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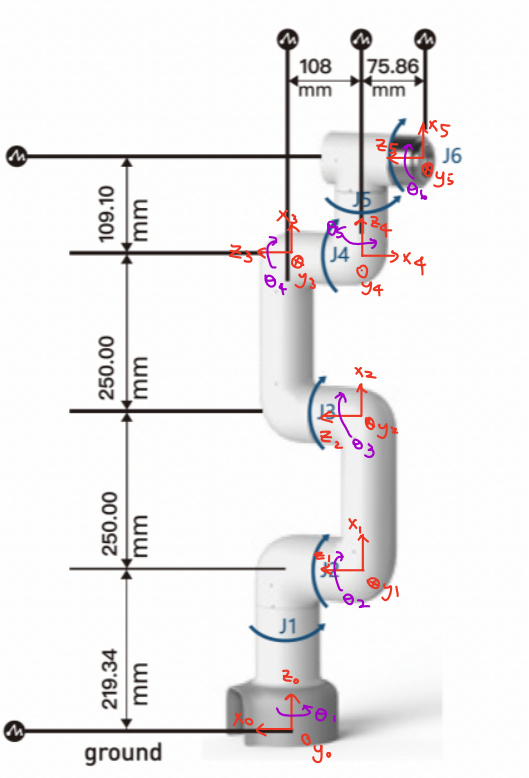
- Projection Matrix:



starting form ↑ ($\forall \theta_i = 0$)

· 0-1:

$$R_0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad PM_{01} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow R_{01} = R_{01} \times PM_{01} = \begin{bmatrix} 0 & s\theta_1 & c\theta_1 \\ 0 & -c\theta_1 & s\theta_1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow {}^0T_1 = \begin{bmatrix} 0 & s\theta_1 & c\theta_1 & -108\,c\theta_1 \\ 0 & -c\theta_1 & s\theta_1 & -108\,s\theta_1 \\ 1 & 0 & 0 & 219.34 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

· 1-2:

$$R_1 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad PM_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow R_{12} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 250\,c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & 250\,s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

· 2-3:

$$R_2 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad PM_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow R_{23} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 250\,c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & 250\,s\theta_3 \\ 0 & 0 & 1 & 108 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

· 3-4:

$$R_3 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 \\ s\theta_4 & c\theta_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad PM_{34} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow R_{34} = \begin{bmatrix} 0 & s\theta_4 & c\theta_4 \\ 0 & -c\theta_4 & s\theta_4 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow {}^3T_4 = \begin{bmatrix} 0 & s\theta_4 & c\theta_4 & 0 \\ 0 & -c\theta_4 & s\theta_4 & 0 \\ -1 & 0 & 0 & -108 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

· 4-5:

$$R_4 = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 \\ s\theta_5 & c\theta_5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad PM_{45} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow R_{45} = \begin{bmatrix} 0 & s\theta_5 & -c\theta_5 \\ 0 & -c\theta_5 & -s\theta_5 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow {}^4T_5 = \begin{bmatrix} 0 & s\theta_5 & -c\theta_5 & 75.86\,c\theta_5 \\ 0 & -c\theta_5 & -s\theta_5 & 75.86\,s\theta_5 \\ 1 & 0 & 0 & 109.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_5 = {}^0T_{EE} = {}^1T_2 \times {}^2T_3 \times {}^3T_4 \times {}^4T_5$$

$$= \begin{bmatrix} 0 & 0 & 1 & -183.86 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 828.44 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(From MATLAB)

- Final Position of E.E. to ground: (mm)

$P_x = -183.86$, $P_y = 0$, $P_z = 828.44$

- MATLAB Result - Projection Matrix :

· Output :

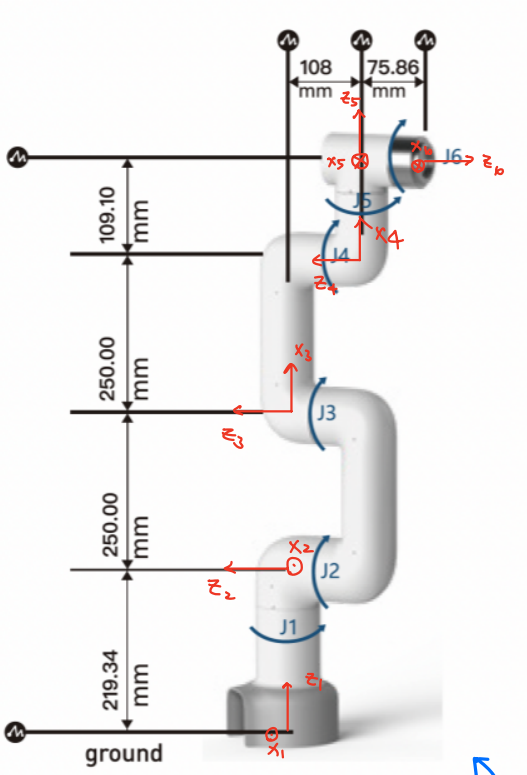
```
Homogeneous Transformation of E.E. to the ground:
T_final =

         0         0    1.0000 -183.8600
         0   -1.0000         0         0
    1.0000         0         0  828.4400
         0         0         0    1.0000

Final position of E.E. to the ground: (mm)
Px = -183.86
Py = 0.00
Pz = 828.44
```

