

Fitting Paillard's Three State Model Parameters via CMA-ES and GA.

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The explanatory value of simple climate models relies upon their ability to mechanistically explain past variation while providing a reasonable fit to available data. We investigate the proper fitting of Paillard's three state model with past data to test its accuracy directly. The parameter fit, our main effort, is achieved via a covariance matrix adaptation evolution strategy (CMA-ES) and a general genetic algorithm (GA). We find that Paillard's model's explanatory value is mainly empirical, as the model is robust to parameter changes and the fit to data from $\delta^{18}O$ records has minimum RMSE 1e-5.

PACS numbers:

INTRODUCTION

Understanding the past climate of the earth allows for us to begin to understand what effects we are having upon it, and to predict what the future will bring. From the analysis of isotope ratios in ice cores and marine sediment cores, we obtain knowledge of past temperature from the ratio of $\delta^{18}O$, a proxy for combined temperature and global ice volume. This time series is constructed over the past 2 Myr, with higher values indicating a colder climate as the ocean is enriched with $\delta^{18}O$ during the colder conditions. The variation in the orbital parameters of our planet affect the amount of solar insolation recieved, and the timescale variation of these quantities is one possible forcing for past climatic change. This was hypothesized as early as the nineteenth century, and further elucidated by Milankovitch, whose name the study of these variations bears. The eccentricity of our elliptical orbit is changing slowly in time, as is the orientation of the planet's spin axis and the longitude closest to the sun at the vernal equinox. We refer to these parameters as e , ϵ and $\bar{\omega}$, respectively.

The main questions arising about this time period from Dijkstra [1] that any satisfactory theory of the Pleistocene Ice Ages should explain are:

- Why did the glacial-interglacial cycles appear in the Pleistocene?
- Which processes in the climate system caused the glacial-interglacial changes in global mean temperature, $p_{CO_2}^o$, and ice sheet extent?
- What caused the transistion (the MPT) from the 41-kyr world to the 100-kyr world about 700 kyr ago?

The Milankovitch theory of orbital forcing is now the 'null hypothesis' answer to these questions, and various models are available that relate this orbital forcing to global mean temperature and ice sheet extent have been proposed. The fit between the changing

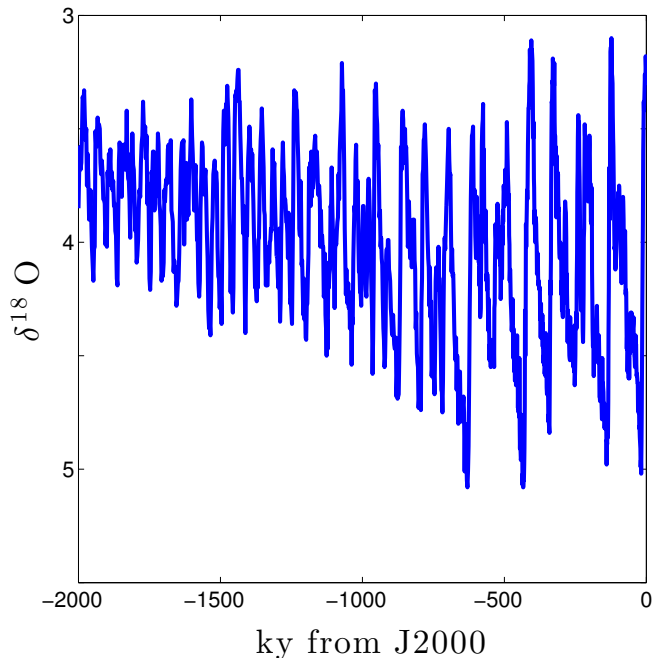


FIG. 1: The LR04 benthic $\delta^{18}O$ stack over the Pleistocene, constructed by the graphic correlation of fifty-seven globally distributed benthic $\delta^{18}O$ records versus time (data from Lisieki [2]).

solar insolation and $\delta^{18}O$ has been demonstrated not to be linear, opening the door for more complex explanations. The potentially important feedbacks of ice-albedo, height-mass balance, load-accumulation, and precipitation-temperature have all been combined into quasi-equilibrium and internal oscillation models, with the latter successfully demonstrating a reasonable fit to the data. In addition to these more complex models is a switching equilibria model from Didier Paillard that fits the data empirically, and does "remarkable" considering the simplicity of the model.

