

ECON_1 TASK

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1. THEORY: DISCRETE CHOICE MODELS

An agent (n) faces a choice of technology alternatives (j). We assume that each agent is a utility maximizer (U_{nj}) in the selection of a technology alternative. However, U_{nj} is unobservable and consequently unknown to the researcher. Instead, we can observe information about the available choice set (x_{nj}), as well as the agent (s_n). Using this information we can specify a representative utility function, which is $V_{nj} = V(x_{nj}, s_n)$.¹

We can write each agents utility function as a function of observed and unobserved utility, $U_{nj} = V_{nj} + \epsilon_{nj}$, where the distribution of the error term (ϵ_{nj}) depends on the researchers specification of the observed utility (V_{nj}). Given that we cannot observe ϵ_{nj} directly we instead derive the probability of a technology choice. The probability that the agent chooses technology j (i.e., the expected value of the indicator function) is $Pr(j|x) = Pr(I[h(x, \epsilon) = j] = 1) = \int I[h(x, \epsilon) = j] \cdot f(\epsilon) d\epsilon$.²

The probability of an individual choosing the alternative that they actually chose is: $\prod_i (P_{ni})^{y_{ni}}$, where y_{ni} is either one for the selected alternative or zero for the remaining unselected options. Given that the exponents are either zero or one the result reduces to the term P_{ni} , which is just the probability of the chosen alternative. The likelihood function then is $\mathcal{L}(\beta) = \prod_{n=1}^N \prod_i (P_{ni})^{y_{ni}}$, which can be rewritten as $\ln(\mathcal{L}(\beta)) = \sum_n \sum_i y_{ni} \ln(P_{ni})$, where the specification of P_{ni} depends on the specification of the model.

2. DATA

We assume the predominant operation cost for a fossil fuel generator is the fuel cost. For renewable sources we assume Fuel price data for the period (1/1986-5/2016) is from the Energy Information Administration (EIA) sources. We use the investment cost information (i.e., overnight cost) by electricity production technology from the Global Change Assessment Model (GCAM) to inform our model.

We assume that fuel costs are the predominant operating cost for fossil fuel generation plants. The Energy Information Administration (EIA) in the Electric Power Monthly report publishes electric utility receipts of and average cost for fossil fuels used in the power generation sector. We generate a monthly time-series dataset for the period 1/1986-5/2016 using the Electric Power Monthly reports from June 1996 (Table 26), January 2010 (Table 4.2), and May 2016 (Table 4.2).

Date: July 22, 2016.

¹ $U_{nj} \neq U_{nj}$ given the unobservable aspects of the utility function.

²The unobserved parameters ϵ_{nj} follow a probability density distribution $f(\epsilon)$, which in the logistic framework we assume to be distributed extreme value with variance of $\frac{\pi^2}{6}$.

The data includes price information for coal, petroleum liquids, and natural gas inputs. The prices listed are averages and in nominal units (i.e., unadjusted for inflation). For continuity purposes we use the total petroleum price from the June 1996 report instead of the heavy oil price.

The decision faced by decision-makers in our analysis is the choice amongst power generation input fuels (not yet focusing on the technology choice). We use the monthly installed net summer capacity by fuel type as the dependent variable of our analysis. Existing nameplate and net summer capacity by energy source, producer type, and state is provided by the EIA. Net summer capacity represents the maximum capacity as determined by testing during the summer peak demand period (i.e., June 1st to September 30th) and is lower than the nameplate capacity since it includes electricity consumption by auxiliaries and the station itself (EIA Glossary, 2016). We consider the capacity change in the electric power industry (i.e., we ignore commercial and industrial power). Also, we are only considering the positive change in net summer capacity in our discrete choice model (i.e., we ignore net summer capacity reductions).

Investment cost data is from the GCAM model input data files. The model has overnight cost estimates in 2010 USD per kW for the time periods 1975, 1990, 2005, 2010, and 2015. We use the following simple linear regression model to interpolate between these data points for each technology (k).

$$(1) \quad \ln(IC_{kt}) = \beta_{0k} + \beta_{1kt} \cdot Year_t + \beta_{2kt} \cdot tech \cdot year \epsilon$$

3. MODEL

These types of models are usually specified as a set of individuals (n) choosing between some alternatives (j). We propose that the total net change in capacity, which varies year by year, is the result of an exogenous process that exists outside the model. The purpose of this model is to determine the factors that effect the technology choice, which is a discrete choice problem.

