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# ExpIt-OOS: Towards Learning from Planning in Imperfect Information Games

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## Abstract

1       The current state of the art in playing many important perfect information games,  
2       including Chess and Go, combines planning and deep reinforcement learning with  
3       self-play. We extend this approach to imperfect information games and present  
4       ExIt-OOS, a novel approach to playing imperfect information games within the  
5       Expert Iteration framework and inspired by AlphaZero. We use Online Outcome  
6       Sampling, an online search algorithm for imperfect information games in place of  
7       MCTS. While training online, our neural strategy is used to improve the accuracy  
8       of playouts in OOS, allowing a learning and planning feedback loop for imperfect  
9       information games.

## 10   1 Introduction

11   Many recent gains in game playing skill for perfect information games have come from combining  
12   planning, deep reinforcement learning and self play. In the Expert Iteration and AlphaZero framework,  
13   a powerful online planning/search algorithm, usually Monte Carlo Tree Search (MCTS) [Browne  
14   et al., 2012] is combined with a learnable heuristic value and policy function, represented with a deep  
15   neural network. During self-play, the learned heuristics are used to guide the planner. The action  
16   recommended by the time consuming planning process is used as feedback to train the heuristic  
17   function. As this heuristic improves, so does the quality of actions chosen by the planner. When  
18   executed carefully this leads to a cycle of mutual improvement and very strong play. Notably these  
19   approaches require no expert domain knowledge of the game to be manually incorporated. The  
20   recent success of the AlphaZero [Silver et al., 2017] and ExpIt [Anthony et al., 2017] exemplifies this  
21   approach.

22   The success of these methods can be partially explained by the complementary nature of MCTS and  
23   Deep Neural Networks. DNNs and Convolutional Neural Networks are powerful pattern recognisers  
24   which can learn strategies that generalise well between states. However they cannot roll out the  
25   combinatorial consequences of hypothetical decisions. MCTS can bring consistency between state  
26   values and policy choices by smoothly propagating the consequences of hypothetical future decisions  
27   backwards up the game tree to previous nodes.

28   We present ExIt-OOS, an instance of Expert Iteration using Online Outcome Sampling [Lanctot  
29   et al., 2014], an online planning algorithm for imperfect information games, in place of MCTS. Thus  
30   extending the learning-from-planning paradigm to imperfect information games. This allows the  
31   playing of a wide class of imperfect information games without modification, while making use of  
32   the rich information provided by planning. During search the current neural strategy is used during  
33   the rollout phase of OOS to improve the quality of the search. We present experimental data on  
34   exploitability and head to head matches between OOS and neural nets trained with ExIt-OOS.

## 35 2 Background

### 36 2.1 Imperfect Information in Extensive-Form Games

37 Extensive-form games are annotated trees that model sequential decision making with multiple  
38 players. Each node in the tree represents a game state and is labelled with a player. The edges leading  
39 out from each node represent actions that can be taken by the acting player in that state. Aside from  
40 the active players, there is also an auxiliary chance player who always takes actions with a fixed  
41 probability, modelling randomness in the game. The leaf nodes represent terminal states and carry a  
42 vector of utilities, one for each player. Players conventionally choose their strategies to maximise  
43 utility. In a zero-sum game, the utilities at each terminal node sum to zero.

44 In an imperfect information game, many states are indistinguishable to a given player and are grouped  
45 into information sets (infosets). We only consider games with perfect recall. Where players always  
46 remember all information that has been revealed to them and every action they have taken in the  
47 past. States with different observable histories must be in different infosets. A behaviour strategy is a  
48 mapping from infosets to probabilities over actions. A player must behave in the same way for every  
49 state in an infoset. Games are provided to our ExIt-OOS implementation as programs that implicitly  
50 define the extensive form game tree.

