NOTES AND CORRESPONDENCE

On the Role of Equilibrium Atmospheric Climate Models in the Theory of Long-Period Glacial Variations

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1. Introduction

It has been noted that, because of the extremely small *net* rates of mass and energy flux responsible for the main long-period changes in global ice amount during the late Quarternary, it will be difficult, if not impossible, to use traditional equilibrium atmospheric climate models to calculate these net fluxes (Saltzman, 1983; Saltzman and Sutera, 1984). By "equilibrium atmospheric models" we mean those from which the multi-year average condition of the atmosphere and surface boundary and mixed layers are deduced either analytically or numerically as an asymptotically steady state [e.g., energy balance models (EBMs), statistical-dynamical models (SDMs), general circulation models (GCMs)]. For example, the uncertainties in the physical parameterizations that are normally used in these models to achieve acceptable statistically steady atmospheric climate distributions are large enough to control the evolution of ice accumulation over long time intervals. It seems unreasonable to expect that GCMs will have the same property relative to long-term climatic change that the primitive equations have relative to weather variations (i.e., of yielding proper net quasi-geostrophic changes over many gravity wave cycles). Whereas the time derivatives in weather variation are only one order of magnitude less than the measurable magnitude of known terms representing geostrophic balance, the time derivatives of ice mass and mean ocean temperature are at least two orders of magnitude less than the measurable magnitude of the flux terms that produce them and these flux terms involve parameterizations the forms of which are not even agreed upon.

In recognizing this state of affairs we are led to question what has generally been assumed to be a fundamental role of such models in the overall theory of long term climatic change, namely, to provide the physically deterministic coupling between atmospheric climatic processes and models of the ice sheets and deep ocean. Such a role seems implicit, for example, in Lorenz' (1970) discussion of a hypothetical "super-model" from which "we shall necessarily obtain changes in climate, including the great ice ages." While this statement may be true *in principle*, from a practical standpoint it is difficult at present to imagine that we will ever be able to achieve such super-model solutions.

The more general implication of the above is that it will not be possible to pursue a purely deductive approach to a theory of the major ice ages proceeding directly from fundamental conservation principles of hydro-thermodynamics to a solution for long-term climatic change. Instead, an *inductive* (or inverse) approach, involving qualitative physics, will probably be required. In its simplest form such qualitative physics is perhaps best expressed in the form of a stochastic–dynamical system of equations governing multi-year running average climatic variables that are independent of the space coordinates (e.g., selected Fourier modes or global integrals). Aside from the simplicity of analysis afforded by the reduction of the order of the system, the use of global variables is largely dictated by the fact that present geological evidence provides a continuous record only of *global* ice mass changes as revealed by the sedimentary core measurements of $\delta(^{18}O)$.

As noted in the above references, such a dynamical system can be conveniently resolved into 1) an essentially *diagnostic* system of equations that constitutes an "equilibrium" model relating the running average, fast-response, variables $X_i^{(f)}$ to each other and the slow-response variables $X_i^{(s)}$, and 2) an essentially *prognostic* system of equations that expresses the qualitative physics governing the slow response variables such as ice mass. Thus, if we neglect stochastic forcing of the fast response variables, we have

$$\frac{dX_i^{(f)}}{dt} = \phi_i(X_j^{(f)}, X_j^{(s)}) + F_i^{(f)}(t) \approx 0 \tag{1a}$$

$$\rightarrow X_i^{(f)} \approx \Phi_i(X_j^{(s)}, F_i^{(f)}), \tag{1b}$$

$$\frac{dX_i^{(s)}}{dt} = \psi_i(X_j^{(f)}, X_j^{(s)}) + F_i^{(s)}(t) + R_i \tag{2a}$$

$$= \psi_i[\Phi_j(X_k^{(s)}, F_i^{(f)}), X_j^{(s)}] + F_i^{(s)}(t) + R_i$$
$$\approx \Psi_i(X_j^{(s)}) + F_i(t) + R_i, \tag{2b}$$

where i, j and k are integer index numbers identifying the various dependent variables, $F(t)$ represents external forcing of periods which are long in comparison with the fast response times, and R denotes stochastic forcing. Equation (1b) can represent the solution obtained from an equilibrium atmospheric climate model (e.g., a GCM), or a semi-empirical relationship.

