ECE 100: Linear Electronic Systems

Professor: Drew Hall

Lab 2:
Active Circuit Design

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Abstract

In this lab we are aiming to separate the digital DSL signal from the voice signal on a telephone line. In order to do this we must design and simulate a low-pass filter with certain specifications. We used MATLAB and Pspice to simulate our calculated transfer function to find a fundamental frequency f_0 that falls between our 4kHz pass-band and 32kHz stop-band. These boundaries were given with the required filter specifications. Building the physical circuit allowed us to see how the input/output waves behaved on the oscilloscope. This allowed us to calculate gain values and compare them to simulated results.

Experimental Procedure

Tools:

- MATLAB
- PSpice/Orcad
- Oscilloscope
- Function Generator
- DC Power Supply (Dual Output)
- Breadboard
- Electrical Components (Wires, Resistors, Capacitors, LF411 Op-amp)

The specifications for the filter are sketched below.

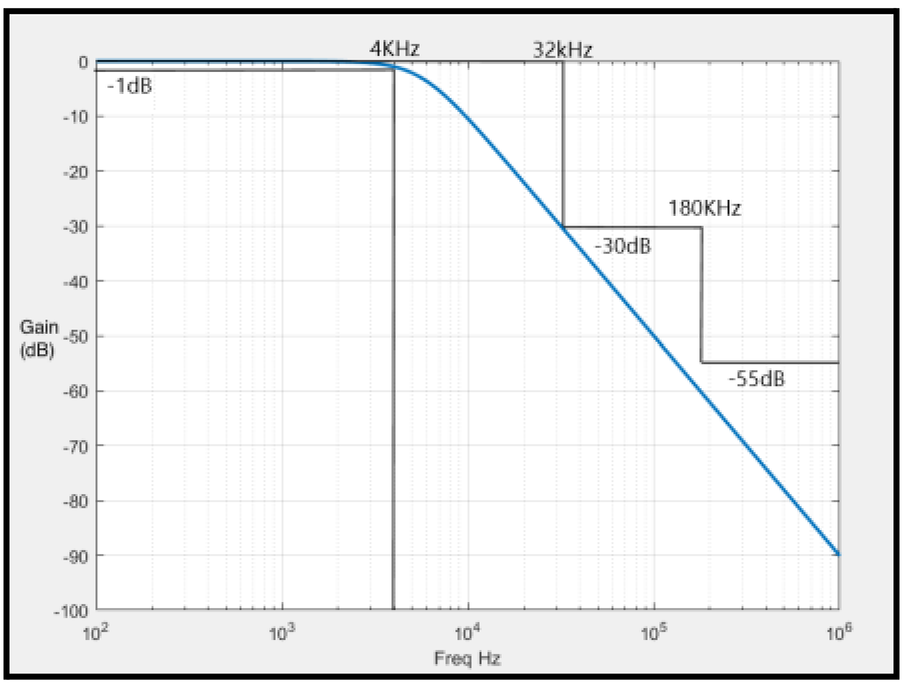


Figure 1: Filter Specs

The function begins to roll off at 4kHz, the gain falls to < -30dB at 32kHz, and then the gain falls to < -55dB at 180kHz.

Section 1: Approximation

Part A)

The magnitude squared of the frequency response of a Butterworth filter is $|H(f)|^2 = \frac{1}{1+\left(\frac{f}{f_0}\right)^4}$.

To see if the transfer function meets the spec, we choose f0 so the transfer function just hits the passband edge. This is the lowest possible value of f_0. Then we check if $|H(f)|^2$ clears the stopband edge. Find this value of f_0 analytically. Calculate the resulting $|H(f)|^2$ at 32 and 180 kHz. Does the filter meet the spec?

Part B)

You will find that the maximally flat transfer function (the Butterworth filter's characteristic) meets the spec, but it is a very tight fit. This leaves little room for component errors. We could increase the order from n = 2 to n = 3, but this will increase the cost by roughly 50%. So instead, we will design and use another filter transfer function. We will allow the $|H(f)|^2$ in the passband to "ripple" from -1 dB at dc up to 0 dB, then down to -1 dB at 4 kHz. Thus, $|H(f)|^2$ will have a peak 1 dB greater than the dc value. At dc, 10 log $10log_{10}|H(f\ =\ 0)|^2 = \ -\ 1dB$ which means $|H(f\ =\ 0)|^2\ =\ 0.7943$.

Following the standard transfer function for a second-order low-pass filter with DC gain of -1dB, $H(s)\ = \frac{(0.7943)^{1/2}}{1+\frac{2\zeta s}{\omega_0}+\frac{s^2}{\omega_0{}^2}}$, find an expression for the frequency at which |H(jω)| peaks. Also, find an expression for $\frac{|H(f_{peak})|^2}{|H(0)|^2}$ as a function of ζ. Use MATLAB to plot $\frac{|H(f_{peak})|^2}{|H(0)|^2}$ in dB vs ζ in the range 0.1 < ζ < 0.7 and find the value of ζ that will provide a 1 dB peak.

Part C)

Now that we have the necessary ζ, we can correct the dc gain using a voltage divider to get $|H(f)|^2 = \frac{0.7943}{\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(2\zeta\frac{f}{f_0}\right)^2}$. All that remains is to choose the natural frequency f_0 so $|H(f)|^2 = \ -1dB$ at 4 kHz. It is not easy to invert this equation for f_0, although MATLAB can do that. One possible way to find f_0 is as follows. Guess f_0, say $f_0 = \ 5000$ Hz, and plot $|H(f)|^2$ in dB vs. f over the range 1,000 < f < 10,000. Find the frequency at which $|H(f)|^2$ drops to -1 dB, say this value is x.

Part D)

Finally, we need to adjust f_0 upwards a bit so $|H(f)|^2$ clears the passband edge and the stopband edge by the same factor. What is your final value of f_0? Save a copy of this graph to show that it meets the spec.

<u>Hint:</u> you can check the distance between the passband's edge and the stopband's edge in both frequency and gain.

Option 1: check the distance in gain	**Option 2: check the distance in frequency**
(-1 dB) - Gain @ 4 kHz = X1	$\frac{(f\ @gain = -1\ dB)}{4\ kHz} = \ X1$
Gain @ 32 kHz - (-30 dB) = X2	$\frac{32\ kHz}{(f\ @gain = -30\ dB)} = \ X2$
|X1 – X2| < 0.1dB is acceptable	$\frac{\|X1-X2\|}{X1} < \ 10\%$ is acceptable

Section 2: Realization / Simulation

Build and test the active circuit. We will use the LF411 op-amp with the pins shown below.

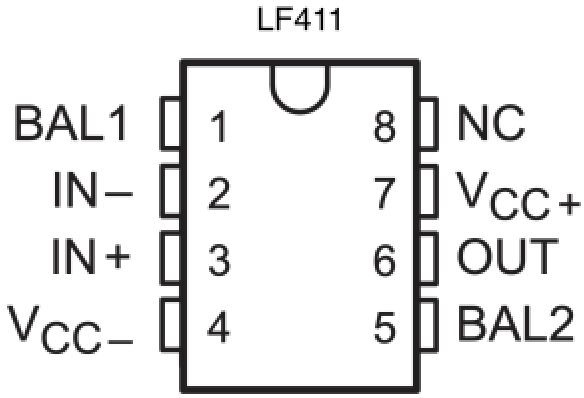


Figure 2: LF411 Pinouts

Part A)

Analyze the Sallen-Key circuit as shown on the following page. Put the transfer function in the general form. For simplicity, we usually set $R1 = R2$. Assume that the resistors are $R1 = R2 = 100k\Omega$. Calculate the component values needed to obtain the necessary values of f_0 and ζ. You found the necessary f_0 and ζ in parts 1(d) and 1(b), respectively. Finally, we can correct the dc gain, setting it to -1 dB, by converting R2 into a voltage divider with a dc gain of -1 dB and a Thevenin resistance of 100kΩ. Do this too.

Part B)

The most important "non-ideal" op-amp effect for this circuit is the output resistance of the op-amp itself. This is typically about 50Ω. At the highest frequencies, you can assume that the capacitors are short circuits and the opamp gain is zero. Show that under these Conditions $|H| \rightarrow \frac{R_{OUT}}{R}$. (Assume $R1 = R2 = R$ in the schematic shown below.)

Part C)

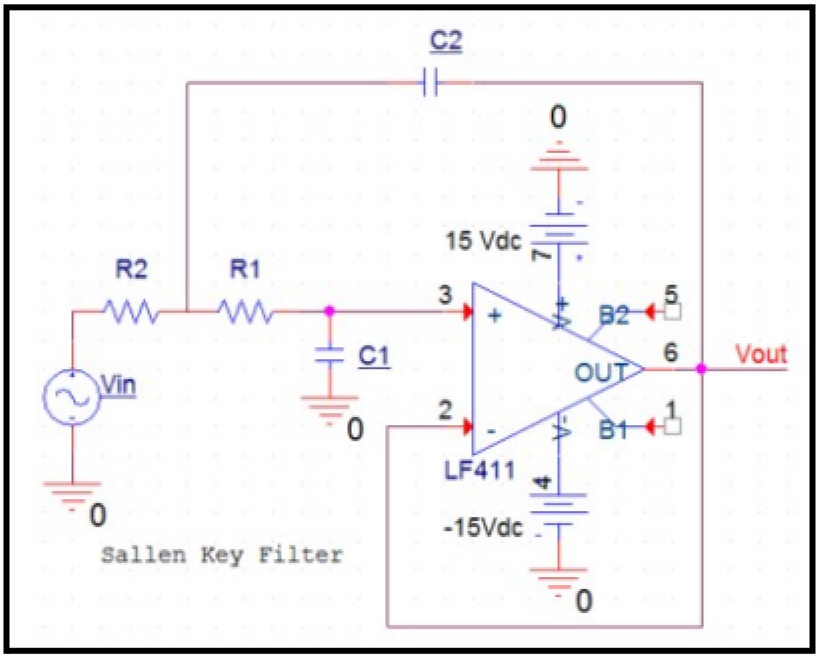


Figure 3: Sallen Key Filter Schematic

Simulate the circuit using an LF411 op-amp. Read the Notes included in this document to learn more about finding this component. Using your simulation results, confirm that your design meets the spec. If you cut all the resistors exactly in half and double all the capacitors, it won't change the theoretical transfer function, but it will increase the effect of the output resistance. Try it. Does the transfer function still meet the spec? Find the lowest resistance for which the simulation still meets the spec. Ensure the gain does not go above -55 dB from 180 kHz to 1 MHz.

Save the simulated $|H(f)|^2$ as an ascii text file so that you can compare it with the theoretical filter using MATLAB. Read the Notes included in this document to learn more about this. Create a MATLAB script that will plot your theoretical $|H(f)|^2$, then read and plot your simulated $|H(f)|^2$ using distinct symbols (i.e., not connected by a line). Overlay two plots. If you want to be really fancy, have it plot the specs, too, as in the first figure in these instructions. Remember to include this script as well as the final plot containing the theoretical and simulated data sets in your final report.

Section 3: Experiment

Part A)

Put probes on both the input and the output and trigger a sine waveform using a signal generator at the input probe. This will allow you to measure low-level signals at the output without losing the trigger. Measure the voltage gain at the spec frequencies: DC (use 5 or 10 Hz instead of 0 Hz), f_{PEAK}, 4 kHz, 32 kHz, 180 kHz, and 1 Mhz. Create a table to report your measured input and output peak-to-peak voltage amplitudes and the calculated voltage gain in dB. Compare this with your theoretical and simulated values and comment on any discrepancies.