In the present work we attempt a rigorous fit of Paillard's model to data using the tools of evolutionary com-

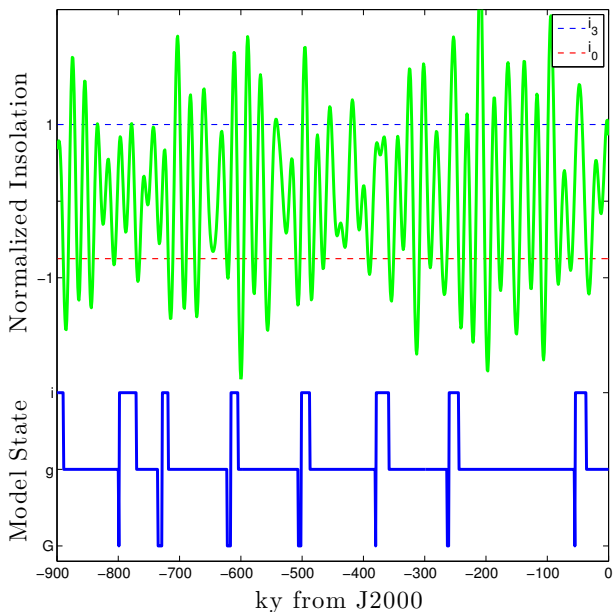


FIG. 2: The discrete Paillard model.

putation.

PAILLARD'S MODEL

With some difficulty understanding his original paper [3], we reconstruct Paillard's model for both versions that he provides. The following two subsections paraphrase [3] directly.

Discrete model

The model has three states called **i** (interglacial), **g** (mild glacial) and **G** (full glacial). This model undergoes an **i-g** transition as soon as the insolation falls below i_0 . An **g-G** transition happens when the ice volume exceeds a threshold v_{\max} and finally a transition **G-i** occurs when the insolation increases above i_1 . These transitions are assumed to be one-way, and the only ones allowed are those just discussed. The **g-G** transition has the constraints that it will happen after some time t_g and with insolation lower than some level i_3 . If t_g is exceeded but i_3 is also exceeded, then the **g-G** transition occurs as the next insolation decrease: when insolation falls below i_2 .

The part which was made least clear was whether the insolation exceeding i_3 necessitates a reset of the time in state **g** until t_g is reached. To achieve similar results, we assume that the answer is yes, the time does reset.

Differential equation model

This model is an extension of the discrete model. There are still three regimes but now we define the ice volume v as a differential equation

$$\frac{dv}{dt} = (v_R - v)/\tau_R - F/\tau_F.$$

where R is the current climate regime ($R = \mathbf{i}, \mathbf{g}, \mathbf{G}$), v_R are the reference ice volumes for the different regimes, F is the forcing and $\tau_{R,F}$ are the time constants. The transitions are the same for **i-g** and **G-g** but **g-G** now happens when $v \geq v_{\max}$, some threshold value (without any condition on the insolation). Paillard also "normalizes the ice volume to unity": $v_g = v_G = v_{\max} = 1$ and $v_i = 0$. For the finale of model weirdness, the forcing F is defined as a truncated insolation signal with truncation function f as

$$f(x) = \frac{1}{2} \left(x + \sqrt{4a^2 + x^2} \right)$$

and setting $a = 1$.

COMPUTING INSOLATION

The only input to the model is the solar insolation, so we are particularly careful to obtain accurate values. Increasingly accurate integrations of the state of the solar system have been performed dating from the time of Lagrange in 1781 through Milankovich's use of Pilgrim's 1904 computations to establish his theory of past climate change to today's direct numerical integrations [4].

Using the orbital parameters of eccentricity e , obliquity ϵ , and longitude of the perihelion $\bar{\omega}$ computed from an integration function `insola.f` provided from Laskar [4] we can reconstruct the insolation at any time. The following derivation is due to Berger [5].

The insolation Q at a given location and time can be written as a function of the solar constant S_0 , the distance from the Earth to sun relative to the average R_0/R_E the solar zenith angle Θ :

$$Q = S_0 \frac{R_0^2}{R_E^2} \cos(\Theta).$$

To solve for the average insolation during one day at a given latitude ϕ , we need to express the solar zenith angle Θ in terms of ϕ and the known orbital parameters. Taking δ as the latitude of the point directly subsolar and h as the hour angle, we use the spherical law of cosines to write

