Uncertainty of Rogue Wave Formation in the Nonlinear Schrödinger Equation

by Andy Reagan

October 2, 2012

Figure 0: Rogue wave?

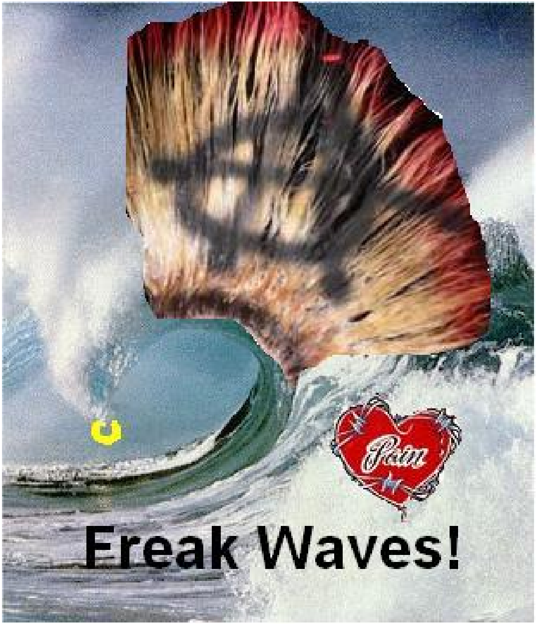

Figure 1: Definitely rogue

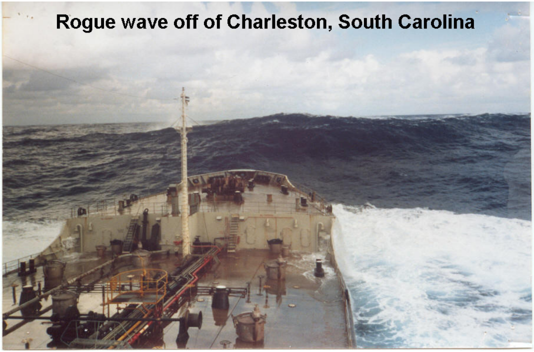

Figure 2: Observed rogue wave

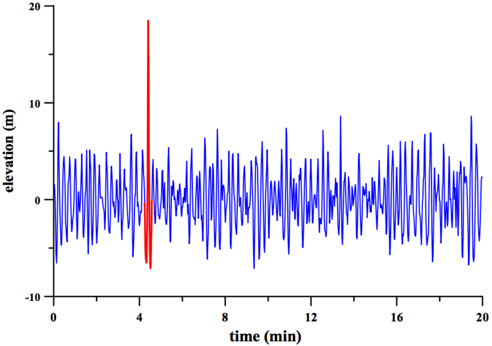

Figure 3: The "Draupner Wave"

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- The structure and nature of these waves is imperative in our understanding of how and why they occur.
- Occur in both water and optical waves.

- In oceanography, the pragmatic approach is to define a rogue wave whenever

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where H is the wave height (distance from trough to crest), ω_c is the crest height (distance from mean sea level to crest), and H_s is the significant wave height, here defined as four times the standard deviation of the surface elevation.

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- Known to cause extensive damage, and are even life-threatening, when they come into contact with ocean liners and passenger ships in the open waters.
- Between 1964 and 1994, it is estimated that more than 22 super-carriers have been lost at sea as a direct result of rogue waves (Kharif & Pelinovsky 2003).

Physical mechanisms of rogue wave formation

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- Linear and non-linear spatial focusing
- Non-linear modulation instability

Rogue Wave Models

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$$iu_t + u_{xx} + 2|u|^2 u = 0$$

- Rogue wave like solution first discovered by Peregrine in 1983

- This Peregrine Soliton is a first order rational soliton, and limiting case of the known Akmediev and Ma breathers

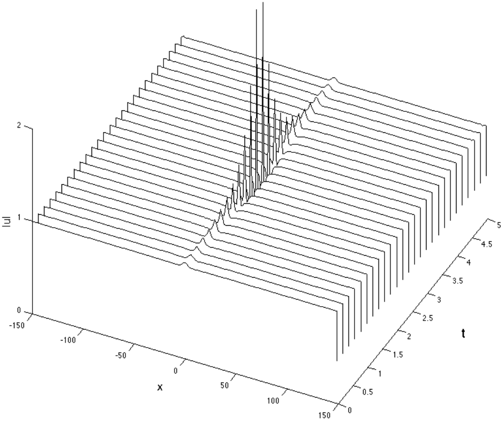

Figure 4: The Peregrine Soliton

UQ

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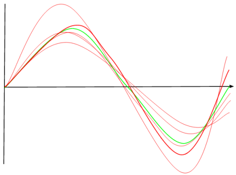
- Methods:
  - Monte Carlo sampling
  - Polynomial chaos expansions
  - Most probable point methods
  - Perturbation method
  - Dimension reduction

