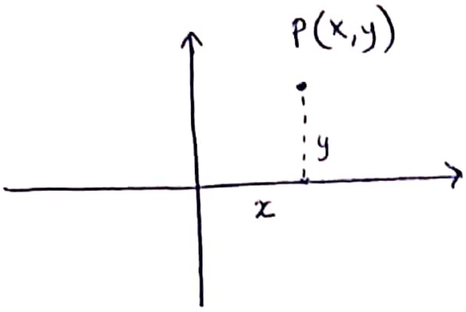
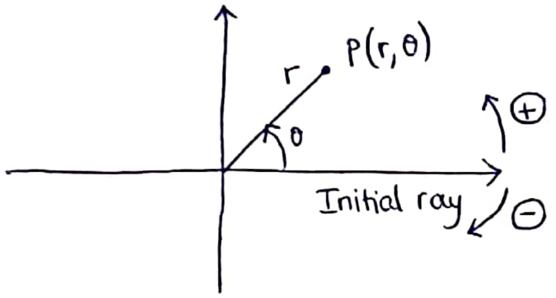
Chapter 11 - Parametric Equations and Polar Coordinates

Section 11.3 - Polar Coordinates



Cartesian coordinates
Rectangular coordinates.



$\boxed{r}$ directed distance from origin to P.

r positive: moving forwards

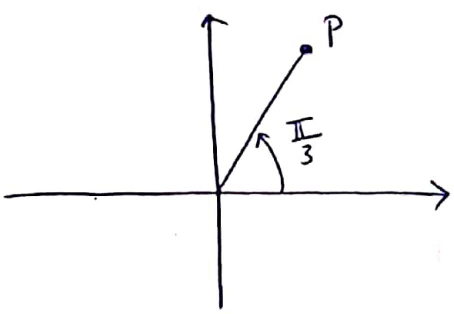
r negative: moving backwards

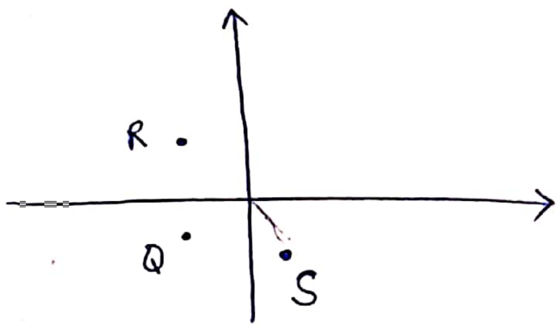
$\boxed{\theta}$ directed angle from the initial ray (the positive x-axis) to [OP)

θ positive: counterclockwise

θ negative: clockwise.

<u>Example:</u> $P\left(2, \frac{\pi}{3}\right)$; $Q\left(-1, \frac{\pi}{6}\right)$; $R\left(-1, \frac{-\pi}{4}\right)$; $S\left(1, -\frac{\pi}{3}\right)$





$P\left(-2, -\frac{2\pi}{3}\right)$

$P\left(-2, \frac{4\pi}{3}\right)$

$P\left(2, -\frac{5\pi}{3}\right)$

Note: Polar Coordinates of a point are not unique.

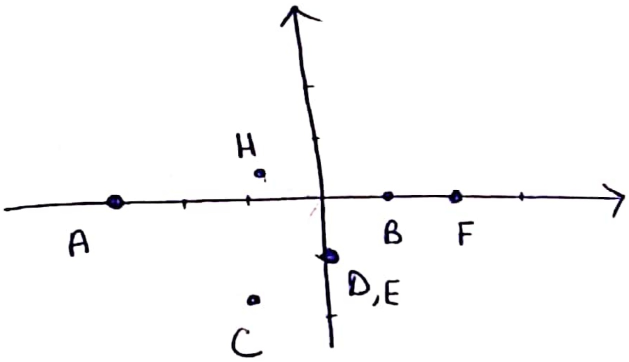
Polar Graphing:

$\boxed{r=a}$: circle centered at the origin with radius $=|a|$.

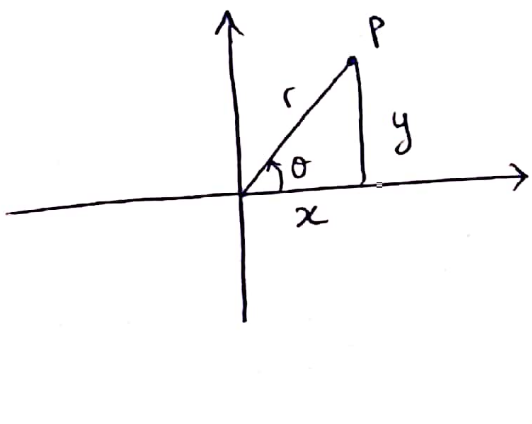
$\boxed{\theta=\alpha}$: straight line through the origin making an angle α with the initial ray.

