ECON 504 Spring 2017 Yilmaz KOCER 1

CHAPTER 15 PROBLEM SOLUTIONS

15.1 Consider first all pure strategy profiles (3x3=9) "$\rightarrow$" shows the best response by the

a). opponent; $A \rightarrow L \rightarrow B \rightarrow R \rightarrow A$ and $M \rightarrow C \rightarrow \begin{smallmatrix} L \\ M \\ R \end{smallmatrix}$ hence (C, M) is the unique pure BNE.

Note that $\frac{1}{2}L + \frac{1}{2}R \underset{\text{strictly dominates}}{>} M$, hence M cannot be a part of any mixed str. in a <u>mixed</u> BNE.

When pl. 2 plays $\alpha L + (1-\alpha)R \longrightarrow \alpha > \frac{1}{2} \longrightarrow B \longrightarrow R > L \longrightarrow$

$\alpha < \frac{1}{2} \rightarrow A \longrightarrow L > R \rightarrow$

player 2 would not randomize in these cases as he is <u>not</u> indifferent; so $\alpha = \frac{1}{2}$

Against $\frac{1}{2}L + \frac{1}{2}R$, player 1 prefers $A \cong B > C$

For player 2 to be indifferent btw L & R (so he plays $\frac{1}{2}L + \frac{1}{2}R$)

player 1 has to play $\frac{1}{2}A + \frac{1}{2}B$.

Hence $\{ \underset{1}{(\frac{1}{2}A + \frac{1}{2}B, \frac{1}{2}L + \frac{1}{R})}, \underset{2}{(C, M)} \}$ is the set of BNE.

b). As in 1 all information sets are visited with positive probability (actually prob. 1)

1 qualifies as a PBE by proposition 15.1 (just augment the strategies with beliefs

$\mu_2(x_2) = \frac{1}{2} = \mu_2(x_3)$). 2 is not PBE as for <u>any</u> belief $(\mu, 1-\mu)$ $\mu \in [0,1]$

either $u(L) = 0\mu + 4(1-\mu) = 4(1-\mu) > 1$ if $\mu < \frac{3}{4}$

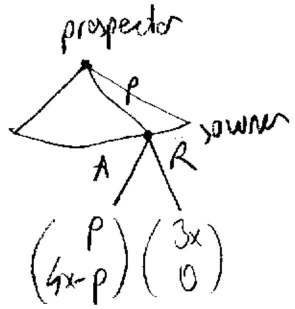
or $u(R) = 4\mu + 0(1-\mu) = 4\mu > 1$ if $\mu > \frac{1}{4}$

hence for <u>no</u> μ $M > L$ and R, M is never a best response,

M is <u>not</u> sequential rational for player 2.

for any belief.

45.2 a)-



prospector offers the price $4x$, and owner accepts.
$4x-4x=0\geq 0$

b)- Suppose in a PBE some set of types choose N, and they ask for a price p, which is either accepted or rejected by owner. Then in this subgame all types of prospectors receive the same payoff, say v.

As type x' prospector prefers certification and getting $4x'$ to v (because we are considering a PBE hence prospectors are sequentially rational); $4x'\geq v \Rightarrow 4x''>v$; x'' choses the certification as well. Hence it is a cutoff/threshold type equilibrium.

Note that all types $[0,x^*)$ chooses N and all $(x^*,1]$ types chooses certification, for some x^*. (x^* is indifferent between the two actions)

Assume on the contrary $x=1$ does not get certification. Hence all $x<1$ should not either, by above logic. The value of a N (not certified) prospector is then $4\cdot E(x\mid x\leq 1)=4E(x)=4\frac{1}{2}=\boxed{2}$ to the owner, hence he is willing to accept $p\leq 2$ only.

$x=1$ gets $\underset{\text{he offers } p=1}{4\cdot 1}-\underbrace{\tfrac{1}{2}}_{\text{cost of certification}}=\boxed{3.5}$ if certificate and at most $\max\{\underset{\text{N gets at most } p=2}{2},\ \underset{\text{goes back home to mine himself.}}{3}\}=\boxed{3}$ if not.

