Data Assimilation and Genetic Algorithms for the Parameter Estimation Problem in Simple Climate Models

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Abstract

Given observations of an atmospheric phenomenon and a well-principled model of that phenomenon, the parameters for the model must be properly tuned if the model is to mimic the data. We investigate the use of genetic algorithm in comparison to data assimilation as a means of performing parameter estimation when tuning models to data. We compare results while tuning chaotic dynamics, observation noise and frequency, and system dimensionality while performing parameter estimation for the Lorenz '63 and Lorenz '96 systems.

1 Introduction

Weather forecasting has become an expected part of everyday life in the modern society. Things like air-travel, disaster preparation, and daily planning rely on effective predictions [1]. However, predicting future states of the atmosphere proves to be difficult as chaotic systems exhibit sensitive dependence of initial conditions [2–4]. This hurdle is overcome by utilizing computationally expensive global climate models (GCMs), but scientists working to improve weather forecasting often lack the time or computational power to execute many GCMs. Instead, climate scientists often use simple models that account for particular aspects of the weather forecasting problem.

Edward Lorenz has made major contributions to the fields of dynamical systems and atmospheric prediction [5–7]. Two such contributions are the wildly popular Lorenz '63 system [8] and the Lorenz '96 system [9]. The Lorenz '63 system (L63), which yields the widely known Lorenz Attractor, is a simple three-variable model with highly tunable dynamics, allowing researchers a computationally tractable means to experiment in the predictability of chaotic systems. The Lorenz '96 system (L96) exhibits tunable chaotic dynamics as well, while additionally providing a computationally tractable way to change the system dimensions and tune the accuracy of data observations. Both systems provide interesting and computationally manageable test beds for the parameter estimation problem across several different types of systems. Figure 1 shows example trajectories for each system.

2 Methods

2.1 The Lorenz '63 Model

2.2 The Lorenz '96 Model

In 1995, Edward Lorenz introduced the following I-dimensional model [5, 7]. The key characteristics of this model include tunable chaotic behavior when subject to enough forcing, and tunable dimensionality. The predecessor to the current model is given by

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F \tag{1}$$

where $i = 1, 2, \ldots, I$ and F is the forcing parameter. Each x_i represents observations of some atmospheric atmospheric quantity, like temperature, evenly distributed about a given latitude of the globe. This implies a modularity in the indexing that is described by $x_{i+I} = x_{i-I} = x_i$.

This early model failed to produce realistic growth rate of the large-scale errors along with lacking tenability in observation reliability. Lorenz went on to introduce

