

Mega Millions: When to Buy a Ticket?

Andy Chen

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1 What is Mega Millions?

Mega Millions is one of America’s two largest lotteries, the other being Powerball. Each week six numbers are drawn at 11pm Eastern Time on Tuesdays and Fridays. Five of the numbers are drawn from a pool of white balls labeled from 1 to 70. The sixth number (henceforth referred to as the “megaball”) is drawn from a separate pool of balls labeled from 1 to 25 [MegaMillions, 2018]. Normal tickets are priced at \$2 each where the purchaser can either choose the 6 numbers and identifying the megaball number, or by selecting the numbers randomly. The payout is determined by the way the numbers on the ticket match the drawn numbers (c.f. Table 1).

# of White Ball Matches	# of Megaball Matches	Payout (dollars)
5	1	Jackpot
5	0	1 000 000
4	1	10 000
4	0	500
3	1	200
3	0	10
2	1	10
1	1	4
0	1	2

Figure 1: Possible ways to win. Winning combinations of white balls and megaballs are listed with their payouts.

When a player wins the jackpot, he or she can choose between receiving the money through an annuity where payments are delivered annually for 30 consecutive years, each payment 5% larger than the last, or through a lump sum whose amount is less than the advertised jackpot amount. Historically, winners have tended to opt for the latter option: many believe they can invest the money and increase the value of their winnings. Since the difference in jackpot value between these two options is large, be sure to consider the correct amount when using the results of this study.

Although winning the lottery is usually left to random chance, games such as Mega Millions allow for larger payouts at different times. The jackpot amount grows after every drawing where the jackpot numbers are not purchased on a ticket. Thus, one strategy to increase the expected payout for Mega Millions involves waiting for the jackpot to accrue to a sizable amount before purchasing a ticket. This strategy will be the main focus of this paper.

Furthermore, along with the normal play ticket Mega Millions also offers a Megaplier mode that increases the payout for winning tickets. This method of play is priced at an additional dollar, totaling at \$3 per ticket. Before each six ball drawing, a multiplier ball is drawn from a pool of 15 marked balls: five with 2X, six with 3X, three with 4X, and one with 5X (c.f. Table 2). When playing this mode, winning tickets have payouts multiplied by the value of the multiplier ball. The Megaplier effect does not apply to tickets that win the jackpot.

# of White Ball Matches	# of Megaball Matches	Payout with Megaplier (dollars)			
		2X	3X	4X	5X
5	0	2 000 000	3 000 000	4 000 000	5 000 000
4	1	20 000	30 000	40 000	50 000
4	0	1000	1500	2000	2500
3	1	400	600	800	1000
3	0	20	30	40	50
2	1	20	30	40	50
1	1	8	12	16	20
0	1	4	6	8	10

Figure 2: Payouts when using the Megaplier mode. Combinations of white balls and megaballs are listed with their payouts when different Megapliers are applied.

2 Expected Values

2.1 The Theory

To find the expected return value for each winning combination, we calculate the possibility of each combination occurring:

$$P(x, y) = \left[\frac{\binom{65}{5-x} \cdot \binom{5}{x}}{\binom{70}{5}} \right] \cdot \left[\frac{\binom{25}{1-y} \cdot \binom{1}{y}}{\binom{25}{1}} \right] \quad (1)$$

where x is the number of white ball matches and y is the number of megaball matches. The first factor in Equation 1 corresponds to the chance that a given white ball combination occurs while the second corresponds to that for the megaball. The different probabilities of obtaining each combination is listed in Table 3.

# of White Ball Matches	# of Megaball Matches	Odds of Winning
5	1	302 575 350.00
5	0	12 607 306.25
4	1	931 001.08
4	0	38 791.71
3	1	14 546.89
3	0	606.12
2	1	692.71
1	1	89.38
0	1	36.63

Figure 3: Possible ways to win. Winning combinations of white balls and megaballs are listed with odds of winning. The odds of a given combination occurring is 1 to the value listed.

