Surplus Labor in a Utility-Maximization Model Both the production and consumption behavior of the farm household must be studied simultaneously if one is to predict whether or not withdrawing agricultural workers will lead to a decrease in output. There exist a number of such general equilibrium models (Nakajima 1969; Jorgenson and Lau 1969; Yotopoulos and Lau 1974). For the study of surplus labor, a model that concentrates on the marginal valuations of labor (the marginal productivity of labor) and income is sufficient.

Sen (1966*b*) provided a cogent defense of surplus labor by distinguishing between the marginal productivity of a laborer in agriculture and the marginal product of a man-hour. The latter being zero is not a necessary condition for surplus labor to exist. More specifically, define household labor, L, as the product of the number of working members of the household, α, times the quantity of work, l, contributed by each individual. For output to decrease, the quantity of household labor, L, should decrease after workers, α, are withdrawn. But this may not necessarily happen if l, the amount of work contributed by each remaining worker, increases at the same time. The analysis, as a result, focuses specifically on agricultural underemployment which, as we suggested earlier, may play an important role in swelling the rolls of the openly unemployed. The empirical question becomes: will l increase as a result of a decrease in α? If so, agricultural underemployment exists, and the provision of new jobs for the workers withdrawn from agriculture is not likely to decrease the magnitude of measured unemployment. To answer this question, both subjective (marginal utility of leisure) and objective (marginal productivity of work) considerations must be combined.

An individual will decrease the quantity of work he supplies, l, when the real cost of work rises. The real cost of work is the marginal disutility of work divided by the marginal utility of income. Consider a decrease in the number of workers in the household,

α by farm exodus. In order to maintain the quantity of household labor, L, unchanged, the quanity of work of each remaining worker, l, has to increase; as a result, the marginal disutility of work to each individual increases. In the meantime, the reduction in α leads to a rise in income for each remaining household member; as a result, the marginal utility of income decreases. Consequently, the real cost of labor rises, with both L and output decreasing. Nonreduction in output (surplus labor) is therefore possible if, and only if, both the marginal disutility of work curve and the marginal utility of income curve are flat in the relevant region. We can formalize this argument following Sen (1966*b*).

Define family labor supply, L, and family output, Q, respectively as

$$L = \alpha l \tag{1}$$

$$Q = \beta q \tag{2}$$

Here α is the number of workers in the household, β is the total number of members in the household ($\beta \geqq \alpha$), l is the amount of work contributed by each individual worker, and q is the consumption per household member. The family output, Q, at a given point of time, is a function of L given the stock of capital and land, and the function is assumed to be smooth and normal, that is, with dimin-

$$Q = Q(L) \tag{3}$$

with

$$Q'(L) > 0 \qquad Q''(L) < 0$$

Assume that the peasant family maximizes a welfare function, w, that is a linear combination of the individual utility of consumption, U_i, and the individual disutility of work, V_i, for all members of the household

$$\begin{aligned} w &= \sum_{i=1}^{\beta} U_i(q_i) - \sum_{i=1}^{\alpha} V_i(l_i) \\ &= \beta U - \alpha V \end{aligned} \tag{4}$$

The utility and disutility curves (identical for each household member) have the normal properties:

$$U = U(q) \tag{5}$$

with

$$U'(q) > 0 \quad \text{and} \quad U''(q) \leqq 0$$

that is, the marginal utility of consumption is positive, and

$$V = V(l) \tag{6}$$

with

$$V'(l) \geqq 0 \quad \text{and} \quad V''(l) \geqq 0$$

that is, the marginal disutility of work is nonnegative.

Family welfare is maximized when[5]

$$Q'(L) = \frac{V'(l)}{U'(q)} \equiv x \tag{7}$$

where x is the "real cost of labor," determined by the individual marginal rate of substitution between income and labor. The maximization rule suggests that labor is applied up to the point where its marginal product is equal to the real cost of labor.

