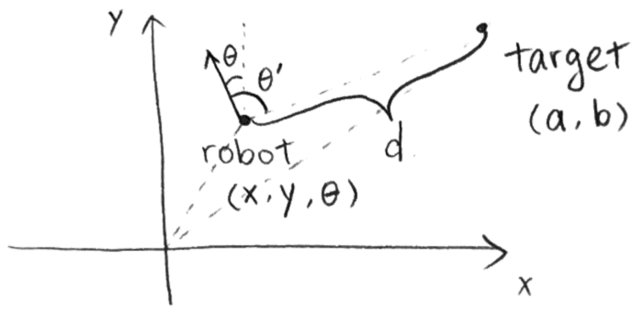


Known:

position of the robot relative to the origin : (x, y)

orientation of the robot relative to the y-axis : θ

position of the target point : (a, b)

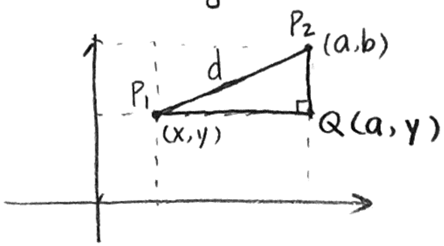
Unknown :

The robot should turn toward the target (by how much? $\Rightarrow \theta'$) and move to the target (by how much? $\Rightarrow d$)

So the unknowns are θ' and d.

Solution :

① How to get the distance d :



Let $P_1 = (x, y)$

$P_2 = (a, b)$

$Q = (a, y)$

We can see that d = hypotenuse of the right triangle P_1P_2Q

with sides $P_1Q = a - x$ and $P_2Q = b - y$.

By Pythagoras' theorem:

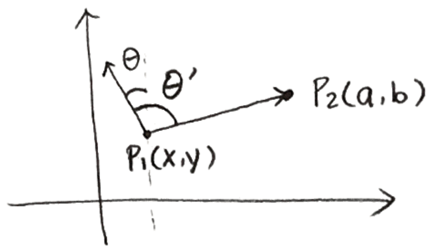
$$d^2 = P_1Q^2 + P_2Q^2$$

so

$$d = \sqrt{P_1Q^2 + P_2Q^2}$$

$$= \sqrt{(a-x)^2 + (b-y)^2}$$

② How to get the angle θ' :



If you remember "vectors" from high school, then you might recall that the angle contained in two vectors can be obtained by <something> x <cosine of angle contained>.

This is the basic strategy I will employ below

The full formula is :

(1) $\boxed{\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}| \cos\alpha}$, where "•" represents the dot product, and $|\vec{v}|$ represents the "length" of vector $\vec{v}$.

In particular, if $\vec{v} = \langle a, b \rangle$ $\boxed{|\vec{v}| = \sqrt{a^2+b^2}}$, from Pythagoras' theorem.

But if we know the coordinates of $\vec{v}, \vec{w}$, there's another way to get the dot product :

if $\vec{v} = \langle a, b \rangle$, $\vec{w} = \langle c, d \rangle$, then

(2) $\boxed{\vec{v} \cdot \vec{w} = ac + bd}$

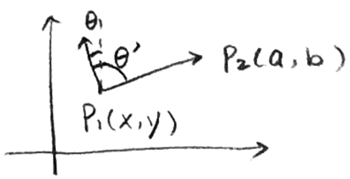
Therefore, we can reorganize (1) to get :

$$\cos\alpha = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}|\,|\vec{w}|}$$

← we know $\boxed{\vec{v} \cdot \vec{w} = ac + bd}$

← we also know $|\vec{v}| = \sqrt{a^2+b^2}$, $|\vec{w}| = \sqrt{c^2+d^2}$

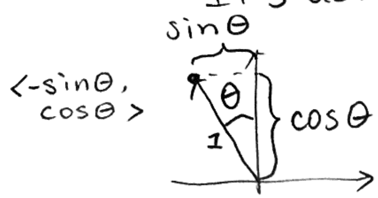
So $\alpha = \arccos\left(\frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}\right) = \arccos\left(\frac{ac + bd}{\sqrt{a^2+b^2} \times \sqrt{c^2+d^2}}\right)$, Therefore, the angle contained between two vectors can be computed, if we know the coordinate representation of the two vectors.



Let's look at this graph again.
We want to know θ', and luckily, θ' is indeed contained between two vectors, one of which is $\overrightarrow{P_1P_2} = \langle a-x, b-y \rangle$, and the other of which is the orientation vector.

But we don't have the coordinate representation of the orientation vector! (We only know the angle θ it forms with the y-axis)

It's actually easy to get a coordinate repr.



From basic trigonometry, if the hypotenuse $= 1$, then the side opposite to θ is $\sin\theta$, and the side adjacent to θ is $\cos\theta$.

Since the vector is in the 2nd quadrant (where $x < 0$, $y > 0$), we can construct a vector $\langle -\sin\theta, \cos\theta \rangle$ to represent the orientation.

Now, we have all the information needed to compute the angle contained between

P_1P_2 and the orientation vector.

However, there's still one more thing we need to know in order to turn the robot : in which direction should it turn?

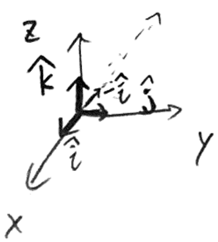
For any θ', there two possibilities:

① the robot can turn clockwise by θ'

② - - - - - - counter-ccw by θ'

Then, which way ? It's intuitive clear that if the target point is on the right w.r.t. the orientation, then the robot should turn ccw, and vice versa

To compute whether the target point is on the right, recall from vector algebra that there's an operation called the <u>cross product</u>, which indicates the relative "righthandedness" of two vectors.

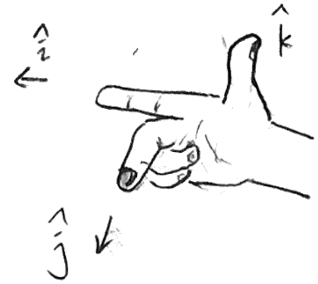


Let $\hat{i}, \hat{j}, \hat{k}$ be the unit vectors in the positive direction of x-, y-, z- axis, respectively.

We know $\hat{i} \times \hat{j} = \hat{k}$ $(k_z > 0)$, and $\hat{i}$ is on the righthand side of $\hat{j}$.

$-\hat{i} \times \hat{j} = -\hat{k}$ $(-k_z < 0)$, and $-\hat{i}$ is on the LH side of $\hat{j}$.

This is generally referred to as the <u>right hand rule</u>.



Therefore, for the orientation vector (let's call it $\vec{r}$) and the target vector $\overrightarrow{P_1P_2}$,

we can pretend they're in the x-y plane in 3D.

Their cross product $\vec{r} \times \overrightarrow{P_1P_2}$ is on the z-axis.

b/c $\hat{i}$ is on the righthand side of $\hat{j}$ iff $k_z > 0$,

it follows that $\overrightarrow{P_1P_2}$ is on the righthand side of $\vec{r}$ iff

$$\overrightarrow{P_1P_2} \times \vec{r} > 0.$$

But if $\overrightarrow{P_1P_2}$ is on the right hand side of $\vec{r}$,

then the robot should turn <u>clockwise</u> by θ' (and vice versa)

$\Downarrow$

($\theta' < 0$ by our definition)