# HOMEWORK - II

***Submit as HARD COPIES (on paper, no Email!) to your grader Jeonghwan Yun by Thursday March 23, 4pm, it will be coarsely graded over 5 points.***

**[1] (a)** Suppose 3 players **A, B** and **C** are playing a game, where each player in turn chooses one of 3 actions, for a total of 3 turns (a player plays 3 rounds in total). How many strategies does A, B and C have? How many terminal nodes are there in the extensive form game?

**(b)** In the above game, suppose we are checking whether a particular strategy profile is a SPNE or not, using the one-shot deviation principle. Assume each comparison of two expressions (like U(x)>U(y) for some suitable x,y strategies in a subgame) constitutes a test. How many "tests" is needed?

**[2]** (***Building trust in Relationships***) **A** and **B** are trying to facilitate cooperation among themselves. **A** has **$1 (million)** and **B** has a business idea. **A** can send **a** $\in$ **[0,1]** dollars to **B**, and **B** invests the money, which triples it (and **B** ends up with **3a** dollars). He can then choose to send back **b** $\in$ **[0,3]** dollars to **A**. And the game ends.

**(a)** What is the set of Pareto optimal payoffs in this situation for the two agents (What payoff couples for the two agents can they achieve the best, in terms of Pareto)? Find the pure NE and pure SPNE of this game. Is the SPNE (show it is unique) outcome Pareto optimal?

**(b)** Assuming rationality is common knowledge, you should have found in (a) there will be <u>no cooperation</u> in the unique SPNE. To facilitate cooperation, instead of sending the **$1 (million)** lump sum in one round, suppose A sends it in (an infinite number of) pieces, each piece followed by some possible returns from **B** after being invested and tripled. Each period, A sends to B the **p**-proportion of the remaining part of the **$1** that is not yet sent, as follows;
In round 0, **A** sends $x_0 \in [0,p]$ dollars. **B** can then return the favor by sending some money back $y_0 \in [0,3p]$. In round 1, **A** sends another $x_1 \in [0,(1-p)p]$ dollars. Then **B** may invest this portion and return **A** some money $y_1 \in [0, 3(1-p)p]$ back. In round 2, **A** sends yet another $x_2 \in [0, (1-p)^2p]$ dollars, and B returns some money, and the game goes on. There are infinitely many rounds, and the agents care about the total (undiscounted) sum of money they end up with.
Call the unique **T-** period history of **A** sending $(1-p)^t p$ and **B** returning $(1-p)^t pk$ **to A** for each period **t = 0,1,..T a "cooperative history"**(things are good so far). Any other history is a **"non-cooperative history"** (sth went wrong and cooperation broke down already). Consider the following strategy profile:
**A:** Send $(1-p)^t p$ dollars on period **t** on a **cooperative history** and send nothing otherwise.
**B:** Return $(1-p)^t pk$ dollars on period **t** if on a **cooperative history,** and return nothing back otherwise.
Give conditions on **p** and **k**, so that these strategies constitute a SPNE, where the equilibrium path is an infinite cooperative history where **A** sends all **$1** to **B** eventually. What is the equilibrium payoffs of **A** and **B** in terms of **p** and **k**? What subset of Pareto optimal outcomes can be achieved by choosing a suitable **p** and **k**?

**[3]** Consider the following multistage game. Player 1 first has to choose how to divide $2 between himself and player 2 (with only integer divisions being possible). Both players observe the division, and they then play the simultaneous move game with the dollar payoffs shown below.

| | A | B | C |
|---|---|---|---|
| Up | x,x | 0,0 | -2,-2 |
| Down | 0,0 | 1,1 | -2,-2 |

Assume that each player is risk neutral and has utility equal to the sum of the number of dollars he or she receives in the divide the dollar game and the dollar payoff he receives in the second stage game

**(a)** Draw a tree diagram to represent the extensive form of this game. How many pure strategies does each player have in the normal form representation of this game?

