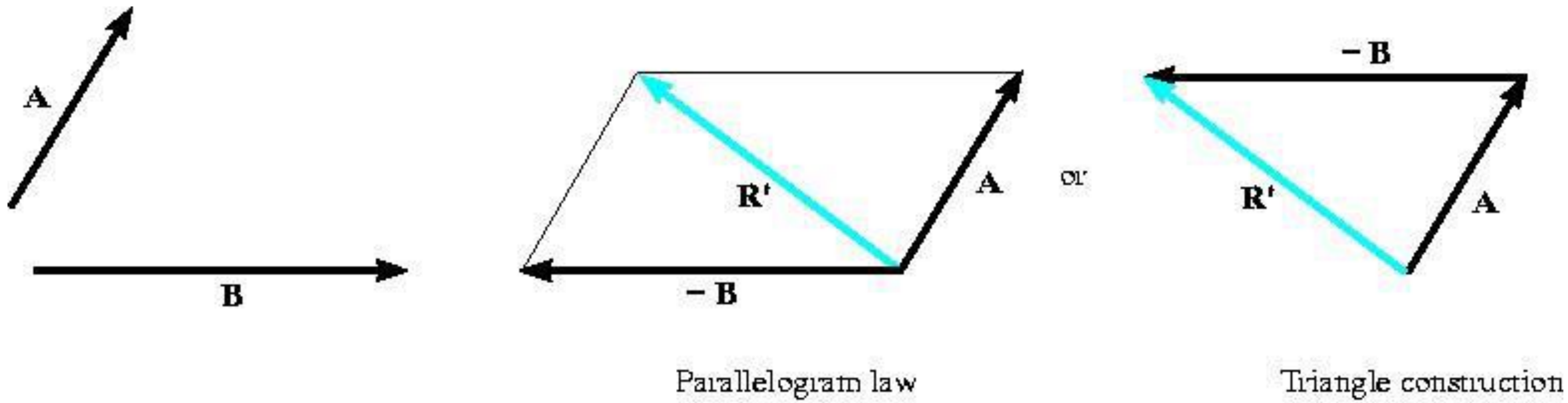
Vector Operations

Vector Subtraction

- Special case of addition

e.g. R’ = A – B = A + (- B)

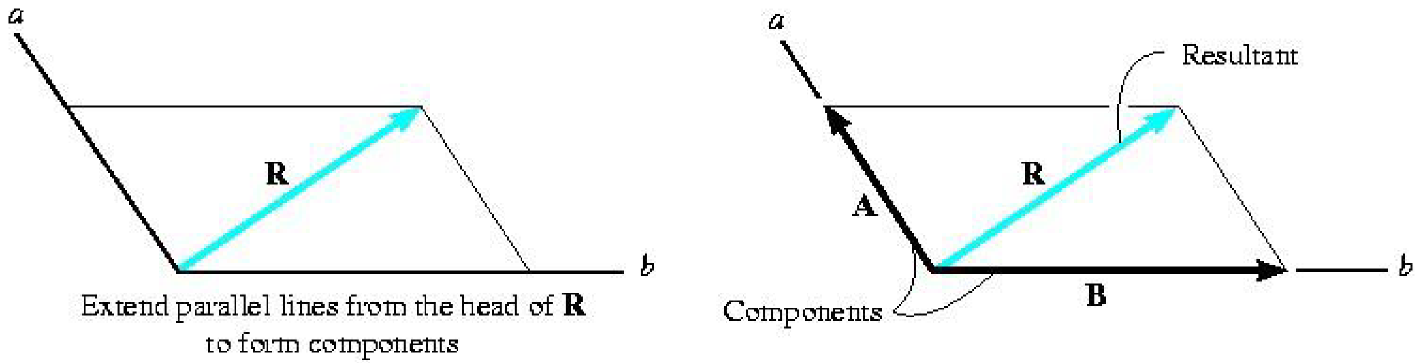
- Rules of Vector Addition Applies



Vector Operations

Resolution of Vector

- Any vector can be resolved into two components by the parallelogram law
- The two components **A** and **B** are drawn such that they extend from the tail or **R** to points of intersection



