What Do Samples Tell Us?

3

Blend Images/REB Images/Getty Images

CASE STUDY According to the Centers for Disease Control and Prevention (CDC), there were 173 cases of measles reported between June 1 and May 29, 2015. About 87% of the cases were related to five outbreaks during the same time period. The CDC also reported that the "United States experienced a record number of measles cases during 2014, with 668 cases from 27 states reported to CDC's National Center for Immunization and Respiratory Diseases (NCIRD). This is the greatest number of cases since measles elimination was documented in the U.S. in 2000." According to the same report by the CDC, "the majority of people who got measles were unvaccinated." Vaccinating children against diseases like measles is controversial.

The debate about childhood vaccinations became a major news issue in late 2014 and early 2015. As of June 2015, Mississippi and West Virginia are the only two states that require vaccines for children with only medical exemptions. California may become the third state. Other states allow exemptions for personal and religious beliefs. A Gallup Poll conducted from February 28–March 1, 2015, asked the following question: "How important is it that parents get their children vaccinated—extremely important, very important, somewhat important, or not at all important?" The Gallup Poll found that 54% of respondents said "extremely important" (down from 64% who responded to a similar 2001 Gallup Poll). Can we trust this conclusion?

Reading further, we find that Gallup talked with 1015 randomly selected adults to reach these conclusions. We're happy Gallup chooses at random—we wouldn't get unbiased information about the importance of childhood vaccinations by asking people attending a conference of the American Medical Association. However, the U.S. Census Bureau said that there were about 258 million adults in the United States in 2013. How can 1015 people, even a random sample of 1015 people, tell us about the opinions of 258 million people? Is 54% who feel that it is extremely important for parents to get their children vaccinated evidence that, in fact, the majority of Americans feel this way? By the end of this chapter, you will learn the answers to these questions.

From sample to population

Gallup's 2015 finding that "a slight majority of Americans, 54%, say it is extremely important that parents get their children vaccinated" makes a claim about the population of 258 million adults. But Gallup doesn't know the truth about the entire population. The poll contacted 1015 people and found that 54% of them said it is extremely important for parents to vaccinate their children. Because the sample of 1015 people was chosen at random, it's reasonable to think that they represent the entire population pretty well. So Gallup turns the *fact* that 54% of the *sample* feel childhood vaccinations are extremely important into an *estimate* that about 54% of *all American adults* feel childhood vaccinations are extremely important. That's a basic move in statistics: use a fact about a sample to estimate the truth about the whole population. To think about such moves, we must be clear whether a number describes a sample or a population. Here is the vocabulary we use.

Parameters and statistics

A **parameter** is a number that describes the **population**. A parameter is a fixed number, but in practice we don't know the actual value of this number.

A **statistic** is a number that describes a **sample**. The value of a statistic is known when we have taken a sample, but it can change from sample to sample. We often use a statistic to estimate an unknown parameter.

So parameter is to population as statistic is to sample. Want to estimate an unknown population parameter? Choose a sample from the population and use a sample statistic as your estimate. That's what Gallup did.

EXAMPLE 1 Should children be vaccinated?

The proportion of all American adults who feel childhood vaccinations are extremely important is a *parameter* describing the population of 258 million adults. Call it p, for "proportion." Alas, we do not know the numerical value of p. To estimate p, Gallup took a sample of 1015 adults. The proportion of the sample who favor such an amendment is a *statistic*. Call it $\hat{p}$, read as "p-hat." It happens that 548 of this sample of size 1015 said that they feel childhood vaccines are extremely important, so for this sample,

$$\hat{p} = \frac{548}{1015} = 0.5399 \text{ (This decimal rounds to 0.54 and can be expressed as 54\%.)}$$

Because all adults had the same chance to be among the chosen 1015, it seems reasonable to use the statistic $\hat{p} = 0.54$ as an estimate of the unknown parameter p. It's a *fact* that 54% of the sample feel childhood vaccines are extremely important—we know because we asked them. We don't know what percentage of all American adults feel this way, but we *estimate* that about 54% do.

Sampling variability

If Gallup took a second random sample of 1015 adults, the new sample would have different people in it. It is almost certain that there would not be exactly 548 respondents who feel that childhood vaccines are extremely important. That is, the value of the statistic $\hat{p}$ will *vary* from sample to sample. Could it happen that one random sample finds that 54% of American adults feel childhood vaccines are extremely important and a second random sample finds that only 41% feel the same? Random samples eliminate *bias* from the act of choosing a sample, but they can still be wrong because of the variability that results when we choose at random. If the variation when we take repeated samples from the same population is too great, we can't trust the results of any one sample.

We are saved by the second great advantage of random samples. The first advantage is that choosing at random eliminates favoritism. That is, random sampling attacks bias. The second advantage is that if we took lots of random samples of the same size from the same population, the variation from sample to sample would follow a predictable pattern. This predictable pattern shows that results of bigger samples are less variable than results of smaller samples.

EXAMPLE 2 Lots and lots of samples

Here's another big idea of statistics: to see how trustworthy one sample is likely to be, ask what would happen if we took many samples from the same population. Let's try it and see. Suppose that, in fact (unknown to Gallup), exactly 50% of all American adults feel childhood vaccines are extremely important. That is, the truth about the population is that $p = 0.5$. What if Gallup used the sample proportion $\hat{p}$ from an SRS of size 100 to estimate the unknown value of the population proportion p?

Figure 3.1 illustrates the process of choosing many samples and finding $\hat{p}$ for each one. In the first sample, 56 of the 100 people felt childhood vaccines are extremely important, so $\hat{p} = 56/100 = 0.56$. Only 36 in the next sample felt childhood vaccines are extremely important, so for that sample $\hat{p} = 0.36$. Choose 1000 samples and make a plot of the 1000 values of $\hat{p}$ like the graph (called a histogram) at the right of Figure 3.1. The different values of the sample proportion $\hat{p}$ run along the horizontal axis. The height of each bar shows how many of our 1000 samples gave the group of values on the horizontal axis covered by the bar. For example, in Figure 3.1 the bar covering the values between 0.40 and 0.42 has a height of slightly over 50. Thus, more than 50 of our 1000 samples had values between 0.40 and 0.42.

Of course, Gallup interviewed 1015 people, not just 100. Figure 3.2 shows the results of 1000 SRSs, each of size 1015, drawn from a population in which the true sample proportion is $p = 0.5$. Figures 3.1 and 3.2 are drawn on the same scale. Comparing them shows what happens when we increase the size of our samples from 100 to 1015.