3.1. Logistic implementation. If a random variable Y_i can take one of several discrete values $1, 2, \dots, J$ then the probability that the i -th response is in the j -th category is λ_{ij} (i.e., $\lambda_{ij} = \Pr\{Y_i = j\}$). Assuming the categories are both mutually exclusive and exhaustive then $\sum_{j=1}^J \lambda_{ij} = 1$.³ The log-odds as estimated by the multinomial logit model then is $\log \frac{\lambda_{ij}}{\lambda_{iJ}} = x_i^T \beta_j$. The multinomial logit model estimates $J - 1$ equations that compare each of the $J - 1$ categories to the baseline category J (i.e., the multinomial logit model reduces to the standard logistic regression in the case of $J = 2$).

The dependent variable of interest is the market share of each technology (or fuel type) S_1, S_2, \dots, S_B with each share defined as $S_b = \frac{Q_b}{\sum_B Q_B}$. We assume that Q_b (i.e., total sales, production, etc.) follows some Cobb-Douglas type production function, $Q_B = e^{\alpha_b} p_b^{\beta_b}$, which can be rewritten as $Q_B = \exp(\alpha_b + \beta_b \cdot \ln(p_b))$. We can rewrite the market share (S_{bt}) as a function of the factors of production (see Equation 4), where S_{bt} is interpreted as the likelihood that a technology b is selected at time period t .

³The i in this case would represent each time period so it should actually be λ_{tj} .

For the logit specification the log likelihood function is $\ln(\mathcal{L}(\beta)) = \sum_n \sum_i y_{ni} (\beta^T x_{ni}) - \sum_n \sum_i y_{ni} \ln \left(\sum_j e^{\beta^T x_{nj}} \right)$. The maximum likelihood estimates of the β parameter is then $\frac{\partial \ln(\mathcal{L}(\beta))}{\partial \beta} = 0$, which is $\sum_n \sum_i (y_{ni} - P_{ni}) \cdot x_{ni}$, where $P_{ni} = \frac{e^{\beta^T x_{ni}}}{\sum_j e^{\beta^T x_{nj}}}$.

3.2. Logit model specification assumptions. In the case of the logistic model the probability of choosing technology choice i is equal to $Pr_{ni} = Pr(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj} \quad \forall \quad j \neq i)$. This can be rewritten as the following $Pr_{ni} = (\epsilon_{nj} < \epsilon_{ni} + V_{ni} - V_{nj} \quad \forall \quad j \neq i)$. Given that the density of the unobserved utility is $f(\epsilon_{nj}) = e^{-\epsilon_{nj}} e^{-e^{-\epsilon_{nj}}}$ and the cumulative distribution function is $F(\epsilon_{nj}) = e^{-e^{-\epsilon_{nj}}}$ then the probability of choosing technology i is $Pr_{ni} = \int \left(\prod_{j \neq i} e^{-e^{-(\epsilon_{nj} + V_{ni} - V_{nj})}} \right) e^{\epsilon_{ni}} e^{-e^{\epsilon_{ni}}} d\epsilon_{ni}$.⁴ The integral simplifies to $Pr_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}}$.⁵

We can normalize the model for the scale of the utility by setting the variance equal to $\frac{\pi^2}{6}$. the scale of the utility is irrelevant to the alternative chosen needs to be normalized as the alternative with the highest utility is the same no matter the scale (i.e., the alternative chosen is the same in these two formulations, $U_{nj}^0 = V_{nj} + \epsilon_{nj}$ and $U_{nj}^1 = \lambda V_{nj} + \lambda \epsilon_{nj}$). Researchers usually do this through the normalization of the variance of the error terms. If we assume the error terms are iid then we can implement this by dividing the model coefficients by the standard deviation of the unobserved portion of utility. The coefficients now represent the 'effect of the observed variables relative to the standard deviation of the unobserved factors' (Train 2009, p. 24). The error variances in a traditional logit are normalized to $\frac{\pi^2}{6}$ (Train 2009). If we feel that we have heteroskedastic errors at any point then we need to review the discussion on p25 (basically just normalize one of the subpopulations and then the variance estimates for the other groups will be relative to that baseline).

The absolute value of U_{nj} does not matter only the difference in utility between alternatives since $Pr(U_{ni} - U_{nj} > 0, \quad \forall \quad j \neq i)$. This fact has implications on variable selection, in that we only include parameters that capture differences across the alternatives. The absolute value of the constant term (k_j) does not matter only the difference. To do this we need to normalize the absolute value of one constant by setting it equal to zero. The remaining $J - 1$ constants are then the difference in constant values for each alternative, as compared to the baseline.⁶ The same holds true for any variable that doesn't vary for an individual between alternatives (i.e., individual income). This issue can be addressed through the normalization of the variable or by interacting it with attributes of the alternatives.