Part B)

Explore the remainder of the frequency range up to the generator limit and ensure there are no unexpected "features." Since this is an active circuit, you must ensure that the signal does not drive the op-amp into a nonlinear operating condition. So long as the output voltage still looks like a sine wave, you are probably OK. A good way to confirm that the system is being tested in a linear range is to change the input voltage by a factor of two and re-measure$|H(f)|^2$; if it is the same, you are OK. Take a scope image before and after changing the input voltage. If they differ, reduce the input voltage by another factor of two and try again. Put your measurements in an ascii file so you can read them into MATLAB and plot them on top of the simulation

Results

Section 1

Part A)

a) $|H(4kHz)|^2 = -1dB$ $\quad \omega_o$

$$|H(f)|^2 = \frac{1}{1+\left(\frac{f}{f_o}\right)^4}$$

$$-1dB = 10^{-.01} = 0.7943 = |H(f)|^2$$

$$0.7943 = \frac{1}{1+\frac{(4KHz)^4}{f_o^4}}$$

$$\frac{4KHz^4}{f_o^4} = \frac{1}{.7943} - 1$$

$$\frac{4KHz^4}{\left(\frac{1}{.7943}-1\right)} = f_o^4$$

$$\boxed{f_o = 5.61\ KHz}$$

$$|H(32KHz)|^2 = \frac{1}{1+\frac{(32KHz)^4}{(5.61KHz)^4}}$$

$$|H(32KHz)|^2 = 9.4\times10^{-4}$$

$$\boxed{10\log_{10}(9.4\times10^{-4}) = -30.26\,dB \checkmark \quad \text{meets spec}}$$

$$|H(180KHz)|^2 = \frac{1}{1+\frac{(180KHz)^4}{(5.61KHz)^4}}$$

$$|H(180KHz)|^2 = 943.54\times10^{-9}$$

$$\boxed{10\log_{10}(943.54\times10^{-9}) = -60.25\,dB \checkmark \quad \text{meets spec}}$$

By looking at the filter specifications, when the frequency is at 32kHz, the gain must reach at least below -30dB, we reach -30.26dB. At 180kHz we must drop below -60dB and we reach -60.25dB. In both cases, **we pass the required specs.**

Part B)

We need to find an expression for the frequency at which |H(jω)| peaks. As well as an expression for $\frac{|H(f_{peak})|^2}{|H(0)|^2}$ as a function of ζ.

b)

$$H(s) = \frac{(.7943)^{1/2}}{1 + \frac{2\zeta s}{\omega_o} + \frac{s^2}{\omega_o^2}}$$

$$H(j\omega) = \frac{(.7943)^{1/2}}{1 + 2\zeta\frac{j\omega}{\omega_o} + \left(\frac{j\omega}{\omega_o}\right)^2}$$

$$H(j\omega) = \frac{(.7943)^{1/2}}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + j\left(\frac{2\zeta\omega}{\omega_o}\right)}$$

$$|H(j\omega)|^2 = \frac{.7943}{\left(1-\left(\frac{\omega}{\omega_o}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_o}\right)^2}$$

$$\omega_p = \frac{\partial}{\partial\omega}\left(|H(j\omega)|^2\right) = \frac{\partial}{\partial\omega}\left(\frac{1}{|H(j\omega)|^2}\right) \quad ☆$$

$$\frac{\partial}{\partial\omega}\left(\frac{1}{|H(j\omega)|^2}\right) = \frac{\partial}{\partial\omega}\left[\left(1-\left(\frac{\omega}{\omega_o}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_o}\right)^2\right]$$

$$0 = 2\left(1-\left(\frac{\omega}{\omega_o}\right)^2\right)\left(-\frac{2\omega}{\omega_o^2}\right) + 2\left(\frac{2\zeta\omega}{\omega_o}\right)\left(\frac{2\zeta}{\omega_o}\right)$$

$$0 = \left(1-\left(\frac{\omega}{\omega_o}\right)^2\right)\left(-\frac{4\omega}{\omega_o^2}\right) + 2\left(\frac{4\omega\zeta^2}{\omega_o^2}\right)$$

$$0 = \left(\cancel{\frac{4\omega}{\omega_o^2}}\right)\left[\left(1-\left(\frac{\omega}{\omega_o}\right)^2\right)(-1) + 2\zeta^2\right]$$

$$0 = \frac{\omega^2}{\omega_o^2} - 1 + 2\zeta^2$$

$$\frac{\omega^2}{\omega_o^2} = 1 - 2\zeta^2$$

$$\frac{\omega}{\omega_o} = \sqrt{1-2\zeta^2}$$

$$\boxed{\omega_p = \omega_o\sqrt{1-2\zeta^2}}$$

$$|H(j\omega)|^2 = \frac{(.7943)^{(1/2)^2}}{\left(1-\left(\frac{\omega}{\omega_o}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_o}\right)^2} = \frac{(.7943)}{\left(1-\frac{\omega^2}{\omega_o^2}\right)^2 + \left(\frac{4\zeta^2\omega^2}{\omega_o^2}\right)}$$

$$|H(\omega_p)|^2 = \frac{(.7943)}{\left[1-\frac{\left(\omega_o\sqrt{1-2\zeta^2}\right)^2}{\omega_o^2}\right]^2 + \frac{4\zeta^2}{\omega_o^2}\left(\omega_o\sqrt{1-2\zeta^2}\right)^2}$$

$$= \frac{(.7943)}{\left[1-\frac{\cancel{\omega_o}^2(1-2\zeta^2)}{\cancel{\omega_o}^2}\right]^2 + \frac{4\zeta^2\cancel{\omega_o^2}}{\cancel{\omega_o^2}}(1-2\zeta^2)}$$

$$= \frac{(.7943)}{(1-1-2\zeta^2)^2 + 4\zeta^2(1-2\zeta^2)}$$

$$= \frac{(.7943)}{4\zeta^4 + 4\zeta^2 - 8\zeta^4}$$

$$= \frac{(.7943)}{(-4\zeta^4 + 4\zeta^2)} = \frac{.7943}{4(\zeta^2-\zeta^4)}$$

$$\boxed{|H(\omega_p)|^2 = \frac{.199}{\zeta^2-\zeta^4}}$$

$$|H(0)|^2 = \frac{.7943}{\left[1-\frac{(0)}{\omega_o^2}\right]^2 + \frac{4\zeta^2}{\omega_o^2}\cdot(0)} = \frac{.7943}{1^2-0+0}$$

$$\boxed{|H(0)|^2 = .7943}$$

$$\frac{|H(\omega_p)|^2}{|H(0)|^2} = \frac{\left(\frac{.7943}{4(\zeta^2-\zeta^4)}\right)}{.7943}$$

$$\boxed{\frac{|H(\omega_p)|^2}{|H(0)|^2} = \frac{1}{4(\zeta^2-\zeta^4)}}$$

Now that we have the necessary equations, we will plot them using MATLAB.

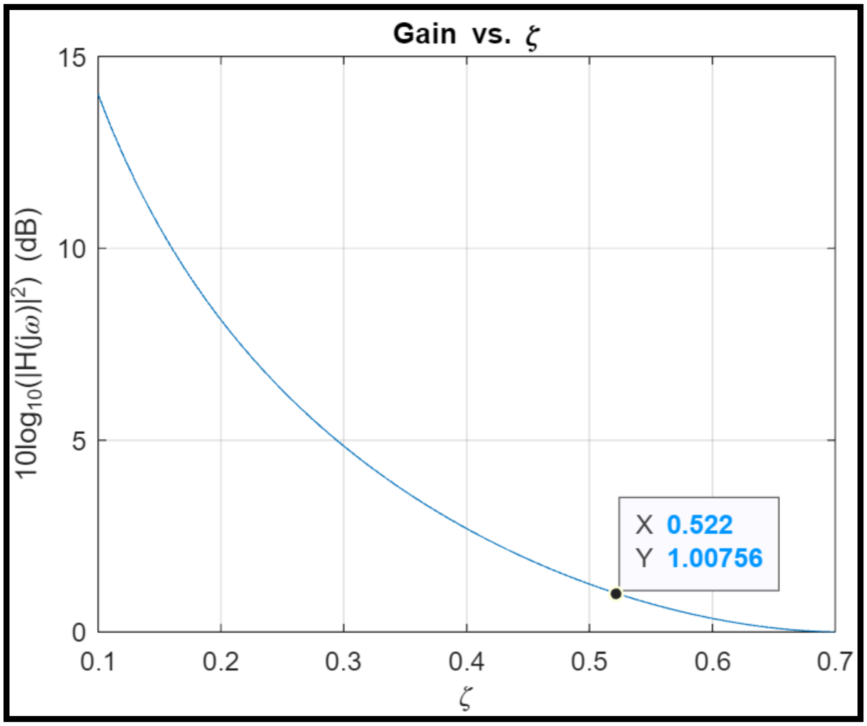


Figure 4: Plot for Gain vs. ζ

After plotting, we look for the point at which our gain is +1dB. This x-coordinate for this point is our desired ζ value. We found $\zeta = 0.522$. Now we insert that ζ value into our frequency response $|H(f)|^2 = \frac{1}{1+\left(\frac{f}{f_0}\right)^4}$. We can use a voltage divider to convert it into $|H(f)|^2 = \frac{0.7943}{\left(1-\left(\frac{f}{f_0}\right)^2\right)^2 + \left(2\zeta\frac{f}{f_0}\right)^2}$. Since we found ζ we are able to now find f_0 by plotting a graph and repeating a similar process.

Part C)

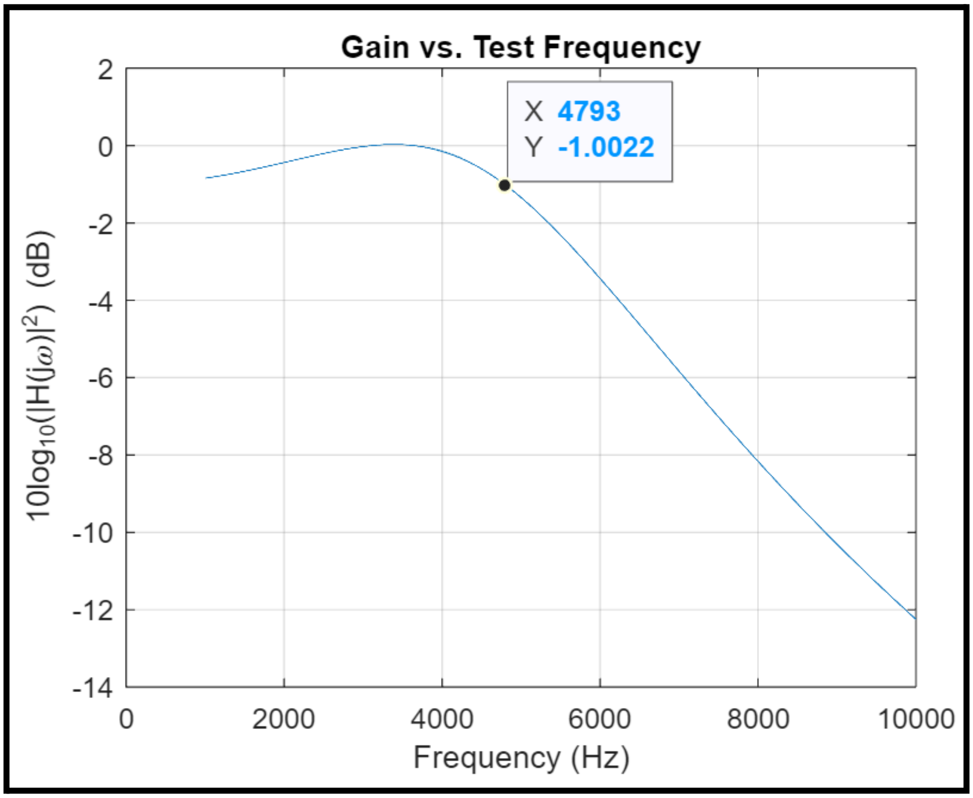


Figure 5: Plot for Gain vs. Test Frequency

Using MATLAB we found the frequency that gives us a gain of -1dB to be 4,794 Hz. Since we guessed the scale factor to be 5,000 Hz, we must convert back to find the real value of f_0.

$$f_0 = 5,000 * \left(\frac{4,000}{X}\right)$$

$$f_0 = 5,000 * \left(\frac{4,000}{(4793)}\right)$$

$$f_0 = 4,171.88\ Hz$$

This f_0 allows us to clear the stop/pass-bands. However, we want our transfer function to lie evenly between both boundaries (the distance from function to pass-band must be almost equal to distance from function to stop-band).