Example UQ Problems

- Broad: weather prediction



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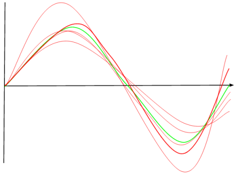
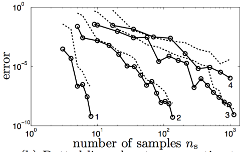


- Specific: recovering flight data from HyShot II experiment



(b) Dotted lines denote error estimate

UQ of PS formation

- Want to characterize formation of PS with uncertain initial condition

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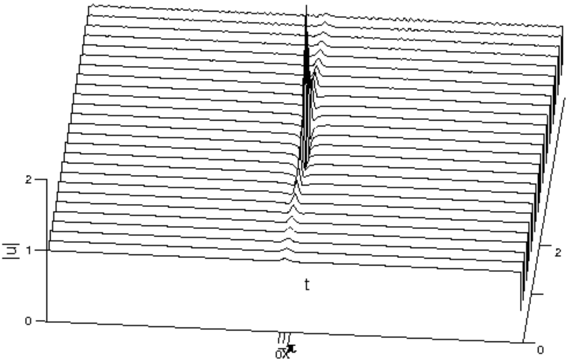
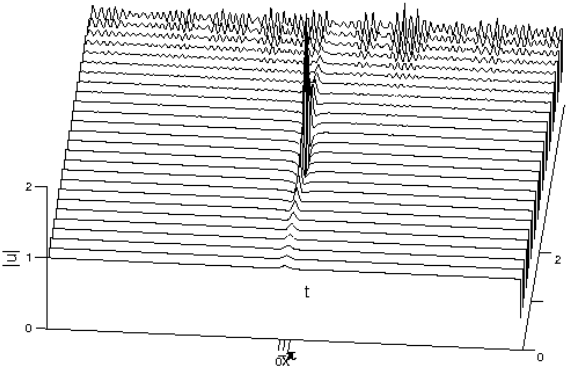
- Can control parameters of noise
  - Random (gaussian) noise: amplitude
  - Superposition of random waves: steepness, wavelength, amplitude



peregrine 16384pts 256plotted point0001eps



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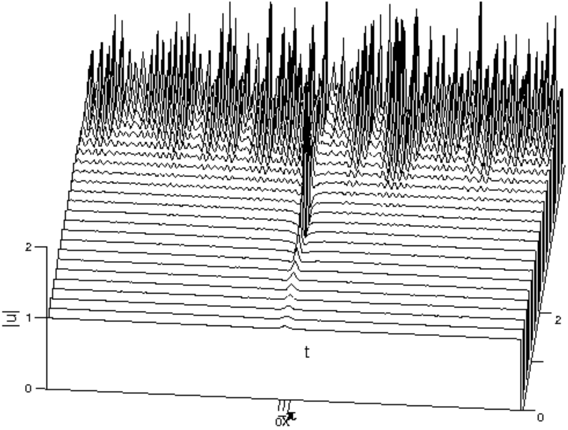

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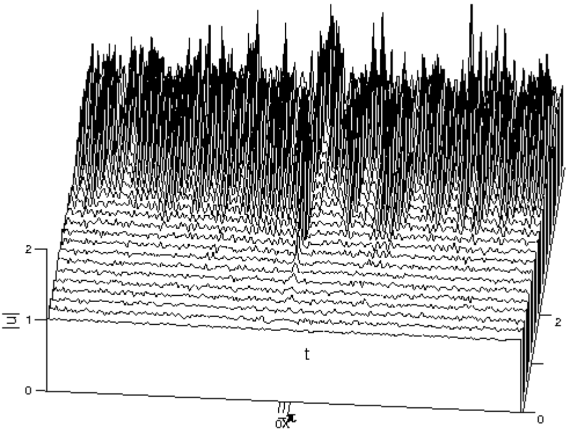

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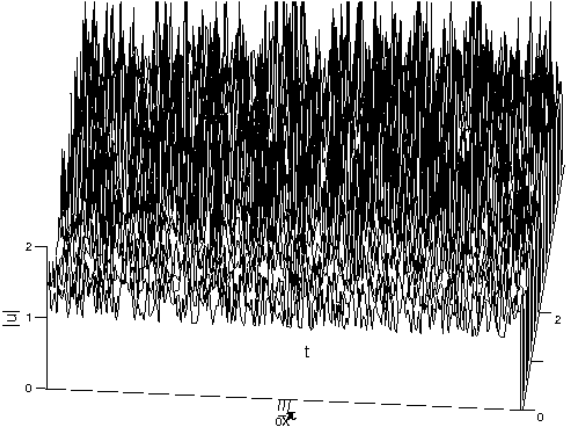

