Spring 2017 USC Yilmaz Kocer **ECON 504 GAME THEORY**

F I N A L   E X A M

You have <u>110 minutes</u>. Total=115 points, 15 pts BONUS.   GOOD LUCK !!

[1] (20pts) (<u>Curving the grades</u>) Yilmaz has two students **X** and **Y**, in a class. Each student can choose an effort level $\mathbf{x \geq 0}$and $\mathbf{y \geq 0}$ studying towards the exam.

a) (4pts) Suppose Yilmaz <u>does not apply a "curve"</u> to the grading of the students; a student exerting **e** amount of effort gets a net payoff of $\mathbf{-e^2}$, <u>independent</u> of the other student's effort.

If each student is maximizing their net utility, what is the optimal level of effort **e** each student chooses, and what is the net payoff of each?

ENTER GAME THEORY… Now, assume instead Yilmaz decides to <u>apply a "curve"</u> to the students' grading. If they choose (**x** , **y**) levels of effort respectively, students get net payoffs

$$\mathbf{u_X = -x^2 + x(y+2) - y^2} \quad \text{and} \quad \mathbf{u_Y = -y^2 + y(x+2) - x^2}$$

b) (8pts) If each student studies the amount as you found in (a), are they better off or worse off under the new curve system? Will they choose the amounts in (a), that is, are the optimal effort choices in (a) make a pure NE of the effort choice game under the curve system? If not, show there is a profitable deviation by one of the players (find it).

c) (8pts) Find the symmetric pure strategy NE of the effort choice game between the two students (where each student chooses the same effort level). Compared to (a), do they study more under the curve system? Do they get higher net grades/payoffs?

[2] (25pts) Suppose that two players play each other for two periods. In the first period they play the game on the left below, and in the second period they play the game on the right. There is no discounting between periods. Players observe the action their opponent took in the first period before choosing their second period actions.

	L	**R**
U	2,2	-10,x
D	y,0	0,0

	L	**R**
U	8,4	0,0
D	0,0	4,8

a) (12pts) For **x = -2** and **y = 5**, find a pure subgame perfect equilibrium in which player 1 receives a payoff of **10 in total**.

b) (13pts) For **x = y = 4** show that there is no pure subgame perfect equilibrium in which **(U,L)** is played in the first period.

[3] (25pts) (All pay auction) A seller sells an item to 2 bidders each with independent identical valuations drawn from uniform distribution on [0,1] (that is drawn i.i.d. from U[0,1]), to maximize the auction revenue. Each bidder learns their valuations and submit a bid in the auction; the highest bid wins but <u>both players pay their bids</u> (The seller thinks this is going to increase sales revenue).

a) (13pts) Show that there is a symmetric pure BNE where the optimal bid strategy for a player is of the form $\mathbf{s(\theta_i) = k\theta_i^2}$ (find the suitable constant k, too).

b) (12pts) Remember that in the first price auction (with N players each with iid valuations from U[0,1]) the optimal bid for a player of type $\boldsymbol{\theta_i}$ is (N-1)/N $\boldsymbol{\theta_i}$ and the expected value of the highest valuation is $E(\boldsymbol{\theta_i}^{[1]}) = N/(N+1)$. Do the players bid less or more, compared to the optimal bids in the first price auction?

Now calculate the expected payment a player makes in this auction, integrating over all types he can be. Multiplying this by 2, find the expected revenue the seller makes in total from the 2 bidders in the all pay auction. Does the seller make more expected revenues under the all pay auction, or the standard first price auction?

[4] (20pts) **(Perfect Bayesian Equilibrium)** Suppose X is deciding whether to make a friendship request to Y on facebook. They don't know each other. Y thinks that (prior probability) X is either an **I**nteresting person (with probability p) or a **B**oring person (with probability 1-p); but X knows whether he himself is Interesting or Boring. First X chooses whether to make a **F**riendship request to Y or **N**ot. Then Y decides whether to **A**ccept friendship or **R**eject it. X gets a payoff of **1** when his request is accepted and **-1** if rejected, whether he is Interesting or Boring; and gets **0** if he does not make a request. Y gets a payoff of **0** from rejecting any request, or not receiving a request in the first place. Accepting any request gives Y a payoff of **1** if X is Interesting, but **-1** if he is Boring. Assume p>1/2.

a) (15pts) Drawing the extensive form game tree, find a pure BNE of this game that cannot be part of a PBE (remember a PBE is a strategy profile together with a belief system).

b) (5pts) Is there a pure BNE where one type of X makes a Friendship request and the other type does not?

[5] (25pts)

a) (15pts) Alice and Bob seek each other. Simultaneously, Alice puts effort $1 \geq \mathbf{a} \geq \mathbf{0}$ and Bob puts effort $\mathbf{1} \geq \mathbf{b} \geq \mathbf{0}$ to search. The probability of meeting is $\mathbf{ab}$ and the value of the meeting is **1** for each player. The search effort costs $\mathbf{a}^2$ to Alice and $\mathbf{b}^2$ to Bob. (The net utility for each player is the expected benefit minus costs he enjoys/incurs). Find the set of all rationalizable strategies for each player.

b) (10pts) In class, we have seen examples like the Cournot game, find the average game (players try to guess 2/3 of the average) where the unique pure NE was the only outcome that survives IESDS. Find a game (try 3x3) which does have a unique pure strategy Nash equilibrium but has other rationalizable outcomes too (IESDS surviving set of outcomes is not a singleton).