More test practice!

Math 425a, Fall 2015

1. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and periodic, i.e. $\exists c \in \mathbb{R}$ such that $f(x + c) = f(x)$ for all $x \in \mathbb{R}$. Show f is uniformly continuous on $\mathbb{R}$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous, and suppose $E \subseteq \mathbb{R}$ is bounded. Show $f(E)$ is bounded.

3. Show that two sets A and B are separated in X if and only if there exists a continuous function $f : X \to \mathbb{R}$ such that $f(x) = 0$ for $x \in A$ and $f(x) = 1$ for $x \in B$.

4. Find an example of two metric spaces X and Y and a continuous bijection f between them such that f^{-1} is not continuous.

5. Rudin, ch. 3, #14.
6. Rudin, ch. 4, #8.
7. Rudin, ch. 4, #20.
8. Rudin, ch. 4, #21.

9. If $s_{n+1} = \sqrt{1 + s_n}$, $s_1 = 1$, does s_n converge? If so, find its limit.

10. Show that if a real number has a repeating decimal expansion, then it is rational.
True or false? Furnish a PROOF if it is true, and a COUNTEREXAMPLE if it is false.

11. Suppose X is a metric space. Then $f : X \to X$ given by $f(x) = x$ is uniformly continuous.

12. If $f : \mathbb{R} \to X$ is continuous and $n \in \mathbb{Z}$, then $f([-n, n])$ is compact in X.

13. $f : A \to \mathbb{R}$ given by $f(x) = \frac{1}{x}$ is uniformly continuous for:
a) $A = (0, 1)$
b) $A = (1, \infty)$
c) $A = (0, \infty)$

14. If $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous, then there exists an a in $\mathbb{R}$ such that $f(a) = \sup_{x \in \mathbb{R}} f(x)$.

15. Let S be infinite. Then S is the range of a convergent sequence in X if and only if S has a unique limit point in X.

16. If there exists an $L > 0$ such that $d(f(x), f(y)) < L \cdot d(x, y)$ for all x, y, then f is uniformly continuous.

17. If $\sum c_n z^n$ converges for $z = 1 + 3i$, then it also converges for $z = 2 + 2i$.

18. If $\lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$ exists, then it is equal to the radius of convergence of $\sum c_n z^n$.

19. If $\liminf s_n = \alpha$, then $\forall \epsilon > 0$, $\exists n$ such that $\alpha < s_n < \alpha + \epsilon$.

20. A sequence in a metric space is convergent if and only if it is Cauchy.