Session 1b notes

Causal Methods in Public health

Counterfactual notation

Consider Zeus and Hera who both have a heart transplant, but Zeus dies and Hera lives. We also *know* that if Zeus had not received the heart transplant, he would have lived, but Hera would have lived either way:

$$[Y_Z^{a=1} = 1] \neq [Y_Z^{a=0} = 0]$$

$$[Y_H^{a=1} = 1] = [Y_H^{a=0} = 1]$$

Individual cause vs. Average cause

Individual causal effects:

$$Y_i^{a=1} - Y_i^{a=0} \neq 0$$

Average or marginal causal effects:

$$P(Y_i^{a=1}) - P(Y_i^{a=0}) \neq 0$$
$$E(Y_i^{a=1}) - E(Y_i^{a=0}) \neq 0$$

Identifiability assumptions 1: Consistency

- For each individual, one of the counterfactual outcomes - the one that corresponds to the treatment value that the individual did receive - is *actually factual*
- That is, an individual with observed treatment A equal to a, has observed outcome Y equal to his counterfactual outcome Y^a
- This equality can be succinctly expressed as $Y = Y^A$ where Y^A denotes the counterfactual Y^a evaluated at the value a corresponding to the individual's observed treatment A
- The equality $Y = Y^A$ is referred to as *consistency*

Identifiability assumptions 2: Positivity

$P(A = a|L = l) > 0$
(for all values of a $\in$ A and l $\in$ L.)

For example, $(Y^{obs}, Y^{mis}) = (Y(1), Y(0))$ where $Y(1)$ is the observed Y when $A = 1$, and $Y(0)$ is unobserved.

Identifiability assumptions 3a: exchangeability

Treated and untreated individuals are *exchangeable* when the assignment of treatment does not depend on the potential outcomes, i.e., when there is no unmeasured confounding that is driving both treatment and outcome.
In randomized experiments, the treated and the untreated are exchangeable because the treated, had they remained untreated, would have experienced the same average outcome as the untreated did, and vice versa.

$$P(Y^a = y | A = 1) = P(Y^a = y | A = 0) = P(Y^a = y)$$

$$Y^a \perp\!\!\!\perp A$$

Identifiability assumptions 3b: conditional exchangeability

In observational studies, when treatment is not randomly assigned by the investigators, the reasons for receiving treatment are likely to be associated with some outcome predictors (L).
If the conditional probability of receiving every value of treatment depends only on measured covariates L:

$$P(Y^a = y|A = 1, L) = P(Y^a = y|A = 0, L) = P(Y^a = y|L)$$

$$Y^a \perp\!\!\!\perp A|L$$