

## HOMework

### LU1: RANDOM VARIABLES

**I) In the following multiple-choice questions, choose only one option.**

1. You decide to collect a bunch of cans of soda and measure the volume of soda in each can. Let  $X$  be the amount of ml of soda in each can. What type of variable is  $X$ ?
  - a)  $X$  is a discrete random variable.
  - b)  $X$  is a continuous random variable.
  - c)  $X$  is a constant.
  - d)  $X$  is not a random variable.
2. A random variable is
  - a) a hypothetical list of the possible outcomes of a random phenomenon.
  - b) any number that changes in a predictable way in the long run.
  - c) any phenomenon in which outcomes are equally likely.
  - d) a variable whose values are numerical outcomes associated with a random phenomenon.
3. You decide to conduct a survey of families with two children. You are interested in counting the number of boys (out of 2 children) in each family. Is this a random variable, and if it is, what are all its possible values?
  - a) Yes, it is a random variable, and its values can be 1 or 2.
  - b) Yes, it is a random variable, and its values are 0, 1, and 2.
  - c) Yes, it is a random variable, and its values can be 2 or 4.
  - d) No, it is not a random variable, since it is not random.

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4. Which of the following is NOT a property of a random variable?
- a) The sum of the probabilities of a random variable is equal to 1.
  - b) A random variable cannot be negative.
  - c) A random variable represents numerical outcomes for different situations or events.
  - d) A random variable can be discrete or continuous.
5. Which of the following statements is true about a probability distribution?
- a) A probability distribution summarizes all possible experimental outcomes and their probability.
  - b) A probability distribution equals one.
  - c) A continuous probability distribution is used to summarize a discrete random variable.
  - d) A probability distribution shows the frequency distribution of a random variable.
6. Suppose a survey was conducted across the country regarding the number of firearms that people had in their households. If the sample size was 25,000 and 17,000 households reported that they had no firearms in their home, what would be the empirical probability that a randomly selected household had no firearms in their home?
- a) 17%
  - b) 25%
  - c) 68%
  - d) 100%
  - e) 0%
7. Suppose a survey was conducted across the country regarding the number of firearms that people had in their households. Let  $X$  = the number of firearms in a household. From the survey of 30,000 households it was determined that the empirical probability of  $X = 1$  was 0.2. How many of the households in the survey had one firearm in their home?
- a) 6 000
  - b) 9 000
  - c) 11 000
  - d) 20 000
  - e) 20%
8. Consider the following probability distribution (see table below) of the number of firearms in a household, constructed from a survey of 25,000 randomly selected households. Let  $X$  be the number of firearms in a

household, and assume that the probability of a household having more than 6 firearms in the home is negligible. If a household is selected at random, then how many firearms would you expect them to have?

$x_i$	$P(X = x_i)$
0	0.680
1	0.200
2	0.098
3	0.019
4	0.001
5	0.001
6	0.001

**Hint:** The expected value (i.e. the mean) of a discrete random variable is computed by summing the values of  $X$  multiplied by the corresponding probabilities:  $E(X) = \sum x_i P(X = x_i)$ .

- a) 0.68
  - b) 0.20
  - c) 0.753
  - d) 0.468
  - e) 1.148
9. In general, a discrete probability distribution requires that:
- a) all outcomes must have the same probability of success.
  - b) the probabilities for all possible outcomes must sum to 1.00.
  - c) the probability of success must be less than 0.1.
  - d) the events must be dependent.
10. In a continuous probability distribution, the random variable:
- a) is limited to certain values.
  - b) may be any value within a certain range.
  - c) may have a probability greater than 1.
  - d) always takes non-negative values.
11. For any probability distribution, the expected value is:
- a) equal to the random variable.
  - b) equal to the mean of the distribution.
  - c) equal to the variance of the distribution.
  - d) cannot be greater than 1.

**12.** Which of the following random variables is NOT continuous?

- a) Amount of gasoline in a car.
- b) Number of goals scored by a hockey team.
- c) Time it takes to commute to work.
- d) Lifetime of a AAA battery.

**13.** Which of the following is not a random variable?

- a) The heights of buildings in New York City.
- b) The number of children in households in the United States.
- c) The amount of money won (or lost) by the next person to walk out of a casino in Las Vegas.
- d) All the above are random variables.

**14.** What would be the correlation between study time and test grades?

- a) Positive correlation
- b) Negative correlation
- c) No correlation
- d) Cannot be determined for random variables

**15.** Correlation refers to

- a) the causal relationship between two variables.
- b) the association between two variables.
- c) the proportion of variance that two variables share.
- d) a statistical method that can only be used with a correlational research design.