The argument at this point will be simplified if we introduce two other auxiliary relationships. Not only is the marginal product of labor, $Q'(L)$, a function of L, but labor it-

[5]Rewrite equation 4 as $w = \beta U(Q/\beta) - \alpha V(L/\alpha)$, where $Q/\beta = q$ and $L/\alpha = l$. Differentiating with respect to L and setting equal to zero, we have

$$\frac{\partial w}{\partial L} = \beta U'(q) \frac{1}{\beta} Q'(L) - \alpha V'(l) \frac{1}{\alpha} = 0$$

Hence, a necessary condition for maximization is

$U'(q)Q'(L) = V'(l)$

self is a function of $Q'(L)$.[6] Furthermore, since from equation 7 $Q'(L)$ is the real cost of labor, this relationship has a negative sign, that is, if $Q'(L)$ increases, L decreases. We write

$$L = \phi[Q'(L)] = \phi(x) \tag{8}$$

with

$$\frac{dL}{d\phi} < 0$$

Since from equation 3 output is a positive function of labor, output is also a negative function of real labor cost

$$Q = \psi[Q'(L)] = \psi(x) \tag{9}$$

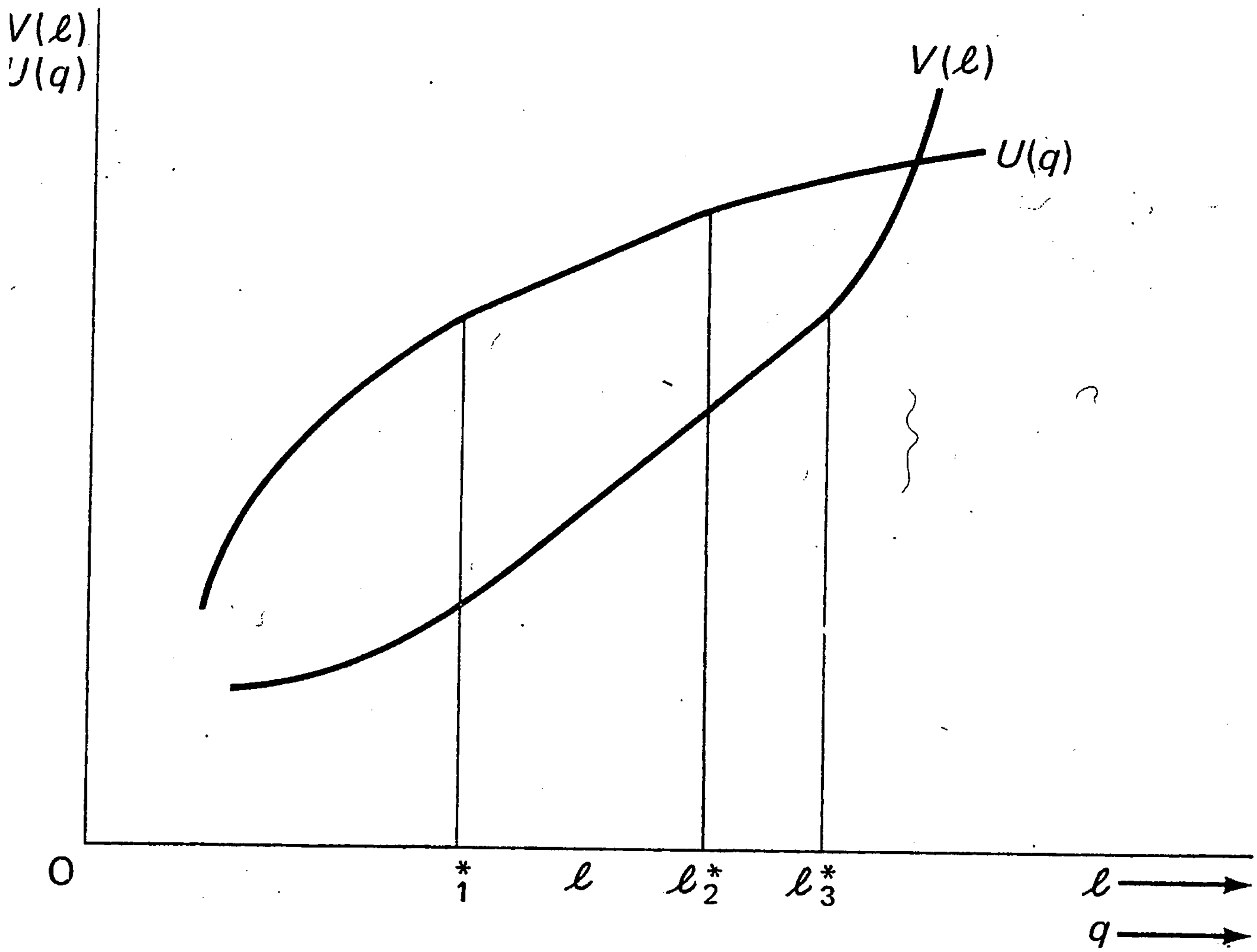
with

$$\frac{dQ}{d\psi} < 0$$

Equation 3, together with the positive marginal productivity of labor condition, indicates that if family labor decreases, output also decreases and surplus labor is ruled out. Therefore, we have to approach the possibility of the existence of surplus labor with L constant: as α decreases, l increases to compensate for any decrease in α. By increasing l, the individual marginal disutility of work, $V'(l)$, is increased. The reduction in α leads to a rise in per capita income for the total, β, family members, and the marginal utility of income, $U'(q)$, decreases. As a result, the real cost of labor, x, increases, which, from equation 9, leads to a reduction in output. Nondecrease in output and hence surplus labor are possible only if both $V(l)$ and $U(q)$ are flat in the relevant region. Only under these circumstances will an increase in the individual's work leave the marginal disutility of work unchanged, and a rise in the individual's income leave the marginal utility of income unchanged. In this case, then, the real cost of labor is insensitive to the withdrawal of workers from the household and the possibility of surplus labor arises. This situation is depicted in Figure 12.1. If l, the actual quantity of work contributed by the individual worker of the household, is in the range l_1^* to l_2^*, the constancy of the real cost of labor is satisfied because the slopes of both $V(l)$ and $U(q)$ are constant. The withdrawal of workers will be compensated by the increase in individual

