HOMEWORK

LU5: INTERVAL ESTIMATION

Suggestion: *Use the form to select the appropriate formula, and solve the exercises using statistical tables. Then, use Excel to find the exact solutions provided here.*

I) In the following multiple-choice questions, choose only one option.

1. A 95% confidence interval for the population mean is calculated to be (75.29 ; 81.45). If the confidence level is reduced to 90%, the confidence interval will:
   a) Become narrower
   b) Become wider
   c) Remain the same
   d) Most likely no longer include the true value of the population mean

2. Which of the following statements is FALSE with regards to the width of a confidence interval?
   a) The sample mean from which the interval is constructed is located half way between the boundaries of the confidence interval.
   b) The width of the interval increases when the sample size is decreased.
   c) The width of the interval decreases when the significance level is increased.
   d) The width of the interval decreases when the sample mean is decreased.

3. A confidence interval is best interpreted as a measure of
   a) The accuracy of the estimate
   b) The bias of the estimate
   c) The truthfulness of the estimate
   d) The origin of the estimate

4. What value of z would you use to calculate the 80% confidence interval for a population mean, given that you know the population standard deviation, the sample size and the sample mean of your sample?

    a) z = 1.282

    b) z = 1.645

    c) z = 1.96

    d) z = 2.576

5. An auditor is trying to estimate the average size of an invoice. A sample of 64 invoices from a large population of invoices yields a mean of $300 with a standard deviation of $40. What is the 95% confidence interval?

    a) [$287, $312]

    b) [$290, $310]

    c) [$291, $309]

    d) [$299, $301]

6. Your company manufacturers Bluetooth chips. To estimate the fraction of defective chips, you sample 100 chips, and find 5 are defective. You are 95% sure the actual percentage of defective Bluetooth chips is within which of the following ranges?

    a) [0.002 ; 0.060]

    b) [0.007 ; 0.093]

    c) [0.013 ; 0.072]

    d) [0.014 ; 0.086]

7. A sample of 100 American males has an average weight of 170 pounds and a standard deviation of 30 pounds. Assuming that the weight of American males is normally distributed, you are 90% confident that the average weight of an American male is approximately between which two values?

    a) 150 and 190 pounds

    b) 160 and 180 pounds

    c) 165 and 175 pounds

    d) 167 and 170 pounds

8. On the basis of a random sample of 100 men from a particular province in South Africa, a 95% confidence interval for the mean height of men in the province is found to be (177.22 cm ; 179.18 cm). What is the value of the sample mean for this sample?

    a) 178.2 cm

    b) 177.38 cm

    c) 176.58 cm

    d) 196 cm

9. A student conducted a study and reported that the 95% confidence interval for the population mean was (46; 54). He was sure that the population standard deviation was 16. What was the sample size (rounded up to the nearest whole number) used to calculate this confidence interval?

    a) 40

    b) 62

    c) 97

    d) 110

10. A large company wants to estimate the average commuting time it takes to employees to get to work. They believe the standard deviation of the commuting time of employees is 30 minutes. If they want to be 95% confident that there estimate of mean commuting time is accurate within 5 minutes, what sample size is needed?

    a) 103

    b) 112

    c) 122

    d) 139

II) Solve the following problems.

1. The weight of the packages of sweets of a certain brand is a random variable with normal distribution. With the purpose of studying the variability of the weight of the mentioned packages, it was gathered a 9 packets sample whose values (in grams) were the following:

98	101	102	97	104	99	96	105	98

   a) Indicate an estimate for the weight variability of the packets of sweets (in *squared grams*).

   b) Suppose that it had been obtained the following confidence interval for the mean weight of those packages: [96.46 ; 103.54[. Determine the confidence level of this interval.

