GLOBAL BIOGEOCHEMICAL CYCLES, VOL. 2, NO. 2, PAGES 177-185, JUNE 1988

CARBON CYCLE INSTABILITY AS A CAUSE OF THE LATE PLEISTOCENE ICE AGE OSCILLATIONS: MODELING THE ASYMMETRIC RESPONSE

Barry Saltzman and Kirk A. Maasch

Department of Geology and Geophysics
Yale University, New Haven, Connecticut

Abstract. A dynamical model of the Pleistocene ice ages, incorporating many of the qualitative ideas advanced recently regarding the possible role of ocean circulation, chemistry, temperature, and productivity in regulating long-term atmospheric carbon dioxide variations, has been constructed. This model involves one additional term (and free parameter) beyond that included in a previous model (B. Saltzman and A. Sutera, 1987), providing the capacity for an asymmetic (for example, "saw-toothed") response. It is shown that many of the main features exhibited by the δ^{18}O–derived ice record and the Vostok core/δ^{13}C–derived carbon dioxide record in the late Pleistocene can be deduced as a free oscillatory solution of the model, including a rapid deglaciation during which a spike of high CO_2 and a rapid surge in North Atlantic deep water production occurs. It is expected that the addition of reasonable levels of external (for example, Earth orbital) forcing will enable the model to account for a significant amount of the remaining observed variance and covariance of the slow response climatic variables over the full Pleistocene including the mid-Pleistocene transition.

Paper number 8J0374.
0886-6236/88/008J-0374$10.00

1. INTRODUCTION

In previous papers we described a dynamical model aimed at accounting for the main features of the observed Pleistocene record of climatic change, primarily as a free oscillatory response [Saltzman, 1987; Saltzman and Sutera (herinafter SS), 1987]. This model is based on a set of qualitative, physically plausible, postulates regarding the behavior of three slow response "prognostic" variables believed to be of prime relevance: (1) global ice mass, (2) atmospheric carbon dioxide, and (3) a measure of the strength of the oceanic "CO_2 pump" believed to be related to the magnitude of North Atlantic deep water (NADW) production. Mean surface temperature, and permanent sea ice extent are additional "diagnostic" variables determined by the above prognostic variables. In the present paper we seek (1) to restate and clarify the nature of these postulates as embodied in SS [1987], particularly with regard to CO_2, (2) to expand upon this previous minimal model by adding one term allowing for effects that may give rise to the rapid deglaciation and accompanying rapid CO_2 rise, and (3) to compare the predictions regarding CO_2

variations with the recent Vostok ice core measurements [Barnola et al., 1987] and recent $\delta^{13}C$ measurements [Curry and Crowley, 1987].

2. QUALITATIVE MODELING OF THE PLEISTOCENE CO_2 VARIATIONS

It is now widely believed on the basis of biogeochemical arguments and box-model-type flux calculations that the control of the natural atmospheric CO_2 variations resides in the state of the upper layer of the world ocean as influenced by the circulation and mass exchanges with the deeper levels [e.g., Broecker, 1982a,b; Oeschger et al., 1984; Ennever and McElroy, 1985; Toggweiler and Sarmiento, 1985; Wenk and Siegenthaler, 1985; Berger, 1982a,b]. Here, we shall try to distill the main hypothesized feedbacks implicit in the above and other studies in an attempt to construct a qualitatively plausible equation for the slow changes in atmospheric CO_2 now being revealed by $\delta^{13}C$ and ice core measurements. Such a construction can only be a preliminary one in view of the hypothetical and incomplete nature of our knowledge of the processes involved. Nonetheless, it is desirable to begin the process of learning the nature and magnitudes of the feedbacks necessary to account for the observed CO_2 changes and the associated changes of the other climatic variables (for example, ice mass and extent, and surface temperature) as revealed by the proxy indicators. As an aside, we note that, in light of general circulation model (GCM) experiments [Manabe and Bryan, 1985], even in the absence of a $\delta^{18}O$ ice record the $\delta^{13}C$ and ice core records of CO_2 would suffice to serve as indicators of major climatic (ice extent) variations in the past.

