HOMEWORK 1: Solutions

Econ 501: Macroeconomic Analysis and Policy

Spring 2016

1.

a) First we show that it is homogeneous of degree 1:

$$A(cK)^{\alpha}(cL)^{1-\alpha} = Ac^{\alpha}K^{\alpha}c^{1=\alpha}L^{1-\alpha} = cAK^{\alpha}L^{1-\alpha}$$

Next show that both factors are necessary:

$$A0(L)^{1-\alpha} = 0$$

$$AK^{\alpha}0 = 0$$

Both factors contribute to output:

$$\frac{\partial F}{\partial K} = A\alpha K^{\alpha-1}L^{1-\alpha}$$

$$\frac{\partial F}{\partial L} = A(1-\alpha)K^{\alpha}L^{-\alpha}$$

Since $0 < \alpha < 1$, each of these is a string of strictly positive numbers being multiplied together. So, each is positive.

Next, we show that F is concave

$$\frac{\partial^2 F}{\partial K^2} = A\alpha(\alpha-1)K^{\alpha-2}L^{1-\alpha}$$

All of these factors are positive with the exception of _ − 1. A negative number multiplied by a positive number produces a negative number, so this is negative.

$$\frac{\partial^2 F}{\partial L^2} = A(1-\alpha)(-\alpha)K^{\alpha}L^{-\alpha-1}$$

All of these factors are positive with the exception of $-\alpha$. So this quantity is negative.

Next we show that the Inada conditions hold. First we rearrange:

$$\frac{\partial F}{\partial K} = \frac{A\alpha L^{1-\alpha}}{K^{1-\alpha}}$$

when L is positive and finite, the numerator is also positive and finite. When K $\rightarrow$0, the denominator also approaches zero, so the entire expression is approaching infinity (Inada condition#1). When K $\rightarrow \rightarrow \propto$, the denominator approaches infinity, so the entire expression approaches zero (Inada condition #2).

b) Let $\hat{A} = A^{\frac{1}{1-\alpha}}$, then $Y = K^{\alpha}(\hat{A}L)^{1-\alpha}$

c) $w_t = \frac{\partial F}{\partial L} = A(1-\alpha)K_t^{\alpha}L_t^{-\alpha} = A(1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}$

d) $R_t = \frac{\partial F}{\partial K} = A(\alpha)K_t^{\alpha-1}L_t^{1-\alpha} = A(\alpha)\left(\frac{L_t}{K_t}\right)^{1-\alpha}$

e) This model implies that wages are increasing in the capital/labor ratio and that interest rates are decreasing in that ratio. Therefore, we should expect to see low wages and high interest rates in poorer countries.

f) $\left(\frac{w_t L_t}{Y_t}\right) = \frac{A(1-\alpha)K_t^{\alpha}L_t^{-\alpha}L_t}{AK_t^{\alpha}L_t^{1-\alpha}} = 1-\alpha$

g) $\left(\frac{R_t K_t}{Y_t}\right) = \frac{A(\alpha)K_t^{\alpha-1}L_t^{1-\alpha}K_t}{AK_t^{\alpha}L_t^{1-\alpha}} = \alpha$

2.

a) If a < w
   (i) Profit-maximizing output Y is zero
   (ii) Profit-maximizing labor demand L is zero
   (iii) Total Profit is zero

   If a = w

(i) Any level of output is Profit-maximizing
(ii) Any level of labor demand is Profit-maximizing
(iii) Total Profit is zero

If $a > w$

(i) Profit-maximizing output is infinity. It's actually not right to say this, since infinity isn't a real number. A better way of putting this is that there does not exist a profit-maximizing level of output, because any arbitrarily large amount of profit can be made. The same language applies to labor demand and total output.
(ii) Profit-maximizing labor demand is infinity
(iii) Total Profit is infinity

b) As long as there is a positive supply of labor, markets will clear only if firms demand that level of labor. The market-clearing wage is a. At that wage, firms are indifferent as to the amount of output they sell and the amount of labor they buy. Firm profits at that wage are zero.
c) For any (finite) wage, profit maximizing output, labor demand, and profit will be infinity.