PART IV

Inference

To *infer* is to draw a conclusion from evidence. *Statistical inference* draws a conclusion about a population from evidence provided by a sample. Drawing conclusions in mathematics is a matter of starting from a hypothesis and using logical argument to prove without doubt that the conclusion follows. Statistics isn't like that. Statistical conclusions are uncertain because the sample isn't the entire population. So statistical inference has to not only state conclusions but also say how uncertain they are. We use the language of probability to express uncertainty.

Because inference must both give conclusions and say how uncertain they are, it is the most technical part of statistics. Texts and courses intended to train people to do statistics spend most of their time on inference. Our aim in this book is to help you *understand* statistics, which takes less technique but, often, more thought. We will look only at a few basic techniques of inference. The techniques are simple, but the ideas are subtle, so prepare to think. To start, think about what you already know and don't be too impressed by elaborate statistical techniques: even the fanciest inference cannot remedy basic flaws such as voluntary response samples or uncontrolled experiments.

What Is a Confidence Interval?

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CASE STUDY According to the Centers for Disease Control and Prevention (CDC), adults who engage in less than 30 minutes of moderate physical exercise per day should consume 1.5 to 2.0 cups of fruits per day and 2 to 3 cups of vegetables per day. Are adults meeting these recommendations? Not exactly, according to recent fruit and vegetable intake information obtained from the 2013 Behavioral Risk Factor Surveillance System (BRFSS).

The BRFSS is an ongoing, state-based, random-digit-dialed telephone survey of adults who are at least 18 years of age. It is the world's largest ongoing telephone health survey system, tracking health conditions and risk behaviors in the United States since 1984. Data are gathered monthly from all 50 states, the District of Columbia (DC), Puerto Rico, the U.S. Virgin Islands, and Guam. In terms of gathering information about fruit and vegetable intake, survey respondents were asked to think about the previous month and to indicate how many times per day, week, or month they consumed whole fruit, 100% fruit juice, dried beans, dark green vegetables, orange vegetables, and other vegetables. Although close to 500,000 individuals were originally contacted to be a part of the survey in 2013, 118,193 respondents were excluded from analysis because they either resided in other countries, had missing responses to one or more questions, or had implausible reports of fruit and vegetable consumption (e.g., eating fruit more than 16 times per day or eating vegetables more than 23 times per day). The results reported in the 2013 BRFSS are based on the final sample of 373,580 adults.

Across the states, results varied. The state with the largest number of survey respondents reporting meeting the daily fruit and vegetable intake recommendations was California. Of the 9,011 California residents who were surveyed, 17.7% met the daily fruit intake recommendations, and 13% met the daily vegetable intake recommendations. We know these numbers won't be exactly right for the entire population of adults in California, but are they close?

In this chapter, we study confidence intervals. These are intervals that help us see how accurate numbers like 17.7% and 13% are. By the end of this chapter, you will be able to construct such intervals for proportions and means, and you will be able to interpret what such intervals represent.

Estimating

Statistical inference draws conclusions about a population on the basis of data about a sample. One kind of conclusion answers questions like, "What percentage of employed women have a college degree?" or "What is the mean survival time for patients with this type of cancer?" These questions ask about a number (a percentage, a mean) that describes a population. Numbers that describe a population are **parameters**. To estimate a population parameter, choose a sample from the population and use a **statistic**, a number calculated from the sample, as your estimate. Here's an example.

EXAMPLE 1 Soda consumption

In July 2015, Gallup conducted telephone interviews with a random sample of 1009 adults. The adults were at least 18 years of age and resided either in one of the 50 U.S. states or in the District of Columbia (DC). Survey respondents were asked to consider several different foods and beverages and to indicate if these were things they actively tried to include in their diet, actively tried to avoid in their diet, or didn't think about at all. Of the 1009 adults surveyed, 616 indicated that they actively tried to avoid drinking regular soda or pop. Based on this information, what can we say about the percentage of all Americans 18 years or age or older who actively try to avoid drinking regular soda or pop?

Our population is adults at least 18 years of age or older who reside in the United States or District of Columbia. The parameter is the proportion who actively tried to avoid drinking regular soda or pop in 2015. Call this unknown parameter p, for "proportion." The statistic that estimates the parameter p is the **sample proportion**

$$\hat{p} = \frac{\text{count in the sample}}{\text{size of the sample}} = \frac{616}{1009} = 0.611$$

A basic move in statistical inference is to use a sample statistic to estimate a population parameter. Once we have the sample in hand, we estimate that the proportion of all adult Americans who actively tried to avoid drinking regular soda or pop in 2015 is "about 61.1%" because the proportion in the sample was exactly 61.1%. We can only estimate that the truth about the population is "about" 61.1% because we know that the sample result is unlikely to be exactly the same as the true population proportion. A confidence interval makes that "about" precise.

95% confidence interval

A **95% confidence interval** is an interval calculated from sample data by a process that is guaranteed to capture the true population parameter in 95% of all samples.

We will first march straight through to the interval for a population proportion, and then we will reflect on what we have done and generalize a bit.

Estimating with confidence

We want to estimate the proportion p of the individuals in a population who have some characteristic—they are employed, or they approve the president's performance, for example. Let's call the characteristic we are looking for a "success." We use the proportion $\hat{p}$ of successes in a simple random sample (SRS) to estimate the proportion p of successes in the population. How good is the statistic $\hat{p}$ as an estimate of the parameter p? To find out, we ask, "What would happen if we took many samples?" Well, we know that $\hat{p}$ would vary from sample to sample. We also know that this sampling variability isn't haphazard. It has a clear pattern in the long run, a pattern that is pretty well described by a Normal curve. Here are the facts.

Sampling distribution of a sample proportion

The **sampling distribution** of a statistic is the distribution of values taken by the statistic in all possible samples of the same size from the same population.

Take an SRS of size n from a large population that contains proportion p of successes. Let $\hat{p}$ be the **sample proportion** of successes,

$$\hat{p} = \frac{\text{count of successes in the sample}}{n}$$

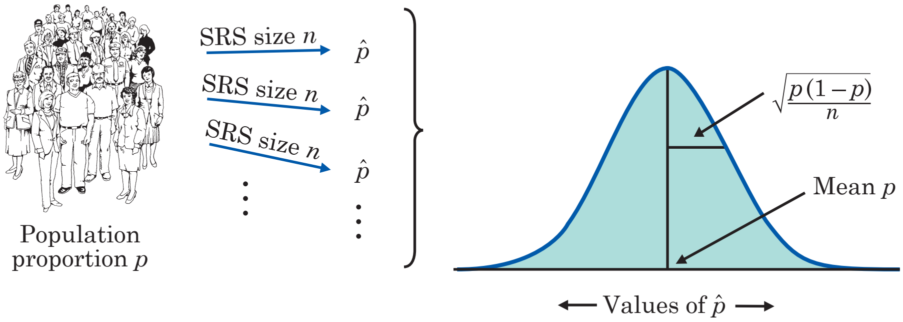

Figure 21.1 Repeat many times the process of selecting an SRS of size n from a population in which the proportion p are successes. The values of the sample proportion of successes $\hat{p}$ have this Normal sampling distribution.

Then, if the sample size is large enough:

- The sampling distribution of $\hat{p}$ is **approximately Normal**.
- The **mean** of the sampling distribution is p.
- The **standard deviation** of the sampling distribution is

$$\sqrt{\frac{p(1-p)}{n}}$$

These facts can be proved by mathematics, so they are a solid starting point. Figure 21.1 summarizes them in a form that also reminds us that a sampling distribution describes the results of lots of samples from the same population.

