Econ 537
2017-S

Suggested Solution _ Homework 1

Summary:

Productivity: $\Theta \sim unif(1, 2)$
Agent's Outside Option: $\mu(\theta) = \frac{4}{3}\theta$
Principal's Revenue: 2θ
Profit: $\pi = 2\theta - w$

1. If the agent productivity is Θ, and the offer is w, what is the condition that the offer is accepted? (Write an inequality in Θ and w)

Accepted when $w \geq \frac{4}{3}\theta$ ……(1)

2. If the principal offers the wage w= $\frac{4}{3}$, argue that the offer is accepted with probability 0.

When $w = \frac{4}{3}$,
for any $\theta \in [1,2]$, $w \leq \frac{4}{3}\theta$.
Thus, $\Pr\left(w \geq \frac{4}{3}\theta\right) = \Pr\left(w > \frac{4}{3}\theta\right) + \Pr\left(w = \frac{4}{3}\theta\right) = 0$

3. If the principal offers the wage w $= \frac{8}{3}$, argue that the offer is accepted with probability 1.
When $w = \frac{8}{3}$,
for any $\theta \in [1,2]$, $w > \frac{8}{3}\theta$.
Thus, $\Pr(w \geq \frac{8}{3}\theta) = 1$

4. Given w, $\frac{4}{3} \leq$ w $\leq \frac{8}{3}$, what is the range of θ so that the offer is accepted?

For some given wage offer, w $\in [\mathbf{1}, \mathbf{2}]$,
since inequality (1) must hold for an agent to accept the offer,
the condition is $\theta \leq \frac{3}{4}w$,
i.e. cutoff productivity, $\theta_0 = \frac{3}{4}w$
Thus, offer w accepting range of θ is [1, $\frac{3}{4}w$]

5. Given w, $\frac{4}{3} \leq$ w $\leq \frac{8}{3}$, what is the probability that the offer is accepted?

$$\Pr\left(1 \leq \theta \leq \frac{3}{4}w\right) = \frac{3}{4}w - 1$$

*6. Given w, $\frac{4}{3} \leq$ w $\leq \frac{8}{3}$, and assume that the offer is accepted, what is the expected profit of the principal?

Conditional on some wage offer, w, is accepted by an agent,

$$\mathrm{E}\left(\pi \ \mid \ 1 \leq \theta \leq \frac{3}{4}w\right)$$

$$= \mathrm{E}\left(2\theta - \mathrm{w} \ \mid \ 1 \leq \theta \leq \frac{3}{4}w\right)$$

$$= \int_1^{\frac{3}{4}w} 2\theta\left(\frac{1}{\frac{3}{4}w - 1}\right)d\theta - \mathrm{w}$$

$$= \left(\frac{1}{\frac{3}{4}w - 1}\right)[\![\theta^2]\!]_1^{\frac{3}{4}w} - w$$

$$= 1 - \frac{1}{4}w$$

*7. The expected profit of the principal (whether or not the offer is accepted or not) is the probability of acceptance (answer from question 5) times the profit after the offer is accepted (answer from equation 6). Write down the expected profit as a function of w.

$$\text{(Unconditional) Expected Profit(w)} = \mathrm{E}\left(\pi \ \middle|\ 1 \le \theta \le \frac{3}{4}w\right)\mathrm{Pr}\left(1 \le \theta \le \frac{3}{4}w\right)$$

$$= \left(1 - \frac{1}{4}w\right)\left(\frac{3}{4}w - 1\right) \ldots\ldots\ldots\ldots (2)$$

* 8. Assume that the first order condition is sufficient for the optimal solution, find out the optimal wage offer by maximizing the function in question 7.

From function presented in (2)

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$$0 = -\frac{1}{4}\left(\frac{3}{4}w - 1\right) + \frac{3}{4}\left(1 - \frac{1}{4}w\right)$$

$$\rightarrow w^* = \frac{8}{3}$$