Yilmaz KOCER

ECON 504 MIDTERM 2 ANSWER KEY

[1] See solution manual.

[2] (a) (B)artender chooses x, (C)ustomer chooses $y=0$ = not buy or 1 = buy ; with

$$u_B(x,0) = -cx \quad u_B(x,1) = p - cx \quad u_C(x,0) = 0 \quad u_C(x,1) = vx - p$$

Note that for B $x=0$ str. dominates any $x \in (0,1]$, $u_B(0,y) > u_B(x,y)$ $\forall x \in (0,1]$ for $y = 0$ or 1 (buy or not buy). Given $x=0$, C wouldn't buy $y=0$. The unique IESDS outcome hence unique NE (no other NE pure or mixed)

(b) For any finite repetitions, as there is unique NE in the stage game and there IS an explicit final game ; the game unravels : no threats.. no carrots / sticks.. punishments are possible : The unique SPNE is the $(x,y) = (0,0)$ play after any history ; and it is the eqm outcome.

(c). Consider the grim-trigger strategies : B : $x=1$ at the first period and after any history with all $(x,y) = (1,1)$ being played all along. Otherwise $x=0$. And symmetrically for C ; $y=1$ at the first period & after all histories composed of all $(1,1) = (x,y)$. Otherwise $y=0$.

Check that these constitute SPNE ;

- After histories when cooperation is broken, play of $(0,0)$ forever after any history is surely NE.
- For a history with cooperation so far (all $(x,y) = (1,1)$ play so far) ;

(B)artender checks the one-shot deviation

$$\overbrace{(p - cx)\frac{1}{1-\delta}}^{x=1} \geq \overbrace{p + \delta \frac{1}{1-\delta} \cdot 0}^{\text{deviate to } x=0}$$

all $(0,0)$ afterwards.

$$\Rightarrow \quad p - cx \geq (1-\delta)p \qquad \boxed{p \geq \frac{cx}{\delta} = \frac{c}{\delta}}$$

(Note that we checked deviation to $x=0$, as it is the best deviation; any deviation will trigger $(x,y) = (0,0)$ forever anyways ; the best is to be the most beneficial one shot deviation $x=0$.)

(C) deviation ;

$$\overbrace{(vx - p)\frac{1}{1-\delta}}^{y=1} \geq \overbrace{0 + \delta\frac{1}{1-\delta} \cdot 0}^{y=0} \quad \Rightarrow \quad \boxed{vx = v \geq p}$$

Hence for $\boxed{v \geq p \geq \frac{c}{\delta}}$ $\exists$ SPNE where the eqm path is $(x,y) = (1,1)$ everyday forever

[3] (a) $u_A(a,b) = b^2+b-ab$, maximizing it over a; FOC $\Rightarrow$ $-b \le 0$ means that $u_A(\cdot)$ is decreasing in "a" for all b; hence $a=0$ maximizes u_A for all b.

$BR_A(b) = 0 \quad \forall b$ (0 is dominant for A)

Similarly for B; hence unique NE = $(a=0, b=0)$

Is $u_A(0,0), u_B(0,0) = (0,0)$ pareto optimal? Is there (a,b) with $u_A(a,b) > 0$ $u_B(a,b) > 0$; take $a=b=1$ $u_A = 1 = u_B > 0$

Hence, not P.O.

(b) Consider the grim trigger strategies with effort level $= k$, unless somebody has cheated and not supplied k effort in the past, in which case effort is set to 0.

By one-shot deviation principle;

$$\underbrace{(k^2+k-k\cdot k)\frac{1}{1-\delta}}_{\text{set } a=k} \geq \underbrace{k^2+k-0\cdot k + \delta\cdot 0 + \delta^2 0 + \ldots}_{\text{deviate to } a=0}$$

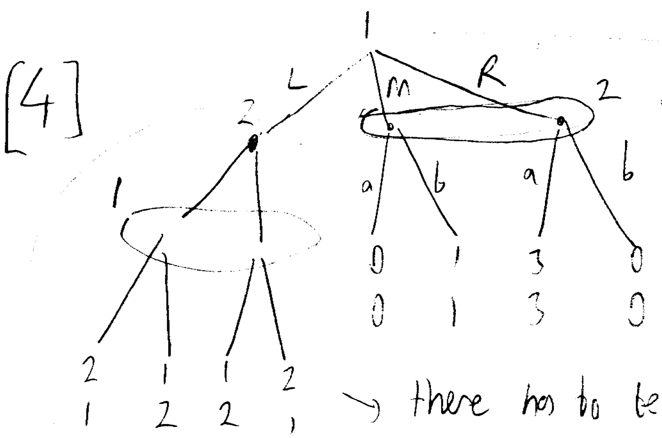
$$\Rightarrow \quad k/_{1-\delta} \geq k^2+k \qquad \cancel{k}\left(\frac{1}{1-\delta}-1\right) \geq k^{\cancel{2}} \qquad \frac{1-1+\delta}{1-\delta} \geq k \qquad \boxed{\frac{\delta}{1-\delta} \geq k}$$

If this holds, effort of k by each player is supplied on the eqm path each period, forever.