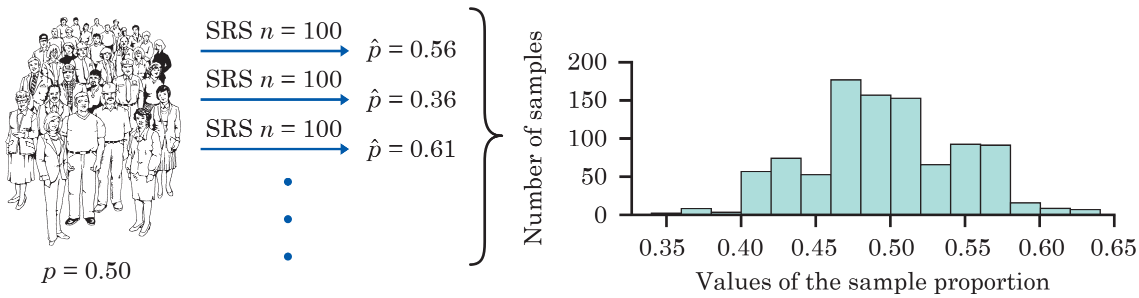


Figure 3.1 The results of many SRSs have a regular pattern. Here, we draw 1000 SRSs of size 100 from the same population. The population proportion is $p = 0.5$. The sample proportions vary from sample to sample, but their values center at the truth about the population.

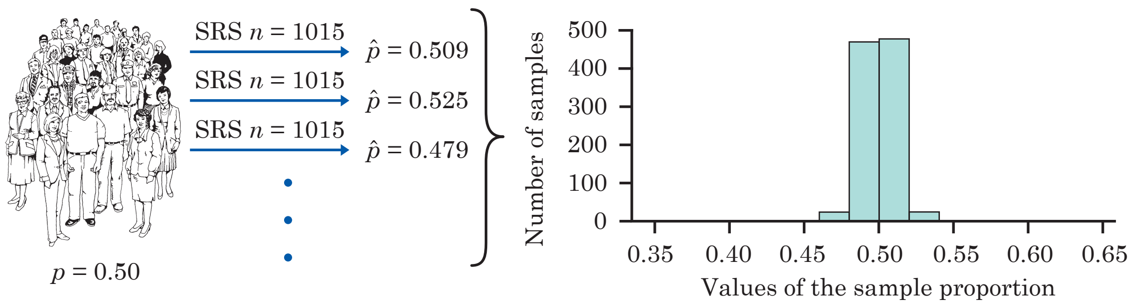


Figure 3.2 Draw 1000 SRSs of size 1015 from the same population as in Figure 3.1. The 1000 values of the sample proportion are much less spread out than was the case for smaller samples.

Look carefully at Figures 3.1 and 3.2. We flow from the population, to many samples from the population, to the many values of $\hat{p}$ from these many samples. Gather these values together and study the histograms that display them.

- In both cases, the values of the sample proportion $\hat{p}$ vary from sample to sample, but the values are centered at 0.5. Recall that $p = 0.5$ is the true population parameter. Some samples have a $\hat{p}$ less than 0.5 and some greater, but there is no tendency to be always low or always high. That is, $\hat{p}$ has no **bias** as an estimator of p. This is true for both large and small samples.
- The values of $\hat{p}$ from samples of size 100 are much more variable than the values from samples of size 1015. In fact, 95% of our 1000 samples of size 1015 have a $\hat{p}$ lying between 0.4692 and 0.5308. That's within 0.0308 on either side of the population truth 0.5. Our samples of size 100, on the other hand, spread the middle 95% of their values between 0.40 and 0.60. That goes out 0.10 from the truth, about three times as far as the larger samples. So larger random samples have less **variability** than smaller samples.

The result is that we can rely on a sample of size 1015 to almost always give an estimate $\hat{p}$ that is close to the truth about the population. Figure 3.2 illustrates this fact for just one value of the population proportion, but it is true for any population proportion. Samples of size 100, on the other hand, might give an estimate of 40% or 60% when the truth is 50%.

Thinking about Figures 3.1 and 3.2 helps us restate the idea of bias when we use a statistic like $\hat{p}$ to estimate a parameter like p. It also reminds us that variability matters as much as bias.

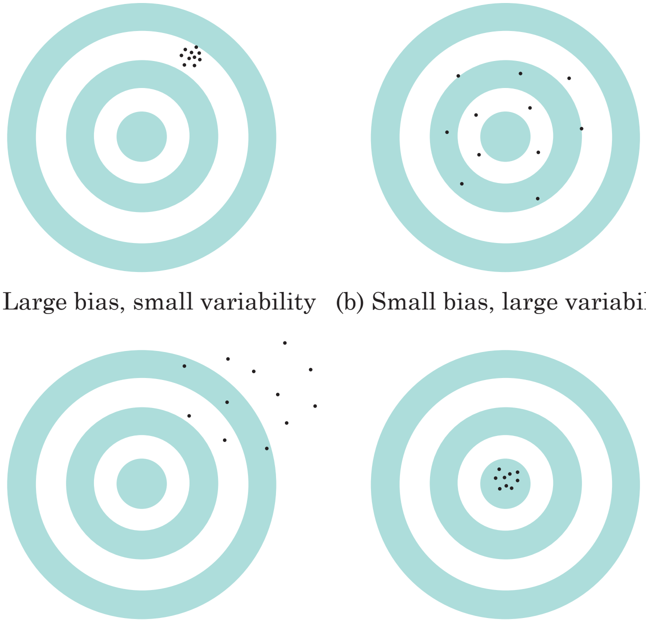
Two types of error in estimation

Bias is consistent, repeated deviation of the sample statistic from the population parameter in the same direction when we take many samples. In other words, bias is a systematic overestimate or underestimate of the population parameter.

Variability describes how spread out the values of the sample statistic are when we take many samples. Large variability means that the result of sampling is not repeatable.

A good sampling method has both small bias and small variability.

