Master Degree in Artificial Intelligence for Science and Technology

Anomaly Detection: Proximity Based Approaches

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OUTLOOK

- Proximity Based Approaches
  - distance based
  - density based
- Local Outlier Factor
- Connectivity Outlier Factor

PROXIMITY BASED APPROACHES

- KEY ASSUMPTION: normal data objects have close neighbors while anomalies are located far from other data objects

- GENERAL TWO-STEP APPROACH
  - compute neighborhood for each data object
  - analyze the neighborhood to determine whether a data object is anomaly or not

- CATEGORIES:
  - distance based methods
    - anomalies are data object most distant from other data objects
  - density based methods
    - anomalies are data objects in low density regions

PROXIMITY BASED APPROACHES

- **ADVANTAGE**
  - — can be used in unsupervised or semi-supervised setting (do not make any assumptions about data distribution)

- **DRAWBACKS**
  - — if normal data object do not have sufficient number of neighbors the techniques may fail
  - — computationally expensive
  - — in high dimensional spaces, data is sparse and the concept of similarity may not be meaningful anymore

    due to the sparseness, distances between any two data objects may become quite similar $\Rightarrow$ each data object may be considered as potential outlier!

PROXIMITY BASED APPROACHES

- **DISTANCE BASED APPROACHES**
  - a point O^* in a dataset is a distance based $DB(p, d)$ outlier/anomaly if at least a fraction p of the data objects in the data set lies greater than distance d from the data object O^*

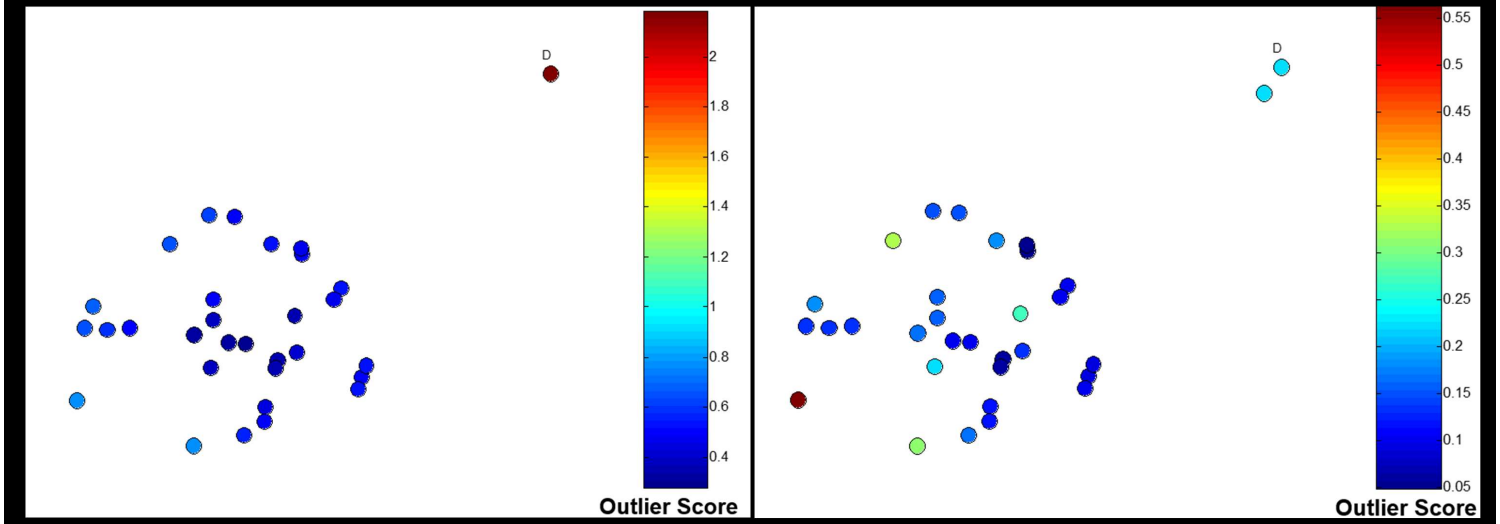
- **DENSITY BASED APPROACHES**
  - compute local densities of particular regions and declare instances in low density regions as potential outlier/anomaly
  - approaches
    - Local Outlier Factor (LOF)
    - Connectivity Outlier Factor (COF)
    - Multi-Granularity Deviation Factor (MDEF)

