Note 12 with HW12

In the public good allocation problem, information about consumer's valuations of the public good is needed to allocate efficiently.

However, such information is private information and is unknown to a government decision maker.

We will consider different types of mechanisms on how to deal with this problem.

A town with a population of 100 wants to decide whether to spend \$104.50 to improve the landscaping in the municipal park. It is too costly to charge a user fee on those who walk through or drive by.

Consumers value the project differently. It is efficient to implement it only if the total valuations exceed the cost \$104.50. For example, if 40 citizens value it at \$2, and 30 citizens value it at \$1. The total valuation is \$110 which exceeds \$104.5. Hence it is efficient to carry out the project.

Each consumer values it at \$2,\$1, or \$0, and this valuation is independent from others' valuations.

One possibility is to put the matter to vote. If it is approved by a majority, then a tax amount of \$1.045 per person will be imposed to carry it out.

Since only persons with a valuation \$2 will value it above the tax amount they have to pay, only those people will vote yes. Using the example of 40 citizens valuing it at \$2, only 40 people will vote for it. The voting outcome is not to carry out the project, which, we have just shown, is not efficient.

The voting process does not lead to efficient outcome, because the valuation of consumers below \$2 is not counted.

Another possibility is to ask citizens to state their valuations for the project (not just saying yes or no), and implement the project only if total valuation exceeds \$104.50.

If the project is carried out, the tax amount is split evenly among the population.

In this case, only citizens with \$2 will report truthfully. The total reported value is only \$80. The project is rejected. This does not lead to efficient outcome.

Another idea is to impose tax differently on different citizens depending on their reported values. For example, if people who valued it at \$1 are taxed less than \$1, they may be willing to report truthfully.

However, in this case people who value it at \$2 may prefer to report value \$1 in order to reduce their tax obligations. Again, it is not a Nash equilibrium for every one to report truthfully.

There is a free-rider problem. If the total valuations of others already exceeds \$104.50, then a consumer will not report a positive value, as it brings no additional benefit, only causes a positive tax amount.

We need a method to induce the citizens to report truthfully, but at the same time we want to avoid the free-rider problem. To induce truthful revelation, people who lower valuation should be allowed to pay less. However this should be done without causing the free-rider problems.

In order to induce truthful revelation, one way is to impose a tax only when the total valuations of all others is less than \$104.50, but exceeds it when we add the consumer's reported value (pivotal consumer). For such pivotal consumers, the tax amount is just enough to add up to \$104.50.

For two pivotal citizens, the one with a higher valuation will pay a higher tax. For non-pivotal citizens, no incentive is needed for them to report a positive value, as the total value without counting theirs already exceeds the cost, and is sufficient for the project to be adopted to achieve an efficient outcome.

Such a tax is always less than the reported value, hence a pivotal citizen is willing to report true valuation to the authorities. The tax amount is independent of the citizen's reported value. Therefore it is a weakly dominant strategy to state your true valuation.

When a citizen is not pivotal, no tax is imposed. This is called Clark-Groves mechanism. This mechanism gives the incentive for everyone to state their true valuations for the project, and allows the efficient one to be chosen.

For example, suppose that it is known that 27 citizens value it at \$2, and 51 citizens value it at \$1, and others value it at \$0. The total value is \$105, hence it is efficient to carry out the project.

A citizen with value \$2 is a pivotal citizen because the sum of all others' value is $26*2+51=103$ and $103+2 > 104.5$. A citizen with value \$1 is also pivotal because the sum total of all others' value is $27*2+50=104$, and $104+1>104.5$.

A citizen with value 0 is not pivotal. A citizen with value \$2 pays tax \$1.5, while a citizen with value \$1 pays tax \$0.5. Thus citizens with higher value pay higher taxes.

One important idea in the mechanism is that the tax you paid depends on others' reported values, not on your reported value. This makes the manipulation ineffective.

Similarly ideas are used in many places, such as multi-object second price auctions to induce bidders to bid truthfully.

Optimal Contractual Breach, an example from production contracts

Assumption on the (stochastic) costs of production: \$20 (low) with probability 0.3, \$60 (moderate) with probability 0.5, and \$200 (high) with probability 0.2

Buyer valuation \$100 (non stochastic).

An optimal complete contract specifies whether the good should be produced in each contingency, and also a price to be paid (the price can be contingent upon the cost of production, or a (non-contingent) single price whenever delivery occurs) upon delivery.

What is an optimal contract? An optimal complete contract calls for production and delivery whenever the cost of production does not exceed \$100, meaning no production and delivery when the cost is \$200.

This property of optimal complete contract does not depend on the price of transaction.

Incomplete contract is much more common than complete contract. Under complete contract, you want to have severe sanctions against violation of the terms of the contract.

