Andy Vo

6) Proof:

Suppose n_1 and n_2 are positive integers, and there exists integer values a, a_1, a_2 such that n_1 divides $(a-a_1)$ and n_2 divides $(a-a_2)$. Let x be $\gcd(n_1, n_2)$ where $x \in \mathbb{Z}$. Then there exists $u, v \in \mathbb{Z}$ such that $n_1 = xu$ and $n_2 = xv$ and $\gcd(u,v) = 1$.

If n_1 divides $(a-a_1)$, then there exists c, such that $c \in \mathbb{Z}$ and $a - a_1 = n_1 c$.

If n_2 divides $(a-a_2)$, then there exists d, such that $d \in \mathbb{Z}$ and $a - a_2 = n_2 d$.

Thus, $(a-a_2)-(a-a_1) = n_2 d - n_1 c$. So, $a_1 - a_2 = n_2 d - n_1 c = xvd - xuc$.

Simplified, $a_1 - a_2 = x(vd - uc)$. Therefore, since x is $\gcd(n_1, n_2)$ and a_1 and a_2 are integers so that $a_1 - a_2$ is an integer, x divides $a_1 - a_2$, or rather, $\gcd(n_1, n_2)$ divides $(a_1 - a_2)$. ∎