### 51 2.2 Nash Equilibrium

52 When playing imperfect information games some care is required in the definition of an optimal  
53 strategy as discussed in Billings et al. [2004], from a game theoretic perspective, optimal usually  
54 means playing a Nash equilibrium strategy, where no player has any incentive to unilaterally change  
55 their strategy, given the strategy of the other players. However this can be problematic, because this  
56 approach can be overly defensive; the opponent is often fallible and has flaws that can be exploited.  
57 For example, in a poker tournament, players with strategies far from equilibrium — but able to exploit  
58 weaker entrants — may well finish ahead of equilibrium players (in another sense, the ‘meta-game’  
59 of changing strategies between hands against realistic opponents is not modelled correctly). However,  
60 this work focuses on finding equilibrium strategies for single game instances and leaves online  
61 adaptation, opponent modelling and exploitation to future work.

### 62 2.3 Online Outcome Sampling and Targeting

63 Online Outcome Sampling is the first published imperfect information search algorithm that converges  
64 to equilibrium strategies in two-player zero-sum games [Lanctot et al., 2014]. It is a sampling  
65 algorithm that uses regret matching to minimise the counterfactual regret locally at each infoset in a  
66 game tree. It is essentially MCCFR with incremental tree growth and targeting.

67 In the Online Outcome Sampling algorithm, all simulations are run from the initial state to a terminal  
68 state, simulations are dynamically targeted to infosets observed during play with boosted probability.  
69 When a simulation is targeted, importance sampling is used to reweight targeted simulations to  
70 keep collected statistics unbiased. The OOS authors propose two kinds of targeting. Firstly, Public  
71 Set Targeting (PST) where simulations are forced to be consistent with the public subgame so far  
72 i.e. players always take their observed public actions and public chance nodes take their observed  
73 outcomes. But private information such as the current hand in poker may differ. Secondly, Information  
74 Set Targeting (IST) where simulation actions are forced to be consistent with all current observations  
75 of the game in progress, including private information. Implementing targeting may require providing  
76 additional complex domain knowledge, beyond just the dynamics of the game to be played. Observed  
77 information may constrain the actions an opponent could have taken in complex ways, for example,  
78 targeting in II Goofspiel requires constraint solving.

79 Strong play in imperfect information games presents challenges not present when playing perfect  
80 information games. Firstly, concealing private information, playing unpredictability, and even  
81 misdirection and deception become important factors. If a player is too predictable the opponent may  
82 exploit this weakness. Furthermore, the value at a given infoset, depends on the distribution over  
83 states conditioned on reaching that infoset, which in turn is dependent on the strategy both players  
84 have been using since the initial state. In imperfect information games counterfactuals matter. Past  
85 games that did not happen, but could have happened affect current strategic choices. OOS and other  
86 counterfactual regret minimisation algorithms employ careful weighting to ensure these factors are

87 taken into account. Changes in strategy in one place in the game tree, can cause the best response in  
88 another far away area to change, this is called non-locality [Lanctot et al., 2014]. OOS keeps statistics  
89 and continually updates strategies even for infosets that are no longer reachable.

## 90 2.4 Expert Iteration

91 Many powerful search and planning algorithms, such as MCTS are table driven and collect statistics  
92 per state or infoset during planning. They cannot generalise and share information between states  
93 even when they are functionally very similar in the game. Contrastingly, Deep Neural Networks can  
94 learn function approximations that generalise well, but creating high quality training data is difficult,  
95 if the correct strategic choice could already be calculated precisely this procedure could be used to  
96 play the game well and the problem would be solved. If computation speed was the only concern,  
97 Imitation Learning could be used to train a neural network to imitate the results of a much slower  
98 search, but it could not surpass the quality of the base search that it was learning from. The key idea  
99 of Expert Iteration (ExIt) [Anthony et al., 2017] is that the search procedure can be improved with  
100 feedback from a partially trained apprentice neural network, creating a cycle of mutual improvement.  
101 A better heuristic, creates more accurate training targets which further improves the heuristic.

