ECE2700J SU23 RC2

Boolean Algebra & Logic Optimization

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Boolean Algebra

Terminology

Take $F = A' \cdot B \cdot C + A \cdot (B + C')$ as an example:

- Variable: represent a value
  A, B, C

- Literal: appearance of a variable or its complement
  A', B, C, A, B, C' (Repetition of a literal is allowed)

- Product term & Sum term : a product of literals & a sum of literals
  $A \cdot B C$ $A + B$,

- Sum-of-products (SOP) form: a sum of product terms
  $F = A' \cdot B \cdot C + A \cdot B + A \cdot C'$

- Product-of-sums (POS) form: a product of sum terms
  $F = (A + B) \cdot (A + C) \cdot (B + C')$

Boolean Algebra

Theorems

(a) $x + 0 = x$;	(b) $x \cdot 0 = 0$;	(theorem 1)
(a) $x + x' = 1$;	(b) $x \cdot x' = 0$;	(theorem 2)
(a) $x + x = x$;	(b) $x \cdot x = x$;	(theorem 3)
(a) $x + 1 = 1$;	(b) $x \cdot 1 = x$;	(theorem 4)
$(x')' = x$;		(involution)

(a) $x + y = y + x$;	(b) $xy = yx$;	(commutative)
(a) $x + (y + z) = (x + y) + z$;	(b) $x(yz) = (xy)z$;	(associative)
(a) $x(y + z) = xy + xz$;	(b) $x + yz = (x+y)(x+z)$;	(distributive)
(a) $x + xy = x$;	(b) $x(x + y) = x$;	(absorption)
(a) $xy + xy' = x$;	(b) $(x + y)(x + y') = x$	(theorem 5)
(a) $x + x'y = x + y$	(b) $x(x' + y) = xy$	(theorem 6)

Boolean Algebra

Important Theorems

$(D')' = D$

- **DeMorgan's Law**

(a) $(x + y)' = x'y'$

(b) $(xy)' = x' + y'$

$(x + y)' = x' \cdot y'$

$(xyz)' = (xy)' + z'$

$(xy + z)' = x' + y' + z'$

- **Exercise**

$\boxed{SOP}$

$((AB' + C)D' + E)' = A'C'E' + BC'E' + DE'$

$= ((AB' + C) \cdot D')' \cdot E'$

$= [(AB' + C)' + D] \cdot E'$

$= [((AB')' \cdot C') + D] \cdot E'$

$= [(A' + B) \cdot C' + D] \cdot E'$

Boolean Algebra

$A \quad B \quad C$

$AB'C$

$A \cdot B' \cdot C$

Minterm and Maxterm

- Minterm is a product of n literals in which each literal appears exactly once in either true or complemented form, but not both.

– Minterm is represented by $\boldsymbol{m_i}$

- Maxterm is a sum of n literals in which each literal appears exactly once in either true or complemented form, but not both.

– Maxterm is represented by $\boldsymbol{M_i}$

Boolean Algebra

Minterm and Maxterm

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

Minterm in Truth Table

$F =$

$x'y'z$

x con1	y con2	z con3	F result	
0	0	0	0	m_0
0	0	1	1	m_1
0	1	0	0	m_2
0	1	1	1	m_3
1	0	0	1	m_4
1	0	1	1	m_5
1	1	0	0	m_6
1	1	1	0	m_7

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$$F = \boxed{x'y'z} + x'yz + xy'z' + xy'z$$
$$= m_1 + m_3 + m_4 + m_5$$
$$= \Sigma\ m(1, 3, 4, 5)$$

Boolean Algebra

Exercise

Q1, • **Find minterm logic equation from these truth table** Sum.

x	y	z	F	
0	0	0	1	m_0
0	0	1	0	m_1
0	1	0	0	m_2
0	1	1	1	m_3
1	0	0	0	m_4
1	0	1	1	m_5
1	1	0	1	m_6
1	1	1	0	m_7

$F = x'y'z' + x'yz + xy'z + xyz'$

$= m_0 + m_3 + m_5 + m_6$

$= \Sigma m(0,3,5,6)$

W	X	Y	Z	F		
0	0	0	0	1	m0	W'X'Y'Z'
0	0	0	1	0	m1	W'X'Y'Z
0	0	1	0	0	m2	W'X'YZ'
0	0	1	1	1	m3	W'X'YZ
0	1	0	0	0	m4	W'XY'Z'
0	1	0	1	0	m5	W'XY'Z
0	1	1	0	0	m6	W'XYZ'
0	1	1	1	1	m7	W'XYZ
1	0	0	0	1	m8	WX'Y'Z'
1	0	0	1	0	m9	WX'Y'Z
1	0	1	0	0	m10	WX'YZ'
1	0	1	1	0	m11	WX'YZ
1	1	0	0	0	m12	WXY'Z'
1	1	0	1	0	m13	WXY'Z
1	1	1	0	0	m14	WXYZ'
1	1	1	1	1	m15	WXYZ

$F = m_0 + m_3 + m_7 + m_8 + m_{15}$.

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$x' \cdot y \cdot z'$

Incompletely Specified Functions

- In a circuit, some input conditions may never happen, then the output is not completely specified
- The corresponding output is designated as "x", called don't care
- A don't care output could be either 0 or 1

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	X
0	1	1	1
1	0	0	1
1	0	1	X
1	1	0	0
1	1	1	0

- **$F = \Sigma m(1, 3, 4)$ with $d(2, 5)$**

Incompletely Specified Functions

Example: bell rings, F = 1.

$A'B'C' + A'B'C + A'BC' + A'BC = A'$

There is a bell with 3 switch A, B and C. Switch A is to make the bell ring, and switch B and C are for the fuses to make sure safety. We know that switch B and C **can't be off together** otherwise the system will be dangerous. If switch A is **off**, the bell F will make sound. If all 3 switches are **on**, energy will be wasted and the bell make sound to remind people not wasting so much energy. Complete the truth table and create logic equations describing the desired behavior for F.

A	B	C	F
0	0	0	X
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

$\Sigma m(1,2,3,7)+d(0,4)$, set $m_0=1$

$= \bar{A} + ABC$. $m_4=0$.