Mid term answers

essays

1 yes, different beliefs may lead to different equilibria.

2. yes, because if the information has no social benefit, then there is no need to require acquisition of information.

3. no, as the political preferences are not uniform.

4. no, as it only requires one side not to force disclosure.

5. yes, as initial cards are not shown, and some cards are not shown during play.

6. yes, as unraveling may imply voluntary disclsoure and revelation of information.

7. yes, it does not depend on your own bid. This makes it easier to have truthful bidding.

8. It is a one-sided law because potential job applicants may voluntarily reveal their lack of disability, leading to unravelling. To make it two-sided, we may allow applicants to lie without penalty to prevent unravelling.

computations

1. (a) The profit is given by $2\theta - \frac{4}{3}\theta = \frac{2}{3}\theta$. The expected profit in the complete information case is $\frac{2}{3}\frac{a+b}{2} = \frac{a+b}{3}$.

(b) If the solution is interior, the optimal wage is $w = \frac{8}{3}a$. Substitute this value to the profit function, we get the profit of the principal

$$\frac{\frac{3}{4}w - a}{b-a}(a - \frac{1}{4}w) = \frac{1}{b-a}(2a-a)(a - \frac{2}{3}a) = \frac{a^2}{3(b-a)}.$$

(c) We need to show

$$\frac{a+b}{3} > \frac{a^2}{3(b-a)}$$

or

$$(b+a)(b-a) > a^2,$$

or

$$b^2 - a^2 > a^2.$$

For the interior solution to hold, we must have $\frac{8}{3}a < \frac{4}{3}b$, or $b > 2a$. Hence we have $b^2 - a^2 > 4a^2 - a^2 = 3a^2 > a^2$. We are done.

2. (a) the probability of winning with no switch is $\frac{1}{5}$. (b) $\frac{4}{5} * \frac{1}{3} = \frac{4}{15}$ is the probability of winnning if you switch. (c) you should switch as the probability of winning is higher. (d) the optimal decision is independent of the prize as the benefit of switching is always higher independent of the size of the prize.

3. (a) If GF uses high test, dl will use low test, as it reduces cost, and gives the firm profit \$5. The diary firm dh will use low test too, and get the same profit \$5. Both use low test.

(b) Since dl only sells to GF with probability 1/2, and dh always sells, the conditional probability when it sells is 2/3 from dh, and 1/3 from dl.

(c) The profit from high test is always $1 whether it is from dh or dl. The profit from low test is 2/3*3=2. Hence it is optimal to use the low test.

(d) The high test is not optimal because it is more likely that the milk is from dh.

(e) it is not a perfect Bayesian equilibrium for General Foods to use the high test when the milk is sold by the diary firm, as (d) shows that it is not optimal for General Food to use high test.

4. (a) $0.25\phi(b)(v-b)$

(b) The first-order condition

$$\frac{\phi'(b)}{\phi(b)} = \frac{1}{\phi(b)-b}.$$

(c) The bidding strategy $b(v) = 0.5v$ has the inverse bidding function $\phi(b) = 2b$. This satisfies the first-order condition because

$$\phi'(b) = 2,$$

hence each side of the first-order condition is equal to $\frac{1}{b}$.

(d) The highest equilibrium bid in this example is $4{*}0.5 = 2$, and equilibrium bid distribution is $G(b) = (0.25{*}2b)^2 = 0.25b^2$. Hence the expected seller revenue is given by

$$\int_0^2 bd(0.25b^2) = 0.5\int_0^2 b^2 db = \frac{4}{3}.$$

(e) When the reservation price is $\rho = 1$, we have equilibrium bidding strategy

$$b(v) = v - \frac{1}{0.25v}\int_1^v 0.25x dx = v - \frac{1}{v}\int_1^v x dx$$

$$= v - \frac{1}{v}\frac{1}{2}(v^2-1) = 0.5v + \frac{1}{2v}.$$

(f) The seller's expected total revenue is given by

$$\int_1^4 b(v)dF^2(v) = \int_1^4 (0.5v + \frac{1}{2v})d(0.25v)^2$$

$$= 0.25 * 0.5\int_1^4 (0.5v^2 + \frac{1}{2})dv = 1.5$$

4. old (a) the demand function of the consumer is (as shown in the notes)

$$q(p) = 1 - \frac{1}{\theta}p$$

Hence the overall demand is

$$D(P) = 0.6(1 - \frac{P}{3}) + 0.4(1 - \frac{P}{4}) = 1 - 0.3P$$

The firm maximizes the profit $D(P)(P-1) = (1-0.3P)(P-1)$, with the first order condition $1.3 = 0.6P$, or $P = \frac{13}{6}$.