## 102 2.5 AlphaZero

103 The AlphaZero [Silver et al., 2017] algorithm trains a deep neural network to approximate a value  
104 function and policy using targets generated from MCTS and self-play. Demonstrating state-of-the-art  
105 play in Chess, Go and Shogi with no human provided heuristic functions. Although AlphaZero did  
106 require extensive meta-domain knowledge in the choice of neural network architecture and hyper-  
107 parameter selection. AlphaZero uses a sophisticated CNN architecture to encode board positions and  
108 move selection, also bringing some significant structural prior information.

109 AlphaZero learns to approximate a value and policy function with a deep neural network:

$$(\mathbf{p}, v) = f_{\theta}(s)$$

110 Where  $s$  is an encoding of the current state,  $v$  is a scalar state value,  $p$  is a probability vector over  
111 actions and  $\theta$  is a vector of neural network parameters. The policy network is trained on a distribution  
112 proportional to the visit counts at the root of MCTS searches, the value network is trained with the  
113 utility of the completed game. AlphaZero uses the neural approximation to improve the MCTS search  
114 in two ways. First, during the expansion step, the neural network is evaluated and used to compute  
115 prior action probabilities for new nodes. Second, instead of a rollout phase, the neural network value  
116 function at the tree fringe is used as a surrogate and backpropagated up the tree. Another perspective  
117 is that the Monte Carlo search is a policy improvement operator applied to the current neural policy.  
118 It is evaluated at states sampled from self play, while the value network takes the role of policy  
119 evaluation making AlphaZero a unconventional and elaborate, policy iteration algorithm.

## 120 3 ExIt-OOS

121 ExIt-OOS is instance of Expert Iteration where Online Outcome Sampling is used for planning and  
122 takes the role of the expert, The apprentice neural network is used as a playout policy to improve the  
123 decisions of the expert. The use of Online Outcome Sampling allows planning online in imperfect  
124 information games, and can converge to a Nash equilibrium.

125 One major difference between the operation of MCTS and OOS is that in the latter, node strategies  
126 are constantly changing and never converge to a specific value, only the average strategy at the node  
127 converges. Unfortunately this protean aspect prevents the simple introduction of node priors, although  
128 work in this area is ongoing. In our implementation the neural network policy is used to provide  
129 accurate rollouts. We learn a neural strategy:

$$\mathbf{p} = f_{\theta}(I_s) \quad l = \sigma^T(\log \sigma - \log \mathbf{p})$$

Where  $\mathbf{p}_a$  is the probability of taking action  $a$  given the info set encoding  $I_s$  at state  $s$ ,  $f_\theta(I_s)$  is a deep neural network with parameters  $\theta$ . The loss  $l$  is a KL Divergence between the OOS expert provided target and the neural neural network output.

Our high performance implementation runs many concurrent simulations which are represented by state machines. Each simulation’s state machine is progressed until it enters a state that requires neural net evaluation, at which point it is added to a evaluation waiting list. When there a no simulations that can progress any further, the waiting list is turned into a batch which is evaluated by the neural network. All simulations can then continue to progress until the next evaluation cycle.

Our experience generation is also run in parallel, where one process is launched for every core. Each process plays one game from start to finish and collects experience and planning results in a list of experience tuples:

$$(I_s, \sigma_{\text{oos}})_i \quad i = 1, \dots, N$$

Which are composed of the info set encoding  $I_s$  and the action probabilities  $\sigma_{\text{oos}}$  found by OOS.  $N$  is the number of experience tuples generated. These are collected in a training process which adds these tuples to the experience reservoir and does a number of steps with gradient descent before sending the new updated neural network parameters to each simulation processes. We use an exponentially weighted reservoir of experience similar to Heinrich and Silver [2016] where new incoming experiences replace old experiences with a certain fixed probability. After each batch of experience from the game playing processes comes in, minibatches are sampled from the reservoir for training.

