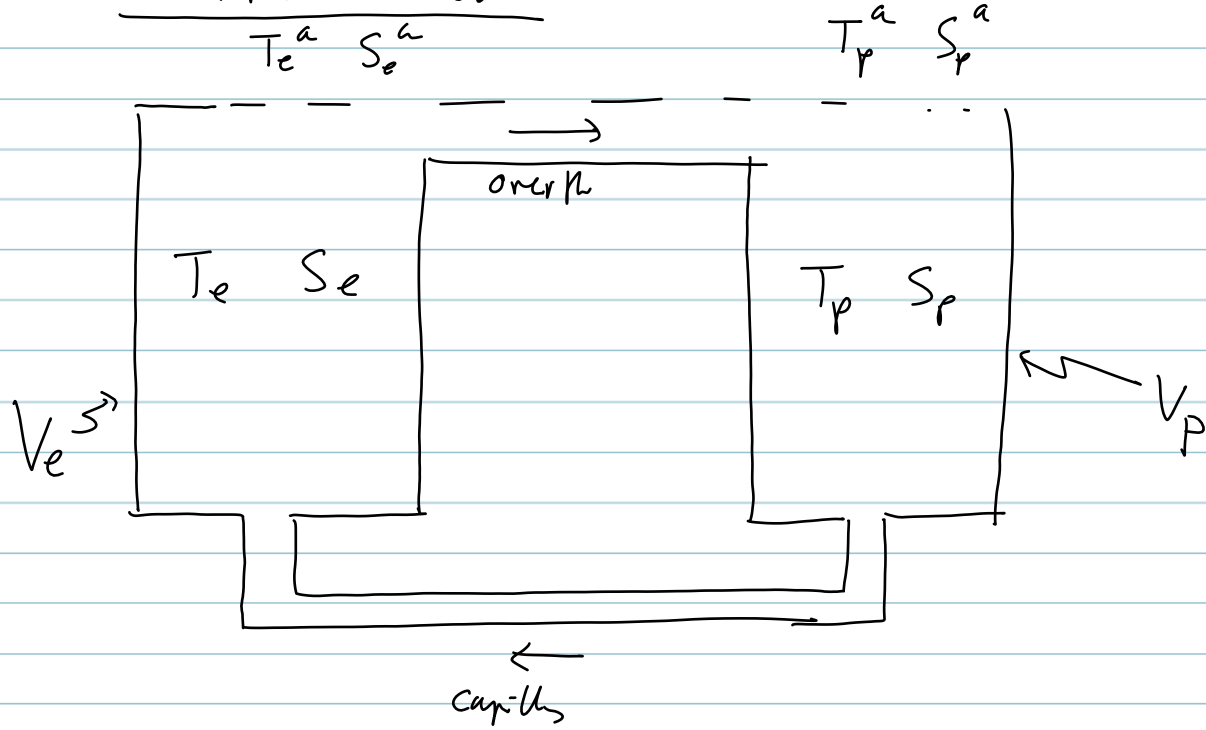
Stommel Model



Flow $\Psi = \gamma \dfrac{\rho_p - \rho_e}{\rho_0}$

ρ_0 reference density
ρ_p pole "
ρ_e equatorial "

$$\rho = \rho_0 \left(1 - \alpha_T (T - T_0) + \alpha_S (S - S_0)\right)$$

T_0 reference temp
S_0 " salinity

Model temperature & salinity in each box under external forcing from atmosphere.

2 effects: (1) left alone, relaxes to atmospheric conditions

(2) flow in overflow & capillary carries heat & salt

Heat & salt balances:

$$V_p \frac{dT_p}{dt} = C^T_p \left(T^a_p - T_p\right) + |\psi| \left(T_e - T_p\right)$$

$$V_e \frac{dT_e}{dt} = C^T_e \left(T^a_e - T_e\right) + |\psi| \left(T_p - T_e\right)$$

$$V_p \frac{dS_p}{dt} = C^S_p \left(S^a_p - S_p\right) + |\psi| \left(S_e - S_p\right)$$

$$V_e \frac{dS_e}{dt} = C^S_e \left(S^a_e - S_e\right) + |\psi| \left(S_p - S_e\right)$$

Realistic forcing:

$$T^a_e - T^a_p > 0$$

$$S^a_e - S^a_p > 0$$

Assume:

$$C_p^T \propto V_p \qquad C_e^T \propto V_e$$

$$C_p^S \propto V_p \qquad C_e^S \propto V_e$$

$$R_T = C^T_p / V_p = C_e^T / V_e$$

$$R_S = C^S_p / V_p = C^S_e / V_e$$

$$\Psi = \gamma\, \frac{\rho_0\left(1-\alpha_T(T_p-T_0)+\alpha_S(S_p-S_0)\right)-\rho_0\left(1-\alpha_T(T_e-T_0)+\alpha_S(S_e-\right.}{\cancel{\rho_0}}$$

$$= \gamma\left[\alpha_T(T_e-T_p)+\alpha_S(S_p-S_e)\right]$$

Note under (relaxed) atmospheric conditions, these two terms are competing.

