Problems **Assume one violin can generate a sound of 55dB. What is the sound level (in dB) generated by 3 such violins?**

$$\textbf{Sound level for one violin: } 10\log_{10} r_1 = 55 \quad \textbf{Sound level for three violins: } 10\log_{10} 3r_1 = X$$

$$\text{Apply log rules: } 10\log_{10} 3r_1 = X \rightarrow 10\log_{10} 3 + 10\log_{10} r_1 = X$$

Suppose a sound from source A is 80dB, and another sound from source B is 40dB. How much is the source A sound larger than the source B sound in terms of power? In terms of voltage?

$$\textbf{Source A: } 10\log_{10}\Gamma_A = 80 \quad \textbf{Source B: } 10\log_{10}\Gamma_B = 40$$

$$\gamma_A = \frac{P_A}{P_{\text{noise}}} \quad \gamma_B = \frac{P_B}{P_{\text{noise}}}$$

$$10\log_{10}\Gamma_A = 80 \rightarrow \log_{10}\Gamma_A = 8 \rightarrow \Gamma_A = 10^8 \quad \Gamma_B = 10^4$$

$$\frac{\Gamma_A}{\Gamma_B} = \frac{\frac{P_A}{P_{\text{noise}}}}{\frac{P_B}{P_{\text{noise}}}} \rightarrow \frac{\Gamma_A}{\Gamma_B} = \frac{P_A}{P_B} = \frac{10^8}{10^4} = 10^4$$

$$\text{Power ratio} = \frac{P_A}{P_B} = 10^4, \text{Voltage ratio} = \boxed{\sqrt{\frac{P_A}{P_B}}} = \sqrt{10^4} = 10^2$$

In a very quiet room, a mic's output voltage is 0.5V. When this mic is moved to a busy street, what will be the output voltage?

$$\textbf{Very quiet room: } 20\text{dB} \quad \textbf{Busy street: } 70\text{dB}$$

$$20\log_{10} r_{\text{room}} = 20 \rightarrow \log_{10} r_{\text{room}} = 1 \rightarrow r_{\text{room}} = 10^1$$

$$20\log_{10} r_{\text{street}} = 70 \rightarrow \log_{10} r_{\text{street}} = 3.5 \rightarrow r_{\text{street}} = 10^{3.5}$$

$$\text{Voltage ratio} = \frac{r_{\text{street}}}{r_{\text{room}}} \rightarrow \frac{10^{3.5}}{10^1} = 10^{2.5}$$

$$\text{Therefore, the output voltage of the mic when moved is } 10^{2.5} \times 0.5V$$

Dithering Process

$$\text{Starting pixel value: } V_s = 111 \text{ (invert value)} \rightarrow 255 - V_s \rightarrow 255 - 111 = 144$$

$$\text{Dithering matrix: } \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$$

Because the dither matrix is 2x2 and the grayscale is 0..255 we need to map the values into 0..4

$$256 \div 5 = 51.2 \text{ therefore } 0..51.2 \rightarrow 0, 51.2..102.4 \rightarrow 1, 102.4..153.6 \rightarrow 2, 153.6..204.8 \rightarrow 3, 204.8..256 \rightarrow 4$$

The inverted pixel value = 144, which falls into 102.4..153.6, therefore it maps to an intensity of 2

We then check against the dither-matrix, if the intensity is > the matrix entry, print a dot, otherwise print nothing

$$144\text{(intensity 2)}, \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2>0 & 2=2 \\ 2<3 & 2>1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ where } 1 = \text{print dot and } 0 = \text{no dot}$$

$$\text{another example, dither matrix: } \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \text{ inverted and mapped image: } \begin{pmatrix} 0 & 2 \\ 3 & 2 \end{pmatrix}$$

now compare each entry in the image to every entry in the dither matrix generating a 2x2 matrix for each image entry

$$0, \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - 2, \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} - 3, \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} - 2, \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\text{putting it all together into the new dithered image: } \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Ordered Dithering Process

Instead of replacing each pixel with a matrix, we slide the matrix over the image and compare one to one pixel value and entry

$$\begin{pmatrix} 0 & 1 & 1 & 3 \\ 4 & 2 & 1 & 3 \\ 2 & 2 & 4 & 1 \\ 1 & 2 & 3 & 0 \end{pmatrix} \text{ this matrix represents the image already inverted and mapped to intensity levels}$$

$$\begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} \text{ this matrix represents the ordered-dithering matrix}$$

we now slide the OD matrix over the image chunk by chunk and compare entry to entry

$$\begin{pmatrix} 0 & 1 & 1 & 3 \\ 4 & 2 & 1 & 3 \\ 2 & 2 & 4 & 1 \\ 1 & 2 & 3 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0=0 & 1<2 & 1 & 3 \\ 4>3 & 2>1 & 1 & 3 \\ 2 & 2 & 4 & 1 \\ 1 & 2 & 3 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \mathbf{0} & \mathbf{0} & 1 & 3 \\ \mathbf{1} & \mathbf{1} & 1 & 3 \\ 2 & 2 & 4 & 1 \\ 1 & 2 & 3 & 0 \end{pmatrix} \rightarrow$$

$$\text{and so on until then result } \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$