## 4 Experiments

We tested our algorithm on Leduc poker [Southey et al., 2012] and II Goofspiel with 6 cards and 13 cards [Lanctot et al., 2014]. Leduc poker is a highly simplified research poker variant. Leduc poker has 6 cards, two suits and three ranks King, Queen, Jack. Each player has a one card hand and there is one community card. There are two betting rounds one before and after the community card is revealed. There is an ante of 1, the first round has a fixed bet/raise of 2 followed in the second by 4. There is a maximum of 2 raises per round. There are only two hand types: high card, or pair formed with the community card. Draws are worth 0, otherwise the utility is the bet value gained or lost after the showdown or when a player folds. All the info sets in Leduc poker are the same size and have 3 elements. Leduc poker has 288 distinct info sets. We use Public Set Targeting in experiments with Leduc Poker.

II Goofspiel( $N$ ) is a variant of Goofspiel. Each player starts with a hand of cards with rank  $0, 1, \dots, N - 1$ . There are  $N$  rounds, at each round a card with point value  $P$  is revealed and both players bid simultaneously on that card by choosing a card from their hand. The player with the higher bid is awarded  $P$  points and both bid cards are removed from their players hands. If both bids are the same rank, no points are awarded. In II Goofspiel, the value cards are revealed in a fixed increasing order,  $P = 0, 1, \dots, N - 1$  and only the winners of each bidding round are revealed, not the rank of the bid. Info sets in II Goofspiel have differing sizes. The info sets increase rapidly in size as the game goes on, while tapering closer to the end, they contain every possible combination of remaining opponent cards, given the rank constraints observed on previous rounds. II Goofspiel(6) has order  $10^5$  info sets and II Goofspiel(13) has approximately  $10^9$  info sets. Information Set Targeting is used for Goofspiel experiments, there is no public game tree.

We use hyper parameters from Lanctot et al. [2014], in all experiments, The search exploration parameter  $\epsilon = 0.4$ , the targeting probability  $\delta = 0.9$ , and the opponent mistake probability  $\gamma = 0.01$ . The only change we make is that instead of manually reweighting our samples when they are targeted, an exponential moving average with decay  $\beta = 0.99$  is used to estimate  $r = \mathbb{E}[s_1/s_2]$  where  $r$  is the ratio of  $s_1$  the probability that a state is sampled with targeting and  $s_2$  the probability that a state is sampled without targeting with expectation taken over all simulations. Multiplying importance weights by  $1/r$  puts heavier weight on samples from later in the game where there is heavier targeting. Our Leduc poker neural network has a single hidden layer with 128 hidden neurons. Our II Goofspiel(6) net 2 had hidden layers with 128 and 64 neurons, in II Goofspiel(13) experiments we used 3 hidden layers of 128, 128 and 64. ReLu activation functions are used for all nets. We use a