· Code :

```
clear all, clc;

% Define Theta
% syms th1 th2 th3 th4 th5 th6;
th1 = 0;
th2 = 0;
th3 = 0;
th4 = 0;
th5 = 0;
th6 = 0;
theta = [th1 th2 th3 th4 th5 th6];

% A function that create rotation matrix
function R = createRotaionMatrix(th)
    R = [cos(th) -sin(th) 0; sin(th) cos(th) 0; 0 0 1];
end

% A function that create homogeneous transformation
function T = createT(R, Trans)
    T = [R Trans; 0 0 0 1];
end

% Create a cell array to store rotation matrices (R)
R = {};

% Create a cell array to store Projection Matrices (PM)
PM = {};
PM{1} = [0 0 1; 0 -1 0; 1 0 0];
PM{2} = eye(3, 3);
PM{3} = eye(3, 3);
PM{4} = [0 0 1; 0 -1 0; -1 0 0];
PM{5} = [0 0 -1; 0 -1 0; 1 0 0];

% Create a cell array to store translation
Trans = {};
Trans{1} = [-108*cos(th1); -108*sin(th1); 219.34];
Trans{2} = [250*cos(th2); 250*sin(th2); 0];
Trans{3} = [250*cos(th3); 250*sin(th3); 108];
Trans{4} = [0; 0; -108];
Trans{5} = [75.86*cos(th5); 75.68*sin(th5); 109.1];

% Create a cell array to store homogeneous transformation (T)
T = {};

% Create the final homogeneous transformation
for i = 1:5

    % Create rotation matrix
    R{i} = createRotaionMatrix(theta(i)) * PM{i};

    % Create homogeneous transformation
    T{i} = createT(R{i}, Trans{i});

    % Create homogeneous transformation
    if i==1
        T_final = T{i};
    else
        T_final = T_final * T{i};
    end
end

% Display the result
fprintf('Homogeneous Transformation of E.E. to the ground:');
T_final
fprintf('Final position of E.E. to the ground: (mm)\n');
fprintf('Px = %.2f\n', T_final(1, end));
fprintf('Py = %.2f\n', T_final(2, end));
fprintf('Pz = %.2f\n', T_final(3, end));
```

- DH Table :

	θ	d	a	α
1-2	θ_1	219.34	0	90°
2-3	$90°+\theta_2$	0	250	0
3-4	θ_3	-108	250	0
4-5	$90°+\theta_4$	0	109.1	90°
5-6	θ_5	75.86	0	-90°



J_2 & J_4 is -90° in this image.

- MATLAB Result - DH Table

· Output:

```
Homogeneous Transformation of E.E. to the ground:
T =

    1.0000         0         0  609.1000
         0    0.0000   -1.0000  108.0000
         0    1.0000    0.0000  143.4800
         0         0         0    1.0000

Final position of E.E. to the ground: (mm)
Px = 609.10
Py = 108.00
Pz = 143.48
fx >>
```

· Code:

/home/andy/ASU/RAS545_Robotics/HW/HW2/HW2_DH.m

untitled | HW2_DH.m | HW2.m

```
clear all, clc;

% Define Theta
% syms th1 th2 th3 th4 th5 th6;
th1 = 0;
th2 = -pi/2;
th3 = 0;
th4 = -pi/2;
th5 = 0;
th6 = 0;

function T = createT(th, d, a, alpha)
    T = [cos(th) -sin(th)*cos(alpha) sin(th)*sin(alpha) a*cos(th);
         sin(th) cos(th)*cos(alpha) -cos(th)*sin(alpha) a*sin(th);
         0 sin(alpha) cos(alpha) d;
         0 0 0 1];
end

% Create a matrix that store all values in DH table
DH_factors = [th1 219.34 0 pi/2;
              pi/2+th2 0 250 0;
              th3 -108 250 0;
              pi/2+th4 0 109.1 pi/2;
              th5 75.86 0 -pi/2;
              ];


for i = 1:length(DH_factors)
    if i==1
        T = createT(DH_factors(1, 1), DH_factors(1, 2), DH_factors(1, 3), DH_factors(1, 4));
    else
        T = T * createT(DH_factors(i, 1), DH_factors(i, 2), DH_factors(i, 3), DH_factors(i, 4));
    end
end


% Display the result
fprintf('Homogeneous Transformation of E.E. to the ground:');
T
fprintf('Final position of E.E. to the ground: (mm)\n');
fprintf('Px = %.2f\n', T(1, end));
fprintf('Py = %.2f\n', T(2, end));
fprintf('Pz = %.2f\n', T(3, end));
```