2.2 Breakeven Points

Breaking even refers to the conditions at which the average winnings of any random sequence of Mega Millions numbers will return the amount at which each ticket is bought. To find the expected return value, the results for tickets with Megaplier and those without were calculated separately. The following calculations are made based off of entirely randomly generated sequences of the six numbers.

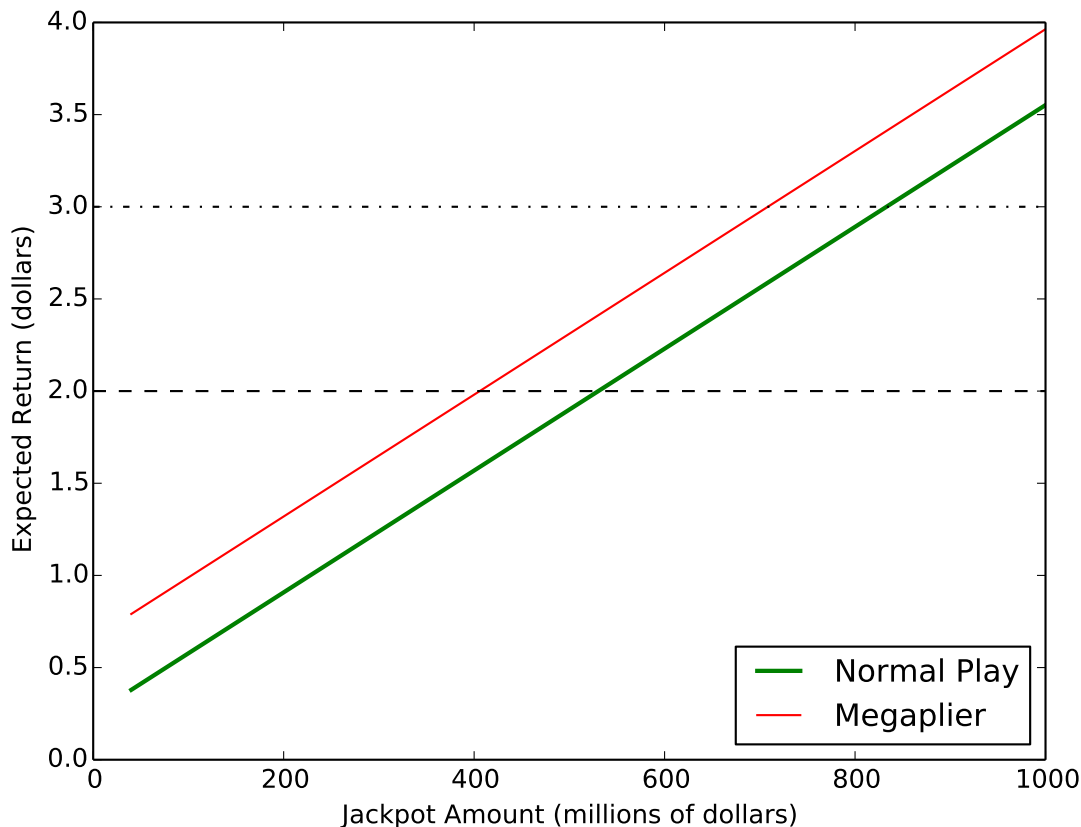


Figure 4: Mega Millions Payout. The expected return value for both normal play and Megaplier tickets are plotted with dependence on the current jackpot amount. The dashed and dash-dotted lines represent the cost of each ticket respectively.

From Figure 4, the expected linear trend between the expected return value and the current jackpot amount is evident. For normal play tickets, the breakeven point is at a jackpot amount of approximately \$530.4 million. For tickets with Megaplier, the breakeven point is at approximately \$708.4 million.

2.3 Taxes

When a winning ticket is redeemed, the recipient must report the prize in their annual income tax report [Wattles, 2016]. When the jackpot is redeemed, one fourth of the payout is immediately withheld by the government; the rest of the federal tax is paid at tax time (c.f. Table 5). The remainder of the prize is then subject to the state taxes ranging up to 8.82% in New York. However, some states, such as California and Florida, do not tax lottery prizes [Mega, 2018]. The state tax will be set at 8% for all subsequent calculations.

