ECE 100: Linear Electronic Systems

Professor: Drew Hall

Lab 1:
Passive RLC Circuits

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Abstract

The purpose of this lab was to analyze various RLC circuits and their properties as they behave as different frequency filters. By using MATLAB we analyzed four different filters (low-, band-, high-pass, and mystery), testing each with different ζ (zeta) values. We created Bode plots comparing the magnitude and phase for each filter with respect to frequency. This showed us the effects of rearranging an RLC circuit without adding or removing parts.

Experimental Procedure

Tools: MATLAB software

Section 1: Low-Pass RLC Circuit

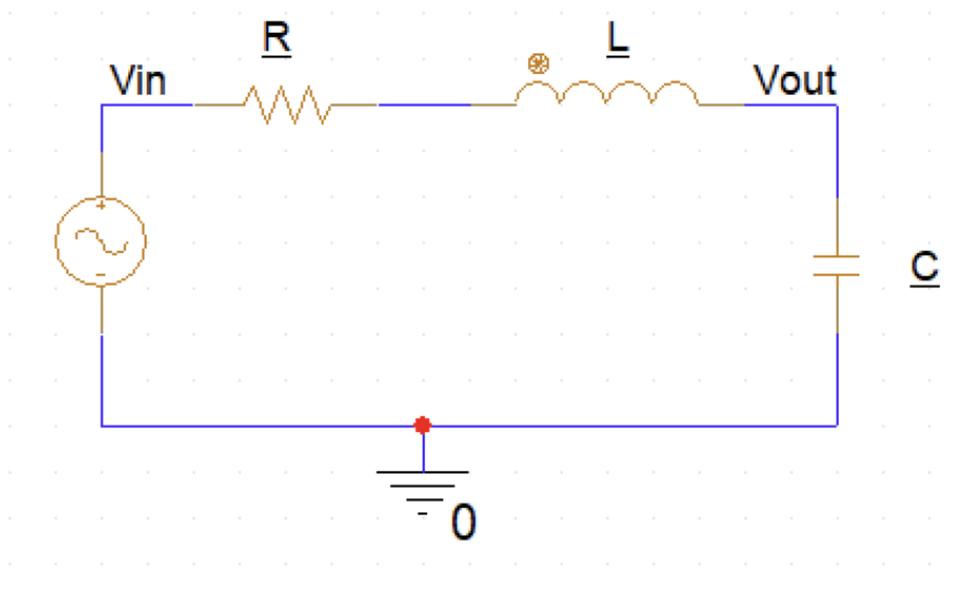


Figure 1: Schematic of Low-Pass filter

Using a voltage divider, the voltage output over voltage input is the transfer function of the circuit. Then we had to find the frequency response of the circuit by writing ω_0 and ζ in terms of R, L, and C. We used MATLAB to plot every ζ value for $|H(j\omega)|$ vs ω. Then we used MATLAB to plot two types of plots, magnitude squared in dB vs. log frequency and degrees vs. log frequency.

Section 2: High-pass and Band-pass RLC circuits:

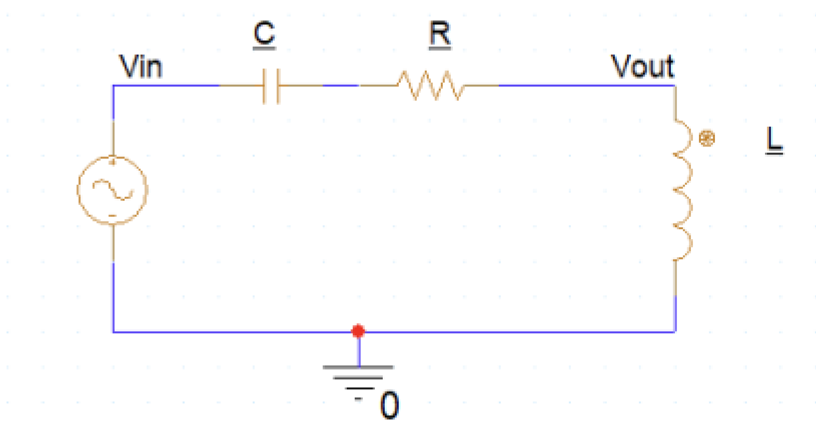


Figure 2: Schematic of High-Pass filter

We found the transfer function of the high-pass filter in terms of R, L, and C. The denominator is the same as the low-pass filter so we have to change the numerator by writing ω_0 and ζ in terms of R, L, and C. Using MATLAB, we modified our scripts and made the Bode plots of $H(j\omega)$ for the high-pass filter.

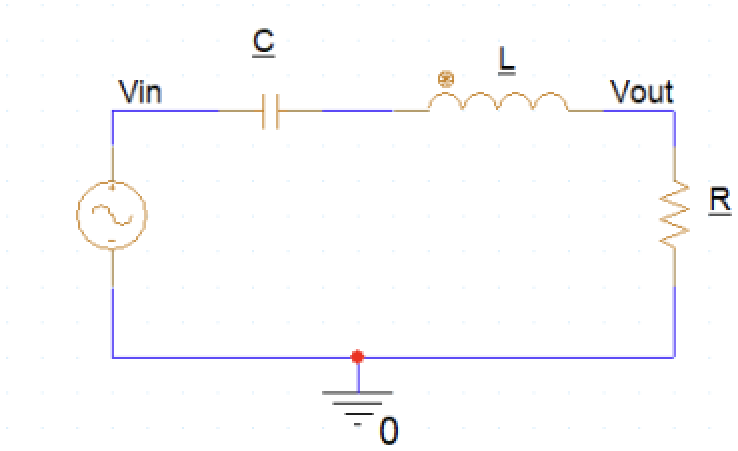


Figure 3: Schematic of Band-Pass filter

The procedure was almost identical to the high-pass filter. This time we had a different numerator which can also be called the gain and changed it by writing ω_0 and ζ in terms of R, L, and C.