Standard error

The standard deviation of the sampling distribution of a sample statistic is commonly referred to as the **standard error**.

EXAMPLE 2 More on soda consumption

Suppose, for example, that the truth is that 62% of Americans actively tried to avoid drinking regular soda or pop in 2015. Then, in the setting of Example 1, $p = 0.62$. The Gallup sample of size $n = 1009$ would, if repeated many times, produce sample proportions $\hat{p}$ that closely follow the Normal distribution with

$$\text{mean} = p = 0.62$$

Eric Raptosh Photography/Getty Images

and

$$\text{standard error} = \sqrt{\frac{p(1-p)}{n}}$$

$$= \sqrt{\frac{(0.62)(0.38)}{1009}}$$

$$= \sqrt{0.0002334} = 0.01528$$

The center of this Normal distribution is at the truth about the population. That's the absence of bias in random sampling once again. The standard error is small because the sample is quite large. So, almost all samples will produce a statistic $\hat{p}$ that is close to the true p. In fact, the 95 part of the 68–95–99.7 rule says that 95% of all sample outcomes will fall between

$$\text{mean} - 2 \text{ standard errors} = 0.62 - 0.0306 = 0.5894$$

and

$$\text{mean} + 2 \text{ standard errors} = 0.62 + 0.0306 = 0.6506$$

Figure 21.2 displays these facts.

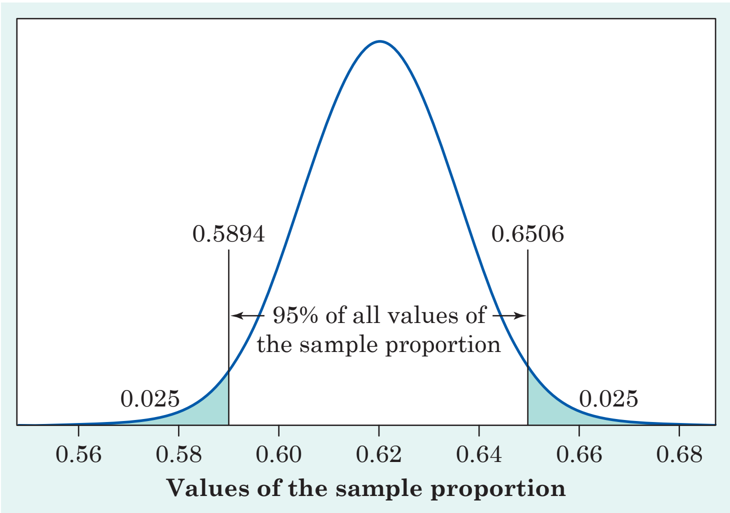

Figure 21.2 Repeat many times the process of selecting an SRS of size 1009 from a population in which the proportion $p = 0.62$ are successes. The middle 95% of the values of the sample proportion $\hat{p}$ will lie between 0.5894 and 0.6506.

So far, we have just put numbers on what we already knew: we can trust the results of large random samples because almost all such samples give results that are close to the truth about the population. The numbers say that in 95% of all samples of size 1009, the statistic $\hat{p}$ and the parameter p are within 0.0306 of each other. We can put this another way: 95% of all samples give an outcome $\hat{p}$ such that the population truth p is captured by the interval from $\hat{p} - 0.0306$ to $\hat{p} + 0.0306$.

The 0.03 came from substituting $p = 0.62$ into the formula for the standard error of $\hat{p}$. For any value of p, the general fact is:

When the population proportion has the value p, 95% of all samples catch p in the interval extending 2 standard errors on either side of $\hat{p}$.

That's the interval

$$\hat{p} \pm 2\sqrt{\frac{p(1-p)}{n}}$$

Is this the 95% confidence interval we want? Not quite. The interval can't be found just from the data because the standard deviation involves the population proportion p, and in practice, we don't know p. In Example 2, we used $p = 0.62$ in the formula, but this may not be the true p.

What can we do? Well, the standard deviation of the statistic $\hat{p}$, or the standard error, does depend on the parameter p, but it doesn't change a lot when p changes. Go back to Example 2 and redo the calculation for other values of p. Here's the result:

Value of p:	0.60	0.61	0.62	0.63	0.64
Standard error:	0.01542	0.01535	0.01528	0.01520	0.01511

We see that, if we guess a value of p reasonably close to the true value, the standard error found from the guessed value will be about right. We know that, when we take a large random sample, the statistic $\hat{p}$ is almost always close to the parameter p. So, we will use $\hat{p}$ as the guessed value of the unknown p. Now we have an interval that we can calculate from the sample data.

95% confidence interval for a proportion

Choose an SRS of size n from a large population that contains an unknown proportion p of successes. Call the proportion of successes in this sample $\hat{p}$. An **approximate 95% confidence interval** for the parameter p is

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

EXAMPLE 3 A confidence interval for soda consumption

The Gallup random sample of 1009 adult Americans found that 616 reported actively trying to avoid drinking regular soda or pop in 2015, a sample proportion $\hat{p} = 0.611$. The 95% confidence interval for the proportion of all Americans who actively tried to avoid drinking regular soda or pop in 2015 is

$$\begin{aligned}\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.611 \pm 2\sqrt{\frac{(0.611)(0.389)}{1009}} \\ &= 0.611 \pm (2)(0.01535) \\ &= 0.611 \pm 0.0307 \\ &= 0.5803 \text{ to } 0.6417\end{aligned}$$

Interpret this result as follows: we got this interval by using a recipe that catches the true unknown population proportion in 95% of all samples. The shorthand is: we are **95% confident** that the true proportion of all Americans who actively tried to avoid drinking regular soda or pop in 2015 lies between 58.03% and 64.17%.

Statistics in Your World

Who is a smoker? When estimating a proportion p, be sure you know what counts as a "success." The news says that 20% of adolescents smoke. Shocking. It turns out that this is the percentage who smoked at least once in the past month. If we say that a smoker is someone who smoked in at least 20 of the past 30 days and smoked at least half a pack on those days, fewer than 4% of adolescents qualify.

NOW IT'S YOUR TURN

21.1 Gambling. A May 2011 Gallup Poll consisting of a random sample of 1018 adult Americans found that 31% believe gambling is morally wrong. Find a 95% confidence interval for the proportion of all adult Americans who believe gambling is morally wrong. How would you interpret this interval?

Understanding confidence intervals

Our 95% confidence interval for a population proportion has the familiar form

$$\text{estimate} \pm \text{margin of error}$$

News reports of sample surveys, for example, usually give the estimate and the margin of error separately: "A new Gallup Poll shows that 65% of women favor new laws restricting guns. The margin of error is plus or minus four percentage points." News reports usually leave out the level of confidence, although it is almost always 95%.

The next time you hear a report about the result of a sample survey, consider the following. If most confidence intervals reported in the media have a 95% level of confidence, then in about 1 in 20 poll results that you hear about, the confidence interval does *not* contain the true proportion.

Here's a complete description of a confidence interval.

Confidence interval

A **level *C* confidence interval** for a parameter has two parts:

- An **interval** calculated from the data.
- A **confidence level *C***, which gives the probability that the interval will capture the true parameter value in repeated samples.

There are many recipes for statistical confidence intervals for use in many situations. Not all confidence intervals are expressed in the form "estimate ± margin of error." Be sure you understand how to interpret a confidence interval. The interpretation is the same for any recipe, and you can't use a calculator or a computer to do the interpretation for you.

Confidence intervals use the central idea of probability: ask what would happen if we were to repeat the sampling many times. The 95% in a 95% confidence interval is a probability, the probability that the method produces an interval that does capture the true parameter.

EXAMPLE 4 How confidence intervals behave

The Gallup sample of 1009 adult Americans in 2015 found that 616 reported actively trying to avoid drinking regular soda or pop, so the sample proportion was

$$\hat{p} = \frac{616}{1009} = 0.611$$

and the 95% confidence interval was

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.611 \pm 0.0307$$

Draw a second sample from the same population. It finds that 700 of its 1009 respondents reported actively trying to avoid drinking regular soda or pop. For this sample,

$$\hat{p} = \frac{700}{1009} = 0.694$$

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.694 \pm 0.0290$$

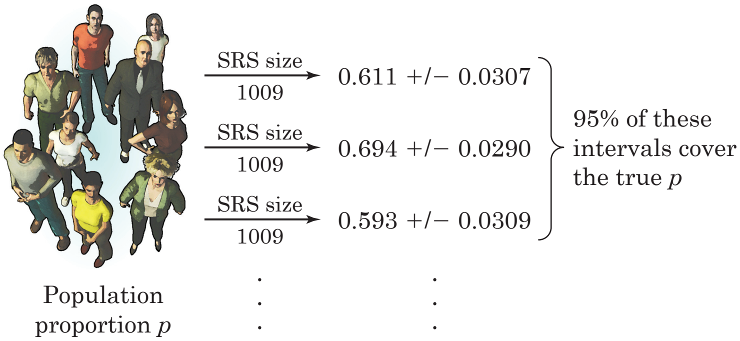

Figure 21.3 Repeated samples from the same population give different 95% confidence intervals, but 95% of these intervals capture the true population proportion p.

Draw another sample. Now the count is 598 and the sample proportion and confidence interval are

$$\hat{p} = \frac{598}{1009} = 0.593$$

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.593 \pm 0.0309$$

Keep sampling. Each sample yields a new estimate $\hat{p}$ and a new confidence interval. *If we sample forever, 95% of these intervals capture the true parameter*. This is true no matter what the true value is. Figure 21.3 summarizes the behavior of the confidence interval in graphical form.

Example 4 and Figure 21.3 remind us that repeated samples give different results and that we are guaranteed only that 95% of the samples give a correct result. On the assumption that two pictures are better than one, Figure 21.4 gives a different view of how confidence intervals behave. Figure 21.4 goes behind the scenes. The vertical line is the true value of the population proportion p. The Normal curve at the top of the figure is the sampling distribution of the sample statistic $\hat{p}$, which is centered at the true p. We are behind the scenes because, in real-world statistics, we usually don't know p.

The 95% confidence intervals from 25 SRSs appear below the graph in Figure 21.4, one after the other. The central dots are the values of $\hat{p}$, the centers of the intervals. The arrows on either side span the confidence interval. In the long run, 95% of the intervals will cover the true p and 5% will miss. Of the 25 intervals in Figure 21.4, there are 24 hits and 1 miss. (Remember that probability describes only what happens in the long run—we don't expect exactly 95% of 25 intervals to capture the true parameter.)

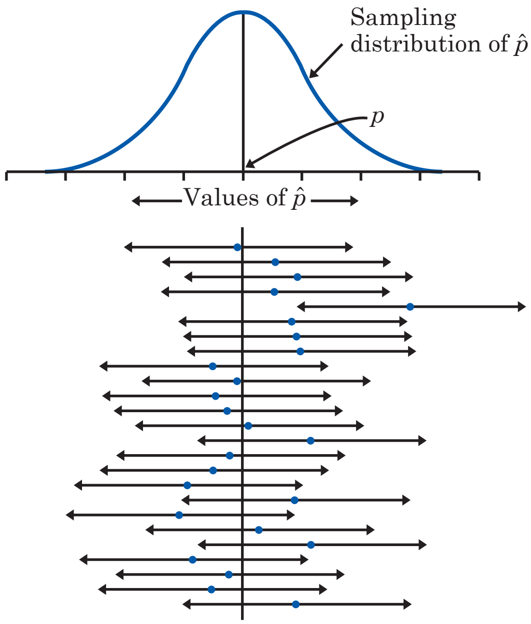

Figure 21.4 Twenty-five samples from the same population give these 95% confidence intervals. In the long run, 95% of all such intervals cover the true population proportion, marked by the vertical line.

Don't forget that our interval is only *approximately* a 95% confidence interval. It isn't exact for two reasons. The sampling distribution of the sample proportion $\hat{p}$ isn't exactly Normal. And we don't get the standard deviation, or the standard error, of $\hat{p}$ exactly right because we used $\hat{p}$ in place of the unknown p. We use a new estimate of the standard deviation of the sampling distribution every time, even though the true standard deviation never changes. Both of these difficulties go away as the sample size n gets larger. So, our recipe is good only for large samples. What is more, the recipe assumes that the population is really big—at least 10 times the size of the sample. Professional statisticians use more elaborate methods that take the size of the population into account and work even for small samples. But our method works well enough for many practical uses. More important, it shows how we get a confidence interval from the sampling distribution of a statistic. That's the reasoning behind any confidence interval.

More on confidence intervals for a population proportion*

We used the 95 part of the 68–95–99.7 rule to get a 95% confidence interval for the population proportion. Perhaps you think that a method that

*This section is optional.

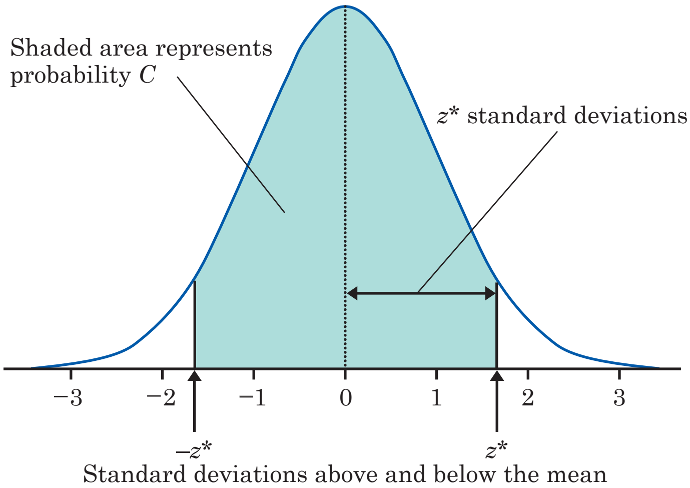

Figure 21.5 Critical values z^* of the Normal distributions. In any Normal distribution, there is area (probability) C under the curve between $-z^*$ and z^* standard deviations away from the mean.

works 95% of the time isn't good enough. You want to be 99% confident. For that, we need to mark off the central 99% of a Normal distribution. For any probability C between 0 and 1, there is a number z^* such that any Normal distribution has probability C within z^* standard deviations of the mean. Figure 21.5 shows how the probability C and the number z^* are related.

Table 21.1 gives the numbers z^* for various choices of C. For convenience, the table gives C as a confidence level in percent. The numbers z^* are called **critical values** of the Normal distribution. Table 21.1 shows that any Normal distribution has probability 99% within ± 2.58 standard deviations of its mean. The table also shows that any Normal distribution has probability 95% within ± 1.96 standard deviations of its mean. The 68–95–99.7 rule uses 2 in place of the critical value $z^* = 1.96$. That is good enough for practical purposes, but the table gives the more exact value.