Rate	Income Over (dollars)
10%	0
12%	9525
22%	38 700
24%	82 500
32%	157 500
35%	200 000
37%	500 000

Figure 5: Tax Brackets and Rates for 2018. Reported income for unmarried individuals is subject to federal taxation according the rates listed [El-Sibaie, 2018] [Bankrate, 2018]. The US government uses a progressive taxation system such that each bracket rate only applies to income amounts greater than the last bracket.

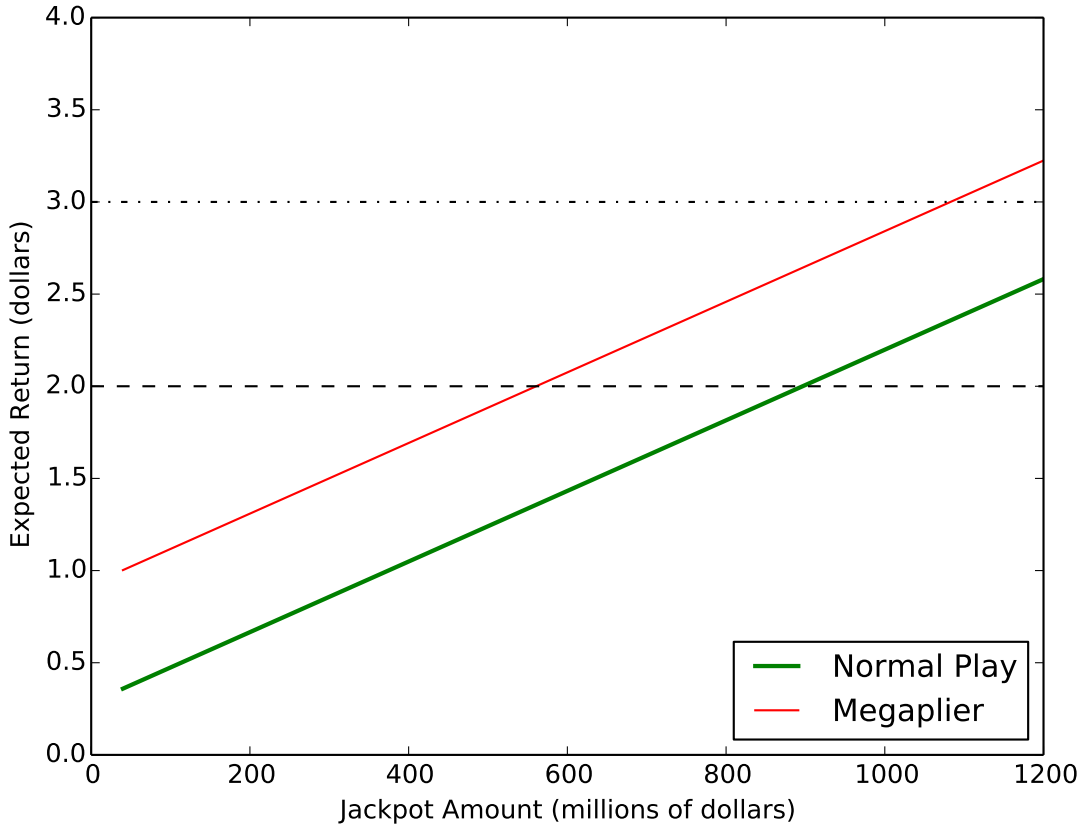


Figure 6: Mega Millions Payout After Taxation. The expected return value for both normal play and Megaplier tickets after accounting for both federal and state tax of 8% are plotted with dependence on the current jackpot amount. The dashed and dash-dotted lines represent the cost of each ticket respectively.