Section 3: Mystery Filter

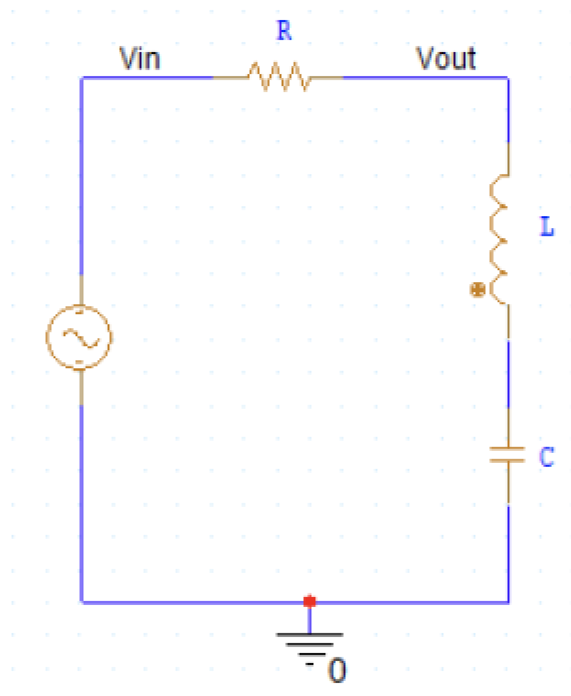


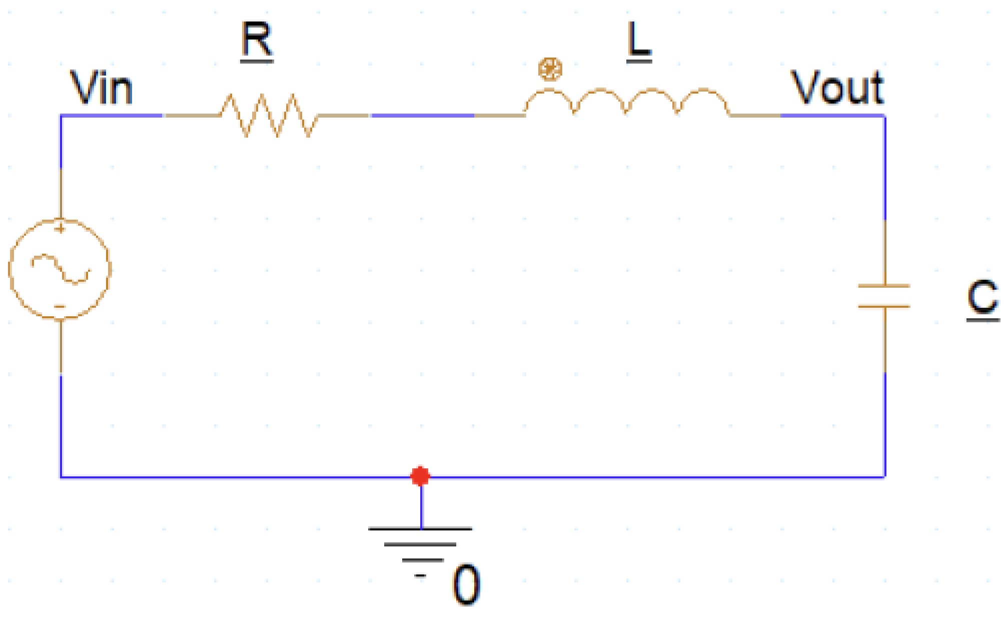
Figure 4: Schematic of Mystery Filter

The procedure was to do the same thing we did for problems 1 and 2. We found the transfer function and the frequency response. Then we proceeded to plot on MATLAB.

Results

* For hand-calculations, refer to ‘Z’ as Zeta (ζ) *

Section 1: Low-Pass RLC Circuit



Part A)

Find the transfer function, $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$, in the above low-pass RLC filter circuit.

$$V_{OUT} = \frac{\left(\frac{1}{sC}\right)}{\left(R + sL + \frac{1}{sC}\right)} \cdot V_{in} \cdot \left(\frac{sC}{sC}\right)$$

$$\boxed{H(s) = \frac{V_{OUT}}{V_{in}} = \frac{1}{(RSC + s^2CL + 1)}}$$

Part B)

The frequency response of the circuit can be written as $H(j\omega) = \frac{1}{1 + 2\zeta\frac{j\omega}{\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2}$. Write ω_0 and ζ in terms of R, L, and C.

$$H(j\omega) = \frac{1}{1 + 2\zeta\frac{j\omega}{\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2}$$

$$H(s) = \frac{1}{1 + \underline{2Z\left(\frac{s}{\omega_0}\right)} + \underline{\left(\frac{s^2}{\omega_0^2}\right)}} = \frac{1}{1 + \underline{RSC} + \underline{s^2CL}}$$

$$2Z\frac{1}{\left(\frac{1}{\sqrt{CL}}\right)} = RC$$

$$2Z\sqrt{CL} = RC$$

$$\boxed{Z = \frac{RC}{2\sqrt{CL}}}$$

$$\frac{s^2}{\omega_0^2} \longrightarrow s^2CL$$

$$\omega_0^2 = \frac{1}{CL}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{CL}}} = \frac{\sqrt{CL}}{CL}$$

Part C)

Write a MATLAB script to calculate and plot the magnitude and phase of $H(j\omega) = \frac{1}{1 + 2\zeta\frac{j\omega}{\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2}$ for $\omega_0 = 1$ and $\zeta = [\ 0.1, 0.3, 0.707, 1.0, 3.0, 10.0]$.

ii. Plot of Magnitude and Phase

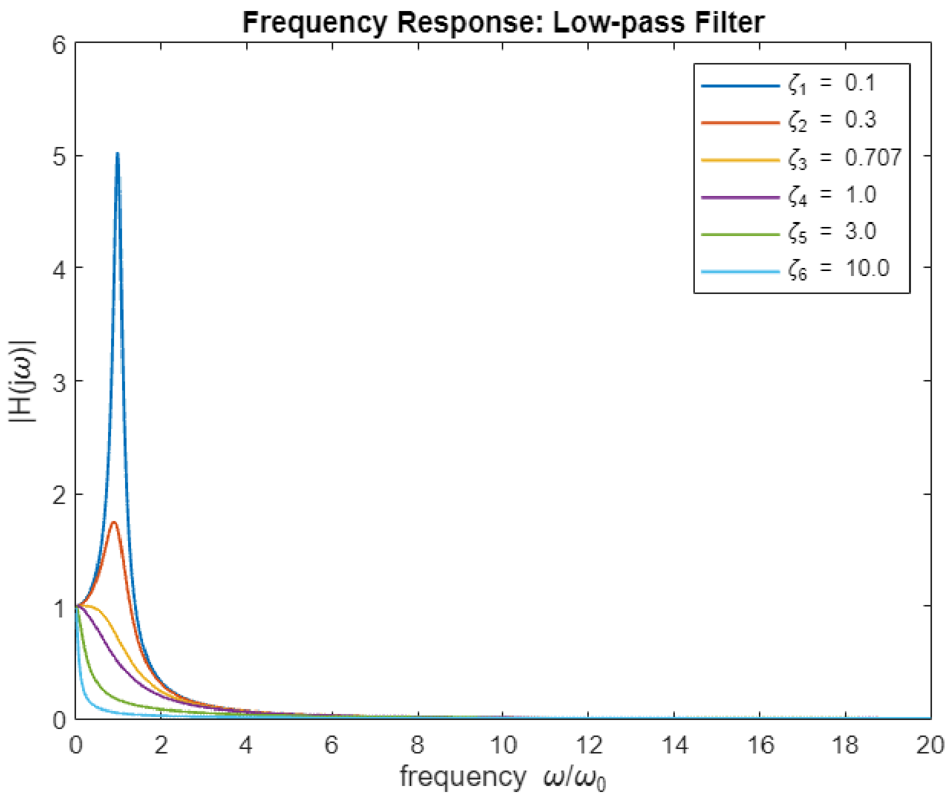


Figure 5: Magnitude plot of low-pass in Linear Scale

We see that this plot is hard to read on linear axes. We can see the magnitude vs frequency based on the ζ value.

