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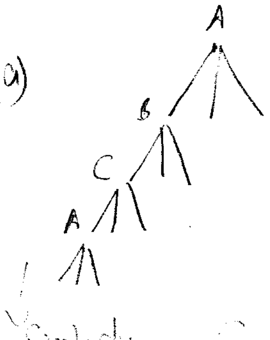
ECON 504 HOMEWORK 2 ANSWER KEY

[1] (a)



A has $1+3^3+3^6$ information sets / decision nodes, at each of which he picks one action out of 3; $3^{1+3^3+3^6}$ strategies!

B has $3^1+3^4+3^7$ info sets / decision nodes $\rightarrow$ $3^{3^1+3^4+3^7}$ strategies.

Similarly, C has $3^2+3^5+3^8$ info sets $\rightarrow$ $3^{3^2+3^5+3^8}$ strategies.

There are $3^{3\times 3} = 3^9$ terminal nodes.

(b) From one-shot deviation principle, to test whether a strategy profile is SPNE, we need to check that at each subgame, the agent who is playing at the initial node (and possibly at other info sets / decision nodes down in that subgame) would not want to deviate to another action in the initial action only (keeping all other decisions at downstream info sets constant). As there are 3 actions at each info set; we need to make two ($3-1=2$) comparisons;

$$\# \text{tests / inequalities} = 2\times\left(1+3^3+3^6+3^1+3^4+3^7+3^2+3^5+3^8\right)$$
$$= 2\left(1+3+\ldots+3^8\right) = 2\cdot\frac{3^9-1}{3-1} = \underline{\underline{3^9-1}}$$

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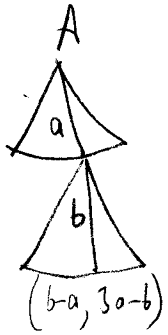
[2] (a) As B can triple any investment, a total of \$3 can be created and ultimately shared in any way; Pareto Set $= \{(x,y) : x,y\geq 0 \quad x+y\leq 3\}$

Note that A can get less than 1 in a Pareto optimal allocation; as long as no money is wasted between the two, any division is P.O. (for example $\left(\frac{1}{2},\frac{5}{2}\right)$ is P.O. because to improve A's money share; one should hurt B, which is not okay; hence $\left(\frac{1}{2},\frac{5}{2}\right)$ is not pareto dominated hence it is P.O.)

SPNE:



B has to keep all the tripled money to himself and not return anything; $b=0$

Hence by backward induction, A doesn't send any money $a=0$, on the eqm. path

$$\text{SPNE} = \Big(\underbrace{a=0}_{\text{A's strategy}},\ \underbrace{b(a)=0 \text{ for all } a}_{\text{B's strategy}}\Big)$$

NE = Suppose the strategies are A $\Rightarrow$ send $a \in [0,1]$, B $\Rightarrow$ return $b(a) \in [0, 3a]$

For $NE = (a^*, b^*(a))$ we should have a^* maximizes $\underbrace{b^*(a) - a}_{\text{net return to A}}$ for A best-responding

and for B best responding to A; $b^*(a^*) = 0$; that is, whatever amount a A is sending, B should send none back, to best respond he would keep all of it.

Then from A best-responding $\quad b^*(a^*) - a^* \geq b^*(a) - a \quad \forall a$

$$0 - a^* \geq b^*(a) - a \quad \forall a$$

$$\geq b^*(0) - 0 = 0 \quad \text{as } b^*(0) \in [0, 3 \times 0] = 0$$

$\rightarrow a^* = 0$ Hence $NE = \{(a^*, b^*(a)) : a^* = 0,\ b^*(a) \leq a\}$

That is A sends \$0, B's strategy is to send back, for each $a \geq 0$, some amount <u>weakly</u> less than a.

Note that all NE outcomes & the unique SPNE outcome is $(1,0)$; <u>not Pareto optimal</u>!

(b) The main idea is this: If you give <u>all</u> the money at once, or indeed, in any finite number of steps; in the <u>last</u> period none of A's money will be returned; knowing this A wouldn't be able to send any money in the second-to-last period because he cannot punish B for not sending that one either; hence all the scheme unravels; no money is sent bc. none will be returned; $(1,0)$ is attained as in (a); SPNE.

However, if the money is divided into infinite pieces such that, at any point in time; if B doesn't return the required chunk of money back; A will not send him any more money so B will lose all future money flow; over which he would have been able to keep some.

To check the suggested strategy profile is SPNE or not, we have to check by one-shot deviation property, for each subgame (= history of play until t, for each t), if A or B would want to <u>deviate</u> just for that period, keeping to the strategy tomorrow onwards

There are 2 kinds of histories (subgames): 1) Cooperation is broken, either A hasn't sent the required amount $(1-p)^t p$ at date t in the past for some t; or B hasn't returned the required payment (or both). 2) Cooperation is not broken yet; both A & B has played according to the scheme thus far.

For "Broken coop" histories: given B's behavior of "not returning anything anymore forever on" A would indeed not send any money anymore, and given A isn't sending anything anymore nomatter what future happens, B would as well play "not return anything anymore", as no money is being sent to him (B) anymore anyways.

For "Cooperation not broken" histories: Suppose today is period t; $(1-p)^t$ dollars left to send, and so far all money was returned k-fold as planned.

For A:

$$\underline{\text{send } (1-p)^t p} \qquad \underline{\text{not send}}$$
$$(1-p)^t \cdot k \quad \geq (1-p)^t$$

→ if A doesn't send his amount, cooperation breaks down, no money transfer anymore.

if he sends his portion today, B returns k fold, and cooperation goes on infinitely, and all $(1-p)^t$ money is sent ultimately & k-fold of that amount is received back by A.

$$\Rightarrow \boxed{k \geq 1} \qquad (1)$$

For B: Suppose B has just received $(1-p)^t p$ at time t, considering whether to return his amount or not;

$$\underline{\text{return } (1-p)^t p k} \qquad \underline{\text{not return}}$$
$$(1-p)^t (3-k) \qquad (1-p)^t p \cdot 3$$

→ B would triple the last batch of money.

if he sends back the correct amount, cooperation will go on each period forever, and including today's batch; all the money sent to B will total $(1-p)^t$ and $(3-k)$ portion of it will be kept by B. (he is going to triple it and send k-fold of it back to A; remaining with $(3-k)$ times of it).

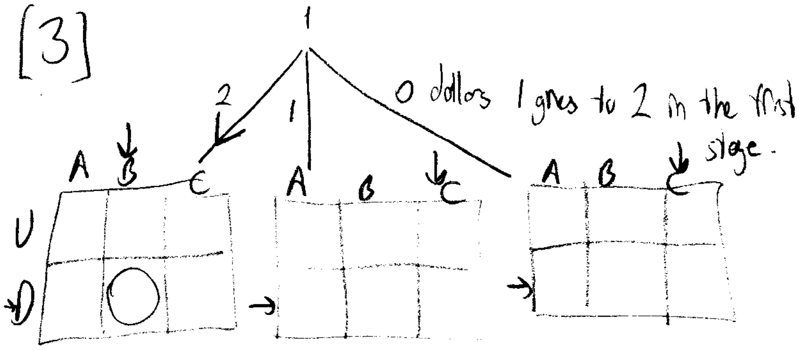
$$3-k \geq 3p \quad \Rightarrow \boxed{3(1-p) \geq k} \qquad (2)$$

Notice that (1) means A ultimately gets (weakly) more than what he has sent. (2) means B will never want to run away with today's money (not returning k-fold of it) in expectation of receiving the rest of the money that is to be sent in the future.

Notice that the larger A wants to keep the fruits of the investment (k large), the lower the bits he has to be sending each period (p small), as large chunks of money tempts B to run away with the money if he is already getting little of it (small $3-k$) in the equilibrium.

SPNE payoffs are $(k, 3-k)$ for $3 \geq k \geq 1$ is the subset of P.O allocations attained. As $k+3-k=3$ all SPNE are P.O.

