FPFS: Revision for noise bias and selection bias

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ABSTRACT

Key words: cosmology: observations – gravitational lensing: weak

1 INTRODUCTION

2 METHOD

We first introduce the second generation of FPFS shear estimator in Section 2.1 and the revision of selection bias in Section 2.3. Then, we summarize the difference between the updated FPFS algorithm with its first generation.

2.1 FPFS2 Shear estimator

Zhang (2008) first proposed to measure shear from the Fourier power function of galaxy images. The Fourier power function of a galaxy is defined as

$$\begin{aligned}\tilde{f}_o(\vec{k}) &= \int f_o(\vec{x}_o) e^{-i\vec{k}\cdot\vec{x}_o} d^2x_o,\\ \tilde{F}_o(\vec{k}) &= |\tilde{f}_o(\vec{k})|^2.\end{aligned} \tag{1}$$

The Fourier power function defined in equation (1) is contaminated by the Fourier power function of noise. Although noise on CCD images (single exposures) does not correlate across pixels (Zhang et al. 2015), noise on coadd exposures correlates across pixels as an ad hoc warping kernel is used to convolve CCD images before re-pixelazing onto common coordinates in the coadding procedure. In this paper, we assume the noise's Fourier power function is fully known, which is denoted as $\tilde{F}_n$. The noise's Fourier power function is subtracted from the galaxy's Fourier power function

$$\tilde{F}_r(\vec{k}) = \tilde{F}_o(\vec{k}) - \mathcal{P}. \tag{2}$$

Subsequently, the PSF's Fourier power function, which is denoted as $\tilde{G}(\vec{k})$, is deconvolved to revise for the smearing from PSF, and the deconvolved galaxy's Fourier power function is projected onto the polar Shapelet basis vectors (Massey & Refregier 2005) as

$$M_{nm} = \int \chi^{\dagger}_{nm} \frac{\tilde{F}_r}{\tilde{G}} \rho d\rho d\phi, \tag{3}$$

where ρ and ϕ are the radius and position angle in the polar coordinates. The polar Shapelet basis vectors are defined as

$$\begin{aligned}\chi_{nm}(\rho,\phi) = &\frac{(-1)^{(n-|m|)/2}}{\sigma^{|m|+1}} \left\{\frac{[(n-|m|)/2]!}{\pi[(n+|m|)/2]!}\right\}^{\frac{1}{2}}\\ &\times \rho^{|m|} L^{|m|}_{\frac{n-|m|}{2}} \left(\frac{r^2}{\sigma^2}\right) e^{-\rho^2/2\sigma^2} e^{-im\phi},\end{aligned}$$

where L^p_q are the Laguerre Polynomials, n is the radial number and m is the spin number, and σ determines the scale of Shapelet functions. We denote the ratio between σ and the scale radius of PSF Fourier power function ($r_{\rm pp}$) as (Li et al. 2018)

$$\beta = \frac{\sigma}{r_{\rm pp}}. \tag{4}$$

Note β determines the effective scale in Fourier space. Li et al. (2018) proposed to set $\beta = 0.85$.

The transformations ((Massey & Refregier 2005)) of many useful shapelet modes under the influence of shear are laid out as follows

$$\begin{aligned}M_{22c} &= \bar{M}_{22c} - \frac{\sqrt{2}}{2} g_1(\bar{M}_{00} - \bar{M}_{40})\\ &\quad + \sqrt{3} g_1 \bar{M}_{44c} + \sqrt{3} g_2 \bar{M}_{44s},\\ M_{22s} &= \bar{M}_{22s} - \frac{\sqrt{2}}{2} g_2(\bar{M}_{00} - \bar{M}_{40})\\ &\quad - \sqrt{3} g_2 \bar{M}_{44c} + \sqrt{3} g_1 \bar{M}_{44s},\\ M_{00} &= \bar{M}_{00} + \sqrt{2}(g_1 \bar{M}_{22c} + g_2 \bar{M}_{22s}),\\ M_{20} &= \bar{M}_{20} + \sqrt{6}(g_1 \bar{M}_{42c} + g_2 \bar{M}_{42s}),\\ M_{40} &= \bar{M}_{40} - \sqrt{2}(g_1 \bar{M}_{22c} + g_2 \bar{M}_{22s})\\ &\quad + 2\sqrt{3}(g_1 \bar{M}_{62c} + g_2 \bar{M}_{62s}),\end{aligned} \tag{5}$$

where $\bar{M}_{nm}$ represent the intrinsic shapelet modes and M_{nm} represent the sheared shapelet modes.