This illustrates a first important role for equilibrium models in a theory of the long-term ice age variations; namely, to provide, or at least to suggest, the form of the relationships (1b) that are necessary for closure of the dynamical system (2a) governing the slow-response variables that are the main signatures and carriers of the climatic change. Thus, the effects of all the fast response variables (e.g., atmospheric wind, temperature, cloudiness and CO_2 content) on the slow response variables (e.g., ice mass) are *implicit* in the coefficients appearing in the dynamical system (2b). For this reason, the "response times" of the components of the dynamical system (2b) are conditioned by the nature of the feedbacks in (1b) as well as in (2a), and need not be the same as those associated with the intrinsic properties of the components of the system such as ice or ocean water. Note that if the period of the main external forcing $F_i(t)$ is larger than these response times, even (2b) can reduce to diagnostic equilibrium equations with the longer-term climatic changes being in the nature of a relatively "fast" response to this forcing. This does not seem to be the case in the late Quaternary when the observed near-100 000 year (y) period in ice mass corresponds to only very weak earth eccentricity forcing, but may be true for much longer-period variations (of the order of tens of millions of years) when tectonic forcing is dominant.

We come now to a second potentially important role for the equilibrium models. A complete theory of the ice ages must account for more than the time evolution of *globally-integrated* quantities. Although at this time only such global measures of climate variations are available on a continuous basis, in the long run the regional geographical distribution of ice thickness and other climatic variables (e.g., temperature, wind, humidity and precipitation rates) as represented by atlases of present climate and reconstructions of past states (such as that laboriously compiled for one point in time, 18 000 y BP, by CLIMAP Project Members, 1976) must also be accounted for. The problem posed is whether, given the past and present values of such slow-response variables as those considered by Saltzman and Sutera (1984), e.g., global ice mass ζ and ocean ice mass χ, we can determine the geographical distribution of continental ice thickness $h_\zeta(x, y)$ and shelf ice thickness $h_\chi(x, y)$, as well as all the other slow- and fast-response climatic variables describing the state of the atmosphere and surficial boundary layers as a function of the space coordinates (x, y, z). It is not at all certain that this inverse calculation can be made, and, if it can, what the uniqueness properties of such a calculation would be, but we shall in the following section propose one line of attack that may be worth pursuing.

2. Cryospheric adjustment

Let us suppose now that we have succeeded in designing a physically plausible dynamical system, the solution of which can account for the main features of the variations of the global slow response variables (such as ζ and χ) as revealed by the geologic evidence. As we have noted, a next problem is to determine the geographical distribution of the ice mass corresponding to the total mass $(\zeta + \chi)$ at any particular point in time. For the sake of argument, as a naive first guess, let us assume that the continental ice is distributed uniformly over all the continental area exposed above sea level commensurate with ζ, and all the marine ice is distributed uniformly over the polar oceans equatorward to some arbitrary latitude, say $\phi_\chi = 30°$N and S. This would constitute an initial assumed surface boundary condition for the application of the fast-response atmospheric–oceanic mixed layer model. It is almost certain that the solution obtained for the fast response parts of the system (e.g., surface air temperature) would be inconsistent with this naively prescribed surface boundary condition; e.g., temperatures well above freezing will prevail over the low latitude ice fields, which from even the simplest heat flux representations will imply huge rates of melting. Thus, in the next iteration toward an assumed equilibrium between ice coverage and atmospheric state it would be necessary to remove ice from many areas, especially in the tropics and subtropics. Under the constraint of constant total ice mass $(\zeta + \chi)$, however, this ice must be placed in higher latitude areas, increasing its thickness there. A new equilibrium solution would then be obtained for this new distribution, with the hope that by successive similar iterations, convergence to a quasi-equilibrium would be obtained. As in the present application of GCMs, the tolerance level accepted for "equilibrium" would be much larger than the accuracy required to compute the disequilibrium rate of change of *total* ice mass involved in long-period climatic change. Thus we would accept a statistically steady state that meets the presently adopted criteria for "convergence," even though this might imply a net rate of ice freezing or melting of the order of the errors in the parameterization used in the model (e.g., the equivalent of several W m^{-2}). The solution so obtained would specify all the fast-response variables including ice coverage. Given the prescribed values of ζ and χ, it might further be hoped that the thickness distribution of ice could be estimated from equilibrium theory for ice sheets and shelves. We can term this complete iterative process "cryospheric adjustment."