From Figure 6, the breakeven point for normal play tickets is at a jackpot amount of approximately \$896.5 million. For tickets with Megaplier, the breakeven point is at approximately \$1082.8 million.

3 Other Considerations

3.1 Splitting the Pot

Figures 4 and 6 reflect the expected values for situations where all jackpot winning tickets receive the advertised jackpot amount. However according to the rules of Mega Millions, if more than one jackpot winning ticket is redeemed for a given drawing, the jackpot amount is split amongst the winners in proportions agreed upon by all parties. For this study, it is assumed that all parties agree to split the jackpot amount evenly. The probability of sharing the jackpot amount is calculated for one, two, three, and four winners:

$$P(w, s) = (P_{\text{jackpot}})^w \cdot (P_{\text{jackpot}})^{s-w} \cdot \binom{s}{w} \quad (2)$$

where P_{jackpot} is the probability of winning the jackpot, w is the number of winning tickets, and s is the number of tickets sold.

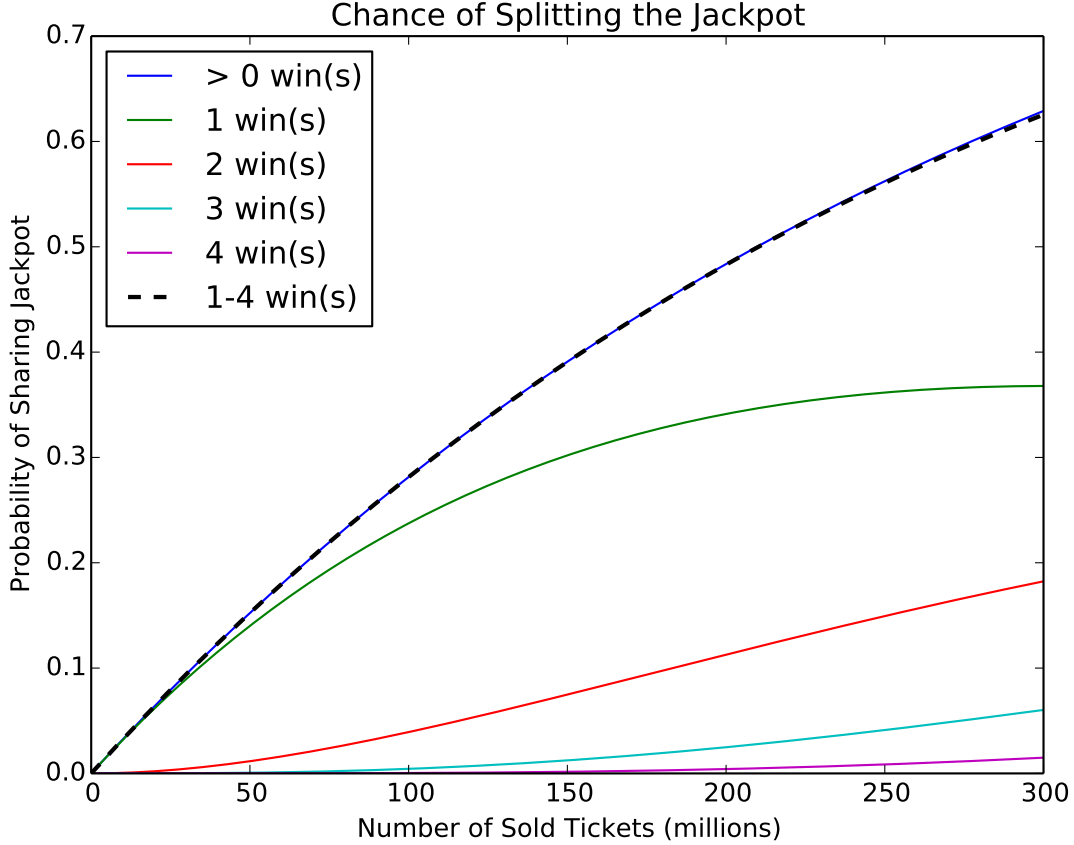


Figure 7: Probability of Splitting the Jackpot. The likelihood of having one or more tickets with the jackpot combination as the number of ticket sales varies.