We define the dimensionless FPFS ellipticity and the corresponding shear response with these shapelet modes as

$$e_1 = \frac{M_{22c}}{M_{20}+C}, \qquad e_2 = \frac{M_{22s}}{M_{20}+C}, \tag{6}$$

$$R_1 = \frac{\sqrt{2}}{2}\frac{M_{00}-M_{40}}{M_{20}+C} + \sqrt{6}\frac{M_{22c}}{M_{20}+C}\frac{M_{42c}}{M_{20}+C}, \tag{7}$$

$$R_2 = \frac{\sqrt{2}}{2}\frac{M_{00}-M_{40}}{M_{20}+C} + \sqrt{6}\frac{M_{22s}}{M_{20}+C}\frac{M_{42s}}{M_{20}+C}, \tag{8}$$

M_{nmc} and M_{nms} are used to denote the real and imaginary part of M_{nm} when $m > 0$. The constant $C = \nu\sigma(M_{20})$ the weighting parameter which adjusts weight between galaxies with different luminosity and reduces noise bias, and ν is termed weighting ratio (Li et al. 2018).

With the definition of average response $R = (R_1 + R_2)/2$, the final shear estimator is

$$\gamma_{1,2} = -\frac{\langle e_{1,2}\rangle}{\langle R\rangle}. \tag{9}$$

2.2 Noise bias

2.2.1 pure noise

The Gaussian noise field is denoted as $n(\vec{x})$ and its Fourier transform is denoted as $\tilde{n}(\vec{k})$. According to the Isserlis' theorem, we have

$$\begin{aligned}\tilde{n}_2(\vec{k}) &= \left\langle \tilde{n}\tilde{n}^\dagger \right\rangle = \mathcal{P}(\vec{k}),\\ \tilde{n}_4(\vec{k}) &= \left\langle \tilde{n}\tilde{n}\tilde{n}^\dagger\tilde{n}^\dagger \right\rangle = \begin{cases} 3\mathcal{P}^2(\vec{k}) & \text{if } \vec{k}=0,\\ 2\mathcal{P}^2(\vec{k}) & \text{else.}\end{cases}\end{aligned} \tag{10}$$

The residual of noise power function is defined as

$$\tilde{\epsilon}(\vec{k}) = \tilde{n}\tilde{n}^\dagger - \mathcal{P}(\vec{k}), \tag{11}$$

and the standard deviation of the residual can be calculated by combining eq.(11) with eq. (10):

$$\sigma^2_{\tilde{\epsilon}}(\vec{k}) = \tilde{n}_4(\vec{k}) - \mathcal{P}^2(\vec{k}) \tag{12}$$

2.2.2 noisy galaxy

However, when we have a galaxy in the noise field, the combined field is

$$\tilde{f}_o(\vec{k}) = \tilde{f}(\vec{k}) + \tilde{n}(\vec{k}), \tag{13}$$

and the total power function is

$$\tilde{F}_o(\vec{k}) = \tilde{F} + \tilde{n}\tilde{n}^\dagger + \tilde{f}\tilde{n}^\dagger + \tilde{f}^\dagger\tilde{n}, \tag{14}$$

with the definition of residuals on the power function:

$$\tilde{\epsilon}(\vec{k}) = \tilde{n}\tilde{n}^\dagger + \tilde{f}\tilde{n}^\dagger + \tilde{f}^\dagger\tilde{n} - \mathcal{P}(\vec{k}), \tag{15}$$

the power function of galaxy after removing the noise power function is

$$\tilde{F}_r(\vec{k}) = \tilde{F}(\vec{k}) + \tilde{\epsilon}(\vec{k}). \tag{16}$$

With eq. (10), the variance of the residual is calculated as

$$\sigma^2_{\tilde{\epsilon}}(\vec{k}) = \begin{cases} 2\mathcal{P}^2 + 4\tilde{F}\mathcal{P} & \text{if } \vec{k}=0,\\ \mathcal{P}^2 + 2\tilde{F}\mathcal{P} & \text{else.}\end{cases} \tag{17}$$