(c) $\delta \uparrow$ means $k \uparrow$ in $\frac{\delta}{1-\delta} = k$ hence the more patient the players are, the higher the maximum effort level they could maintain in an eqm.

---

[4]



strategy = (, ; ,)

pl 1 initial node

pl. 1 second info set

pl. 2's decision node on the left subgame

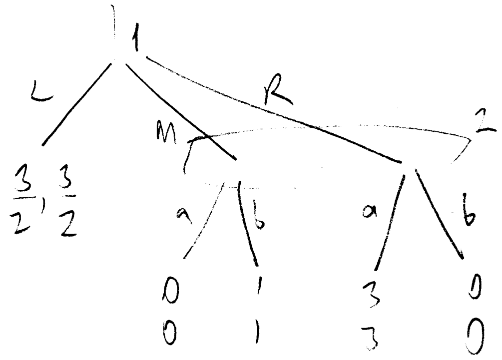
pl 2's info set on the right

→ there has to be Nash eqm in this single subgame (other than the whole game itself.)

Hence player 1 → $\frac{1}{2}x + \frac{1}{2}y$, player 2 → $\frac{1}{2}l + \frac{1}{2}r$ in this subgame. (unique NE)

And each player gets "$\frac{3}{2}$" payoff in this subgame.

	1/2	1/2
1/2	2,1	1,2
1/2	1,2	2,1



→ this is not a subgame, however;

	a	b
M	0,0	1,1
R	3,3	0,0

Player 1 R, Player 2 a gives $(3,3) > (\frac{3}{2},$

Hence (Player 1: $R, \frac{1}{2}x + \frac{1}{2}y$) Player 2: $(\frac{1}{2}l + \frac{1}{2}r, a)$) is SPNE

Given player 1's strategy, player 2 is definitely maximising, in the big game.

Given pl. 2 strategy, player 1 //

Also, the only subgame has NE play with $(\frac{1}{2}x + \frac{1}{2}y, \frac{1}{2}l + \frac{1}{2}r)$ ✓

<u>Second SPNE</u>:

(Player 1: $(L, \frac{1}{2}x + \frac{1}{2}y)$ Player 2: $(\frac{1}{2}l + \frac{1}{2}r, b)$))

Notice that, given player 2 plays b, indeed player 1 plays L $(\frac{3}{2} > 1)$

Most importantly; player 2 playing b is sequentially rational if he assumes player 1 would have played M if he had to play M or R. (It is not a seperate subgame.)

[redacted]

[5] With the one-shot deviation principle, we need to check at each history, whether it would pay to deviate today only, given the opponent's strategy (tit-for-tat) and given you're going to play according to your original strategy tomorrow onwards (tit-for-tat)

There are 4 types of histories as the play today depends on yesterday's play being (C,C), (C,D), (D,C) or (D,D).

(1)- At a history where last period play is (C, C) $\Rightarrow$

$$\underset{\text{play } C}{\frac{1}{1-\delta}} \geq \underset{\text{deviate} \Rightarrow \text{play } D}{2 + \underset{DC}{\delta(-1)} + \underset{CD}{\delta^2(2)} + \underset{DC}{\delta^3(-1)} + \dots}$$

$$\frac{1}{1-\delta} \geq 2(1+\delta^2+\delta^4+\dots) - \delta(1+\delta^2+\ \ )$$

$$\frac{1}{1-\delta} \geq \frac{2}{1-\delta^2} - \frac{\delta}{1-\delta^2} \Rightarrow 1+\delta \geq 2-\delta \qquad \boxed{\delta \geq \frac{1}{2}}$$

(2)- Last period play is (C, D) $\Rightarrow$

$$\underset{\text{play } D}{2+\delta(-1)+\delta^2(+2)+\dots} \geq \underset{\text{deviate} \Rightarrow \text{play } C}{\frac{1}{1-\delta}} \Rightarrow \boxed{\delta \leq \frac{1}{2}}$$

opposite ineq in (1)

(3)- Last period play is (D, C) $\Rightarrow$

$$\underset{\text{play } C}{-1+\delta 2+\delta^2(-1)+\dots} \geq \underset{\text{deviate} \Rightarrow \text{play } D}{0+\delta 0+\dots}$$

$$\frac{-1}{1-\delta^2} + \frac{2\delta}{1-\delta^2} \geq 0 \Rightarrow 2\delta - 1 \geq 0$$

$$\delta \geq \frac{1}{2}$$

(4)- Last period play is D, D $\Rightarrow$

$$\underset{\text{play } D}{0} \geq \underset{\text{deviate} \Rightarrow \text{play } C}{-1+\delta 2+\delta^2(-1)+\dots}$$

opposite ineq in (3) $\Rightarrow \boxed{\delta \leq 1/2}$

Hence tit-for-tat for both players is SPNE only when $\delta = \frac{1}{2}$ exactly.