In Figure 7, the chance of sharing the jackpot increases as the number of tickets sold. It is also evident that the chance of splitting the jackpot at all can be well approximated by the sum of the probabilities for one, two, three, and four winning tickets for the our study. On the other hand, the average number of winning tickets for a given number of ticket sales is easily represented linearly:

$$w = P_{\text{jackpot}} \cdot s. \quad (3)$$

3.2 Ticket Demand

As the jackpot remains unclaimed through several consecutive drawings, the amount grows at an increasing rate. This acceleration in the grand winnings results from the increased demand for tickets as the advertised amount increases. On October 31st of 2017, the cost of each normal play ticket increased from \$1 to \$2, among other changes. To maintain a consistent demand model, only the ticket sales data of drawing dates between November 2017 and August 2018 were used.

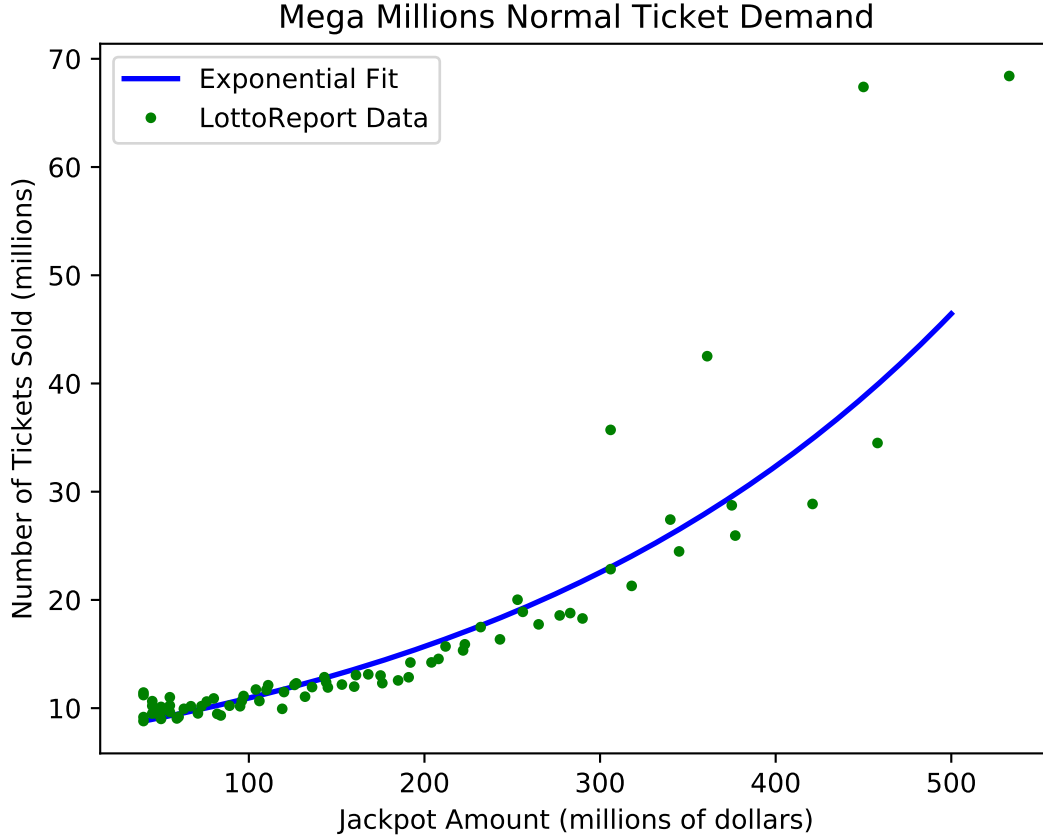


Figure 8: Mega Millions Normal Ticket Sales. The number of normal tickets sold for varying jackpot sizes between November 2017 and July 2018 [LottoReport, 2018].