[6] The inverse functi...

Figure 12.1 Surplus Labor Without Zero Marginal Product



work. Outside this range, however, for example, to the right of l_2^*, the increase in individual work will not be sufficient to compensate for the withdrawal of workers, because although the marginal disutility of labor is constant up to l_3^*, the marginal utility of income is not; nor will it be sufficient when l is to the left of l_1^* because neither curve is flat in that range.

Of the two conditions for the existence of surplus labor, constant marginal disutility of labor and constant marginal utility of income, the former is less objectionable than the latter. How can the flatness of the utility of income curve be rationalized? At a level of living close to the minimum subsistence, when a "decent" livelihood has not yet been achieved, nondiminution of the desire to earn more income as income rises may not be implausible. Even if the "decent" living standard has been achieved or, whether or not it has been achieved, we are still operating on the part of the marginal utility of income curve that is diminishing, one could devise a system of taxes that would wipe away the rise in income per head that resulted from the departure of some workers from the household. Then there would be no net increase in per capita income, and the invariance of the marginal utility with respect to the variation in income would not arise. Given the possibility of designing and administering such a tax system, surplus labor would exist, of course, if the marginal disutility of work were also constant within the same range.

Under certain assumptions, we can generalize the model of the equilibrium of the peasant household to apply to the agricultural economy as a whole. Suppose we have a large number of identical families in the agricultural economy. Furthermore, assume constant returns to scale, assume that nonlabor resources can be reallocated among the households, and assume that the ratio $k = \beta/\alpha$ of the total members to workers in each household remains constant. The question is to determine the quantitative response of peasant output to a withdrawal of workers. In terms of elasticities, we want to determine the elasticity of output with respect to working members of the economy.

