Homework 3, Due Tuesday, Feb. 28

1. For the linear equation

$$u_t = (L + M)u,$$

where L and M are linear time-independent operators, prove that the Strang splitting scheme

$$u_{n+1} = e^{Lh/2} e^{Mh} e^{Lh/2} u_n$$

has second-order accuracy in time (here h is the time-step size).

2. Use the second-order (Strang) split-step method to solve the Gross-Pitaevskii equation

$$iU_t + U_{xx} - x^2 U + |U|^2 U = 0,$$

on the x interval $[-40,\ 40]$ for $0 \le t \le 2$ with the following initial condition

$$U(x, 0) = e^{-x^2/2}.$$

Choose your Δx and Δt values so that your numerical solution has accuracy no less than 10^{-4}. Turn in your numerical code as well as the numerical results.