PROXIMITY BASED APPROACHES: DISTANCE BASED OUTLIER DETECTION

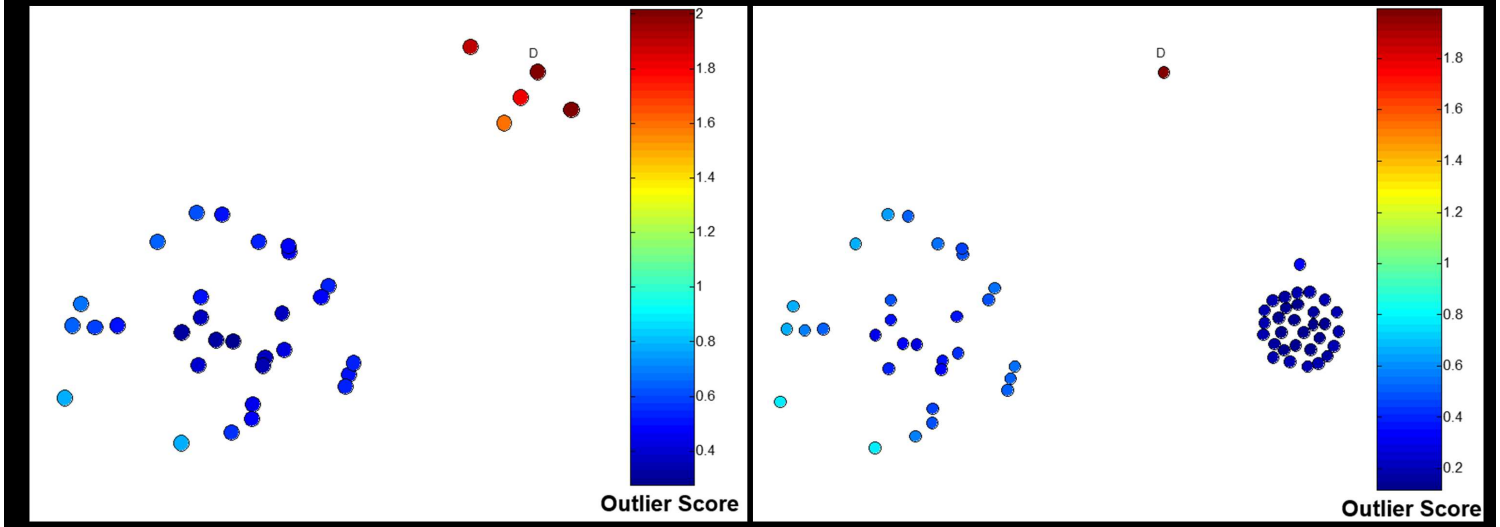
- **NEAREST NEIGHBOR (NN) APPROACH**[*,**]
  - for each data object O compute the distance to the k^{th} nearest neighbor d_k
  - sort all data object s according to the distance d_k
  - outliers are data objects that have the largest distance d_k and therefore are located in the more sparse neighborhoods
  - usually data objects that have top $n\%$ distance d_k are identified as outliers
    - n is a user parameter
  - not suitable for datasets that have modes with varying density

* Knorr, Ng, Algorithms for Mining Distance-Based Outliers in Large Datasets, VLDB98
** S. Ramaswamy, R. Rastogi, S. Kyuseok: Efficient Algorithms for Mining Outliers from Large Data Sets, ACM SIGMOD Conf. On Management of Data, 2000.