The logit model assumes that $\epsilon_{ni} \quad \forall i$ (i.e., technology choice selected) are iid extreme value (i.e., unobserved portion of the utility is uncorrelated over the choice set and have the same variance).⁷ The dependence of decisions on prior decision making can also violate the independence assumption inherent to the logit model. The logistic implementation makes the strong assumption that the unobserved portion of the utility function are independent across alternatives. Though

⁴The Pr_{ni} can be interpreted as the joint probability of not choosing alternative i multiplied by the probability of choosing alternative i .

⁵ V_{ni} is usually specified to be linear in parameters (i.e., not multiplicative).

⁶This can be implemented in the mlogit R package through the 'reflevel' command.

⁷The assumption of independence can be quite limiting.

it should be noted that through proper model specification we could reduce the importance of the independence assumption. If we specify the utility model well (i.e., $U_{nj} = V_{nj} + \epsilon_{nj}$), accounting for the important differences between alternatives, then the error term should be white noise (i.e., the error for one alternative provides little to no information about the error for another alternative). Should we believe correlation exists across alternatives then we either respecify the model to account for the correlation structure; explore other alternatives later that do not have this restrictive assumption (i.e., GEV, probit, and mixed logit); or continue with the logit specification as an approximation. The generalized extreme value model (GEV) allows for correlation in the unobserved factors over alternatives (if the correlation amongst these factors is zero then the GEV is equivalent to the logit model).

We implement the model using the mlogit R package. We implement this using the weights command in the mlogit R package.

- (1) set reflevel
- (2) the logit model is a misspecification if taste variation is at least partly random (a probit or mixed logit is a better alternative)
- (3) an implication of independence from irrelevant alternatives (IIA) is that the logit model substitutes proportionally across alternatives (nested logits, probits and mixed logit offer solutions to this problem); tests are available to test the IIA assumption
- (4) How to treat the capacity factors? Net summer capacity for wind equals nameplate capacity
- (5) as cross sectional or panel data?
- (6) For a discrete choice model the choice set must be mutually exclusive, exhaustive, and finite from the decision makers point of view. In our particular circumstance, we can redefine the choice set, as it is possible to use more than one technology, to be any combination of the technology choices such that the set is then mutually exclusive (i.e., A only, B only, or both A and B). To ensure that the list is exhaustive might require that we include a none of the above option as well.
- (7) net change in capacity can be negative
- (8) if a technology has no chance of being selected it can be removed from the logit specification
- (9) the relationship between the logit probability and representative utility is S-shaped which means that if representative utility is very low (relatively), a small increase in the utility of the alternative has little effect on the probability of its being chosen (from a policy perspective this would be like choosing between investing in an area with poor bus service or an area with sufficient bus service: more impact in the area with sufficient service on bus ridership)

$$(2) \quad \text{logit}(p) = \beta_0 + \beta_1 \cdot \text{time}_i$$

$$(3) \quad \text{logit}(p) = \beta_0 + \beta_1 \cdot \text{time}_i + \beta$$

$$(4) \quad S_{bt} = \frac{\exp(\alpha_b + \gamma_t + \beta_b \cdot \ln(p_{bt}))}{\sum_{k=1}^B \exp(\alpha_k + \gamma_t + \beta_k \cdot \ln(p_{kt}))}$$

In the literature this model is referred to as the multiplicative competitive interaction (MCI) model (Cooper 1988, 1993). The MCI model assumes that a proportional increase in all prices (i.e., all fuel types in this case) would cause no change in the distribution of market shares (assuming equality in the β_b parameters). An alternative model formulation is the multinomial logit (MNL) model, which includes the actual prices (instead of the logged prices). The MNL model assumes that an absolute price increase for all market participants (i.e., all fuel types) would result in no change to the distribution of market shares (assuming all β_b parameters are equal). Of the two assumptions the MCI seems the more realistic one.

We can expand the previous model (see Equation 4) to include cross-price elasticities.

$$(5) \quad Q_B = \exp \left(\alpha_b + \gamma_t + \beta_b \cdot \ln(p_{bt}) + \sum_{k \neq b} \eta_{bk} \cdot \ln(p_{kt}) \right)$$

4. NEXT STEPS

We might consider the nested logit models to incorporate the specific technology choice once the fuel choice has been made. glm implementation with percentage dependent variable and a weight equal to the change in installed MW because the percentages are not equivalent between the years (i.e., the absolute value is different than the relative percentage).