## ECON 604 GAME THEORY, Spring 2016 Professor Yilmaz Kocer

## ASSIGNMENT II

Due: Friday, May 6<sup>th</sup> 5pm SUBMIT via EMAIL ONLY!!! Either type your answers, or carefully scan legible handwriting! 35 points bonus! Have a great summer!

## 1) (30pts)

(a) (10pts) Provide an example in which the folk theorem fails under perfect monitoring, and  $\dim V = I - 1$ , i.e. the set of feasible and IR payoffs is of dimension one less than the number of players. If not, prove that the folk theorem holds in this case.

(Hint: Consider the standard case: finite action environment, with pure minmax payoff as the minimum IR payoff. Consider the limit set of SPE payoffs when discount factor tends to 1). The example we solved in class was 1 dimensional where there were 3 players.

(b) (20pts) Monotonicity of Public Perfect Equilibrium payoffs with respect to the discount factor. Remember we couldn't prove it in class that "the more patients the players are, the bigger the set of payoffs achievable as PPE payoffs". Assume  $0 < \beta_1 < \beta_2 < 1$  and consider the infinitely repeated stage game with finite actions, finite players and finite public signals. Is the set of PPE payoffs E( $\beta_1$ ) contained in E( $\beta_2$ )? Consult Mailath and Samuelson pp248 for a sufficient condition that guarantees this monotonicity.

2) (35pts) Suppose the following game is played T times with no discounting:

	A	В	C
A	3, 3	-1, 4	0, 0
В	4, -1	0,0	0, -1
C	0,0	-1, 0	x, x

(a)(10pts) Show that if  $x \ge 1/(T-1)$ , there is a subgame perfect equilibrium where (A, A) is played in the first period.

**(b) (25pts)** Consider the following recursive definition of a *renegotiation-proof equilibrium* (RPE). For a single period game, define the set of RPE to be the set of Pareto undominated Nash equilibria (NE that is not Pareto dominated by some other NE). For a game of length T, consider the set of Nash equilibria that specify an RPE for the continuation game following any first period history. An equilibrium in this set is an RPE if it is not Pareto dominated by another equilibrium in this set. Now, suppose x = 1, and explain why repetition of **(C, C)** in every period will be the only renegotiation-proof equilibrium.

**3) (35pts)** Consider an infinitely repeated version of the following game with perfect monitoring:

	L	M	R
U	1, 1	3, 0	-2, 0
M	0, 3	2, 2	-2, 0
D	0, -2	0, -2	-4, -4

- (a) (10pts) Suppose  $\delta \ge 1/2$ . Show that the set  $\{(1, 1), (2, 2)\}$  is self-generating.
- (b) (15pts) Suppose  $\delta = 1/3$ . Find a self-generating set that contains the point (2, 2).
- (c) (10pts) Show that there is no self-generating set containing the point (-3, -3).

## 4) (35pts)

- a) (20pts) Fudenberg & Tirole Game Theory, problem 7.1 in chapter 7, Mechanism Design.
- **b)** (15pts) Mas Colell, Whinston Green, problem 23.D.2 in chapter 23.