An example of incomplete contracts is: production and delivery of the good always and price is P (has to be higher than \$76, the average cost of production, for the supplier to agree to it).

Under incomplete contract, you may not want severe sanctions against breach. The right amount of breach damage measure should be adopted. When the damage measure is properly chosen, we can achieve the optimal outcome as in a complete optimal contract. This is called the optimal damage measure.

The optimal damage measure is the expectation measure – this is the amount that will compensate the buyer so that he has the same payoff from performance.

For instance, in our example, suppose that the incomplete contract calls for performance (production and delivery) without contingency (that is under all situations), and the buyer pays the price \$50.

The buyer payoff from performance \$100-50=50. If the seller breaches when cost is \$200, he has to pay the amount \$50 to the buyer. That puts the buyer in the situation he would have enjoyed as if there is performance (the buyer expects performance). The amount of damage measure \$50 is called the expectation damage.

If the seller performs, the payoff is -200+50=-150 (difference between cost and price). If the seller breaches the payoff is -50 (paying the damage measure). Clearly it is better for the seller to breach. The buyer still enjoys the same payoff as if there is performance.

Rubinstein Bargaining Model

Two players bargain over a fixed sum of money. Assume that in the beginning, player one makes an offer to player two who can accept or reject the offer. If the offer is accepted, the game ends. If the offer is rejected, in the following period, player two makes an offer to player one. Again player one can accept or reject the offer. If the offer is accepted, the game ends. If not, player one then makes another offer to player two, and the bargaining continues as before with possibly infinitely many rounds of offers. The discount factor is assumed to be $\delta < 1$ for both players.

This is an alternating form of bargaining. Because of the discount factor, both players have an incentive to conclude the bargaining as soon as possible. There is indeed a unique subgame perfect equilibrium in which the bargaining ends in the first period.

To compute the equilibrium, we can use the following arguments. Let s be the share of the total amount player one offers to player two in equilibrium. Player two can accept which yields the payoff s. If player two rejects the offer, the bargaining continues with the player two now replacing player one. The same offer s would have to be made to player one, and player two payoff will be $(1-s)\delta$. In equilibrium, we would expect both payoffs to be equal. Hence

$$s = (1-s)\delta$$

and we get the equilibrium solution

$$s = \frac{\delta}{1-\delta}.$$

This means player one gets $\frac{1}{1+\delta}$ share of the total, while player two gets the share $\frac{\delta}{1+\delta}$.

The same equilibrium can be arrived at by analyzing a finite bargaining game and let the number of periods of bargaining goes to infinity.

Homework 12

Due April 24

1. A town with a population of 100 wants to decide whether to spend \$104.50 to improve the landscaping in the municipal park. It is too costly to charge a user fee on those who walk through or drive by.

Citizens value the project differently. It is efficient to implement it only if the total valuations exceed the cost \$104.50. Each citizen values it at \$2,\$1, or \$0, and this valuation is independent from others' valuations. We know that the Clark-Groves mechanism induces the citizens to report their valuations truthfully. This question is about the tax revenues collected by the mechanism. It is known that 27 citizens value it at \$2, and 51 citizens value it at \$1, and others value it at \$0 (based on population survey).

(a) Describe the tax payment of the Clark-Groves mechanism.

(b) Who is a pivotal citizen?

(c) How much tax revenue is collected? Is it enough to cover the cost of the project \$104.5?

2. In the Rubinstein bargaining model over the split of a dollar, it is relatively easy to figure out the unique subgame perfect equilibrium of the bargaining problem. We can use the solution of the finite game to find the solution of the infinite horizon game. This has been shown in the classroom discussions when the number of bargaining periods n is odd. Here you need to work out the solution when n is even (player two makes the offer in the last period).

(a) Find out the subgame perfect equilibrium when $n = 8$.

(b) Extrapolate the solution in (a) to the case of a general even number n.

(c) Let n goes to infinity, and find the solution of the infinite period bargaining problem.

3. Consider the following normal form game of the two firms engaging in price competition. Each firm can choose either high or low price strategy. The payoffs are given by

	H	L
H	(15,15)	(0,30)
	(30,0)	(5,5)

Consider the supergame with a discount factor δ.

(a) What is the condition for the Grim strategy to be a Nash equilibrium?

(b) Determine the range of δ to have a Nash equilibrium with Grim strategy for each firm.

(c) Why is this equilibrium a subgame perfect Nash equilibrium?

4. In problem 3, assume that the two firms decide to follow the following patterns of pricing behavior: (H,L), (L,H) in the first two periods then followed by (H,H) forever. Any time there is a deviation, they will follow a grim punishment strategy after the deviation. Show that this is a subgame perfect Nash equilibrium for δ which is sufficiently close to 1. Determine the exact interval of δ for which the result holds.