In Figure 8, the available data for normal ticket sales presented an exponential trend between ticket sales and the jackpot size for amounts between \$40 million and \$500 million dollars:

$$S(p) = S_0 \exp(I \cdot p) \quad (4)$$

where S is the number of tickets sold and p is the jackpot amount. S_0 can be interpreted as the number of tickets sold if Mega Millions did not offer a jackpot prize. Additionally, I represents the incentive factor for individuals to purchase a ticket: as the jackpot amount increases, the willingness to buy a ticket also increases. The fitted curve calculates an S_0 value of approximately 7.6 million tickets and an incentive factor of 0.00362.

In Figures 9 and 10, the data for Just the Jackpot and Megaplier ticket sales both indicated exponential trends as well. The fitted curves estimate S_0 values of approximately 6.9 thousand and 447.1 thousand tickets with incentive factors of 0.00324 and 0.00293, respectively. The low sales of Just the Jackpot tickets can be

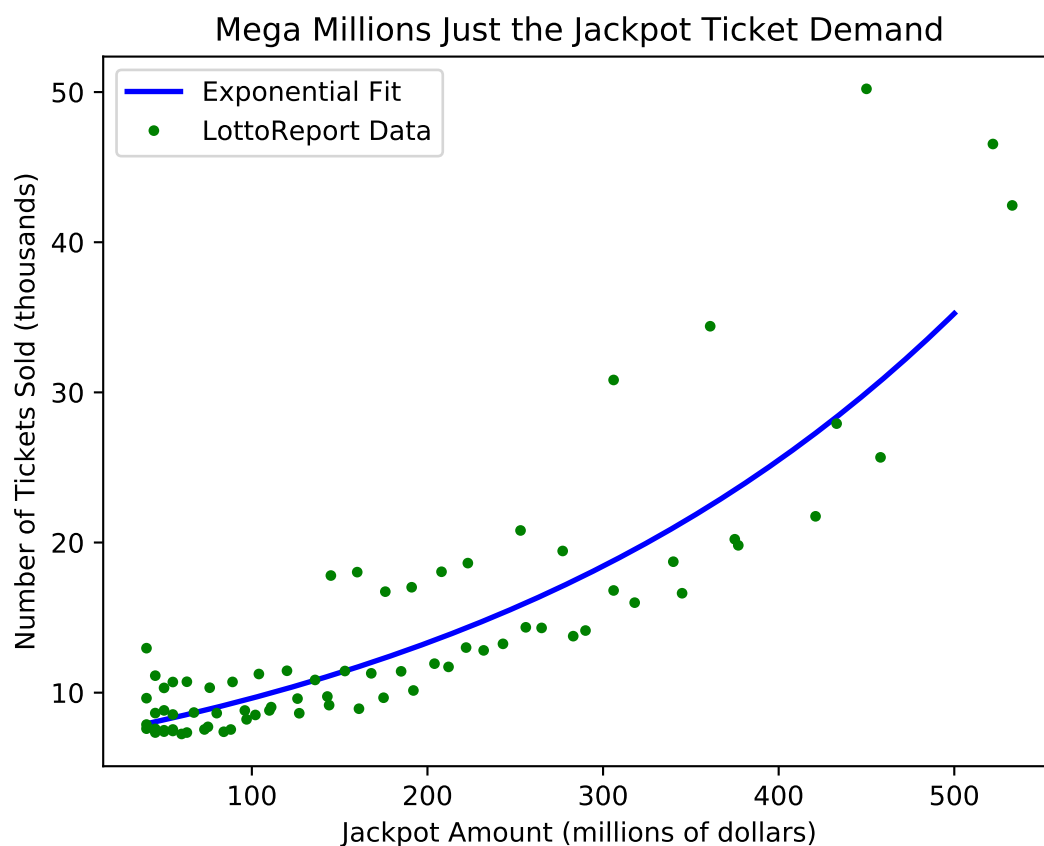


Figure 9: Mega Millions Normal Ticket Sales. The number of Just the Jackpot tickets sold for varying jackpot sizes between December 2017 and August 2018 [LottoReport, 2018].

