Spring 2017 USC Yilmaz Kocer **NAME:**
ECON 504 Game Theory **ID#:**

MIDTERM EXAM II

You have <u>100 minutes</u>. Total=120 points, 20 pts BONUS. Show all your work! GOOD LUCK !!

[1] (20pts) **<u>(Chapter 8 problem 6)</u> Investment in the Future**: Consider two firms that play a simultaneous Cournot competition game with demand $\mathbf{p = 100 - q}$ and costs for each firm given by $\mathbf{c_i(q_i) = 10q_i}$ for $\mathbf{i \in \{1, 2\}}$. Before the two firms play the Cournot game firm 1 can invest in cost reduction. If it invests the costs of firm 1 will drop to $\mathbf{c_1(q_1) = 5q_1}$. The cost of investment is $\mathbf{F > 0}$. Firm 2 does not have this investment opportunity and Firm 1's investment decision is commonly observed before the Cournot competition.

a) (10pts) Find the critical value $\mathbf{F^*}$ below which (that is, for all $\mathbf{F < F^*}$) the unique subgame-perfect equilibrium involves firm 1 investing.

b) (10pts) Assume that $\mathbf{F > F^*}$. Find a Nash equilibrium of the game that is not subgame perfect.

[2] (25pts) (Chapter 10 Problem 7) Diluted Happiness: Consider a relationship between a bartender and a customer. The bartender serves bourbon to the customer and chooses $\mathbf{x} \in [0, 1]$, which is the proportion of bourbon in the drink served, while $\mathbf{1-x}$ is the proportion of water. The cost of supplying such a drink is $\mathbf{cx}$, where $\mathbf{c > 0}$. The customer, without knowing $\mathbf{x}$, decides on whether or not to buy the drink at the market price $\mathbf{p}$ (*p is a fixed price, <u>not</u> determined by the players*). If he buys the drink his payoff is $\mathbf{vx - p}$, and the bartender's payoff is $\mathbf{p - cx}$. Assume that $\mathbf{v > c}$ and all payoffs are common knowledge. If the customer does not buy the drink he gets $\mathbf{0}$ and the bartender gets $\mathbf{-cx}$. Because the customer has some experience, once the drink is bought and he tastes it, he learns the value of $\mathbf{x}$, but this is only after he pays for the drink. (HINT: Think of this as the simultaneous move stage game).

a) (5pts) Find all the (pure and mixed) Nash equilibria of this game.

b) (10pts) Now assume that the customer is visiting town for 10 days, and this "bar game" will be played on each of the 10 evenings that the customer is in town. Assume that each player tries to maximize the (non-discounted) sum of his stage payoffs. Find all the subgame-perfect equilibria of this game.

c) (10pts) Now assume that the customer is a local, and the players perceive the game as repeated infinitely many times. Assume that each player tries to maximize the discounted sum of his stage payoffs, with $\boldsymbol{\delta} \in \mathbf{(0,1)}$ What is the range of prices $\mathbf{p}$ (expressed in the parameters of the problem) for which there exists a subgame perfect equilibrium in which every day the bartender chooses $\mathbf{x = 1}$ and the customer buys at the price $\mathbf{p}$?

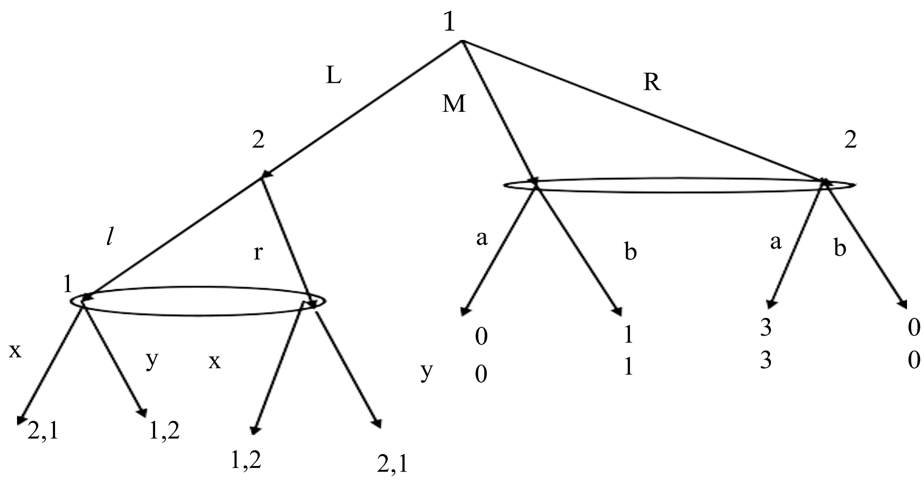
[3] (25pts) A and B are forming a firm, and the value of their relationship (the firm) depends on their efforts $\mathbf{a \geq 0}$ and $\mathbf{b \geq 0}$, respectively. Suppose **A**'s utility from the relationship is $\mathbf{u_A(a,b) = b^2 + b - ab}$, and symmetrically B's utility is $\mathbf{u_B(a,b) = a^2 + a - ab}$.

a) (10pts) Compute the partners' best-response functions and find the Nash equilibrium of this game. Is the Nash equilibrium Pareto optimal?

b) (10pts) Now suppose that the partners interact over time, which we model with the infinitely repeated version of the game. Let $\boldsymbol{\delta \in (0, 1)}$ denote the discount factor of the players. Under what conditions can the partners sustain some positive effort level $\mathbf{k = a = b}$ over time? (**Hint:** consider a grim trigger strategy equilibrium where each player exerts **k** units of effort unless cooperation is broken, in which case returns to the NE effort.)

b) (5pts) How does the *maximum* sustainable effort depend on the partners' patience $\boldsymbol{\delta}$?

[4] (25pts)



a) (15pts) Find a (pure or mixed!) SPNE of the following game, carefully describing the equilibrium strategies of the 2 players, and showing it is indeed SPNE

b) (10pts) Find another (pure or mixed!) SPNE and show it is indeed SPNE.

[5] (25pts) Consider the infinitely repeated prisoners' dilemma with stage payoffs given. Assume that both players discount payoffs by $\boldsymbol{\delta}$. The "**tit-for-tat**" strategy is formulated as follows.

	Cooperate	**D**efect
Cooperate	1,1	-1,2
Defect	2,-1	0,0

Start out by playing C. After that, choose the action that the other player used in the previous period.

Is the strategy profile where *both* players use tit-for-tat constitute a SPNE for some $\boldsymbol{\delta} \in \mathbf{(0, 1)}$?

(HINT: Use the one shot deviation principle for *each possible history*. There are 4 different types of histories that you need to check the deviations; ones that have (C,C) as last period's play, (C,D) as last period's play.. etc.)