CS 131 - Spring 2020, Assignment 1 Answers

Andy Vo

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Problem 1

a) $(a_3.\overline{b_3}) + (a_3.b_3.a_2.\overline{b_2}) + (a_3.b_3.a_2.b_2.a_1.\overline{b_1}) + (a_3.b_3.a_2.b_2.a_1.b_1.a_0.\overline{b_0})$

b) $c_0 = a_0 \oplus b_0$
$c_1 = (a_0.b_0) + ((a_1 \oplus b_1).(\overline{a_0.b_0})) + (a_0.b_0.a_1.b_1)$
$c_2 = (a_1.b_1)+((a_2 \oplus b_2).(\overline{a_1.b_1}))+((a_2 \oplus b_2).(\overline{a_1.(a_0.b_0)}))+((a_2 \oplus b_2).(\overline{b_1.(a_0.b_0)}))+(a_1.b_1.a_2.b_2)+(a_0.b_0.a_1.a_2.b_2)+$ $(a_0.b_0.b_1.a_2.b_2)$

c) $\neg(((((((((a_9 \oplus a_8) \oplus a_7) \oplus a_6) \oplus a_5) \oplus a_4) \oplus a_3) \oplus a_2) \oplus a_1) \oplus a_0)$

Problem 2

x, y, z, v

a) True,False,True,True

b) True,False,True,True

c) Function is not satisfiable.

$$\underline{x\overline{x} + y\overline{y} + z\overline{z} + v\overline{v}}$$

Complement Law

$$0 + 0 + 0 + 0$$

Idempotent Law

$$0$$

d) Function is not satisfiable.

$$\underline{(x + y)\overline{(x + y + z)} + (x + y + z)\overline{(x + y + z + v)}}$$

De Morgan's Law

$$(x + y)(\overline{x}.\overline{y}.\overline{z}) + (x + y + z)(\overline{x}.\overline{y}.\overline{z}.\overline{v})$$

Distributive Law

$$(x\overline{x} + y\overline{x})(x\overline{y} + y\overline{y})(x\overline{z} + y\overline{z}) + (x\overline{x} + y\overline{x} + z\overline{x}) + (x\overline{y} + y\overline{y} + z\overline{y}) + (x\overline{z} + y\overline{z} + z\overline{z}) + (x\overline{v} + y\overline{v} + z\overline{v})$$

Complement Law

$$(0 + y\overline{x})(x\overline{y} + 0)(x\overline{z} + y\overline{z}) + (0 + y\overline{x} + z\overline{x}) + (x\overline{y} + 0 + z\overline{y}) + (x\overline{z} + y\overline{z} + 0) + (x\overline{v} + y\overline{v} + z\overline{v})$$

Identity Law

$$(y\overline{x})(x\overline{y})(x\overline{z} + y\overline{z}) + (y\overline{x} + z\overline{x}) + (x\overline{y} + z\overline{y}) + (x\overline{z} + y\overline{z}) + (x\overline{v} + y\overline{v} + z\overline{v})$$

Associative Law

$$(x\overline{x})(y\overline{y})(x\overline{z}+y\overline{z})+(y\overline{x}+z\overline{x})+(x\overline{y}+z\overline{y})+(x\overline{z}+y\overline{z})+(x\overline{v}+y\overline{v}+z\overline{v})$$

Complement Law

$$(0)(0)(x\overline{z}+y\overline{z})+(y\overline{x}+z\overline{x})+(x\overline{y}+z\overline{y})+(x\overline{z}+y\overline{z})+(x\overline{v}+y\overline{v}+z\overline{v})$$

Domination Law

$$0+(y\overline{x}+z\overline{x})+(x\overline{y}+z\overline{y})+(x\overline{z}+y\overline{z})+(x\overline{v}+y\overline{v}+z\overline{v})$$

Identity Law

$$(y\overline{x}+z\overline{x})+(x\overline{y}+z\overline{y})+(x\overline{z}+y\overline{z})+(x\overline{v}+y\overline{v}+z\overline{v})$$

Distributive Law

$$(y\overline{x}x\overline{y}+y\overline{x}z\overline{y}+z\overline{x}x\overline{y}+z\overline{x}z\overline{y})(x\overline{z}+y\overline{z}(x\overline{v}+y\overline{v}+z\overline{v})$$

Complement Law

$$(0+0+0+z\overline{x}z\overline{y})(x\overline{z}+y\overline{z}(x\overline{v}+y\overline{v}+z\overline{v})$$

Identity Law

$$(z\overline{x}z\overline{y})(x\overline{z}+y\overline{z}(x\overline{v}+y\overline{v}+z\overline{v})$$

Distributive Law

$$(z\overline{x}z\overline{y}x\overline{z}+z\overline{x}z\overline{y}y\overline{z})(x\overline{v}+y\overline{v}+z\overline{v})$$

Complement Law

$$(0+0)(x\overline{v}+y\overline{v}+z\overline{v})$$

Identity Law

$$0(x\overline{v}+y\overline{v}+z\overline{v})$$

Domination Law

$$0$$

Problem 3

a) Simplify $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ to five operators.

$$\underline{((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))}$$

Distributive Law

$$(\neg q \wedge \neg p) \vee (\neg q \wedge \neg r) \vee (\neg p \wedge q) \vee (\neg p \wedge r)$$

Commutative Law

$$(\neg p \wedge \neg q) \vee (\neg q \wedge \neg r) \vee (\neg p \wedge q) \vee (\neg p \wedge r)$$

