Answers to HW 12

1. (a) When the total valuations of all others is less than \$104.50, but exceeds it after we add the consumer's reported value, then the consumer pays the tax amount which is just enough to add up to \$104.50. In all other cases, the consumer pays no tax.

(b) A consumer is pivotal if the project is adopted only when his valuation is added to the total. A consumer with value \$2 is a pivotal consumer because the sum of all others' value is 26*2+51=103 and 103+2>104.5. A consumer with value \$1 is also pivotal because the sum total of all others' value is 27*2+50=104, and 104+1>104.5. A consumer with value 0 is not pivotal.

(c) A consumer with value \$2 pays tax 1.5, while a consumer with value pays tax \$0.5. The total tax paid is then 27*1.5+51*0.5= 66.0 which is less than the total cost needed to finance the project.

2. (a) In the subgame perfect equilibrium, player one offers

$$\delta - \delta^2 + \delta^3 - \delta^4 + \delta^5 - \delta^6 + \delta^7 = \frac{\delta + \delta^8}{1 + \delta}$$

to player two which is accepted right away, and player two gets

$$1 - \frac{\delta(1 + \delta^7)}{1 + \delta} = \frac{1 - \delta^8}{1 + \delta}.$$

(b) More generally, for even n, in equilibrium, player one offers $\frac{\delta + \delta^n}{1+\delta}$ to player two, which is accepted right away, and player one gets $\frac{1-\delta^n}{1+\delta}$.

(c) As $n \to \infty$, the offers converges to $\frac{\delta}{1+\delta}$ which is the solution of the Rubinstein model with infinite horizon.

3. (a) The payoff from following the Grim strategy without deviation for each firm is 15 each period. This yields total discounted sum of payoffs $\frac{15}{1-\delta}$. If a firm deviates at period 1, then it gets 30 in the first period, and 5 for the rest of the periods. The total discounted sum of payoffs is

$$30 + \frac{5\delta}{1 - \delta}.$$

Deviation is not profitable if

$$30 + \frac{5\delta}{1 - \delta} < \frac{15}{1 - \delta}.$$

(b) Solving the inequality, we get

$$30 - 25\delta < 15,$$

or $\delta > \frac{15}{25} = 0.6$.

If this is true, then the Grim strategy is a Nash equilibrium.

(c) It is a perfect Nash equilibrium because, the same computation applies to deviation at any period shows that there is no profit from deviation at any period. The punishment strategy itself is a Nash equilibrium. Hence it is a subgame perfect Nash equilibrium.

4. If player one follows the prescribed play, the payoff is $30\delta + \frac{15}{1-\delta}\delta^2$.

If player one deviates in the first period, the play becomes (L,L) forever. The payoff from deviation is $\frac{5}{1-\delta}$. Equilibrium condition requires that

$$30\delta + \frac{15}{1-\delta}\delta^2 \geq \frac{5}{1-\delta}.$$

This condition becomes $30\delta - 29\delta^2 - 5 \geq 0$, or $\delta \geq 0.183\,503$. If player one deviates in the second period, the play becomes $(H, L), (H, H)$, followed by (L, L) forever. The payoff from deviation is $15\delta + \frac{5}{1-\delta}\delta^2$. Equilibrium requires

$$30\delta + \frac{15}{1-\delta}\delta^2 \geq 15\delta + \frac{5}{1-\delta}\delta^2.$$

It becomes $15 - 5\delta \geq 0$, which is always true. If player one deviates in the third period, the play becomes $(H, L), (L, H), (L, H)$, followed by (L, L) forever. The payoff from deviation is $30\delta + 30\delta^2 + \frac{5}{1-\delta}\delta^3$. Equilibrium condition requires

$$30\delta + \frac{15}{1-\delta}\delta^2 \geq 30\delta + 30\delta^2 + \frac{5}{1-\delta}\delta^3.$$

It becomes $5\delta\,(5\delta - 3) \geq 0$, or $\delta \geq 0.6$. Deviation in any period after the third leads to the same result.

Similarly, the payoff of player two from the prescribed candidate equilibrium play is $30 + \frac{15}{1-\delta}\delta^2$. If player two deviates in the first period, the play is (H, H) followed by (L, L) forever. The payoff is $15+\frac{5\delta}{1-\delta}$. Equilibrium condition requires that

$$30 + \frac{15}{1-\delta}\delta^2 \geq 15 + \frac{5\delta}{1-\delta}.$$

The above inequality holds for all δ. If player two deviates in the second period, the play is $(H, L), (L, L)$ followed by (L, L) forever. The payoff from deviation is $30 + \frac{5}{1-\delta}\delta^2$. Equilibrium condition requires

$$30 + \frac{15}{1-\delta}\delta^2 \geq 30 + \frac{5}{1-\delta}\delta.$$

It becomes $5\delta\,(3\delta - 1) \geq 0$, or $\delta \geq \frac{1}{3}$. If player two deviates in the third period or later, the computation is similar to that of the player, and we have the condition $\delta \geq 0.6$. In conclusion, the condition for equilibrium play is $\delta \geq 0.6$.