CS 131 - Spring 2020, Assignment 1 Answers

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Problem 1.

a) Given: If she finished her homework, then she went to the party.
Inverse: Is she didn't finish her homework, then she didn't go to the party.
Converse: If she went to the party, then she finished her homework.
Contrapositive: If she didn't go to the party, then she didn't finish her homework.

b) Given: If he trained for the race, then he finished the race.
Inverse: If he didn't train for the race, then he didn't finish the race.
Converse: If he finished the race, then he trained for the race.
Contrapositive: If he didn't finish the race, then he didn't train for the race.

Problem 2.

a) We wish to show that $(p \to q) \wedge (r \to q)$ and $(p \wedge r) \to q$ are not logically equivalent.

p	q	r	$(p \to q)$	$(r \to q)$	$(p \wedge r)$	$(p \to q) \wedge (r \to q)$	$(p \wedge r) \to q$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	F	F	T	F	F
T	F	F	F	T	F	F	T
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	F	T	T

Table 1: Truth table to prove $(p \to q) \wedge (r \to q)$ and $(p \wedge r) \to q$ are logically not equivalent.

b) As seen in the truth table above in **Table 1**, $(p \to q) \wedge (r \to q)$ and $(p \wedge r) \to q$ are not logically equivalent. In the last two columns, there are two instances in which the two propositions are not logically equivalent one another: when "p is true, q is false, and r is false" and when "p is false, q is false, and r is true."

Problem 3.

Given that "L_1: First Path Leads to Being Lost Forever", "L_2: Seconds Path leads to Being Lost Forever", "T_1: First Path Leads to Treasure", and "T_2: Second Path Leads to Treasure"

a)

1. $L_1 \wedge T_2$ means that the first path will lead to being lost forever and the second path will lead to treasure.
2. $(L_1 \wedge T_2) \vee (L_2 \wedge T_1)$ means that the first path will lead to being lost forever and the second path will lead to treasure, or The second path leads to being lost forever and the first path will lead to treasure.

b) If we assume the lying inscription is true and the truthful inscription is false, this will lead to a contradiction. Let's assume the first inscription is lying so we set that to true, and assume the second is truthful so we add $\neg$ to the second expression.

First Method

$$\underline{(L_1 \land T_2) \land \neg((L_1 \land T_2) \lor (L_2 \land T_1))}$$

De Morgan's Law

$$(L_1 \land T_2) \land \neg(L_1 \land T_2) \land \neg(L_2 \land T_1)$$

Complement Law

$$F \land \neg(L_2 \land T_1)$$

Domination Law

$$F$$

Second Method

$$\underline{(L_1 \land T_2) \land \neg((L_1 \land T_2) \lor (L_2 \land T_1))}$$

De Morgan's Law

$$(L_1 \land T_2) \land \neg(L_1 \land T_2) \land \neg(L_2 \land T_1)$$

De Morgan's Law

$$(L_1 \land T_2) \land (\neg L_1 \lor \neg T_2) \land \neg(L_2 \land T_1)$$

Distributive Law

$$(L_1 \land T_2 \land \neg L_1) \lor (L_1 \land T_2 \land \neg T_2) \land \neg(L_2 \land T_1)$$

Commutative Law

$$(L_1 \land \neg L_1 \land T_2) \lor (L_1 \land T_2 \land \neg T_2) \land \neg(L_2 \land T_1)$$

Complement Law

$$(F \land T_2) \lor (L_1 \land T_2 \land \neg T_2) \land \neg(L_2 \land T_1)$$

Complement Law

$$(F \land T_2) \lor (L_1 \land F) \land \neg(L_2 \land T_1)$$

Domination Law

$$(F) \lor (L_1 \land F) \land \neg(L_2 \land T_1)$$

Domination Law

$$(F) \lor (F) \land \neg(L_2 \land T_1)$$

Idempotent Law

$$F \land \neg(L_2 \land T_1)$$

Domination Law

$$F$$

c) As shown by the proof above, when we assume the first inscription is lying and the second is truthful by setting the former as true and latter as false, we prove a contradiction. Therefore, by proof by contradiction, it can be concluded that the first inscription is indeed false and that the second must be true. Furthermore, because $(L_1 \land T_2)$ is False, then $(L_1 \land T_2) \lor (L_2 \land T_1)$ is $F \lor (L_2 \land T_1)$. By the Identity Law, $F \lor (L_2 \land T_1)$ is simply $(L_2 \land T_1)$. Since we've established that the second inscription is True, then $(L_2 \land T_1)$ is True. Thus, the path of being lost forever is the second path, and the treasure lies in the first path.