iii. Bode Plot

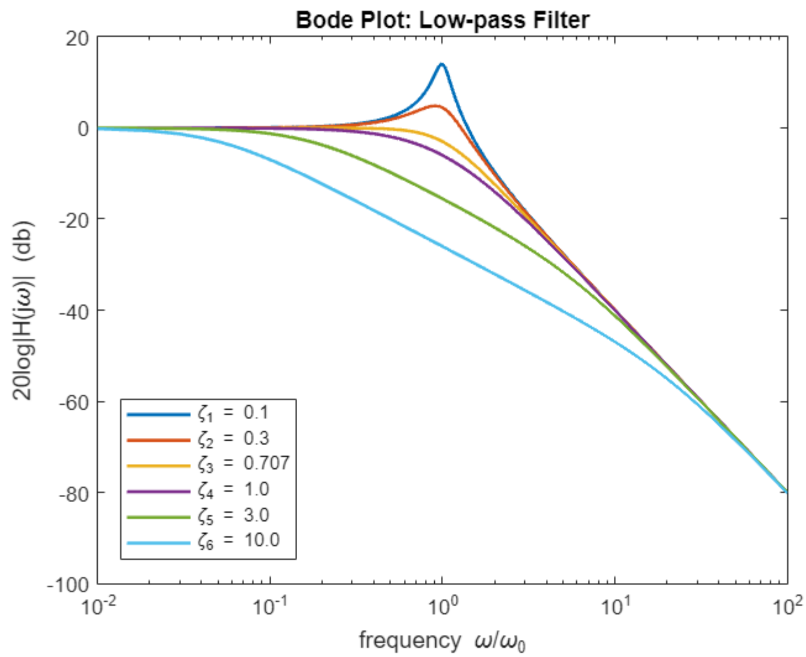


Figure 6: Magnitude plot of low-pass filter in dB vs log frequency

iv. Phase Plot

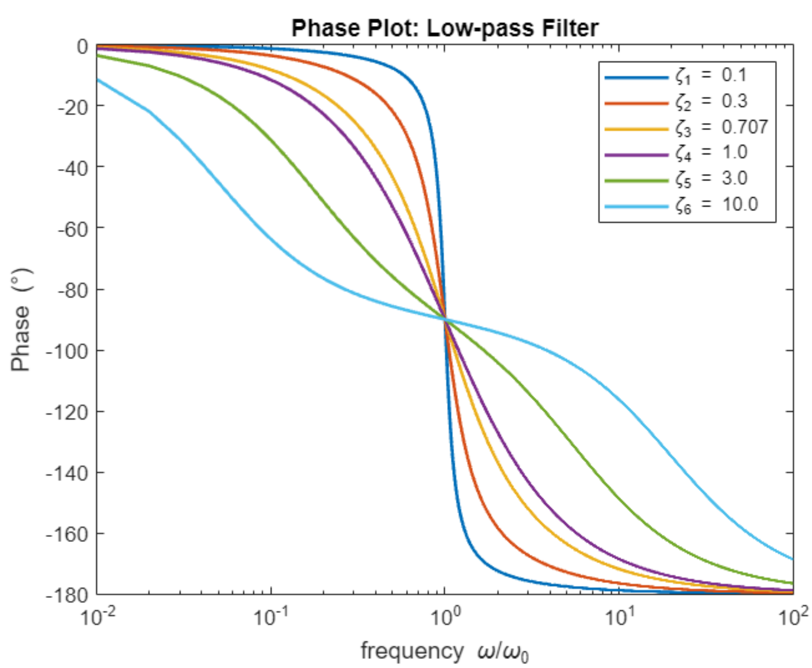


Figure 7: Phase plot of low-pass filter

We can see that lower frequencies are passed through and all the higher frequencies are rejected.

Part D)

When the damping factor ζ is less than 0.707, a peak occurs in $|H(j\omega)|$. Analytically, find expressions for the peak frequency, ω_{peak}, and $\left|H(\omega_{peak})\right|$ in term of ω_0 and ζ. Assuming $\omega_0 = 1$, calculate the values of ω_{peak} for $\zeta = 0.1$ and 0.3. Also find $\left|H(\omega_{peak})\right|$ at these two ω_{peak} values. Do these values agree with the peaks in your MATLAB figures for $\zeta = 0.1$ and 0.3?

$$H(j\omega) = \frac{1}{1 + 2\zeta\frac{j\omega}{\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2}$$

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\left(\frac{2\zeta\omega}{\omega_0}\right)}$$

$$|H(j\omega)|^2 = \frac{1}{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_0}\right)^2}$$

$$\omega_p = \frac{\partial}{\partial\omega}\left(|H(j\omega)|^2\right) = \frac{\partial}{\partial\omega}\left(\frac{1}{|H(j\omega)|^2}\right) \quad \star$$

$$\frac{\partial}{\partial\omega}\left(\frac{1}{|H(j\omega)|^2}\right) = \frac{\partial}{\partial\omega}\left[\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_0}\right)^2\right]$$

$$0 = 2\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)\left(-\frac{2\omega}{\omega_0^2}\right) + 2\left(\frac{2\zeta\omega}{\omega_0}\right)\left(\frac{2\zeta}{\omega_0}\right)$$

$$0 = \left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)\left(-\frac{4\omega}{\omega_0^2}\right) + 2\left(\frac{4\omega\zeta^2}{\omega_0^2}\right)$$

$$0 = \left(\frac{4\cancel{\omega}^{\,0}}{\omega_0^2}\right)\left[\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)(-1) + 2\zeta^2\right]$$

$$0 = \frac{\omega^2}{\omega_0^2} - 1 + 2\zeta^2$$

$$\frac{\omega^2}{\omega_0^2} = 1 - 2\zeta^2$$

$$\frac{\omega}{\omega_0} = \sqrt{1 - 2\zeta^2}$$

$$\boxed{\begin{array}{l} \omega_p = \omega_0\sqrt{1 - 2\zeta^2} \\ \omega_p = 0.9899 \\ \omega_p = 0.9055 \end{array}}$$

@ $\zeta = 0.1 \longrightarrow \omega_p = 0.9899$

@ $\zeta = 0.3 \longrightarrow \omega_p = 0.9055$

$$|H(j\omega)| = \sqrt{\frac{1}{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_0}\right)^2}}$$

$$= \sqrt{\frac{1}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{4\zeta^2\omega^2}{\omega_0^2}\right)}}$$

$$|H(\omega_p)| = \sqrt{\frac{1}{\left[1 - \frac{\left(\omega_0\sqrt{1-2\zeta^2}\right)^2}{\omega_0^2}\right]^2 + \frac{4\zeta^2}{\omega_0^2}\cdot\left(\omega_0\sqrt{1-2\zeta^2}\right)^2}}$$

$$= \sqrt{\frac{1}{\left[1 - \frac{\cancel{\omega_0}^2(1-2\zeta^2)}{\cancel{\omega_0}^2}\right]^2 + \frac{4\zeta^2\cancel{\omega_0}}{\cancel{\omega_0}^2}\cdot(1-2\zeta^2)}}$$

$$= \sqrt{\frac{1}{(1 - 1 - 2\zeta^2)^2 + 4\zeta^2(1 - 2\zeta^2)}}$$

$$= \frac{\sqrt{1}}{\sqrt{4\zeta^4 + 4\zeta^2 - 8\zeta^4}}$$

$$= \frac{1}{\sqrt{-4\zeta^4 + 4\zeta^2}} = \frac{1}{\sqrt{4\zeta^2(-\zeta^2 + 1)}}$$

$$\boxed{|H(\omega_p)| = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}}$$