[3]



a). A has $3 \times 2 \times 2 \times 2 = 24$ strategies
B has $3 \times 3 \times 3 = 27$

b). Consider the strategy profile;

$$pl\,1 \Rightarrow \left(\text{give } \$2,\ \underset{\text{if } \$2}{\text{Down}},\ \underset{\$1}{\text{Down}},\ \underset{\$0}{\text{Down}}\right)$$

$$pl\,2 \Rightarrow \left(B \text{ if } \$2,\ C \text{ if } \$2,\ C \text{ if } \$2\right)$$

("if $2" means the decision node after player 1 gives player 2 $2 in the first stage game)

Check they are best responses to each other;

Given pl 2 "threatens" pl 1, to play C if he doesn't give him $2 in the first stage game; pl 1 would rather give $2 $\Rightarrow U_1 = 1$ give $1 $\Rightarrow 1 + (-2) = -1$ $1 > -1, 0$
give $0 $\Rightarrow 2 + (-2) = 0$

note that as C is strictly dominated for pl 2, so the threat is not credible, yet Nash eqm doesn't rule them out. Note we didn't use (x, x) box in this NE.

c). Note that second stage play should be NE, in a SPNE.

If $x < 0 \longrightarrow (D, B)$ is the unique NE, $x \geq 0$ (U, A) & (D, B) are NE.

If $x < 0 \longrightarrow (D, B)$ is played in second stage, and by backward induction pl 1 gives $0 to pl. 2. SPNE $= \left(pl1 \Rightarrow (\text{give } \$0, D, D, D),\ pl2 \Rightarrow (B, B, B)\right)$ unique.

If $x \geq 0$ pl 1 giving $0 at stage I, and players coordinating on one of the equilibria $((U, A)$ or $(D, B))$ for all 3 second stage information sets each make a SPNE; hence SPNE is unique only when $x < 0$.

$\hookrightarrow$ $\left(Pl\,1 \Rightarrow (\text{give } \$0, D, D, D),\ Pl\,2 \Rightarrow (B, B, B)\right)$ and $\left(Pl\,1 \Rightarrow (\text{give } \$0, U, U, U),\ Pl\,2 \Rightarrow (A, A, A)\right)$ are both SPNE.

d). In a SPNE, second stage play is NE; the payoffs are either $\underset{(x,\,x)}{(U, A)}$ or $\underset{(1,\,1)}{(D, B)}$ (if $x < 0$ the unique SPNE in (b) has pl 1 giving $0; so $x \geq 0$ hence there are 2 NE in second stage game). For pl 1 giving $2 in stage I (rather than $0) could be justified as part of SPNE only if the followup second stage game NE payoff depends on the first stage action; and in such a way that pl 1 payoff is at least $2 higher in the NE following giving $2, compared to the NE after giving $0;

Hence ; $0 + x \geq 2 + 1 \Rightarrow \boxed{x \geq 3}$ should hold for the SPNE.

pl 1 $\Rightarrow$ (give \$2 , U , D , D) pl 2 $\Rightarrow$ (A , B , B)

(Notice that $0 + 1 \geq 2 + x$ cannot happen for our case $x \geq 0$, hence
(pl 1 $\Rightarrow$ (give \$2 , D , U , U) pl2 $\Rightarrow$ (B , A , A)) is not SPNE)

e)- NO. Notice that a SPNE is a NE for the game. pl 1 can always choose (give \$0 , D , D , D) and guarantee a payoff of \$2 , given pl 2 doesn't choose C (it's strictly dominated) in any SPNE. Hence when best responding to a strategy of pl 2, he cannot perform worse than that guaranteed payoff of \$2.

[4] a). Consider the strategies: pl 1 : Always Yield , after any history.
pl 2 : Always Fight on , after any history.