$$\cos(\Theta) = \sin(\phi) \sin(\delta) + \cos(\phi) \cos(\delta) \cos(h).$$

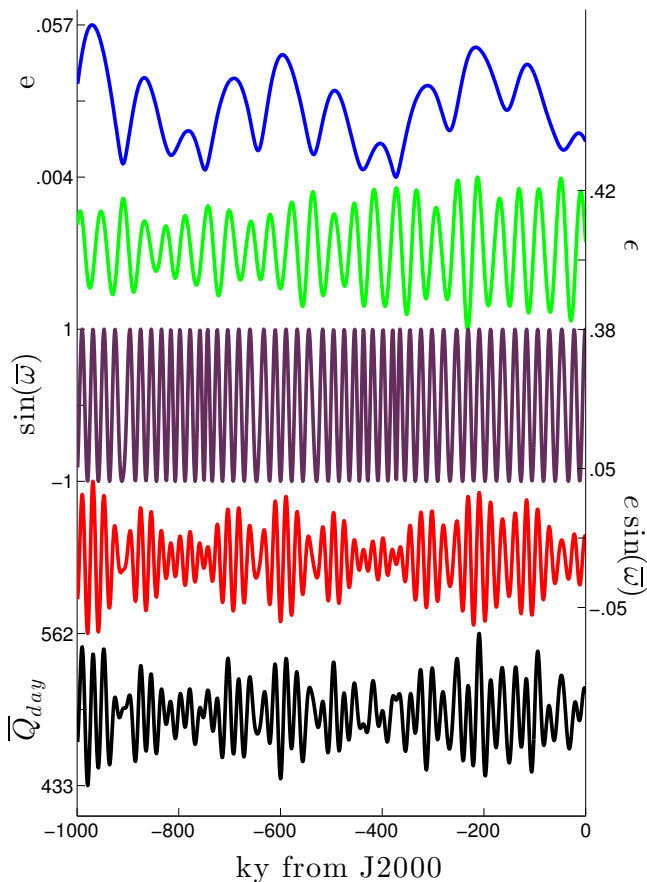


FIG. 3: The average daily solar insolation \bar{Q}_{day} is computed on the summer solstice at 65N with the provided e , ϵ , and $\bar{\omega}$.

Now to have the average daily insolation \bar{Q}_{day} we take the integral of Q from h_o to $-h_o$ where h_o is the hour angle for which the sun first rises. Since we are concerned with the $\phi = 65\text{N}$, the sun always rises and sets so we won't consider the cases for the sun neither rising nor setting. Integrating, we have

$$\bar{Q}_{\text{day}} = \frac{S_0}{\pi} \frac{R_0^2}{R_E^2} [h_o \sin(\phi) \sin(\delta) + \cos(\phi) \cos(\delta) \sin(h_o)].$$

Defining $\theta = 0$ to be the vernal equinox, we can write δ and R_0/R_E as a function of the orbital parameters in a simple form at the summer solstice ($\theta = \pi/2$):

$$\delta = \epsilon \quad \text{and} \quad \frac{R_0}{R_E} = 1 + e \sin(\bar{\omega}).$$

EVOLUTIONARY ALGORITHM METHODS

Morgan

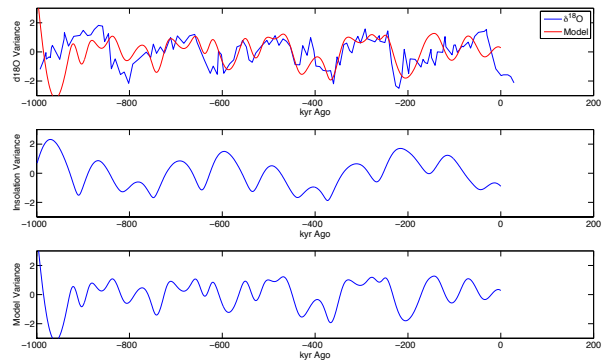


FIG. 4: A first run of the Paillard ODE model, over the $\delta^{18}\text{O}$ record show a reasonable empirical fit.

RESULTS

Figure 3 depicts the reconstruction of insolation data back in time.

Figure 4 is a direct representation of the second figure of Paillard's original paper [3].

CONCLUSION

The fitted Paillard-Reagan-Frank model of climatic variability wholly explains the temperature variation in the $\delta^{18}\text{O}$ record. With only forcing from changing solar insolation, this verifies that past climatic variation of larger time scales is a result of the variation of the Earth's orbital parameters.

The authors would like to thank the poor chaps who were out in the arctic drilling ice cores for their sacrifices of comfort in the name of science.

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