Example: $A(3,\pi)$ $B(1,0)$ $C(-2,\frac{\pi}{3})$ $D(1,-\frac{\pi}{2})$

$E(1,\frac{3\pi}{2})$ $F(-2,\pi)$ $H(1,\frac{5\pi}{6})$.



Relating Polar to Cartesian Coordinates



$\cos\theta = \frac{x}{r} \Rightarrow \boxed{x = r\cos\theta}$

$\sin\theta = \frac{y}{r} \Rightarrow \boxed{y = r\sin\theta}$

$\boxed{\tan\theta = \frac{y}{x}}$

$\boxed{x^2+y^2=r^2}$

Examples:

1) $r^2 \cos\theta \sin\theta = 2$

$xy = 2$

2) $r = 4\sec\theta$

$r = \frac{4}{\cos\theta}$

$r\cos\theta = 4$

$x = 4$

3) $r = -2\csc\theta$

$r = \frac{-2}{\sin\theta}$

$r\sin\theta = -2$

$y = -2$

Remark: $\boxed{r = a\sec\theta}$ vertical line $x = a$

$\boxed{r = b\csc\theta}$ Horizontal line $y = b$.

4) $r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x$

$x^2 - 4x + y^2 = 0$

$x^2 - 4x + 4 - 4 + y^2 = 0$

$(x-2)^2 + y^2 = 4$

Center $(2,0)$, radius $= 2$.

$\boxed{r = a\cos\theta}$ circle center $(\frac{a}{2}, 0)$ and radius $\frac{|a|}{2}$

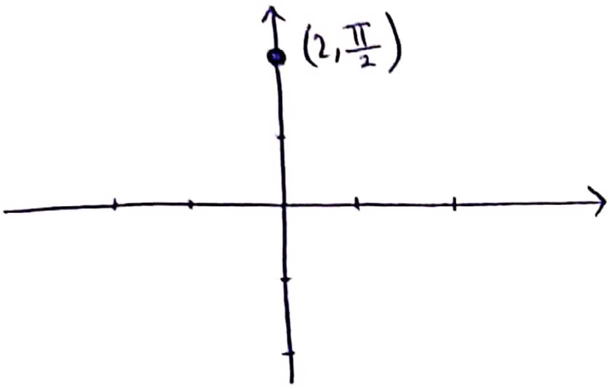
$\boxed{r = a\sin\theta}$ circle center $(0, \frac{a}{2})$ and radius $\frac{|a|}{2}$

$\boxed{r = a}$ circle center $(0,0)$ and radius $|a|$.

Exercises:

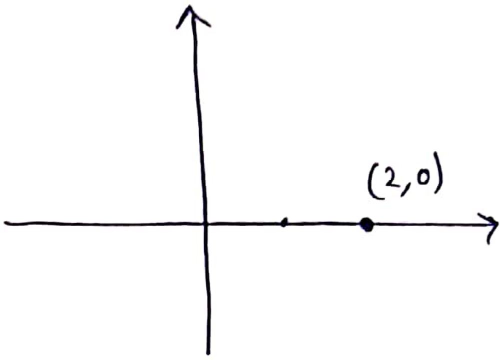
3) Plot the following points (given in polar coordinates)
Then find all the polar coordinates of each point.

a) $(2, \frac{\pi}{2})$



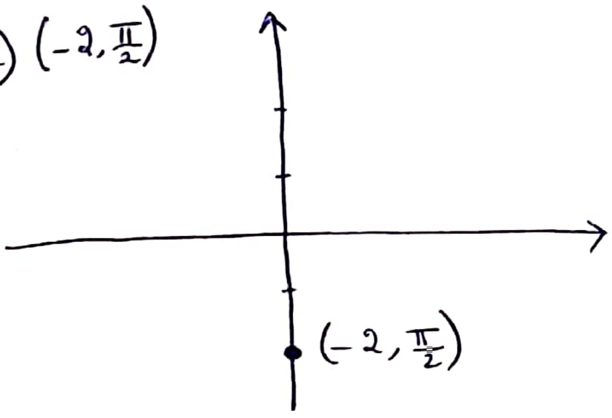
$(-2, -\frac{\pi}{2})$ $(2, \frac{3\pi}{2})$ $(2, -\frac{3\pi}{2})$