From Figure 21.5 we see that, with probability C, the sample proportion $\hat{p}$ takes a value within z^* standard deviations of p. That is just to say

TABLE 21.1 Critical values of the Normal distributions

Confidence level C	Critical value z^*	Confidence level C	Critical value z^*
50%	0.67	90%	1.64
60%	0.84	95%	1.96
70%	1.04	99%	2.58
80%	1.28	99.9%	3.29

that, with probability C, the interval extending z^* standard deviations on either side of the observed $\hat{p}$ captures the unknown p. Using the estimated standard deviation of $\hat{p}$ produces the following recipe.

Confidence interval for a population proportion

Choose an SRS of size n from a population of individuals of which proportion p are successes. The proportion of successes in the sample is $\hat{p}$. When n is large, an **approximate level C confidence interval for p** is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where z^* is the critical value for probability C from Table 21.1.

EXAMPLE 5 A 99% confidence interval

The Gallup random sample of 1009 adult Americans in 2015 found that 616 reported actively trying to avoid drinking regular soda or pop. We want a 99% confidence interval for the proportion p of all adult Americans who actively avoided drinking regular soda or pop. Table 21.1 says that for 99% confidence, we must go out $z^* = 2.58$ standard deviations. Here are our calculations:

$$\hat{p} = \frac{616}{1009} = 0.611$$

$$\begin{aligned}\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.611 \pm 2.58\sqrt{\frac{(0.611)(0.389)}{1009}} \\ &= 0.611 \pm (2.58)(0.0153) \\ &= 0.611 \pm 0.0395 \\ &= 0.5715 \text{ to } 0.6505\end{aligned}$$

We are 99% confident that the true population proportion is between 57.15% and 65.05%. That is, we got this range of percentages by using a method that gives a correct answer 99% of the time.

Compare Example 5 with the calculation of the 95% confidence interval in Example 3. The only difference is the use of the critical value 2.58 for 99% confidence in place of 2 for 95% confidence. That makes the margin of error for 99% confidence larger and the confidence interval wider. Higher confidence isn't free—we pay for it with a wider interval. Figure 21.5 reminds us why this is true. To cover a higher percentage of the area under a Normal curve, we must go farther out from the center. Figure 21.6 compares the lengths of the 90%, 95%, and 99% confidence intervals.

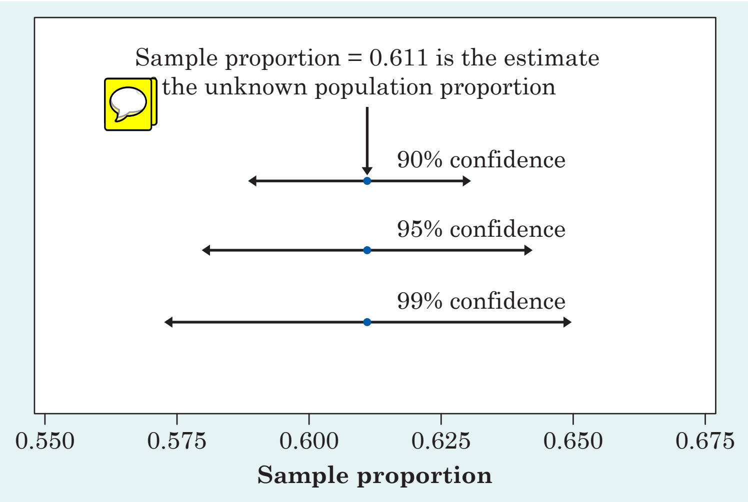

Figure 21.6 The lengths of three confidence intervals for the soda consumption example. All three are centered at the estimate $\hat{p}$ = 0.611. When the data and the sample size remain the same, higher confidence results in a larger margin of error.

NOW IT'S YOUR TURN

21.2 Gambling. A May 2011 Gallup Poll consisting of a random sample of 1018 adult Americans found that 31% believe that gambling is morally wrong. Find a 99% confidence interval for the proportion of all adult Americans who believe that gambling is morally wrong. How would you interpret this interval?

The sampling distribution of a sample mean*

What is the mean number of hours your college's first-year students study each week? What was their mean grade point average in high school? We often want to estimate the mean of a population. To distinguish the population mean (a parameter) from the sample mean $\bar{x}$, we write the population mean as μ, the Greek letter mu. We use the mean $\bar{x}$ of an SRS to estimate the unknown mean μ of the population.

Like the sample proportion $\hat{p}$, the sample mean $\bar{x}$ from a large SRS has a sampling distribution that is close to Normal. Because the sample mean of an SRS is an unbiased estimator of μ, the sampling distribution of $\bar{x}$ has μ as its mean. The standard deviation, or standard error, of $\bar{x}$ depends on the standard deviation of the population, which is usually written as σ, the Greek letter sigma. By mathematics we can discover the following facts.

*This section is optional.

Sampling distribution of a sample mean

Choose an SRS of size n from a population in which individuals have mean μ and standard deviation σ. Let $\bar{x}$ be the mean of the sample. Then:

- The sampling distribution of $\bar{x}$ is **approximately Normal** when the sample size n is large.
- The **mean** of the sampling distribution is equal to μ.
- The **standard deviation** or the **standard error** of the sampling distribution is $\sigma/\sqrt{n}$.

It isn't surprising that the values that $\bar{x}$ takes in many samples are centered at the true mean μ of the population. That's the lack of bias in random sampling once again. The other two facts about the sampling distribution make precise two very important properties of the sample mean $\bar{x}$:

- The mean of a number of observations is less variable than individual observations.
- The distribution of a mean of a number of observations is more Normal than the distribution of individual observations.

Figure 21.7 illustrates the first of these properties. It compares the distribution of a single observation with the distribution of the mean $\bar{x}$ of 10

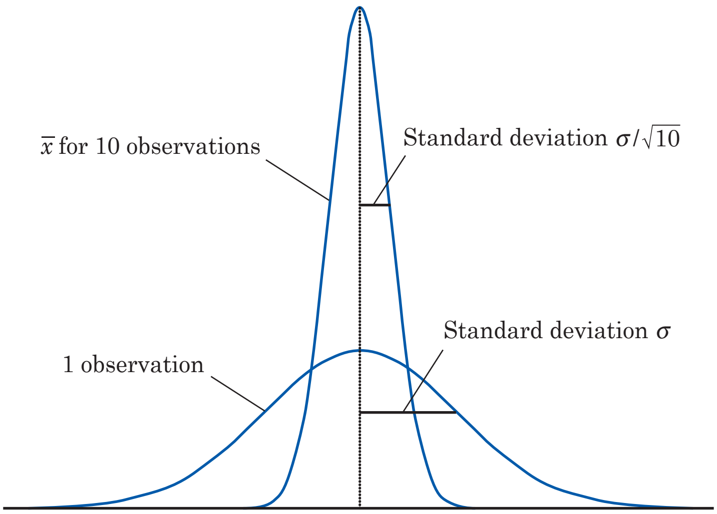

Figure 21.7 The sampling distribution of the sample mean $\bar{x}$ of 10 observations compared with the distribution of individual observations.

observations. Both have the same center, but the distribution of $\bar{x}$ is less spread out. In Figure 21.7, the distribution of individual observations is Normal. If that is true, then the sampling distribution of $\bar{x}$ is exactly Normal for any size sample, not just approximately Normal for large samples. A remarkable statistical fact, called the **central limit theorem**, says that as we take more and more observations at random from *any* population, the distribution of the mean of these observations eventually gets close to a Normal distribution. (There are some technical qualifications to this big fact, but in practice, we can ignore them.) The central limit theorem lies behind the use of Normal sampling distributions for sample means.