Trivially the equilibrium path (outcome) of this strategy profile is pl 1 Y pl 2 F in the first period. Now we'll show it's a SPNE :

As opposed to the prisoner's dilemma, dynamic Cournot ... etc examples , there is only ONE type of history to check the one-shot deviations (in those examples there were 2 ; 1) cooperation going on 2) cooperation had broken down , types of histories.) Those are the histories where the game is still going on (and hasn't ended yet.).

pl 1 's possible deviations given pl 2 's strategy and given that pl 1 himself is going to follow his strategy tomorrow onwards ; he is just contemplating today to deviate and " Fight on " :

	yield		Fight on	
pl 1 :	0	$\geq$	$\underset{(F,F)}{-1} + \delta \cdot \underset{(Y,F)}{0}$	the game ends tomorrow. he's going to yield tomorrow anyways.
	Fight on		**yield**	
pl 2	10	$\geq$	x	no payoffs for tomorrow onwards b.c. the game ends today.

b). Suppose in a symmetric MIXED SPNE, each agent Yields with probability p each period the game is going on. (when ultimately someone yields, game ends)

For the only type of history where the game hasn't ended, each player should be indifferent between Y & F, given the other's strategy & given his future behavior by the one-shot deviation principle.

$$\underbrace{px + (1-p)\cdot 0}_{\text{yield}} = \underbrace{p\cdot 10 + (1-p)(-1) + (1-p)\,\delta \cdot \overbrace{(\text{payoff from tomorrow onwards})}^{\pi}}_{\text{Fight on}}$$

notice that π = yield = Figh on as the game tomorrow is exactly the same game from stationarity.

$$px = 11p - 1 + (1-p)\delta px \qquad (c)$$

For $x = 0$ (b) $0 = 11p - 1$ $\boxed{p = \frac{1}{11}}$

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[5] (a) The manufacturer's profit $\pi_{man} = q(x-10)$ and the retailer's profit is

$$\pi_{ret} = \left(200 - \frac{q}{100}\right) q - xq = 200q - \frac{q^2}{100} - xq$$

Then, given that manufacturer sets price x, $\max_{q} \pi_{ret} = 200q - \frac{q^2}{100} - xq$

$$\frac{\partial \pi_{ret}}{\partial q} = 0 = 200 - 2q/100 - x \qquad q^*(x) = 10{,}000 - 50x$$

Hence manufacturer knows that if he sets price $= x$, the retailer will buy $q^*(x)$ units from him and $\pi_{man} = q^*(x)(x-10) = (10{,}000 - 50x)(x-10) = 10{,}500x - 50x^2 - 100{,}000$

π_{man} is maximized where $\frac{\partial \pi_{man}}{\partial x} = 0 = 10{,}500 - 100x$ $\underline{x = 105}$

$q = 10{,}000 - 50\cdot 105 = \underline{4{,}750}$ Hence $p = 200 - \frac{q}{100} = 200 - \frac{4750}{100} = \underline{152.50}$

(b) Manufacturer's profits $\pi_{man} = \left(200 - \frac{q}{100}\right) q - 10q$ maximized at

$$\frac{\partial \pi}{\partial x} = 0 = 200 - \frac{2q}{100} - 10 \qquad \underline{q = 9{,}500} \qquad \underline{p = 105}$$

c)- In part (a) the total profit for man & retailer is $(152.5-10)\cdot 4750 = 676{,}875$

In part (b) the manufacturer's payoff is $95 \cdot 9{,}500 = 902{,}500$

Remember monopoly cuts production below competitive level ($p = MC$) and <u>overcharges</u> for the good. In part (a) this inefficiency occurs twice; from man $\to$ retailer $\to$ consumers rather, in part (b) it happens only once manufacturer $\to$ consumer.

As part (b) price $105 < 152.5$, one would prefer the single monopoly world.