At the most basic level, neglecting inputs due to volcanism, anthropogenic sources, and the apparently minor exchange with the continental biosphere, carbon dioxide will increase in the atmosphere if the flux of CO_2 from oceanic sources (particularly from warm lower latitude waters) exceeds the oceanic uptake (primarily in cold higher latitude waters). From all indications this balance is a "delicate" one, highly vulnerable to changes in ocean circulation and mixing and associated changes in the chemical-biological-thermal state of the upper layer waters with which the atmosphere tends to (but probably never does) equilibrate.

From the above and other studies we are led to assume that the main controls on this delicate balance are the following (not necessarily in order of importance).

I. Sea surface temperature— Although there should be enhanced solubility and uptake of CO_2 in colder water, this effect alone appears to be relatively small, particulary if cold temperatures are accompanied by high salinity [Broecker, 1982b].

II. Permanent ice coverage as measured by summer sea ice extent— The following arguments have been proposed:

a) Large ice extent should enhance the efficiency of high-latitude bioproductivity by moving the main phytoplankton bloom zones to lower latitudes where sunlight is stronger– the photic effect [Knox and McElroy, 1984]. This effect might be small, however, because the main radiative differences occur in winter rather than in the high-productivity summer months [e.g., Smithsonian Institution, 1951].

b) Enhanced surface seasonal meltwater volume from extensive, thick, sea ice fields provides a more stable cap on high-latitude waters. This may affect productivity and CO_2 downdraw, but more importantly, would tend to inhibit deep convective mixing in high latitudes that can cause high CO_2 levels in polar waters [Worthington, 1968]. Such enhanced stability presently prevails in the Okhotsk Sea and appears to have prevailed in glacial Antarctic waters [Morley and Hays, 1983; Toggweiler and Sarmiento, 1985].

c) The increased atmospheric and oceanic baroclinicity engendered by a large equatorward ice extent should: (1) drive large horizontal surface water exchange between low and high latitudes, an effect that tends to decrease atmospheric CO_2 [e.g., Ennever and McElroy, 1985; Wenk and Siegenthaler, 1985], (2) increase storminess and precipitation over higher-latitude waters tending to decrease surface salinity, thereby augmenting the stability effect of meltwater described in IIb above, and (3) enhance shallow mechanical mixing in high-latitude surface waters leading to an increase in the mixing coefficient for the CO_2 flux from the atmosphere to the ocean [Liss and Merlivat, 1986].

III. The strength of the main thermohaline circulation, particularly the production of North Atlantic deep water (NADW)— Several major effects are probable:

a) The first-order effects of a large mass and flow of NADW in the world ocean would appear to be a lowering of atmospheric CO_2 as a result of (1) the intensification of the pycnocline by filling the deeper levels of the ocean with dense saline water, thereby stablizing the ocean with respect to in situ convective overturnings that would otherwise tend to bring up carbon-rich water and increase atmospheric CO_2 and (2) the accompanying strong downwelling of CO_2 in the NADW production zone, with horizontal replacement from lower latitudes by carbon-poor waters, i.e., an increase in the "solubility pump" [Volk and Hoffert, 1985]. In addition, the delivery of nutrients throughout the world ocean by the NADW "conveyor" might under certain conditions illustrated in box models lead to increased global productivity and downdraw of CO_2. These effects, as well as the effects of in situ deep convection mentioned in IIb and above, are discussed by Oeschger et al. [1984], Ennever and McElroy [1985], Toggweiler and Sarmiento [1985], and Wenk and Siegenthaler [1985].

b) As a secondary effect of a strong intrusion of a relatively warmer, though saltier, NADW under cold Antarctic surface waters, however, is the possibility for locally enhanced instability of these waters to in situ deep convection tending to diminish the effect of the above NADW-type "solubility pump" in these waters. That is, small increases in NADW from an equilibrium state should tend to lower CO_2 in accordance with IIIa, but further increases in NADW might start increasing surface-deep vertical mixing in the Antarctic waters thus tending to increase CO_2 in accordance with IIb. This hypothesis regarding the effect of NADW-type overturning on the Antarctic Circumpolar-type vertical mixing is discussed qualitatively by Toggweiler and Sarmiento [1985, p. 178] but remains to be tested by further observational and modeling studies; here we adopt this as one hypothesis that can account for the asymmetry of the climatic response including the rapid changes of CO_2, NADW, and ice mass during deglaciation.