**16.** A random variable, time to load a delivery truck, is uniformly distributed. The distribution is defined by the

- a) minimum and maximum times to load a delivery truck.
- b) maximum time to load a delivery truck.
- c) minimum time to load a delivery truck.
- d) standard deviation.

**17.** A uniform distribution's shape is:

- a) bell shaped.
- b) positively skewed.
- c) negatively skewed.
- d) rectangular.

**18.** For a uniform distribution with a minimum = 2 and a maximum = 10, what is the probability that the uniformly distributed random variable is between six and ten?

- a) 4.0
- b) 0.5
- c) 1.0
- d) 0.125

**19.** A normal distribution can be characterized as a/an \_\_\_\_\_ distribution.

- a) bell-shaped
- b) symmetric
- c) asymptotic
- d) All of the above.

**20.** If a random variable is normally distributed,

- a) the random variable cannot assume negative numbers.
- b) the distribution of the random variable is defined by a mean and standard deviation.
- c) the variance of the random variable must be greater than or equal to one.
- d) the probability that the random variable is greater than the mean is 0.60.

**21.** If the continuous random variable  $X$  has distribution function  $F(x)$  and probability density function  $f(x)$ , then

- a)  $f(x)$  never decreases
- b)  $F(x)$  never decreases
- c)  $f(x)$  is always constant
- d)  $F(x)$  is always constant
- e)  $F(x) = f(x)$

- 22.** The distribution function  $F(x)$  of a discrete random variable  $X$  is always
- a) continuous
  - b) continuous to the right of the values of  $X$
  - c) constant
  - d) increasing
  - e) decreasing
- 23.** The probability density function curve of a continuous random variable shows the probability as being
- a) the height of the curve
  - b) the area under the curve
  - c) the slope of the curve
  - d) a range on the horizontal axis
  - e) a range on the vertical axis
- 24.** The continuous random variable  $X$  has distribution function  $F(x)$  and probability density function  $f(x)$ , thus
- a)  $F(x)$  is equal to zero if  $x < 0$
  - b)  $F(x)$  is non-decreasing
  - c)  $f(x)$  is always constant
  - d)  $F(x)$  is always constant
  - e) None of the above
- 25.** A random variable is said to be discrete if
- a) you can count or list all possible values
  - b) it can assume any real value in an interval
  - c) the probability rules apply
  - d) it can be expressed graphically
  - e) the probability function is positive
- 26.** A random variable is said to be continuous if
- a) you can count or list all possible values
  - b) it can assume any real value in an interval
  - c) the probability rules apply
  - d) it can be expressed graphically
  - e) the probability function is positive

**27.** Let  $X$  be a random variable that takes the values 1, 2 and 3. If  $P(X=1)=0.1$  and  $P(X=2)=0.2$ , what is the value of  $P(X=3)$ ?

- a) 0.1
- b) 0.3
- c) 0.5
- d) 0.7
- e) 0.9

**28.** The number of defects on the surface of a certain DVD mark is a random variable  $X$  with probability function:

$x$	0	1	2	3	4
$f(x)$	0.40	0.20	0.15	0.15	0.10

What is the probability of finding at least three defects on a DVD?

- a) 0.10
- b) 0.15
- c) 0.25
- d) 0.75

**29.** The number of defects on the surface of a certain DVD mark is a random variable  $X$  with probability function:

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What is the probability of finding less than three defects on a DVD?

- a) 0.10
- b) 0.15
- c) 0.25
- d) 0.75

**30.** The number of defects on the surface of a certain DVD mark is a random variable  $X$  with probability function:

$x$	0	1	2	3	4
$f(x)$	0.45	0.35	0.14	0.05	0.01

The distribution function of  $X$  is given by:

$$\text{a) } F(x) = \begin{cases} 0 & , x < 0 \\ 0.45 & , 0 \leq x < 1 \\ 0.80 & , 1 \leq x < 2 \\ 0.94 & , 2 \leq x < 3 \\ 0.99 & , 3 \leq x < 4 \\ 1 & , x \geq 4 \end{cases}$$

$$\text{c) } F(x) = \begin{cases} 0 & , x < 0 \\ 0.45 & , 0 \leq x \leq 1 \\ 0.80 & , 1 \leq x \leq 2 \\ 0.94 & , 2 \leq x \leq 3 \\ 0.99 & , 3 \leq x \leq 4 \\ 1 & , x \geq 4 \end{cases}$$

$$\text{e) } F(x) = \begin{cases} 0 & , x \leq 0 \\ 0.45 & , 0 < x < 1 \\ 0.80 & , 1 \leq x < 2 \\ 0.94 & , 2 \leq x < 3 \\ 0.99 & , 3 \leq x \leq 4 \\ 1 & , x > 4 \end{cases}$$

$$\text{b) } F(x) = \begin{cases} 0 & , x < 0 \\ 0.45 & , 0 < x < 1 \\ 0.80 & , 1 < x < 2 \\ 0.94 & , 2 < x < 3 \\ 0.99 & , 3 < x < 4 \\ 1 & , x \geq 4 \end{cases}$$

$$\text{d) } F(x) = \begin{cases} 0 & , x < 0 \\ 0.45 & , 0 < x \leq 1 \\ 0.80 & , 1 < x \leq 2 \\ 0.94 & , 2 < x \leq 3 \\ 0.99 & , 3 < x \leq 4 \\ 1 & , x \geq 4 \end{cases}$$



**II) Solve the following problems.**

1. Let  $X$  be a random variable with the following probability function:

$$X \begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.132 & 0.329 & 0.329 & 0.165 & 0.041 & 0.004 \end{cases}$$

- Find the distribution function of  $X$ .
- Draw a graph of the distribution function of  $X$ .

2. Let  $X$  be a random variable with the following distribution function:

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1/6 & , 0 \leq x < 2 \\ 1/4 & , 2 \leq x < 4 \\ 1/2 & , 4 \leq x < 6 \\ 1 & , x \geq 6 \end{cases}$$

- Draw a graph of the distribution function of  $X$ .
- Find the probability function of  $X$ .
- Draw a graph of the probability function of  $X$ .
- Calculate:

- $P(X \leq 1)$
- $P(2 \leq X < 6)$
- $P(0 < X \leq 2)$
- $P(X > 5)$

3. Let  $X$  be a random variable with the following probability function:

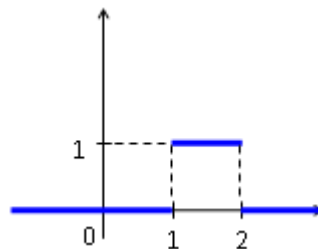
$$X \begin{cases} -2 & -1 & 0 & 1 & 2 \\ 0.1 & 0.25 & 0.3 & 0.25 & 0.1 \end{cases}$$

- Draw a graph of the probability function.
- Which is the mean value of  $X$ ? Why?
- Compute  $E(2X+3)$ .
- Given that  $V(X)=1.3$ , compute  $V(2X+3)$ .

4. Let  $X$  be the random variable that represents the number of errors per page in a book. The probability function of  $X$  is given by ( $a$  and  $b$  are constants):

$$X \begin{cases} 0 & 1 & 2 & 3 & 4 \\ 0.7 & a & b & 0.05 & 0.05 \end{cases}$$

- a) Knowing that in 80% of the pages there is a maximum of 1 error, calculate the values of  $a$  and  $b$ .  
 b) Find the distribution function of  $X$ .
5. Suppose the lifetime (in hours) of a certain type of bacteria is a random variable  $X$  with probability density function given by the following graph:



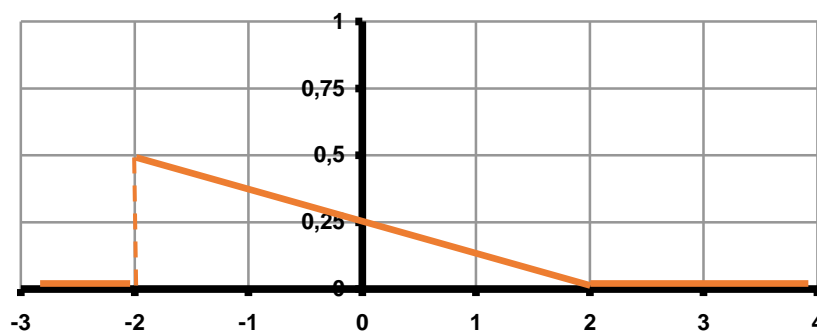
- a) What is the name of this probability distribution?  
 b) What is the average life span of this type of bacteria?  
 c) Find the probability of a bacteria living longer than 1h30m.  
 d) What is the probability of a bacteria living less than 1h15m?
6. Let  $X$  be a random variable with probability density function given by

$$f(x) = \frac{1}{3}, \quad 0 < x < 3$$

- a) Draw a graph of  $f(x)$ .  
 b) What is the name of this probability distribution?  
 c) What is the mean value of  $X$ ? Justify your answer.  
 d) Calculate:
- $P(X < 1)$
  - $P(X \geq 2)$
  - $P(1 < X < 2)$
7. Let  $X$  be a random variable with probability density function given by

$$f(x) = \frac{1}{4} - \frac{1}{8}x, \quad -2 < x < 2; f(x) = 0, \text{ otherwise}$$