The optimal price is $\frac{13}{6} = 2.1667$. The optimal profit isThe maximum profit is

$$(P-c)D(P) = (\frac{13}{6} - 1)(1 - \frac{3}{10}\frac{13}{6}) = \frac{49}{120} = 0.40833$$

(b) First we compute the low-demander surplus

$$S_L(P) = \theta_L v(D_L(P)) - PD_L(P) = 3((1-\frac{P}{3}) - \frac{1}{2}(1-\frac{P}{3})^2) - P(1-\frac{P}{3}) = \frac{1}{6}P^2 - P + \frac{3}{2}.$$

Hence

$$S_L'(P) = \frac{1}{3}P - 1.$$

The expected demand is

$$D(P) = 1 - \frac{3}{10}P,$$

and the optimal price is the solution of the equation

$$P = c - \frac{D(P) + S_L'(P)}{D'(P)}$$

or

$$P = 1 + \frac{1 - \frac{3}{10}P + \frac{1}{3}P - 1}{\frac{3}{10}} = \frac{1}{9}P + 1.$$

We have $P = \frac{9}{8}$. The fixed fee is

$$S_L(P) = \frac{1}{6}P^2 - P + \frac{3}{2} = \frac{1}{6}(\frac{9}{8})^2 - \frac{9}{8} + \frac{3}{2} = \frac{75}{128} = 0.58594.$$

The equilibrium expected profit of the monopolist is

$$S_L(P^*) + (P^* - c)D(P^*) = 0.58594 + \frac{1}{8}(1 - \frac{3}{10}\frac{9}{8}) = 0.66875.$$

(c) In this case, $P = 1$, and fixed fee is the consumer surplus of the high demander

$$S_H(P) = 4((1 - \frac{P}{4}) - \frac{1}{2}(1 - \frac{P}{4})^2) - P(1 - \frac{P}{4}) = \frac{1}{8}(P-4)^2 = \frac{9}{8}.$$

The profit is given by $\frac{4}{10}\frac{9}{8} = \frac{9}{20} = 0.45$.

(d) Profit in (b) is higher, so that it is better to include both the low and high demander.

(e) The participation constraints

$$\theta_i v(q_i) - T_i \geq 0 \text{ for } i = H, L$$

and the incentive constraints

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L$$

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H$$

The binding constraints are the participation constraint for the low demander and the incentive constraint of the high demander, or

$$\theta_H v(q_H) - T_H = \theta_H v(q_L) - T_L,$$

$$3v(q_L) = T_L.$$

We have

$$T_H = 4v(q_H) - 4v(q_L) + T_L = 4v(q_H) - v(q_L)$$

Substitute into the following objective function

$$0.6(T_L - q_L) + 0.4(T_H - q_H)$$

we get the new objective function

$$0.8q_L - 0.7q_L^2 + 0.4(3q_H - 2q_H^2)$$

This is an unconstrained problem. Take the derivatives, and we have

$$q_L^* = \frac{4}{7}, q_H^* = \frac{3}{4}$$

Hence we can compute the payments

$$T_L = \theta_L v(q_L) = 3(\frac{4}{7} - \frac{8}{49}) = \frac{60}{49}$$

and

$$T_H = 4(\frac{3}{4} - \frac{1}{2}(\frac{3}{4})^2) - (\frac{4}{7} - \frac{8}{49}) = \frac{575}{392} = 1.4668.$$

The optimal profit is

$$0.4(T_H - q_H) + 0.6(T_L - q_L) = 0.4(1.4668 - \frac{3}{4}) + 0.6(\frac{60}{49} - \frac{4}{7}) = \frac{19}{28} = 0.67856.$$

(f) The profit here is highest because it allows consumer taste to be used in the design of contracts.

5(a) If the seller finds a bad furnace and reports the true information, he gets \$180 from the house. However if he hides the information, the buyer does not necessarily conclude that it is a bad one, because it is possible that it may be a good one. Therefore, the house will be worth more than \$180. Hence it is better that he hides information.