We can think of the true value of the population parameter as the bull's-eye on a target and of the sample statistic as an arrow fired at the bull's-eye. Bias and variability describe what happens when an archer fires many arrows at the target. *Bias* means that the aim is off, and the arrows



(a) Large bias, small variability (b) Small bias, large variability

(c) Large bias, large variability (d) Small bias, small variability

Figure 3.3 Bias and variability in shooting arrows at a target. Bias means the archer systematically misses in the same direction. Variability means that the arrows are scattered.

land consistently off the bull's-eye in the same direction. The sample values do not center about the population value. Large *variability* means that repeated shots are widely scattered on the target. Repeated samples do not give similar results but differ widely among themselves. Figure 3.3 shows this target illustration of the two types of error.

Notice that small variability (repeated shots are close together) can accompany large bias (the arrows are consistently away from the bull's-eye in one direction). And small bias (the arrows center on the bull's-eye) can accompany large variability (repeated shots are widely scattered). A good sampling scheme, like a good archer, must have both small bias and small variability. Here's how we do this:

Managing bias and variability

To reduce bias, use random sampling. When we start with a list of the entire population, simple random sampling produces *unbiased* estimates: the values of a statistic computed from an SRS neither consistently overestimate nor consistently underestimate the value of the population parameter.

To reduce the variability of an SRS, use a larger sample. You can make the variability as small as you want by taking a large enough sample.

In practice, Gallup takes only one sample. We don't know how close to the truth an estimate from this one sample is because we don't know what the truth about the population is. But *large random samples almost always give an estimate that is close to the truth*. Looking at the pattern of many samples shows how much we can trust the result of one sample.

Margin of error and all that

The "margin of error" that sample surveys announce translates sampling variability of the kind pictured in Figures 3.1 and 3.2 into a statement of how much confidence we can have in the results of a survey. Let's start with the kind of language we hear so often in the news.

What margin of error means

"Margin of error plus or minus 4 percentage points" is shorthand for this statement:

If we took many samples using the same method we used to get this one sample, 95% of the samples would give a result within plus or minus 4 percentage points of the truth about the population.

Take this step-by-step. A sample chosen at random will usually not estimate the truth about the population exactly. We need a margin of error to tell us how close our estimate comes to the truth. But we can't be *certain* that the truth differs from the estimate by no more than the margin of error. Although 95% of all samples come this close to the truth, 5% miss by more than the margin of error. We don't know the truth about the population, so we don't know if our sample is one of the 95% that hit or one of the 5% that miss. We say we are **95% confident** that the truth lies within the margin of error.

EXAMPLE 3 Understanding the news

Here's what the TV news announcer says: "A new Gallup Poll finds that a slight majority of 54% of American adults feel it is extremely important that parents vaccinate their children. The margin of error for the poll was 4 percentage points." Plus or minus 4% starting at 54% is 50% to 58%. Most people think Gallup claims that the truth about the entire population lies in that range.

This is what the results from Gallup actually mean: "For results based on a sample of this size, one can say with 95% confidence that the error attributable to sampling and other random effects could be plus or minus 4 percentage points for American adults." That is, Gallup tells us that the margin of error includes the truth about the entire population for only 95% of all its samples—"95% confidence" is shorthand for that. The news report left out the "95% confidence."

Finding the margin of error exactly is a job for statisticians. You can, however, use a simple formula to get a rough idea of the size of a sample survey's margin of error. The reasoning behind this formula and an exact calculation of the margin of error are discussed in Chapter 21. For now, we introduce an approximate calculation for the margin of error.

A quick and approximate method for the margin of error

Use the sample proportion $\hat{p}$ from a simple random sample of size n to estimate an unknown population proportion p. The margin of error for 95% confidence is approximately equal to $1/\sqrt{n}$.

EXAMPLE 4 What is the margin of error?

The Gallup Poll in Example 1 interviewed 1015 people. The margin of error for 95% confidence will be about

$$\frac{1}{\sqrt{1015}} = \frac{1}{31.8591} = 0.031 \text{ (that is, 3.1\%)}$$

Gallup announced a margin of error of 4%, and our quick method gave us 3.1%. Our quick and approximate method can disagree a bit with Gallup's for two reasons. First, polls usually round their announced margin of error to the nearest whole percent to keep their press releases simple. Second, our rough formula works for an SRS. We will see in the next chapter that most national samples are more complicated than an SRS in ways that tend to slightly increase the margin of error. In fact, Gallup's survey methods section for this particular poll included the statement, "Each sample of national adults includes a minimum quota of 50% cellphone respondents and 50% landline respondents, with additional minimum quotas by time zone within region. Landline and cellular telephone numbers are selected using random-digit-dial methods." While these methods go beyond what we will study in the next chapter, the complexity of collecting a national sample increases the value of the margin of error.

Our quick and approximate method also reveals an important fact about how margins of error behave. Because the sample size n appears in the denominator of the fraction, larger samples have smaller margins of error. We knew that. Because the formula uses the square root of the sample size, however, *to cut the margin of error in half, we must use a sample four times as large.*

EXAMPLE 5 Margin of error and sample size

In Example 2, we compared the results of taking many SRSs of size $n = 100$ and many SRSs of size $n = 1015$ from the same population. We found that the variability of the middle 95% of the sample results was about three times larger for the smaller samples.

Our quick formula estimates the margin of error for SRSs of size 1015 to be about 3.1%. The margin of error for SRSs of size 100 is about

$$\frac{1}{\sqrt{100}} = \frac{1}{10} = 0.10 \text{ (that is, 10\%)}$$

Because 1015 is roughly 10 times 100 and the square root of 10 is 3.1, the margin of error is about three times larger for samples of 100 people than for samples of 1015 people.

NOW IT'S YOUR TURN

3.1 Voting and personal views. In May 2015, the Gallup Poll asked a random sample of 1024 American adults, "Thinking about how the abortion issue might affect your vote for major offices, would you only vote for a candidate who shares your views on abortion or consider a candidate's position on abortion as just one of many important factors or not see abortion as a major issue?" It found that 21% of respondents said they will only vote for a candidate with the same views on abortion that they have. What is the approximate margin of error for 95% confidence?

Confidence statements

Here is Gallup's conclusion about the views of American adults about vaccinating children in short form: "The poll found that a slight majority of Americans, 54%, feel it is extremely important to vaccinate children. We are 95% confident that the truth about all American adults is within plus or minus 4 percentage points of this sample result." Here is an even shorter form: "We are 95% confident that between 50% and 58% of all American adults feel it is extremely important to vaccinate children." These are *confidence statements.*

Confidence statements

A **confidence statement** has two parts: a **margin of error** and a **level of confidence**. The margin of error says how close the sample statistic lies to the population parameter. The level of confidence says what percentage of all possible samples satisfy the margin of error.