Part D)

By increasing/decreasing our f_0, we are able to shift the transfer function right/left respectively.

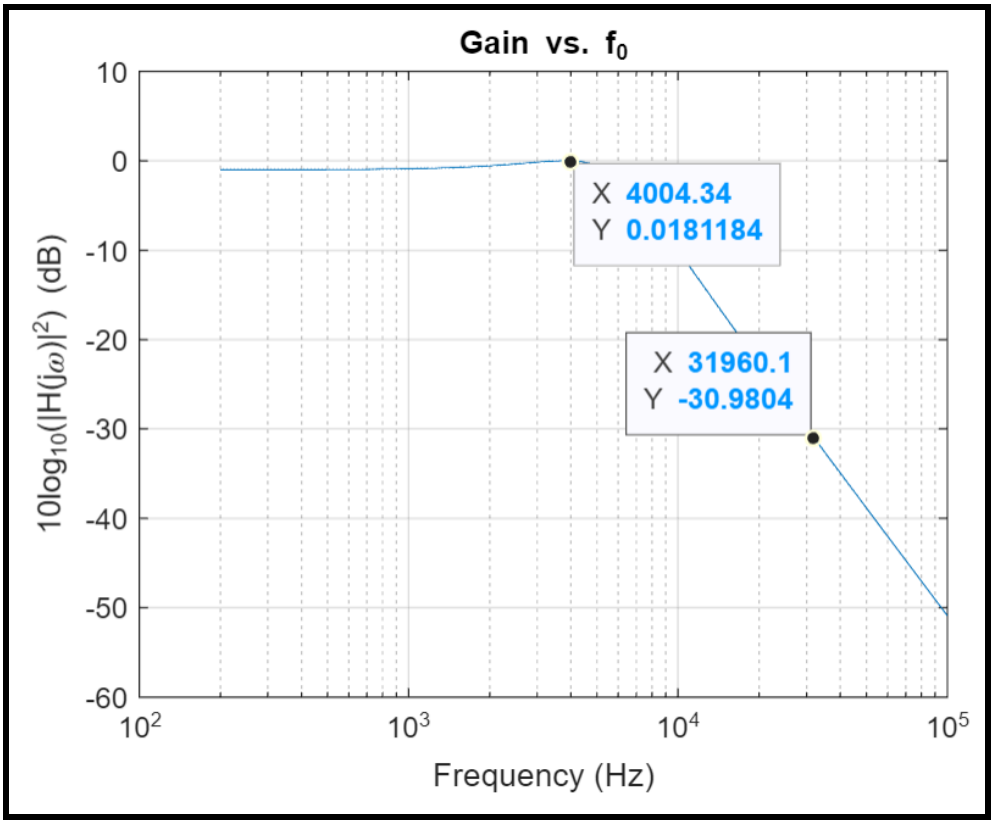


Figure 6: Plot of Gain vs. f_0

After adjusting f_0, our new value is $f_0 = 5,650\ Hz$.

Using this frequency we are able to clear the stop/pass-band edges with equal distance on both sides. We verified this by using option 1.

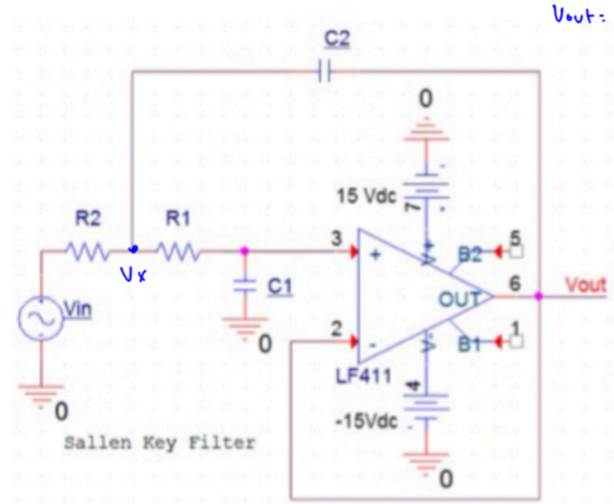
Option 1: check the distance in gain	**Measured Results**
(-1 dB) - Gain @ 4 kHz = X1	(-1) - (0.0181) = **-1.0181 = X1**
Gain @ 32 kHz - (-30 dB) = X2	(-30.9804) - (-30) = **-0.9804 = X2**
|X1 – X2| < 0.1dB is acceptable	(-1.0181) - (-0.9804) = **-0.0377 < 0.1dB**

Section 2

Part A)

We are going to find the transfer function of the op-amp circuit in order to find our required resistances and capacitance values. We split R_1 into two parallel resistors, R_a and R_b.

a)



Vout:

KCL at V_x:

$$(1)\quad \frac{V_x - V_i}{Z_1} + \frac{V_x - V_+}{Z_2} + \frac{V_x - V_o}{Z_3} = 0$$

KCL at V_+:

$$(2)\quad \frac{V_+ - V_x}{Z_2} + \frac{V_+}{Z_4} = 0$$

$$(3)\quad V^- = V^+ = V_o$$

$$(1)\quad V_x\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) - V_i\left(\frac{1}{Z_1}\right) - V_o\left(\frac{1}{Z_2} + \frac{1}{Z_3}\right) = 0$$

$$(2):\quad V_o\left(\frac{1}{Z_2} + \frac{1}{Z_4}\right) - V_x\left(\frac{1}{Z_2}\right) = 0$$

$$V_x = V_o\left(1 + \frac{Z_2}{Z_4}\right)$$

$$V_o\left(1 + \frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) - V_i\left(\frac{1}{Z_1}\right) - V_o\left(\frac{1}{Z_2} + \frac{1}{Z_3}\right) = 0$$

$$\frac{V_o}{V_I} = \frac{\frac{1}{Z_1}}{\left(1 + \frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) - \frac{1}{Z_2} - \frac{1}{Z_3}} = \frac{\frac{1}{Z_1}}{\frac{1}{Z_1} + \cancel{\frac{1}{Z_2}} + \cancel{\frac{1}{Z_3}} + \frac{Z_2}{Z_1 Z_4} + \frac{1}{Z_4} + \frac{Z_2}{Z_3 Z_4} \cancel{- \frac{1}{Z_2}} \cancel{- \frac{1}{Z_3}}}$$

$$H(s) = \frac{V_o}{V_i} = \frac{Z_3 Z_4}{Z_3 Z_4 + Z_2 Z_3 + Z_1 Z_3 + Z_1 Z_2}$$

$Z_1 = R_2$

$Z_2 = R_1$

$Z_3 = 1/sC_2$

$Z_4 = 1/sC_1$

$$H(s) = \frac{\frac{1}{sC_2}\frac{1}{sC_1}}{\frac{1}{sC_2}\cdot\frac{1}{sC_1} + R_1\left(\frac{1}{sC_2}\right) + \left(R_2\frac{1}{sC_2}\right) + R_1 R_2}$$

$$= \frac{1}{1 + sR_1C_1 + sR_2C_1 + s^2 R_1 R_2 C_1 C_2}$$

$$= \frac{1}{1 + s(R_1C_1 + R_2C_2) + s^2 R_1 R_2 C_1 C_2}$$

$\omega \to \infty$

$$H(s) = \frac{1}{1 + sC_1(R_1 + R_2) + s^2 C_1 C_2 R_1 R_2}$$

$$\frac{2\zeta}{\omega_o} = C_1(R_1 + R_2)$$

$\zeta = 0.5227$

$\omega_o = 2\pi f$

$f = 5650\ Hz$

$$C_1 = \frac{2\zeta}{\omega_o(R_1 + R_2)} = \frac{2(0.522)}{2\pi(5650Hz)(200K)}$$

$$C_1 = 147.04\ pF$$

$$V_i\ \frac{R_1}{R_1 + R_2}$$

$R_a \| R_b = 100K\Omega$

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\sqrt{R_1 R_2 C_1 C_2} = \frac{1}{\omega_o} \Rightarrow R_1 R_2 C_1 C_2 = \frac{1}{\omega_o^2}$$

$$C_2 = \frac{1}{\omega_o^2 (R_1 R_2 C_1)} = \frac{1}{(2\pi 5650)^2 (100K \cdot 100K \cdot 147.04p)}$$

$$C_2 = 639.64\ pF$$

$$V_o = \frac{R_b}{R_a + R_b} V_i$$

$20\log(x) = -1 \quad x = 0.89$

$=$

$$\frac{V_o}{V_i} = \frac{R_b}{R_a + R_b}$$

$$-1dB = \frac{R_b}{R_b + R_a} \Rightarrow 0.89 = \frac{R_b}{R_b + R_a}$$

$R_x = R_a \| R_b$

$$R_x = \frac{R_a R_b}{R_a + R_b}$$

$$0.89 = \frac{R_b}{R_a + R_b}$$

$$R_x = \frac{R_b}{R_a + R_b}(R_a)$$

$R_x = 100K$

$100K = 0.89(R_a)$

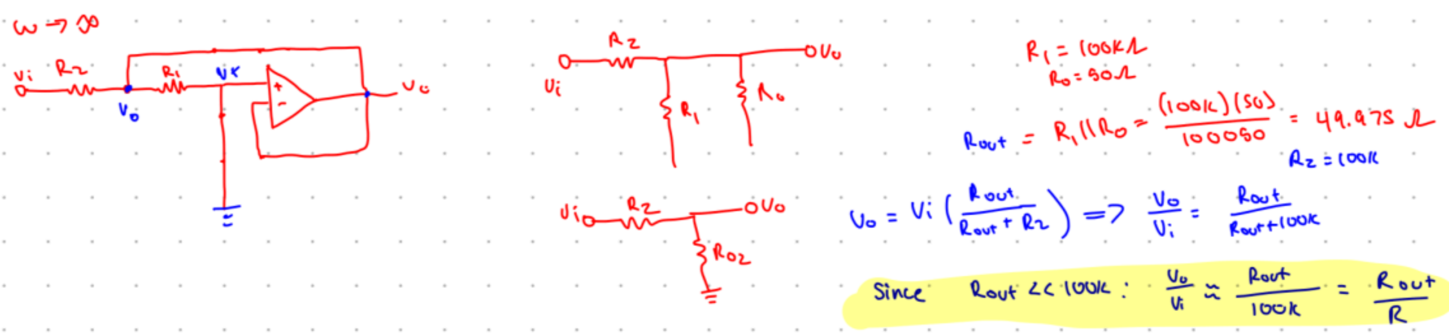
$$R_a = 112.36\ K\Omega$$

$$0.891 = \frac{R_a}{R_a + R_b} \Rightarrow 0.891R_b + 0.891R_a = R_a \Rightarrow 0.11R_b = 0.891R_a = R_b = \frac{0.891R_a}{0.11}$$

$$R_b = 919.69\ K\Omega$$

Part B)

We want to prove that $|H| \rightarrow \frac{R_{OUT}}{R}$ at the highest frequencies. We assume that the capacitors are short circuits and the opamp gain is zero (Assume $R1 = R2 = R$).