explained by the minuscule chance of winning the jackpot despite having two sets of numbers. Although the sales of Megaplier tickets is far greater than that for Just the Jackpot, the low number in comparison to a normal ticket can be attributed to the higher cost per ticket. The incentive factor for Just the Jackpot tickets and normal tickets are comparable since both tickets are valid for jackpot winning combination. However, the presence of a lower cost substitute as well as the greater chance of winning a smaller payout likely drive customers towards the normal ticket instead. Since a customer who purchases a Megaplier ticket is likely aiming for a smaller payout winning combination, the more significant decrease in incentive for a Megaplier ticket is to be expected.

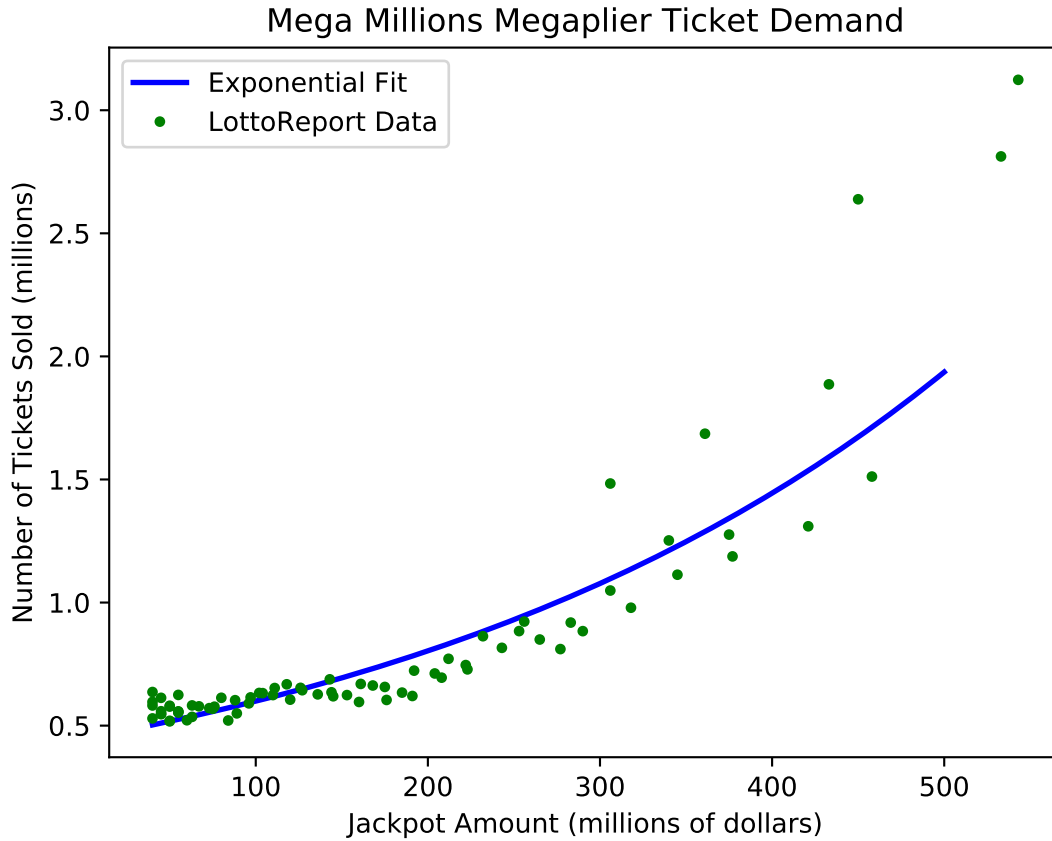


Figure 10: Mega Millions Megaplier Ticket Sales. The number of megaplier tickets sold for varying jackpot sizes between December 2017 and August 2018 [LottoReport, 2018].