PROXIMITY BASED APPROACHES: DISTANCE BASED OUTLIER DETECTION



PROXIMITY BASED APPROACHES: DISTANCE BASED OUTLIER DETECTION

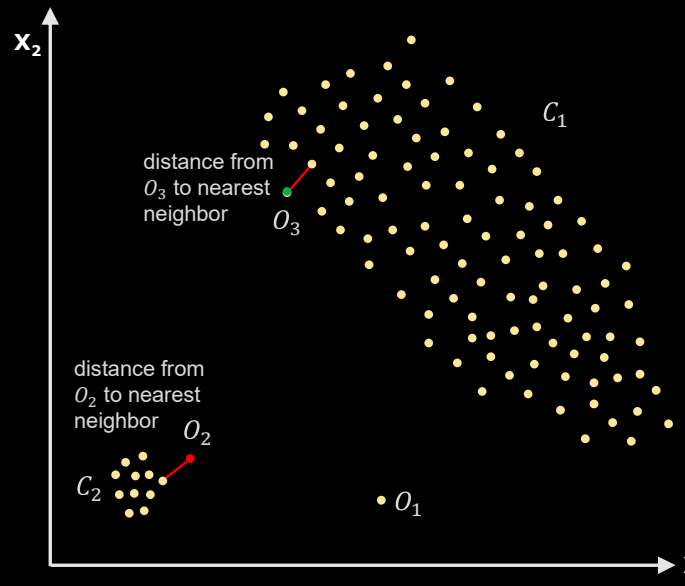


PROXIMITY BASED APPROACHES: DISTANCE BASED OUTLIER DETECTION

- **NEAREST NEIGHBOR (NN) APPROACH: STRENGTHS/WEAKNESSES**
  - simple
  - expensive – $O(n^2)$
  - sensitive to parameters (k and $n\%$)
  - sensitive to variations in density
  - distance becomes less meaningful in high-dimensional space

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- **LOCAL OUTLIER FACTOR (LOF) APPROACH**
  - — Example
    - in the distance based approach, O_2 is not considered as outlier, while the LOF approach finds both O_1 and O_2 as outliers
    - distance based approach may consider O_3 as outlier, but LOF approach does not



PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- **LOCAL OUTLIER FACTOR (LOF) APPROACH**
  - for each data object p compute the distance to the k[th] nearest neighbor d_k
  - compute reachability distance for each data object q with respect to data object p as

$$reach - dist(p,q) = max\{d_k, d(p,q)\}$$

  - compute local reachability density $lrd(q)$ of data object q as inverse of the average reachability distance based on the $MinPts$ nearest neighbors of data object q

$$lrd(q) = \frac{MinPts}{\sum_p reach - dist_{MinPts}(p,q)}$$

  - $LOF(q)$ is the ratio of average local reachability density of q's k-nearest neighbors and local reachability density of the data object q

$$LOF(q) = \frac{1}{MinPts}\sum_p \frac{lrd(p)}{lrd(q)}$$

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- **LOCAL OUTLIER FACTOR (LOF) APPROACH: STRENGTHS/WEAKNESSES**
  - simple
  - expensive – $O(n^2)$
  - sensitive to parameters k and $MinPts$
  - density becomes less meaningful in high-dimensional space

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

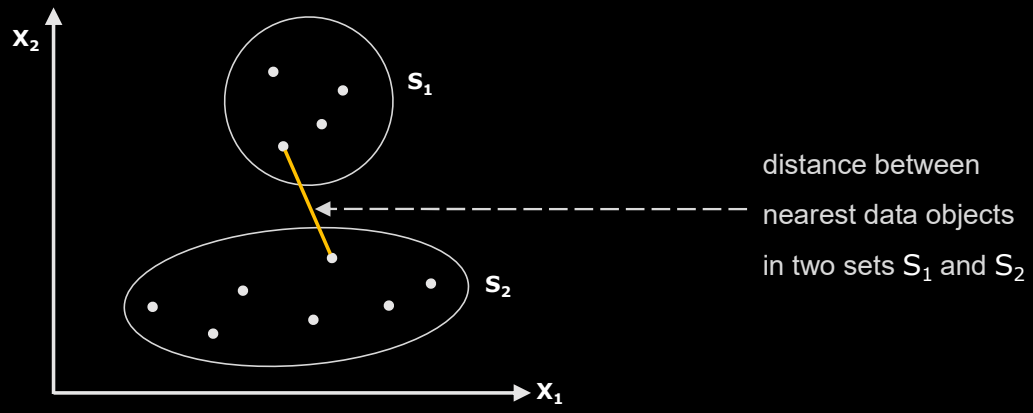
- **CONNECTIVITY OUTLIER FACTOR (COF)**
  - outliers are data objects p where average chaining distance $ac - dist_{kNN(p)}(p)$ is larger than the average chaining distance of their k-nearest neighborhood $kNN(p)$