**(b)** Show that for any **x** the game has a Nash equilibrium in which player chooses to give both dollars to player 2 in the initial divide-the-two-dollars game.

**(c)** For what values of **x** will the game have a unique subgame perfect equilibrium?

**(d)** For what values of **x** is there a subgame perfect equilibrium in which player 1 gives both dollars to player 2 in the initial divide-the-two-dollars game.

**(e)** Can the game have a subgame perfect equilibrium in which player 1's total payoff is less than 2?

**[4] (War of Attrition)** In each period, two players simultaneously choose between "Yield" (Y) and "Fight on" (F) and they receive payoffs given by the following stage-game matrix:

| | **Yield** | **Fight on** |
|---|---|---|
| **Yield** | x,x | 0,10 |
| **Fight on** | 10,0 | -1,-1 |

The length of the game depends on the players' behavior. Specifically, if one or both players select Y in a period, then the game ends at the end of this period. Otherwise, the game continues into the next period. Players discount payoffs between periods according to discount factor $\boldsymbol{\delta} \in$ **(0,1)**. Assume $\mathbf{x < 10}$.
**(a)** Show that this game has a subgame perfect equilibrium in which player 1 chooses **Y** and player 2 chooses **F** in the first period. (Hence the game ends at the end of period 1).
**(b)** Assume $\mathbf{x = 0}$. Compute the symmetric SPNE of this game. (Hint: In each period, the players randomize between **Y** and **F**. Let **p** denote the probability that each player selects **Y** in a given period.)
**(c)** Write an expression for the symmetric equilibrium value of **p** for the case $\mathbf{x \neq 0}$.

**[5] (15pts) (An IO application: Double Marginalization)** A manufacturer produces smartphones, then sells them to a retailer who in turn sells them to consumers. The retailer has a demand curve $\mathbf{p = 200 - (q/100)}$. The manufacturer has a unit cost of **$10** per smartphone, the retailer has no cost of production, other than whatever it must pay to the manufacturer for the smartphones he buys to sell to the consumers.
**(a)** Consider the following timing on the pricing decisions and interactions between the manufacturer and the retailer. First, the manufacturer sets a unit price **x** that the retailer must pay for each smartphone. Then, the retailer decides how many smartphones **q** to purchase, from the manufacturer and sell to consumers. Calculating the profits of both the manufacturer and retailer as a function of possibly **q** and **x**, calculate the subgame perfect equilibrium of this game. How much does the manufacturer charge to the retailer, and how much does the retailer charge the consumers?
**(b)** Now assume instead that the manufacturer sells to consumers directly, bypassing the middleman (the retailer). The manufacturer faces the same demand the retailer faces in (a). Calculate the manufacturer's profit-maximizing choice of **q** and the associated monopoly price **p** in this case.
**(c)** Compare the joint profit of the manufacturer and retailer in part (a) with the manufacturer's profit in part (b). Why are they different? As a consumer, would you like to live in a world with two sequential (vertical, as it is called in the literature) monopolies as in (a) or in a single "*vertically integrated*" monopoly as in (b)?

**[6]** **Chapter 8 Problem 13**

**[7]** **Chapter 9 Problem 2**

**[8]** **Chapter 9 Problem 5**

**[9]** **Chapter 10 Problem 3**

**[10]** **Chapter 10 Problem 8**

EXTRA Practice PROBLEMS to PREPARE FOR THE EXAMS:

https://ocw.mit.edu/courses/economics/14-12-economic-applications-of-game-theory-fall-2012/index.htm contains the freely accessible MIT Undergraduate Game Theory Class, with all the lecture notes, assignments and exams. Among many other Game Theory classes you might find online, or other Game Theory texts with practice problems, this seems to closely match our level in this class. Some material we have covered and will cover in the last segment is not covered in this MIT class (dynamic incomplete games), so do not solely rely on this MIT class' material.