From another, more general, viewpoint this same asymmetric effect would result from the assumption that a negative anomaly of NADW implies greater static instability of the world ocean, and a positive anomaly implies greater static stability. Because fluxes due to deep convective overturnings (for example, of carbon) are a highly nonlinear function of the density stratification, bifurcating to a relatively high value when a critical point of instability is approached or exceeded, we should expect a much larger flux increase for a negative anomaly of NADW than a flux decrease for a positive anomaly of the same magnitude.

IV. Sea level change associated with global ice mass changes— Three hypotheses have been advanced:

a) A rising sea level is accompanied by coral reef growth ($CaCO_3$ deposition) releasing excess CO_2 to the surface waters and hence the atmosphere according to the reaction,

$$Ca^{++} + 2HCO_3^- \rightleftharpoons CaCO_3 + CO_2 + H_2O$$

with the reverse reaction operative during falling sea level [Berger, 1982a,b; Keir and Berger, 1983].

b) During falling sea level nutrients (for example, phosphate) stored in organic detritus deposited on the continental shelves during sea level rise are eroded back into the oceans where they enhance productivity and carbon burial in the deeper ocean [Broecker, 1982a].

c) Exposure of new land masses in tropical areas as sea level falls enhances the global vegetative sink for CO_2, possibly a small effect.

V. Negative feedbacks— In all natural systems strong departures from equilibrium are opposed by dissipative effects. The likely enhanced growth of continental vegetation and enhanced dissolution in surface water under high atmospheric CO_2 partial pressure are examples of such negative feedbacks. We may further speculate that when the entire climatic system is far from its equilibrium state this tendency for damping of CO_2 extremes is maximized. For example, if the stabilizing effect of a large volume of NADW is very strong, large horizontal gradients of the pCO_2 of surface water might be expected; i.e., we might expect anomalously low values of pCO_2 where the reduced convective overturning occurs in nutrient-rich waters, and anomalously high values where the reduced overturning occurs in nutrient-limited waters. Thus, in such an enhanced NADW-type circulation state, if the atmospheric CO_2 concentration were anomalously low an unusually large partial pressure difference would exist between the nutrient-limited areas and the atmosphere tending to raise the atmospheric value, whereas if the atmospheric concentration were high a reverse gradient over the nutrient-rich (low surface pCO_2) regions would exist tending to reduce atmospheric CO_2.

We shall next examine the implications of the set I-V of qualitatively plausible feedback controls on atmospheric CO_2, keeping in mind that this set is certainly not exhaustive or definitive. In particular, we aim to express these qualitative or "conceptual" arguments in more formal mathematical manner based on the following definitions: $(\ )'$ is departure of $(\ )$ from an equilibrium state; τ is global temperature of the water surface; η is global mean extent of permanent sea ice; I is global ice mass; μ is atmospheric CO_2 concentration; and N is the amount and extent of NADW, assumed to be a measure of the global thermohaline circulation and the mean static stability of the world ocean (i.e., the Brunt-Vaisala frequency). Departures of this quantity from the equilibrium state are opposite in sign from those of the quantity θ discussed by Saltzman [1987] and SS [1987] (i.e., $N' \sim -\theta'$).

Thus, with reference to the above mechanisms, I through V, the rate of change of carbon dioxide ($d\mu/dt \equiv \dot{\mu}$) can be formulated in the following simple manner:

$$\dot{\mu} = \underbrace{r_1\tau'}_{\text{(I)}} - \underbrace{r_2\eta'}_{\text{(IIa,b,c)}} - \underbrace{(r_3 - b_3N')N'}_{\text{(IIIa,b)}} - \underbrace{r_5\dot{I}'}_{\text{(IVa,b,c)}} - \underbrace{(r_4 + b_4N'^2)\mu'}_{\text{(V)}} + \mathcal{F}_\mu \qquad (1)$$

where r_1, r_2, r_3, r_5, b_3 and b_4 are assumed to be positive rate constants and $\mathcal{F}_\mu$ denotes external forcing due to direct inputs of CO_2 (for example, volcanic effects unbalanced by weathering).

Note that if we were to consider that μ represents not only the concentration of atmospheric CO_2, but, rather, the combined concentration of two of the main greenhouse gases, CO_2 and methane, the contributions from the processes described under (I) and (IV) in equation (1) would probably be augmented. This is because of the dependence of the methane flux on temperature and on organic processes in "wet tundra" [Guthrie, 1986; Mooney et al., 1987].