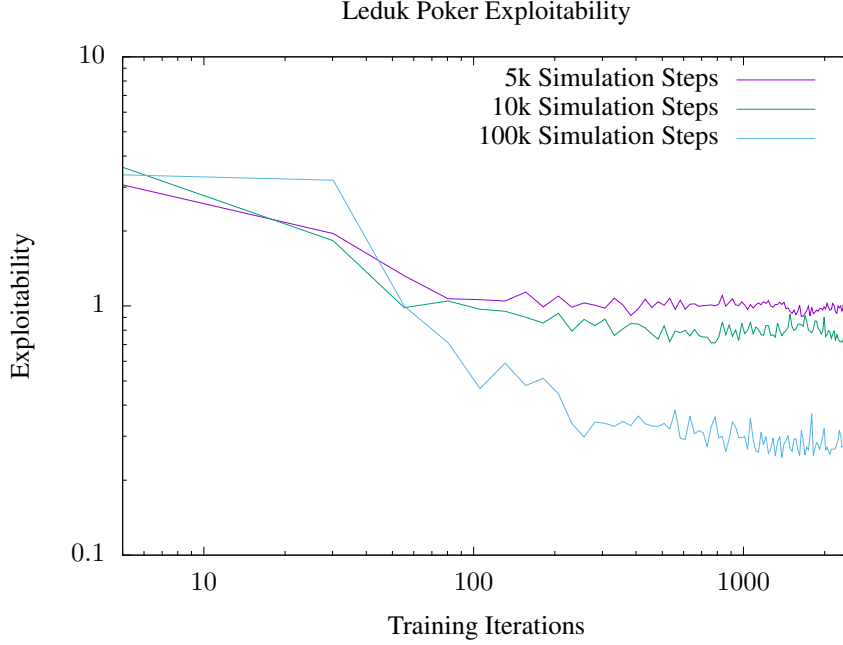


Figure 1: Leduk Poker Exploitability During Training

softmax output layer large enough to contain all legal actions. Illegal actions have their probabilities set to zero while the rest are normalised, they are not considered in the loss calculation. We use a reservoir that can hold experience from 32,000 full games with a decay ratio of  $\beta_{\text{reservoir}} = 2$ . In Leduk and Goof. 6 we train 128 steps every 32 games, for Goof. 13 we train 512 steps every 32 games. All head-to-head match ups were run for 5000 games. One iteration corresponds to 32 episodes, and 128 or 512 gradient descent steps, we use Adam as our optimisation algorithm and a learning rate of  $10^{-3}$ . We used 20 concurrent searches in all experiments. In the Goof. 6 and Goof. 13 experiments, we ran 10k simulation steps.

In figure 1 we see the exploitability during training in Leduk poker, performance is very dependent on how many search simulations are run, however Leduk Poker has such a small number of infosets that the whole tree is created very quickly, so the prior knowledge from rollouts doesn't help much. In table 1 with Goof. 6 we again see performance reaching a ceiling in head to head match ups with the teacher. In Goof 13. tables 2 and 3 we see increasing performance, eventually slightly surpassing the teacher, we believe this is due to the much greater value provided by rollouts in the larger game. All neural networks are feed forward and execute orders of magnitude faster than running the search.

## 5 Related Work

Expert Iteration (ExIt) was independently developed from the similar AlphaZero. The algorithm was used to improve the state-of-the-art in game Hex [Anthony et al., 2017]. The authors of the algorithm propose an analogy to human psychology and "thinking fast and slow" where system 1 and system 2 thinking integrate to solve problems. Our algorithm is heavily inspired by AlphaZero and is an instance of ExIt, so therefore is related to AlphaGo and AlphaGo Zero. These algorithms all combine planning, self-play and deep learning. However AlphaGo also used supervised learning from human games and did not use the outcome of MCTS planning as training targets. While very general, without some modification, none of these algorithms are suitable for playing imperfect information games.

The state of the art for playing imperfect information games, are exemplified by the AI poker playing programs Libratus [Brown and Sandholm, 2017] and DeepStack [Moravčík et al., 2017], both use

II Goof.(6)	Iter.	Random	OOS (10k)
ExpIt-OOS	100	66% (2.5)	30% (2.7)
	200	63% (2.6)	34% (2.7)
	1000	69% (2.5)	26% (2.2)
	2000	68% (2.5)	27% (2.2)
	2500	67% (2.5)	25% (2.2)

Table 1: Win rates of NN trained with ExpIt-OOS for II Goofspiel(6)

II Goof.(13)	Iter.	Random	OOS (10k)
ExpIt-OOS	100	61% (2.7)	40% (2.7)
	200	69% (2.5)	44% (2.7)
	500	74% (2.4)	<b>53% (2.6)</b>

Table 2: Win rates of NN trained with ExpIt-OOS for II Goofspiel(13)

safe subgame re-solving as a core component. Libratus, uses a hybrid offline and online method. Where a relatively coarse abstract game is solved offline with Monte Carlo Counter Factual Regret minimisation to create a base strategy. The base strategy is refined online using safe subgame re-solving with a fine grained abstraction. The subgame fixes all betting actions and community cards so far, with modified hand probabilities reflecting the relative chance of having a certain hand conditioned on reaching this subgame. DeepStack never computes and stores a full strategy and instead uses continual re-solving for every decision. Deep counterfactual value networks are used to approximate search results after a certain search depth limit. Their neural network was trained supervised on a dataset of randomly generated games that were solved offline. DeepStack and Libratus are built from general building blocks, but rely on the availability of a rich subgame structure. This is present in poker because all actions are public, forming a very informative public game tree. They also gain efficiency from the fact that all Heads-Up Texas Hold'em Poker infosets are the same size, have the same structure, and are relatively small (the only unknown information is a combination of 2 opponent playing cards). DeepStack does not have a feedback loop between planning and training, the deep counterfactual value network is trained on fixed offline solutions. Libratus is table driven and does not utilise neural networks for generalisation.

