Note 6 with homework assignment #6

We now consider the perfect Bayesian equilibrium when the cost for all types of sellers (or just one type of seller) is below \$5. In this discussion, we will allow mixed strategies, and show that there is only one equilibrium, in which the seller acquires information. (Again assuming that the acquisition of information is not observable by the buyer).

We want to show that It is not possible to have an equilibrium in which a percentage q%, q<1, of sellers acquire information, while others don't.

To show this, assume that a percentage q% of the sellers acquire information, and others don't. When the reported information is good, the price the buyer will pay is \$200.

When the sellers remain silent, with probability q/2 it is a bad furnace after the acquisition of information. With probability (1-q)/2, it is good with no acquisition. With probability (1-q)/2, it is bad with no acquisition of information. Therefore the total probability of silence is $\frac{q}{2} + 1 - q = 1 - \frac{q}{2}$. The conditional probability that it is good when the seller is silent is $\frac{\frac{1}{2}(1-q)}{1-\frac{q}{2}} = \frac{1-q}{2-q}$. Hence the buyer is willing to pay

$$200\frac{1-q}{2-q} + 180(1 - \frac{1-q}{2-q}) = \frac{380-200q}{2-q}.$$

This price is between 180 and 190 when q=1,0 respectively.

To compute the value of information, note that with information, the seller either gets 200 or (380-200q)/(2-q). Each has equal probability. Hence with information the expected price is

$$0.5 * 200 + \frac{190-100q}{2-q} = \frac{390-200q}{2-q}.$$

The difference of this amount with the price without information is

$$\frac{390-200q}{2-q} - \frac{380-200q}{2-q} = \frac{10}{2-q} \geq 5.$$

The value of information is therefore larger than or equal to \$5.

Hence if the cost of information is below \$5, it is optimal to acquire information. Hence we must have q=1.

This shows that in equilibrium, it must be true that all sellers acquire information when the cost is lower than \$5.

Now consider regime (2) in which there is mandatory disclosure, but the seller can decide whether to acquire information or not.

In this regime, if the seller acquires no information, the price he receives is 190. If the seller acquires information, he either reports it is good or it is bad (verifiable information), and on average also gets \$190.

The seller is not required to acquire information, and there is a cost of acquisition. In this case, no seller will acquire information.

Since the information has no social benefit, this is more efficient than the case with acquisition of information.

Under regime (3), there is mandatory acquisition of information and mandatory disclosure of information.

This is the worst possible outcome because there is no social benefit from the information, but the cost is incurred whether it is high or low.

## Information with social benefit

Assume that the information is about how the house was built. It affects the value of the house, but it also has a social value in the sense the buyer then knows how to fix the house properly.

The information may also has social value if it helps the sellers and buyers to match more efficiently.

We maintain the same information about the value of the good and bad house (200 and 180 respectively). However when the information is disclosed, it is worth \$5 whether the house is good or bad.

We maintain earlier assumptions about costs. Half of the sellers have high costs (\$8) in acquiring information, half of the sellers have low cost (\$4). Because the value of information is \$5, it is efficient if only low cost sellers gather information.

Under the voluntary regime (1), it is an equilibrium for all to gather information and then disclose it. To support this equilibrium, we assume an out-of-equilibrium belief of the buyer: when there is no report, the furnace is bad. Disclosing information has the benefit of receiving additional value due to social benefit.

After gathering information, the furnace is either good or bad equally likely. With the out-of-equilibrium belief, the seller can collect \$195 on average by disclosure, and only gets \$180 with no disclosure. Therefore there is always disclosure.

If there is no gathering, there is no information to report, and the seller only gets \$180. The value of information is \$15, but the cost of information is either \$4 or \$8. Hence it is optimal for all sellers acquire information.

Note that in equilibrium all sellers acquire and disclose information. Therefore, in equilibrium, there is always a report: it is either good or bad. When there is no report, it is out-of-equilibrium. This is why we say that the belief assumption we make is an out-of-equilibrium belief. What if the out-of-equilibrium belief is: when there is no report, it is equally likely that the furnace is either good or bad?

Under this assumption, there will be no disclosure when the seller acquires information and finds a bad furnace. Disclosing it, he gets only \$185, while no disclosure yields \$190. There will be no disclosure. Therefore the belief system does not support the candidate equilibrium as an equilibrium. It is possible to support the same equilibrium with a different out-of-equilibrium belief. Note that the out-of-equilibrium belief affects the value of information.