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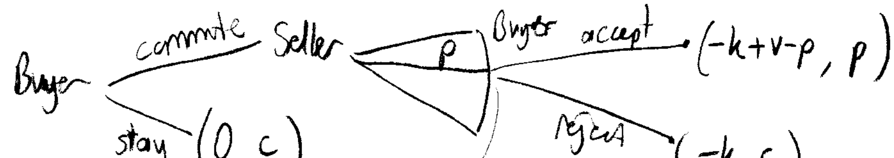
[6] problem 8.13

a)-



Buyer at the last decision node/subgame - accepts iff $v \geq p$.

b)- The next to last decision node/subgame, the seller sets the price $p = v$ (maximize p such that $v \geq p$ results in $p = v$; given $v > c$ the seller would prefer to sell at this price rather than not selling and getting c payoff.)

Then, the buyer at the initial node would decide commute $\to -k+v-p = -k+v-v = -k$
stay $\to 0 > -k$

Buyers stays home. This is the unique $\boxed{\text{SPNE}}$; Buyer: stay home, accept $p \leq v$ seller: offer $p = v$, as we have unraveled by backward induction uniquely what buyer & seller would do at each step.

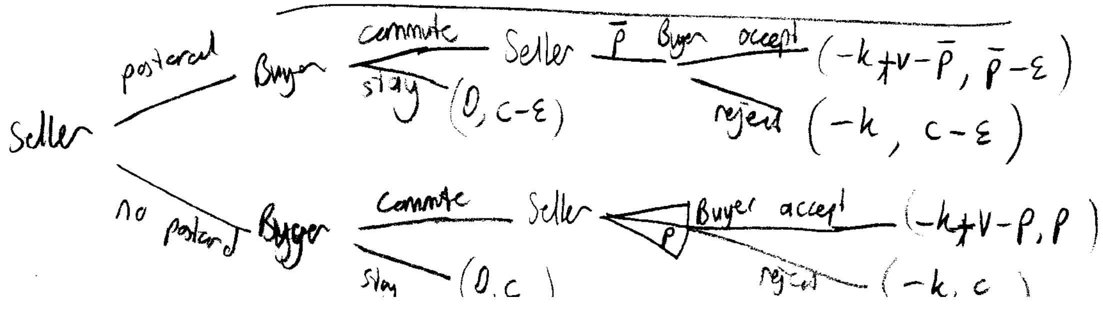
No, it's not Pareto Optimal as $(0, c) < (-k+v-p, p)$ for some suitable p. (that is $p > c$ and $v-p-k > 0$, that is $p < v-k$ is a feasible price as $v-k > c$)

c)- Yes, take a price as above in (b) $c < \bar{p} < v-k$ so that selling the good at $\bar{p}$ is better for <u>both</u> players than $(0, c)$ (buyer staying at home)

We're going to support seller selling and buyer buying at this price p in a NE;

Buyer: commute, accept only if price is $\bar{p}$; reject otherwise
Seller: offer price $\bar{p}$. Clearly the strategies are best responses to each other; NE

d)-



Consider the price $\bar{p}$ above, and the extended game here where "postcard" means seller is committed to price offer $\bar{p}$.

The SPNE of this extended game is as follows:
the lower arm (seller chooses "no postcards") is identical to the original game, hence the SPNE should require Buyer → stays home, accepts $p \leq v$
Seller → offer $p = v$.
In the upper arm subgame (seller sends postcard) SPNE requires
Buyer → commute, accept $\bar{p}$
Seller → has unique strategy
Hence the SPNE of the overall game has seller sends postcard in addition to above as $\bar{p} - \varepsilon \geq c$.

The idea: Once the customer pays k and comes to the shop, as k is already sunk, the seller will offer $p = v$ to get all the surplus from the customer (and he will accept). But knowing this that he will be stripped off any surplus once at the shop and not compensated for the transp. cost k; he decides to stay home. Now they are both worse off; there are prices that a sale would leave both of them happier but the seller cannot commit not to exploit the customer once he already paid the cost k and came to the shop; which prevents the customer coming to the shop in the first place.
In (d) the seller has a "commitment device", the postcard, that makes trade possible

