Preface

The analysis of auctions as games of incomplete information originates in the work of William Vickrey (1961). In this book I discuss the theory of auctions in this tradition. The goal is to give an account of developments in the field in the 40 years since Vickrey's pioneering paper.

I do not attempt to provide a comprehensive survey of auction theory. The field has burgeoned, especially in the past couple of decades, and a comprehensive survey would be nearly impossible. The Econ Lit database alone has more than a thousand entries with the word "auction" or "auctions" in their titles, and about one-half of these papers are theoretical. Instead, I have opted to concentrate on selected themes that I consider to be central to the theory. I adopt the point of view that a detailed consideration of a few basic models is more fruitful than a perfunctory discussion of a large number of variations. I can only hope that my choice of themes is not too arbitrary.

The models that are considered are discussed in some detail, and, with a few minor exceptions, complete proofs of all propositions are provided. It is my contention that the strengths and weaknesses of the theory can be appreciated only by examining the inner workings of the propositions with some care.

Game Theory. The theory of games, especially concerning games of incomplete information, constitutes the basic apparatus of the book. Most modern graduate texts in microeconomics now have a substantial emphasis on game theory (see Kreps, 1990; Mas-Colell, Whinston, and Green, 1995); any of these can acquaint the reader with the basic notions needed to follow the material in this book. More advanced texts on game theory include Fudenberg and Tirole (1991) and Osborne and Rubinstein (1994). To assist the reader, Appendix F contains the basic game theoretic definitions used in this book but is by no means an adequate substitute for consulting one of the texts mentioned.

Appendices. Auxiliary matters are relegated to a series of appendices. In particular, Appendices A through D contain some essential material concerning continuous probability distributions.

Notational Conventions. The notation is more or less standard. Real-valued random variables are denoted by uppercase letters—say, X or Y—and their realizations by the corresponding lowercase letters, x or y. Sets are denoted by scripted letters, $\mathcal{X}$ or $\mathcal{Y}$. Thus, for instance, the random variable X takes on values $x \in \mathcal{X}$.

Boldface characters denote vectors, so $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ is an N-vector whose ith component is x_i. If $\mathbf{x}$ and $\mathbf{y}$ are N-vectors, then $\mathbf{x} \geq \mathbf{y}$ denotes that for

all i, $x_i \geq y_i$, and $\mathbf{x} \gg \mathbf{y}$ denotes that for all i, $x_i > y_i$. The vector $\mathbf{x}_{-i}$ is obtained from $\mathbf{x}$ by omitting the ith component—that is, $\mathbf{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$ and we identify $(x_i, \mathbf{x}_{-i})$ with $\mathbf{x}$.

Vector valued random variables are denoted by bold uppercase letters, $\mathbf{X}$ or $\mathbf{Y}$.

The symbol ■ denotes the end of a proof, and ▲ denotes the end of an example.

Definitions of recurring symbols—for example, the symbol m is used throughout the book to denote a monetary payment—can be found via the index.

References. At the end of each chapter is a section titled "Chapter Notes." This contains bibliographic references to the works on which the material in the chapter is based. So as to not interrupt the flow, the body of the chapter itself contains no bibliographic references.

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Pronouns. Although there is a single author, the remainder of the book uses the plural "we"—as in "we see that …"—rather than the singular "I." This is not to indicate any royal lineage but only that the book is intended to be a conversation between the author and the reader.

Notes on the 2nd Edition

The basic structure of the book remains unchanged. The new material mainly concerns developments in the theory of auctions of multiple nonidentical objects (in the form of a new Chapter 17). A distinguishing feature of the second edition is that I have added some problems to accompany most chapters. These are of varying levels of difficulty: Some ask the reader to verify aspects of the theory in specific examples, and others are meant to extend the material in the chapter.

Without naming them, I thank the many people who sent comments, corrections, and suggestions. I am also grateful to Matthew Wampler-Doty, Vikram Kumar, and especially Alexey Kushnir for assistance. As was the case for the first edition, this project would not have been possible without Scott Bentley's encouragement and gentle nudging.