Our approach is similar to Neural Fictitious Self Play (NFSP) [Heinrich and Silver, 2016] in the use of deep neural networks to represent policies and training on experiences generated during self play. NFSP uses a Deep Q-learning like algorithm to approximate a best response in a model-free way. It does not use any planning. The benefit of using Q-learning is that the NFSP does not require access to the dynamics of the environment. However if model dynamics are available, NFSP is unable to benefit. We do not use fictitious self play, and only keep one current best estimate of the Nash equilibrium strategy learned from the online search.

Alg.	Iter.	II Goof.(13) Elo
ExpIt-OOS	100	1500 ( <i>fixed</i> )
	200	1531 (4.1)
	500	<b>1600 (3.5)</b>
OOS 10k		1572 (3.7)

Table 3: Elo rating of NN trained with ExpIt-OOS for II Goofspiel(13)

SmoothUCT [Heinrich and Silver, 2015], and MCTS with Exp3 or Regret Matching for action selection [Lisy et al., 2013] are planning methods for imperfect information games that can converge to Nash equilibria in large complex games, but must be run in an offline setting with each simulation run from the root. Online variants of these algorithms may later be created and could be incorporated into ExIt-OOS. Information Set Monte Carlo Tree Search (ISMCTS) [Cowling et al., 2012] is an extension of MCTS which can be run online in imperfect information settings, however ISMCTS samples hidden states uniformly from the current info set, and may produce highly exploitable strategies, however ISMCTS works well in practice for many games. All these planning methods keep statistics per info set and have no generalisation between states. Hand crafted abstractions are often used when solving large games to reduce the search space by aliasing similar states and actions together, thus manually sharing information between states and actions — instead of learning from data.

## 6 Future Work

Online Outcome Sampling is an effective online search algorithm. However even with high quality rollouts provided by the companion deep neural network, the exploitability in seems to be somewhat limited by the number of search iterations. An intriguing alternative is to use fictitious self play in a similar vein to NFSP, but replace the Neural Q-learning learning with a best response computed using a one sided MCTS search (where the other player strategy is kept fixed at the average strategy). Although, even with the opponent strategy fixed computing a best response still requires solving a POMDP online. It is possible the distribution over opponent hidden states could be learned and sampled for each search iteration.

There is also the possibility that due to the highly non-local nature of equilibrium play; small changes in strategy in one place in the game tree can cause large changes at distant nodes in response. Local search systems may have their limits. Global black box, optimisation algorithms like evolutionary strategies may be more suitable and general. Perhaps these algorithms could be adapted to converge to a robust and balanced population of strategies. This could lead to better play even for games without a well defined Nash equilibria such as full table poker. Having a balanced population of agents with interlocking strengths and weaknesses, chosen in the correct way could allow a meta-game playing strategy to choose the best one to exploit opponents over repeated rounds.

Ultimately there may be some combination of techniques similar to ExIt-OOS and AlphaZero that allow building an AI computer program that can reach super-human performance on any given abstract strategy game even with imperfect information. Using a single learning algorithm. Without any human domain knowledge. This is a small step in that direction. The authors are excited to continue this work.

## 7 Conclusion

The ExIt-OOS algorithm presents a novel approach to learn neural strategies in a large and general class of imperfect information games, even when there is no subgame structure. ExIt-OOS can utilise the high quality strategic information derived from planning with Online Outcome Sampling and known environment dynamics.

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