2.2.3 Shapelet modes revision

We define the shapelet modes of residuals as $\mathcal{E}_{nm}$, and the expectation of these residual shapelet modes equal zero. The second-order statistical properties of these modes are

$$\begin{aligned}\left\langle \mathcal{E}_{nm}\mathcal{E}^\dagger_{n'm'}\right\rangle &= \int \frac{\chi^\dagger_{nm}\chi_{n'm'}}{\tilde{G}^2(\vec{k})}\sigma^2_{\tilde{\epsilon}}(\vec{k})d^2k\\ &= \int \frac{\chi^\dagger_{nm}\chi_{n'm'}}{\tilde{G}^2(\vec{k})}\left(\mathcal{P}^2 + 2\tilde{F}\mathcal{P}\right)\end{aligned} \tag{18}$$

The observed FPFS ellipticity is revise as

$$\begin{aligned}\langle e^o_1\rangle &= \left\langle \frac{M_{22c}+\mathcal{E}_{22c}}{M_{20}+C+\mathcal{E}_{20}}\right\rangle\\ &= \langle e_1\rangle + \left\langle \frac{M_{22c}\left\langle \mathcal{E}_{20}\mathcal{E}_{20}\right\rangle}{(M_{20}+C)^3}\right\rangle - \left\langle \frac{\left\langle \mathcal{E}_{20}\mathcal{E}_{22c}\right\rangle}{(M_{20}+C)^2}\right\rangle\end{aligned} \tag{19}$$

$$\begin{aligned}\left\langle \frac{M^o_{00}-M^o_{40}}{M^o_{20}+C}\right\rangle = \left\langle \frac{M_{00}-M_{40}}{M_{20}+C}\right\rangle + \left\langle \frac{(M_{00}-M_{40})\left\langle \mathcal{E}_{20}\mathcal{E}_{20}\right\rangle}{(M_{20}+C)^3}\right\rangle\\ - \left\langle \frac{\langle \mathcal{E}_{20}(\mathcal{E}_{00}-\mathcal{E}_{40})\rangle}{(M_{20}+C)^2}\right\rangle\end{aligned} \tag{20}$$

$$\begin{aligned}\left\langle \frac{M^o_{22c}M^o_{42c}}{(M^o_{20}+C)^2}\right\rangle &= \left\langle \frac{M_{22c}M_{42c}}{(M_{20}+C)^2}\right\rangle\\ &+ 3\left\langle \frac{M_{22c}M_{42c}\left\langle \mathcal{E}_{20}\mathcal{E}_{20}\right\rangle}{(M_{20}+C)^4}\right\rangle + \left\langle \frac{\left\langle \mathcal{E}_{22c}\mathcal{E}_{42c}\right\rangle}{(M_{20}+C)^2}\right\rangle\\ &- 2\left\langle \frac{M_{42c}\langle \mathcal{E}_{22c}\mathcal{E}_{20}\rangle}{(M_{20}+C)^3} + \frac{M_{22c}\langle \mathcal{E}_{42c}\mathcal{E}_{20}\rangle}{(M_{20}+C)^3}\right\rangle\end{aligned} \tag{21}$$

2.3 Selection Bias

Selection bias refers to the multiplicative bias or additive bias caused by selection. Such bias emerges if the observable used for selection correlates with the shear signal (multiplicative bias) or the ellipticities of PSF (additive bias). We denote the observable used for selection as X and the correlation with the shear or PSF ellipticities has the form

$$X = \bar{X} + g_1Y_1 + e_{\mathrm{PSF},1}Z_1 + ... \tag{22}$$

Here $\bar{X}$ refers to the intrinsic property before the shear distortion and PSF smearing, Y refers to the response of the property $\bar{X}$ to the shear which is a spin-2 property, and Z quantifies the correlation between the observed property X and the shape of PSF which is caused by the imperfect PSF revision in the observation of X. We follow **?** and give out the changes in the first component of ellipticities due to the selection

$$\begin{aligned}\Delta\langle e_1\rangle = g_1 &\int \frac{dn}{Nd\Omega}|_{X=X_{\mathrm{edge}}}Y_1e_1d\Omega\\ &+ e_{\mathrm{PSF},1}\int \frac{dn}{Nd\Omega}|_{X=X_{\mathrm{edge}}}Z_1e_1d\Omega.\end{aligned} \tag{23}$$

