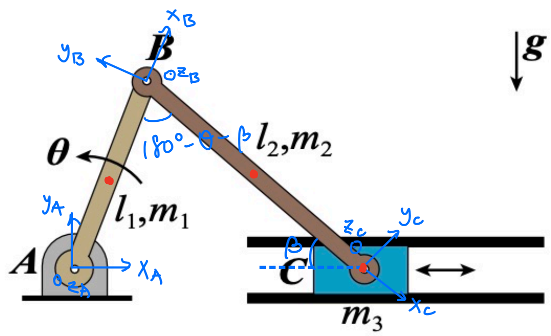
Name: Andy Tsai, ASUID: 1233653410, RAS545 HW4



$$l_2 \sin\beta = l_1 \sin\theta \Rightarrow \beta = \sin^{-1}\left(\frac{l_1}{l_2}\sin\theta\right)$$

$$\sin\beta = \frac{l_1}{l_2}\sin\theta \Rightarrow \cos\beta = \frac{\sqrt{l_2^2 - l_1^2 s^2\theta}}{l_2}$$

a). DH- Table:

	θ	d	a	α
A−B	θ	0	l_1	0
B−C	$-(\theta+\beta)$	0	l_2	0

$$^{A}T_B = \begin{bmatrix} c\theta & -s\theta & 0 & l_1 c\theta \\ s\theta & c\theta & 0 & l_1 s\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{B}T_C = \begin{bmatrix} c(\theta+\beta) & s(\theta+\beta) & 0 & l_2 c(\theta+\beta) \\ -s(\theta+\beta) & c(\theta+\beta) & 0 & -l_2 s(\theta+\beta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{A}T_C = \begin{bmatrix} c\beta & s\beta & 0 & l_2 c\beta + l_1 c\theta \\ -s\beta & c\beta & 0 & l_1 s\theta - l_2 s\beta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

· Position of masses:

$$Pos_{m_1} = \begin{bmatrix} \frac{l_1}{2}c\theta \\ \frac{l_1}{2}s\theta \\ 0 \end{bmatrix} ; \quad Pos_{m_2} = \begin{bmatrix} l_1 c\theta + \frac{l_2}{2}c\beta \\ \frac{l_2}{2}s\beta \\ 0 \end{bmatrix} ; \quad Pos_{m_3} = \begin{bmatrix} l_1 c\theta + l_2 c\beta \\ 0 \\ 0 \end{bmatrix}$$

b) · Potential Energy:

$$P_{tot} = P_{m_1} + P_{m_2} + P_{m_3} = m_1 g\left(\frac{l_1}{2} s\theta\right) + m_2 g\left(\frac{l_1}{2} s\theta\right) + 0 = (m_1+m_2) g\left(\frac{l_1}{2} s\theta\right)$$

· Kinetic Energy:

$$K_{tot} = K_{m_1} + K_{m_2} + K_{m_3}$$

$$= \left(\frac{1}{2} m_1 V_{m_1}^2 + \frac{1}{2} I_{m_1} W_{m_1}^2\right) + \left(\frac{1}{2} m_2 V_{m_2}^2 + \frac{1}{2} I_{m_2} W_{m_2}^2\right) + \left(\frac{1}{2} m_3 V_{m_3}^2 + 0\right)$$

$$V_{m_1}^2 = |\dot{Pos}_{m_1}|^2 = \left(\frac{l_1}{2}\dot\theta\right)^2$$

$$V_{m_2}^2 = |\dot{Pos}_{m_2}|^2 = \left(l_1 s\theta\dot\theta + \frac{l_2}{2} s\beta\dot\beta\right)^2 + \left(\frac{l_2}{2} c\beta\dot\beta\right)^2$$

$$V_{m_3}^2 = |\dot{Pos}_{m_3}|^2 = \left(l_1 s\theta\dot\theta + l_2 s\beta\dot\beta\right)^2$$

$$W_{m_1}^2 = \dot\theta^2 \ ; \ W_{m_2}^2 = \dot\beta^2$$

$$K_{tot} = \frac{l_1^2}{6} m_1 \dot\theta^2$$

$$+ \frac{1}{2} m_2 \left[\left(l_1 s\theta\dot\theta + \frac{l_2}{2} s\beta\dot\beta\right)^2 + \left(\frac{l_2}{2} c\beta\dot\beta\right)^2 + \frac{l_2^2}{12}\dot\beta^2\right]$$

