HOMEWORK 3: Solutions

Econ 501: Macroeconomic Analysis and Policy

Spring 2016

a) We have covered this part in class, refer to your notes.
b) Leontief preferences imply $c_{1t} = c_{2t+1}$. Using the budget constraints, you can get savings, $s_t = \dfrac{w_t}{1 + R_{t+1}}$. Thus, the savings rate, $s = \dfrac{1}{1 + R_{t+1}}$, which is clearly decreasing in the interest rate.
c) Substitute for R_{t+1} and w_t from the Firm's FOC and get $\left(1 + \alpha k_{t+1}^{\alpha-1}\right)k_{t+1} = (1-\alpha)k_t^{\alpha}$
d) The steady state capital stock is: $k^* = (1 - 2\alpha)^{\frac{1}{1-\alpha}}$
e) We covered the log preferences case in class. Refer to your notes. The golden rule saving rate, $s_{gold} = \dfrac{\alpha}{1-\alpha}$.
The equilibrium savings rate will exceed the golden rule level if $\dfrac{\beta}{1+\beta} > \dfrac{\alpha}{1-\alpha}$. You can see here that the equilibrium saving rate is increasing in β while the golden rule rate is increasing in α.

1.

a) Consumer 1's problem: $u(c_1, c_1') = Max \log(c_1) + \beta \log(c_1')$

s.t. $c_1 + \dfrac{c_1'}{1+r} = y_1 + \dfrac{y_1'}{1+r}$

Consumer 2's problem: $u(c_2, c_2') = Min\{c_2 + \beta c_2'\}$

s.t. $c_2 + \dfrac{c_2'}{1+r} = y_2 + \dfrac{y_2'}{1+r}$

FOC for consumer 1: $\dfrac{\beta c_1}{c_1'} = \dfrac{1}{1+r}$

FOC for consumer 2: Note that this is a kinked utility function: $\frac{c_2}{c_2'} = \beta$

Goods market clearing: $\begin{aligned} c_1 + c_2 &= y_1 + y_2 \\ c_1' + c_2' &= y_1' + y_2' \end{aligned}$

b) Substitute the FOC into the budget constraints to get the optimal consumption levels.

$$\begin{aligned}
c_1^* &= \frac{1}{1+\beta}\left(y_1 + \frac{y_1'}{1+r}\right) \\
c_1'^* &= \beta(1+r)c_1^* \\
c_2^* &= \beta c_2'^* \\
c_2'^* &= \frac{1+r}{1+\beta(1+r)}\left(y_2 + \frac{y_2'}{1+r}\right)
\end{aligned}$$

Lending/borrowing is defined as $s_t=y_t-c_t$

$$s_1^* = y_1 - c_1^* = y_1 - \frac{1}{1+\beta}\left(y_1 + \frac{y_1'}{1+r}\right)$$

$$s_2^* = y_2 - c_2^* = y_2 - \frac{\beta(1+r)}{1+\beta(1+r)}\left(y_2 + \frac{y_2'}{1+r}\right)$$

Equlibirum r can be found by market clearing condition, $s_1+s_2=0$.

c) Since preferences are convex and there are no distortionary taxes or externality present in the endowment economy, both the first and second welfare theorems hold. Thus, the competitive equilibrium is pareto optimal.