$$ac - dist_{kNN(p)}(p) = \sum_{i=1}^{r} \frac{2(r-i)}{r(r-1)} dist(e_i)$$

  - COF identifies outliers as data objects whose neighborhoods is sparser than the neighborhoods of their neighbors

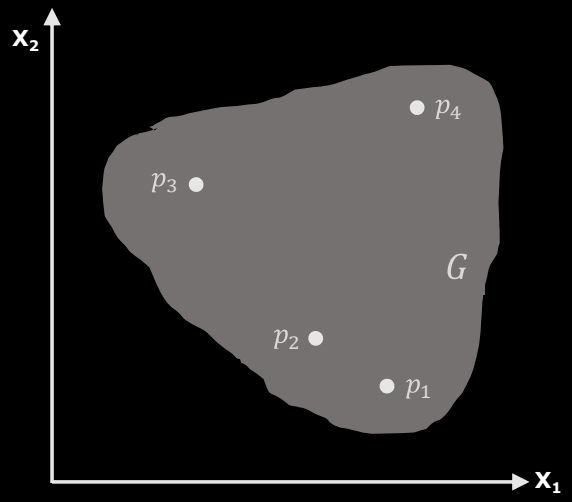
PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- **CONNECTIVITY OUTLIER FACTOR (COF): DISTANCE BETWEEN TWO SETS**
  - distance between nearest data objects in two sets



PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- CONNECTIVITY OUTLIER FACTOR (COF): SET BASED PATH



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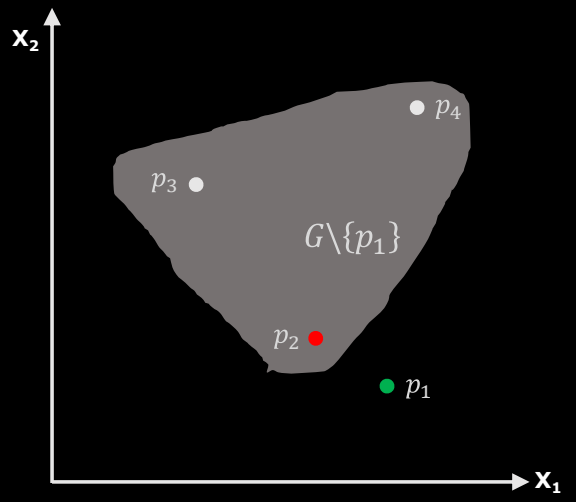
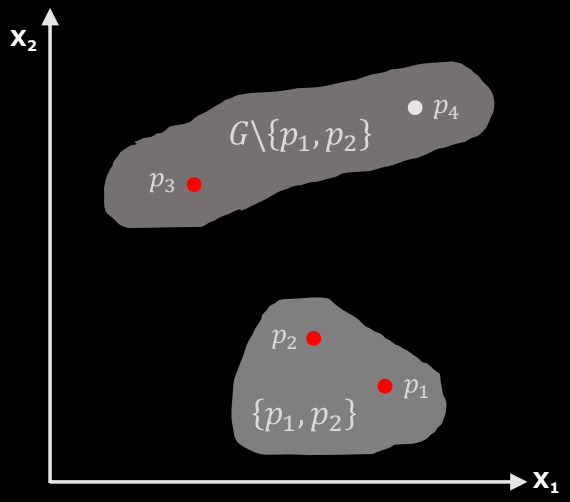