· Lagrangian of the system:

$$L = K_{tot} - P_{tot}$$

$$= \frac{l_1^2}{6} m_1 \dot\theta^2 + \frac{1}{2} m_2 \left[\left(l_1 s\theta\dot\theta + \frac{l_2}{2} s\beta\dot\beta\right)^2 + \left(\frac{l_2}{2} c\beta\dot\beta\right)^2 + \frac{l_2^2}{12}\dot\beta^2\right]$$

$$+ \frac{1}{2} m_3 \left(l_1 s\theta\dot\theta + l_2 s\beta\dot\beta\right)^2 - (m_1+m_2) g\left(\frac{l_1}{2} s\theta\right)$$

c)

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) - \frac{dL}{d\theta} = 0$$

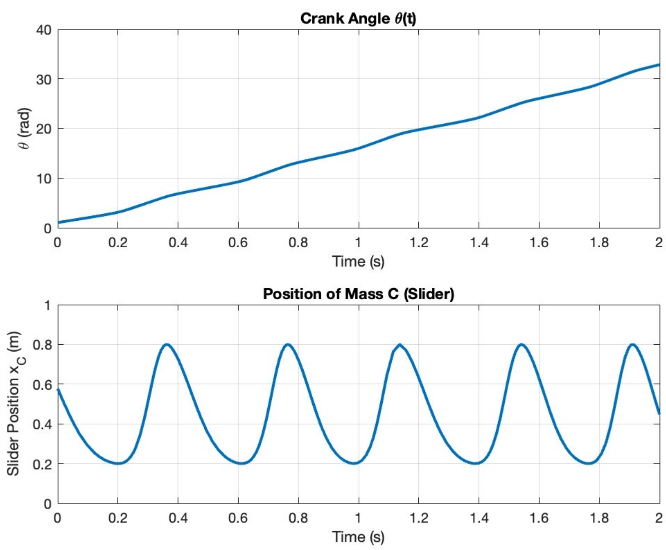
$$\Rightarrow \frac{l_1^2}{3} m_1 \ddot{\theta} + l_1 m_3 s\theta \left(l_2 c\beta \dot{\beta}^2 + l_1 c\theta \dot{\theta}^2 + l_2 s\beta \ddot{\beta} + l_1 s\theta \ddot{\theta} \right)$$

$$+ \frac{1}{2} (l_1 m_2 s\theta (l_2 s\beta \dot{\beta}^2 - l_2 c\beta \ddot{\beta} + 2 l_1 c\theta \dot{\theta}^2 + 2 l_1 s\theta \ddot{\theta}) + \frac{l_1}{2} g c\theta (m_1 + m_2) = 0$$

where $\beta = s^{-1}\left(\frac{l_1}{l_2} s\theta\right)$, $\dot{\beta} = \frac{l_1 c\theta}{\sqrt{l_2^2 - l_1^2 s^2\theta}} \dot{\theta}$, $\ddot{\beta} = \frac{l_1 c\theta \ddot{\theta} - l_1 s\theta \dot{\theta}^2}{\sqrt{l_2^2 - l_1^2 s^2\theta}} + \frac{l_1^3 c^2\theta \, s\theta \, \dot{\theta}^2}{\left(\sqrt{l_2^2 - l_1^2 s^2\theta}\right)^3}$

d)

MATLAB:



Simulink: