Geophysical Fluid Dynamics

Both atmosphere & ocean governed the eqns. of GFD.

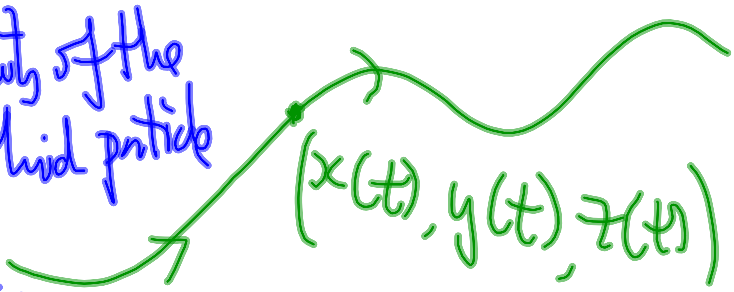
Derivation based on

1. Newton's laws of motion $F = ma$
2. Two changes of reference-frame.

Equations govern the motion of a fluid particle: @

$(x, y, z) \in \mathbb{R}^3$

for velocity of the fluid particle



Eqns. give $u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$, $w = \frac{dz}{dt}$

3 other key physical quantities:

Density: $\rho = \rho(x, y, z, t)$

Pressure: $p = p(x, y, z, t)$

Temperature: $T = T(x, y, z, t)$

Ocean: Salinity: $S = S(x, y, z, t)$

Atmos: Humidity: $q = q(x, y, z, t)$

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Frames of Reference:

① Lagrangian vs. Eulerian.

Lagrangian: live @ fluid particle & flow with it

Eulerian: fixed position & watching fluid go by

② Rotating vs. Inertial

Rotating: rotating w/ Earth

Inertial: absolute frames watch the Earth rotate.

US: Eulerian & Rotating.
Laws of physics: Lagrangian & inertial

Transformations:

Lagrangian → Eulerian

Consider a scalar quantity

$f = f(x, y, z, t)$

$\left(\frac{df}{dt}\right)_L$ = derivative wrt time in Lagrangian frame

$\left(\frac{df}{dt}\right)_E$ = " " " " Eulerian frames

$$f = f(x, y, z, t)$$

$$(x(t), y(t), z(t)) \Leftarrow$$ fluid particle (moving in time)

Consider $f = f(x(t), y(t), z(t), t)$

$$\left(\frac{df}{dt}\right)_L = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} + \frac{\partial f}{\partial t}$$

$$= U \cdot \nabla f + \left(\frac{df}{dt}\right)_E$$

$$U = (u, v, w)$$

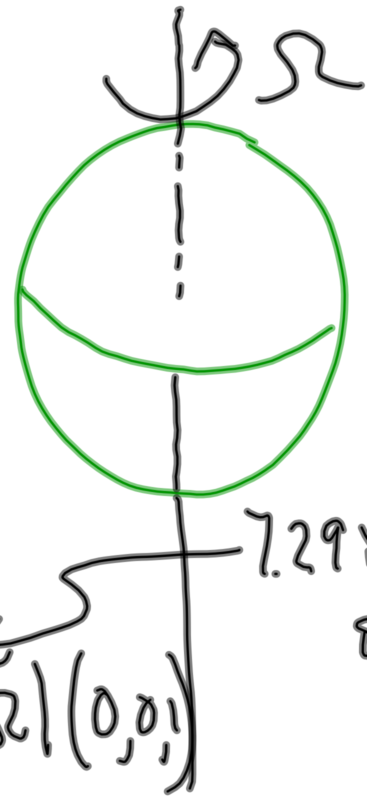
$$\left(\frac{df}{dt}\right)_E$$

Inertial $\to$ Rotating.

$F = F(x,y,z,t) \in \mathbb{R}^3$

e.g. $\vec{F} = (u, v, w)$

$$\left(\frac{dF}{dt}\right)_I = \left(\frac{dF}{dt}\right)_R + \Omega \times F$$



$7.29 \times 10^{-5}\ s^{-1}$

$\Omega = |\Omega|(0,0,1)$

Newton's Law:

$$F = ma$$

Since thinking in a continuum limit of point (masses) of fluid, look at eqns. in terms of mass / unit volume, i.e. density.

$$\rho \left(\frac{d}{dt} v_I \right)_{t,I} = \text{sum of forces acting on fluid particle}$$

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Forces on RHS:

$$- F_{gravity} - F_{pressure} + F_{friction}$$

gravitational force of Earth

pressure force of ambient fluid.

more mysterious!