[7] Problem 9.2
a). For the centipede game, on a NE outcome/play path the last person choosing C (or c) would rather have chosen N (or n); so NE outcome should be pl 1 choosing N.
If pl 2 chooses c at the first decision node; pl 1 would have rather chosen C and got 2 rather than 1 hence pl 2 has to be choosing A at his first decision node
So $(\underbrace{N, x}_{pl\ 1}, \underbrace{n, y}_{pl\ 2})$ are all NE for $x = N$ or C
$y = n$ or c

The second stage game has 2 pure $((A, a)$ & $(B, b))$ and 1 mixed NE
$\left(\frac{3}{4}A + \frac{1}{4}B, \frac{3}{4}a + \frac{1}{4}b\right)$

b)- pl $\Rightarrow$ $\underbrace{2 \times 2}_{\text{centipede decision nodes}} \times 2^5$ pl 2 $\Rightarrow$ $2 \times 2 \times 2^5$

centipede game has 5 different histories/terminal nodes, for each of which players choose what to do in second stage game.

c)- $\delta = 0$ means there is no second stage game played. In the centipede game, unique SPNE is ;
or that those payoffs are immaterial.

pl 1 = (N, N) pl 2 = (n, n)

Augment this with ANY second stage game strategies, and it is a SPNE of the whole game.

d)- Consider the strategies: pl 1: $\underbrace{(C, N)}_{\text{centipede game strategy}}$, (B if centipede outcome is (2,2) and A otherwise)

pl 2: (c, n), (b if centipede outcome is (2,2) and a otherwise)

Definitely the outcome (play path) of this strategy profile is ($\underbrace{C, c, N}_{\text{centipede}}$ and (B, b))

Is it SPNE? Second stage play is definitely NE always. Consider the centipede game strategy (given the continuation & given opponent's strategy)

The last decision node behavior (pl 2) doesn't change continuation game's coordinated NE (Aa or Bb); hence pl 2 chooses n. In the next to last node; pl 1 chooses:

$C \Rightarrow$ $1 + \delta 1 = 2$ $\rightarrow$ second stage game NE payoff; 1 $\rightarrow$ centipede payoff $\quad N \Rightarrow 2 + \delta 3 = 5$

$5 \geq 2$ chooses N. $\quad \delta = 1$

First decision node for pl 2:

$c \Rightarrow 2 + \delta 3 = 5 \quad n \rightarrow 3 + \delta 1 = 4 \quad 5 \geq 4$ chooses c

First decision node for pl 1:

$C \Rightarrow 2 + \delta \cdot 3 = 5 \quad N \rightarrow 1 + \delta 1 = 2 \quad 5 \geq 2$ chooses C hence SPNE.

e)- Above we needed: $2 + \delta \cdot 3 \geq 3 + \delta \cdot 1 \quad \delta \geq \frac{1}{2}$

f)- Note that the "threat" of a "bad" followup stage game NE, facilitated cooperation in the centipede game. Note that the only issue is to incentivize players to play C (or c) in the centipede game. By playing c (C) rather than n (N), players forego at most 1 unit of payoff. But they facilitate coordinating on the good equilibrium (rather than bad) gaining $3 - 1 = 2$ units; hence as long as $\delta \geq \frac{1}{2}$, it pays off.

[8] Problem 9.6 (instead of 9.5 which has some errors & problems so <u>SKIP IT</u>!) 10

a). $(100-q_1-q_2)q_1 - 10q_1 \Rightarrow$ FOC $\Rightarrow 100-2q_1-q_2-10=0$ $\quad q_1 = \frac{90-q_2}{2}$

by symmetry $q_2 = \frac{90-q_1}{2}$ $\longrightarrow$ solving together $q_1 = q_2 = 30 \rightarrow$ profits $(900, 900)$

Second stage game has 2 NE (A,a) & (B,b)

b). For the Cournot game, $q_1 = q_2 = q$ maximize $(100-q-q)q - 10q$

FOC $\Rightarrow 100 - 4q - 10 = 0 \quad q = 22.5 \rightarrow$ profits $= (1012.5, 1012.5)$

Second stage game; (A,a) & (B,b) are the symmetric PO allocations.

