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ABSTRACT

Key words: cosmology: observations – gravitational lensing: weak

1 INTRODUCTION

2 METHOD

2.1 High order shapelets

In order to reduce the uncertainty caused by pixel noise, FPFS algorithm defines the boundary of each galaxy using a circular top-hap aperture around the centroid of the galaxy (**?**) and sets the pixels outside the boundary to zero. The ratio between the aperture radius and the galaxy's half-light radius is set to a constant and denoted as α. The half-light radius of each galaxy is calculated from the second order adaptive moment matrix measured by the re-Gaussianization algorithm (**?**) and the centroid is set to the center of the galaxy footprint. The Fourier power function of the galaxy is calculated as

$$\begin{aligned} \tilde{f}_o(\vec{k}) &= \int f_o(\vec{x}_o) e^{-i\vec{k}\cdot\vec{x}_o} d^2x_o, \\ \tilde{F}_o(\vec{k}) &= |\tilde{f}_o(\vec{k})|^2. \end{aligned} \tag{1}$$

The Fourier power function defined in equation (1) is contaminated by the Fourier power function of noise. The Fourier power function of noise depends on the correlation function of noise which is the weighted inverse Fourier transform of the noise Fourier power function (**?**). Although noise on CCD images (single exposures) does not correlate across pixels (**?**), noise on coadd exposures correlates across pixels since an ad hoc warping kernel is used to convolve CCD images before re-pixelazation on common coordinates in the coadding procedure. The noise correlations on coadd exposures are mainly determined by the shape of the warping kernels. The PSF Fourier power function, denoted as $\tilde{G}(\vec{k})$, is subsequently deconvolved from the observed galaxy's Fourier power function to remove the influence of PSF

$$\tilde{F}(\vec{k}) = \frac{\tilde{F}_r(\vec{k})}{\tilde{G}(\vec{k})}. \tag{2}$$

Next, the deconvolved galaxy Fourier power function is projected onto the polar Shapelet basis vectors (**?**) as

$$M_{nm} = \int \chi^*_{nm} \tilde{F}(\rho,\phi) \rho d\rho d\phi. \tag{3}$$

The polar Shapelet basis vectors are defined as

$$\begin{aligned} \chi_{nm}(\rho,\phi) = &\frac{(-1)^{(n-|m|)/2}}{\sigma^{|m|+1}} \left\{ \frac{[(n-|m|)/2]!}{\pi[(n+|m|)/2]!} \right\}^{\frac{1}{2}} \\ &\times \rho^{|m|} L^{|m|}_{\frac{n-|m|}{2}} \left(\frac{r^2}{\sigma^2}\right) e^{-\rho^2/2\sigma^2} e^{-im\phi}, \end{aligned}$$

where L^p_q are the Laguerre Polynomials, n is the radial number and m is the spin number, and σ determines the scale of Shapelet functions. We denote the ratio between σ and the scale radius of PSF Fourier power function ($r_{\rm pp}$) as (**?**)

$$\beta = \frac{\sigma}{r_{\rm pp}}. \tag{4}$$

Note that α determines the effective scale in configuration space whereas β determines the effective scale in Fourier space. **?** proposed to set $\alpha = 4, \beta = 0.85$ and showed that the systematic bias for such setup is below one percent of the shear signal.

The transformation formulas of the shapelet modes under the influence of shear have been given by **?**, which are

$$\begin{aligned} M_{22c} &= \bar{M}_{22c} - \frac{\sqrt{2}}{2} g_1 (\bar{M}_{00} - \bar{M}_{40}) \\ &\quad + \sqrt{3} g_1 \bar{M}_{44c} + \sqrt{3} g_2 \bar{M}_{44s}, \\ M_{22s} &= \bar{M}_{22s} - \frac{\sqrt{2}}{2} g_2 (\bar{M}_{00} - \bar{M}_{40}) \\ &\quad - \sqrt{3} g_2 \bar{M}_{44c} + \sqrt{3} g_1 \bar{M}_{44s}, \\ M_{00} &= \bar{M}_{00} + \sqrt{2}(g_1 \bar{M}_{22c} + g_2 \bar{M}_{22s}), \\ M_{40} &= \bar{M}_{40} - \sqrt{2}(g_1 \bar{M}_{22c} + g_2 \bar{M}_{22s}) \\ &\quad + 2\sqrt{3}(g_1 \bar{M}_{62c} + g_2 \bar{M}_{62s}), \end{aligned} \tag{5}$$

where $\bar{M}_{nm}$ represent the intrinsic shapelet modes and M_{nm} represent the sheared shapelet modes.