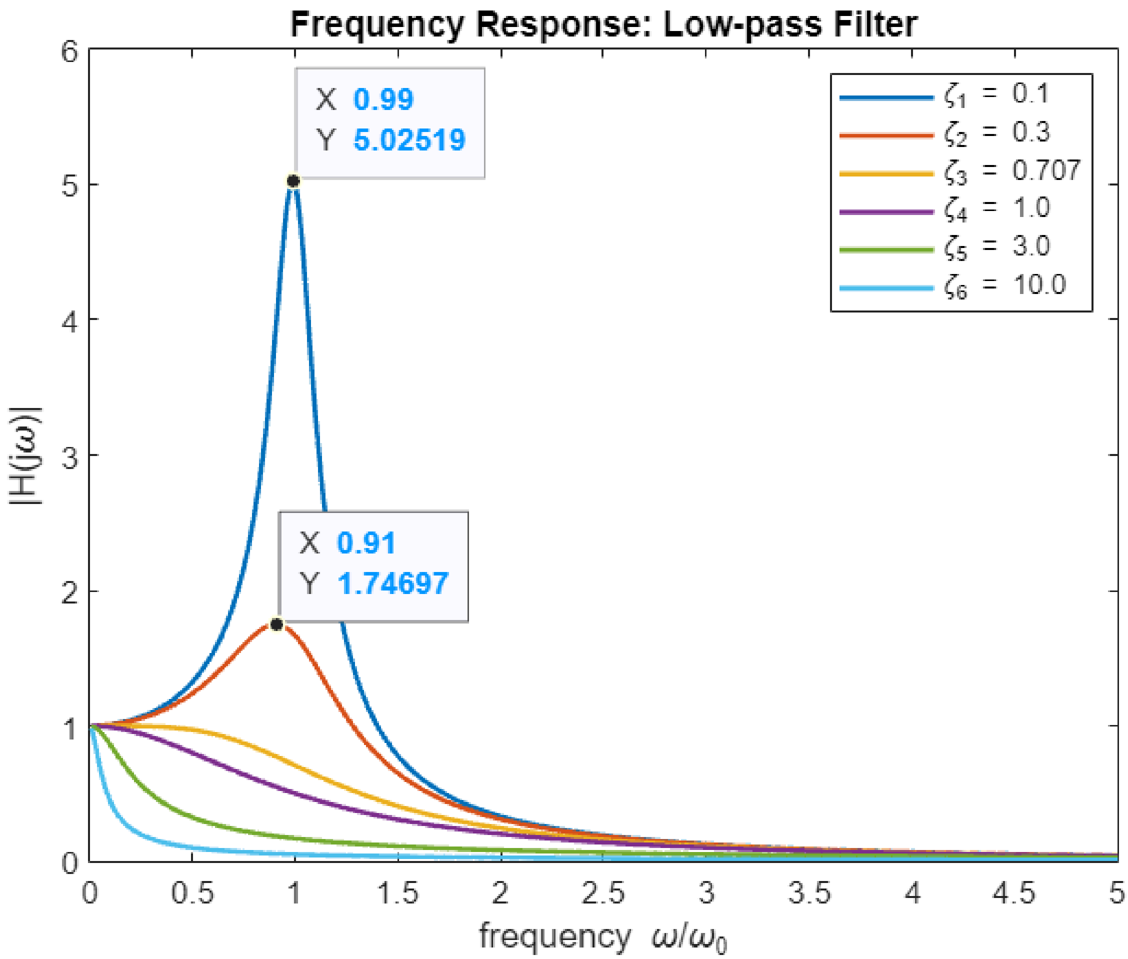


Figure 8: Magnitude plot of low-pass filter with Peak Values Labeled

Yes, these numbers match up with the results we calculated above. We noticed that the peak gets sharper and more narrow as ζ decreases.

Section 2: High-pass and Band-pass RLC Circuits

Part A)

Write the transfer functions $H_{HP}(s)$ and $H_{BP}(s)$ in terms of R, L, and C. Put them in the generalized form so the denominator is the same as the one for $H_{LP}(s)$ and the numerator is written in terms of s, ω_0 and ζ.

$$V_{OUT} = \frac{sL}{\frac{1}{sC} + R + sL} V_{in}$$

$$H(s) = \frac{V_{OUT}}{V_{in}} = \frac{sL}{\frac{1}{sC} + R + sL} \cdot \left(\frac{sC}{sC}\right)$$

$$\boxed{H_{HP}(s) = \frac{s^2 CL}{1 + sRC + s^2 LC}}$$

$$\boxed{H(s) = \frac{1}{1 + 2\zeta\left(\frac{s}{\omega_0}\right) + \left(\frac{s^2}{\omega_0^2}\right)}}$$

$$\begin{cases} RC = 2\zeta\left(\frac{1}{\omega_0}\right) & (1) \\ LC = \frac{1}{\omega_0^2} & (2) \end{cases}$$

$$H(s) = \frac{s^2\left(\frac{1}{\omega_0^2}\right)}{1 + sRC + s^2 LC}$$

$$= \frac{(j\omega)^2\left(\frac{1}{\omega_0^2}\right)}{1 + j\omega RC + (j\omega)^2 LC}$$

$$\boxed{H(j\omega) = \frac{\left(-\frac{\omega^2}{\omega_0^2}\right)}{1 + j\omega RC - \omega^2 LC}}$$

$$V_{OUT} = \frac{R}{\frac{1}{sC} + sL + R}(V_{in})$$

$$\boxed{H_{BP}(s) = \frac{V_{OUT}}{V_{in}} = \frac{sCR}{1 + s^2 CL + sCR}}$$

$$\begin{cases} RC = 2\zeta\left(\frac{1}{\omega_0}\right) \\ LC = \frac{1}{\omega_0^2} \end{cases}$$

$$H(s) = \frac{s\left(2\zeta\left(\frac{1}{\omega_0}\right)\right)}{1 + s^2\left(\frac{1}{\omega_0^2}\right) + s\left(2\zeta\left(\frac{1}{\omega_0}\right)\right)}$$

$$H(j\omega) = \frac{(j\omega)\left[2\zeta\left(\frac{1}{\omega_0}\right)\right]}{1 + (j\omega)^2\left(\frac{1}{\omega_0^2}\right) + (j\omega)\left(2\zeta\left(\frac{1}{\omega_0}\right)\right)}$$

$$= \frac{\left(\frac{2j\omega\zeta}{\omega_0}\right)}{\left(1 - \frac{\omega^2}{\omega_0^2} + \frac{2j\omega\zeta}{\omega_0}\right)} \cdot \left(\frac{\omega_0^2}{\omega_0^2}\right)$$

$$\boxed{H(j\omega) = \frac{2j\omega\zeta\omega_0}{\omega_0^2 - \omega^2 + 2j\omega\zeta\omega_0}}$$

Part B)

