SET-UP

① BVD model:

$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi, \nabla^2\psi + f + \gamma h) = -C\nabla^2\psi + G \qquad (1)$$

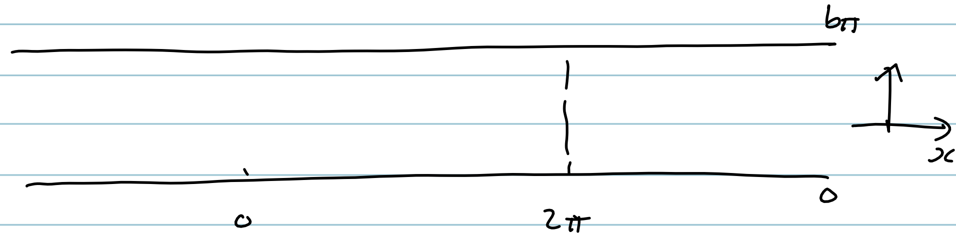
$-C\nabla^2\psi$: Ekman damping

G: forcing (e.g. for radiation field creating "thermal wind")

Domain. $[0,2\pi]\times[0,b\pi]$

B.C. $\psi(0,y,t) = \psi(2\pi,y,t)$

$\frac{\partial\psi}{\partial x}(x,0,t) = \frac{\partial\psi}{\partial x}(x,b\pi,t)$



② Spectral expansion:

Operator $L = \nabla^2$ B.C. $\varphi(0,y) = \varphi(2\pi,y)$

$\varphi_x(x,0) = \varphi_x(x,b\pi) = 0$

$$\nabla^2\varphi_{nm} = \lambda_{nm}\varphi_{nm} \qquad |n| = 0,1,2,\dots;\ m = 1,2,\dots$$

$$(2)\quad \varphi_{0m}(y) = \sqrt{2}\cos\frac{my}{b}$$

$$(3)\quad \varphi_{nm}(x,y) = \sqrt{2}\,e^{inx}\sin\frac{my}{b}$$

Orthonormal expansion

$$\psi(x,y,t) = \sum_{n,m}\psi_{nm}(t)\,\varphi_{nm}$$

$$\psi(x,y,t) = \sum_{n,m} \cancel{\phi}_{nm}(t)\,\varphi_{nm}$$

Work w/ L^2 inner product over $\mathbb{C}$.

$$\langle \psi(x,y,t), \varphi_{\ell k} \rangle = \sum_{n,m} \psi_{nm}(t) \langle \varphi_{nm}, \varphi_{\ell k} \rangle$$

$\langle \varphi_{nm}, \varphi_{\ell k} \rangle = 0$ unless $n = \ell$ & $m = k$.

(4) $$\Rightarrow \quad \psi_{nm}(t) = \langle \psi(x,y,t), \varphi_{nm} \rangle$$
$$= \int_0^{b\pi}\int_0^{2\pi} \psi(x,y,t)\,\overline{\varphi_{nm}}\,dx\,dy$$

③ Projection (Galerkin):

6-mode $\quad |n| = 0, 1 \quad ; \quad m = 1, 2$

$$\psi_{01},\ \psi_{02},\ \psi_{-11},\ \psi_{-12},\ \psi_{11},\ \psi_{12}$$

3-mode $\quad |n| = 0, 1 \quad ; \quad m = 1$

$$\psi_{01},\ \psi_{-11},\ \psi_{11}$$

topography $\quad h(x,y) = \cos x \sin \frac{y}{b}, \Rightarrow h = \frac{1}{2\sqrt{2}}(\varphi_{11} + \varphi_{-11})$

forcing $\quad g = g(y), \Rightarrow g = \sum_{m=1}^{\infty} g_{0m}\varphi_{0m}$

$\boxed{\text{GOAL}}$

Derive the 3-mode Charney-deVore model

i.e., equations for

$$\psi_{01}(t), \quad \psi_{-11}(t), \quad \psi_{11}(t)$$

which are the coefficients of the lowest 3 modes in the spectral expansion.

Expect: ODEs for these terms, i.e.

$$\psi_{01}' = \ldots$$

$$\psi_{-11}' = \ldots$$

$$\psi_{11}' = \ldots$$

where the RHS are functions of $\psi_{01}, \psi_{-11}, \psi_{11}$

Note that these equations will be complex & we have to find the "real" versions.

Steps: (1) expand h & g

(2) calculate λ_{nm}

(3) calculate eqn. (1) in series expansion

(4) project onto modes.