It is not an equilibrium for all sellers not to acquire information, and nothing to disclose. When buyers believe that no sellers gather information, buyers pay \$190 when there is silence. In this case, gathering information yields \$205 for the house when it is good, and \$190 for the house when it is bad (here it is optimal not to disclose bad furnace). The value of information is 197.5 – 190=7.5. sellers.

The cost of information is less than this for low cost sellers. Therefore low cost sellers find it profitable to deviate. Therefore, it is not an equilibrium.

It is not an equilibrium for low cost sellers to acquire information, and high cost sellers to remain uninformed.

To see this, consider a high cost seller and we want to determine the value of information for such a seller.

If a seller acquires information and finds a bad furnace, and discloses it, he gets $185. If he hides it, then he gets $186.67 (because he thinks the furnace is bad with probability 2/3). Therefore a seller would not disclose information when he finds a bad furnace. Note that this can change with a difference belief of the buyer. We also know that when there is no disclosure, the furnace need not be a bad one.

If there is no acquisition of information, there is no report, and the buyer thinks that the house is worth $186.67 as commented earlier.

If there is acquisition of information, the seller gets $205 when he finds a good furnace and discloses it. When he finds a bad one, he will hide it, and gets $186.67. Each has equal probability, hence he gets $195.84 on average.

The value of information is $195.84 – 186.67= 9.17 which is higher than the cost of acquisition $8. Therefore a high cost seller will acquire information, violating the equilibrium condition. This proves that it is not an equilibrium for high cost sellers to remain uninformed.

It is not an equilibrium either for all sellers to acquire information, and the information is disclosed only when it is good. Note that we have commented earlier, disclosure of information is always better in this case, because the buyer can infer that no disclosure means bad furnace. Disclosure has the added benefit of the $5 (social value)

The only equilibrium in this regime is for all to acquire and disclose. However this is not efficient.

In conclusion, in this game there is only one perfect Bayesian equilibrium and the equilibrium outcome is not efficient.

Under regime (2) of mandatory disclosure, the high cost seller does not acquire information. Acquiring information, the seller gets $205 with probability 0.5, and $185 with probability 0.5. On average he gets $195.

If he acquires no information, he gets $190 (no social value provided). The value of information is $5.

Therefore, a high cost seller will not acquire information, but low cost seller do.

Under regime (2), the equilibrium outcome is an efficient outcome.

Under regime (3) of mandatory acquisition and disclosure, the outcome is not efficient because high cost sellers incur costs.

When buyers decide whether to acquire information, additional issues arise. If the buyer finds good information and disclose it, the seller may raise the price, and the buyer will have less incentive to acquire information.

HW 6

Due Oct 20

1. Under regime (2), we have voluntary acquisition of information but there is mandatory disclosure of information after acquisition. The information has no social value, and is verifiable. Assume that the cost of acquiring information is \$8 for high cost sellers, and \$4 for low cost sellers. The probability of a low cost seller is p (and that of a high cost seller is 1-p). The house is worth \$200 when it has a good furnace, and \$180 when it has a bad furnace. A seller does not know whether the furnace is good (neither does the buyer). The probability of finding a good furnace when the seller incurs the cost and acquires information is q. This acquisition of information is not observable by the buyer. Assume that the buyer believes that only low cost sellers acquire information.

(a) What is the price of the house when the seller acquires information?

(b) What is the price of the house when the seller acquires no information?

(c) What is the perfect equilibrium outcome?

(d) Is the equilibrium outcome efficient?

2. Now back to the regine (1) in the problem above. Now assume that the information revealed by the seller has social value and it is worth \$6 whether the furnace is good or bad. Furthermore, assume that the probability of a low cost seller is 0.6. All other assumptions in the notes are maintained (including the probability 0.5 of finding a good or bad furnace).

(a) Check whether it is a perfect Bayesian equilibrium for all sellers to acquire information and disclose it. Show your computations.

(b) Check whether it is a perfect Bayesian equilibrium for low cost sellers to acquire information, and high cost sellers to remain uninformed, and the seller always discloses the information when he finds a bad furnance.

(c) Check whether it is a perfect Bayesian equilibrium for low cost sellers to acquire information, and high cost sellers to remain uninformed, and the seller hides the information when he finds a bad furnance.

(d) Check whether it is a perfect Bayesian equilibrium for low cost sellers to acquire information, and high cost sellers to remain uninformed, and the seller discloses the information with probability $\pi$, $0 < \pi < 1$, when he finds a bad furnance.

3. In the problem 2 above, now consider the regime (2).

(a) What is the value of acquiring information?

(b) Would a low cost seller acquires information? Would a high cost seller acquires information?

(c) In the perfect Bayesian equilibrium outcome efficient?