HOMEWORK 2

Econ 501: Macroeconomic Analysis and Policy

1. Consider the optimal growth model with Cobb-Douglas utility, Cobb-Douglas production, and 100% depreciation every period. The social planner maximizes

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to the constraints:

$$k_{t+1} = Ak_t^{\alpha} - c_t$$
$$k_t \geq 0$$
$$k_0 > 0 given$$

a) Derive the Euler equation.
b) Find the steady state values of k_t, c_t, and y_t.
c) Find the steady state savings rate.
d) Suppose that α= 1, so that we have an "AK" model. Ignore for the time being the possibility that there is no solution and assume that the Euler equation applies. What is the growth rate of consumption?

2. What if household preferences were given by a linear utility function:

$$\sum_{t=0}^{\infty} \beta^t c_t$$

subject to the constraints:

$$k_{t+1} = Ak_t^{\alpha} - c_t$$
$$c_t \geq 0$$
$$k_t \geq 0$$
$$k_0 > 0 given$$

a) Prove that there exists a solution to this optimal growth problem. You can do this by finding any finite number which is bigger than the highest possible utility level.

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b) Describe as best you can the solution to this planner's problem. You should note that mechanical application of the first-order conditions and the Euler equation is unlikely to find the solution.
c) Suppose instead that $\alpha = 1$, so that we have an "AK" model. Identify conditions under which there is a solution to the planner's problem, and describe the solution under these conditions.

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