3. THE COMPLETE SYSTEM

From similarly qualitative arguments [Saltzman, 1987] we can express the rate of change of global ice mass in the form

$$\dot{I}' = -s_1\tau' - s_2\mu' + s_3\eta' - s_4I' + \mathcal{F}_I \qquad (2)$$

and the rate of change of North Atlantic deep water (or the thermohaline circulation) in the form

$$\dot{N}' = -c_0I' - c_2N' + \mathcal{F}_N \qquad (3)$$

where s_1, s_2, s_3, s_4, c_0 and c_2 are assumed to be positive rate constants and $\mathcal{F}_I$ and $\mathcal{F}_N$ denote external forcing. In writing (3) we are giving formal expression to the often-stated suggestion that glacial ice exercises a dominant control of NADW production. According to this scenario, during extensive ice coverage the major North Atlantic downwelling zones are frozen over leading to a "cutoff" or reduction in production. Thus a weak thermohaline circulation resembling more closely that of the present Pacific Ocean tends to prevail in times of maximum glaciation [e.g., Boyle and Keigwin 1987]. The damping rate constant c_2 represents all the effects of eddy viscous and diffusive processes that tend to reduce the presence of NADW.

We assume, further, that as a consequence of a series of diagnostic "sensitivity" experiments with a GCM we can express the fast response variables τ and η in terms of prescribed slow response variables I, μ, and N. These expressions should have the first-order forms

$$\tau' = -\alpha I' + \beta\mu' + \mathcal{F}_\tau \quad (4)$$

$$\eta' = e_I I' - e_\mu \mu' + \mathcal{F}_\eta \quad (5)$$

where α, β, e_I, and e_μ are the equilibrium sensitivity coefficients assumed to be positive (for example, $\beta = \partial\tau/\partial\mu = O(10^{-2}\mathrm{K(ppm)}^{-1})$, Broccoli and Manabe [1987]) and $\mathcal{F}_\tau$ and $\mathcal{F}_\eta$ denote the effects of external (for example, Earth orbital) forcing. The possible dependence of τ and η on N is uncertain; here we have simply neglected any such dependence (though it may be plausible in a future study to assume that NADW production represents a mechanism for transfering salt to deeper ocean levels permitting colder temperatures to prevail stably in surface waters, as appears to be the case in high-latitude Southern Hemisphere waters [e.g., Deacon, 1984]).

Substituting (4) and (5) into (1), (2) and (3), we obtain the following dynamical system governing the slow response variables:

$$\dot{I}' = -a_0 I' - a_1 \mu' + \mathrm{F}_I \quad (6)$$

$$\dot{\mu}' = -b_0 I' + b_1 \mu' - (r_3 - b_3 N')N' - b_4 N'^2 \mu' + \mathrm{F}_\mu \quad (7)$$

$$\dot{N}' = -c_0 I' - c_2 N' + \mathrm{F}_N \quad (8)$$

where $a_0 = (s_4 - s_1\alpha - s_3 e_I)$, $a_1 = (s_1\beta + s_3 e_\mu + s_2)$, $b_0 = (r_1\alpha + r_2 e_I - r_5 a_0)$, $b_1 = (r_1\beta + r_2 e_\mu + r_5 a_1 - r_4)$, $\mathrm{F}_I = (\mathcal{F}_I - s_1\mathcal{F}_\tau + s_3\mathcal{F}_\eta)$, $\mathrm{F}_\mu = (\mathcal{F}_\mu + r_1\mathcal{F}_\tau - r_2\mathcal{F}_\eta - r_5\mathcal{F}_I)$, and $\mathrm{F}_N = \mathcal{F}_N$.

We have very little a priori knowledge regarding the appropriate values of the rate constants in these equations. We assume here that they are all positive and that the time constants, a_0^{-1} and c_2^{-1}, are no greater than ~ 10 kyr. In particular, by setting $b_1 > 0$ we are assuming that all the positive feedback processes involved in the rate constants r_1, r_2, and r_5 dominate over the negative feedback measured by r_4, so that a basic instability is introduced in the system that can drive the free oscillatory behavior. Further, to reduce the system to a simpler form that is still capable of oscillating, we shall set $b_0 = 0$. Then the system becomes almost identical to the one discussed in SS [1987], the only difference being the addition of a single term in (7), $b_3 N'^2$. This new term introduces a basic asymmetry in the response, permitting a more rapid CO_2 buildup than decline. (Note that the coefficients "r_3" and "b_4" in (7) were called "b_5" and "b_6", respectively, in SS [1987]).

