MACHINE LEARNING 2018

Homework 1

November 10, 2018

- This homework is due at 2 PM, November 17, 2018.
- You may write your answers in Vietnamese or English or a mix of both languages.
- You may consult textbooks and print and online materials.

- Please show all of your work. Answers without appropriate justification will receive very little credit. For programming questions, please submit all the code.

Scores

1. Problem 1 (_____ /30 pts.)

2. Problem 2 (_____ /20 pts.)

3. Problem 3 (_____ /50 pts.)

Total: (_____ /100 pts.)

Problem 1. *(30 points)*
Prove the following properties:
(a) Suppose A is a square matrix, λ is an eigenvalue of A, and $s \geq 0$ is an integer. Then, λ^s is an eigenvalue of A^s.
(b) If A and B are square, nonsingular matrices and X is a square matrix, then $X(A + XBX^T)^{-1} = A^{-1} - A^{-1}X(B^{-1} + X^T A^{-1} X)^{-1} X^T A^{-1}$.

Problem 2 *(20 points)*
Prove that all isolated local minimizers are strict.

Problem 3 *(50 points)*
Consider a convex quadratic function in n-dimensional space of the form

$$f(x) = \frac{1}{2} x^T A x + b^T x,$$

where A is a symmetric, positive semidefinite matrix of size n x n and b is a vector of size n. Write a Python program that takes A and b as inputs and optimizes function f using Gradient Descent algorithm. Note: initialization is important.