For the elasticity of output with respect

[7]This is the argument of Nurkse (1953, p. 43) in connection with the necessity of enforced savings for

to workers, we write

$$\frac{dQ}{d\alpha}\frac{\alpha}{Q}=\frac{dQ}{dL}\frac{dL}{d\alpha}\frac{\alpha}{Q} \tag{10}$$

Solving first for $dL/d\alpha$ we have[8]

$$\frac{dL}{d\alpha}=\frac{\left(\frac{V''(l)}{V'(l)}\cdot L\cdot k-\frac{U''(q)}{U'(q)}Q\right)}{\left(\beta\frac{V''(l)}{V'(l)}-\alpha\beta\frac{Q''(L)}{Q'(L)}-\alpha Q'(L)\frac{U''(q)}{U'(q)}\right)} \tag{11}$$

Then by substitution of equation 11 for $dL/d\alpha$

[8]The calculations, outlined in Sen (1966*b*, p. 433), are as follows:

$\beta=k\cdot\alpha$ by assumption (1*n*)

$$\frac{dx}{d\alpha}=\left(\frac{\frac{dV'(l)}{d\alpha}U'(q)-\frac{dU'(q)}{d\alpha}V'(l)}{[U'(q)]^2}\right) \quad \text{from equation 7} \tag{2n}$$

$$\frac{dV'(l)}{d\alpha}=V''(l)\left(\frac{\left(\frac{dL}{d\alpha}\cdot\alpha-L\right)}{\alpha^2}\right) \tag{3n}$$

from equations 6 and 1

$$\frac{dU'(q)}{d\alpha}=U''(q)\left(\frac{Q'(L)\left(\frac{dL}{d\alpha}\right)\beta-Q\cdot k}{\beta^2}\right) \tag{4n}$$

from equations 5, 2, and 1*n*

$$\frac{dx}{d\alpha}=\frac{dL}{d\alpha}\frac{1}{\phi'(x)}=\frac{dL}{d\alpha}Q''(L) \tag{5n}$$

from equations 7 and 8.

Therefore, from equations 2*n* to 5*n*, we have equation in equation 10, we have

$$\frac{dQ}{d\alpha}\frac{\alpha}{Q}$$

$$= \frac{Q'(L)L}{Q}\left(\frac{\dfrac{V''(l)l}{V'(l)} - \dfrac{U''(q)q}{U'(q)}}{\dfrac{V''(l)l}{V'(l)} - \left(\dfrac{U''(q)q}{U'(q)}\dfrac{Q'(L)L}{Q}\right) - \dfrac{Q''(L)}{Q'L}}\right)$$

or

$$E = G\left(\frac{n+m}{n+(mG)+g}\right) \tag{12}$$

where we define as follows:

$E = \dfrac{dQ}{d\alpha}\dfrac{\alpha}{Q}$ for the elasticity of output with respect to the number of workers

$G = \dfrac{Q'(L)L}{Q}$ for $dL = 1$ unit; the elasticity of output with respect to labor

$n = \dfrac{V''(l)l}{V'(l)}$ for $dl = 1$ unit; the elasticity of the marginal disutility of work with respect to hours of work

$m = -\dfrac{U''(q)q}{U'(q)}$ for $dq = 1$ unit; the absolute value of the elasticity of the marginal utility of income with respect to income

$g = -\dfrac{Q''(L)L}{Q'(L)}$ for $dL = 1$ unit; the absolute value of the elasticity of the marginal product of labor with

With equation 12, we can investigate the different conditions under which surplus labor may appear. Surplus labor implies that $E = 0$, that is, output remains unchanged with the withdrawal of workers. The following cases arise:

1. The extreme case occurs when $n = m = 0$. This coincides with the case in which the utility and the disutility curves are both flat within a certain region, as presented in Figure 12-1.
2. The case of $G = 0$ is the extreme classical case where the marginal product of labor and therefore the elasticity of output with respect to labor are zero. This case does not strictly follow.[9] Heuristically, however, one may conceive of making G smaller and smaller, which leads to infinitesimally small E.

The elasticity of output with respect to workers, E, can readily be determined empirically by fitting a production function. What relationship does this elasticity bear to the elasticity of output with respect to labor, G? By reference to equation 12, the relationship between E and G can be analyzed in terms of the following cases:

1. If the hours of work, l, are institutionally fixed, this leads to $E = G$. This, however, is not a realistic assumption for the peasant economy.
2. If n is very large, we have G approaching E. Heuristically, this corresponds to the case in which the marginal disutility schedule becomes vertical. It implies constancy of hours worked and thus coincides with the previous case.
3. A special case of $E = G$ arises when $m = g/(1 - G)$. We can visualize this case as follows:

> When some people are withdrawn from the peasant economy, with an unchanged number of hours of work per person, the marginal physical return from work will increase. On the other hand, since the people left behind will now enjoy a higher income, the utility value of a unit of output will now be lower. The condi-

[9]Marginal productivity of labor being zero requires, by equation 7, that the marginal disutility of labor be zero, too. This, however, has been ruled

tion quoted corresponds to the specia case when the two forces just cancel ou each other. (Sen 1966*b*, p. 435.)

