Introduction to Algorithms | (2nd Edition)

Chapter 31.1, Problem 12E | Bookmark | Show all steps: ON

Problem

Give an efficient algorithm to convert a given β-bit (binary) integer to a decimal representation. Argue that if multiplication or division of integers whose length is at most β takes time $M(\beta)$, then binary-to-decimal conversion can be performed in time $\Theta(M(\beta) \lg \beta)$.(*Hint:* Use a divide-and-conquer approach, obtaining the top and bottom halves of the result with separate recursions.)

Step-by-step solution

Step 1 of 3

• Firstly, until the original number is in form of power to 2, the length of the number bumps up. Doing this will not affect the asymptotic.

• The value of the number will not be changed, if the most significant side is padded with zeros.

• Spilt the binary integer into two parts as the less and the most significant half.

• The less significant is represented by *l* and the most significant half is represented by *m* to make the input equal as $m2^{\frac{\beta}{2}}+l$.

• After that, the *l* and the *m* is converted into decimal, recursively.

• Since, there is the need of above later, then compute the decimal version of all the values of 2^{2^i} upto 2^{β}.

• The count of the numbers is $\lg(\beta)$, so the time taken by the straightforward approach is $O(\lg^2(\beta))$.

• Therefore, this will be overshadowed by the rest of the algorithm.

• After this, compute the value of $m2^{\frac{\beta}{2}}+l$, this include both addition and the multiplication of the two numbers. Therefore, the recurrence relation would be as follows:

$$T(\beta)=2T\left(\frac{\beta}{2}\right)+M\left(\frac{\beta}{2}\right)$$

Comment

Step 2 of 3

• For some value of epsilon, it is difficult to separate *M* from the linear by n^{ϵ}.

• The analysis become easier, if the fact that the computation for the multiplication going down in the subcases will be ignored.

As per this concession, the runtime is as follows through the master theorem:

$$T(\beta)\in O(M(\beta)\lg(\beta))$$

Comment

Step 3 of 3

There is some procedure too to convert the binary number to the decimal. This will take only $\Theta(\beta)$ time while compared with the algorithm provided by the theory of automata. This algorithm of automata takes the following time:

$$\Theta(M(\beta)\lg(\beta)) \in \Omega(\beta \lg(\beta))$$

A deterministic finite transducer can be constructed between the two languages, since it will take only that number of steps as the number of bits present in the input.

This will take only linear time.

Also, keep track on the carryover from each of the digit to the next one.

Comment

Was this solution helpful? 0 0

Recommended solutions for you in Chapter 31.1

Chapter 31.1, Problem 9E

Show that the gcd operator is associative. That is, prove that for all integers a, b,and c,gcd (a, gcd (b, c))= gcd(gcd(a,b), c).

See solution

Chapter 31.1, Problem 11E

Give efficient algorithms for the operations of dividing a β-bit integer by a shorter integer and of taking the remainder of a ß-bit integer when divided by a shorter integer. Your algorithms should run in time O(ß2).

See solution

See more problems in subjects you study