

## HOMework 1

Econ 501: Macroeconomic Analysis and Policy

Spring 2016

1. Suppose that the world is described by the Solow model and the production function is

$$F(K, L) = AK^\alpha L^{1-\alpha}$$

where  $0 < \alpha < 1$ . This is the Cobb-Douglas function we discussed in class.

- a) Show that this production function is “neoclassical” (in the sense that it obeys the conditions I outlined in the lecture).
- b) In the formulation above, technological progress is neutral. Show that with a suitable transformation of  $A$  we can write the function so that technological progress is labor augmenting.
- c) Find the equilibrium wage ( $w_t$ ).
- d) Find the equilibrium rental rate on capital ( $R_t$ ).
- e) Poor countries have a low capital-to-labor ratio, where rich countries have a high capital-to-labor ratio. What does this model imply about relative wages and interest rates (assume the same level of technology in both countries) in rich and poor countries?

f) What is labor’s share of output  $\left( \frac{w_t L_t}{Y_t} \right)$

g) What is capital’s share of output  $\left( \frac{R_t K_t}{Y_t} \right)$

2. In the Solow model, markets are competitive and production exhibits constant returns to scale. As a result, applying the Euler theorem, we see that the firm profits are zero. Here’s a simple problem to see why.

Suppose you are a firm with production function  $Y = aL$ . You buy labor at wage  $w$ , and sell your output on the open market at a price of  $p$ . You are a price taker in all markets. Without loss of generality, we normalize prices so that  $p = 1$ .

a) For a given wage rate  $w$ , find the:

- (i) Profit-maximizing output  $Y$
- (ii) Profit-maximizing labor demand  $L$
- (iii) Total Profit

Note “Infinity” is a potentially valid answer

b) Now suppose that labor markets are competitive. Find the:

- (i) Market-clearing wage
- (ii) Firm profits at that wage

c) Now we allow for increasing returns to scale. The production function is  $Y = aL^2$ . For a given wage rate  $w$ , find the:

- (i) Profit-maximizing output  $Y$
- (ii) Profit-maximizing labor demand  $L$
- (iii) Total profit