d) Based on the answer, the first path carries the treasure. Now we want to get the treasure. But what if we will get lost AND get the treasure. Is there anything that can make me 100% sure that I will not get lost and get the treasure? Is there a way to prove these two things are mutually exclusive? We proved that L1 and T2 are wrong by contradiction. The answer is no. We now know that the first inscription is lying and the second is truthful. With that in mind, logically, we should assume that we can make the same expression but have the first inscription as false and the second be true by adding a negation to the first, and that this expression should be equivalent to True. However, when we try to "prove by truth", we see that we cannot get True. We end up with a statement. Furthermore, there is nothing that says $(\neg L_1 \wedge T_1)$. We will never be sure that being lost and finding treasure are mutually exclusive simply because we cannot logically deduce that it is True.

Problem 4.

a) Creating an expression equivalent to $p \oplus q$ using $\neg$, $\wedge$, and $\vee$.

p	q	$(p \vee q)$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$	$(p \oplus q)$
T	T	T	F	F	F
T	F	T	T	T	T
F	T	T	T	T	T
F	F	F	T	F	F

Table 2: Truth table to prove $(p \vee q) \wedge \neg(p \wedge q)$ is a logically equivalent way to express $(p \oplus q)$ using only $\neg$, $\wedge$, and $\vee$.

b) Creating an expression equivalent to $p \wedge q$ using only $\neg$ and $\vee$.

$$\underline{\neg(\neg p \vee \neg q) \equiv p \wedge q}$$

$$\neg(\neg p \vee \neg q)$$

De Morgan's Law

$$\neg\neg p \wedge \neg\neg q$$

Double Negation Law

$$p \wedge q$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$	$p \wedge q$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

Table 3: Truth table to prove $\neg(\neg p \vee \neg q)$ is a logically equivalent way to express $p \wedge q$ using only $\neg$ and $\vee$.

c) Creating an expression equivalent to $p \vee q$ using only $\neg$ and $\rightarrow$.

$$\underline{\neg p \rightarrow q \equiv p \vee q}$$

$$\neg p \rightarrow q$$

Conditional Identity Law

$$\neg\neg p \vee q$$

Double Negation Law

$$p \vee q$$

p	q	$\neg p$	$\neg p \to q$	$p \lor q$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

Table 4: Truth table to prove $\neg p \to q$ is a logically equivalent way to express $p \lor q$ using only $\neg$ and $\to$.

As shown above, we proved that $\neg p \to q$ is a logically equivalent way to express $p \lor q$ using only $\neg$ and $\to$ through a truth table in Table 4 and by using laws of propositional logic. With that logic, we can also express the $\land$ (and) and $\oplus$ (xor) operator using only $\neg$ and $\to$ as well. How? Well we now know how to prove the *or* operator with $\neg$ and $\to$. In part b, we were able to prove the *and* operator using only the $\neg$ and $\lor$ operators. We can substitute the $\lor$ operator with $\neg$ and $\to$ to prove the *and* operator. Backtracking further to part a, we used both the $\lor$ and $\land$ and $\neg$ operators to prove for *xor* ($\oplus$). Thus, we can use $\neg$ and $\to$ to substitute for both *or* ($\lor$) and *and* ($\land$) operators to write an expression equivalent to $p \oplus q$.

d) There are 16 total possible binary operators. This is because when you have a binary operation, you will have two variables, which means there are four possible rows (2^2). Four each row, you can have two possible outputs. Therefore, you can have $(2^2)^2$, which is 2^4, which is 16 possible binary operators for two variables.

e) NAND.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

Table 5: Truth table of $p \uparrow q$.

f) Expressing $\neg p$ and $p \lor q$.

p	q	$(p \uparrow p)$	$\neg p$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	T

Table 6: Truth table to prove $(p \uparrow p)$ is a logically equivalent way to express $\neg p$ using only NAND operator($\uparrow$) and NOT operator ($\neg$).

p	q	$\neg p$	$\neg q$	$(\neg p \uparrow \neg q)$	$p \vee q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Table 7: Truth table to prove $(\neg p \uparrow \neg q)$ is a logically equivalent way to express $p \vee q$ using only NAND operator($\uparrow$) and NOT operator ($\neg$).