(a) (b)

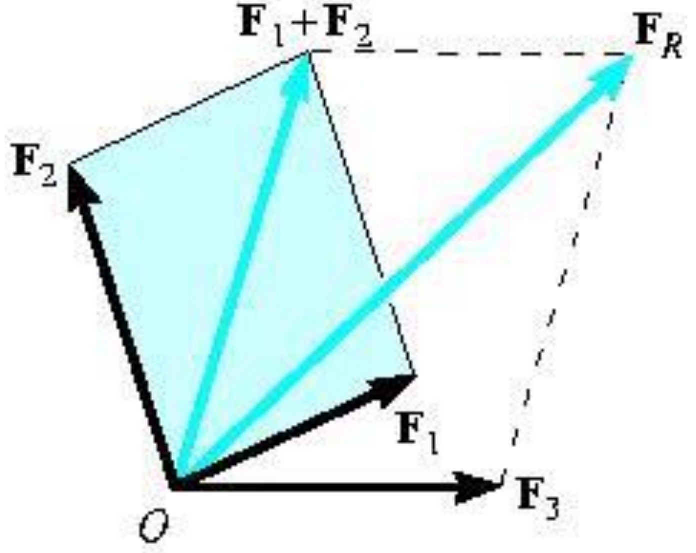
Vector Addition of Forces

- When two or more forces are added, successive applications of the parallelogram law is carried out to find the resultant

e.g. Forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ acts at a point O

First, find resultant of $\mathbf{F_1} + \mathbf{F_2} \Rightarrow$

Resultant, $\mathbf{F_R} = (\ \mathbf{F_1} + \mathbf{F_2}\ ) + \mathbf{F_3}$



Vector Addition of Forces

Procedure for Analysis:

Parallelogram Law

- Make a sketch using the parallelogram law
- Two components forces add to form the resultant force
- Resultant force is shown by the diagonal of the parallelogram
- The components is shown by the sides of the parallelogram

Vector Addition of Forces

Procedure for Analysis:

Trigonometry

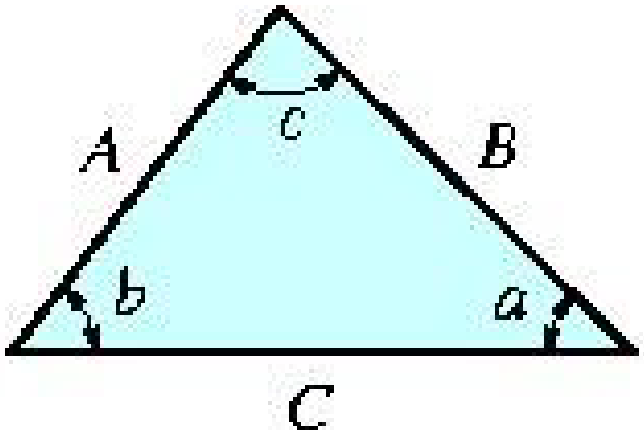
- Redraw half portion of the parallelogram
- Magnitude of the resultant force can be determined by the **law of cosines**
- Direction if the resultant force can be determined by the **law of sines**
- Magnitude of the two components can be determined by the **law of sines**

Vector Addition of Forces

Procedure for Analysis:

Trigonometry

- Magnitude of the two components can be determined by the **law of sines**.



