MACHINE LEARNING 2018

Homework 2

November 17, 2018

- This homework is due at 2 PM, November 24, 2018.

- Please submit the HW via Google Form (Link will be sent out shortly). Code for programming problems should be submitted as .py files.

- You can discuss HW problems with the instructor, TAs, classmates, or others, but the work you submit must be your own work.

- You may write your answers in Vietnamese or English or a mix of both languages.

- You may consult textbooks and print and online materials.

- Please show all of your work. Answers without appropriate justification will receive very little credit. For programming questions, please submit all the code.

Problem 1. (Adapted from Ross [1]) *(10 points)*
In the course project at a Machine Learning class, 15 students are to be divided
into 3 groups of respective sizes 4, 5, and 6. How many divisions are possible?

Problem 2 (Adapted from Ross [1]) *(15 points)*
A box contains 4 marbles, 2 red, 1 green, and 1 blue. Consider an experiment
that consists of taking 1 marble from the box, then replacing it in the box and
drawing a second marble from the box. Describe the sample space. Repeat when
the second marble is drawn without first replacing the first marble.

Problem 3 (Adapted from Ross [1]) *(10 points)*
Two cards are chosen at ramdom from a deck of 52 playing cards. What is the
probability that they
(a) are both Jacks (J's);
(b) have the same value?

Problem 4 (Adapted from Ross [1]) *(10 points)*
Two fair dice are rolled. What is the probability that at least one die lands on
5, given that the sum of the dice is i, $i = 3, 4, \ldots, 12$?

Problem 5 (Ross [1]) *(15 points)*
Let X be a Poisson random variable with parameter λ. Show that $P\{X = i\}$
increases monotonically and then decreases monotonically as i increases, reaching
its maximum when i is the largest integer not exceeding λ.
HINT: Consider $P\{X = i\}/P\{X = i - 1\}$

Problem 6 (Ross [1]) *(10 points)*
The standard deviation of X, denoted $SD(X)$, is given by

$$SD(X) = \sqrt{Var(X)}$$

Find $SD(aX + b)$ if X has variance σ^2.

Problem 7 (Ross [1]) *(30 points)*
The joint density function of X and Y is

$$f(x, y) = \begin{cases} x + y & 0 < x < 1, \ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Are X and Y independent?
(b) Find the density function of X.
(c) Find $P\{X + Y < 1\}$.

References

[1] S. Ross, A First Course in Probability, 6th Ed, Prentice Hall, 2002