We can assume that R1 is negligible.

Part C)

We will use PSpice simulations to find the lowest resistance for which the simulation still meets the spec. We must ensure the gain does not go above -55 dB from 180 kHz to 1 MHz. For each schematic we tested different resistor and capacitor values and repeated the simulation.

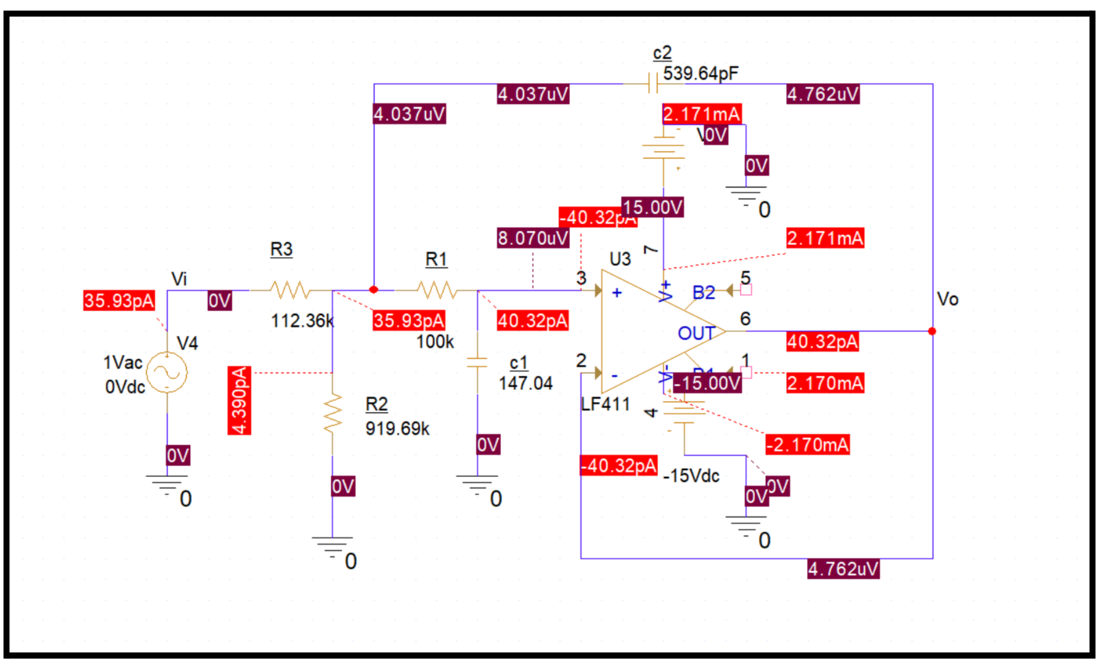


Figure 7: Initial Simulation Schematic (1)

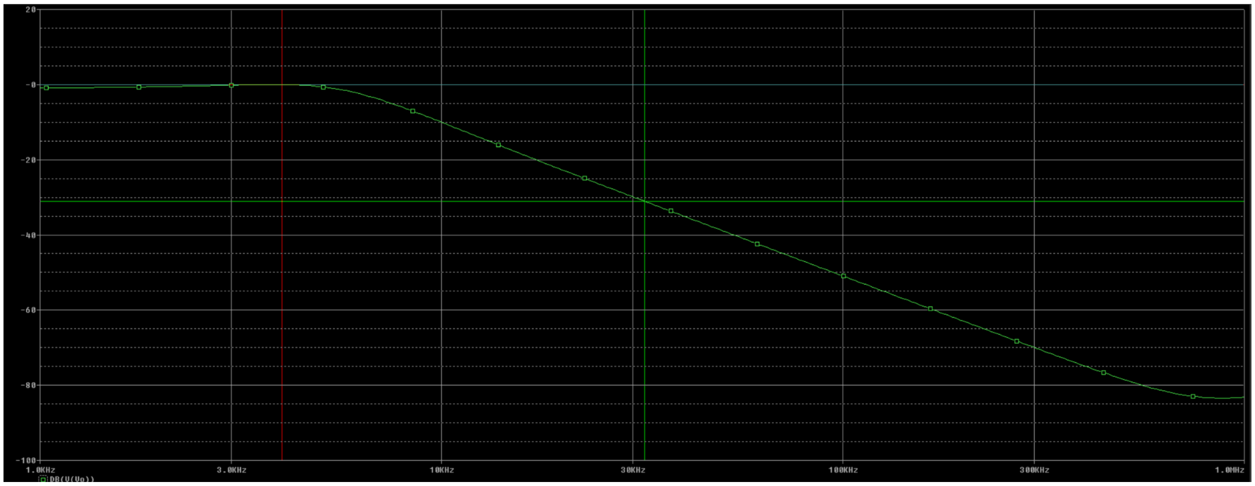


Figure 8: Simulation results for gain $\left(20log_{10}\left(\frac{Vout}{Vin}\right)\right)$ between 1kHz to 1MHz

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	4.0069K	32.080K	-28.073K
CURSOR 1,2	DB(V(Vo))	-3.5568m	-31.071	31.068

Figure 8 Legend

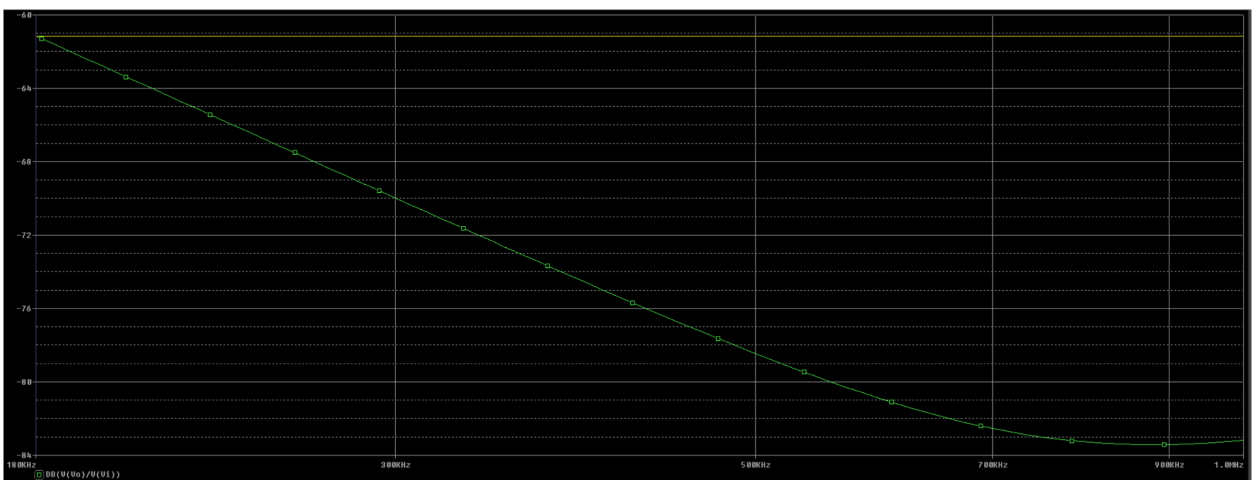


Figure 9: Simulation results for gain $\left(20log_{10}\left(\frac{Vout}{Vin}\right)\right)$ between 180kHz to 1MHz

