MATH 425a ASSIGNMENT 3
FALL 2015 Prof. Alexander
Due Wednesday September 23.

Rudin Chapter 2 #5, 7b, 8, 9ac, 11 (omit d_5) plus the problems (I) - (V) below:

(I) Suppose $A_1, \ldots A_n$ are subsets of a metric space.

(a) If x is not a limit point of any of the sets A_i (that is, $x \notin \cup_{i=1}^n A_i'$), show that x is not a limit point of $\cup_{i=1}^n A_i$.

(b) Let $B = \cup_{i=1}^n A_i$. Restate (a) as a statement about the relation between B' and $\cup_{i=1}^n A_i'$. Justify, at least informally.

(c) Show that $\overline{B} \subset \cup_{i=1}^n \overline{A_i}$.

(II) Suppose x, y are two distinct points of a metric space. Show that there exist radii $r, s > 0$ such that the neighborhoods $N_r(x)$ and $N_s(y)$ are disjoint. How do the values r, s and $d(x, y)$ have to be related, in order for your proof to work?

(III) Suppose X is a metric space, and every point of some set $F \subset X$ is an isolated point of F. Show that you can choose a neighborhood $N_{r_x}(x)$ of each $x \in F$ such that none of the $N_{r_x}(x)$'s overlap, that is, $N_{r_x}(x) \cap N_{r_y}(y) = \phi$ for all $x, y \in F$.

Here r_x is some radius that may be different for different x's.

(IV) Suppose $A \subset F$ and F is closed. Show that $\overline{A} \subset F$.

(V)(a) Give an example of nonempty closed sets $E_1 \supset E_2 \supset \ldots$ such that $\cap_{n \geq 1} E_n = \emptyset$.

(b) Give an example of nonempty closed sets $E_1 \subset E_2 \subset \ldots$ such that $\cup_{n \geq 1} E_n$ is open.

HINTS:

(5) First construct a set with just one limit point, say 0.

(9)(a) If $x \in E^\circ$, then by definition, $N_r(x) \subset E$ for some r. Show that all points $y \in N_r(x)$ are interior points of E. See Theorem 2.19.

(11) d_2 is the tricky one. First prove this general fact about nonnegative numbers: $\sqrt{b+c} \leq \sqrt{b} + \sqrt{c}$. Then relate this to the triangle inequality.

(II) Draw a picture of two points x, y and their disjoint neighborhoods, in the plane. Use this to help you understand how r, s and $d(x, y)$ have to be related.

(III) This is a more difficult one. Each x being isolated means each x has a neighborhood containing no other points of x. But unless we do more, these neighborhoods might overlap. How can you specify the radii of these balls, so that they *don't* overlap? See problem (II).

(V) You can use (possibly unbounded) intervals in $\mathbb{R}$.