MATH 425b MIDTERM EXAM 1
February 19, 2016
Prof. Alexander

Last Name: ______________________________

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Signature: ______________________________

Problem	Points	Score
1	23	
2	24	
3	28	
4	25	
Total	100	

Notes:

(1) Use the backs of the sheets if you need more room.

(2) If you can't do part (a) of a problem, you can assume it and do part (b).

(1)(23 points) Suppose $\{\phi_n, n \geq 1\}$ is an orthonormal system on $[a, b]$, and $f \sim \sum_{n=1}^{\infty} c_n \phi_n$, meaning the Fourier coefficients of f with respect to $\{\phi_n\}$ are the c_n. Let $\{a_n\}$ be any *other* sequence in $\mathbb{C}$ (that is, $a_n \neq c_n$ for at least one n.) Show that $\sum_{n=1}^{N} a_n \phi_n(x)$ does not converge in L^2 to $f(x)$, as $N \to \infty$. HINT: Use something from the proof of Theorem 8.11.

(2)(24 points) You may take the following as given: the function $f(x) = (1-x)^{-2/3}$ has Taylor series $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ and the coefficients $a_n = \frac{f^{(n)}(0)}{n!}$ satisfy $a_n \to 0$. For every x inside the radius of convergence, the sum is $f(x)$.

(a) Show that the radius of convergence is at least 1, that is, the series converges for $|x| < 1$.

(b) *Without calculating the coefficients* a_n, show that $\sum_n a_n$ diverges. HINT: Consider $x \nearrow 1$. There's very little to do if you apply the right theorem.

(3)(28 points) Let $\mathcal{F}$ be an equicontinuous family of functions from $[0, \infty)$ to $\mathbb{R}$, [and suppose $f(0) = 0$ for all $f \in \mathcal{F}$. *Added at exam.*] Show that there is a uniform linear bound for the functions in $\mathcal{F}$, that is, there exist $A, B > 0$ such that

$$|f(x)| \leq A + Bx \quad \text{for all } f \in \mathcal{F} \text{ and } x \in [0, \infty).$$

HINT: Let $\epsilon = 1$ and take δ from the definition of equicontinuity. Prove it first when x is one of the values $0, \delta, 2\delta, 3\delta, \ldots$

(4)(25 points) Let

$$C_B[0,1) = \{\text{all continuous functions } f : [0,1) \to \mathbb{R} \text{ with} \|f\|_\infty < \infty\}.$$

with distance given by the sup norm, $d(f,g) = \sup_{x\in(0,1]} |f(x) - g(x)|$, and let

$$\mathcal{A} = \{f \in C_B[0,1) : \lim_{x\to 1} f(x) = 0\}.$$

You may take as given that $\mathcal{A}$ is an algebra.

(a) Show that $\mathcal{A}$ separates points on $[0,1)$ and vanishes at no point of $[0,1)$.

(b) Show that $\mathcal{A}$ is not dense in $C_B[0,1)$. Explain why this does not violate the Stone-Weierstrass Theorem, 7.32.