EXAMPLE 6 The central limit theorem in action

Figure 21.8 shows the central limit theorem in action. The top-left density curve describes individual observations from a population. It is strongly right-skewed. Distributions like this describe the time it takes to repair a household appliance, for example. Most repairs are quickly done, but some are lengthy.

The other three density curves in Figure 21.8 show the sampling distributions of the sample means of 2, 10, and 25 observations from this population. As the sample size n increases, the shape becomes more Normal The mean remains fixed and the standard error decreases, following the

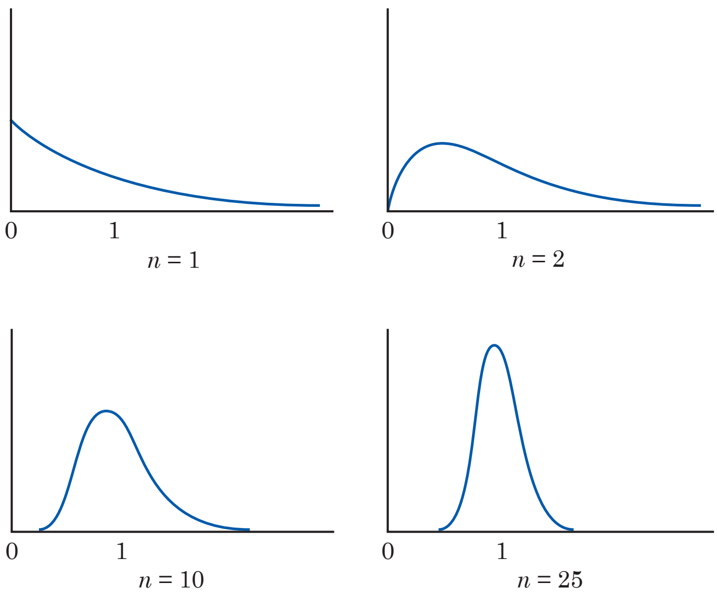

Figure 21.8 The distribution of a sample mean $\bar{x}$ becomes more Normal as the size of the sample increases. The distribution of individual observations ($n = 1$) is far from Normal. The distributions of means of 2, 10, and finally 25 observations move closer to the Normal shape.

pattern $\sigma/\sqrt{n}$. The distribution for 10 observations is still somewhat skewed to the right but already resembles a Normal curve. The density curve for $n = 25$ is yet more Normal. The contrast between the shapes of the population distribution and of the distribution of the mean of 10 or 25 observations is striking.

Confidence intervals for a population mean*

The standard error of $\bar{x}$ depends on both the sample size n and the standard deviation σ of individuals in the population. We know n but not σ. When n is large, the sample standard deviation s is close to σ and can be used to estimate it, just as we use the sample mean $\bar{x}$ to estimate the population mean μ. The estimated standard error of $\bar{x}$ is, therefore, $s/\sqrt{n}$. Now we can find confidence intervals for μ following the same reasoning that led us to confidence intervals for a proportion p. The big idea is that to cover the central area C under a Normal curve, we must go out a distance z^* on either side of the mean. Look again at Figure 21.5 to see how C and z^* are related.

Confidence interval for a population mean

Choose an SRS of size n from a large population of individuals having mean μ. The mean of the sample observations is $\bar{x}$. When n is reasonably large, an **approximate level *C* confidence interval for μ** is

$$\bar{x} \pm z^* \frac{s}{\sqrt{n}}$$

where z^* is the critical value for confidence level C from Table 21.1.

The cautions we noted in estimating p apply here as well. The recipe is valid only when an SRS is drawn and the sample size n is reasonably large. How large is reasonably large? The answer depends upon the true shape of the population distribution. A sample size of $n \geq 15$ is usually adequate unless there are extreme outliers or strong skewness. For clearly skewed distributions, a sample size of $n \geq 40$ often suffices if there are no outliers.

The margin of error again decreases only at a rate proportional to $\sqrt{n}$ as the sample size n increases. And it bears repeating that $\bar{x}$ and s are strongly influenced by outliers. Inference using $\bar{x}$ and s is suspect when outliers are present. Always look at your data.

*This section is optional.

EXAMPLE 7 NAEP quantitative scores


Roger Bamber/Alamy

The National Assessment of Educational Progress (NAEP) includes a mathematics test for high school seniors. Scores on the test range from 0 to 300. Demonstrating the ability to use the Pythagorean theorem to determine the length of a hypotenuse is an example of the skills and knowledge associated with performance at the Basic level. An example of the knowledge and skills associated with the Proficient level is using trigonometric ratios to determine length.

In 2009, 51,000 12th-graders were in the NAEP sample for the mathematics test. The mean mathematics score was $\bar{x} = 153$, and the standard deviation of their scores was $s = 34$. Assume that these 51,000 students were a random sample from the population of all 12th-graders. On the basis of this sample, what can we say about the mean score μ in the population of all 12th-grade students?

The 95% confidence interval for μ uses the critical value $z^* = 1.96$ from Table 21.1. The interval is

$$\bar{x} \pm z^* \frac{s}{\sqrt{n}} = 153 \pm 1.96 \frac{34}{\sqrt{51{,}000}}$$
$$= 153 \pm (1.96)(0.151) = 153 \pm 0.3$$

We are 95% confident that the mean score for all 12th-grade students lies between 152.7 and 153.3.

NOW IT'S YOUR TURN

21.3 Blood pressure of executives. The medical director of a large company looks at the medical records of 72 executives between the ages of 35 and 44 years. He finds that the mean systolic blood pressure in this sample is $\bar{x} = 126.1$ and the standard deviation is $s = 15.2$. Assuming the sample is a random sample of all executives in the company, find a 95% confidence interval for μ, the unknown mean blood pressure of all executives in the company.

STATISTICS IN SUMMARY

Chapter Specifics

- **Statistical inference** draws conclusions about a population on the basis of data from a sample. Because we don't have data for the entire population, our conclusions are uncertain.

- A **confidence interval** estimates an unknown parameter in a way that tells us how uncertain the estimate is. The interval itself says how closely we can pin down the unknown parameter. The **confidence level** is a probability that says how often in many samples the method would produce an interval that does catch the parameter. We find confidence intervals starting from the **sampling distribution** of a statistic, which shows how the statistic varies in repeated sampling.
- The standard deviation of the sampling distribution of the sample statistic is commonly referred to as the **standard error**.
- We estimate a population proportion p using the sample proportion $\hat{p}$ of an SRS from the population. Confidence intervals for p are based on the **sampling distribution** of $\hat{p}$. When the sample size n is large, this distribution is approximately Normal.
- We estimate a population mean μ using the sample mean $\bar{x}$ of an SRS from the population. Confidence intervals for μ are based on the **sampling distribution** of $\bar{x}$. When the sample size n is large, the **central limit theorem** says that this distribution is approximately Normal. Although the details of the methods differ, inference about μ is quite similar to inference about a population proportion p because both are based on Normal sampling distributions.