Trace Color	Trace Name	Y1
	X Values	180.000K
CURSOR 1,2	DB(V(Vo)/V(Vi))	-61.146

Figure 9 Legend

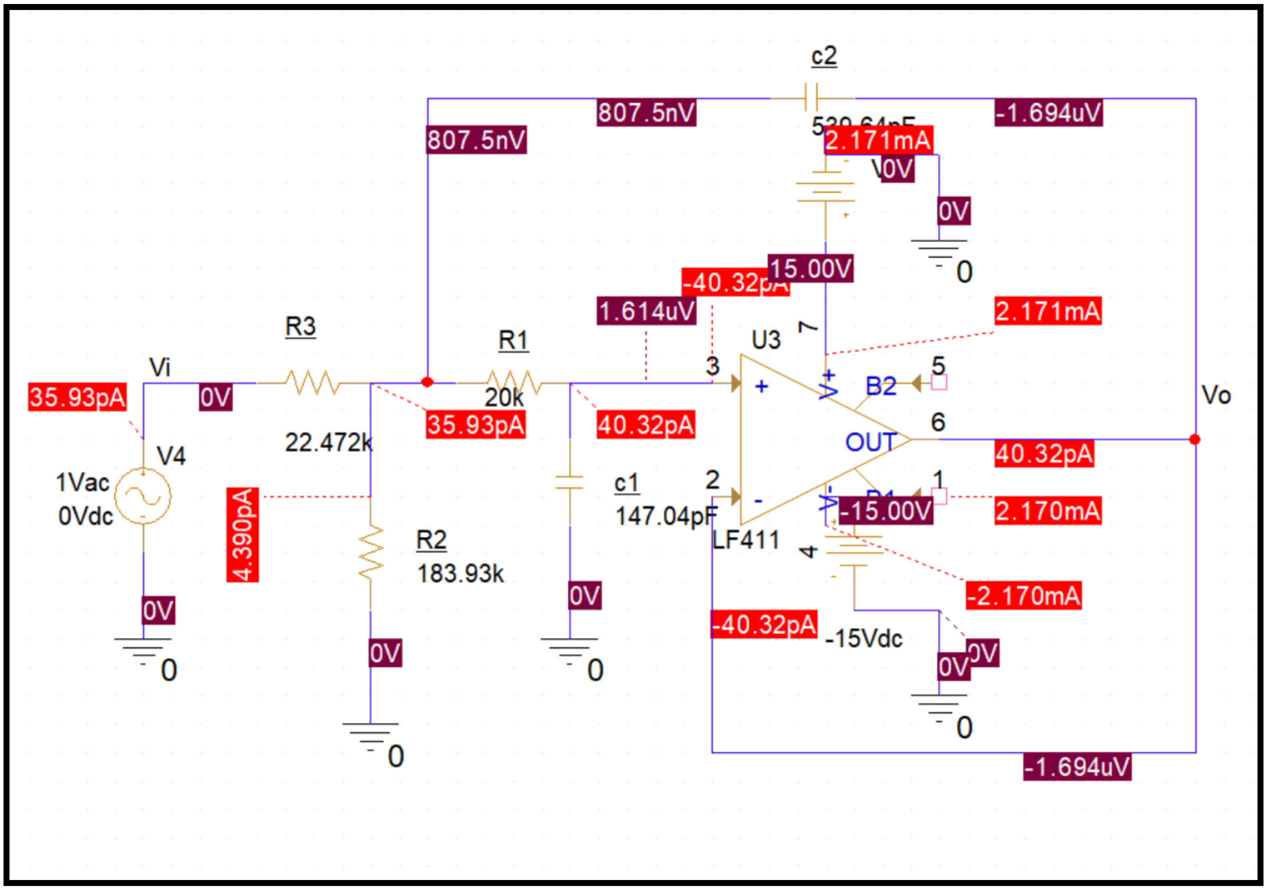


Figure 10: Simulation schematic (2) after decreasing Resistance 80%

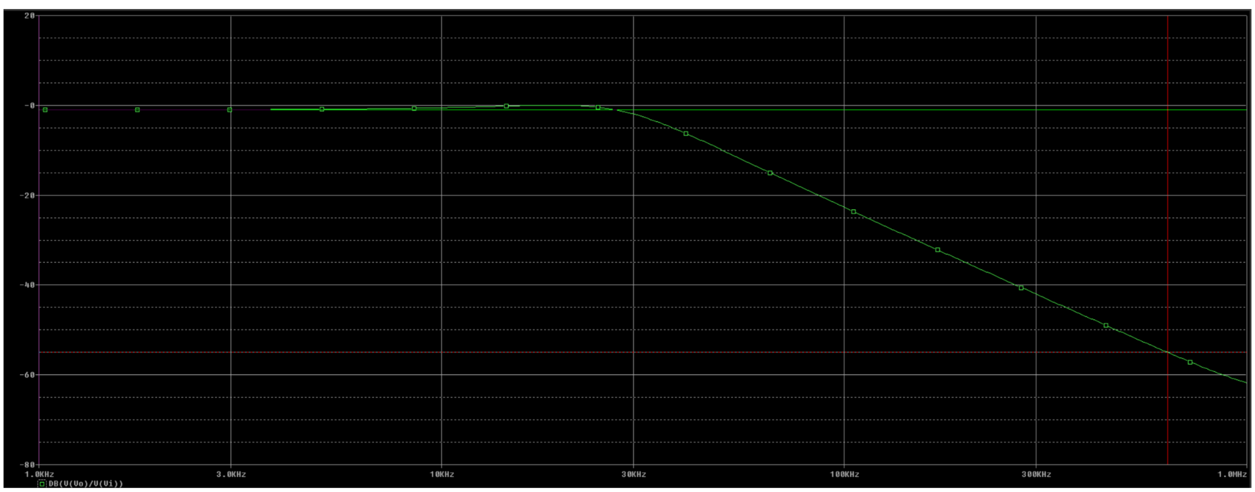


Figure 11: Plot for simulation schematic: 2

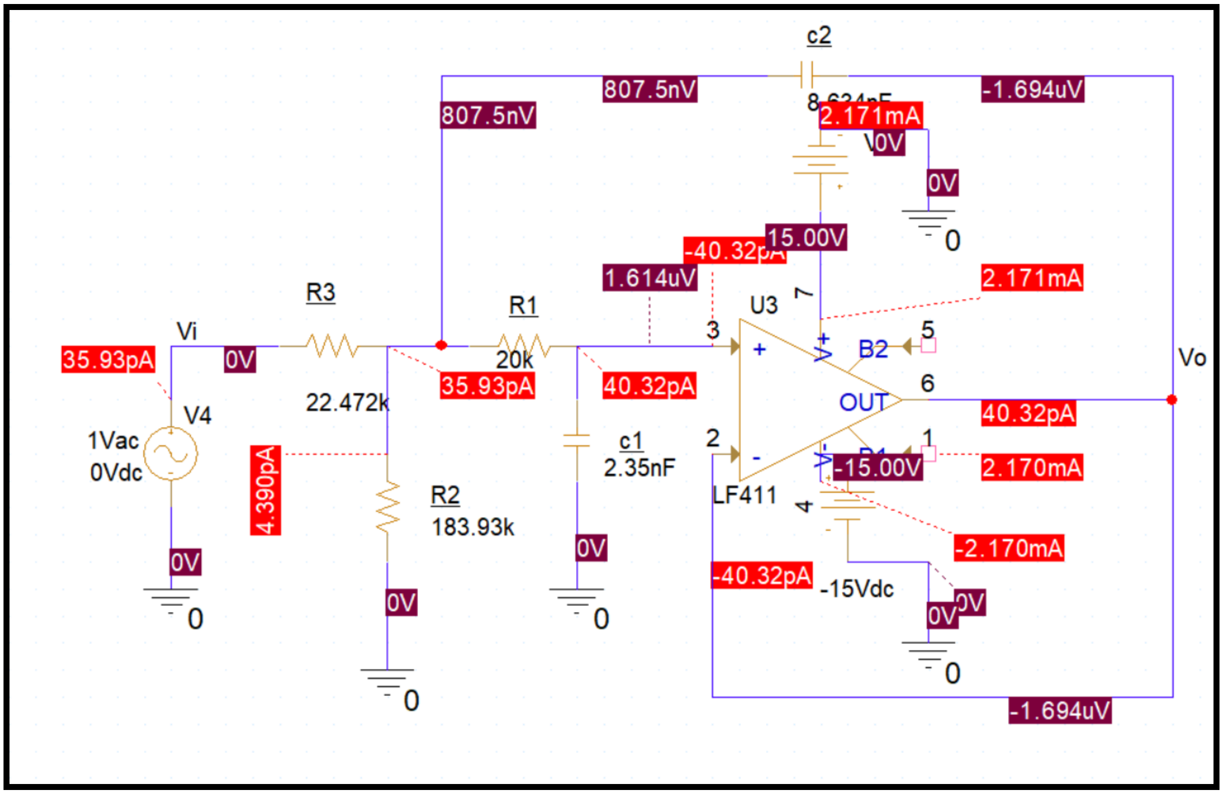


Figure 12: Simulation schematic (3) after increasing Capacitance 1600%

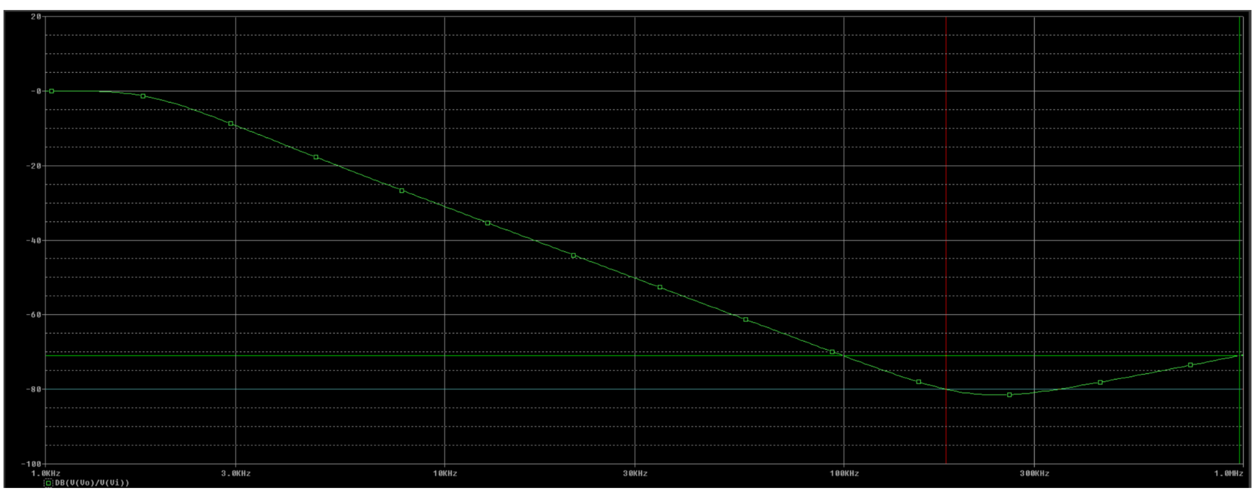


Figure 13: Plot for simulation schematic: 3

Trace Color	Trace Name	Y1	Y2	
	X Values	180.074K	977.237K	-7
CURSOR 1,2	DB(V(Vo)/V(Vi))	-79.995	-70.981	-9

Figure 13 Legend

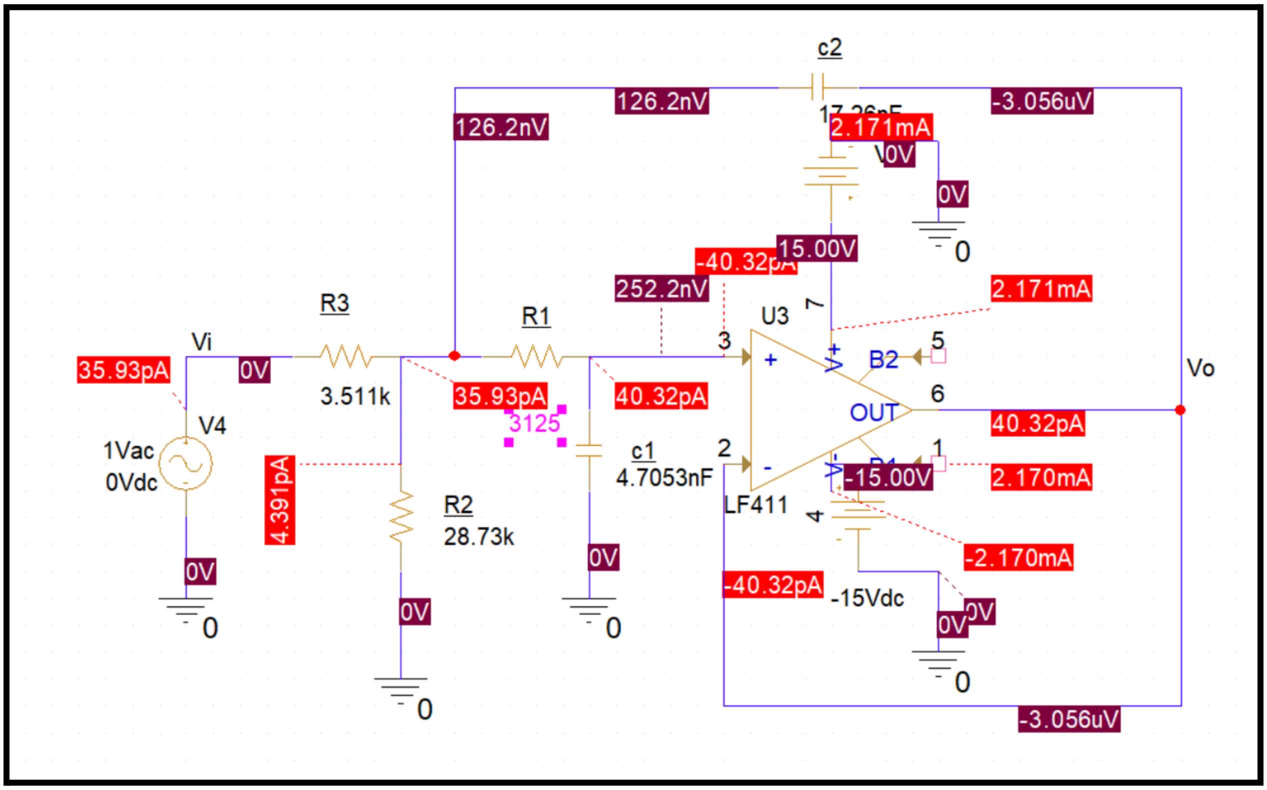


Figure 14: Simulation schematic (4) after increasing Capacitance 3200% and decreasing Resistance 3.125%

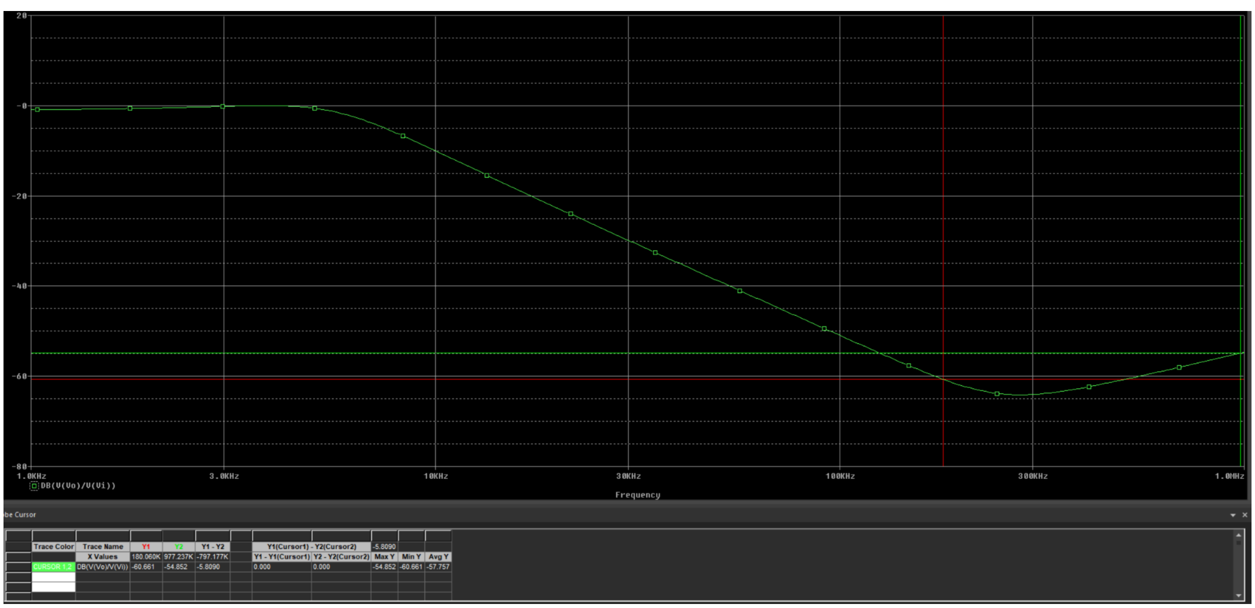


Figure 15: Plot for simulation schematic: 4

We saw that this works best for our boundary conditions. This is what we will use to compare our lab results.