A confidence statement is a fact about what happens in all possible samples and is used to say how much we can trust the result of one sample. The phrase "95% confidence" means "We used a sampling method that gives a result this close to the truth 95% of the time." Here are some hints for interpreting confidence statements:

- *The conclusion of a confidence statement always applies to the population, not to the sample*. We know exactly the opinions of the 1015 people in the sample because Gallup interviewed them. The confidence statement uses the sample result to say something about the population of all American adults.
- *Our conclusion about the population is never completely certain*. Gallup's sample *might* be one of the 5% that miss by more than 4 percentage points.
- *A sample survey can choose to use a confidence level other than 95%*. The cost of higher confidence is a larger margin of error. For the same sample, a 99% confidence statement requires a larger margin of error than 95% confidence. If you are content with 90% confidence, you get in return a smaller margin of error. Remember that our quick and approximate method gives the margin of error only for 95% confidence.
- *It is usual to report the margin of error for 95% confidence*. If a news report gives a margin of error but leaves out the confidence level, it's pretty safe to assume 95% confidence.
- *Want a smaller margin of error with the same confidence? Take a larger sample*. Remember that larger samples have less variability. You can get as small a margin of error as you want and still have high confidence by paying for a large enough sample.

Statistics in Your World

The telemarketer's pause People who do sample surveys hate telemarketing. We all get so many unwanted sales pitches by phone that many people hang up before learning that the caller is conducting a survey rather than selling vinyl siding. Here's a tip. Both sample surveys and telemarketers dial telephone numbers at random. Telemarketers automatically dial many numbers, and their sellers come on the line only after you pick up the phone. Once you know this, the telltale "telemarketer's pause" gives you a chance to hang up before the seller arrives. Sample surveys have a live interviewer on the line when you answer.

EXAMPLE 6 2012 election polls

In 2012, shortly before the presidential election, SurveyUSA, a polling organization, asked voters in several states who they would vote for. In Minnesota they asked a random sample of 574 likely voters, and 50% said they would vote for Barack Obama and 43% said Mitt Romney. SurveyUSA reported the margin of error to be plus or minus 4.2%. In Georgia they sampled 595 likely voters, and 44% said they would vote for Obama and 52% said Romney. The margin of error was reported to be plus or minus 4.1%.

There you have it: the sample of likely voters in Georgia was slightly larger, so the margin of error for conclusions about voters in Georgia is slightly smaller (4.1% compared to 4.2%). We are 95% confident that between 39.9% (that's 44% minus 4.1%) and 48.1% (that's 44% plus 4.1%) of likely voters in Georgia would vote for Obama. Note that the actual 2012 election results for Georgia were 45.5% for Obama, which is within the margin of error.

NOW IT'S YOUR TURN

3.2 Voting and personal views. In May 2015, the Gallup Poll asked a random sample of 1024 American adults, "Thinking about how the abortion issue might affect your vote for major offices, would you only vote for a candidate who shares your views on abortion or consider a candidate's position on abortion as just one of many important factors or not see abortion as a major issue?" It found that 21% of respondents said they will only vote for a candidate with the same views on abortion that they have. Suppose that the sample size had been 4000 rather than 1024. Find the margin of error for 95% confidence in this case. How does it compare with the margin of error for a sample of size 1024?

Sampling from large populations

Gallup's sample of 1015 American adults was only 1 out of every 254,200 adults in the United States. Does it matter whether 1015 is 1-in-100 individuals in the population or 1-in-254,200?

Population size doesn't matter

The variability of a statistic from a random sample is essentially unaffected by the size of the population as long as the population is at least 20 times larger than the sample.

Why does the size of the population have little influence on the behavior of statistics from random samples? Imagine sampling soup by taking a spoonful from a pot. The spoon doesn't know whether it is surrounded by a small pot or a large pot. As long as the pot of soup is well mixed (so that the spoon selects a "random sample" of the soup) and the spoonful is a small fraction of the total, the variability of the result depends only on the size of the spoon.

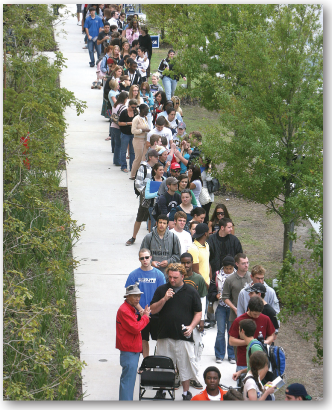
This is good news for national sample surveys like the Gallup Poll. A random sample of size 1000 or 2500 has small variability because the sample size is large. But remember that even a very large voluntary response sample or convenience sample is worthless because of bias. Taking a larger sample can never fix biased sampling methods.

However, the fact that the variability of a sample statistic depends on the size of the sample and not on the size of the population is bad news for anyone planning a sample survey in a university or a small city. For example, it takes just as large an SRS to estimate the proportion of Arizona State University undergraduate students who call themselves political conservatives as to estimate with the same margin of error the proportion of all adult U.S. residents who are conservatives. We can't use a smaller SRS at Arizona State just because there were 67,500 Arizona State undergraduate students and more than 258 million adults in the United States in 2013.

STATISTICAL CONTROVERSIES

Should Election Polls Be Banned?

Preelection polls tell us that Senator So-and-So is the choice of 58% of Ohio voters. The media love these polls. Statisticians don't love them because elections often don't go as forecasted, even when the polls use all the right statistical methods. Many people who respond to the polls change their minds before the election. Others say they are undecided. Still others say which candidate they favor but won't bother to vote when the election arrives. Election forecasting is one of the less satisfactory uses of sample surveys because we must ask people now how they will vote in the future.



George Skene/MCT/Newscom

Exit polls, which interview voters as they leave the voting place, don't have these problems. The people in the sample have just voted. A *good* exit poll, based on a national sample of election precincts, can often call a presidential election correctly long before the polls close. But, as was the case in the 2004 presidential election, exit polls can also fail to call the results correctly. These facts sharpen the debate over the political effects of election forecasts.

Can you think of good arguments *against* public preelection polls? Think about how these polls might influence voter turnout. Remember that voting ends in the East several hours before it ends in the West. What about arguments against exit polls?

What are arguments *for* preelection polls? Consider freedom of speech, for example. What about arguments for exit polls?