The reason we collect data is not to learn about the individuals that we observed but to infer from the data to some wider population that the individuals represent. Chapters 1 through 6 tell us that the way we produce the data (sampling, experimental design) affects whether we have a good basis for generalizing to some wider population—in particular, whether a sample statistic provides insight into the value of the corresponding population parameter. Chapters 17 through 20 discuss probability, the formal mathematical tool that determines the nature of the inferences we make. Chapter 18 discusses sampling distributions, which tell us how statistics computed from repeated SRSs behave and hence what a statistic (in particular, a sample proportion) computed from our sample is likely to tell us about the corresponding parameter of the population (in particular, a population proportion) from which the sample was selected.

In this chapter, we discuss the basic reasoning of statistical estimation of a population parameter, with emphasis on estimating a population proportion and population mean. To an estimate of a population parameter, such as a population proportion, we attach a margin of error and a confidence level. The result is a confidence interval. The sampling distribution, first introduced in Chapter 3 and discussed more fully in Chapter 18, provides the mathematical basis for constructing confidence intervals and understanding their properties. We will provide more advice on interpreting confidence intervals in Chapter 23.

CASE STUDY EVALUATED Going back to the survey results presented in the Case Study at the beginning of the chapter, the report presented by the BRFSS gave confidence intervals. With 95% confidence, the true percentage of California residents who meet the daily recommendations for fruit consumption is between 17.3% to 18.1%, and the true percentage of California residents who meet the daily recommendations for vegetable consumption is between 12.6% and 13.4%. Interpret these intervals in plain language that someone who knows no statistics will understand.

LaunchPad macmillan learning **Online Resources**

- The Snapshots video *Inference for One Proportion* discusses confidence intervals for a population proportion in the context of a sample survey conducted during the 2012 presidential elections.
- The StatClips Examples video, *Confidence Intervals: Intervals for Proportions Example C* provides an example of how to calculate a 90% confidence interval for a proportion.
- The Snapshots video *Confidence Intervals* discusses confidence intervals for a population mean in the context of an example involving birds killed by wind turbines.

CHECK THE BASICS

For Exercise 21.1, see page 499; for Exercise 21.2, see page 505; and for Exercise 21.3, see page 509.

21.4 Bike riding. In 2012, the National Highway Traffic Safety Administration (NHTSA) of the U.S. Department of Transportation conducted the National Survey of Bicyclist and Pedestrian Attitudes and Behaviors. A total of 7509 individuals aged 16 years and older participated in this survey. Of these 7509 individuals, 2580 individuals reported that they rode a bicycle within the past year. Of these self-reported bicycle riders, 1187 said they had never worn a helmet when riding a bicycle. Based on this information, we know the sample proportion, $\hat{p}$, of bicycle riders from this survey who have never worn a helmet is

(a) 0.460.
(b) 0.344.
(c) 0.158.
(d) 0.013.

21.5 Saving for retirement. In 2014, a leading financial services provider conducted a survey in order to determine how Americans are planning for retirement. Of the 1017 adults aged 18 years and older who were included in the survey, 214 said they were currently not saving anything for retirement. Based on this information, the 95% confidence interval for the proportion of all adults who are currently not saving for retirement would be

(a) 0.155 to 0.265.

(b) 0.185 to 0.235.

(c) 0.197 to 0.223.

(d) 0.200 to 0.220.

21.6 Computer use. A random sample of 197 12th-grade students from across the United States was surveyed, and it was observed that these students spent an average of 23.5 hours on the computer per week, with a standard deviation of 8.7 hours. If we plan to use these data to construct a 99% confident interval, the margin of error will be approximately

(a) 0.07.

(b) 0.62.

(c) 1.6.

(d) 8.7.

21.7 Anxiety. A particular psychological test is used to measure anxiety. The average test score for all university students nationwide is 85 points. Suppose a random sample of university students is selected and a confidence interval based on their mean anxiety score is constructed. Which of the following statements about the confidence interval is true?

(a) The resulting confidence interval will contain 85.

(b) The 95% confidence interval for a sample of size 100 will generally have a smaller margin of error than the 95% confidence interval for a sample of size 50.

(c) For a sample of size 100, the 95% confidence interval will have a smaller margin of error than the 90% confidence interval.

21.8 Pigs. A 90% confidence interval is calculated for a sample of weights of 135 randomly selected pigs, and the resulting confidence interval is from 75 to 90 pounds. Will the sample mean weight (from this particular sample of size 135) fall within the confidence interval?

(a) No.

(b) Yes.

(c) Maybe.

CHAPTER 21 EXERCISES

21.9 A student survey. Tonya wants to estimate what proportion of the students in her dormitory like the dorm food. She interviews an SRS of 50 of the 175 students living in the dormitory. She finds that 14 think the dorm food is good.

(a) What population does Tonya want to draw conclusions about?

(b) In your own words, what is the population proportion p in this setting?

(c) What is the numerical value of the sample proportion $\hat{p}$ from Tonya's sample?

21.10 Fire the coach? A college president says, "99% of the alumni support my firing of Coach Boggs." You contact an SRS of 200 of the college's 15,000 living alumni and find that 66 of them support firing the coach.

(a) What population does the inference concern here?

(b) Explain clearly what the population proportion p is in this setting.

(c) What is the numerical value of the sample proportion $\hat{p}$?

21.11 Are teachers engaged in their work? Results from a Gallup survey conducted in 2013 and 2014 reveal that 30% of Kindergarten through Grade 12 school teachers report feeling engaged in their work. The report from this random sample of 6711 teachers stated that, with 95% confidence, the margin of sampling error was ±1.0%. Explain to someone who knows no statistics what the phrase "95% confidence" means in this report.

21.12 Gun control. A December 2014 Pew Research Center poll asked a sample of 1507 adults whether they thought it was important to control gun ownership. A total of 693 of the poll respondents were in support of controls on gun ownership. Although the samples in national polls are not SRSs, they are similar enough that our method gives approximately correct confidence intervals.

(a) Say in words what the population proportion p is for this poll.

(b) Find a 95% confidence interval for p.

(c) The Pew Research Center announced a margin of error of plus or minus 2.9 percentage points for this poll result. How well does your work in part (b) agree with this margin of error?

21.13 Using public libraries. The Pew Research Center Libraries 2015 Survey examined the attitudes and behaviors of a nationally representative sample of 2004 people ages 16 and older who were living in the United States. One survey question asked whether the survey respondent had visited, in person, a library or a bookmobile in the last year. A total of 922 of those surveyed answered Yes to this question. Although the samples in national polls are not SRSs, they are similar enough that our method gives approximately correct confidence intervals.

(a) Explain in words what the parameter p is in this setting.

(b) Use the poll results to give a 95% confidence interval for p.

(c) Write a short explanation of your findings in part (b) for someone who knows no statistics.

21.14 Computer crime. Adults are spending more and more time on the Internet, and the number experiencing computer- or Internet-based crime is rising. A 2010 Gallup Poll of 1025 adults, aged 18 and over, found that 113 of those in the sample said that they or a member of their household were victims of computer or Internet crime on their home computer in the past year. Although the samples in national polls are not SRSs, they are similar enough that our method gives approximately correct confidence intervals.

(a) Explain in words what the parameter p is in this setting.

(b) Use the poll results to give a 95% confidence interval for p.

21.15 Gun control. In Exercise 21.12, you constructed a 95% confidence interval based on a random sample of $n = 1507$ adults. How large a sample would be needed to get a margin of error half as large as the one in Exercise 21.12? You may find it helpful to refer to the discussion surrounding Example 5 in Chapter 3 (page 47).

