Note 2 with HW2

The Partnership Model

In the principal agent model above, the income generated by the agent belongs to the principal. The agent is only entitled to a fixed wage income. In more complex principal agent models, the agent payoff may also depend on the income generated so that the principal gives a portion of the income to the agent (sometimes called contingent wage). The contingent wage induces the agent to behave more like a co-owner of the project.

In a partnership model, the relationship between the two are more similar. Each agent can have a share of the income generated. For simplicity, we assume that the generated income 2θ is split equally between the two agents, so that each agent gets $\theta,$ if the partnership forms. In more advanced partnership models, the way the total income is split between the agents is determined endogenously through negotiation or bargaining processes, and is part of the contract determination. Here we assume that they are pre-determined.

To become a partner, an agent may be entitled to a wage income for the contribution brought to the partnership by the agent. In other cases, such as franchises, to become a partner of the franchise, often you have to pay to become a partner. For example, if you need the franchise brand name to generate higher income, you may have to pay a franchise fee. The existence of a good outside option matters. You need to pay franchise fees because the franchiser has better outside options than you do, and you have to pay to become a franchisee. We assume no outside option for agent 1, and the franchise fee depends only on the outside option (or productivity) of agent 2.

In this example, we will see how the different outside options lead to different outcomes in equilibrium. There is agent 1 who is considering forming a partnership with agent 2. Agent 1 cannot get any income or profit without the agent 2 working for her (no outside option). The agent 2 has an outside option. The outside option depends on the productivity of the agent 2. If the productivity is represented by $\theta,$ the outside option (or the highest expected income from other firms' offers) is $\mu(\theta) = \frac{4}{3}\theta.$ The productivity also affects the income generated by the agent 2 if the partnership forms. The total generated income is 2θ if the productivity is $\theta.$ The parameter θ is uniformly distributed over the interval $[a,b].$

In a partnership model, which partner pays whom is determined endogenously. In other words, the optimal choice may go both ways, depending on the outside options. A partner who has good outside options and is productive is often the one who gets paid. If a partner has few outside options, or low productivity, then he is likely to pay the fee.

The productivity of the agent 2 is known only to herself. but unknown to the agent 1. The game is as follows. Agent 1 proposes a price p at which he is willing to accept agent 2 as a partner. Here $p > 0$ means that agent 1 pays agent 2 the amount $p.$ If $p < 0,$ it means that agent 2 pays a fee $-p$ to agent 1 to become a partner. Agent 2 accepts or refuses. If agent 2 accepts, the partnership forms, and the price is implemented and the profit for each agent is as described above. If agent 2 refuses, the profit for agent 1 is 0 whereas the profit for agent 2 is $\mu(\theta)$.

First assume that $\mu(\theta) = \frac{4}{3}\theta$. We will follow the same steps to find out the equilibrium outcome of the game. In the first step, we determine the probability of getting the offer p accepted by agent 2. The agent 2 accepts the offer if the total payoff $\theta + p \geq \mu(\theta) = \frac{4}{3}\theta$. This becomes $\theta \leq 3p$. Therefore only the agents with productivity lower than $3p$ will accept the offer. Notice that intuitively, agent 2 has good options, and we expect $p \geq 0$. We can think of p as the wage $w = p$ when it is positive. The probability of the offer accepted is therefore $\frac{3p-a}{b-a}$.

In step 2, we determine the profit of agent 1 after the offer is accepted. Since the agent 1's share of income is θ, and θ is uniformly distributed over the interval $[a, 3p]$, the expected profit for agent 1 after the offer is accepted is $\frac{a+3p}{2} - p = \frac{a+p}{2}$.

In step 3, we determine the expected profit of agent 1. This is just the product of the probability of acceptance and the profit after acceptance. Hence the expected profit is

$$\frac{3p-a}{b-a}\frac{a+p}{2}.$$

In the last step, we find the optimal p. Note that as agent 1 raises p, the profit is higher. Intuitively, the higher wage not only increases the probability of acceptance, it also increases the profit after acceptance. The reason is that higher wage recruits higher productivity agent, and this out weighs the higher wage cost. Hence the optimal offer is the highest possible offer when $3p = b$, or $p = \frac{b}{3}$. This is a corner solution, and agent 2 always accept the offer. The contract outcome is efficient.

We say it is efficient because there is no probability of contract failure so that the two work together to generate a better outcome. When b becomes higher, the optimal wage offer is also higher, unlike the case without profit share in which the optimal wage offer remains the same $\frac{8}{3}a$. The additional more highly productive agent 2 is excluded from the contract, because the principal is not willing to offer a higher wage. This does not apply to the partnership case because the total payment to the agent 2 ($\frac{b}{3} + \theta$) depends on the productivity of agent 2. Thus the cost of increasing p is not as large, and can be compensated by including the more productive agent 2. The incentive feature introduced here is that you don't need to pay everyone the same amount. You pay more when you have a more productive partner.