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

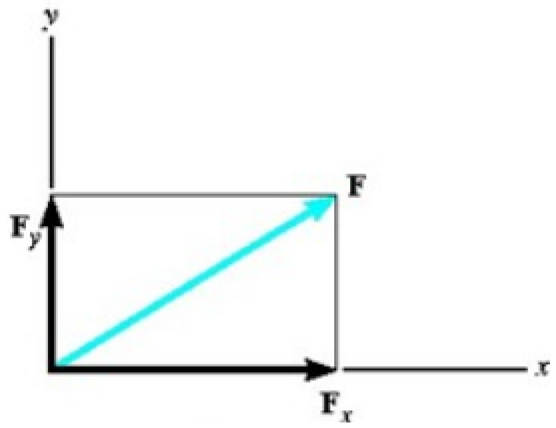
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Addition of a System of Coplanar Forces

- Scalar Notation: Components of forces expressed as algebraic scalars Fx= $|\mathbf{F}|\cos\theta$ and Fy=$|\mathbf{F}|\sin\theta$
- Cartesian Vector Notation in unit vectors i and j

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$



Addition of a System of Coplanar Forces

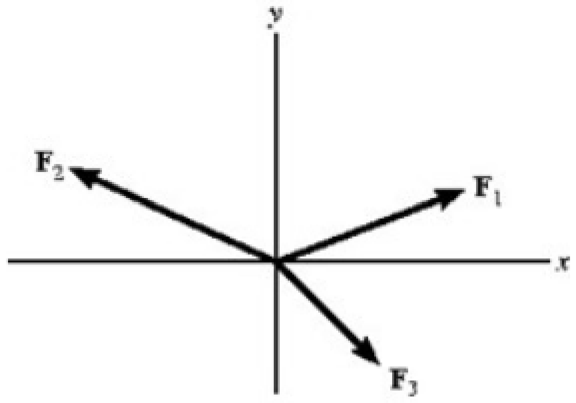
- Coplanar Force Resultants

$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$

$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$

$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$

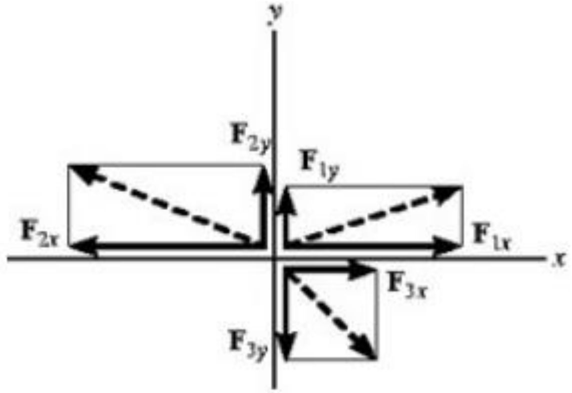
$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}$



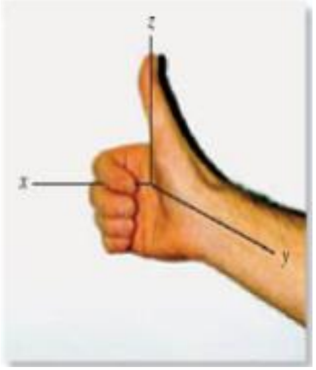
- Scalar Notation

$F_{Rx} = F_{1x} - F_{2x} + F_{3x}$

$F_{Ry} = F_{1y} + F_{2y} - F_{3y}$



Cartesian Vectors



- **Right-Handed Coordinate System**

A right-handed rectangular or Cartesian coordinate system

- **Rectangular Components of a Vector**

A vector A may have one, two or three rectangular components along the x, y and z axes, depending on orientation

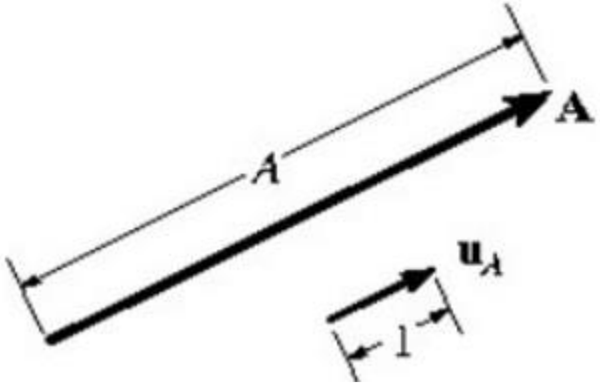
By two successive applications:

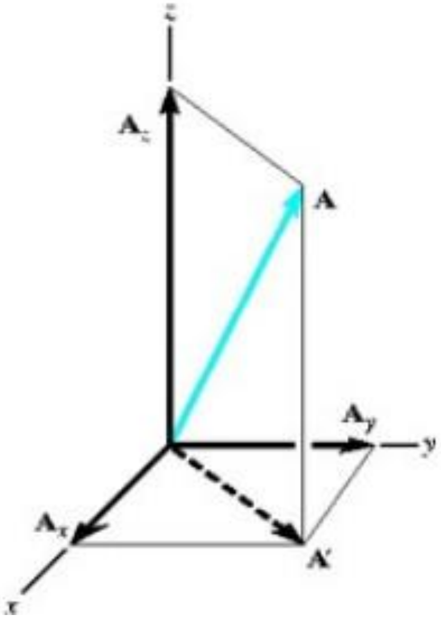
A=A '+ A_z

A'=A_x + A_y

A=A_x + A_y + A_z

A=$|A|\boldsymbol{u}_A$



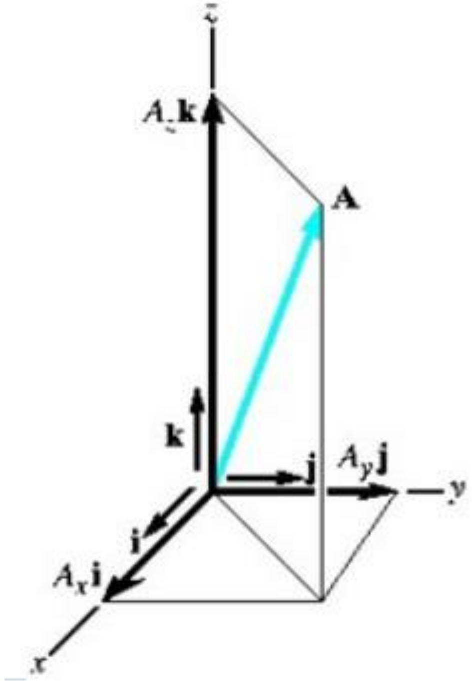


Cartesian Vectors

Cartesian Vector Representation

- Three components of A act in the positive i, j and k directions

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}$$

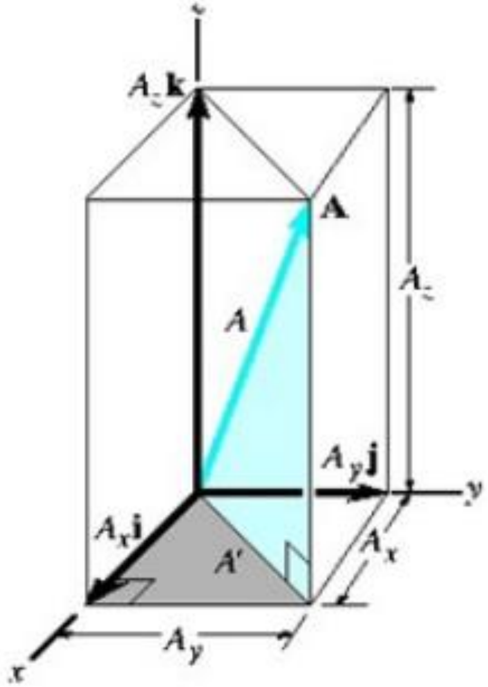


Cartesian Vectors

Magnitude of a Cartesian vector

- From the colored triangle $A = \sqrt{A'^2 + Az^2}$
- From the shaded triangle $A' = \sqrt{Ax^2 + Ay^2}$
- Combining the equations gives magnitude of **A**

$$A = \sqrt{Ax^2 + Ay^2 + Az^2}$$



Cartesian Vectors

- **Coordinate Direction Angles**

Orientation of A is defined as the coordinate direction angles α, β, γ measured between A and the x, y and z axes $0° \leq \alpha, \beta \; and \; \gamma \leq 180°$