For some thought-provoking articles on polls, especially in light of the exit poll failures in the 2004 presidential election, see the following websites:

www.washingtonpost.com/wp-dyn/articles/A47000-2004Nov12.html

www.washingtonpost.com/wp-dyn/articles/A64906-2004Nov20.html

thehill.com/opinion/columnists/dick-morris/4723-those-faulty-exit-polls-were-sabotage

You may also go to www.google.com and search using key words "exit poll failures in the 2004 presidential election."

STATISTICS IN SUMMARY

Chapter Specifics

- The purpose of sampling is to use a sample to gain information about a population. We often use a sample **statistic** to estimate the value of a population **parameter**.
- This chapter has one big idea: to describe how trustworthy a sample is, ask, "What would happen if we took a large number of samples from the same population?" If almost all samples would give a result close to the truth, we can trust our one sample even though we can't be certain that it is close to the truth.
- In planning a sample survey, first aim for small **bias** by using random sampling and avoiding bad sampling methods such as voluntary response. Next, choose a large enough random sample to reduce the **variability** of the result. Using a large random sample guarantees that almost all samples will give accurate results.
- To say how accurate our conclusions about the population are, make a **confidence statement**. News reports often mention only the **margin of error**. Most often, this margin of error is for **95% confidence**.

That is, if we chose many samples, the truth about the population would be within the margin of error 95% of the time.

- We can roughly approximate the margin of error for 95% confidence based on a simple random sample of size n by the formula $1/\sqrt{n}$. As this formula suggests, only the size of the sample, not the size of the population, matters. This is true as long as the population is much larger (at least 20 times larger) than the sample.

LINK IT In Chapter 1, we introduced sample surveys as an important kind of observational study. In Chapter 2, we discussed both good and bad methods for taking a sample survey. Simple random sampling was introduced as a method that deliberately uses chance to produce unbiased data. This deliberate use of chance to produce data is one of the big ideas of statistics.

In this chapter, we looked more carefully at how sample information is used to gain information about the population from which it is selected. The big idea is to ask what would happen if we used our method for selecting a sample to take many samples from the same population. If almost all would give results that are close to the truth, then we have a basis for trusting our sample.

In practice, how easy is it to take a simple random sample? What problems do we encounter when we attempt to take samples in the real world? This is the topic of the next chapter.

CASE STUDY EVALUATED In the Case Study at the beginning of the chapter, 54% of those surveyed in 2015 felt that it is extremely important to vaccinate children. The Gallup Poll stated that childhood vaccinations were considered to be extremely important by a slight majority (54%) of American adults. Is 54% evidence that, in fact, the majority of American adults in 2015 felt that childhood vaccinations are extremely important? Use what you have learned in this chapter to answer this question. Your answer should be written so that someone who knows no statistics will understand your reasoning.

LaunchPad macmillan learning Online Resources

- The video technology manuals explain how to select an SRS using Excel, JMP, Minitab, and the TI 83/84.
- The Statistical Applet *Simple Random Sample* can be used to select a simple random sample when the number of labels is 144 or fewer.

CHECK THE BASICS

For Exercise 3.1, see page 47; for Exercise 3.2, see page 49.

3.3 What's the parameter? An online store contacts 1500 customers from its list of customers who have purchased in the last year and asks the customers if they are very satisfied with the store's website. One thousand (1000) customers respond, and 696 of the 1000 say that they are very satisfied with the store's website. The parameter is

(a) the 69.6% of respondents who replied they are very satisfied with the store's website.

(b) the percentage of the 1500 customers contacted who would have replied they are very satisfied with the store's website.

(c) the percentage of all customers who purchased in the last year who would have replied they are very satisfied with the store's website.

(d) None of the above is the parameter.

3.4 What's the statistic? A state representative wants to know how voters in his district feel about enacting a statewide smoking ban in all enclosed public places, including bars and restaurants. His staff mails a questionnaire to a simple random sample of 800 voters in his district. Of the 800 questionnaires mailed, 152 were returned. Of the 152 returned questionnaires, 101 supported the enactment of a statewide smoking ban in all enclosed public places. The statistic is

(a) $152/800 = 19.0\%$

(b) $101/800 = 12.6\%$

(c) $101/152 = 66.4\%$

(d) unable to be determined based on the information provided.

3.5 Decreasing sampling variability. You plan to take a sample of size 500 to estimate the proportion of students at your school who support having no classes and special presentations on Veteran's Day. To be twice as accurate with your results, you should plan to sample how many students? (*Hint*: Use the quick and approximate method for the margin of error.)

(a) 125 students

(b) 250 students

(c) 1000 students

(d) 2000 students

3.6 What can we be confident about? A May 2015 University of Michigan C.S. Mott Children's Hospital National Poll on Children's Health reported that 90% of adults are concerned that powdered alcohol will be misused by people under age 21. The margin of error was reported to be 2 percentage points, and the level of confidence was reported to be 95%. This means that

(a) we can be 95% confident that between 88% and 92% of adults are concerned that powdered alcohol will be misused by people under age 21.

(b) we can be 95% confident that between 88% and 92% of adults who were surveyed are concerned that powdered alcohol will be misused by people under age 21.

(c) we know that 90% of adults are concerned that powdered alcohol will be misused by people under age 21.

(d) Both (a) and (b) above are true.

3.7 Which is more accurate? The Pew Research Center Report entitled “How Americans value public libraries in their communities,” released December 11, 2013, asked a random sample of 6224 Americans aged 16 and older, “Have you used a Public Library in the last 12 months?” In the entire sample, 30% said Yes. But only 17% of those in the sample over 65 years of age said Yes. Which of these two sample percents will be more accurate as an estimate of the truth about the population?

(a) The result for those over 65 is more accurate because it is easier to estimate a proportion for a small group of people.

(b) The result for the entire sample is more accurate because it comes from a larger sample.

(c) Both are equally accurate because both come from the same sample.

(d) We cannot determine this, because we do not know the percentage of those in the sample 65 years of age or younger who said Yes.

CHAPTER 3 EXERCISES

3.8 Unemployment. The boldface number in the next paragraph is the value of either a **parameter** or a **statistic**. State which it is.

The Bureau of Labor Statistics announces that last month it interviewed all members of the labor force in a sample of 60,000 households; **5.4%** of the people interviewed were unemployed.

