Issue: explaining the MPT

~ 800 kyr BP

Transition 41 kyr cycle
↓
100 kyr cycle.

Mystery of Pleistocene Ice Ages

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Hypothesis (Maasch-Saltzman).
1984-1988 - 1990.
41 kyr cycle - orbital (Milankovitch) forcing
@ MPT
Carbon cycle play a significant role & shifted the period.

On the mathematical level,
the claim is that the changes
in the carbon cycle induced a
HOPF BIFURCATION.

Carbon Cycle (Dijkstra)

(1) Evidence: carbon content during glaciations reduced by

25% 30% 30%

Land (vegetation), atmosphere, upper ocean

Presumably 1,000 GT C → deep ocean

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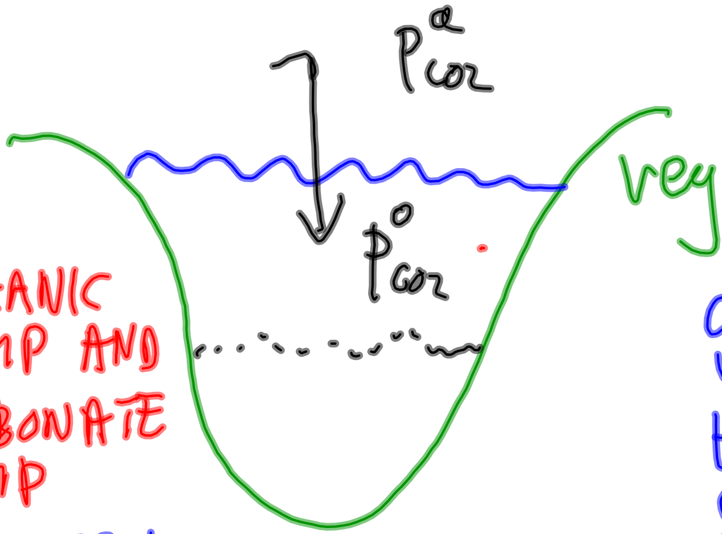

SOLUBILITY PUMP

glacial period
↓
temp dec.
sea level falls
↓
increase in salinity of upper ocean
↓
changes solubility of carbon.

ORGANIC PUMP AND CARBONATE PUMP

upper ocean
↓
low ocean

reduces $P^a_{CO_2}$ ← reduces $P^o_{CO_2}$ ← changes solubility of carbon.

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Prognostic

Global ice mass I

Atmospheric CO_2 μ

(North Atlantic deepwater formation N.

temp & salinity of deepwater in NA. ←

slow

Diagnostic

Mean surface temp T

Permanent sea ice extent η

fast

Assume diagnostic variables expressed in terms of prognostic variables (fast response is averaged!)

M-S eqs.

I, μ, N are actually "anomalies" i.e. deviations from some fixed value

$$\boxed{\begin{aligned} \dot{I} &= -a_0 I - a_1 \mu \quad (+F_I) \\ \dot{\mu} &= -b_0 I + b_1 \mu - (r_1 - b_3 N) N - b_4 N^2 \mu \quad (+F_\mu) \\ \dot{N} &= -c_0 I - c_1 N \quad (+F_N) \end{aligned}}$$

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Non. dimensionalize

Milankovitch

$$\tau = a_0 t$$

$$X = \frac{c_0}{c_1}\sqrt{\frac{b_4}{a_1}}\, T$$

$$Y = \frac{a_1 c_0}{c_2}\sqrt{\frac{b_4}{a_0^3}}\, \mu$$

$$Z = \sqrt{\frac{b_4}{a_0}}\, N$$

$$\dot{X} = -X - Y - uM(\tau)$$

$$\dot{Y} = -pZ + rY + sZ^2 - Z^2 Y$$

$$\dot{Z} = -q(X + Z)$$

$$\dot{} = \frac{d}{d\tau}$$

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$\dot{X} = -X - Y$ ice mass

$\dot{Y} = -pZ + rY - sZ^2 - Z^2 Y$ atmos CO_2

$\dot{Z} = -q(X + Z)$ NADW

p, q, r, s parameters (>0)

F.P.'s $X = -Y$, $X = -Z \Rightarrow Y = Z$.

$(-\alpha, \alpha, \alpha)$ ex. $\alpha = 0$

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F.P.s @ $(-\alpha, \alpha, \alpha)$.

(1) $\alpha = 0$

(2) $\alpha = -\frac{s}{2} + \frac{\sqrt{s^2 - 4(p-r)}}{2}$

(3) $\alpha = -\frac{s}{2} - \frac{\sqrt{s^2 - 4(p-r)}}{2}$

may exist or not depends sign of $s^2 - 4(p-r)$

Linearize eqs. @ $(-\alpha, \alpha, \alpha)$

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$$f(x,y,z) = \begin{pmatrix} -x-y \\ -pz+ry-sz^2-z^2y \\ -q(x+z) \end{pmatrix}$$

$$Df(-\alpha,\alpha,\alpha) = \begin{bmatrix} -1 & -1 & 0 \\ 0 & r-\alpha^2 & -p-2s\alpha-2\alpha^2 \\ -q & 0 & -q \end{bmatrix}$$

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Characteristic polynomial:

$$(\lambda+q)(1+\lambda)(r-\varphi^2-\lambda)-q(p+2s\varphi+2\varphi^2)=0$$

Case: $\varphi=0$

$$(\lambda+q)(1+\lambda)(r-\lambda)-qp=0$$

Q: Does a Hopf bifurcation occur?

Need: a complex conjugate pair w/ real part $=0$.

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Idea: see if cha poly = 0
has purely imaginary soln. for
some set of parameter values

Set $\lambda = i\gamma$

$$(i\gamma+q)(1+i\gamma)(r-i\gamma)-qp=0$$

Take Re & Im parts

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$$\gamma^2(q+1-r) = q(p-r) \qquad \text{Re.}$$

$$\gamma^2 = r(q+1) - q \qquad \text{Im.}$$

$$\Rightarrow \frac{q(p-r)}{q+1-r} = r(q+1) - q$$

$$q(p-r) = (q+1-r)\big(r(q+1)-q\big)$$

Idea: solve quadratic for q in p & r

$$q^2(r-1) - q\left[(r-1)^2 - r + p - r\right] + r(1-r) = 0.$$

To have a real soln.:

$$-4(r-1)r(1-r) = 4r(r-1)^2 \geq 0$$

$\forall\, r > 0$

$\Rightarrow b^2 - 4ac > 0$ & hence real

soln.

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Need to check:

1. Hopf occurs at positive parameter values.
2. @ prospective Hopf point, e-values move thru' imag. axis nondeg. as a relevant parameter varies.
3. supercritical Hopf, periodic orbits attracting (Check!!)

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This all works as there is a
Hopf bifurcation in the MS system.

Maasch - Saltzman:

$p = 0.1, \quad q = 1.5, \quad r = 1.5, \quad s = 0$

$\Rightarrow$ 100 kyr free oscillation

Also show: if initialized aptly at 2 Myr BP
$\Rightarrow$ transition @ 900 kyr BP to 100 kyr ...

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