c). Consider the SPNE: produce $q_i = 22.5$ and in the second stage, play B (or b) if $(22.5, 22.5)$ was played and play A (or a) otherwise.

$$\underbrace{1012.5 + \delta\, 300}_{\substack{\text{conform today}\\ \text{to the strategy and}\\ \text{set } q_1 = 22.5}} \;\text{or}\; \geq u_1\Big(\underbrace{\tfrac{90-22.5}{2}}_{\text{cheat and produce } q_1 = \frac{90-22.5}{2}}, 22.5\Big) + \delta\, 100$$

$$\delta \geq \frac{\underbrace{(43.75 \cdot 33.75 - 10 \cdot 33.75)}_{1139} - 1012.5}{200} = 0.633$$

d). Consider the symmetric SPNE is: produce q and coordinate on good NE (B,b) if both produced q, and bad eqm otherwise.

$$u_1(q,q) + \tfrac{1}{2}\, 300 \geq u_1\left(\tfrac{90-q}{2}, q\right) + \tfrac{1}{2}\, 100$$

$$(100-2q)q - 10q + 150 \geq \left(\left(55 - \tfrac{q}{2}\right) - 10\right)\tfrac{90-q}{2} + 50$$

$$(90-2q)q + 150 \geq \tfrac{1}{4}(90-q)^2 + 50$$

$$360q - 8q^2 + 600 \geq 8100 - 180q + q^2 + 200$$

$$9q^2 - 540q + 7700 \leq 0$$

$$(3q-90)^2 - 400 \leq 0$$

$$-20 < 3q - 90 < 20$$

$$q \in \left(\tfrac{70}{3}, \tfrac{110}{3}\right) \longrightarrow$$

e). One can see from (d)'s equation where $\frac{1}{2}$ is replaced by δ, that as $\delta \to 0$ the q interval shifts right, closer to the NE quantities.

As joint payoffs increase from $(30, 30)$ towards $(22.5, 22.5)$; the best SPNE is $(70/3, 70/3)$

[9] Problem 10.3 a)- As in class, we're checking one-shot deviations at each history for each player. There're $\boxed{3}$ types of histories 1)- (M,m) has been played each period so far 2)- Something else has just been played, so we're supposed to play (F,f) for once. 3)- We just played (F,f) for once, and we are expected to return to cooperative phase, where we play (M,m).

1)- $\frac{4}{1-\delta} \geq \underbrace{5}_{(F,m)} + \delta \cdot \underbrace{1}_{\text{my opponent will play } f\text{, and I'll play } F} + \delta^2 \cdot \frac{4}{1-\delta}$ then we'll revert back to play of (M,m) forever.

2)- $\underbrace{1}_{(F,f)} + \delta\frac{4}{1-\delta} \geq \underbrace{-1}_{(M,f)} + \delta\frac{4}{1-\delta}$ $\rightarrow$ if my opponent is going to play f as part of a $t=1$ punishment phase, I would indeed play F this period too.

3)- Same as (1): $\frac{4}{1-\delta} \geq 5 + \delta \cdot 1 + \delta^2 \frac{4}{1-\delta}$

(1) implies $\frac{4}{\cancel{1-\delta}}(1-\delta^2) \geq 5+\delta$ $\quad 4+4\delta \geq 5+\delta$ $\quad \boxed{\delta \geq \frac{1}{3}}$

b)- Now there are $\boxed{4}$ types of histories. 1)- all (M,m) played so far. 2)- Somebody just cheated & we're supposed to begin a punishment phase. 3)- We're in the middle of a punishment phase; we just played (F,f) and we're supposed to play another (F,f) before reverting to (M,m) forever. 4)- We just finished 2 periods of punishment phase of (F,f) and are expected to return to (M,m) forever now.

1)- $\frac{4}{1-\delta} \geq 5 + \delta \cdot 1 + \delta^2 \cdot 1 + \delta^3 \frac{4}{1-\delta}$ in the punishment phase, the best you can do is join the punishment and play F.

