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Problem 3: Runtime Analysis

a) 0 1 2 3
2 4 16 256
2^1 2^2 2^4 2^8
2^{2^0} 2^{2^1} 2^{2^2} 2^{2^3}

$2^{2^k} \leq \Theta(n)$

$2^k \leq \log(\Theta(n)) = k \leq \log(\log(\Theta(n)))$

$\boxed{T(n) = \log(\log(\Theta(n)))}$

b) if

$$\sum_{i=1}^{n} \left(\Theta(1) + \Theta\left(\sum_{k=0}^{i^3-1} \Theta(1) \right) \right)$$

if statement: $n \% \sqrt{n} = \sqrt{n}$

$$= \sum_{i=1}^{n} \Theta(1) + \sum_{i=1}^{\sqrt{n}} \Theta(i^3)$$

$$= \Theta(n) + \Theta(n^{\frac{7}{2}}) = \boxed{\Theta(n^{\frac{7}{2}})}$$

c) if; worst case if $= \Theta(\log(n)+1)$

$$\sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\Theta(1) + \Theta\left(\sum_{m=1}^{n} \Theta(1) \right) \right) \right)$$

k 0 1 2 3 4
m 0 1 2 4 8 16
2^0 2^1 2^2 2^3 2^4
$\Rightarrow 2^{k-1}$

$2^{k-1} \leq n$ $\quad k \leq \log(n)+1$

$$\sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\Theta(1) + \Theta(\log(n)+1) \right) \right)$$

$\Theta(1)$

$$= \sum_{i=1}^{n} \left(\Theta(n) + \Theta(\log(n)) \right)$$

$T(n) = \Theta(n^2) + \Theta(n \log(n)) = \boxed{\Theta(n^2)}$

d)

initializing vars

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$$\Theta(1)+\Theta(1)+\sum_{i=0}^{n}\left(\Theta\left(\Theta(1)+\Theta(1)+\sum_{j=0}^{k}\left(\Theta(1)+\Theta(1)+\Theta(1)+\Theta(1)+\Theta(1)\right)\right)\right)$$

j	0	1	2	3	4
k	10	15	$\frac{45}{2}$	34	

$$\Rightarrow j = 10\cdot\left(\frac{3}{2}\right)^k$$

$$k\log\left(\frac{3}{2}\right) = \log\left(\frac{j}{10}\right)$$

$$k = \frac{\log\left(\frac{j}{10}\right)}{\log\left(\frac{3}{2}\right)} = \log_{\frac{3}{2}}\left(\frac{j}{10}\right)$$

$$= \Theta(1)+\sum_{i=0}^{n}\left(\Theta(1)+\sum_{j=0}^{\log_{\frac{3}{2}}\left(\frac{j}{10}\right)}\Theta(1)\right)$$

$$= \Theta(1)+\sum_{i=0}^{n}\left(\Theta(1)+\Theta\left(\log_{\frac{3}{2}}\left(\frac{j}{10}\right)\right)\right)$$

$$= \Theta(n)+\Theta\left(n\log_{\frac{3}{2}}\left(\frac{j}{10}\right)\right) \leftarrow < \Theta(n)$$

$$= \boxed{\Theta(n)}$$