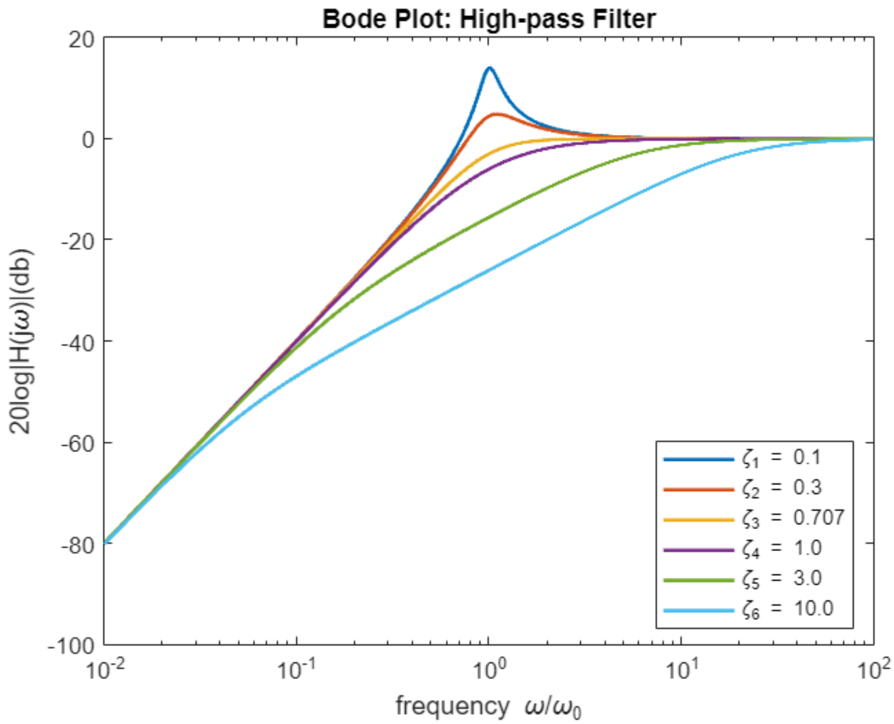


Figure 9: Magnitude plot of high-pass filter in dB vs log frequency

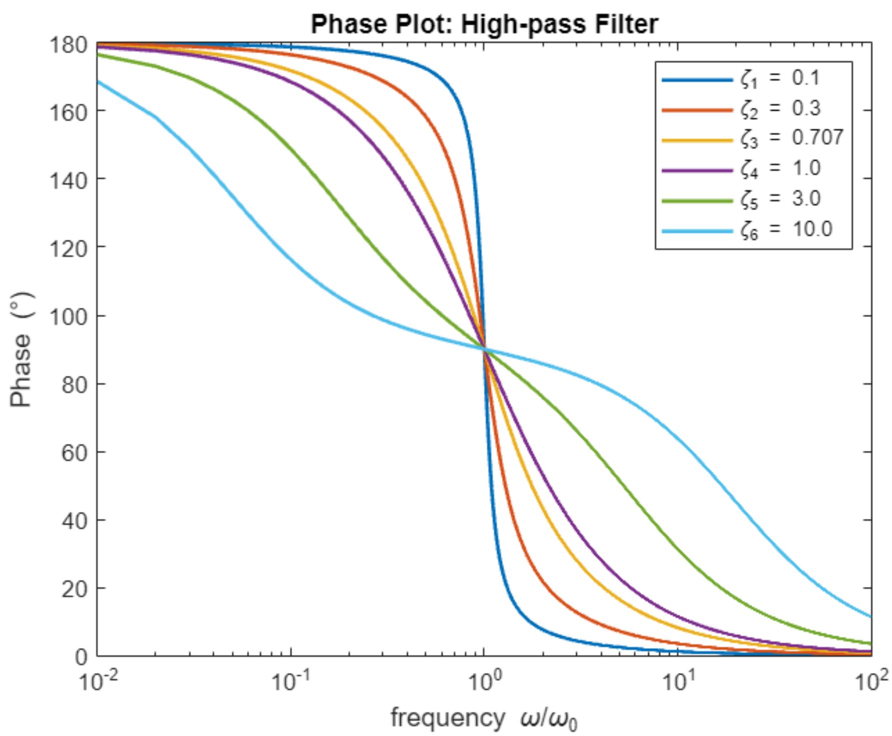


Figure 10: Phase plot of high-pass filter

We can see that lower frequencies are rejected and higher frequencies are allowed to pass through.

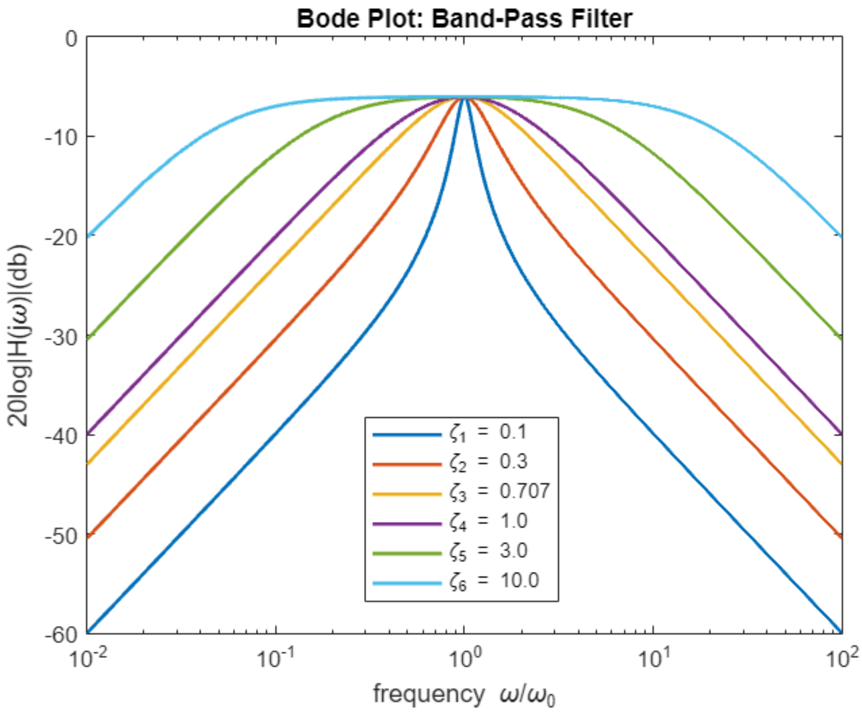


Figure 11: Magnitude plot of band-pass filter in dB vs log frequency

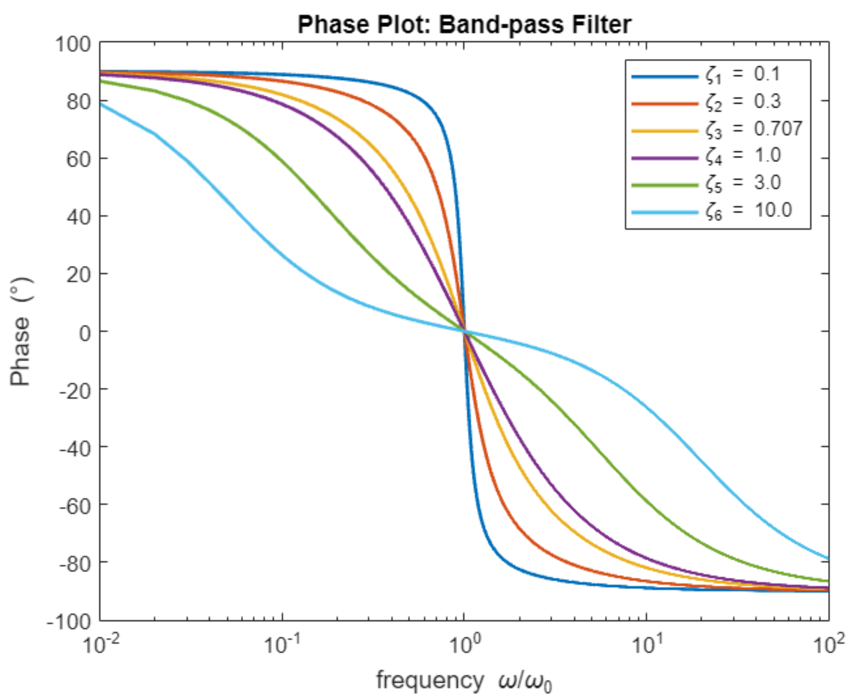


Figure 12: Phase plot of band-pass filter

We see that this allows a selected range of frequencies and rejects everything else that is out of the range.