It can readily be imagined that, depending on the prescribed distribution of ice, a huge imbalance might occur, at least initially, between ice coverage and the fast response state variables of the atmosphere and surface layer of the ocean. In the proposed iterative scheme this imbalance would set in motion large swings in ice coverage and thickness that constitute the climate–cryospheric equivalent of weather–atmospheric "gravity waves." After this initial adjustment, equilibration of a more quasi-static nature could ensue at each succeeding time increment of the slow variable evolution.

In the most primitive case, if we apply a Budyko–Sellers-type EBM to a homogeneous earth required to contain a certain ice mass ($\zeta + \chi$), we might start by initially assuming that this ice is uniformly distributed over the entire earth (Fig. 1, light stippling). According to this simple model, such an "ice-covered earth" might constitute an acceptable equilibrium state (Budyko, 1969). Alternatively, if we started with a uniform distribution in which some thicker ice sheet extended only down to some subtropical latitude, it would probably be necessary for the ice edge to retreat to some more polar latitude ϕ_i in order to reach equilibrium. The thickness distribution within the polar caps would then be required to take some form compatible with equilibrium ice sheet theory, e.g., Weertman (1964) (see Fig. 1, heavy stippling). This example already illustrates one of the main difficulties of this procedure, namely the possible *nonuniqueness* of the solution. However, the potential existence of such multiple equilibria is, in itself, important and worthy of special consideration. It seems reasonable to expect that we will be confronted steadily with the question of whether multiple equilibria are an artifact of an imperfect model or a real feature of the climatic system.

If, as another example, one were to consider the marine ice mass χ to be distributed uniformly to some arbitrary latitude (see Fig. 2, light stippling), a simple model such as that described by Saltzman (1978) would show that a damped oscillation (representing marine cryospheric "gravity waves") might ensue, leading to an equilibrium of the form shown in Fig. 2 (heavy stippling) containing the same fixed mass of sea ice. Of course, to model the true climatic system it would be much better to use a high resolution GCM, in which the atmosphere, mixed-layer ocean, biolithosphere and shallow cryosphere are treated as a combined system, to obtain the best approximation to the equilibrium marine and land ice thickness as a function of geography. Such a GCM would also contain the constraints of the hydrologic cycle that could dictate, for example, that insufficient moisture would be available to support the existence of a large ice mass in Siberia, as is indicated in climatic reconstructions of past ice mass maxima.

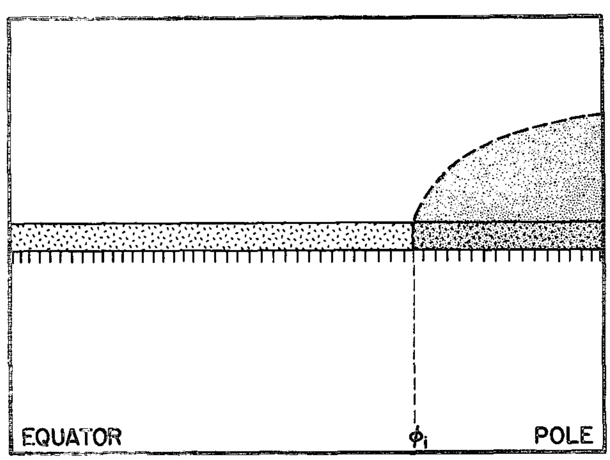

FIG. 1. Possible equilibrium distributions of land ice having the same total mass ζ: ice-covered earth of uniform thickness (light stippling), and polar ice sheet (heavy stippling).

