MIDTERM EXAM I

You have <u>100 minutes</u>. Total=120 points, 20 pts BONUS. Show all your work! GOOD LUCK !!

[1] (30pts) (Diminishing Returns to Information) Suppose the state of nature **x** can either be High (**H**) or Low (**L**); with **%60** prior probability of being **H**. You are trying to guess **x**; you get $1 if you guess correctly and $0 if not. Suppose there is a signal (**s**) available for purchase that advices you on the state and is correct with **%80** probability.

a) (5pts) How much is one signal worth to you?

b) (10pts) If you can get 2 iid draws of the signal and decide accordingly, what is this two-signals combo worth to you?

c) (10pts) Solve (b) for 3 iid signals.

d) (5pts) Draw your “demand” for signals; what you would at most pay for the marginal signal. Is it downward sloping?

[2] (25pts) Heather (**X**) and Yilmaz (**Y**) have a new baby. If each parent invests in the child $\mathbf{x \geq 0}$ and $\mathbf{y \geq 0}$ amounts respectively, the child gets $\mathbf{x + y}$ units of care. Each parent has an ideal amount for the total parental investment in the baby; Yilmaz would want the baby to have total care (investment) at the amount $\mathbf{a > 0}$. As the mother, Heather wants the baby to have total care $\mathbf{a + \varepsilon}$ where $\mathbf{\varepsilon > 0}$. The more the total care baby receives differs from their ideals, the unhappier the parents are; in particular,

$$\mathbf{U_X(x\,,\,y) = -\,(x + y - (a + \varepsilon))^2} \quad \text{and} \quad \mathbf{U_Y(x\,,\,y) = -\,(x + y - a)^2}$$

a) (10pts) Find the best response correspondence/function for both X and Y, $\mathbf{BR_X(y)}$ and $\mathbf{BR_Y(x)}$.

b) (5pts) Using **(a),** find (all, if there are many) the pure strategy Nash Equilibrium of this game.

c) (5pts) Assume **ε = 0.** What would be the pure strategy NE of this game (find all of them, if there are many)?

d) (5pts) Consider the set of pure strategy NE for this game. As a function of **$\varepsilon \in$[0,1]** , plot the NE equilibrium <u>payoffs</u> **NE**x**(ε)** for Heather; for example **NE**x**(0.7)** is the set of all possible payoffs Heather can receive in some pure NE of this game when **ε** = 0.7. Is this correspondence **NE**x**(ε)** "*continuous*" at **ε = 0** ? That is, **NE**x**(0)** "shrinks"(is not *lower semi-continuous*) or "explodes" (is not *upper semi-continuous*) in the neighborhood of **ε = 0** ?

[3] (15pts) a) Find a mixed Nash equilibria of the following game (**5pts**), show that there is no other.

	K	L	M
A	3,1	0,0	1,3
B	0,0	2,2	0,0

b) (5pts) What would be the prediction of game theory for the game on the right, if we assume rationality is common knowledge among the players?

(In this example, you can think of pure strategies when eliminating; you don't need to worry about mixed strategies)

	a	b	c	d
W	5,4	4,4	4,5	12,2
X	3,7	8,7	5,8	10,6
Y	2,10	7,6	4,6	9,5
Z	4,4	5,9	4,10	10,9

[4] (15pts) (Coordination) Suppose there are **100** drivers on the road, each simultaneously and independently choosing driving speeds **$v \in [70, 100]$.** Each driver wants to drive as fast as possible, but does not want to get a speeding ticket at any speed (you don't need this, but assume their net utility equals their net speed minus $100 if they get a ticket). The cops will ticket anyone who is driving strictly faster than **n** other drivers where **$n \in \{1, 2, 3, .. , 99\}$,** where **n** is a parameter of the game. For each n, find the set of pure NE.

[5] (15pts) Suppose we are constructing a 2 player game where each player has 2 actions, by "filling in " any payoff with an i.i.d. draw from **U[0,1]** , the uniform distribution.

a) (5pts) What is the probability that this game has a strictly dominant equilibrium?

b) (10pts) Now solve the problem for **N** players with each having **a** many strategies.

------ BONUS PROBLEM-----

[6] (20pts) (Picking the larger number) Assume two players are playing the following game. Player 1 chooses a random number **x** from the uniform distribution **U[0,1]** and if he likes he can keep that number, if not, he can draw (i.i.d.) a new number , say **y**, from the same distribution, but this time he has to stick with the new number (he cannot go back to the first picked number). The other player separately and independently does the same. Then they simultaneously show their numbers; whoever has a higher number wins a prize worth **$1**. Identify a pure strategy for a player with a threshold **$x^* \in [0,1]$** below which he redraws a new number, above which he doesn't and sticks with the number.

Find the unique symmetric Nash Equilibrium of this game (symmetric means both players will use the same threshold **x^*** in the equilibrium).

HINT: Assume that my opponent chooses cutoff strategy x^* (but I obviously do not know the value drawn by my opponent). Suppose now that my draw in stage 1 is exactly x^*.
(1) What is my expected payoff if I keep this draw?
(2) What is my expected payoff if I do not keep this draw?
(3) For x^* to be my cutoff, I should be indifferent between keeping it or redrawing; hence equate (2) and (3) to find the symmetric equilibrium of the game.