- p_2 is nearest neighbor of set $\{p_1\}$ in $G\backslash\{p_1\}$

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

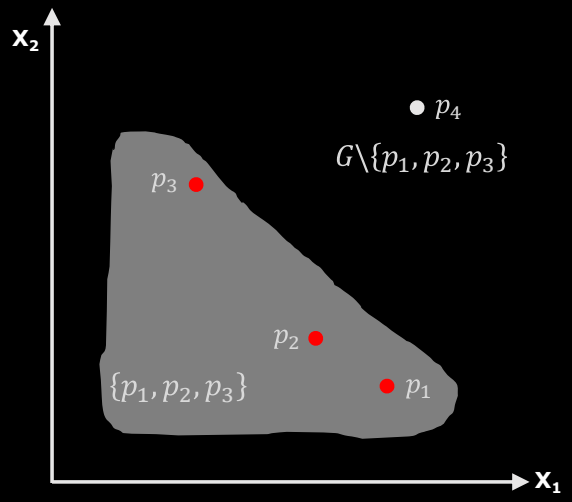
- CONNECTIVITY OUTLIER FACTOR (COF): SET BASED PATH



- p_2 is nearest neighbor of set $\{p_1\}$ in $G\backslash\{p_1\}$
- p_3 is nearest neighbor of set $\{p_1, p_2\}$ in $G\backslash\{p_1, p_2\}$

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

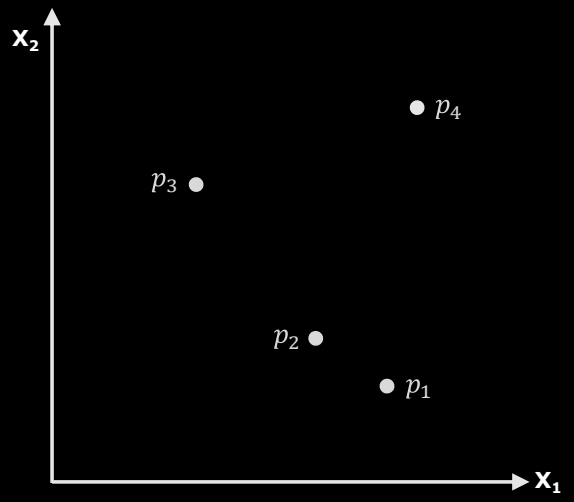
- CONNECTIVITY OUTLIER FACTOR (COF): SET BASED PATH



- p_2 is nearest neighbor of set $\{p_1\}$ in $G\backslash\{p_1\}$
- p_3 is nearest neighbor of set $\{p_1, p_2\}$ in $G\backslash\{p_1, p_2\}$
- p_4 is nearest neighbor of set $\{p_1, p_2, p_3\}$ in $G\backslash\{p_1, p_2, p_3\}$

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

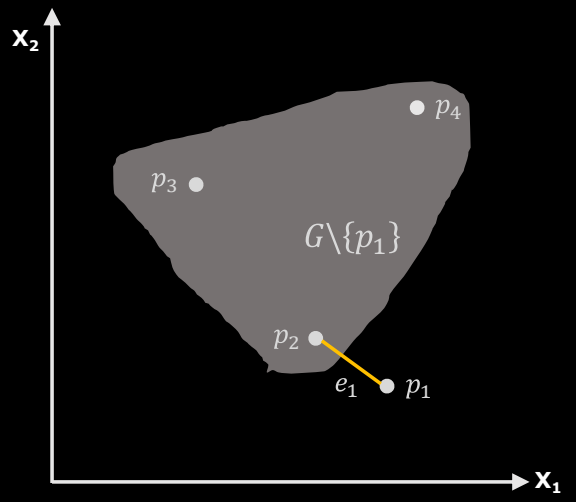
- CONNECTIVITY OUTLIER FACTOR (COF): SET BASED PATH



sequence $\{p_1, p_2, p_3, p_4\}$ is called
Set Based Nearest Path (SBN)
from p_1 on G