3.3 All Things Considered

Using Equations 1, 4, and 3, an all inclusive model can be created to find the expected return value of a normal ticket. In Figure 11, the expected return value never reaches the cost of a normal ticket. Although a breakeven jackpot value does not exist, the expected value reaches a maximum when the jackpot amount is approximately \$840.3 million. Since the Megaplier only applies to non-jackpot winnings, the curve for a Megaplier ticket would only shifted upwards. Although the expected value of the Megaplier ball is 3X which lifts the expected return by a promising amount of \$0.72, the higher cost of the Megaplier ticket extinguishes any hope of surpassing the breakeven point.

4 Future Directions

This work selected to study the expected return value of Mega Millions over any other lottery solely due to its presence in the news at the time of writing (July 19 of 2018). Future work can aim to include the studies of other lotteries, such as Powerball, and compare the expected return values. Take the results of this study with caution, always play responsibly, and best of luck.

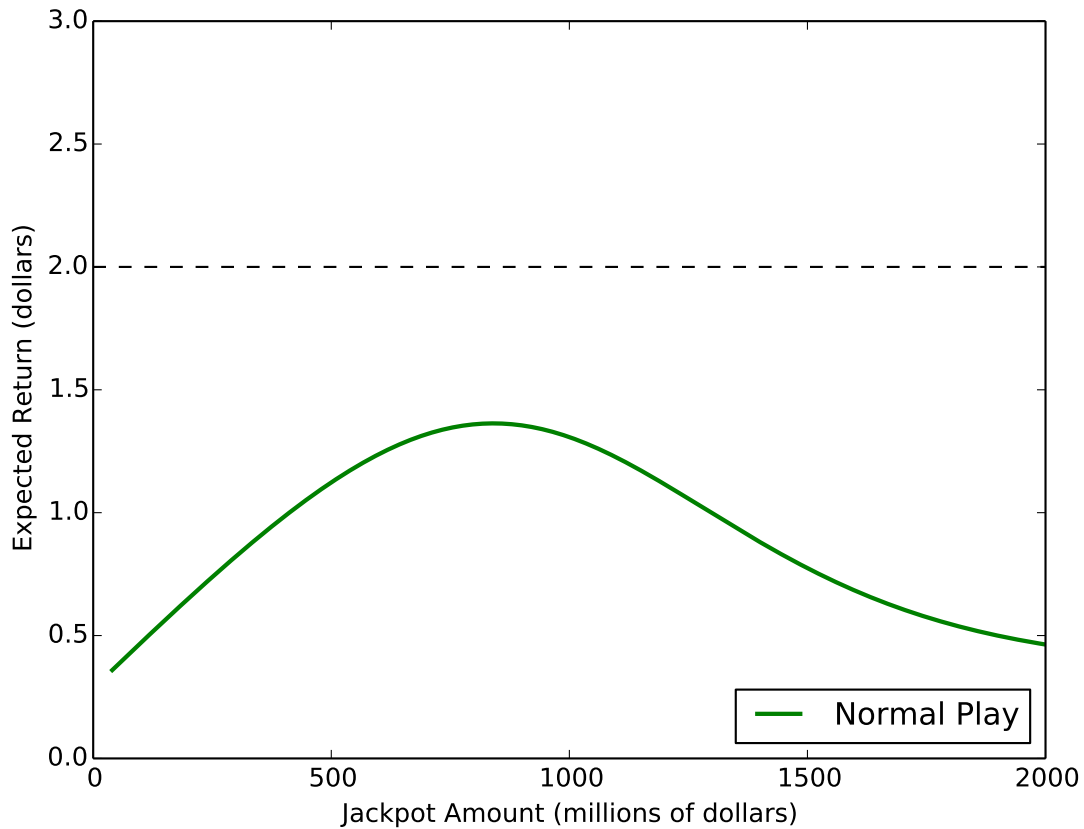


Figure 11: Mega Millions Payout with All Things Considered. The expected return value for normal play tickets are plotted with dependence on the current jackpot amount. The dashed line represents the cost of each ticket.

5 References

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