Note: signs are conventional

Friction forces:

1. internal friction i.e., molecular viscosity in negligible for atmosph and ocean.
2. A non-trivial viscosity term is usually added to account for sub-grid-scale turbulent effects,
3. Friction from boundaries:
   - a) wind @ surface of oceans
   - b) friction from sea floor.

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$$r = (x, y, z)$$

Want $v_I = \left(\frac{dr}{dt}\right)_I = \left(\frac{dr}{dt}\right)_R + \Omega \times r$

$$\left(\frac{d}{dt} v_I\right)_I = \frac{d}{dt}\left(v_R + \Omega \times r\right)_I$$

$$= \left(\frac{d}{dt} v_R\right)_I + \frac{d}{dt}\left(\Omega \times r\right)_I$$

$$= \left(\frac{dU_R}{dt}\right)_R + 2\Omega \times U_R + \Omega \times \Omega \times r$$

N's 2nd law:

$$\rho\left(\left(\frac{du_R}{dt}\right)_R + 2\Omega \times U_R + \Omega \times \Omega \times r\right) = -F_g - F_p + F_f$$

drop the subscripti

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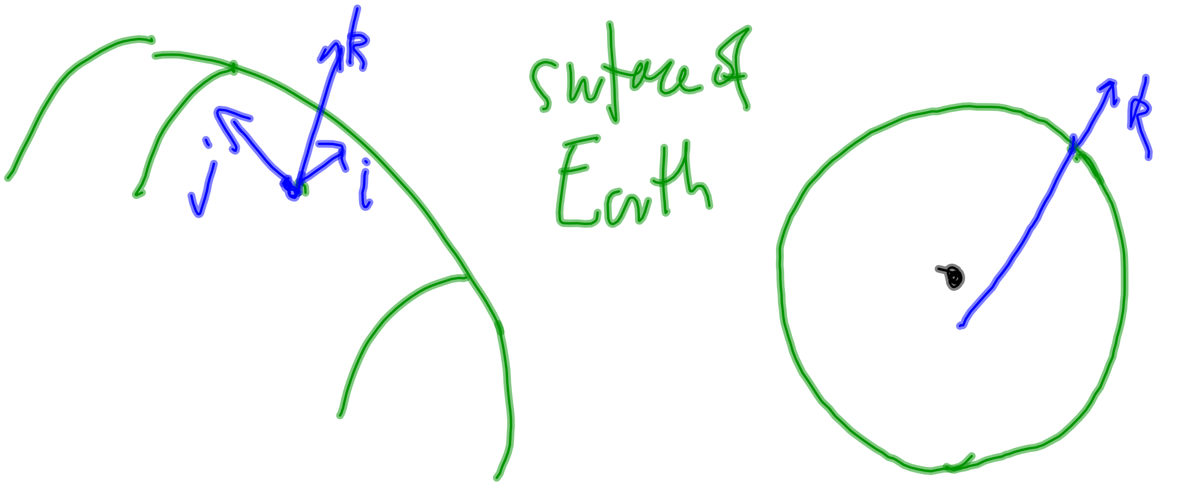
$$\rho\left(\frac{dU}{dt} + 2\Omega\times U + \Omega\times\Omega\times r\right) = \sum \text{forces.}$$

Coriolis Centrifugal force.

In eqs. of GFD, $\Omega\times\Omega\times r$ not present.



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To a good approx:

Force due to gravity $= -g\rho k + \Omega \times \Omega \times r$

account for oblateness of Earth. Thus the centrifugal term is neglected.

$$\rho\left(\frac{dU}{dt} + 2\Omega \times U\right) = -g\rho k - \nabla p + G$$

frictional force

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But $\frac{dU}{dt} = \left(\frac{dU}{dt}\right)_L = \frac{\partial U}{\partial t} + U\cdot\nabla U$

$$\Rightarrow \boxed{\frac{\partial U}{\partial t} + U\cdot\nabla U + 2\Omega\times U = -gk - \frac{\nabla p}{\rho} + \tilde{G} \qquad \tilde{G} = G/\rho}$$

$U = (u, v, w)$ This is 3 eqns. Actually con. of mom.

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Problem: 3 eqns. in 5 unknowns.
(u, v, w, ρ, p)

Need 2 more eqns.

Conservation of mass $\frac{\partial \rho}{\partial t} + \nabla \cdot \rho U = 0$

Conservation of energy. ← brings in temperature.

Need eqn. of state ← temp → density, pressure etc

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