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Emergence of rogue waves from optical turbulence

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Abstract: We show the emergence of rogue wave events from optical turbulence by analyzing the long term evolution of the field. In particular, we identify three turbulent regimes depending on the incoherence in the system.

OCIS codes: (190.0190) Nonlinear optics; (190.5530) Pulse propagation and temporal solitons; (030.1640) Coherence; (030.6600) Statistical optics.

1. Introduction

The understanding of the mechanisms underlying the process of optical rogue wave generation is a subject of growing interest. Several works [1-5] have been focused on the study of two well-known classes of solutions of the NonLinear Schrödinger Equation (NLSE), namely soliton or quasi-soliton solutions and, on the other hand, the Peregrine soliton and its generalizations, namely the Akhmediev Breathers (AB) solutions. Here, we propose an approach for the understanding of optical rogue waves based on the study of the impact of the incoherence in the nonlinear system.

2. Model

We consider the one-dimensional NLSE in the presence of Third-Order Dispersion (TOD):

$$i\frac{\partial u}{\partial z} = -\frac{1}{2}\frac{\partial^2 u}{\partial t^2} + i\sigma\frac{\partial^3 u}{\partial t^3} - |u|^2 u \qquad (1)$$

We normalized the problem with respect to the nonlinear length $L_0 = 1/\gamma P$ and time $\tau_0 = (\beta_2 L_0)^{1/2}$, where γ is the nonlinear coefficient, P the average power of the field and β_2 the second-order dispersion coefficient. In these units, the normalized TOD coefficient reads $\sigma = \beta_3/(6\, L_0^{1/2}\, \beta_2^{3/2})$, β_3 being the TOD coefficient. The NLSE conserves the 'energy' (Hamiltonian) $H = H_L + H_{NL}$, which has a linear kinetic contribution $H_L = \int k(\omega)\, |u(\omega)|^2\, d\omega$ and a nonlinear contribution $H_{NL} = -1/2 \int |u(t)|^4\, dt$, where $k(\omega) = \omega^2/2 - \sigma\, \omega^3$ is the linear dispersion relation. This Hamiltonian is known to provide a natural measure of the amount of incoherence in the system [6]. We study here the transition from the coherent quasi-soliton regime to the fully turbulent regime by increasing the energy H of the system and by keeping constant the power $N = \int |u(t)|^2\, dt$.

3. Results

We performed intensive numerical simulations of Eq.(1) with periodic boundary conditions. Contrary to previous studies [2-4], we are interested here in the long term evolution of the system (typically $z \sim 2000\ L_0$). Figure 1a reports the average of the maximum intensity peak detected in the temporal window in the last stage of propagation (i.e., over $500\ L_0$) as a function of the conserved energy H.

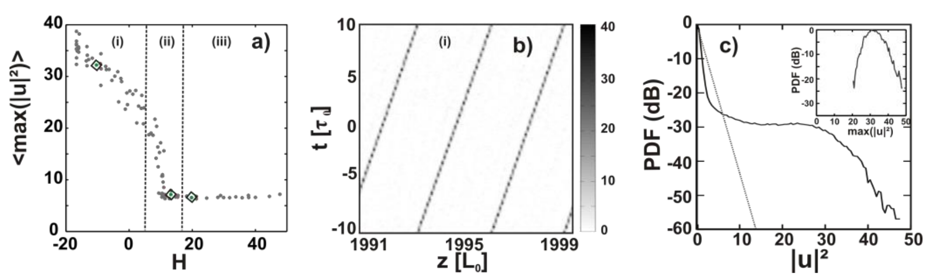

Fig. 1: **(a)** Average of the maximum intensity peak detected in the temporal window during the last stage of propagation ($500\ L_0$) as a function of the Hamiltonian H, for $\sigma = 0.02$. The power is kept constant $N = \int |u(t)|^2\, dt$. The green dots (or diamonds) in (a) are analyzed separately. **(b)** Space-time intensity pattern showing a coherent quasi-soliton propagating in the midst of small-scale fluctuations for H = -10.3 (Regime (i)). **(c)** PDF of the intensity of the field and of the maxima of the intensity (inset). The gray line stands for Gaussian statistics of the field amplitude.

