Group I
[Answer: correct = 1 point; wrong = –0.1 points]

1. Let X be a continuous random variable. Which of the following statements is true?

☐ If x<0 then f(x)=0.

☐ If x<0 then F(x)=0.

☐ f(x) never decreases.

☐ Both f(x) and F(x) cannot take values less than zero.

☐ None of the statements is true.

2. Let X be a random variable that takes the values 0, 2, 4 and 6. Considering that k is a constant, and P(X=0) = P(X=2) = k, and that P(X=4) = P(X=6) = 0.2, what is the value of k?

☐ 0.1

☐ 0.2

☐ 0.3

☐ 0.4

☐ 0.8

3. Let X be a random variable with the following probability density function:

Let **a=0** and **b=4**. Which of the following statements is correct?

☐ c = 0.25 and E(4X) = 2

☐ c = 0.25 and E(4X) = 4

☐ c = 0.25 and E(4X) = 8

☐ c = 2 and E(4X) = 4

☐ c = 2 and E(4X) = 8

4. It is known that, on average, 2.4 firefighters with severe burns arrive at a hospital each year. Which of the following distributions would you use to calculate the probability of at least 2 firefighters with severe burns arriving at the hospital in 2021?

☐ Uniform distribution.

☐ Binomial distribution.

☐ Poisson distribution.

☐ Continuous distribution.

☐ None of those distributions.

5. Which of the following is NOT a condition of the Binomial distributions?

☐ The number of trials must be large.

☐ Two possible outcomes for each trial.

☐ The trials are independent.

☐ The probability of success remains constant from trial to trial.

6. 90% of complaints that reach a company's customer support line are unfounded. If 10 complaints are randomly selected from a large volume of complaints, it is expected that

☐ 0 are unfounded

☐ 1 is unfounded

☐ 5 are unfounded

☐ 9 are unfounded

☐ 10 are unfounded

7. Consider the random variable Z~N(0, 1). If **P(Z > x) = 0.975** then x is equal to

☐ −1.03

☐ 1.03

☐ 0.025

☐ −1.96

☐ 1.96

8. A 95% confidence interval for the population mean is **[12.4; 17.6]**. Which of the following is true?

☐ A point estimate of the population mean is 15 with a corresponding relative precision of 17.33%.

☐ A point estimate of the population mean is 15 with a corresponding relative precision of 0.35%.

☐ A point estimate of the population mean is 5.2 with a corresponding relative precision of 2.6%.

☐ The sample mean is 15 and the sampling error is 5.2.

9. To estimate the λ parameter of the Poisson distribution the following estimator is used:

$$\hat{\lambda} = \bar{X} + \frac{n}{2}$$

This estimator is

☐ Biased and asymptotically unbiased

☐ Biased and asymptotically biased

☐ Unbiased and asymptotically biased

☐ Unbiased and asymptotically unbiased

10. A confidence interval for $p_1 - p_2$ was obtained: [–0.2; 0.6]. This interval…

☐ It is badly built because one of the extremes is negative.

☐ For the degree of confidence considered, it allows to conclude that $p_1 < p_2$.

☐ For the degree of confidence considered, it allows to conclude that $p_1 = p_2$.

☐ For the degree of confidence considered, it allows to conclude that $p_1 > p_2$.

☐ It does not allow to draw conclusions about the relationship between the values of p_1 and p_2.

Group II

(**1.5 points**) Let X_1, X_2, …, X_7 be an iid random sample from a population with mean equal to λ and variance equal to λ^2. Consider the following **unbiased estimator of the population mean**:

$$T = \left(\frac{1}{3}\sum_{i=1}^{6} X_i\right) - X_7$$

TRUE or FALSE ?

T is a consistent estimator of the population mean.

1. (**1 point**) Complete the sentence with a **number with 4 decimal places** (*comma or period separating the decimal places depends on your computer; you will get an error message if you choose the wrong symbol, in which case, change the symbol you used*):

IQ Tests and scores are scaled to have a mean of 100 and a standard deviation of 15. The probability of having an IQ value between 115 and 130 is *approximately* _____.

2. (**1 point**) Complete the sentence with a **number with 1 decimal places** (*comma or period separating the decimal places depends on your computer; you will get an error message if you choose the wrong symbol, in which case, change the symbol you used*):

The following is the distribution function of the random variable X:

$$F(X) = \begin{cases} 0 \, , & x < 1 \\ \dfrac{4}{10} \, , & 1 \le X < 2 \\ \dfrac{6}{10} \, , & 2 \le X < 3 \\ \dfrac{8}{10} \, , & 3 \le X < 4 \\ 1 \, , & X \ge 4 \end{cases}$$

The value of P($2 \le X \le 3$) is equal to _____.

3. (**1 point**) Complete the sentence with a **number without decimal places** (*comma or period separating the decimal places depends on your computer; you will get an error message if you choose the wrong symbol, in which case, change the symbol you used*):

The social services of a university want to estimate the average time it takes to students to get to school. They believe the standard deviation of the commuting time of students is 30 minutes.

If they want to be 95% confident that their estimate of mean commuting time is accurate within 5 minutes, the minimum sample size needed is equal to _____.

Group IV

Suppose that the distribution of the weight of one bag of carrots is **X~N(µ=0, σ=2)**, the distribution of the weight of one bag of potatoes is **Y~N(µ=1, σ=1)**, and the distribution of the weight of one bag of onions is **Z~N(µ=10, σ=3)**. X, Y and Z are independent random variables.

Selecting bags at random, the shop owner wants to find the probability that the sum of two bags of carrots with one bag of onions exceeds the weight of three bags of potatoes.

a) (0.5 points) What is the name of probability distribution of the random variable **W = 2X–3Y+Z**?

b) (1 point) Which are the values of the parameter(s) of the probability distribution of **W = 2X–3Y+Z**?

Group V

Scores on an IQ test are normally distributed with mean equal to 100. Suppose 20 people are randomly selected and tested. The standard deviation in the sample group is 15.

You need to compute the probability that the average test score in the sample group will be smaller than 107.

a) (1 point) Which of the following formulas will you use (**A, B, C or D**)?

A	C
$Z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0; 1)$	$T = \dfrac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{(n-1)}$
B	**D**
$Z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \overset{a}{\sim} N(0; 1)$	$Z = \dfrac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \overset{a}{\sim} N(0; 1)$

b) (1 point) Indicate the final value of that probability.

A restaurant wants to analyse meal prices (in euros) in the city it operates in. It is known that the meal prices follow a Normal distribution. In a random sample of 30 restaurants in that city, the average price was 20 euros and the variance was 10.

You need to find a 95% confidence interval (CI) for the variability of the meal prices in that city.

a) (1 point) In the following, **indicate the values** of **(n–1)**, **1–α/2**, **α/2**, as well as the obtained values of the quantiles of the Chi-square distribution (χ^2) that you will use to compute that CI:

$$Q(n-1; 1-\alpha/2) = ?$$

$$Q(n-1; \alpha/2) = ?$$

b) (1 point) Indicate the 95% confidence interval that you obtained.