When $[a, b] = [1, 2]$, the wage agent 1 pays to agent 2 is $\frac{2}{3}$ which is lower than the wage $\frac{8}{3}$ we show in the case without profit shares. Hence the agent 1 pays less wage, but complements that with profit share. The total income received by agent 2 is $\frac{2}{3} + \theta$ which is in the interval $[\frac{5}{3}, \frac{8}{3}]$. Agent 2 receives lower wage payment, except the one with productivity 2. The expected wage and income payment by agent 1 is $\frac{2}{3} + \frac{3}{2} = \frac{13}{6}$ which is lower than the total wage payment $\frac{8}{3}$ when there is no profit sharing. When b becomes higher, the partnership arrangement is still efficient, but the pure wage payment model is inefficient. Moreover, agent 1 has higher expected profit in the partnership case (compare expected profits we computed before).

Agent 2 may have to pay agent 1 when we are dealing with lower productive agent 2. We now consider the case $\mu(\theta) = 0.5\theta$. Intuitively, agent 2's outside option is not as good as before. Agent 2 may need to pay a fee to become a partner. To find out the equilibrium outcome, we follow the same four step procedure. In the first step, agent 2 accepts the offer if and only if $\theta + p \geq \mu(\theta) = 0.5\theta$, or $\theta \geq -2p$. Clearly p needs to be negative, and we let $f = -p$ represent the fee that needs to be paid by agent 2. So agent 2 accepts the offer if and only if $\theta \geq 2f$. Now agent 1 gets higher productivity agent as a partner. The probability of the offer being accepted is $\frac{b-2f}{b-a}$.

In the second step, the profit of agent 1 after the offer is accepted is $\frac{b+2f}{2} + f = \frac{b+4f}{2}$.

In the third step, the expected profit of agent 1 is $\frac{b-2f}{b-a}\frac{b+4f}{2}$. Now there is a trade-off when the fee is raised. A higher fee lowers the probability of the offer accepted, but recruits higher productivity agent.

To find the optimal fee, we take the first-order condition, and gets

$$2b - 16f = 0,$$

or $f = \frac{b}{8}$. To get an interior solution, we need to have $2f > a$, or $b > 4a$. In this case, the optimal fee is $f = \frac{b}{8}$, or optimal $p = -\frac{b}{8}$. If $b \leq 4a$, then we have a corner solution $f = \frac{a}{2}$. This means that the fee is set so low that agent 2 accepts it for sure, and we have efficient contractual outcome in this case.

We now compute the optimal wage without the profit sharing. Agent 2 accepts the contract if $0.5\theta \leq w$, or $\theta \leq 2w$. The probability of acceptance is $\frac{2w-a}{b-a}$. Once agent 2 accepts the contract, the expected profit of agent 1 is $\frac{0.5a+w}{2} - w = \frac{1}{4}a - \frac{w}{2}$.

Hence the expected profit of agent 1 (without conditioning on acceptance) is

$$\frac{2w-a}{b-a}\left(\frac{1}{4}a - \frac{w}{2}\right).$$

Taking the derivative with respect to w, we get the derivative

$$a - 2w.$$

Hence the first order condition is $w = \frac{a}{2}$. However this is smaller than a. Hence the optimal wage has a corner solution $w = a$, and no one accepts the contract. There is contract failure. The profit sharing arrangement make it possible to restore the efficiency of the outcome.

questions to ask:

What determines high or low franchise fees? How the outside option affects the franchise fee?

When can you get a franchise for free?
When you need to pay to get a partner?
How the income sharing rule affects franchise fee?

HW2

Due Sep 22

1. In the partnership model, let the outside option of agent 2 be $\mu(\theta) = \frac{4}{3}\theta$. Assume that agent 1 decides to give more income sharing to agent 2. Now the sharing arrangement is $\frac{1}{3}$ to agent 1, and $\frac{2}{3}$ to agent 2. This is the only change we make to the partnership model, all other assumptions are kept the same.

(a) If agent 1 pays an amount $p \geq 0$ to agent 2, what is the probability of acceptance?

(b) If agent 1 charges an amount $f \geq 0$ to agent 2, what is the probability of acceptance?

(c) Determine the optimal p under the new sharing arrangement.

(d) What is the expected profit of agent 1 in equilibrium in the new sharing arrangement?

(e) Compare the profit of agent 1 in (d) to that of the equilibrium profit of agent 1 in the original sharing arrangement (equal split) in the notes. Which profit is higher?

(f) Use your intuition to argue why the sharing arrangement $\left(\frac{1}{3}, \frac{2}{3}\right)$ may be the best for agent 1.

2. In the partnership model, let the outside option of agent 2 be $\mu(\theta) = \frac{3}{4}\theta$. All other assumptions are kept the same, including the sharing arrangement (equal split). Assume that $a = 0, b = 1$.

(a) Find the optimal offer p.

(b) Is the optimal p above positive or negative? why?

(c) Compute the profit of agent 1 in the equilibrium?

(d) When you compare the equilibrium outcome to that of the outside option $\mu(\theta) = 0.5\theta$, is it true that higher option of agent 2 leads to higher p?