Chapter 3: Equilibrium of a particle 2D/3D

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Textbook: R.C. Hibbeler, *Engineering Mechanics: Statics*, 14th Edition, Pearson Prentice Hall, ISBN 978-981-06-8134-0, 2010

Plan

- Condition for the equilibrium of a particle
- Free-Body diagram
- Coplanar Force Systems
- Three-Dimensional Force Systems

Condition for the equilibrium of a particle

- **Equilibrium:** particle at rest or moving at constant velocity.
- **Newton's first law of motion**:

$$\Sigma \mathbf{F} = 0$$

Where ΣF is the vector sum of all forces acting on the particle.

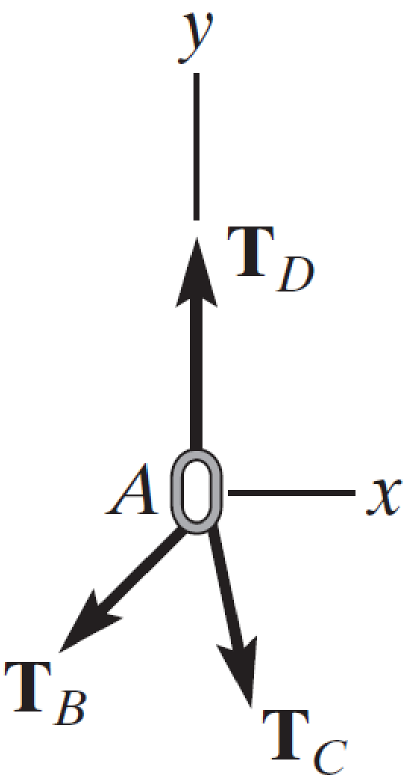
- **Newton's second law of motion:**

$$\Sigma \mathbf{F} = m\mathbf{a}$$

If the forces verify Newton's first law, therefore m**a**=0, **a**=0,
The particle is at rest or at constant velocity

Free-Body Diagram

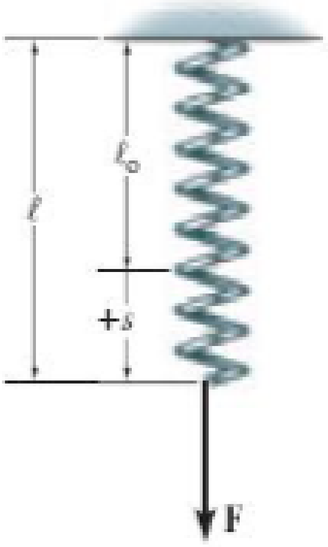
- **Free-Body Diagram (FBD):** a sketch that shows only the forces acting on a body.



Free-Body Diagram

Springs

- Linear elastic spring: change in length is directly proportional to the force acting on it.
- Elasticity of the spring: spring constant or stiffness k
- Magnitude of force when spring is deformed (elongated or compressed) $F=k\,s$



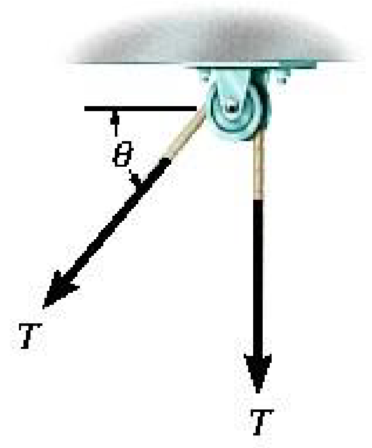
Where s is the difference between spring's deformed length l and its undeformed length l_0: $s=l-l_0$

- If $s > 0$, elongation, **F** pull on the spring
- If $s < 0$, shortening, **F** push on the spring

Free-Body Diagram

Cables and Pulleys

- Cables (or cords) are assumed to have negligible weight and they cannot stretch
- A cable only support tension or pulling force
- Tension always acts in the direction of the cable
- Tension force in a continuous cable must have a constant magnitude for equilibrium
- For any angle , the cable is subjected to a constant tension **T** throughout its length.



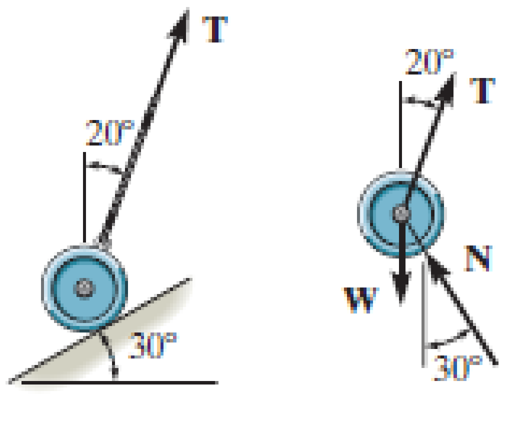
Cable is in tension

Free-Body Diagram

Smooth contact

- If an object rests on a smooth surface, the surface will exert a force on the object that is normal to the surface at the point of contact.

- In the figure below, the three forces (weight W, normal force N, force T of the cord) are concurrent at the center of the cylinder ⇒ equation of equilibrium can be applied to this particle.

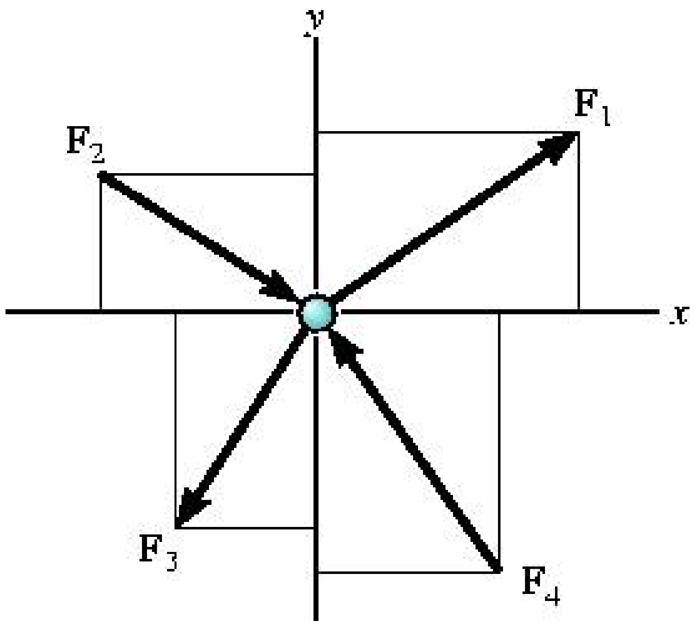


(a)
(b)

Coplanar Force Systems

- If a particle is subjected to coplanar forces in the x-y plane ⇒ for equilibrium, resolve into i and j components: $\Sigma \mathbf{F}=0$ ⇒

$$\Sigma \mathbf{F}_x=0 \ \& \ \Sigma \mathbf{F}_y=0$$



Three-Dimensional Force Systems

- When particle is in equilibrium, the vector sum of all the forces acting on it must be zero: $\Sigma \mathbf{F}=0$
- In case of three-dimensional system,

$\Sigma F_x=0$, $\Sigma F_y=0$ & $\Sigma F_z=0$: **three scalar equations of equilibrium**