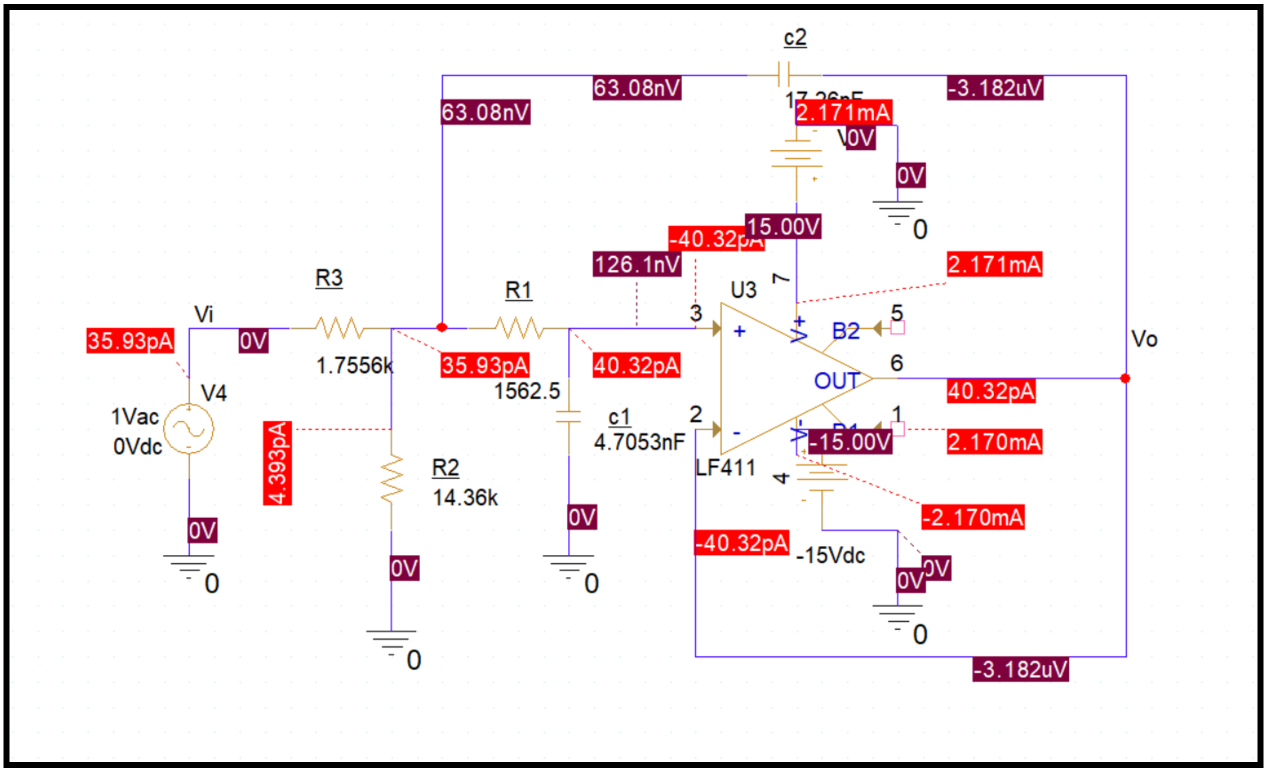


Figure 16: Simulation schematic (5) after increasing Capacitance 3200% and decreasing Resistance 1.5625%

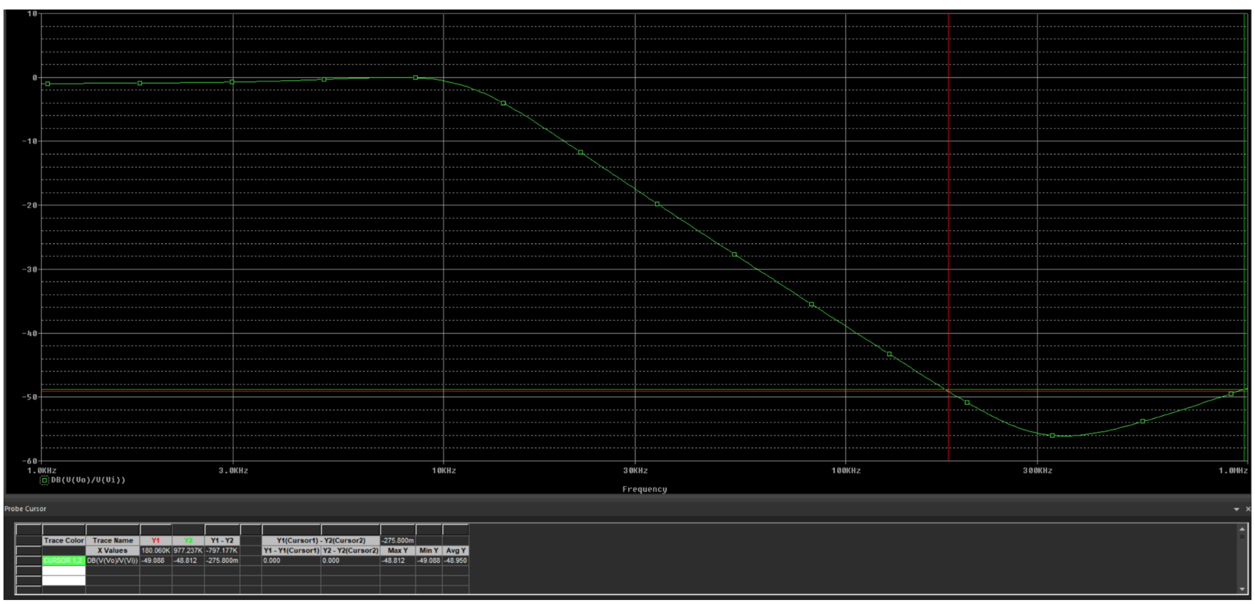


Figure 17: Plot for simulation schematic: 5

This one starts the roll off late so it is not what we want.

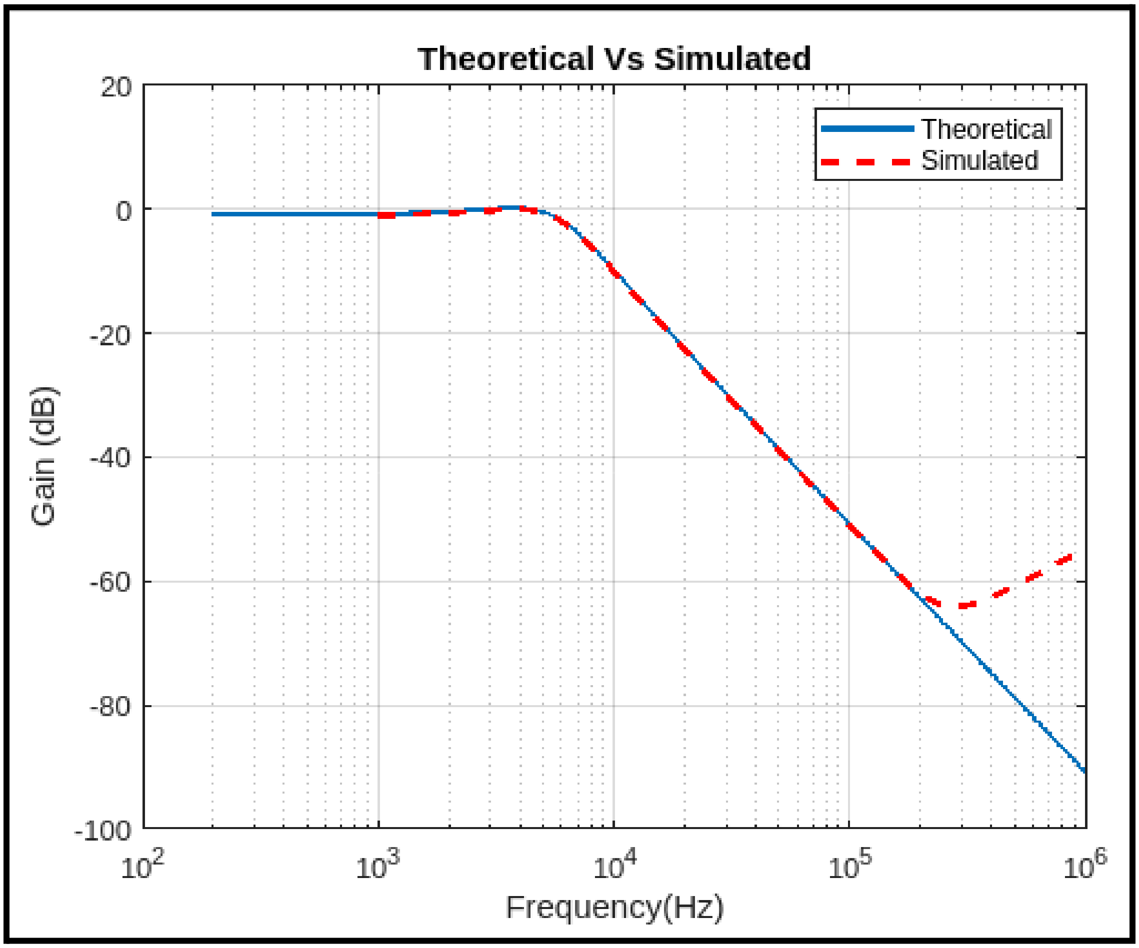


Figure 18: Plot of Theoretical vs. Simulated transfer functions

We see that this transfer function matches the specs that we want.

Section 3

Part A)

After building the circuit, we placed oscilloscope probes at the input and output terminals and measured the peak-to-peak voltages at each point for various frequencies.