2)- $1 + \delta(+1) + \delta^2 \frac{4}{1-\delta} \geq -1 + \delta \cdot 1 + \delta^2 \frac{4}{1-\delta}$ 3)- $1 + \delta\frac{4}{1-\delta} \geq -1 + \delta\frac{4}{1-\delta}$

4)- same as (1): in both type of histories (M,m) is expected to be played forever.

$\frac{4}{1-\delta}(1-\delta^3) \geq 5+\delta+\delta^2$

$4(1+\delta+\delta^2) \geq 5+\delta+\delta^2$ $\quad 3\delta^2 + 3\delta - 1 \geq 0$

$\boxed{\delta_2 = \frac{-3+\sqrt{21}}{6}} < \frac{-3+\sqrt{25}}{6} = \frac{1}{3}$

c)- $\delta_2 < \delta_1$; so when players can punish for 2 periods rather than 1, the punishment is more effective in disciplining the agents and they can support cooperative play ((M, m) each period, forever) for a larger set of discount factors.

[10] Problem 10.8 a) $\max_{q_1} U_1 = (200-(q_1+q_2))q_1 \Rightarrow$

FOC $\Rightarrow \frac{\partial U_1}{\partial q_1} = 200 - 2q_1 - q_2 = 0 \quad q_1 = \frac{200-q_2}{2}$ similarly $q_2 = \frac{200-q_1}{2}$

$\rightarrow \quad 2q_2 = 200 - \left(\frac{200-q_2}{2}\right) \Rightarrow \frac{3q_2}{2} = 100 \quad \boxed{q_2 = \frac{200}{3} = q_1 \quad U_1^c = U_2^c = \left(\frac{200}{3}\right)^2}$

b)- The monopoly output is; maximize $(200-Q)Q \Rightarrow$ FOC $\Rightarrow 200 - 2Q = 0$

$q_1 = q_2 = 50 \quad U_1^{mon} = U_2^{mon} = 5000 \qquad Q = 100$

$U_1(75, 50) = 75 \cdot 75 = \frac{9}{16} 10{,}000$

best response to 50 $\Rightarrow \frac{200-q_2}{2}$

For grim trigger to work: $\underbrace{(5{,}000)\cdot\frac{1}{1-\delta}}_{\text{monopoly sharing}} \geq \underbrace{\frac{9}{16}10{,}000}_{\text{deviate today}} + \underbrace{\delta\frac{1}{1-\delta}\left(\frac{200}{3}\right)^2}_{\text{forever on Cournot eqm.}}$

$\frac{5}{1-\delta} \geq \frac{90}{16} + \frac{\delta}{1-\delta}\frac{40}{9} \qquad 720 \geq 810(1-\delta) + 144\delta + 640(1-\delta)$

$(810 + 640 - 144)\delta \geq 810 + 640 - 720$

$\boxed{\delta \geq \frac{730}{1306}}$

c)- From chapter 5, prop 5.2; $P_1 = P_2 = 0$ is the unique NE.

d)- Charge $p_i = 100$ until one party deviates; then forever charge $p_i = 0$

$5000 \cdot \frac{1}{1-\delta} \geq 100 \cdot 100 + \delta\frac{1}{1-\delta}\cdot 0 \qquad \frac{1}{1-\delta} \geq 2 \qquad \boxed{\delta \geq \frac{1}{2}}$

$P_i = 0 \rightarrow U_i = 0$

when opponent charging p=100 if you charge $100-\varepsilon$, you get whole demand.

e)- Just as in problem 9; $5000\frac{1}{1-\delta} \geq 100\cdot 100 + \delta\cdot 0 + \delta^2 \cdot 0 + \delta^3 \cdot \frac{5000}{1-\delta}$

$5000(1+\delta+\delta^2) \geq 10{,}000 \qquad \delta^2 + \delta - 1 \geq 0 \rightarrow \boxed{\delta \geq \frac{-1+\sqrt{5}}{2}} = 0.618$

Punishing forever (d) is a stronger incentive to keep firms cooperating, holding for a larger set of discount factors.