CS 131 - Spring 2020, Assignment 1 Answers

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Problem 1.

Let's say that **p** = "Mary is in CS131" and **q** = "Kevin is in CS131."

a) $\neg(p \wedge q)$

b) $\neg p \wedge \neg q$

c) $\neg(p \vee q)$

d) $\neg p \vee \neg q$

Problem 2.

Let's say that **A** = "Anna's face is muddy", **B** = "Björn's face is muddy", and **C** = "Cathy's face is muddy."

a) $A \vee B \vee C$. This is the simplest explanation given that a muddy face could be on A or B or C.

b) We know that all three children can see each other's faces but not their own. We also know that *at least* one child has a muddy face according to an adult named David. When we look in the perspective of A, at least B or C has to have a muddy face. Otherwise, A would know A has the muddy face and go home. But A doesn't go home. In fact, no children go home after the light turns white. Therefore, we know that B or C, written as $B \vee C$ must be True. Essentially, in the perspective of each child, one of the two other children has a muddy face. For A, that is B or C. Going through the other children, the same can deduced from their respective perspective, which can be written in Boolean algebra as $(B \vee C) \wedge (A \vee C) \wedge (A \vee B)$.

c) The light turns white a second time and no children go home. At this point, each child knows that there is at least two muddy faces. Why? Because if it was just one, then the one muddy face child would see no muddy faces and as a result, would go home. Now that A knows there has to be two muddy faces, A still doesn't have to go home because A sees that both B and C have muddy faces. But the same can be said respective to B and to C. If I am A, I know that at least B or C has a muddy face, so one of them should have gone home. But in this case, B does not go home. That means that A knows that B sees a muddy face on A or on C. If only C had the muddy face, C would have known from the start and went home. However, because C didn't go home either, A knows C saw that A and/or B has the muddy face. Essentially, in A's perspective, A does not know she herself has a muddy face for sure because at this point, she knows that it has to be at least two people, and it doesn't have to be her. That is why she (A) doesn't go home. The same applies with B and C. That can be written as $(B \wedge C) \wedge (A \wedge C) \wedge (B \wedge C)$ which can be simplified as $A \wedge B \wedge C$.

d) All the children walk when the walk signal turns white the third time because of the common knowledge they share. At this point, each of the children can 100% know that respectively they are the one with the muddy face. From the deductions, it was at least one, then at least two, then all three. To better explain, they realized by the first light that it had to be at least one by David's information. By the second light, they realized it had to be two. However, after the second light, nobody moved. From that (in A's perspective), A knows that B sees two muddy faces and also knows that C sees two muddy faces. A overlaps here and as a result, A knows A has a muddy face. This logic can be applied with the other two children as well.

e) The children would not have walked home if they were not told the initial statement by David because they would have lacked the critical information needed to make the logical conclusions. The children would never know they themselves had the muddy face.

Problem 3.

Given: **p** = "you study computer science"; **q**= "you'll be smart"; **r** = "you'll be happy"

a) In Boolean logic, "if you study computer science, you'll be smart and if you study computer science, you'll be happy" is equivalent to $(p \to q) \wedge (p \to r)$. "If you study computer science, you'll be smart and happy" in Boolean logic is $p \to (q \wedge r)$.

$$\underline{(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)}$$

$$(p \to q) \wedge (p \to r) \tag{1}$$

Distributive Law

$$p \to (q \wedge r) \tag{2}$$

b) Below is the truth table proving tautology. Tautology is proven when the proposition is always true, regardless of the truth values of the individual propositions that compose it.

p	q	r	$(p \to q)$	$(q \to r)$	$(p \to q) \wedge (q \to r)$	$(p \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Table 1: Table 1: Truth table to prove $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.