4. The above are special cases. In general, we would not expect the elasticity of output with respect to workers to be identical with the elasticity of output with respect to labor. In fact one can verify from equation 12 that the condition reduces to

$$E \left\{ \begin{matrix} \leq \\ = \\ \geq \end{matrix} \right\} G \quad \text{according as} \quad m \left\{ \begin{matrix} \leq \\ = \\ \geq \end{matrix} \right\} \frac{g}{1-G}$$

For the case of a Cobb-Douglas production function, we have $g = (1 - G)$, so that the condition reduces to

$$E \left\{ \begin{matrix} \leq \\ = \\ \geq \end{matrix} \right\} G \quad \text{according as} \quad m \left\{ \begin{matrix} \leq \\ = \\ \geq \end{matrix} \right\} 1$$[10]

Extensions of the Model

We have presented a utility-maximization model of surplus labor based on rather conventional assumptions about the marginal productvity of labor and the marginal utility of leisure. This general model can be modified by introducing specific assumptions that enhance its realism. Stiglitz (1969) adds some realistic assumptions and strengthens Sen's results.

Sen does not allow for seasonal labor utilizations. In most agricultural activities (with dairy farming and specialized crops being the likely exceptions), the relationship between peak season and off-peak season labor utilization is one of complementarity rather than of perfect substitutability. It is useful to assume

[10]For the C-D production function with constant returns to scale, we write

$$Q(L) = AL^{a}$$

$$Q'(L) = a\frac{Q}{L}$$

Substituting into g, we have

$$g = -\frac{\frac{dQ'}{dL}L}{a\frac{Q}{L}}$$

$$= -\frac{\frac{d}{dL}\left(a\frac{Q}{L}\right)L^2}{aQ}$$

$$= -\frac{\left(\frac{Q'}{L} - \frac{Q}{L^2}\right)L^2}{}$$

that the marginal productivity of labor supplied at harvest and planting times exceeds by a considerable amount the marginal productivity of labor at other times of the year. As a result, output is not simply a function of total labor supply, L (where $L = \alpha l$). Stiglitz rather assumes that the production function has two labor arguments, with workers being fully utilized in peak seasons (l being at the possible maximum), while at other times of the year the hours worked per week are determined to maximize utility. With this modification, Sen's results become even stronger. Labor can never be in surplus and total output must fall as workers migrate to the urban sector, regardless of the marginal utility of consumption and the marginal disutility of work—which are the crucial factors in Sen's argument. The empirical corollary of Stiglitz' analysis is that the supply price of labor to the urban sector is always upward sloping. Furthermore, assume that urban workers' marginal propensity to consume is close to one. An increase in wage rates in the urban sector that proposes to attract additional workers would result in decreasing the savings available for financing economic development.

Zarembka (1972, Chapter 1) has further elaborated Sen's model by specifying a CES production function in land and hours of work per family member and a CES utility function in hours of work and output per family member. The crucial parameters then become the elasticity of indifferent substitution in the utility function and the elasticity of factor substitution in the production function. If the former is greater than the latter, work hours per worker increase as a family member leaves the farm, and the potential for labor surplus exists. Sen's result is a special case of this. Another special case is the classical position of zero marginal productivity of a laborer, which is implied by an elasticity of indifferent substitution approaching infinity (Berry and Soligo 1968) or by an elasticity of factor substitution equal to zero. If the elasticity of indifferent substitution is equal to the elasticity of factor substitution, work hours are constant with respect to changes in the number of family members and output decreases with the withdrawal of labor. In such a case, it is not necessary to distinguish between labor, *L*, and the number of workers. Jorgenson's (1961) neoclassical model fits this category. Finally, if the elasticity of indifferent substitution is smaller than that of factor substitution, work hours per worker decrease as a family member leaves the farm.