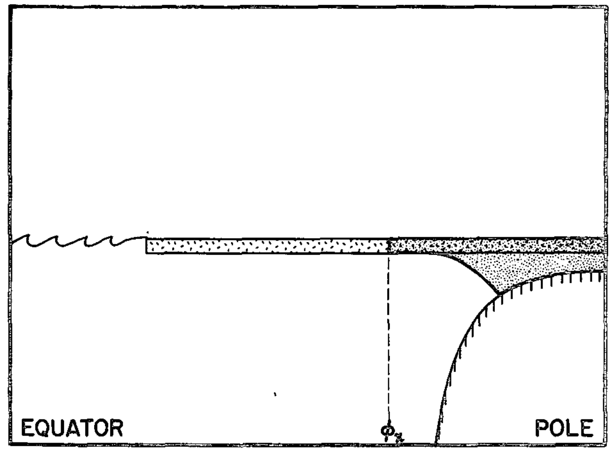

FIG. 2. Assumed initial, nonequilibrium, distribution of marine ice of total mass χ (light stippling), and hypothetical equilibrium marine ice distribution for the same mass of ice χ (heavy stippling).

More formally, if we rewrite the solution of a quasi-equilibrium model for the fast response variables (1b) in the form

$$X_i^{(f)}(x, y) = \Phi_i[\zeta, \chi, \ldots, X_j^{(s)}, \ldots, h(x, y), F_i^{(f)}],$$

where $h(x, y)$ is the geographical distribution of ice thickness, we can summarize the cryospheric adjustment procedure we have described as one in which by successive approximations we obtain the inverse solution

$$(h, X_i^{(f)})_{x,y} = \hat{\Phi}_i(\zeta, \chi, \ldots, X_j^{(s)}, F_i^{(f)}), \qquad (3)$$

where $\zeta, \chi, \ldots, X_j^{(s)}$ are the global values specified from the slow-response solution at any given time. In essence, (3) represents a state of cryospheric "balance."

3. Modeling the ice ages

It is a common article of faith that a deductive explanation of the ice ages based on fundamental thermo-hydrodynamic principles, while difficult to achieve because of the tremendous complexity of the climatic system, can ultimately be attained by a continued creative effort of "super-model" building (e.g., Lorenz, 1970; Gates, 1981). It was shown by Saltzman (1983) and Saltzman and Sutera (1984), however, that such an expectation is unreasonable from a practical viewpoint. Instead, we imply in this discussion that a "theory" of the ice ages can be reduced to two parts: 1) a prognostic part, *inductive* in nature, involving qualitative physics expressed as a forced, dissipative stochastic-dynamical system governing the slow-response global variables, and 2) a diagnostic part, *deductive* in nature, by means of which we recover the local details of the complete climatic state given the global variables. It is this latter part that will depend on the development of a "super-GCM". It is possible, of course, to combine these two parts in a single "coupled" system wherein the rates of energy and mass flux are tuned, inductively, to yield plausible ice mass evolutions; at the same time explicit representations and feedbacks are included that relate the fast and slow response parts (e.g., Birchfield *et al.*, 1982; Pollard, 1983). The point is that in spite of their relatively great complexity and many degrees of freedom, the physical basis for the *long-term evolution* obtained from these "coupled" models is no more quantitative or rigorous than that of the much lower-order dynamical systems models.

In closing, we might add that from the aforementioned coupled models and others that emphasize the role of orbital forcing (see also Saltzman *et al.*, 1984), it seems likely that only if some external "pacemaker" $F(t)$ is of significance will it be possible to account for all the long-term variations of climate and to fix the phase in correct relation to observations (Hays *et al.*, 1976; Nicolis, 1984).

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