- The direction cosines of **A** is

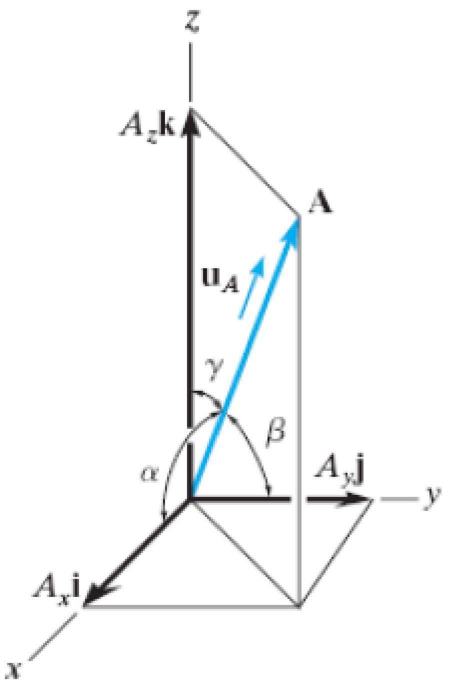
$$\cos\alpha = \frac{A_x}{|\boldsymbol{A}|} \qquad \cos\beta = \frac{A_y}{|\boldsymbol{A}|} \qquad \cos\gamma = \frac{A_z}{|\boldsymbol{A}|}$$

The unit vector $\mathbf{u_A}$=A/$|\boldsymbol{A}|$=(Ax/A)**i**+(Ay/A)**j**+(Az/A)**k**

$\mathbf{u_A}$=$cos\alpha\boldsymbol{i} + cos\beta\boldsymbol{j} + cos\gamma\boldsymbol{k}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

A=$|\boldsymbol{A}|\boldsymbol{u_A} = |\boldsymbol{A}|cos\alpha\boldsymbol{i} + |\boldsymbol{A}|cos\beta\boldsymbol{j} + |\boldsymbol{A}|cos\gamma\boldsymbol{k} = Ax\boldsymbol{i} + Ay\boldsymbol{j} + Az\boldsymbol{k}$

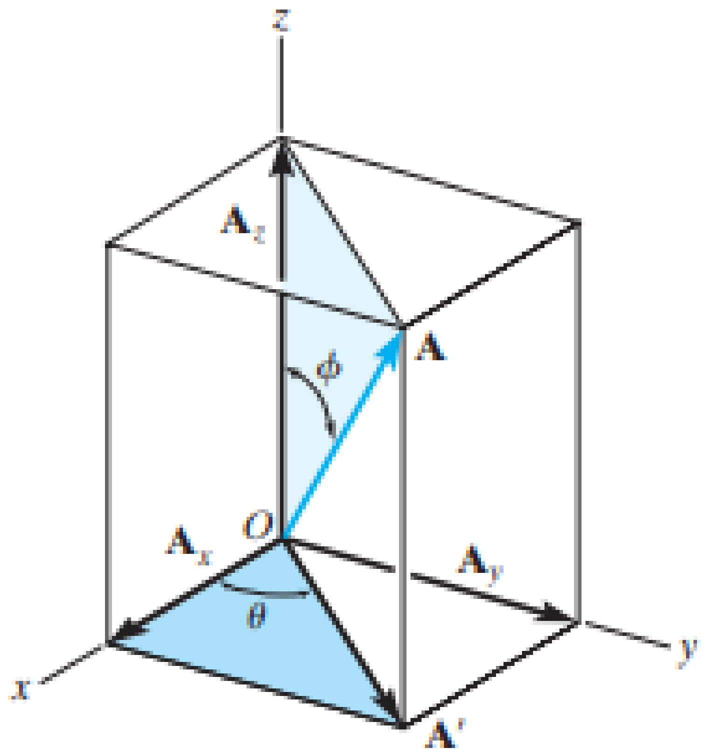


Cartesian Vectors

- **Transverse and Azimuth Angles**

The direction of A can be specified using two angles:

Transverse angle **θ** and Azimuth angle **Φ:**



$A_z = A \cos \Phi$ and $A' = A \sin \Phi \Rightarrow$

$A_x = A' \cos \theta = A \sin \Phi \cos \theta$

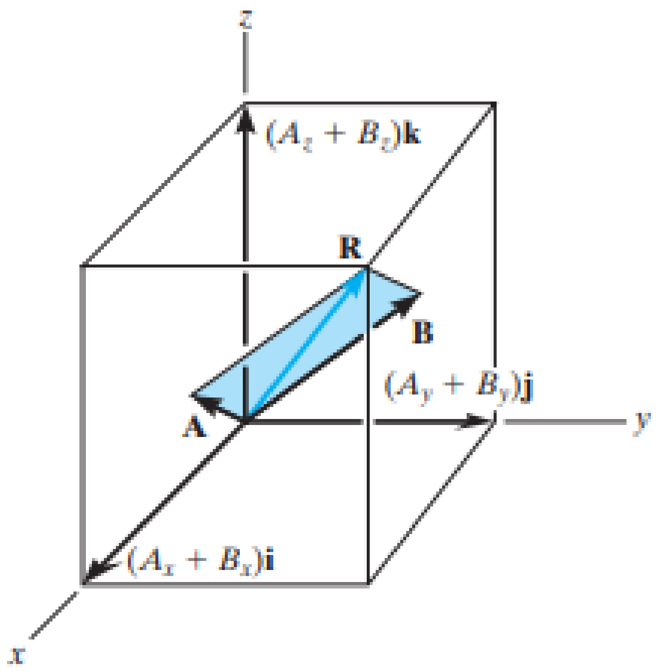
$A_y = A' \sin \theta = A \sin \Phi \sin \theta$

Addition of Cartesian Vectors

- The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B} = (A_x + Bx)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$

- For a system of several concurrent forces, the force resultant :

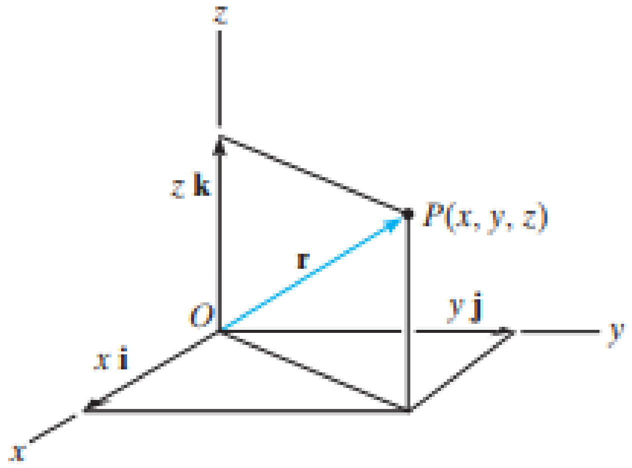
$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$



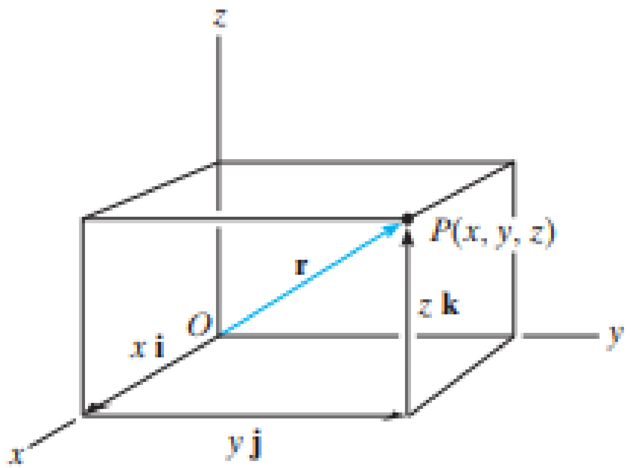
Position Vectors

- Position vector: is a fixed vector which locates a point in space relative to another point ⇒ Position Displacement Vector