21.16 The effect of sample size. A December 2014 CBS News/*New York Times* poll found that 52% of its sample thought that basic medical care in the United States was affordable. Give a 95% confidence interval for the proportion of all adults who feel this way, assuming that the result $\hat{p} = 0.52$ comes from a sample of size

(a) $n = 750$.

(b) $n = 1500$.

(c) $n = 3000$.

(d) Explain briefly what your results show about the effect of increasing the size of a sample on the width of the confidence interval.

21.17 Random digits. We know that the proportion of 0s among a large set of random digits is $p = 0.1$ because all 10 possible digits are equally probable. The entries in a table of random digits are a random sample from the population of all random digits. To get an SRS of 200 random digits, look at the first digit in each of the 200 five-digit groups in lines 101 to 125 of Table A in the back of the book. How many of these 200 digits are 0s? Give a 95% confidence interval for the proportion of 0s in the population from which these digits are a random sample. Does your interval cover the true parameter value, $p = 0.1$?

21.18 Tossing a thumbtack. If you toss a thumbtack on a hard surface, what is the probability that it will land point up? Estimate this probability p by tossing a thumbtack 100 times. The 100 tosses are an SRS of size 100 from the population of all tosses. The proportion of these 100 tosses that land point up is the sample proportion $\hat{p}$. Use the result of your tosses to give a 95% confidence interval for p. Write a brief explanation of your findings for someone who knows no statistics but wonders how often a thumbtack will land point up.

21.19 Don't forget the basics. The Behavioral Risk Factor Surveillance System survey found that 792 individuals in its 2010 random sample of 6911 college graduates in California said that they had engaged in binge drinking in the past year. We can ~~used~~ this finding to calculate confidence intervals for the proportion of all college graduates in California who engaged in binge drinking in the past year. This sample survey may have bias that our confidence intervals do not take into account. Why is some bias likely to be present? Does the sample proportion 11.5% probably overestimate or underestimate the true population proportion?

21.20 Count Buffon's coin. The eighteenth-century French naturalist Count Buffon tossed a coin 4040 times. He got 2048 heads. Give a 95% confidence interval for the probability that Buffon's coin lands heads up. Are you confident that this probability is not 1/2? Why?

21.21 Share the wealth. The *New York Times* conducted a nationwide poll of 1650 randomly selected American adults. Of these, 1089 felt that ~~distributed~~ money and wealth in this country should be more evenly distributed among more people. We can consider the sample to be an SRS.

(a) Give a 95% confidence interval for the proportion of all American adults who, at the time of the poll, felt that ~~distributed~~ money and wealth in this country should be more evenly distributed among more people.

(b) The news article says, "In theory, in 19 cases out of 20, the poll results will differ by no more than 3 percentage points in either direction from what would have been obtained by seeking out all American adults." Explain how your results agree with this statement.

21.22 Harley motorcycles. In 2013, it was reported that 55% of the new motorcycles that were registered in the United States were Harley-Davidson motorcycles. You plan to interview an SRS of 600 new motorcycle owners.

(a) What is the sampling distribution of the proportion of your sample who own Harleys?

(b) How likely is your sample to contain 57% or more who own Harleys? How likely is it to contain at least 51% Harley owners? Use the 68–95–99.7 rule and your answer to part (a).

21.23 Do you jog? Suppose that 10% of all adults jog. An opinion poll asks an SRS of 400 adults if they jog.

(a) What is the sampling distribution of the proportion $\hat{p}$ in the sample who jog?

(b) According to the 68–95–99.7 rule, what is the probability that the sample proportion who jog will be 7.3% or greater?

21.24 The quick method. The quick method of Chapter 3 (page 46) uses $\hat{p} \pm 1/\sqrt{n}$ as a rough recipe for a 95% confidence interval for a population proportion. The margin of error from the quick method is a bit larger than needed. It differs most from the more accurate method of this chapter when $\hat{p}$ is close to 0 or 1. An SRS of 500 motorcycle registrations finds that 68 of the motorcycles are Harley-Davidsons. Give a 95% confidence interval for the proportion of all motorcycles that are Harleys by the quick method and then by the method of this chapter. How much larger is the quick-method margin of error?

21.25 68% confidence. We used the 95 part of the 68–95–99.7 rule to give a recipe for a 95% confidence interval for a population proportion p.

(a) Use the 68 part of the rule to give a recipe for a 68% confidence interval.

(b) Explain in simple language what "68% confidence" means.

(c) Use the result of the Gallup Poll (Example 3, page 499) to give a 68% confidence interval for the proportion of adult Americans who reported actively trying to avoid drinking regular soda or pop in 2015. How does your interval compare with the 95% interval in Example 3?

21.26 Simulating confidence intervals. In Exercise 21.25, you found the recipe for a 68% confidence interval for a population proportion p. Suppose that (unknown to anyone) 60% of Americans actively tried to avoid drinking regular soda or pop in 2015.

(a) How would you simulate the proportion of an SRS of 25 adult Americans?

(b) Simulate choosing 10 SRSs, using a different row in Table A for each sample. What are the 10 values of the sample proportion $\hat{p}$ who actively tried to avoid drinking regular soda or pop in 2015?

(c) Find the 68% confidence interval for p from each of your 10 samples. How many of the intervals capture the true parameter value $p = 0.6$? (Samples of size 25 are not large enough for our recipe to be very accurate, but even a small simulation illustrates how confidence intervals behave in repeated samples.)

The following exercises concern the optional sections of this chapter.

21.27 Gun control. Exercise 21.12 reports a Pew Research Center poll in which 693 of a random sample of 1507 adults were in support of controls on gun ownership. Use Table 21.1 to give a 90% confidence interval for the proportion of all adults who feel this way. How does your interval compare with the 95% confidence interval from Exercise 21.12?

21.28 Using public libraries. Exercise 21.13 reports a Pew Research Center Libraries 2015 Survey that found that 922 in a random sample of 2004 American adults said that they had visited, in person, a library or bookmobile in the last year. Use Table 21.1 to give a 99% confidence interval for the proportion of all American adults who have done this. How does your interval compare with the 95% confidence interval of Exercise 21.13?

21.29 Organic Food. A 2014 Consumer Reports National Research Center survey on food labeling found that 49% of a random sample of 1004 American adults report looking for information on food labels about whether the food they are purchasing is organic. Use this survey result and Table 21.1 to give 70%, 80%, 90%, and 99% confidence intervals for the proportion of all adults who feel this way. What do your results show about the effect of changing the confidence level?

21.30 Unhappy HMO patients. How likely are patients who file complaints with a health maintenance organization (HMO) to leave the HMO? In one year, 639 of the more than 400,000 members of a large New England HMO filed complaints. Fifty-four of the complainers left the HMO voluntarily. (That is, they were not forced to leave by a move or a job change.) Consider this year's complainers as an SRS of all patients who will complain in the future. Give a 90% confidence interval for the proportion of complainers who voluntarily leave the HMO.

21.31 Estimating unemployment. The Bureau of Labor Statistics (BLS) uses 90% confidence in presenting unemployment results from the monthly Current Population Survey (CPS). The September 2015 survey reported that of the 156,715 individuals surveyed in the civilian labor force, 148,800 were employed and 7915 were unemployed. The CPS is not an SRS, but for the purposes of this exercise, we will act as though the BLS took an SRS of 156,715 people. Give a 90% confidence interval for the proportion of those surveyed who were unemployed. (*Note:* Example 3 in Chapter 8 on page 166 explains how unemployment is measured.)