(b) The probability that a low cost seller acquires information and finds a good furnace is $\frac{p}{2}$. This will be reported. When there is no report, either the low cost seller finds bad furnace, or it is a high cost seller who does not have information. The buyer cannot distinguish between the two cases. The total probability is $1 - \frac{p}{2}$. The probability that it is a good furnace even though there is no report is $\frac{1-p}{2}$. The probability that it is a bad furnace when there is no report is 1-$\frac{p}{2}$-$\frac{1-p}{2}$=0.5. Therefore, conditional on no report, the probability that it is bad furnace is $\frac{1}{2-p}$ and the probability that it is a good furnace is $1 - \frac{1}{2-p} = \frac{1-p}{2-p}$.

(c) Acquiring information in this case leads to \$200 with probability 0.5. With probability 0.5, the seller gets

$$180 * \frac{1}{2-p} + 200 * \frac{1-p}{2-p} = \frac{380-200p}{2-p}.$$

Therefore with information, he gets

$$\frac{1}{2} * 200 + \frac{1}{2} * \frac{380-200p}{2-p} = \frac{390-200p}{2-p}.$$

When the low cost seller does not acquire information, (the buyer does not know this) there is no information and no report. The buyer is still willing to pay

$$\frac{380-180p}{2-p}$$

The value of information is then

$$\frac{390-200p}{2-p} - \frac{380-200p}{2-p} = \frac{10}{2-p}.$$

(d). The equilibrium conditions are

$$4 < \frac{10}{2-p} < 8$$

or $p \in [0, \frac{3}{4})$.

6(a) After gathering information, the furnace is either good or bad equally likely. With the out-of-equilibrium belief, the seller can collect \$186 by disclosure when the furnance is bad, and only gets \$180 with no disclosure. Therefore there is always disclosure.

(b) If there is no gathering, there is no information to report, and the seller only gets \$180. The value of information is \$16, but the cost of information is either \$4 or \$8. Hence it is optimal for all sellers acquire information.

(c) If the seller acquires information, and finds a bad furnace, he gets \$186 by disclosing it, but gets \$190 without disclosure. Therefore, there is no incentive to disclose given the buyer's belief. Therefore, the buyer's belief is inconsistent with the seller optimal behavior, and we don't have a perfect Bayesian equilibrium.

(d) The seller is indifferent between disclosing it or hiding it when he finds a bad furnace. If the buyer believes that the seller who finds a bad furnace discloses with a probability π, then a low cost seller acquires information and reports a good furnace (the probability is $\frac{p}{2}$), or a bad furnace (the probability is $\frac{p}{2}\pi$). It is also possible that a low cost seller acquires information and gives no report (the probability is $\frac{p}{2}(1-\pi)$, or it is a high cost seller and no information is available (the probability is $1-p$). The probability of no report is $\frac{p}{2}(1-\pi)+1-p$. The probability of no report but the furnace is a good one is $\frac{1-p}{2}$. Conditional on no report, the furnace is good with probability $\frac{\frac{1-p}{2}}{\frac{p}{2}(1-\pi)+1-p} = \frac{1-p}{2-p(1+\pi)}$, and is bad with probability $1 - \frac{1-p}{2-p(1+\pi)}$. With no report, the buyer is willing to pay

$$200 * \frac{1-p}{2-p(1+\pi)} + 180 * (1 - \frac{1-p}{2-p(1+\pi)}) = \frac{380 - p(200+180\pi)}{2-p(1+\pi)}.$$

If the seller finds a bad furnace, and reports it, he gets \$186. The indifference between report or no report of the seller gives us

$$\frac{380 - p(200+180\pi)}{2-p(1+\pi)} = 186$$

Hence the probability π is determined by the above equation. Solving the equation gives us $\pi = \frac{1}{3}(7 - \frac{4}{p}) > 0$.

(e) To compute the value of information, a seller who acquires information will find a good furnace with probability 0.5 and gets \$206. He will find a bad furnace with probability 0.5, and is indifferent between reporting it or hiding it, because either way he gets \$186. Hence with information he gets 0.5*206+0.5*186=196. With no information, he has nothing to report, and he will get \$186. Thus the value of information is 196-186=10. This value of information is higher than the cost of acquiring it for both types of the seller. Hence it is optimal for all seller to acquire the information. This is inconsistent with the belief that only the low cost seller acquires information. Hence we conclude that it is not a perfect Bayesian equilibrium for low cost sellers to acquire information, and high cost sellers to remain uninformed.