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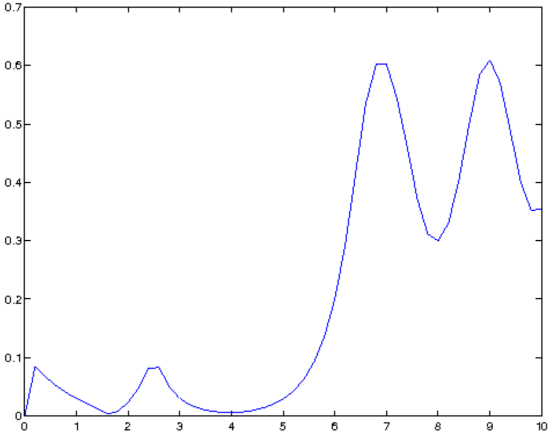
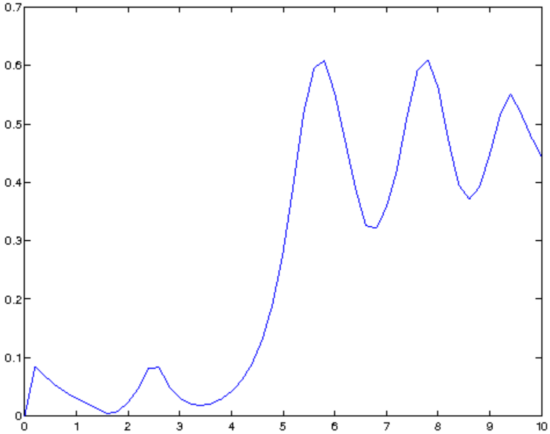


16384pts error point0001eps long



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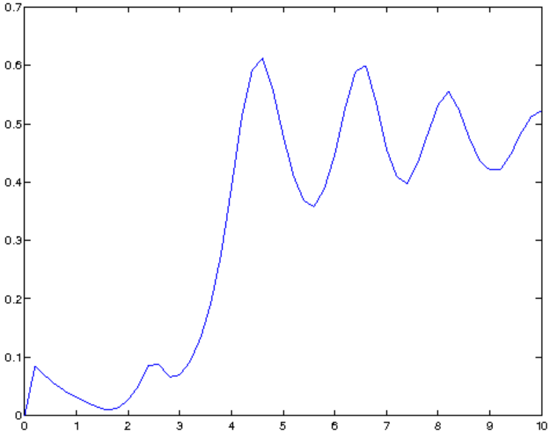

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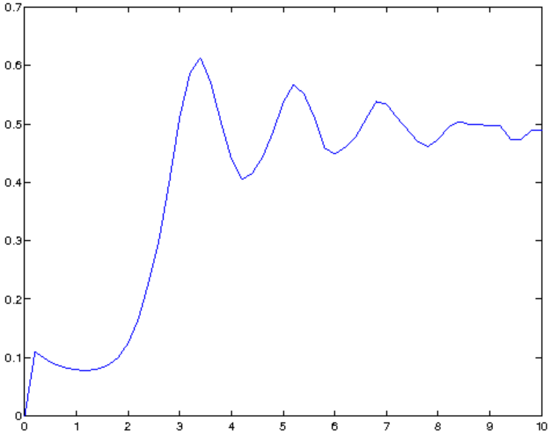

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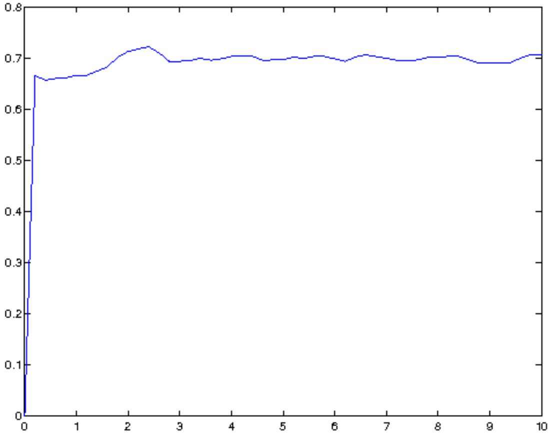

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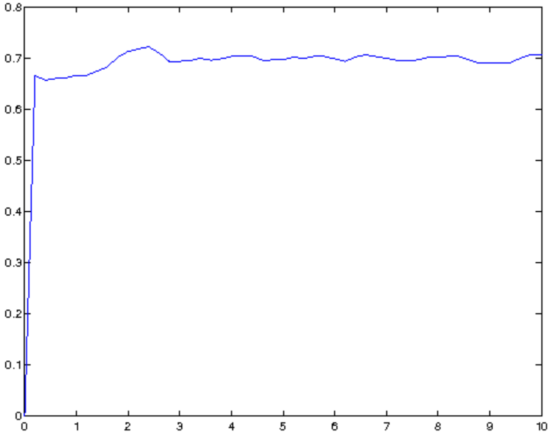

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Next steps

- Create error from wave superposition

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- Extend to 3-Dimensional NLS
  - With random wave superposition, look for parameters leading to more freak waves