2. Two random variables X_1 and X_2, follow a normal distribution with variance 3.64 and 4.03, respectively. Build a 95% confidence interval for the difference between their means ($\mu_1 - \mu_2$), knowing that in the collected samples were obtained the following results:

   $n_1 = 32$ $\bar{x}_1 = 16.2$

   $n_2 = 40$ $\bar{x}_2 = 14.85$

3. The coordinator of two centres of Sports Medicine wants to compare the average weights of the athletes, whose distribution is normal. Two independent random samples of 10 athletes each were collected, and provided the following results: average of 77kg and standard deviation of 6kg for sample #1, and average of 68kg and standard deviation of 10kg for sample #2. Assuming that the variance of the athletes' weights is identical in both centres, determine a 90% confidence interval for the difference between the mean weights of the athletes in the two centres.

4. The manager of an industrial unit wants to test a new method of execution of a given task on the assembly line, being that the time of implementation follows a normal distribution. The manager is convinced that the new method makes the task more quickly and, as such, it is beneficial for the company. In this context, the task was performed by 9 workers, first according to the traditional method, and then, by the new method. The results obtained (in minutes) were the following:

Workers	1	2	3	4	5	6	7	8	9
Traditional method	10.5	10.2	10.0	9.9	9.9	10.2	10.6	10.2	10.4
New method	9.6	9.7	9.6	10.0	9.9	9.6	10.0	9.5	9.5

   a) Build a 99% confidence interval for the difference between the mean times of execution of the mentioned task.

   Hint: note that there is only one *sample of nine workers.*

b) Considering the interval obtained in the previous question, do you think that the manager has a reason to be convinced that the new method is more advantageous for the company? Justify your answer.

5. The weight of electronic components produced by a certain company is a random variable that follows a normal distribution. In order to study the variability of the weight of the components, a sample of 11 elements was collected and the following values were obtained (in grams):

98	97	102	100	98	101	102	105	95	102	100

a) Provide an estimate for the variance of the weight of the components.

b) Build a 95% confidence interval for the variance of the weight of the components.

6. A company wants to launch a new product to a market of 1 million inhabitants. During the market study, 1000 individuals were surveyed of which 800 have stated that they were not planning to use the new product.

a) Build a 95% confidence interval for the proportion of inhabitants of the population that plan to use the new product. Interpret the result.

b) How many people will be deemed necessary to add to the sample initially collected for the sampling error to be at the most 1%?

7. A multinational in the area of chemical consumables wants to buy airtime on a television program. To estimate the rating level of the said program, the multinational intends to build a 99% confidence interval for the proportion of people who watch the program assiduously, with a maximum sampling error of 5%. What suggestion would you give to the multinational for the size of the sample to be collected?

8. Two brands of pills, one of them containing aspirin, are advertised as getting rid of a headache in a record time. Independent experiences were carried out with each brand, and the time people took to get rid of headaches was recorded (in minutes). Consider that the times follow a normal distribution. The samples' results were the following:

Pill 1 (containing aspirin)					Pill 2 (without aspirin)			
9.6	9.4	9.3	11.2		10.6	13.2	11.7	9.6
11.4	12.1	10.4	9.6		8.5	9.7	12.3	12.4
10.2	8.8	13.0	10.2		10.8	10.8		

a) Assuming the equality of population variances, build a 95% confidence interval for the difference between the mean times and state your conclusion on the efficacy of the two brands of pills.

b) Assuming the inequality of population variances, build a 95% confidence interval for the difference between the mean times and state your conclusion on the efficacy of the two brands of pills.

SOLUTIONS

Group I) Multiple choice

1. a
2. d
3. a
4. a
5. b
6. b
7. c
8. a
9. b
10. d

Group II) Problems

Question 1)

   a) $s^2 = 10$

   b) 0.99

Question 2) [0.44 ; 2.26]

Question 3) [2.6; 15.4]

Question 4) a) [0.104 ; 0.896]

Question 5)

   a) 8

   b) [3.9 ; 24.6]

Question 6)

   a) [0.175 ; 0.225]

   b) $\Delta n = 5147$

Question 7) n ≥ 664

Question 8)

   a) [−1.73 ; 0.68]

   b) [−1.76 ; 0.71]