21.32 Safe margin of error. The margin of error $z^*\sqrt{\hat{p}/(1-\hat{p})n}$ is 0 when $\hat{p}$ is 0 or 1 and is largest when $\hat{p}$ is 1/2. To see this, calculate $\hat{p}(1-\hat{p})$ for $\hat{p} = 0, 0.1, 0.2, \ldots, 0.9$, and 1. Plot your results vertically against the values of $\hat{p}$ horizontally. Draw a curve through the points. You have made a graph of $\hat{p}(1-\hat{p})$. Does the graph reach its highest point when $\hat{p} = 1/2$? You see that taking $\hat{p} = 1/2$ gives a margin of error that is always at least as large as needed.

21.33 The idea of a sampling distribution. Figure 21.1 (page 496) shows the idea of the sampling distribution of a sample proportion $\hat{p}$ in picture form. Draw a similar picture that shows the idea of the sampling distribution of a sample mean $\bar{x}$.

21.34 IQ test scores. Here are the IQ test scores of 31 seventh-grade girls in a Midwest school district:

114	100	104	89	102	91	114	114	103	105	
[illegible]	130	120	132	111	128	118	119	86	72	
111	103	74	112	107	103	98	96	112	112	93

(a) We expect the distribution of IQ scores to be close to Normal. Make a histogram of the distribution of these 31 scores. Does your plot show outliers, clear skewness, or other non-Normal features? Using a calculator, find the mean and standard deviation of these scores.

(b) Treat the 31 girls as an SRS of all middle-school girls in the school district. Give a 95% confidence interval for the mean score in the population.

(c) In fact, the scores are those of all seventh-grade girls in one of the several schools in the district. Explain carefully why we cannot trust the confidence interval from (b).

21.35 Averages versus individuals. Scores on the ACT college entrance examination vary Normally with mean $\mu = 18$ and standard deviation $\sigma = 6$. The range of reported scores is 1 to 36.

(a) What range contains the middle 95% of all individual scores?

(b) If the ACT scores of 25 randomly selected students are averaged, what range contains the middle 95% of the averages $\bar{x}$?

21.36 Blood pressure. A randomized comparative experiment studied the effect of diet on blood pressure. Researchers divided 54 healthy white males at random into two groups. One group received a calcium supplement, and the other group received a placebo. At the beginning of the study, the researchers measured many variables on the subjects. The average seated systolic blood pressure of the 27 members of the placebo group was reported to be $\bar{x} = 114.9$ with a standard deviation of $s = 9.3$.

(a) Give a 95% confidence interval for the mean blood pressure of the population from which the subjects were recruited.

(b) The recipe you used in part (a) requires an important assumption about the 27 men who provided the data. What is this assumption?

21.37 Testing a random number generator. Our statistical software has a "random number generator" that is supposed to produce numbers scattered at random between 0 and 1. If this is true, the numbers generated come from a population with $\mu = 0.5$. A command to generate 100 random numbers gives outcomes with mean $\bar{x} = 0.536$ and $s = 0.312$. Give a 90% confidence interval for the mean of all numbers produced by the software.

21.38 Will they charge more? A bank wonders whether omitting the annual credit card fee for customers who charge at least \$2500 in a year will increase the amount charged on its credit cards. The bank makes this offer to an SRS of 200 of its credit card customers. It then compares how much these customers charge this year with the amount that they charged last year. The mean increase in the sample is \$346, and the standard deviation is \$112. Give a 99% confidence interval for the mean amount charges would have increased if this benefit had been extended to all such customers.

21.39 A sampling distribution. Exercise 21.37 concerns the mean of the random numbers generated by a computer program. The mean is supposed to be 0.5 because the numbers are supposed to be spread at random between 0 and 1. We asked the software to generate samples of 100 random numbers repeatedly. Here are the sample means $\bar{x}$ for 50 samples of size 100:

0.532	0.450	0.481	0.508	0.510	0.530	0.499	0.461	0.543	0.490
0.497	0.552	0.473	0.425	0.449	0.507	0.472	0.438	0.527	0.536
0.492	0.484	0.498	0.536	0.492	0.483	0.529	0.490	0.548	0.439
0.473	0.516	0.534	0.540	0.525	0.540	0.464	0.507	0.483	0.436
0.497	0.493	0.458	0.527	0.458	0.510	0.498	0.480	0.479	0.499

The sampling distribution of $\bar{x}$ is the distribution of the means from all possible samples. We actually have the means from 50 samples. Make a histogram of these 50 observations. Does the distribution appear to be roughly Normal, as the central limit theorem says will happen for large enough samples?

21.40 Will they charge more? In Exercise 21.38, you carried out the calculations for a confidence interval based on a bank's experiment in changing the rules for its credit cards. You ought to ask some questions about this study.

(a) The distribution of the amount charged is skewed to the right, but outliers are prevented by the credit limit that the bank enforces on each card. Why can we use a confidence interval based on a Normal sampling distribution for the sample mean $\bar{x}$?

(b) The bank's experiment was not comparative. The increase in amount charged over last year may be explained by lurking variables rather than by the rule change. What are some plausible reasons charges might go up? Outline the design of a comparative randomized experiment to answer the bank's question.

21.41 A sampling distribution, continued. Exercise 21.39 presents 50 sample means $\bar{x}$ from 50 random samples of size 100. Using a calculator, find the mean and standard error of these 50 values. Then answer these questions.

(a) The mean of the population from which the 50 samples were drawn is $\mu = 0.5$ if the random number generator is accurate. What do you expect the mean of the distribution of $\bar{x}$'s from all possible samples to be? Is the mean of these 50 samples close to this value?

(b) The standard error of the distribution of $\bar{x}$ from samples of size $n = 100$ is supposed to be $\sigma/10$, where σ is the standard deviation of individuals in the population. Use this fact with the standard deviation you calculated for the 50 $\bar{x}$'s to estimate σ.

21.42 Plus four confidence intervals for a proportion. The large-sample confidence interval $\hat{p} \pm z^* \sqrt{\hat{p}(1 - \hat{p})/n}$ for a sample proportion p is easy to calculate. It is also easy to understand because it rests directly on the approximately Normal distribution of $\hat{p}$. Unfortunately, confidence levels from this interval can be inaccurate, particularly with smaller sample sizes where there are only a few successes or a few failures. The actual confidence level is usually *less* than the confidence level you asked for in choosing the critical value z^*. That's bad. What is worse, accuracy does not consistently get better as the sample size n increases. There are "lucky" and "unlucky" combinations of the sample size n and the true population proportion p.

Fortunately, there is a simple modification that is almost magically effective in improving the accuracy of the confidence interval. We call it the "plus four" method because all you need to do is *add four imaginary observations, two successes and two failures*. With the added observations, the **plus four estimate** of p is

$$\tilde{p} = \frac{\text{number of successes in the sample} + 2}{n + 4}$$

The formula for the confidence interval is exactly as before, with the new sample size and number of successes. To practice using the plus four confidence interval, consider the following problem. Cocaine users commonly snort the powder up the nose through a rolled-up paper currency bill. Spain has a high rate of cocaine use, so it's not surprising that euro paper currency in Spain often shows traces of cocaine. Researchers collected 20 euro bills in each of several Spanish cities. In Madrid, 17 out of 20 bore traces of cocaine. The researchers note that we can't tell whether the bills had been used to snort cocaine or had been contaminated in currency-sorting machines. Use the plus four confidence interval method to estimate the proportion of all euro bills in Madrid that have traces of cocaine.

EXPLORING THE WEB

Follow the QR code to access exercises.