More precisely, we identified in Fig. 1a three different regimes: For $H \leq 5$, we recover the quasi-soliton turbulent regime (i): a large amplitude quasi-soliton immersed in a sea of small-scale fluctuations eventually emerges from multiple inelastic collisions. Figure 1b represents the space-time intensity pattern over the last 10 nonlinear lengths. The corresponding Probability Distribution Functions (PDF) of the intensity of the field, and of the maxima of the intensity, are reported in Fig. 1c. We point out that, because of the large localization of the power in the quasi-soliton, the field statistics deviates substantially from the Gaussian statistics, i.e. $f_I(I) = \exp(-I)$ with our normalized units. This observation is corroborated by the PDF of the intensity maxima, which is centered at a high value (~ 30) and exhibits a narrow width and symmetric shape, thus confirming the persistence of the high power quasi-soliton structure.

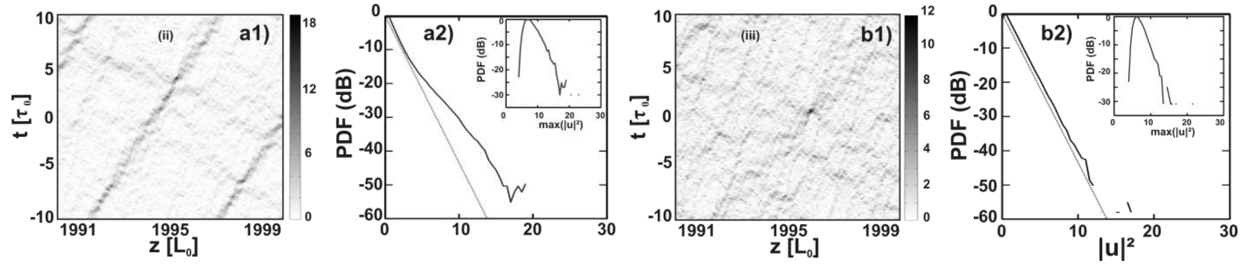

Fig. 2**: (a1,b1)** Space-time intensity pattern for H = 13.3 (Regime (ii)) and H = 19.5 (Regime (iii)). **(a2,b2)** Corresponding PDF of the intensity of the field and of the maxima of the intensity (insets).

For $5 \leq H \leq 15$, the system enters into the second regime (ii), in which the quasi-soliton structures exhibit a kind of intermittent dynamics. It is interesting to note that, despite such intermittent-like behavior, the trajectory of the quasi-soliton may still be identified in the space-time intensity pattern (see Fig. 2a1). We note that, as for the regime (i), the field does not exhibit a Gaussian statistics. However, the PDF of the maxima of the intensity gets relatively broader and asymmetric (see Fig. 2a2). This is due to the intermittent (non-persistent) character of the quasi-soliton structure, whose significant fluctuations tend to favor the high intensity tail of the PDF. The resulting asymmetry in the PDF bears a strong resemblance with the typical L-shaped probability distributions that characterize freak-wave extreme events. Finally, for $H \geq 15$, the system enters into the regime (iii) which refers to the genuine turbulent regime, which can be described in detail by the wave turbulence theory. Note that this kinetic approach provides an analytical description of the evolution of the wave spectrum, which has been found in quantitative agreement with the numerical simulations, without using adjustable parameters. This regime (iii) is fully incoherent and weakly nonlinear therefore quasi-soliton structures are no longer generated. The corresponding spatiotemporal intensity pattern reveals that very short-lived rogue wave events may still emerge from the turbulent field, although these extreme events become rare (see Fig. 2b1). The generation of intense flashes in regime (iii) is not related to coherent quasi-soliton structures. This is corroborated by the statistics of the field, which approaches a Gaussian statistics, as expected for a fully incoherent system of weakly nonlinear waves. For higher values of H (H ~ 30), the system becomes essentially 'linear', i.e. $|H_{NL}| \ll |H_L|$.

4. Conclusion

In conclusion, we provided some general physical insights into the spontaneous emergence of extreme events from turbulent fluctuations. The new optical rogue waves reported in the regime (ii) and, more importantly in regime (iii), seem to exhibit properties analogous to hydrodynamic rogue wave events, a feature that needs to be analyzed in more detail so as to draw a substantiated analogy between them. Actually, recent numerical investigations reveal that the sporadic rogue wave events in regime (iii) exhibit properties reminiscent of Peregrine solitons and high-orders Akhmediev breathers.

5. References

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