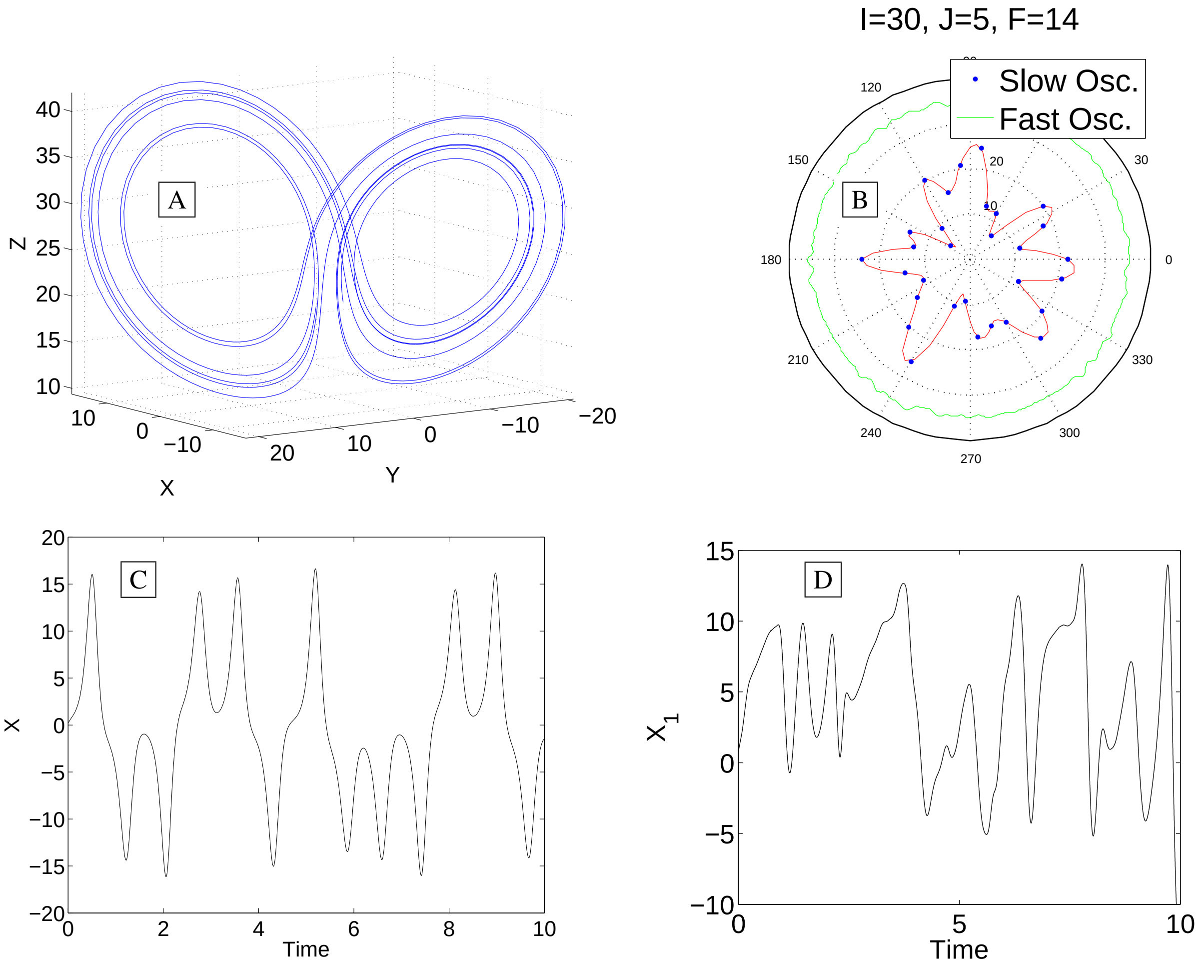

Figure 1. (A) The popular "Lorenz Attractor" produced with the Lorenz '63 system. This three-variable system produces a "butterfly"-like chaotic attractor that is well-known among fractal and chaos enthusiasts. (B) An snapshot of a trajectory of the Lorenz '96 system. Each blue point is a slow oscillator, and the adjacent sections of green represent the fast oscillators coupled with the corresponding slow oscillator. The origin represents the lowest value achieved by any of the slow oscillators on this trajectory. The red line is a cubic spline interpolation of the blue data points. (C) An example trajectory of the X variable from the Lorenz '63 system. (D) An example trajectory for a slow oscillator of the Lorenz '96 system.

a more flexible model in 1996 by coupling two systems similar to the model in equation (1), but differing in time scales. The equations for the Lorenz '96 model [9] are given as

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F - \frac{hc}{b}\sum_{j=1}^{J} y_{(j,i)} \qquad (2)$$

$$\frac{dy_{(j,i)}}{dt} = cby_{(j+1,i)}(y_{(j-1,i)} - y_{(j+2,i)}) - cy_{(j,i)} + \frac{hc}{b}x_i \qquad (3)$$

where $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, J$. The parameters b and c indicate the time scale of solutions to equation (3) relative to solutions of equation (2), and h is the coupling parameter. The coupling term can be thought of as a parameterization of dynamics occurring at a spatial and temporal scale unresolved by the x variables. Again, each x_i represents an atmospheric observation about a latitude that oscillates in slow time, and the set of $y_{(j,i)}$ are a set of J fast time oscillators that act as a damping force on x_i. The y's exhibit a similar modularity described by $y_{(j+IJ,i)} = y_{(j-IJ,i)} = y_{(j,i)}$.

2.3 Data Assimilation

2.4 Genetic Algorithm

3 Results

4 Discussion

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