Section 3: Mystery Filter

What happens if you take the output across C and L together rather than just one? Write the transfer function $H(s)$ and plot the Bode plots for $H(j\omega)$ as you did for the band-pass and high-pass filters. What could you use this circuit for?

$$V_{OUT} = \frac{\left(sL + \frac{1}{sC}\right)}{\left(R + sL + \frac{1}{sC}\right)} \cdot V_{in}$$

$$\boxed{H(s) = \frac{V_{OUT}}{V_{in}} = \frac{s^2CL + 1}{RsC + s^2CL + 1}}$$

$$\begin{cases} RC = 2z\left(\frac{1}{\omega_0}\right) & (1) \\ LC = \frac{1}{\omega_0^2} & (2) \end{cases}$$

$$H(s) = \frac{s^2\left(\frac{1}{\omega_0^2}\right) + 1}{s\left(2z\cdot\left(\frac{1}{\omega_0}\right)\right) + s^2\left(\frac{1}{\omega_0^2}\right) + 1}$$

$$H(j\omega) = \frac{(j\omega)^2\left(\frac{1}{\omega_0^2}\right) + 1}{(j\omega)\left(2z\left(\frac{1}{\omega_0}\right)\right) + (j\omega)^2\left(\frac{1}{\omega_0^2}\right) + 1}$$

$$H(j\omega) = \frac{1 - \frac{\omega^2}{\omega_0^2}}{\left(\frac{2j\omega z}{\omega_0} - \frac{\omega^2}{\omega_0^2} + 1\right)} \cdot \left(\frac{\omega_0^2}{\omega_0^2}\right)$$

$$\boxed{H(j\omega) = \frac{\omega_0^2 - \omega^2}{2j\omega z\omega_0 - \omega^2 + 1}}$$

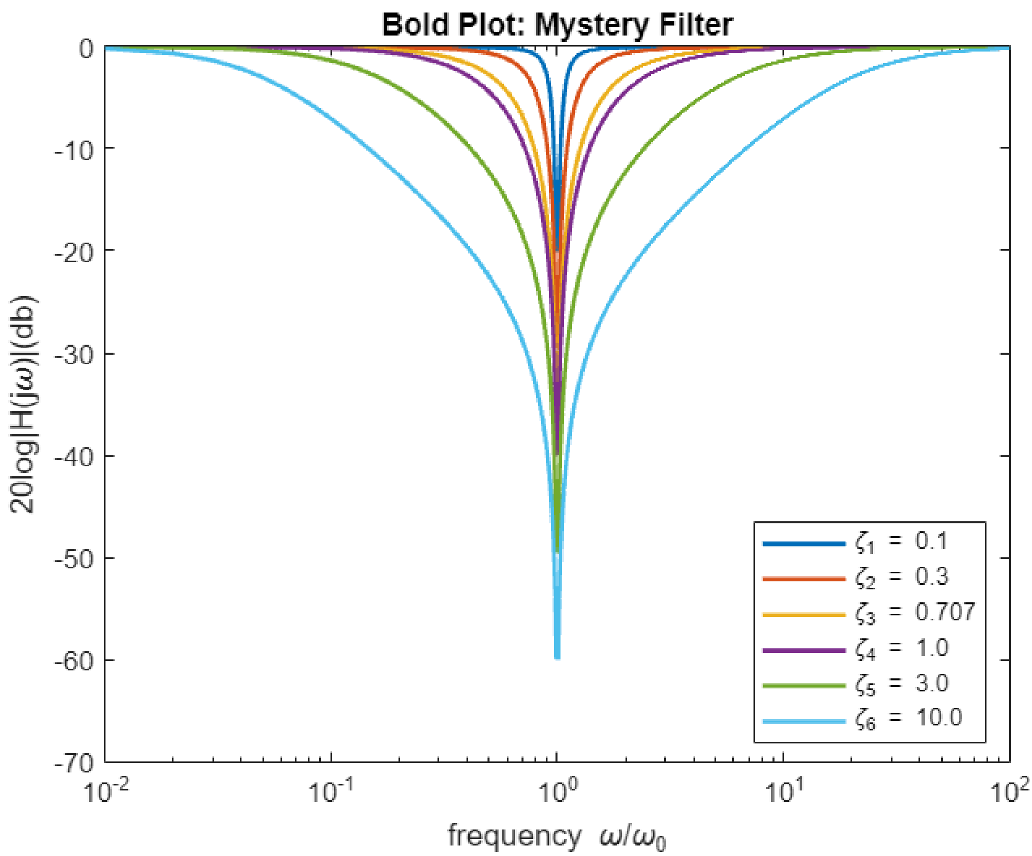


Figure 13: Magnitude plot of mystery filter in dB vs log frequency

We can see that when $\omega = \omega_0$ it rejects the frequency and allows it to pass when they are not equal. This eliminates all frequencies within a band of frequencies

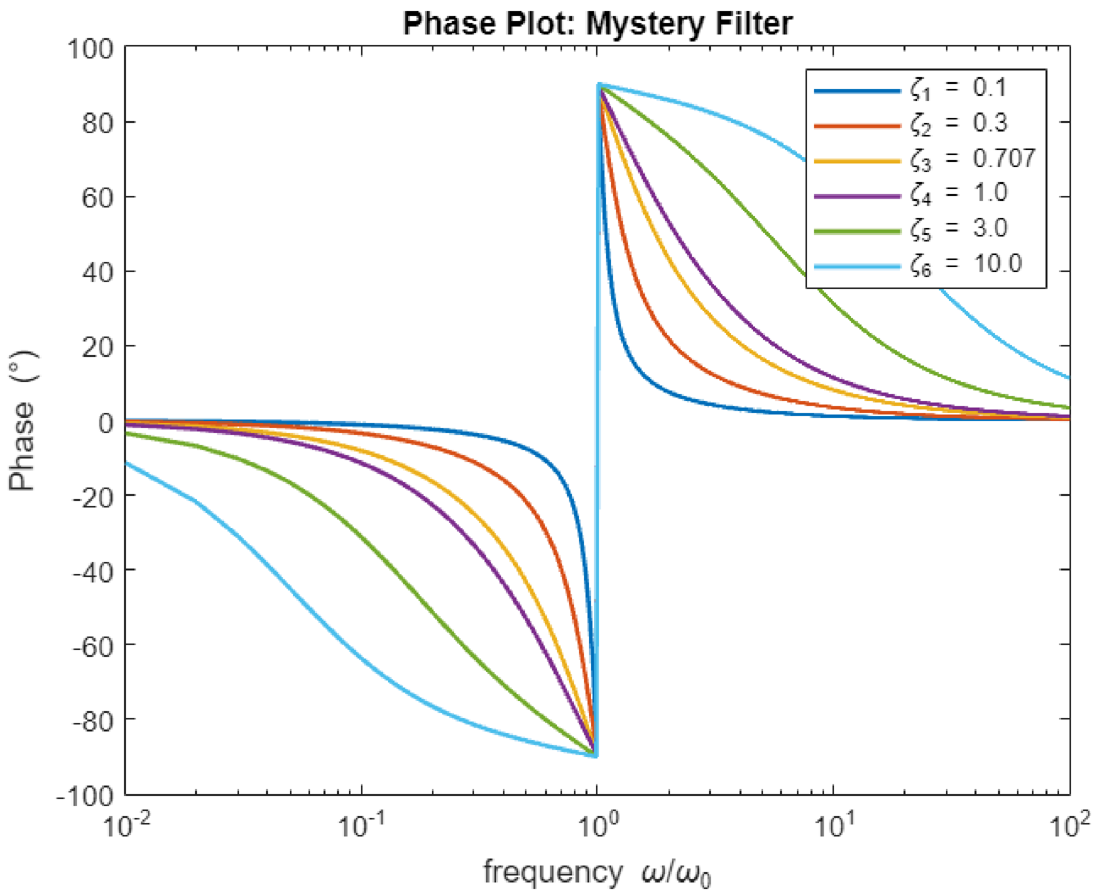


Figure 14: Phase plot of mystery filter

We found that this is a band-stop filter. We came to this conclusion because when $\omega_0 = 1$ and $\omega = 1$ the output would be 0. However, when ω is not equal to ω_0, the filter will allow the signal to pass. We think this would be really useful for noise cancellation because you can select the frequencies you don't want to hear and cancel them out. When we took the output out of both C and L, we noticed that there was a difference in resonance.

