MACHINE LEARNING 2018

Homework 1

November 10, 2018

- This homework is due at 2 PM, November 17, 2018.
- For Problem 1 and Problem 2, please submit a hard copy or hand-written copy of your solutions at the beginning of the Saturday class. If you cannot attend the class, please submit the soft copy through the Google Form (link will be sent out shortly).

- Please submit the code for Problem 3 through the Google Form.
- You can discuss HW problems with classmates or others, but the work you submit must be your own work.
- You may write your answers in Vietnamese or English or a mix of both languages.
- You may consult textbooks and print and online materials.
- Please show all of your work. Answers without appropriate justification will receive very little credit. For programming questions, please submit all the code.

Scores

1. Problem 1 (________ /30 pts.)
2. Problem 2 (________ /20 pts.)
3. Problem 3 (________ /50 pts.)

Total: (________ /100 pts.)

Problem 1. *(30 points)*
Prove the following properties:
(a) Suppose A is a square matrix, λ is an eigenvalue of A, and $s \geq 0$ is an integer. Then, λ^s is an eigenvalue of A^s.
(b) If A and B are square, nonsingular matrices and X is a square matrix, then $(A + XBX^T)^{-1} = A^{-1} - A^{-1}X(B^{-1} + X^T A^{-1} X)^{-1} X^T A^{-1}$.

Problem 2 *(20 points)*
Prove that all isolated local minimizers are strict.

Problem 3 *(50 points)*
Consider a convex quadratic function in n-dimensional space of the form

$$f(x) = \frac{1}{2} x^T A x + b^T x,$$

where A is a symmetric, positive semidefinite matrix of size n x n and b is a vector of size n. Write a Python program that takes A and b as inputs and optimizes function f using Gradient Descent algorithm. Note: initialization is important.