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$



(a)



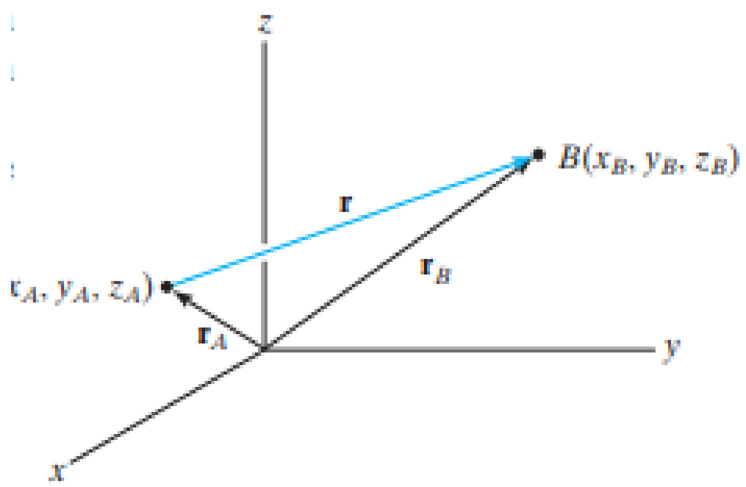
(b)

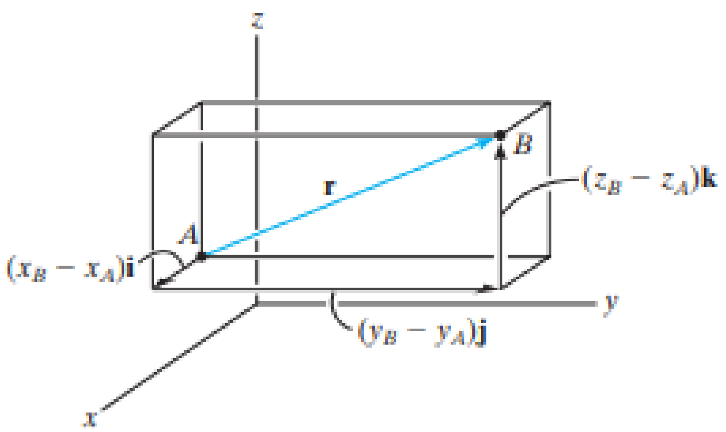
Position Vector: Displacement and Force

- Using the triangle rule, $\mathbf{r_A} + \mathbf{r} = \mathbf{r_B}$

$\mathbf{r} = \mathbf{r_B} - \mathbf{r_A} = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$

$\mathbf{r} = (x_B - x_A)\,\mathbf{i} + (y_B - y_A)\,\mathbf{j} + (z_B - z_A)\mathbf{k}$