Section 4: Linear System Tools

MATLAB can also do symbolic algebra, including Laplace transforms. It has several tools that are very useful for analyzing linear systems. For example, the code below will produce Bode plots and step response plots for the circuits you analyzed above:

```
s = tf('s');                              %define s as a transfer function variable
w0 = 2*pi*15.9e3; zeta = 1/sqrt(2);      %define parameters
h = 1/(1 + 2*zeta*s/w0 + (s/w0)^2);      %define transfer function
bode(h);                                  %create a bode plot with nice scales
figure                                    %new fig so step does not overwrite bode
step(h);                                  %create scaled a step response plot
stepinfo(h)                               %compute rise time, overshoot, etc.
```

Part A)

Use these tools to create a single plot with all the step responses for the transfer functions in Part 1c.

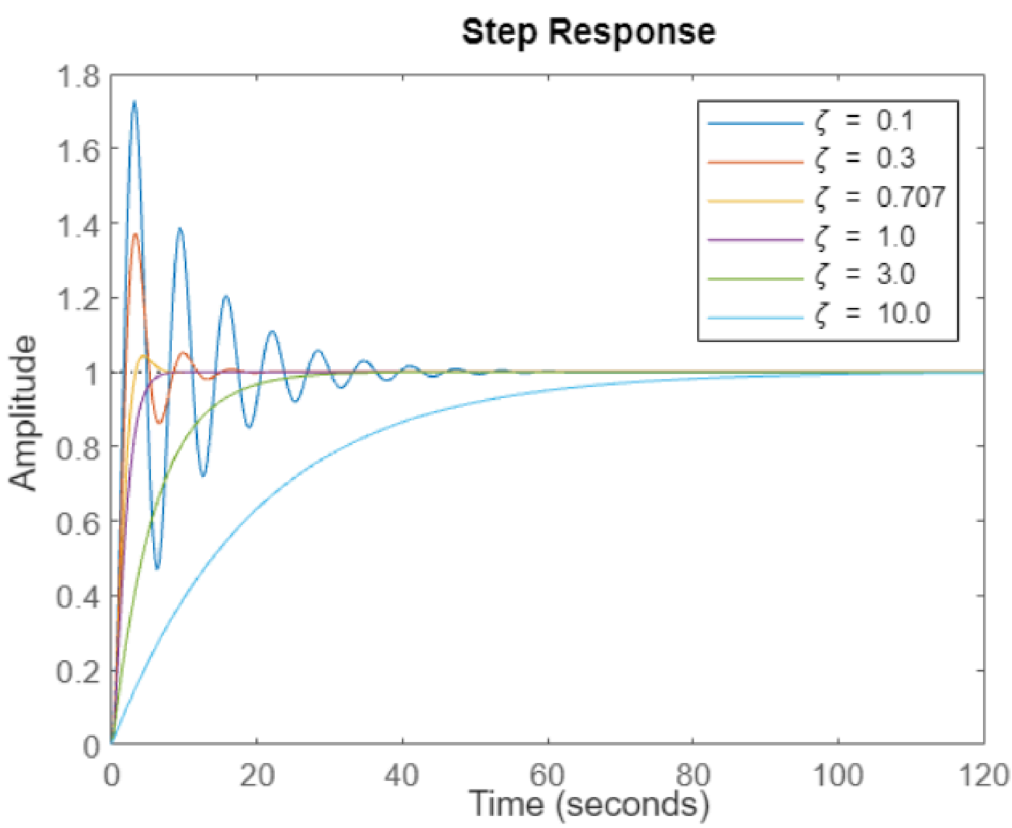


Figure 15: Step Response of low-pass filter

We noticed that if ζ > 1, the step response will have an exponential curve. If ζ < 1, The step response will have an oscillatory part. We also see that ζ = 0.707 represents maximally flat and ζ = 1 is critically damped.

Part B)

Overshoot of the step-response of a second-order low-pass filter: $\%\ overshoot\ =\ 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ for $0\ <\ \zeta\ <\ 1$ and $\%\ overshoot\ =\ 0$ for ζ > 1. Plot the theoretical Overshoot Values vs. ζ for 0.01 < ζ < 10. Use a solid line for plotting the theoretical values. Next, find the overshoot values for each step response in part 1c with the help of the stepinfo function. Overlay the plot of these calculated overshoot values on your current plot. Show these values as a series of symbols, not a solid line.

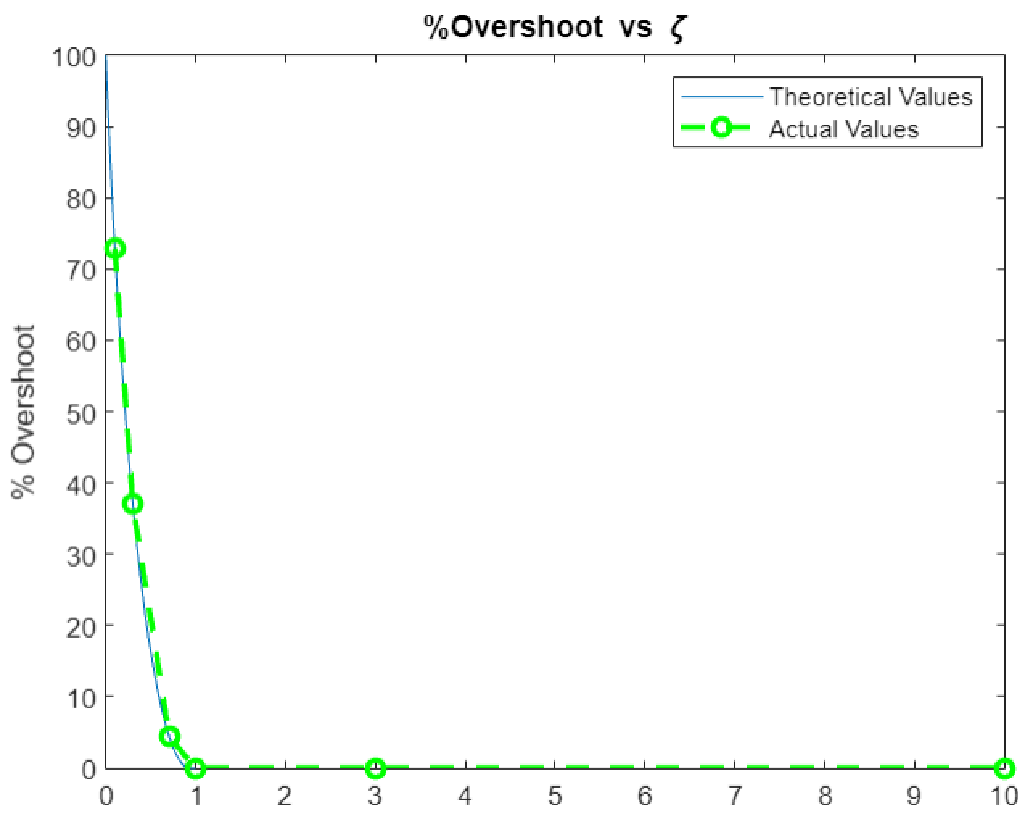


Figure 16: Overshoot % vs ζ

After normalization, we see that as ζ decreases, the rise time decreases and the overshoot increases. From the graph, we see that ζ = has the slowest rise time but no overshoot and ζ = 0.1 has the fastest rise time but high overshoot.

Conclusion

By doing this experiment we were able to show that rearranging an RLC circuit can be used to create different filters and block out unwanted frequencies. The low-pass blocked any high frequencies, the high-pass blocked any low frequencies, and the band-pass only allowed frequencies between specific values to pass through. We saw that the smaller the pole, the longer the rise time and less overshoot. We found that the damping coefficient (ζ) helps control and set the amount of overshoot and rise time. $\zeta = 0.1$ gave us the fastest rise time but the most overshoot and $\zeta = 10$ gave us the slowest rise time but no overshoot.Overall our calculations matched our MATLAB simulations.