As an example, we shall show that ignoring any forcing ($\mathrm{F}_I = \mathrm{F}_\mu = \mathrm{F}_N = 0$) we can assign a set of values to the rate constants in (6) - (8) (including reasonable values of the time constants a_0 and c_2 enumerated above) that give a credible account of the main ~ 100 kyr variations of I and μ as participants in a "natural oscillator" involving a third slow variable, N. Although we shall not deal with this here, the further inclusion of Earth orbital (Milankovitch) forcing would be expected to enhance the capability of the model to account for the ~ 20 and ~ 40 kyr variance as well as to supply the missing time phase information, i.e., the "pacemaker" [Hays et al., 1976] properties of the forcing [Saltzman et al., 1984; Saltzman, 1987].

4. THE SOLUTION

As in SS [1987] we rescale equations (6) - (8) with the following tranformations: $t = [a_0^{-1}]t^*$, $I' = [c_2 c_0^{-1}(a_0/b_4)^{1/2}]X$, $\mu' = [c_2(a_1 c_0)^{-1}(a_0^3/b_4)^{1/2}]Y$, and $N' = [(a_0/b_4)^{1/2}]Z$, whence, after setting $b_0 = 0$, our system becomes

$$\dot{X} = -X - Y \quad (9)$$

$$\dot{Y} = -pZ + rY + sZ^2 - Z^2 Y \quad (10)$$

$$\dot{Z} = -q(X + Z) \quad (11)$$

where $(\dot{\ }) = d(\ )/dt^*$, $p = a_1 c_0 b_2 / a_0^2 c_2$, $q = c_2/a_0$, $r = b_1/a_0$, and $s = a_1 b_3 c_0 (b_4/a_0)^{1/2}/a_0 b_4 c_2$ (the new asymmetry parameter). These four adjustable parameters are constrained to be positive, and in addition if we fix a_0^{-1} to be 10 kyr then we also require that $q > 1.00$.

In SS [1987] we showed that with the values $p = 0.1$, $q = 1.5$, $r = 1.5$ and $s = 0$, and a properly assigned initial condition at 2 m.y. B.P., we could obtain a solution having a transition at ~ 900 kyr B.P. to a dominant ~ 100-kyr-period oscillation as observed [e.g., Maasch, 1988]. Here, with the inclusion of a

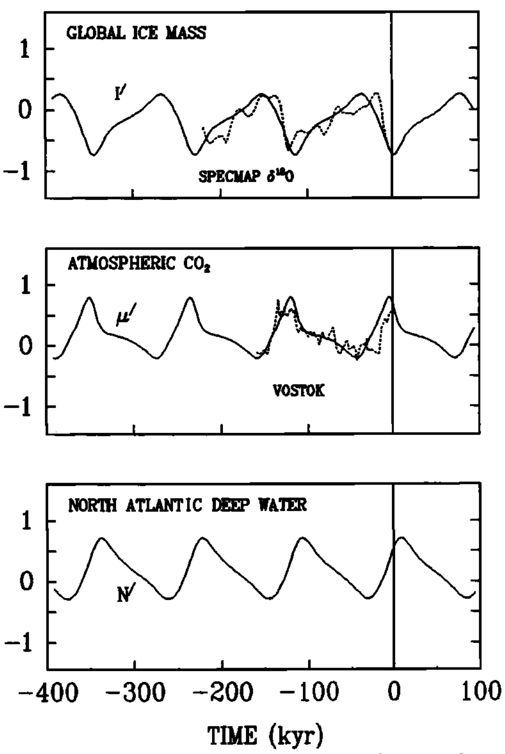

Fig. 1. Time-dependent periodic solution for departures from equilibrium, in nondimensional units scaled to a range of unity, for an arbitrary 500-kyr period. Upper panel: global ice mass (solid) compared with SPECMAP $\delta^{18}O$ curve (dashed) over past 200 kyr. Middle panel: atmospheric carbon dioxide (solid), compared with Vostok CO_2 curve (dashed) over past 160 kyr. Lower panel: North Atlantic deep water. The vertical line is assumed to correspond to conditions at the present interglacial time.