3.9 Ball bearing acceptance. Each boldface number in the next paragraph is the value of either a **parameter** or a **statistic**. In each case, state which it is.

A carload lot of ball bearings has an average diameter of **2.503** centimeters (cm). This is within the specifications for acceptance of the lot by the purchaser. The inspector happens to inspect 100 bearings from the lot with an average diameter of **2.515** cm. This is outside the specified limits, so the lot is mistakenly rejected.

3.10 Registered Republicans. Each boldface number in the next paragraph is the value of either a **parameter** or a **statistic**. In each case, state which it is.

Voter registration records show that **25%** of all voters in the United States are registered as Republicans. However, a national radio talk show host found that of 20 Americans who called the show recently, **60%** were registered Republicans.

3.11 Wireless telephones. Each boldface number in the next paragraph is the value of either a **parameter** or a **statistic**. In each case, state which it is.

The Behavioral Risk Factor Surveillance System (BRFSS) telephone survey is conducted annually in the United States. Of the first 100 numbers dialed, **55** numbers were for wireless telephones. This is not surprising, because, as of the first half of

2014, **52.1%** of all U.S. children lived in households that were wireless only.

3.12 A sampling experiment. Figures 3.1 and 3.2 show how the sample proportion $\hat{p}$ behaves when we take many samples from the same population. You can follow the steps in this process on a small scale.

Figure 3.4 represents a small population. Each circle represents an adult. The white circles are people who favor a constitutional amendment that would define marriage as being between a man and a woman, and the colored circles are people who are opposed. You can check that 50 of the 100 circles are white, so in this population the proportion who favor an amendment is $p = 50/100 = 0.5$.

(a) The circles are labeled 00, 01, . . . , 99. Use line 101 of Table A to draw an

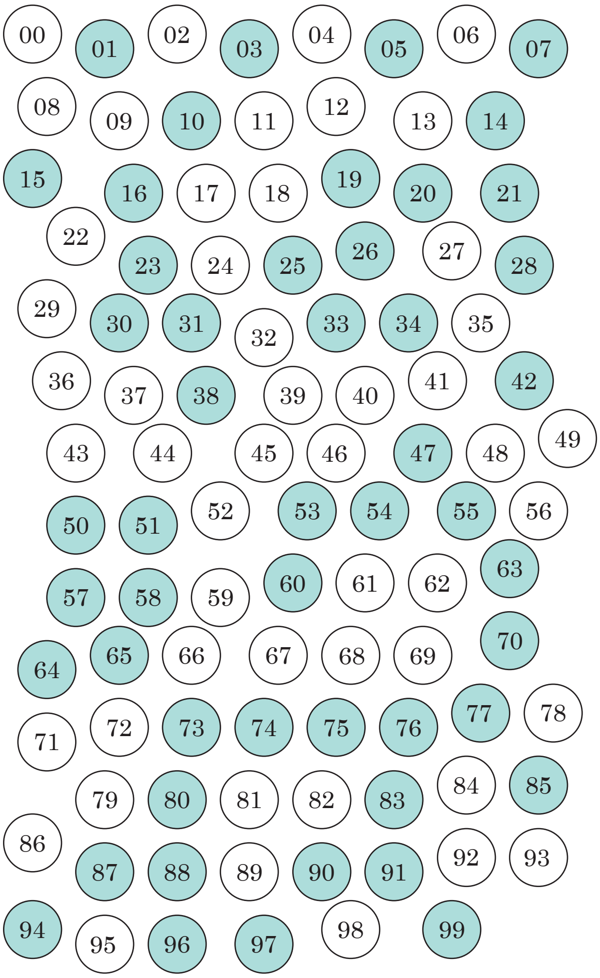


Figure 3.4 A population of 100 individuals for Exercise 3.12. Some individuals (white circles) favor a constitutional amendment and the others do not.

SRS of size 4. What is the proportion $\hat{p}$ of the people in your sample who favor a constitutional amendment?

(b) Take nine more SRSs of size 4 (10 in all), using lines 102 to 110 of Table A, a different line for each sample. You now have 10 values of the sample proportion $\hat{p}$. Write down the 10 values you should now have of the sample proportion $\hat{p}$.

(c) Because your samples have only four people, the only values $\hat{p}$ can take are $0/4$, $1/4$, $2/4$, $3/4$, and $4/4$. That is, $\hat{p}$ is always 0, 0.25, 0.5, 0.75, or 1. Mark these numbers on a line and make a histogram of your 10 results by putting a bar above each number to show how many samples had that outcome.

(d) Taking samples of size 4 from a population of size 100 is not a practical setting, but let's look at your results anyway. How many of your 10 samples estimated the population proportion $p = 0.5$ exactly correctly? Is the true value 0.5 in the center of your sample values? Explain why 0.5 would be in the center of the sample values if you took a large number of samples.

3.13 A sampling experiment. Let us illustrate sampling variability in a small sample from a small population. Ten of the 25 club members listed here are female. Their names are marked with asterisks in the list. The club chooses five members at random to receive free trips to the national convention.

Alonso	Darwin	Herrnstein	Thomson
Binet*	Epstein	Jimenez*	Toulmin
Blumenbach	Ferri	Luo	Vogt*
Chase*	Gonzales*	Moll*	Went
Chen*	Gupta	Morales*	Wilson
		Myrdal	Yerkes
		Perez*	Zimmer
		Spencer*	

(a) Use the *Simple Random Sample* applet, other software, or a different part of Table A to draw 20 SRSs of size 5. Record the number of females in each of your samples. Make a histogram like that in Figure 3.1 to display your results. What is the average number of females in your 20 samples?

(b) Do you think the club members should suspect discrimination if none of the five tickets go to women?

3.14 Another sampling experiment. Let us illustrate sampling variability in a small sample from a small population. Seven of the 20 college softball players listed here are in-state students. Their names are marked with asterisks in the list. The coach chooses five players at random to receive a new scholarship funded by alumni.

Betsa	Richvalsky*
Blanco	Romero
Christner	Sbonek*
Connell	Susalla*
Driesenga*	Swearingen
Falk	Sweet
Garfinkel*	Swift*
Lawrence	Vargas
Montemarano	Wagner
Ramirez	Wald*

(a) Use the *Simple Random Sample* applet, other software, or a different part of Table A to draw 20 SRSs of size 5. Record the number of in-state students in each of your samples. Make a histogram like that in Figure 3.1 to display your results. What is the average number of in-state players in your 20 samples?

