Assignment 2. Andong Yan

1) a) Consider the game:

I

	A	B
A	1,1,1	0,0,0
B	0,0,0	0,0,0

II

	A	B
A	0,0,0	0,0,0
B	0,0,0	1,1,1 .

So $\dim V^* = 2$

$*$: No SPE with average payoff $< \frac{1}{4}$.

First stage, two players play $V_i(A) \geqslant \frac{1}{2}$ or $V_i(B) \geqslant \frac{1}{2}$

so if player 3 plays A.

he gets at least $\frac{1}{4}$ average

for the following time, player 3 plays his ~~equili~~ equilibrium strategy

so his payoff is $(1-\delta)\frac{1}{4} + \delta x$

x is the least average in the second stage.

$x = \inf v \geqslant (1-\delta)\frac{1}{4} + \delta x$

$\therefore x \geqslant \frac{1}{4}$

b)

2) (a) Backward Induction. since no discounting
at T, both play B, so every period player plays B.
pay off $u_i = 0$

Strategy : play A first round, then play C forever.

first period : pay off : $3 + x \cdot (T-1)$
if deviate to B. $4 + 0 \cdots + 0 = 4$
$\because x(T-1) \geq 1$
$\therefore 3 + x(T-1) \geq 4$
$\therefore$ ~~SPE~~ NE

future periods : pay off : $x \cdot (T-1) > 0$
if deviate : $0 \cdot (T-1)$ or $(-1) + 0 + 0 \cdots$
$\therefore$ SPE

(b) ~~RPE = {(3,3) (4,-1) (-1,4)}~~
single period NE = { (C, C), (B, B) }
$\therefore$ RPE = {(C, C)}
for repeated game, repetition of (C,C) gives payoff (T, T)
any deviation will not be profitable, so NE.
if play A, deviation then play B.
~~[illegible]~~. ~~[illegible]~~ since backward induction.
forget first period, both player want to deviate. so end up payoff (0, 0)
$\therefore$ repetition of (C, C) is the only RPE
(repetition of (B, B) is dominated.
play A, deviation then C is not NE)

3) (a) Consider play (M, M) if deviation play (U, L) forever.

if no deviation

M, M $2 + 2\delta + 2\delta^2 + \cdots$

devia. $3 + \delta + \delta^2 + \cdots$

to keep no profit, $\delta \geq \frac{1}{2}$, satisfy

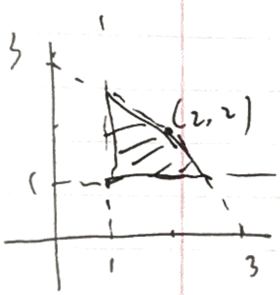
if deviation happens

U, L $1 + \delta + \delta^2 + \cdots$

~~deviation~~ no incentive to deviate.

$\therefore$ (M, M) (U, L) as $\{(1,1), (2,2)\}$ is self-genrating.

(b)



(c) $(-3, -3)$ cannot be self-genrating

since any player will deviate from $(-3, -3)$

and never want to deviate to $(-3, -3)$

This is becawe $(-3, -3)$ is the most punishment to both player so nobody will use this to threat other one.

4. a) $\text{Max } \underline{p}(\underline{x}\,\underline{w} + (1-\underline{x})\underline{L}) + \bar{p}(\bar{x}\,\bar{w} + (1-\bar{x})\bar{L})$

s.t. IR1: $\underline{x}\, u(\underline{\theta}-\underline{w}) + (1-\underline{x})\, u(-\underline{L}) \geq u(0)$

IR2: $\bar{x}\, u(\bar{\theta}-\bar{w}) + (1-\bar{x})\, u(-\bar{w}) \geq u(0)$

IC1: $\underline{x}\, u(\underline{\theta}-\underline{w}) + (1-\underline{x})u(-\underline{L}) \geq \bar{x}\, u(\underline{\theta}-\bar{w}) + (1-\bar{x})\, u(-\bar{L})$

IC2: $\bar{x}\, u(\bar{\theta}-\bar{w}) + (1-\bar{x})\, u(-\bar{L}) \geq \underline{x}\, u(\bar{\theta}-\underline{w}) + (1-\underline{x})\, u(-\underline{L})$

$\bar{x} \leq \underline{p} + \bar{p}/2$

$\underline{p}\,\underline{x} + \bar{p}\,\bar{x} \leq 1/2$

4) b) i) if seller is a_1, buyer is a_2.

seller will trade when $\theta_1 > \theta_2$

so $k(\theta_1, \theta_2) = \begin{cases} 1 & \text{if } \theta_1 > \theta_2 \\ 0 & \text{if } \theta_2 \leq \theta_1 \end{cases}$

$v_1(k) = \theta_1$, ~~$v_2(k)$~~ $= 0$ if $k = 0$

$v_2(k) = \theta_2$ if $k = 1$

$\therefore E_{\theta_2}[v_2(k, \theta_2)] = E_{\theta_2}[\theta_2 \mid \theta_2 > \theta_1]$ For a_1

$$= \int_{\theta_1}^{1} \theta_2 \, d\theta_2$$

$$= \frac{1 - \theta_1^2}{2}$$

$$\gamma_1(\theta_2) = -E_{\theta_1}(v_1(k, \theta_1), \theta_1))$$

$$= -E_{\theta_1}(\theta_1 \mid \theta_1 \geq \theta_2)$$

$$= -\int_{\theta_2}^{1} \theta_1 \, d\theta_1 = \frac{\theta_2^2 - 1}{2}$$

$\therefore$ transfer function $= \frac{1 - \theta_1^2}{2} + \frac{\theta_2^2 - 1}{2} = \frac{\theta_2^2 - \theta_1^2}{2}$

for buyer, it should be $\frac{\theta_1^2 - \theta_2^2}{2}$

ii) for seller, we should solve

$$\max_{\hat{\theta}_1} E_{\theta_2}\left(\theta_1 \cdot Pb(\theta_1 \geq \theta_2) + \frac{\theta_2^2 - \theta_1^2}{2}\right)$$

$$= \theta_1 \cdot \int_0^{\hat{\theta}_1} d\theta_2 - \frac{\hat{\theta}_1^2}{2} + E_{\theta_2}\left(\frac{\theta_2^2}{2}\right)$$

~~$= \theta_1 \hat{\theta}_1 \theta_2 - \frac{\hat{\theta}_1^2}{2} + E_{\theta_2}\left(\frac{\theta_2^2}{2}\right)$~~

F.O.C get $\hat{\theta}_1 = \theta_1$

the same for buyer: $\hat{\theta}_2 = \theta_2$

So it's truth-telling.