nonzero value of s, we fix on another region of parameter space to show that this model can exhibit the rapid deglaciations and CO_2 changes and the observed CO_2 - ice mass phase relation [Shackleton and Pisias, 1985; Curry and Crowley, 1987; Barnola et al., 1987]. In particular, if we take $p = 0.9$, $q = 1.2$, $r = 0.8$, and $s = 0.8$, we obtain the oscillatory solution of period ~ 115 kyr, shown in Figure 1. When combined with observational estimates of the paleoranges of I, μ, and N, these parameter values uniquely determine all of the coefficients in (6) - (8). Thus, for example, if we assume these paleoranges are $\hat{I} = 5 \times 10^{19}$ kg, $\hat{\mu} = 150$ ppm, and $\hat{N} = 3 \times 10^{17}$ m^3 (1/4 the volume of the ocean), and given that the ranges of the solution values of X, Y, and Z are $\hat{X} = 3.026$, $\hat{Y} = 3.969$, and $\hat{Z} = 2.505$, respectively, we find that $a_0 = 10^{-4}$ yr^{-1}, $a_1 = 4.37 \times 10^{13}$ kg (yr ppm)$^{-1}$, $b_1 = 8.00 \times 10^{-5}$ yr^{-1}, $b_2 = 2.84 \times 10^{-20}$ ppm (yr m^3)$^{-1}$, $b_3 = 2.11 \times 10^{-37}$ ppm (yr m^6)$^{-1}$, $b_4 = 6.97 \times 10^{-39}$ (yr m^6)$^{-1}$, $c_0 = 8.70 \times 10^{-7}$ m^3 (yr kg)$^{-1}$, and $c_2 = 1.20 \times 10^{-4}$ yr^{-1}.

Note that, in contrast to the symmetric solution obtained in SS [1987], we now obtain a more "saw-toothed" ice variation with a rapid deglaciation accompanied by both a spike of high atmospheric CO_2 and a resurgent growth of NADW. If we assume the vertical line in this figure can be identified with the present interglacial time, we find a reasonably good correspondence over the last 200 kyr between our model I' variations and the SPECMAP $\delta^{18}O$ variations [Imbrie et al., 1984], and between our μ' variations and the Vostok core CO_2 measurements over this same period. It can be seen, for example, that not only are the trends of these two variables in general agreement with the observations but their relative phases as well. As noted above, there is good reason to expect that when Milankovitch forcing is added a good deal of the ~ 20- and ~ 40-kyr-period variance in the observations will be accounted for also.

According to our model, the slow response climatic system tends to avoid that state in which the global ice mass, atmospheric CO_2, and NADW simultaneously have magnitudes near their mean values averaged over the entire late Pleistocene, this state corresponding to the unstable fixed point at $(X, Y, Z) = (0,0,0)$. Thus, there should be a relatively low probability of observing this point in the paleoclimatic record of the (I, μ, N) phase space. When the system is very far from this unstable equilibrium large stabilizing restorative forces come into play, constraining the system to execute oscillatory variation about the unstable equilibrium. Starting, arbitrarily, at a state of minimum global ice mass similar to that which exists at present, the main sequence of physical processes maintaining this free oscillation can be outlined as follows: The interglacial state is characterized by relatively high concentrations of atmospheric CO_2 and NADW, that are not in equilibrium with each other because the high volume of NADW implies an overall oceanic state that would tend to draw down CO_2 from the atmosphere by the feedbacks represented by the effects IIIa, b discussed in Section 2. As atmospheric CO_2 decreases the global ice mass begins to grow, which accelerates the CO_2 decrease due to the sea level fall (see Section 2, IV), and causes more extensive permanent shelf and sea ice fields over the deep water production areas that gradually begin to reduce NADW. This reduction, in turn, slows the CO_2 solubility and biological pumps and decreases the stability of the pycnocline globally allowing increased upward mixing of carbon ultimately reversing the decrease in atmospheric CO_2. At this stage of near-minimum atmospheric CO_2 and NADW, and near-maximum of global ice mass, the pycnocline stability is reduced to a critical level permitting strong in situ vertical overturnings in the oceans that cause a surge in the carbon flux to the surface waters increasing the atmospheric CO_2 sharply. This rapid CO_2 increase forces a global warming that results in rapid deglaciation, removing the ice cover from the NADW - producing zones and causing a surge in NADW that again begins to draw down CO_2 and stabilizes the world ocean pycnocline. Thus, the system is returned to its interglacial state where it is poised to repeat the cycle.