(b) Do you think the college should suspect discrimination if none of the five scholarships go to in-state players?

3.15 Canada's national health care. The Ministry of Health in the Canadian province of Ontario wants to know whether the national health care system is achieving its goals in the province. Much information about health care comes from patient records, but that source doesn't allow us to compare people who use health services with those who don't. So the Ministry of Health conducted the Ontario Health Survey, which interviewed a random sample of 61,239 people who live in the province of Ontario.

(a) What is the population for this sample survey? What is the sample?

(b) The survey found that 76% of males and 86% of females in the sample had visited a general practitioner at least once in the past year. Do you think these estimates are close to the truth about the entire population? Why?

3.16 Environmental problems. A Gallup Poll found that Americans, when asked about a list of environmental problems, were increasingly worried about problems such as pollution of drinking water, soil, air, and waterways, but were least worried about climate change or global warming. Gallup's report said, "Results for this Gallup poll are based on telephone interviews conducted March 5–8, 2015, with a random sample of 1,025 adults, aged 18 and older, living in all 50 U.S. states and the District of Columbia."

(a) What is the population for this sample survey? What is the sample?

(b) The survey found that 64% of Republicans and those who lean Republican and 76% of Democrats and those who lean Democratic in the sample are most concerned about pollution of drinking water. Do you think these estimates are close to the truth about the entire population? Why?

3.17 Bigger samples, please. Explain in your own words the advantages of bigger random samples in a sample survey.

3.18 Sampling variability. In thinking about Gallup's sample of size 1015, we asked, "Could it happen that one random sample finds that 54% of adults feel that childhood vaccination is extremely important and a second random sample finds that only 42% favor one?" Look at Figure 3.2, which shows the results of 1000 samples of this size when the population truth is $p = 0.5$, or 50%. Would you be surprised if a sample from this population gave 54%? Would you be surprised if a sample gave 42%? Use Figure 3.2 to support your reasoning.

3.19 Health care satisfaction. A November 2014 Gallup Poll of 828 adults found that

472 are satisfied with the total cost they pay for their health care. The announced margin of error is ±4 percentage points. The announced confidence level is 95%.

(a) What is the value of the sample proportion $\hat{p}$ who say they are satisfied with the total cost they pay for their health care? Explain in words what the population parameter p is in this setting.

(b) Make a confidence statement about the parameter p.

3.20 Bias and variability. Figure 3.5 shows the behavior of a sample statistic in many samples in four situations. These graphs are like those in Figures 3.1 and 3.2. That is, the heights of the bars show how often the sample statistic took various values in many samples from the same population. The true value of the population parameter is marked on each graph. Label each of the graphs in Figure 3.5 as showing high or low bias and as showing high or low variability.

3.21 Is a larger sample size always better? In February 2004, *USA Today* conducted an online poll. Visitors to its website were asked the following question: "Should the U.S. pass a constitutional amendment banning gay marriage?" Visitors could vote by clicking a button. The results as of 3:30 P.M. on February 25 were that 68.61% voted No and 31.39% voted Yes. A total of 63,046 votes had been recorded. Using our quick method, we find that the margin of error for 95% confidence for a sample of this size is roughly equal to four-tenths of 1 percentage point. Is it correct to say that, based on this *USA Today* online poll, we are 95% confident that 68.61% ± 0.4% of American adults are opposed to having the United States pass a constitutional amendment banning gay marriage? Explain your answer. Be careful not to confuse your

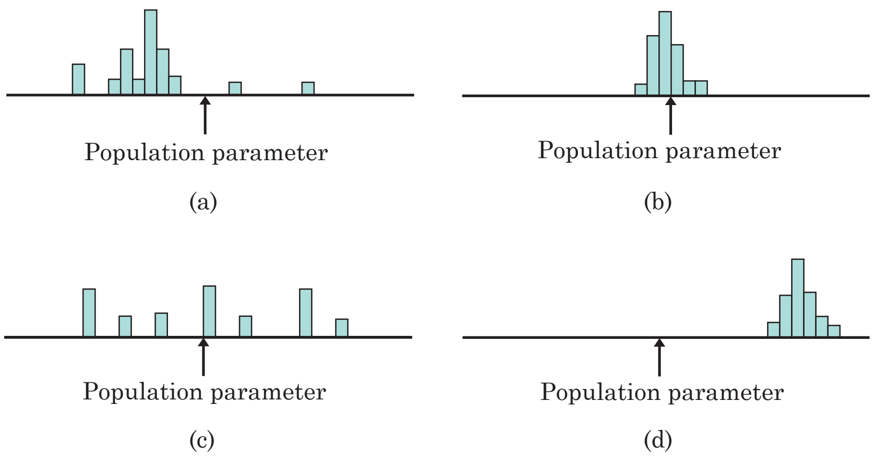


Figure 3.5 Take many samples from the same population and make a histogram of the values taken by a sample statistic. Here are the results for four different sampling methods for Exercise 3.20.

personal opinion with the statistical issues.

3.22 Predict the election. Just before a presidential election, a national opinion poll increases the size of its weekly sample from the usual 1000 people to 4000 people. Does the larger random sample reduce the bias of the poll result? Does it reduce the variability of the result?

3.23 Take a bigger sample. A management student is planning a project on student attitudes toward part-time work while attending college. She develops a questionnaire and plans to ask 25 randomly selected students to fill it out. Her faculty adviser approves the questionnaire but suggests that the sample size be increased to at least 100 students. Why is the larger sample helpful? Back up your statement by using the quick and approximate method to estimate the margin of error for samples of size 25 and for samples of size 100.

3.24 Sampling in the states. An agency of the federal government plans to take an SRS of residents in each state to estimate the proportion of owners of real estate in each state's population. The populations of the states range from about 563,600 people in Wyoming to about 37.3 million in California, according to the 2010 U.S. census.

(a) Will the variability of the sample proportion change from state to state if an SRS of size 2000 is taken in each state? Explain your answer.

(b) Will the variability of the sample proportion change from state to state if an SRS of $1/10$ of 1% (0.001) of the state's population is taken in each state? Explain your answer.

3.25 Polling women. A *New York Times* Poll on women's issues interviewed 1025 women randomly selected from the United States, excluding Alaska and Hawaii. The poll found that 47% of the women said they do not get enough time for themselves.