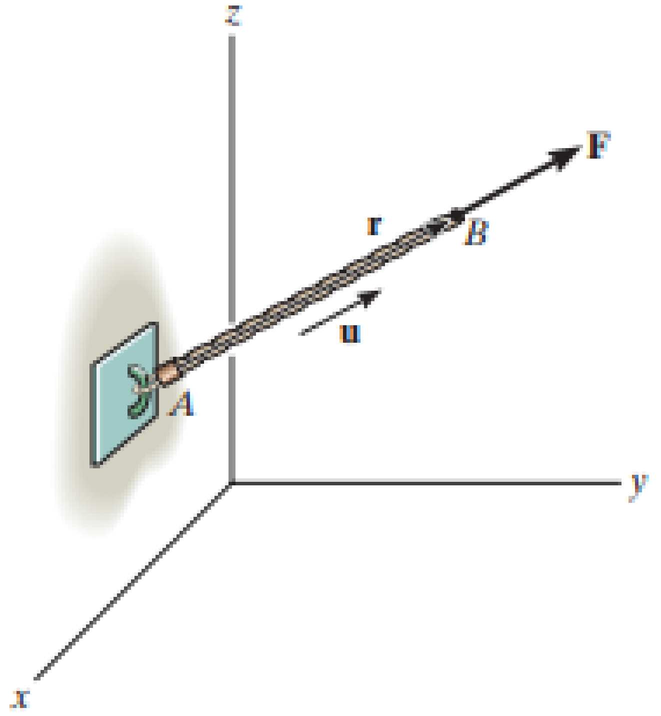




Force vector along a line

- **Force F directed along the cord AB**

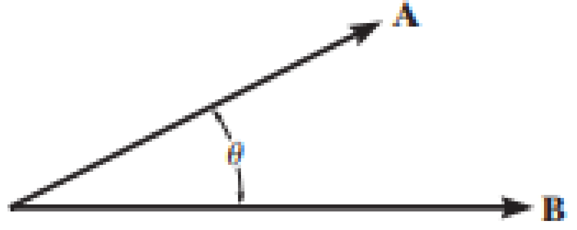
$$\mathbf{F} = F\,\mathbf{u} = F\left(\frac{r}{r}\right) = F\left(\frac{(x_B - xA)\boldsymbol{i} + (y_B - y_A)\boldsymbol{j} + (z_B - z_A)\boldsymbol{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$



Dot Product

- Dot product of vectors A and B:

$$\mathbf{A} \,.\, \mathbf{B} = \mathrm{A}\,\mathrm{B}\cos\theta$$



- **Laws of Operation**

1. Commutative law **A·B** = **B·A**
2. Multiplication by a scalar a(**A·B**) = (a**A**)·**B** = **A**·(a**B**) = (**A·B**)a
3. Distribution law **A**·(**B** + **D**) = (**A·B**) + (**A·D**)

- **Cartesian Vector Formulation**

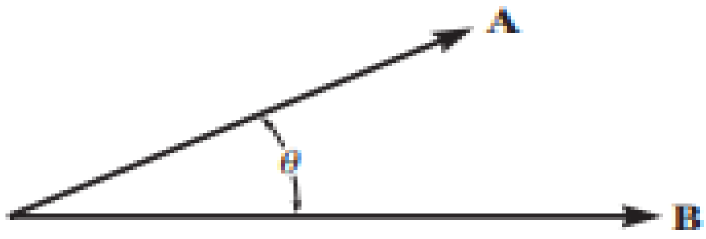
$$\mathbf{A} \,.\, \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Dot Product

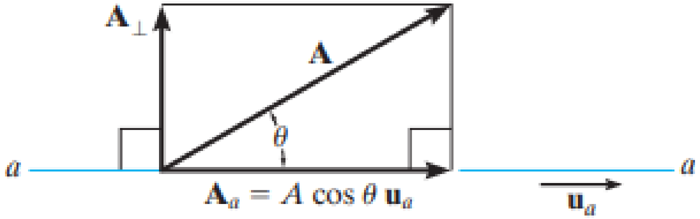
Applications:

- The angle formed between two vectors or intersecting lines:

$$\theta = \cos^{-1}\left(\frac{\boldsymbol{A}.\boldsymbol{B}}{AB}\right),\ 0° \leq \theta \leq 180\ °$$



- The components of a vector parallel and perpendicular to a line:



The parallel component: $A_a = A\,cos\theta = \boldsymbol{A}.\boldsymbol{u_a}$

The perpendicular component:

$A_{\perp} = A\,sin\theta\ with\ \theta = \cos^{-1}(\boldsymbol{A}.\boldsymbol{u_a}/A)$

Or $A_{\perp} = \sqrt{A^2 - {A_a}^2}$