

Problems for 10/1

① If $\phi_{0m}(y) = \sqrt{2} \cos \frac{my}{b}$

and $\phi_{nm}(y) = \sqrt{2} e^{inx} \sin \frac{my}{b}$

then $\nabla^2 \phi_{nm} = \lambda_{nm} \phi_{nm}$, $|n| = 0, 1, \dots; m = 1, 2, \dots$

Calculate λ_{nm} .

② Show that $\int_0^{2\pi} e^{inx} dx = \begin{cases} 0 & \text{if } n \neq 0 \\ 2\pi & \text{if } n = 0 \end{cases} (n \in \mathbb{Z})$

③ Consider the four integrals:

$$\int_0^{\pi} \cos mz \cos nz \cos kz dz$$

$$\int_0^{\pi} \cos mz \cos nz \sin kz dz$$

$$\int_0^{\pi} \cos mz \sin nz \sin kz dz$$

$$\int_0^{\pi} \sin mz \sin nz \sin kz dz.$$

If each of m, n, k are $0, 1$ or -1 , what are different possible values of these integrals? Most importantly, find the combination of values of m, n, k that make them zero (respectively non-zero).