The graph of  $f(x)$  is the following:



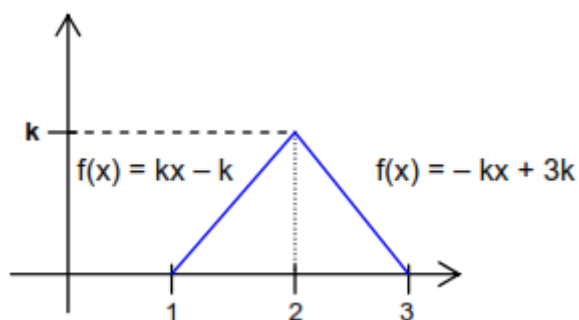
- a) Check that  $f(x)$  satisfies the probability density function conditions.
- b) Calculate  $P(X < 0)$ .

8. Let  $X$  be a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{k}{2} & , \quad a \leq x < b \\ 0 & , \quad \text{otherwise} \end{cases}$$

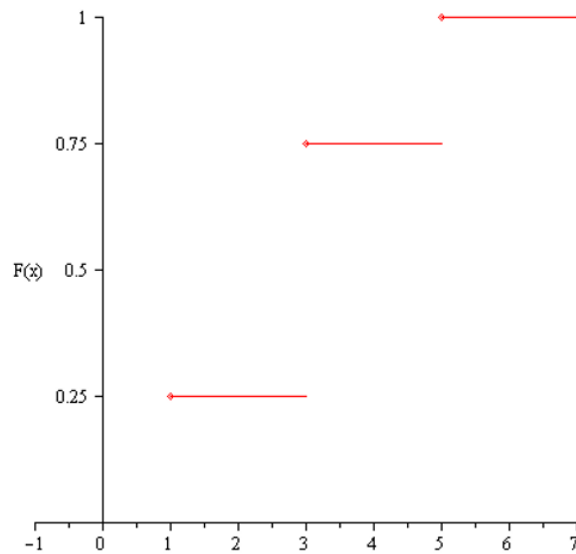
- a) Draw the graph of  $f(x)$  and find the value of  $k$ .
- b) Find  $E(X)$  as a function of the constants  $a$  and  $b$ .
- c) Let  $a=0$  and  $b=2$ ; calculate  $P(X > 3/4)$ .

9. The daily demand for blood of type O– (in litres) in a hospital is a random variable with probability density function given by the following graph:



- a) Show that  $k=1$ .
- b) How many litres of O– blood are expected to be demanded per day?
- c) What is the probability of being demanded more than 2 litres of O– blood per day?

**10.** Below you are given the graph of the distribution function for a random variable  $X$ . Please use it to answer the following questions.



- a) Find  $P(X \leq 2)$ .
- b) Find  $P(X=3)$
- c) Draw the graph of the probability mass function for this random variable. Please clearly label your axes.

## SOLUTIONS

### Group I) Multiple choice

- |       |       |       |
|-------|-------|-------|
| 1. b  | 11. b | 21. b |
| 2. d  | 12. b | 22. b |
| 3. b  | 13. d | 23. b |
| 4. b  | 14. a | 24. b |
| 5. a  | 15. b | 25. a |
| 6. c  | 16. a | 26. b |
| 7. a  | 17. d | 27. d |
| 8. d  | 18. b | 28. c |
| 9. b  | 19. d | 29. d |
| 10. b | 20. b | 30. a |

### Group II) Problems

Question 2

$$b) X \begin{cases} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1/6 & 0 & 1/12 & 0 & 1/4 & 0 & 1/2 \end{cases}$$

- d)
- i)  $1/6$
  - ii)  $1/3$
  - iii)  $1/12$
  - iv)  $1/2$

Question 3

- b) 0
- c) 3
- d) 5.2

Question 4

- a)  $a = b = 0.1$

$$b) F(x) = \begin{cases} 0 & , \quad x < 0 \\ 0.7 & , \quad 0 \leq x < 1 \\ 0.8 & , \quad 1 \leq x < 2 \\ 0.9 & , \quad 2 \leq x < 3 \\ 0.95 & , \quad 3 \leq x < 4 \\ 1 & , \quad x \geq 4 \end{cases}$$

Question 5

- a) Uniform
- b) 1.5 hours
- c) 0.5
- d) 0.25

Question 6

- c) 1.5
- d)
- i) 0.3333
- ii) 0.3333
- iii) 0.3333

Question 7b) 0.75

Question 8

- a)  $k = 2/(b - a)$
- b)  $E(X) = (a+b)/2$
- c)  $5/8$

Question 9

- b)  $E(X) = 2$
- c) 0.5

Question 10

- a) 0.25
- b) 0.50