(a) The poll announced a margin of error of ± 3 percentage points for 95% confidence in its conclusions. Make a 95% confidence statement about the percentage of all adult women who think they do not get enough time for themselves.

(b) Explain to someone who knows no statistics why we can't just say that 47% of all adult women do not get enough time for themselves.

(c) Then explain clearly what "95% confidence" means.

3.26 Polling men and women. The sample survey described in Exercise 3.25 interviewed 472 randomly selected men as well as 1025 women. The poll announced a margin of error of ± 3 percentage points for 95% confidence in conclusions about women. The margin of error for results concerning men was ± 5 percentage points. Why is this larger than the margin of error for women?

3.27 Explaining confidence. A student reads that we are 95% confident that the average score of eighth-grade girls on the writing part of the 2011 National Assessment of Educational Progress is 163.8 to 165.2. Asked to explain the meaning of this statement, the student says, "95% of all eighth-grade girls have writing scores between 163.8 and 165.2." Is the student right? Explain your answer.

3.28 The death penalty. In October 2014, the Gallup Poll asked a sample of 1017 adults, "Are you in favor of the death penalty for a person convicted of murder?" The proportion who said they were in favor was 63%.

(a) Approximately how many of the 1017 people interviewed said they were in favor of the death penalty for a person convicted of murder?

(b) Gallup says that the margin of error for this poll is ±4 percentage points. Explain to someone who knows no statistics what "margin of error ±4 percentage points" means.

3.29 Find the margin of error. Example 6 tells us that a SurveyUSA poll asked 595 likely voters in Georgia which presidential candidate they would vote for; 52% said they would vote for Mitt Romney. Use the quick method to estimate the margin of error for conclusions about all likely voters in Georgia. How does your result compare with SurveyUSA's margin of error given in Example 6?

3.30 Find the margin of error. Exercise 3.28 concerns a Gallup Poll sample of 1017 people. Use the quick and approximate method to estimate the margin of error for statements about the population of all adults. Is your result close to the ±4% margin of error announced by Gallup?

3.31 Find the margin of error. Exercise 3.15 describes a sample survey of 61,239 adults living in Ontario. Estimate the margin of error for conclusions having 95% confidence about the entire adult population of Ontario.

3.32 Belief in God. A Gallup Poll conducted in May 2011 reports that 92% of a sample of 509 adults said Yes when asked "Do you believe in God?"

(a) Use the quick method to estimate the margin of error for an SRS of this size.

(b) Assuming that this was a random sample, make a confidence statement about the percentage of all adults who believe in God.

In the NEWS

3.33 Abortion. A 2014 Gallup Poll of 1028 adults found that 216 thought abortion should be illegal in all circumstances, a decrease of 2 percentage points from the record high in 2009. Make a confidence statement about the percentage of all adults who thought abortion should be illegal in all circumstances, at the time the poll was taken. (Assume that this is an SRS, and use the quick and approximate method to find the margin of error.)

3.34 Moral uncertainty versus statistical uncertainty. In Exercise 3.33 and in the Case Study, we examined polls involving controversial issues, either from a moral or personal liberty perspective (we will call this "moral uncertainty"). In both polls, national opinion was divided, suggesting that there is considerable moral uncertainty regarding both issues. What was the margin of error (the "statistical uncertainty") in both polls? Is it possible for issues with a high degree of moral uncertainty to have very little statistical uncertainty? Discuss.

3.35 Smaller margin of error. Exercise 3.28 describes an opinion poll that interviewed 1017 people. Suppose that you want a margin of error half as large as the one you found in that exercise. How many people must you plan to interview?

3.36 Satisfying Congress. Exercise 3.19 describes a sample survey of 828 adults, with margin of error ±4% for 95% confidence.

(a) A member of Congress thinks that 95% confidence is not enough. He wants to be 99% confident. How would the margin of error for 99% confidence based on the same sample compare with the margin of error for 95% confidence?

(b) Another member of Congress is satisfied with 95% confidence, but she wants a smaller margin of error than ±4 percentage points. How can we get a smaller margin of error, still with 95% confidence?

3.37 The Current Population Survey. Though opinion polls usually make 95% confidence statements, some sample surveys use other confidence levels. The monthly unemployment rate, for example, is based on the Current Population Survey of about 60,000 households. The margin of error in the unemployment rate is announced as about two-tenths of 1 percentage point with 90% confidence. Would the margin of error for 95% confidence be smaller or larger? Why?

3.38 Honesty and Wall Street? In May 2012, the Harris Poll asked a random sample of 1016 adults if they agreed with the following statement: "Most people on Wall Street would be willing to break the law if they believed they could make a lot of money and get away with it." It found that 711 agreed with the statement. Write a short report of this finding, as if you were writing for a newspaper. Be sure to include a margin of error. Be careful not to confuse your personal opinion with the statistical findings.

3.39 Who is to blame? A February 2009 poll conducted by the Marist College Institute for Public Opinion in Poughkeepsie, New York, asked a random sample of 2071 U.S. adults who or what was responsible for a company's failure or success. Of those surveyed, 70% attributed a company's failure or success to the decisions of its top executives. The poll asked the same question of a random sample of 110 business executives. Among executives, 88% said that top executives were responsible for a company's success or failure.

Marist reported that the margin of error for one of these results was ±9 percentage points and for the other it was ±2.5 percentage points. Which result had the margin of error of ±9 percentage points? Explain your answer.

3.40 Simulation. Random digits can be used to *simulate* the results of random sampling. Suppose that you are drawing simple random samples of size 25 from a large number of college students and that 20% of the students are unemployed during the summer. To simulate this SRS, generate 25 random digits using the *Simple Random Sample* applet or let 25 consecutive digits in Table A stand for the

25 students in your sample. The digits 0 and 1 stand for unemployed students, and other digits stand for employed students. This is an accurate imitation of the SRS because 0 and 1 make up 20% of the 10 equally likely digits.

Simulate the results of 50 samples by counting the number of 0s and 1s in the first 25 entries in each of the 50 repetitions of the *Simple Random Sample* applet or in each of the 50 rows of Table A. Make a histogram like that in Figure 3.1 to display the results of your 50 samples. Is the truth about the population (20% unemployed, or 5 in a sample of 25) near the center of your graph? What are the smallest and largest counts of unemployed students that you obtained in your 50 samples? What percentage of your samples had either four, five, or six unemployed?

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Follow the QR code to access exercises.