$3.5>3$ hence $x=1$ gets the certificate

c)- x^* should be indifferent btw certification or N;

$$4x^*-\frac{1}{2}=\max\{\underset{\text{go home}}{3x^*},\ \underset{\text{N gets the price } 4E(x\mid x\leq x^*)}{4\cdot\frac{x^*}{2}}\}=3x^* \qquad \boxed{x^*=\frac{1}{2}}$$

$\frac{x^*}{2}$

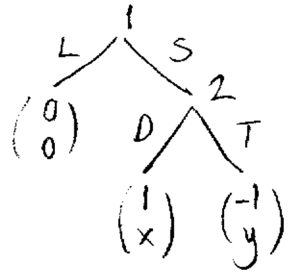
Notice that whatever x^* might have been, in eqm. $[0,x^*]$ chooses N and proposes p. Price p is accepted only when $4E(x\mid x\leq x^*)-p\geq 0$

$\Rightarrow p\leq 4\frac{x^*}{2}=2x^*$. But x^* would rather go home & mine, $3x^*>2x^*$ hence there's no trade in equilibrium (for $x\leq x^*=\frac{1}{2}$)

d)- YES. Looking back at 12.5 problem, $x\in[0,\frac{1}{2}]$ mine themselves as before. But for $x\in(\frac{1}{2},1]$ $\underbrace{(4x-\tfrac{1}{2})}_{\text{eqm payoff}}-\underbrace{3x}_{\text{eqm payoff in 12.5}}=x-\frac{1}{2}>0$ So high value types are helped by certification.

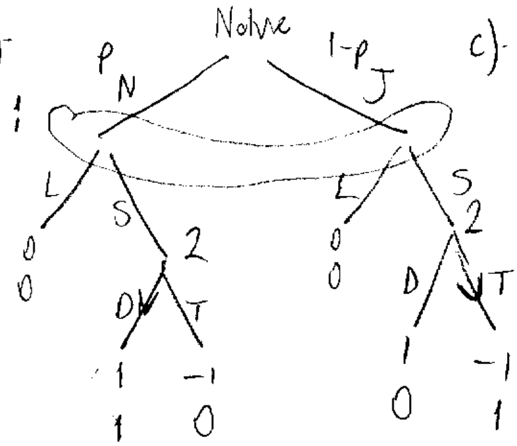
L3

15.3 a).



For choice of S to be on the eqm. path in a SPNE, $x > y$ should hold so that 2 chooses D and hence 1 chooses S $(1 > 0)$

b).



c). Let $p = \frac{3}{4}$. Note that 2's N type chooses D & J type chooses T, by sequential rationality in a PBE.

Next notice that 1's information set is always reached, hence beliefs must be consistent with nature & strategies; $(\mu, 1-\mu) = (p, 1-p) = (\frac{3}{4}, \frac{1}{4})$

For player 1 $\quad L \to 0$

$S \to 1p + (-1)(1-p) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} > 0$

hence chooses S.

Hence unique PBE $\Rightarrow (S, (D, T))$

For other BNE (that is not PBE), consider $(L, (T, T))$ or $(L, (T, D))$

given (T, T) or (T, D), L is indeed optimal $(0 > -1 \quad \frac{3}{4}(-1) + \frac{1}{4}(1) < 0)$ and trivially player 2 strategy is optimal given L (because it is irrelevant)

Player 2's sequential rationality is not checked in a BNE, hence these are BNE.

d). For no p. IGNORE THIS PART of the QUESTION.

(15.6) a). 1 → A, B 2 → C, D 3 → E, F

AC → E → B × 1

AD → E → yes BNE (A, D, E)

BC → F → D × 2 BNE

BD → E or F if E → A (B, D, F)

F

b). 1 (A, D, E) is not part of a PBE becase player 2's D is not sequentially rational! (4 > 1)

2 (B, D, F) is a part " for $(\mu, 1-\mu)$ such that

$$\underbrace{2\mu + 0(1-\mu)}_{E} \leq 0\mu + 1(1-\mu)$$

$$3\mu \leq 1$$

$\boxed{\mu \leq \frac{1}{3}}$ is a PBE.

You're encouraged to solve 15.4 & 15.5 to prodce PBEs, but in the exam I will use more succinct games so that you can construct the game within minutes; these exercises however involve lengthy & complicated game descriptions.