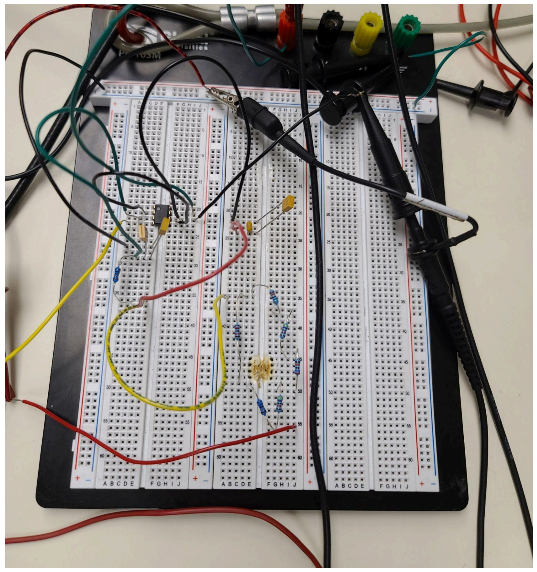


Figure 19: Breadboard Circuit

		DC (10Hz)	f_{PEAK} (5,650 Hz)	4 kHz	32 kHz	180 kHz	1 MHz	2MHz
Measured	V_{in} =	1.04V	10.4V	10.4V	10.3V	10.1V	10.0V	10.4V
	V_{out} =	1.04V	9.4V	7.4V	0.266V	0.004V	0.02V	0.016V
	Gain (dB)	0	-2.956	-0.8781	-31.759	-68.045	-53.979	-55.91
Simulated	Gain (dB)	0	-1.3108	0.0235	-30.959	-60.5936	-55.0256	NaN
Theoretical	Gain (dB)	0	-1.3742	-0.043	-31.04	-61.12	-90.91	-102.9dB

Table of measured and calculated gain

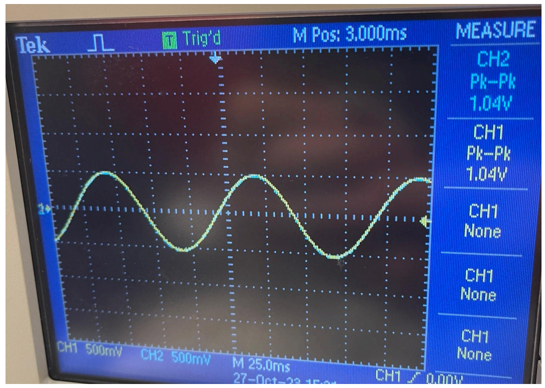


Figure 20: Voltage Input (yellow) and Output (blue) at 10Hz

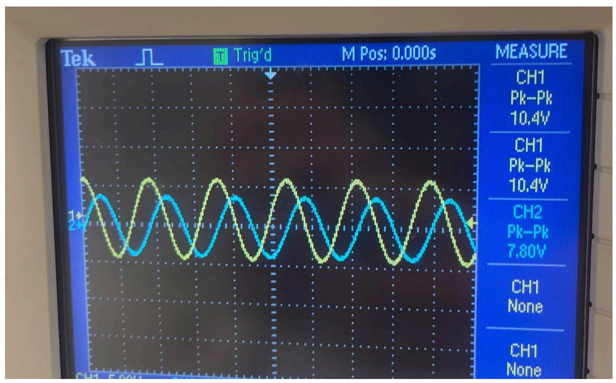


Figure 21: Voltage Input (yellow) and Output (blue) at 5650Hz

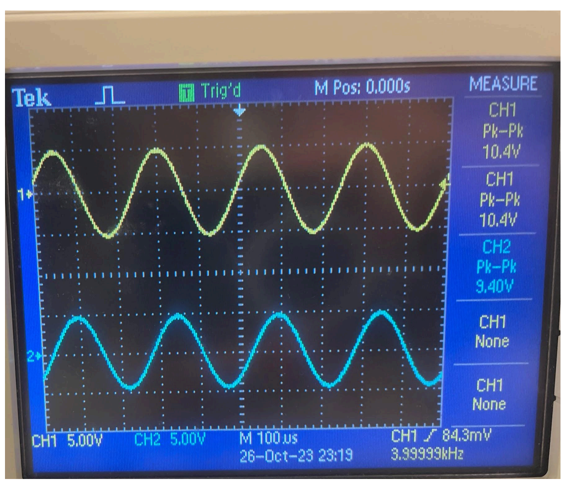


Figure 22: Voltage Input (yellow) and Output (blue) at 4KHz

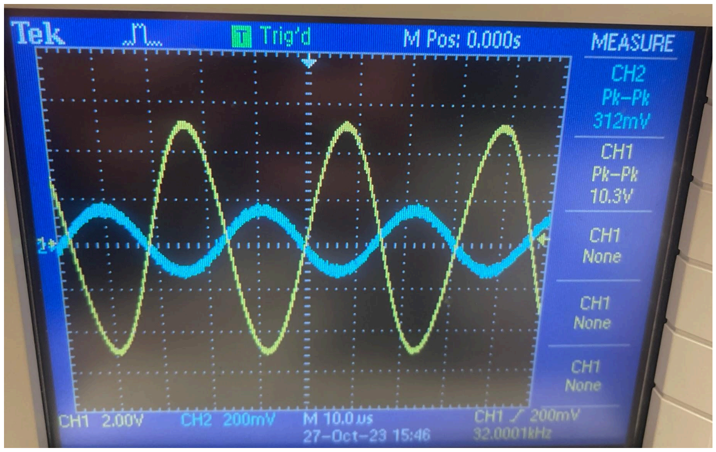


Figure 23: Voltage Input (yellow) and Output (blue) at 32kHz

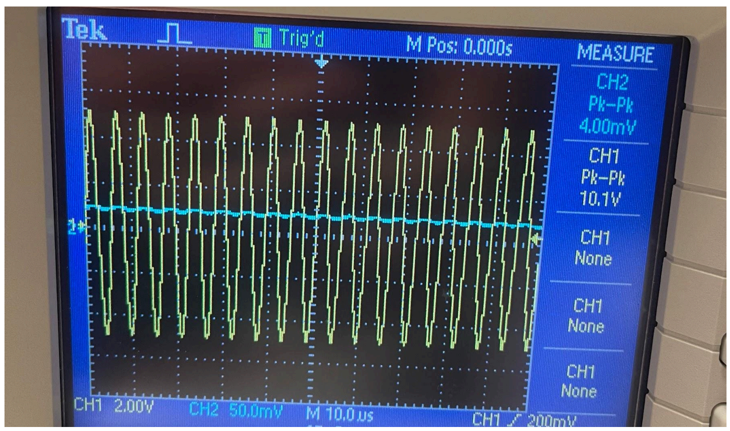


Figure 24: Voltage Input (yellow) and Output (blue) at 180kHz

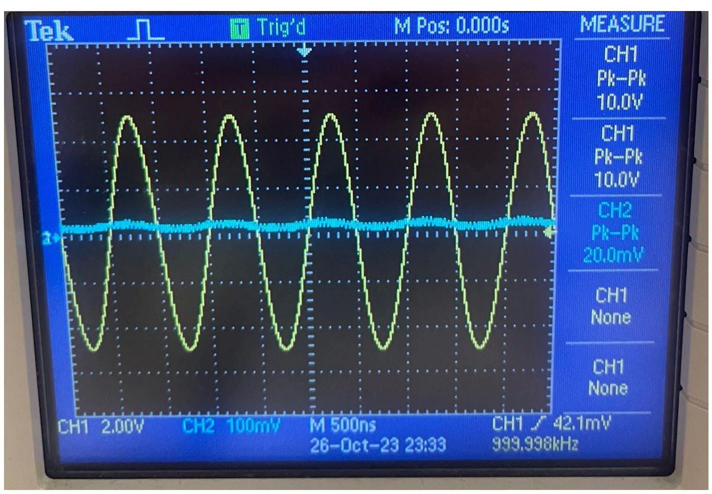


Figure 25: Voltage Input (yellow) and Output (blue) at 1MHz

Figure 26: Voltage Input (yellow) and Output (blue) at 2MHz

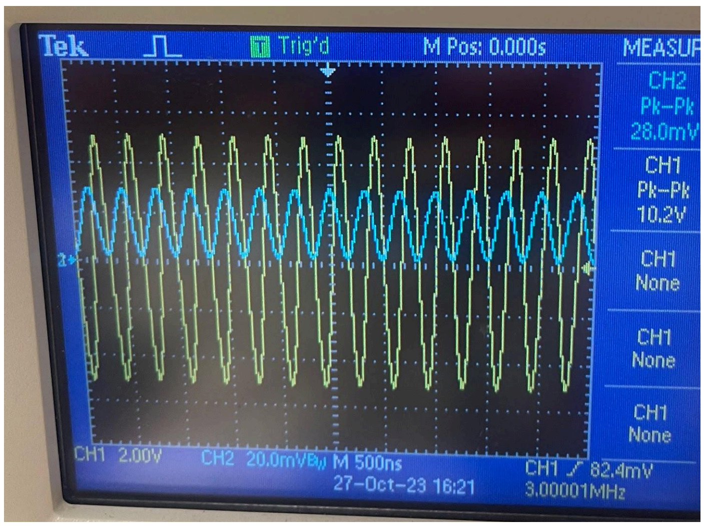


Figure 28: Voltage Input (yellow) and Output (blue) at 3MHz

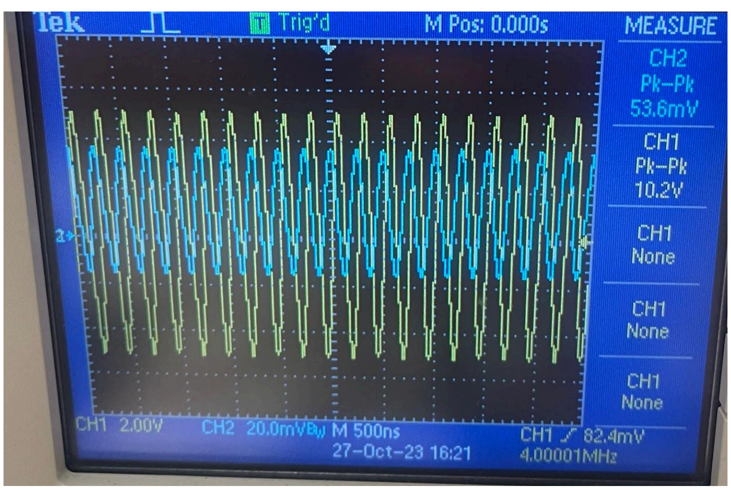


Figure 29: Voltage Input (yellow) and Output (blue) at 4MHz

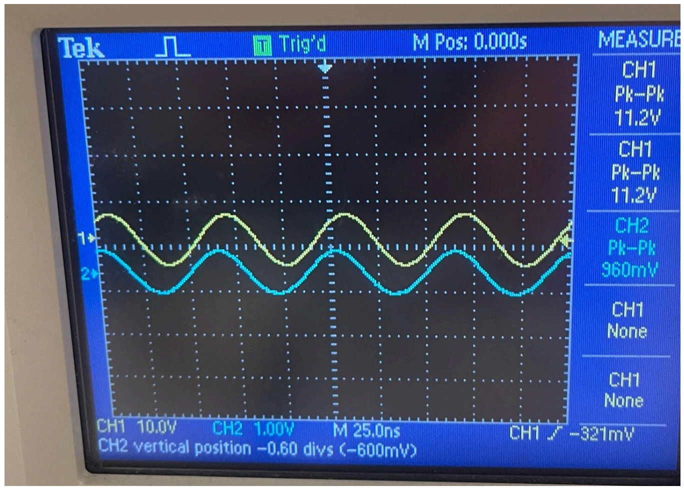


Figure 30: Voltage Input (yellow) and Output (blue) at 16MHz

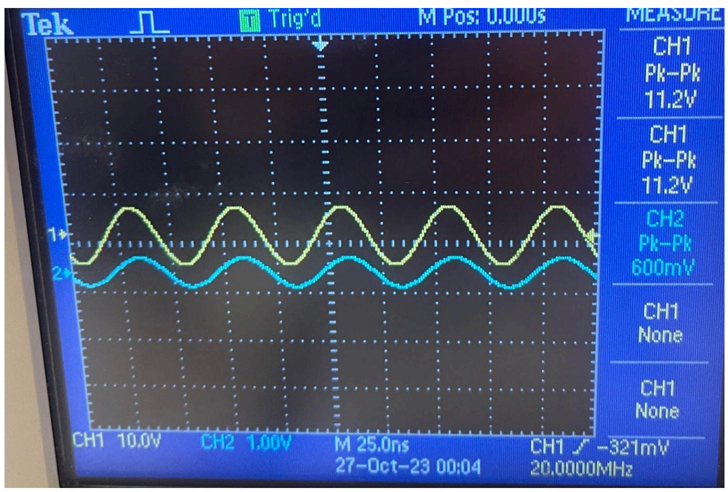


Figure 31: Voltage Input (yellow) and Output (blue) at 20MHz

We see that this signal starts to deform a little bit after 16 MHz and a lot at 20MHz.