Then with the assumption that

$$\frac{dn}{Nd\Omega} = p_a(X)p_b(\Omega') \tag{24}$$

we have

$$\begin{aligned}\Delta\langle e_1\rangle = g_1p_a(X_{\mathrm{edge}}) &\int p_bY_1e_1d\Omega'|_{X=X_{\mathrm{edge}}}\\ + e_{\mathrm{PSF},1}p_a(X_{\mathrm{edge}}) &\int p_bZ_1e_1d\Omega'|_{X=X_{\mathrm{edge}}}.\end{aligned} \tag{25}$$

The multiplicative selection bias and additive selection bias is defined as

$$\begin{aligned}m_{\mathrm{sel}} &= p_a(X_{\mathrm{edge}})\int p_bY_1e_1d\Omega'|_{X=X_{\mathrm{edge}}}\\ &= p_a(X_{\mathrm{edge}})A_m,\end{aligned} \tag{26}$$

$$\begin{aligned}a_{\mathrm{sel}} &= p_a(X_{\mathrm{edge}})\int p_bZ_1e_1d\Omega'|_{X=X_{\mathrm{edge}}}\\ &= p_a(X_{\mathrm{edge}})A_a,\end{aligned} \tag{27}$$

respectively. We denote the ratio between the multiplicative (additive) bias and partial density function at X_{edge} as A_m (A_a).

We define the FPFS flux ratio as

$$s = \frac{M_{00}}{M_{20} + C}, \tag{28}$$

and use the FPFS flux ratio as the selection function.

The FPFS flux ratio is also influenced by the shear and the relationship between the sheared FPFS flux ratio (s) and the intrinsic FPFS flux ratio ($\bar{s}$) is

$$s - \bar{s} = g_1(\sqrt{2}\frac{M_{22c}}{M_{20} + C} - \sqrt{6}s\frac{M_{42c}}{M_{20} + C}) \tag{29}$$

$$+ g_2(\sqrt{2}\frac{M_{22s}}{M_{20} + C} - \sqrt{6}s\frac{M_{42s}}{M_{20} + C}). \tag{30}$$

$\bar{s}$ is isotropic (spin-0) on the intrinsic plane but s is not. Therefore, the selection using s as the selection function is not an isotropic selection on the intrinsic plane. Such selection does not align with the premise that the intrinsic galaxies have isotropic orientations statistically and causes selection bias.

3 IMAGE SIMULATION

The HSC-like Bulge+Disk+Knot (BDK) simulation is an HSC version of the BDK simulation (?). The simulation is generated by Galsim which is an open-source image simulation package (?). We use Sersic models (?) which are fitted to the 25.2 magnitude limited galaxy sample from the COSMOS data [1] to simulate the bulge and disk of galaxies. The fluxes of these galaxies are scaled by a factor of 2.587 to match the fluxes in HSC observation. In order to avoid repeating the exact parameters, we interpolate the joint radius-flux distribution by randomly rescaling the radius and flux of the original Sersic model. To simulate the knots of star formation, we distribute N random points which statistically obey the Gaussian distribution around the center of the galaxies, where N is a random number evenly distributed between 50 and 100. The ellipticity of the Gaussian distribution follows the ellipticity of Sersic model and the half light radius of the Gaussian distribution is fixed to 2.4 pixels. The pixel scale of the simulation is set to the HSC pixel scale, namely $0.168''$. The fraction of the flux of the knots is a random number evenly distributed between 0% and 10%. The galaxies are rotated to random directions and subsequently sheared by the same shear signal ($g_1 = 0.02, g2 = 0.00$). For the HSC-like BDK simulation, we use g_1 to determine the multiplicative bias and use g_2 to determine the additive bias. The galaxy images are convolved with a galaxy model of the first-year HSC data release.

Each convolved galaxy is placed around the center of a 64×64 stamp. The HSC-like BDK simulation generate 4×10^8 galaxies.

4 SUMMARY AND OUTLOOK

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REFERENCES

Li X., Katayama N., Oguri M., More S., 2018, MNRAS, 481, 4445
Massey R., Refregier A., 2005, MNRAS, 363, 197
Zhang J., 2008, MNRAS, 383, 113
Zhang J., Luo W., Foucaud S., 2015, J. Cosmology Astropart. Phys., 1, 24

[1] great3.jb.man.ac.uk/leaderboard/data/public/COSMOS_25.2_training_sample.tar.gz

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