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- CONNECTIVITY OUTLIER FACTOR (COF): COST DESCRIPTION



PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

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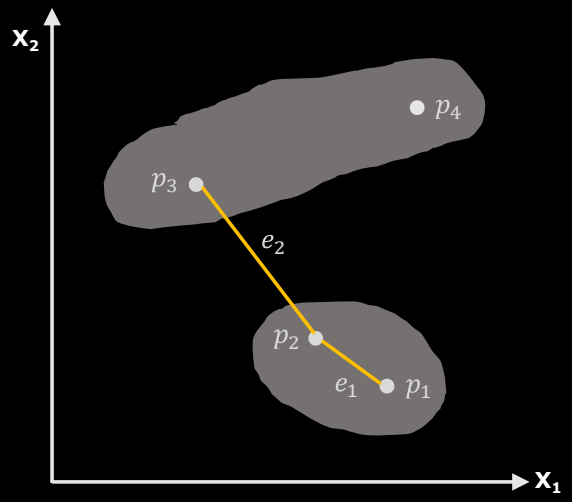

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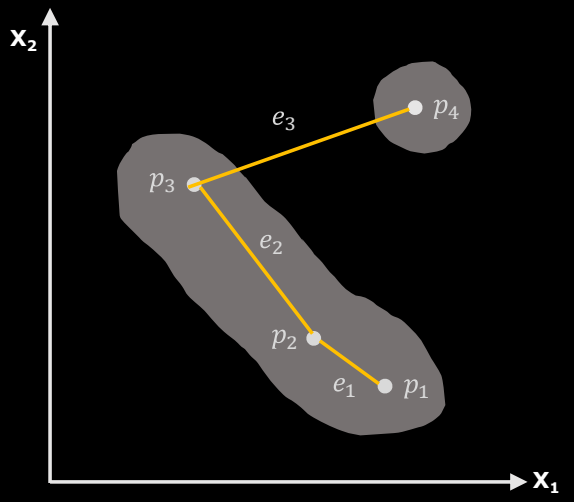
- CONNECTIVITY OUTLIER FACTOR (COF): COST DESCRIPTION



— distances $dist(e_i)$ between two sets $\{p_1, \dots, p_i\}$ and $G\backslash\{p_1, \dots, p_i\}$ for each i are called COST DESCRIPTIONS

— edges e_i for each i are called SBN trail

— SBN trail may not be a connected graph!

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- **CONNECTIVITY OUTLIER FACTOR (COF)**
  - outliers are data objects p where average chaining distance $ac - dist_{kNN(p)}(p)$ is larger than the average chaining distance of their k-nearest neighborhood $kNN(p)$

$$ac - dist_{kNN(p)}(p) = \sum_{i=1}^{r} \frac{2(r-i)}{r(r-1)} dist(e_i)$$

  - we average cost descriptions!
  - we would like to give more weights to data objects closer to the data object p_1
  - the smaller $ac - dist$, the more compact is the neighborhood G of p

PROXIMITY BASED APPROACHES: DENSITY BASED OUTLIER DETECTION

- **CONNECTIVITY OUTLIER FACTOR (COF)**
  - COF is computed as the ratio of the $ac - dist$ (average chaining distance) at the data object and the mean $ac - dist$ at the data object's neighborhood
  - similar idea as LOF approach:
    - a data object is an outlier if its neighborhood is less compact than the neighborhood of its neighbors

$$COF_k(p) = \frac{ac - dist_{kNN(p) \cup p}(p)}{\frac{1}{k}\sum_{o \in kNN(p)} ac - dist_{kNN(o) \cup o}(o)}$$

RECAP

- Proximity Based Approaches
  - distance based
  - density based
- Local Outlier Factor
- Connectivity Outlier Factor