MATLAB Code

Problem 1: Low-pass RLC Circuit:

```
clear all; clc;

%z = [0.1 0.3 0.707 1.0 3.0 10.0];

w = 0.01:0.01:100;

z = [0.1 0.3 0.707 1.0 3.0 10.0];

w_0 = 1;

h2 = ones(6,10000);

%Magnitude vs frequency Graph

for counter = 1:1:6

h1 = 1./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);

h2 = abs (h1);

figure (1);

plot (w,h2,'LineWidth', 1.1);

hold on

end
```

```
title( "Frequency Response: Low-pass Filter")

legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 =
3.0","\zeta_6 = 10.0" )

xlabel ("frequency \omega/\omega_0")

ylabel("|H(j\omega)|")

xlim([0 20])

hold off
```

```
%Magnitude vs frequency Better Visual
for counter = 1:1:6
h1 = 1./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);
h2 = abs (h1);
figure (2);
plot (w,h2,'LineWidth', 1.5);
hold on
end
```

```
title( "Frequency Response: Low-pass Filter")
legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 =
3.0","\zeta_6 = 10.0" )
xlabel ("frequency \omega/\omega_0")
ylabel("|H(j\omega)|")
xlim([0 5])
zeta_101 = findobj(gcf, "DisplayName", "\zeta_1 = 0.1");
datatip(zeta_101,0.99,5.025);
zeta_203 = findobj(gcf, "DisplayName", "\zeta_2 = 0.3");
datatip(zeta_203,0.91,1.747);
hold off
```

```
%Bode Plot :LPF
for counter = 1:1:6
h1 = 1./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);
h2 = abs (h1);
```

```
hdb = 20.*log10(h2);

figure (3);

semilogx (w,hdb,'LineWidth', 1.5);

hold on

end

title( "Bode Plot: Low-pass Filter")

legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 =
3.0","\zeta_6 = 10.0" )

xlabel ("frequency \omega/\omega_0")

ylabel("20log|H(j\omega)| (db)")

legend("Position", [0.15612,0.14203,0.19801,0.25058])

hold off

%Phase Plot: LPF

for counter = 1:1:6

h1 = 1./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);

ph = (angle(h1)).*(180/pi);

figure (4);

semilogx(w,ph,'LineWidth', 1.5);

hold on

end

title( "Phase Plot: Low-pass Filter")

legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 =
3.0","\zeta_6 = 10.0" )

xlabel ("frequency \omega/\omega_0")

ylabel("Phase (\circ)")

hold off
```

Problem 2: High-pass and Band-pass RLC circuits:

```
z = [0.1 0.3 0.707 1.0 3.0 10.0];

w_0 = 1;

h2 = ones(6,10000);

% Bode Plot:high pass filter

for counter = 1:1:6

h1 = ((1i*w)./w_0).^2./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);

h2 = abs (h1);

hdb = 20.*log10(h2);

figure (5);

semilogx (w,hdb,'LineWidth', 1.5);

hold on

end
```

```
title( "Bode Plot: High-pass Filter")

legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 =
3.0","\zeta_6 = 10.0" )

xlabel ("frequency \omega/\omega_0")

ylabel("20log|H(j\omega)|(db)")

legend("Position", [0.68568,0.13177,0.19801,0.25058])

hold off
```

```
%Phase Plot :HPF

for counter = 1:1:6

h1 = ((1i*w)./w_0).^2./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);
```

```
ph = (angle(h1)).*(180/pi);
figure (6);
semilogx(w,ph,'LineWidth', 1.5);
hold on
end
title( "Phase Plot: High-pass Filter")
legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 = 3.0","\zeta_6 = 10.0" )
xlabel ("frequency \omega/\omega_0")
ylabel("Phase (\circ)")
hold off
```

```
%band-pass bode plot
for counter = 1:1:6
h1 = (1i.*w.*z(counter))./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);
h2 = abs (h1);
hdb = 20.*log10(h2);
figure (7);
semilogx (w,hdb,'LineWidth', 1.5);
hold on
end
```

```
title( "Bode Plot: Band-pass Filter")
legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 = 3.0","\zeta_6 = 10.0" )
xlabel ("frequency \omega/\omega_0")
```

```
ylabel("20log|H(j\omega)|(db)")

legend("Position", [0.41609,0.15486,0.19801,0.25058])

hold off
```

```
%Phase Plot :Band-Pass

for counter = 1:1:6

h1 = (1i.*w.*z(counter))./(1 + ((2.*z(counter).*w.*1i)./w_0) + (1i.*w).^2);

ph = (angle(h1)).*(180/pi);

figure (8);

semilogx(w,ph,'LineWidth', 1.5);

hold on

end

title( "Phase Plot: Band-pass Filter")

legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 =
3.0","\zeta_6 = 10.0" )

xlabel ("frequency \omega/\omega_0")

ylabel("Phase (\circ)")

hold off
```

Problem 3: Mystery Filter

```
z = [0.1 0.3 0.707 1.0 3.0 10.0];

w_0 = 1;

h2 = ones(6,10000);

for counter = 1:1:6

h1 = (1-w.^2)./(1 + (2.*z(counter).*w.*1i) + (1i.*w).^2);

h2 = abs (h1);

hdb = 20.*log10(h2);

figure (9);

semilogx (w,hdb,'LineWidth', 1.5);

hold on

end
```

```
title( "Bold Plot: Mystery Filter")

legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 = 3.0","\zeta_6 = 10.0" )

xlabel ("frequency \omega/\omega_0")

ylabel("20log|H(j\omega)|(db)")

legend("Position", [0.68568,0.13177,0.19801,0.25058])

hold off
```

```
%Phase Plot Band-Pass

for counter = 1:1:6

h1 = (1-w.^2)./(1 + (2.*z(counter).*w.*1i) + (1i.*w).^2);

ph = (angle(h1)).*(180/pi);
```

```
figure (10);

semilogx(w,ph,'LineWidth', 1.5);

hold on

end

title( "Phase Plot: Mystery Filter")

legend ("\zeta_1 = 0.1","\zeta_2 = 0.3","\zeta_3 = 0.707","\zeta_4 = 1.0","\zeta_5 =
3.0","\zeta_6 = 10.0" )

xlabel ("frequency \omega/\omega_0")

ylabel("Phase (\circ)")

hold off
```

Problem 4: Linear System Tools

a) Step Responses for Transfer Functions

```
s = tf('s');
omega_0 = 1;
z = [0.1 0.3 0.707 1.0 3.0 10.0];
%bode(h);
for counter = 1:1:6
h = 1/(1 + 2.*z(counter).*(s/omega_0) + (s/omega_0)^2);
figure (11)
step(h);
stepinfo(h,'LineWidth', 1.5);
hold on
end
title("Step Response")
legend ("\zeta = 0.1","\zeta = 0.3","\zeta = 0.707","\zeta = 1.0","\zeta = 3.0","\zeta = 10.0" )
xlabel ("Time")
ylabel("Amplitude")
hold off
```

b) Plot Overshoot

```
zeta = 0:0.01:1;
percent_overshoot = 100.*exp(1).^((-pi.*zeta)./sqrt(1-(zeta.^2)));
figure (13);
plot(zeta,percent_overshoot)
hold on
```

```
title("%Overshoot vs \zeta")

xlabel("\zeta")

ylabel("% Overshoot")

xlim([0 10])

z = [0.1 0.3 0.707 1.0 3.0 10.0];

overshoot = [72.9156 37.1410 4.3251 0 0 0];

plot (z , overshoot, 'g--o','LineWidth', 2);

legend ('Theoretical Values', 'Actual Values')

hold off
```