Although this model is physically plausible, we note at least one deficiency; namely, that there is only one real equilibrium point for our particular parameter set (at $(X, Y, Z) = (0, 0, 0)$ where the eigenvalues are -1.76, $0.18 \pm 0.19i$). Thus, the system cannot exhibit the observed transition from a different early Pleistocene climatic regime that would represent a different equilibrium, as was the case in SS [1987]. In other words, for the present parameter values our model is not general enough to account for the full Pleistocene observations, given the fact that Milankovitch forcing was essentially unchanged in early Pleistocene. Such a transition at ~ 900 kyr B.P. is possible within the framework of a multiple equilibrium regime either as an "autobifurcation" [SS, 1987], or perhaps more likely, as a consequence of a progressive change in at least one of the parameters which can be viewed as an externally forced "control" parameter. Since, as noted, Milankovitch forcing varies only very slightly over the full Pleistocene [Berger and Pestiaux, 1984], some new, possibly tectonic, forcing might be involved. In the context of this scenario it would be this new forcing associated with the control parameter change that "turns on," i.e. "causes," the major 100-kyr-period ice age cycle in the late Pleistocene. One leading possibility, discussed by Ruddiman et al. [1986], is that some progressive ocean floor sill growth or erosion in the North Atlantic and Pacific can gradually modify the effect of ice on NADW production. In a future paper we shall give a fuller discussion of the parameter range for which we can find both the asymmetric time behavior and the transition capability arising from multiple equilibria, as well as a discussion of the possible role of forcing in accounting for more of the details of the full Pleistocene variations.

5. CONCLUDING REMARKS

As discussed in previous papers [e.g., SS, 1984] because of the extremely slow rate at which the main Pleistocene ice variations occurred it is not possible to explain them on the basis of the fundamental fluxes of heat, momentum, and water mass that must be involved. Instead it seems necessary to approach the problem in a more inductive manner by trying to formulate a physically plausible dynamical system that can account for a maximum amount of the observed variance with a minimum number of adjustable parameters, given the known forcing (for example, due to Earth orbital variations).

In this context, we have presented a model involving three slow response variables: the global ice mass, the concentration of atmospheric CO_2 (which can control surface temperature and ice mass through the "greenhouse" effect), and a measure of the physical-chemical-biological state of the whole ocean (perhaps measurable by the amount of North Atlantic deep water) that may control the Earth's natural carbon cycle over the Pleistocene. Particularly with regard to the CO_2 feedbacks, in this study we have tried to formalize more clearly than in Saltzman [1987] and SS [1987] the conceptual arguments that lead to the three ordinary differential equations that connect these slow response variables in a closed dynamical system. With the inclusion of an additional "asymmetric" term expressing the nonlinear effect of extrema in the values of NADW and the implied static stability, this model can account for the main features of the late Pleistocene ice variations as a free ~ 100-kyr oscillation with four semiadjustable parameters. In addition the model predicts variations of carbon dioxide that are in good agreement with the recent Vostok ice core analyses. It can be expected on the basis of previous studies that with the further inclusion of Earth orbital, and perhaps other (tectonic?), forcing, in a parameter regime admitting multiple equilibria, a good deal of the total variance of the full Pleistocene climatic changes can be accounted for.

Acknowledgments. This research has been supported by the Division of Atmospheric Sciences, National Science Foundation under grants ATM-8411195 and ATM-8802630 and by NASA under grant NAS 8-36356. We are grateful to T. Volk and J. R. Togweiller for their constructive reviews, and to R. Oglesby for useful discussions.

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K. A. Maasch and B. Saltzman, Department of Geology and Geophysics, P. O. Box 6666, Yale University, New Haven, CT 06511.

(Received January 29, 1988;
revised May 17, 1988;
accepted May 17, 1988.)