Associative Law

$$(\neg p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge r) \vee (\neg q \wedge \neg r)$$

Distributive Law

$$(\neg p \wedge (\neg q \vee q \vee r)) \vee (\neg q \wedge \neg r)$$

Complement Law

$$(\neg p \wedge (T \vee r)) \vee (\neg q \wedge \neg r)$$

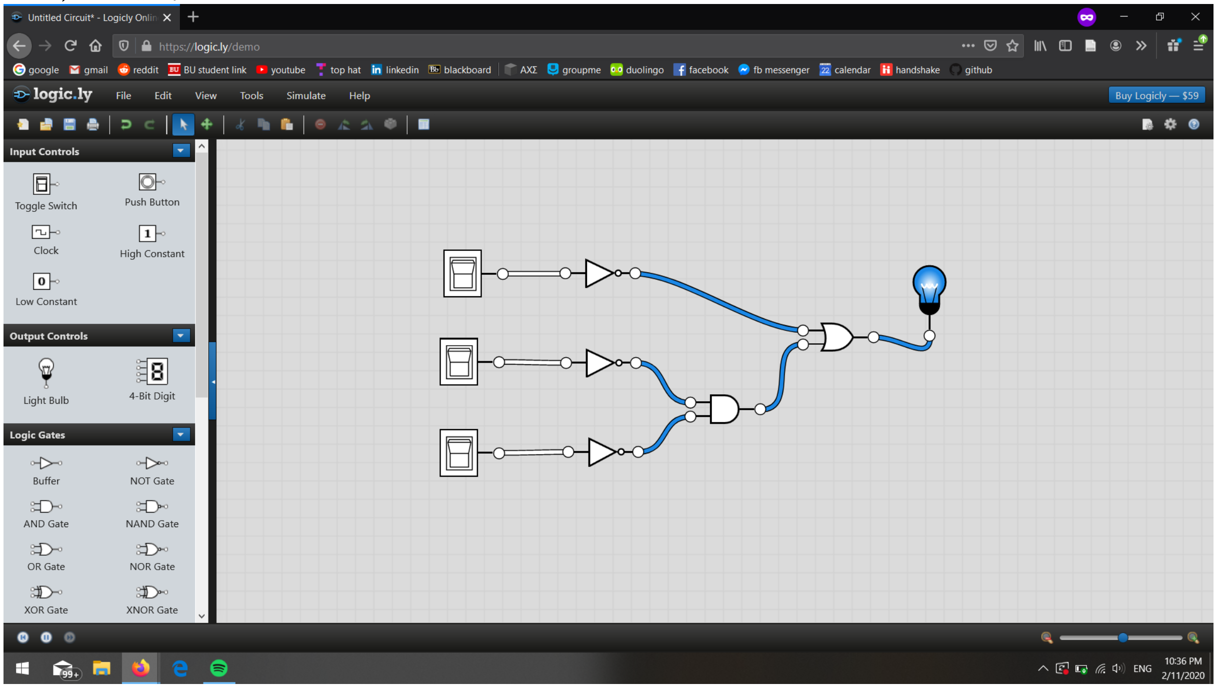
Domination Law

$$(\neg p \wedge T) \vee (\neg q \wedge \neg r)$$

Identity Law

$$\neg p \vee (\neg q \wedge \neg r)$$

b) Display it as a circuit.



Problem 4

a) This is a valid argument as this is using the rule of simplification. Using the Commutative Law, the order of the propositions do not matter. Thus, both "Kangaroos live in Australia" and "Kangaroos are marsupials" can both be p. By simplification, $(p \wedge q) \rightarrow p$, and in this case, the claim can be arranged to kangaroos are marsupials and live in Australia. Therefore, kangaroos are marsupials.

b) This is a valid argument. It displays disjunctive syllogism. One of the claims in the first propositions must be true given that it is an "or" statement. If the first isn't true, then the latter must be true.

c) This is a valid argument, displaying the rule of modus ponens quite straightforwardly.

d) This is a valid argument. It showcases the rule of inference of addition. We know p is true. Therefore, if p is true, then p or q will always be true because p is true.

e) This argument is not valid. It follows the format of the following: $((p \rightarrow q) \wedge q) \rightarrow p$. This is a logical fallacy and not a tautology. The logical problem with this argument is that there is no conditional proposition that would have led to the conclusion of exercising every day. The conclusion of becoming an athlete can be reached. Furthermore, if we try to simplify the statement by using the Conditional Identity Law, we do not reach a tautology.

f) This argument is demonstrating hypothetical syllogism, following the $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ format. Thus, this argument is valid.

Problem 5

a)

1. $p \rightarrow u$ Hypothesis
2. $u \rightarrow s$ Hypothesis

3. $p \to s$ Hypothetical Syllogism 1,2
4. $\neg r \to \neg s$ Hypothesis
5. $s \to r$ Contrapositive Rule 4
6. $p \to r$ Hypothetical Syllogism 3,5
7. $\neg t \to \neg r$ Hypothesis
8. $r \to t$ Contrapositive 7
9. $p \to t$ Hypothetical Syllogism 6,8
10. $t \to q$ Hypothesis
11. $p \to q$ Hypothetical Syllogism 9,10

b)

1. $s \to r$ Hypothesis
2. $r \to p$ Hypothesis
3. $s \to p$ Hypothetical Syllogism 1,2
4. $p \to (q \wedge r)$ Hypothesis
5. $p \to q$ Conditional Simplification
6. $s \to q$ Hypothetical Syllogism 3,5