Look at temperature equations:

$$\frac{dT_p}{dt} = \frac{C_p^T}{V_p}\left(T_p^a - T_p\right) + \frac{\gamma}{V_p}\left|\alpha_T(T_e-T_p)+\alpha_S(S_p-S_e)\right|\left(T_e-T_p\right)$$

$$\Rightarrow \frac{dT_p}{dt} = R_T\left(T_p^a - T_p\right) + \quad ''$$

$$\frac{dT_e}{dt} = R_T\left(T_e^a - T_e\right) + \frac{\gamma}{V_p}\left|\alpha_T(T_e-T_p)+\alpha_S(S_p-S_e)\right|\left(T_p-T_e\right)$$

Set $T = T_p - T_e$, $S = S_p - S_e$

$$\frac{dT}{dt} = R_T(T_p^a - T_e^a) + R_T\left(T_e - T_p\right) + \gamma\left[\frac{|\alpha_T T + \alpha_S S|}{V_p} + \frac{|\alpha_T T + \alpha_S S|}{V_e}\right]$$

$$\frac{dS}{dt} = R_S(S_p^a - S_e^a) + R_S\left(S_e - S_p\right) + \gamma\left(\frac{|\tilde\Psi|}{V_p} + \frac{|\tilde\Psi|}{V_e}\right)\left(S_e - S_p\right)$$

$$\frac{dT}{dt} = R_T(T_p^a - T_e^a) - R_T T - \gamma|\tilde\Psi|\left(\frac{V_e+V_p}{V_eV_p}\right)T \qquad \tilde\Psi = \alpha_T T + \alpha_S S$$

$$\frac{dS}{dt} = R_S(S_p^a - S_e^a) - R_S S - \gamma|\tilde\Psi|\left(\frac{V_e+V_p}{V_eV_p}\right)S$$

$$\frac{dT}{dt} = R_T\eta - R_T T - \gamma|\tilde{\psi}|\left(\frac{V_e+V_p}{V_eV_p}\right)T$$

$$\frac{dS}{dt} = R_S\xi - R_S S - \gamma|\tilde{\psi}|\left(\frac{V_e+V_p}{V_eV_p}\right)S$$

and $\tilde{\psi} = \alpha_T T + \alpha_S S$, $\eta = T_p^a - T_e^a$, $\xi = S_p^a - S_e^a$

Non-dimensionalize:

① change time scale by R_T:

$$\frac{1}{R_T}\frac{dT}{dt} = \frac{dT}{d\tau} \qquad \text{so } \frac{dt}{d\tau} = R_T$$

if $t = R_T\tau$

$$\frac{dT}{d\tau} = -T - \left(\frac{\gamma}{R_T}|\tilde{\psi}|\left(\frac{V_e+V_p}{V_eV_p}\right)T\right) + \eta_1$$

$$\frac{dS}{d\tau} = -\frac{R_S}{R_T}S + \frac{R_\xi}{R_T}\eta_2 - \left(\frac{\gamma}{R_T}|\tilde{\psi}|\left(\frac{V_e+V_p}{V_eV_p}\right)S\right)$$

$$\frac{\gamma}{R_T}|\alpha_T T + \alpha_S S|\left(\frac{V_e+V_p}{V_eV_p}\right)T$$

$$\text{Set} \quad \tilde{T} = \frac{\gamma \sigma_T}{R_T}\left(\frac{V_e + V_p}{V_e V_p}\right) T = \delta_T T$$

above term becomes:

$$\left|\tilde{T} + \tilde{S}\right| \frac{\tilde{T}}{\delta}$$

$$\Rightarrow \frac{dT}{d\tau} = -T + \eta_1 - \left|\tilde{T} + \tilde{S}\right| \frac{\tilde{T}}{\delta_T}.$$

$$\Rightarrow \frac{d\tilde{T}}{d\tau} = -\tilde{T} + \delta_T \eta_1 - \left|\tilde{T} + \tilde{S}\right| \tilde{T}.$$

$$\text{d} \quad \frac{dS}{d\tau} = -\frac{R_S}{R_T} S + \frac{R_S}{R_T}\eta_2 - \left|\tilde{T} + \tilde{S}\right| \frac{\tilde{S}}{\delta_S}$$

$$\frac{d\tilde{S}}{d\tau} = -\frac{R_S}{R_T}\tilde{S} + \frac{R_S}{R_T}\delta_S \eta_2 - \left|\tilde{T} + \tilde{S}\right| \tilde{S}.$$

$\tilde{T} \to T \quad (\bar{T})$ $\qquad \frac{R_S}{R_T} \to \eta_3$

$\tilde{S} \to -S \quad (-\bar{S})$ $\qquad \delta_T \eta_1 \to \eta_1$

$\tau \to t$ $\qquad \frac{R_S}{R_T}\delta_S \eta_2 \to -\eta_2$

$$\boxed{\begin{aligned} \frac{dT}{dt} &= \eta_1 - T\left(1 + |T - S|\right) \\ \frac{dS}{dt} &= \eta_2 - S\left(\eta_3 + |T - S|\right) \end{aligned}}$$

$T = T_e - T_p$

$S = S_e - S_p$