Part B)

Explore the remainder of the frequency range up to the generator limit and ensure there are no unexpected "features." Since this is an active circuit, you must ensure that the signal does not drive the op-amp into a nonlinear operating condition. So long as the output voltage still looks like a sine wave, you are probably OK. A good way to confirm that the system is being tested in a linear range is to change the input voltage by a factor of two and re-measure $|H(f)|^2$; if it is the same, you are OK. Take a scope image before and after changing the input voltage. If they differ, reduce the input voltage by another factor of two and try again. Put your measurements in an ascii file so you can read them into MATLAB and plot them on top of the simulation

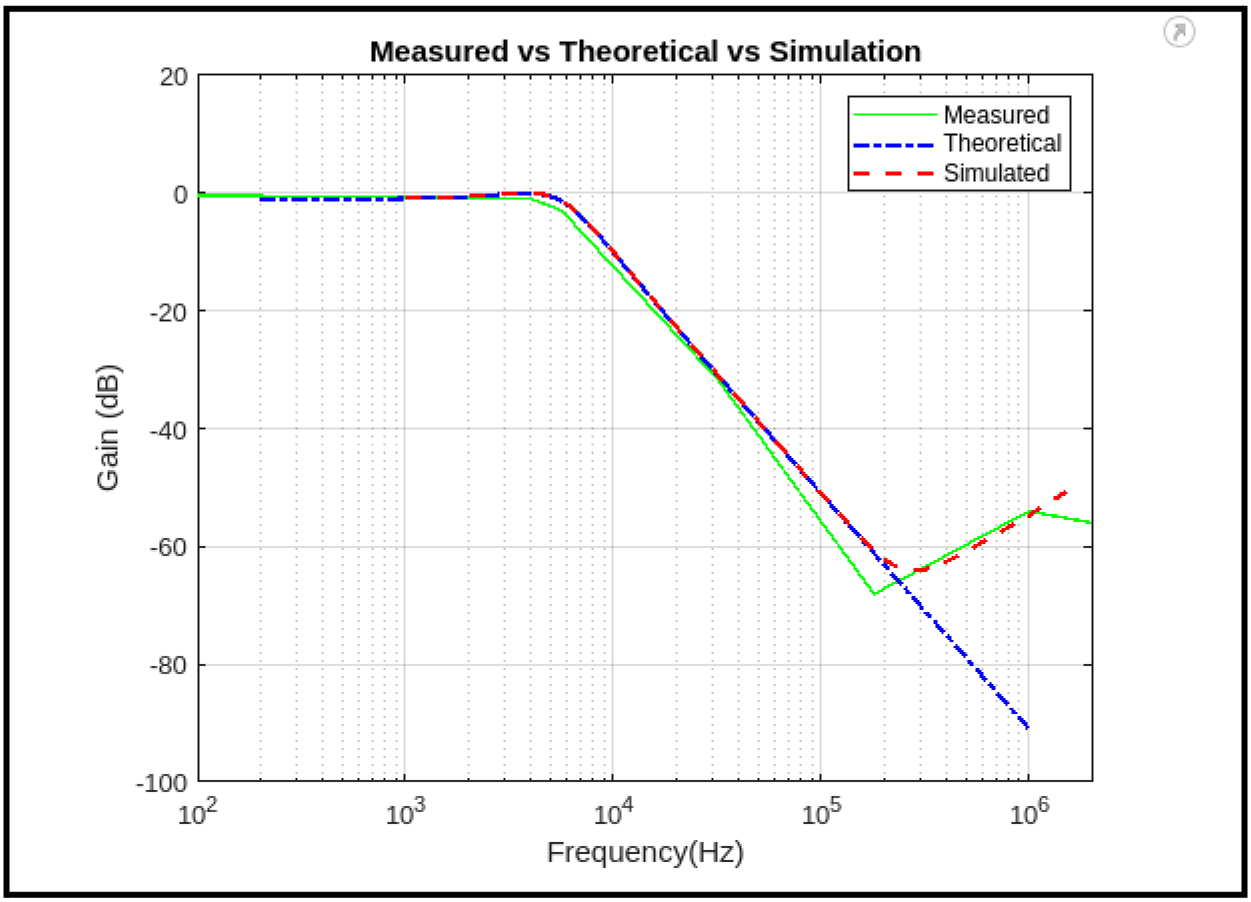


Figure 32: Plot of Measured vs. Theoretical vs. Simulated transfer function

Conclusion

To initiate the design process for an active low-pass filter, we first explored the feasibility of employing a 2nd order Butterworth filter to meet our specified requirements. After careful analysis, we determined that setting the cutoff frequency (Fo) at approximately 4171Hz would enable us to attain our desired peak at 4 kHz. Subsequently, we evaluated the filter's performance by applying Fo and substituting frequencies of 32 kHz and 180 kHz for f, resulting in attenuation levels of -30.26 dB and -50.26 dB, respectively. While these attenuation values technically fell within our acceptable range, we recognized that the component tolerances required to achieve this design might be unmanageably large. As a more suitable alternative, we decided to permit a minor amount of passband ripple, ranging from -1 dB to 0 dB, with a modified design. This adjustment allowed us to more precisely align the amplitude response to our requirements.

In implementing this revised design, we determined a damping factor (zeta) of 0.52 by identifying the point on the squared amplitude response graph where it crosses -1 dB and divided by itself at zero. Finally, by referring to the amplitude response graph, we pinpointed a cutoff frequency (Fo) value of 5650 Hz that would ensure our design's compliance with the desired specifications.

To implement this filter using a Sallen-Key design, we made the assumption that Z1 equals Z2, and R1 equals R2, both set at 100k ohms. We then utilized the given values of Fo and Z to calculate the appropriate capacitor values, resulting in 147pF for C1 and 539pF for C2. The only remaining design parameter was establishing a gain of -1dB, which we achieved by splitting R2 into two parallel resistors with the correct resistance ratio. Once we gathered components with values close to those mentioned, we assembled the circuit. This configuration yielded gain values of -0.9dB, -32 dB, -68 dB, and -54 dB for frequencies of 4kHz, 32kHz, 180kHz, and 1MHz respectively.

While these values differed slightly from our initial simulations, we initially attributed the variation to potentially faulty components. We also did not have exact component values available (i.e. We needed to put various capacitors in parallel to achieve our desired values).

MATLAB Code

Question 1

Part B

```
z1 = .1:.001:.7
H1_mag_sq = 1./(4*z1.^2 - 4*z1.^4)
h1_log = 10*log10(H1_mag_sq)
plot(z1, h1_log)

title("Gain vs. \zeta")
ylabel("10log_{10}(|H(j\omega)|^2) (dB)")
xlabel("\zeta")
ax = gca
chart = ax.Children(1)
datatip(chart, 0.522, 1.008)
grid on
```

Question 1

Part C

```
f1 = 1000:1:10000
f_0 = 5000
z2 = .52
H2_mag_sq = .7943./((1-(f1/f_0).^2).^2 + (2*z2.*(f1/f_0)).^2)
h2_log = 10*log10(H2_mag_sq)
plot(f1, h2_log)
grid on
ax = gca
chart = ax.Children(1)
datatip(chart, 4794, -1.00)

title("Gain vs. Test Frequency")
ylabel("10log_{10}(|H(j\omega)|^2) (dB)")
```

```
xlabel("Frequency (Hz)")


f2 = 200:.01:100000
H3_mag_sq = .7943./((1-(f2/f_02).^2).^2 + (2*z2.*(f2/f_02)).^2)
h3_log = 10*log10(H3_mag_sq)
semilogx(f2, h3_log)
title("Gain vs. f_0")
ylabel("10log_{10}(|H(j\omega)|^2) (dB)")
xlabel("Frequency (Hz)")
grid on

ax = gca
chart = ax.Children(1)
datatip(chart, 4000, .0186)
datatip(chart, 32000, -31.0024)


desiredY = interp1(f2, h3_log, 4000)
desiredY = interp1(f2, h3_log, 32000)
```

Question 2

```
f = 200:10:1000000;
z = 0.522;
figure (4);
f_02 = 5650;
H3_mag_sq = .7943./((1-(f/f_02).^2).^2 + (2*z.*(f/f_02)).^2);
h3_log = 10*log10(H3_mag_sq);
semilogx(f, h3_log,'LineWidth', 2,'Color','b');


results_freq = [10 4000 5650 32000 180000 1000000 2000000];
gain = [0 0 0 0 0 0 0];


for counter = 1:1:7
gain(counter) = interp1(f,h3_log, results_freq(counter))
end
hold on
grid on
xlabel('Frequency(Hz)');
ylabel('Gain (dB)');
title("Theoretical Vs Simulated")
xlim([100 1000000]);
sim = readtable('new.txt');
sim_frequency = sim.Frequency;
sim_out = sim.Vo;


semilogx(sim_frequency,sim_out,'Color','r','LineStyle','--','LineWidth',
2);
desiredY1 = interp1(sim_frequency,sim_out, 180000);
desiredY2 = interp1(sim_frequency,sim_out, 1000000);
legend("Theoretical","Simulated")
```

```
ax3 = gca
chart3 = ax3.Children(1)
datatip(chart3, 180000, desiredY1);
datatip(chart3, 1000000, desiredY2);


dt = findobj(gca,"DataIndex",227);
set(dt,"Location","southwest");
hold off
```

Question 3

```
f0 = 5650;
f = 200:10:1000000;
z = 0.522;
results_freq = [10 4000 5650 32000 180000 1000000 2000000];
v_in = [1.04 10.4 10.4 10.3 10.1 10.0 10.0];
v_out= [1.04 9.4 7.4 .266 .004 .02 .016];
gain = [0 0 0 0 0 0 0];
for counter = 1:1:7
gain(counter) = 20*log10(v_out(counter)/v_in(counter));
End
figure (6);
H3_mag_sq = .7943./((1-(f/f0).^2).^2 + (2*z.*(f/f0)).^2);
h3_log = 10*log10(H3_mag_sq);


%Adding results

semilogx(results_freq, gain,'LineWidth', 1,'Color','g')
hold on
semilogx(f, h3_log,'LineWidth', 1.5,'Color','b','LineStyle','-.')
semilogx(sim_frequency,sim_out,'Color','r','LineStyle','--','LineWidth',
1.5)
legend("Measured","Theoretical","Simulated")
grid on
xlabel('Frequency(Hz)');
ylabel('Gain (dB)');
title("Measured vs Theoretical vs Simulation")
xlim([100 2000000]);
hold off
```