b) $(2, 0)$



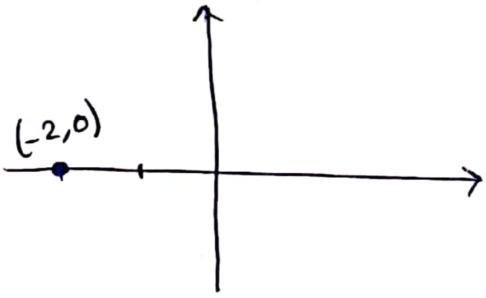
$(-2, \pi)$ $(-2, -\pi)$ $(2, 2\pi)$

c) $(-2, \frac{\pi}{2})$



$(2, -\frac{\pi}{2})$ $(2, \frac{3\pi}{2})$ $(-2, \frac{-3\pi}{2})$

d) $(-2, 0)$



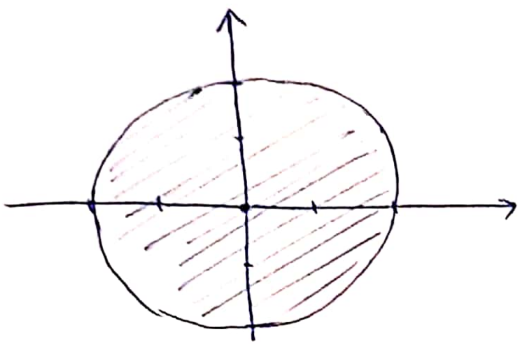
$(-2, 2\pi)$ $(2, \pi)$ $(2, -\pi)$

Graph the sets of points whose polar coordinates satisfy the equations and inequalities.

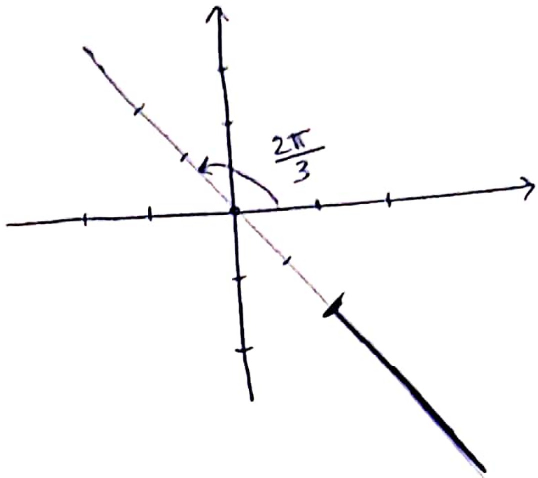
12) $0 \leq r \leq 2$

$r=0$ (circle of center $(0,0)$ and radius 0)

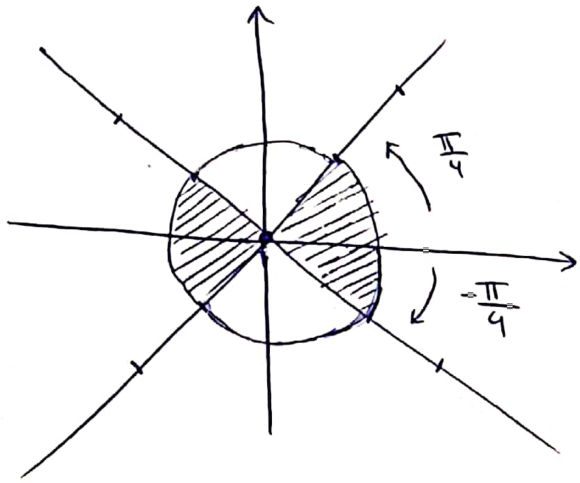
$r=2$ (circle of center $(0,0)$ and radius 2)



16) $\theta = \frac{2\pi}{3}$; $r \leq -2$



24) $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$, $-1 \leq r \leq 1$



Replace the polar coordinates with equivalent Cartesian equations
Then identify the graph.

32) $r = -3\sec\theta$

$$r = \frac{-3}{\cos\theta}$$

$r\cos\theta = -3$

$x = -3$

vertical line $\boxed{x = -3}$

38) $r^2 \sin 2\theta = 2$

$2r^2 \cos\theta \sin\theta = 2$

$2xy = 2$

Hyperbola of equ. $\boxed{xy = 1}$

45) $r^2 = -4r\cos\theta$

$x^2 + y^2 = -4x$

$x^2 + 4x + y^2 = 0$

$x^2 + 4x + 4 - 4 + y^2 = 0$

$(x+2)^2 + y^2 = 4$

circle of center $(-2, 0)$ and radius 2

Replace the Cartesian equations with equivalent polar equations.

60) $xy = 2$

$r\cos\theta \, r\sin\theta = 2$

$$r^2 \frac{\sin 2\theta}{2} = 2$$

$r^2 \sin 2\theta = 4.$

63) $x^2 + (y-2)^2 = 4$

$r^2\cos^2\theta + (r\sin\theta - 2)^2 = 4$

$r^2\cos^2\theta + r^2\sin^2\theta - 4r\sin\theta + 4 = 4$

$r^2(\cos^2\theta + \sin^2\theta) - 4r\sin\theta = 0$

$r^2 - 4r\sin\theta = 0$

$r(r - 4\sin\theta) = 0$

$r = 0$ (special case) <u>or</u> $r = 4\sin\theta$.