Fixed Points of Stommel Model

(1) $\dot{T} = \eta_1 - T(1 + |T - S|)$

(2) $\dot{S} = \eta_2 - S(\eta_3 + |T - S|)$

Note:
$\eta_1 > 0$
$\eta_2 > 0$
$\eta_3 > 0$

Fixed points:

(3) $\eta_1 - T(1 + |T - S|) = 0$

(4) $\eta_2 - S(\eta_3 + |T - S|) = 0$

Set $\varphi = T - S$, $T = T$

(5) $\eta_1 - T(1 + |\varphi|) = 0$

(6) $\eta_2 - (T - \varphi)(\eta_3 + |\varphi|) = 0$.

(5) $\Rightarrow$ $T = \dfrac{\eta_1}{1 + |\varphi|}$
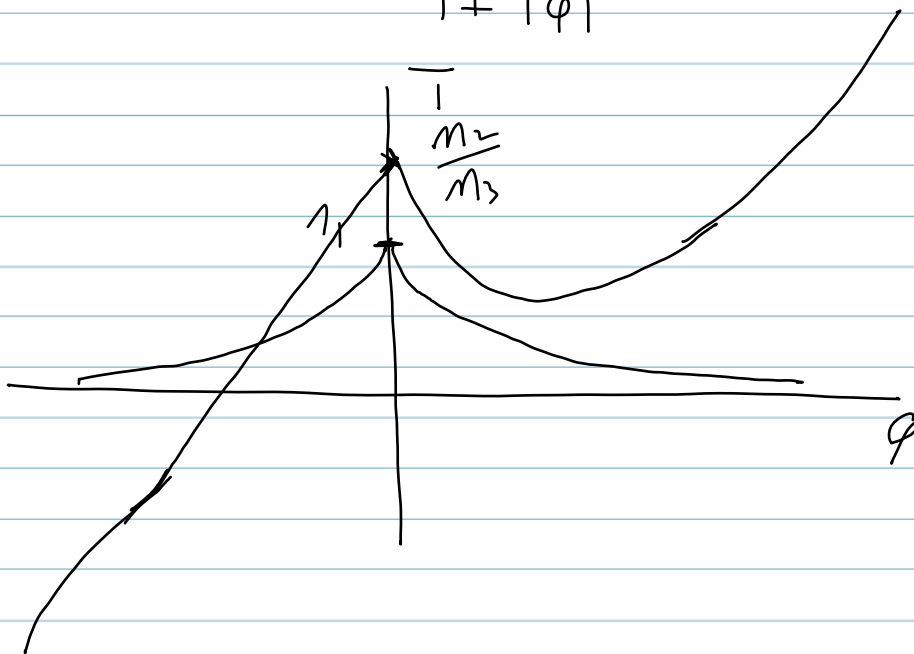
depicted: $\dfrac{\eta_2}{\eta_3} > \eta_1$

$\dfrac{\eta_2}{\eta_3^2} > 1$

$\varphi > 0$: $(T - S > 0)$

$(6) \Rightarrow$ $\eta_2 - (T - \varphi)(\eta_3 + \varphi) = 0$

$\eta_2 - (T\eta_3 + T\varphi - \varphi\eta_3 - \varphi^2) = 0$

$\cancel{\varphi^2 + \varphi / \eta_3 = T}$

$T(\varphi + \eta_3) = \eta_2 + \varphi^2 + \varphi\eta_3$

$$\boxed{T = \frac{\varphi^2 + \varphi\eta_3 + \eta_2}{\varphi + \eta_3}}$$

$$\frac{dT}{d\varphi} = \frac{(2\varphi + \eta_3)(\varphi + \eta_3) - (\varphi^2 + \varphi\eta_3 + \eta_2)}{(\varphi + \eta_3)^2}$$

$$= \frac{2\varphi^2 + 3\varphi\eta_3 + \eta_3^2 - \varphi^2 - \varphi\eta_3 - \eta_2}{(\varphi + \eta_3)^2}$$

$$= \frac{\varphi^2 + 2\varphi\eta_3 + \eta_3^2 - \eta_2}{(\varphi + \eta_3)^2}$$

$$\boxed{\frac{dT}{d\varphi} = 1 - \frac{\eta_2}{(\varphi + \eta_3)^2}}$$
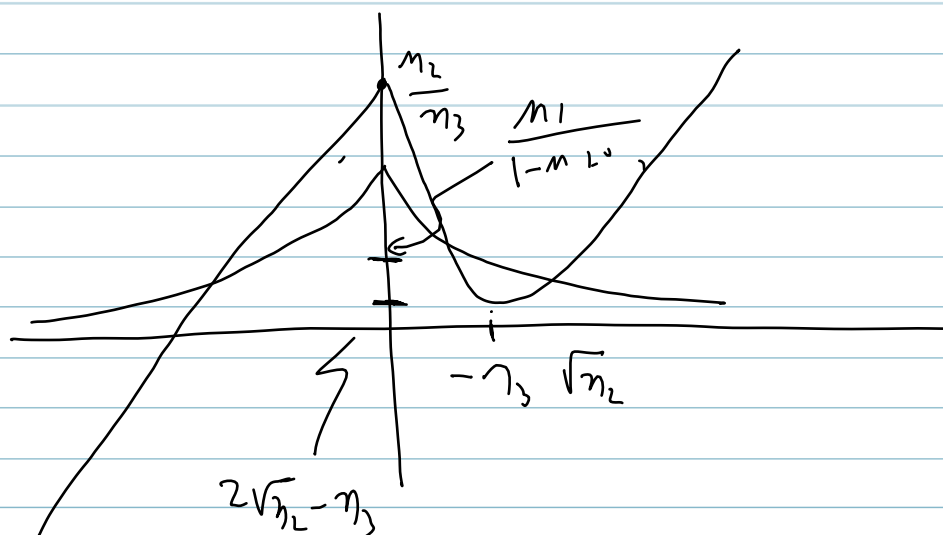
$\varphi < 0$: $(T - S < 0)$

$(6) \Rightarrow$ $\eta_2 - (T - \varphi)(\eta_3 - \varphi) = 0$

$$\boxed{T = \frac{\varphi^2 - \varphi\eta_3 \cancel{\#} \eta_2}{\varphi \cdot \eta_3}}$$

$$\boxed{\frac{dT}{d\varphi} = 1 + \frac{\eta_2}{(\varphi - \eta_3)^2}}$$

... parameters $\qquad$ $0 < \eta_3 < 1 < \eta_1$

and vary $\qquad$ η_2

Claim : can have 3 intersections:

Condit^n for 3 f.p.'s :

$$\eta_1 > \left(2\sqrt{\eta_2} - \eta_3\right)\left(1 - \eta_3 + \sqrt{\eta_2}\right)$$

Note: (1 <u>C/-not</u> have more than 3 f.p's

(2) must have at least one of

$$\frac{\eta_2}{\eta_3} > \eta_1$$

or $\qquad \frac{\eta_2}{\eta_3} = \eta_1$

or $\qquad \frac{\eta_2}{\eta_3} < \eta_1$

$= 1$] $\exists$ at least one f.p.