Finally, using these shapelet modes, we define the dimensionless FPFS ellipticity and the corresponding shear response as

$$e_1 = \frac{M_{22c}}{M_{00}+C}, \qquad e_2 = \frac{M_{22s}}{M_{00}+C}, \tag{6}$$

$$R_{1,2} = \frac{\sqrt{2}}{2} \frac{M_{00}-M_{40}}{M_{00}+C} + \sqrt{2} e^2_{1,2}, \tag{7}$$

M_{nmc} and M_{nms} are used to denote the real and imaginary part of M_{nm} when $m > 0$. The constant C the weighting parameter which adjusts weight between galaxies with different luminosity and reduces noise bias (**?**). According to **?**, we quantify the spread of M_{00} with Δ which is the value of M_{00} at which its histogram drops below $1/8$ of its maximum on the side of higher M_{00} and normalize C with Δ as $\nu = C/\Delta$. We conservatively set $\nu = 4$ and the weight for galaxies with different S/N can be found from the left panel of Figure 7 in **?**.

With the definition of average response $R = (R_1 + R_2)/2$, the final shear estimator is

$$\gamma_{1,2} = -\frac{\langle e_{1,2} \rangle}{\langle R \rangle}. \tag{8}$$

There exists a minus sign in the final shear estimator since the ellipticity are defined in Fourier space.

2.2 Image boundary

2.2.1 periodic boundary

2.2.2 top-hat

2.2.3 Fourier Lanczos

2.3 Selection Bias

3 TEST

3.1 Setups

The HSC-like Bulge+Disk+Knot (BDK) simulation is an HSC version of the BDK simulation (**?**). The simulation is generated by Galsim which is an open-source image simulation package (**?**). We use Sersic models (**?**) which are fitted to the 25.2 magnitude limited galaxy sample from the COSMOS data [1] to simulate the bulge and disk of galaxies. The fluxes of these galaxies are scaled by a factor of 2.587 to match the fluxes in HSC observation. In order to avoid repeating the exact parameters, we interpolate the joint radius-flux distribution by randomly rescaling the radius and flux of the original Sersic model. To simulate the knots of star formation, we distribute N random points which statistically obey the Gaussian distribution around the center of the galaxies, where N is a random number evenly distributed between 50 and 100. The ellipticity of the Gaussian distribution follows the ellipticity of Sersic model and the half light radius of the Gaussian distribution is fixed to 2.4 pixels. The pixel scale of the simulation is set to the HSC pixel scale, namely $0.168''$. The fraction of the flux of the knots is a random number evenly distributed between 0% and 10%. The galaxies are rotated to random directions and subsequently sheared by the same shear signal ($g_1 = 0.02, g2 = 0.00$). For the HSC-like BDK simulation, we use g_1 to determine the multiplicative bias and use g_2 to determine the additive bias. The galaxy images are convolved with a Moffat PSF (**?**)

$$g_m(\vec{x}) = [1 + c(|\vec{x}|/r_p)^2]^{-\beta_m}, \tag{9}$$

where $c = 2^{\frac{1}{\beta_m - 1}} - 1$ is a constant parameter. The profile of the Moffat PSF is determined by β_m, where $\beta_m = 3.5$. The scale of the Moffat PSF is determined by its Full Width Half Maximum (FWHM), where FWHM = 0.6. The ellipticity of the Moffat PSF is set to ($e_1 = 0, e_2 = 0.025$). Each convolved galaxy is placed around the center of a 64×64 stamp. The HSC-like BDK simulation generate 4×10^8 galaxies.

[1] great3.jb.man.ac.uk/leaderboard/data/public/COSMOS_25.2_training_sample.tar.gz

3.2 Results

4 SUMMARY AND OUTLOOK

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