Econ 580 Quiz Solution.

1\. (a) $\pi_i = (200 - Q - 20)\, q_i \quad , \quad i = 1, 2, 3$

FOC: $\quad 200 - Q - 20 - q_i = 0$

Notice that the 3 firms are symmetric, $q_1 = q_2 = q_3, \quad Q = 3q_i$

$\Rightarrow \quad 180 - 4q_i = 0$

$\Rightarrow \quad q_1 = q_2 = q_3 = 45 \quad , \quad P = 200 - 135 = 65$

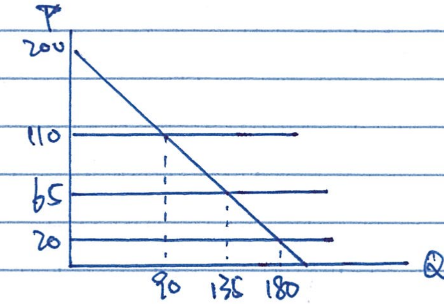
$\pi_1 = \pi_2 = \pi_3 = (200 - 135 - 20) \cdot 45 = 2025$

(b) $\pi_m = (200 - Q_m - 20)\, Q_m$

FOC: $\quad 200 - Q_m - 20 - Q_m = 0$

$\Rightarrow \quad Q_m = 90 \quad , \quad P_m = \cancel{90}\ 110$

$\pi_m = (110 - 20) \cdot 90 = 8100$



(c) $CS_c = \frac{1}{2}(200 - 65) \cdot 135 = 9112.5$

$DWL_c = \frac{1}{2}(65 - 20)(180 - 135) = 1012.5$

$CS_m = \frac{1}{2}(200 - 110) \cdot 90 = 4050$

$DWL_m = \frac{1}{2}(110 - 20) \cdot (180 - 90) = 4050.$

(d) $HHI = s_1^2 + s_2^2 + s_3^2 = 3 \cdot (33.3)^2 = 3326.67$

Technology innovation: it will not change the HHI.

Reason: since each firm's marginal cost is reduced by the same amount, they are still symmetric and the output levels are relatively the same. The market shares are unchanged.

(e) ① $\pi_i = (200 - \frac{1}{2}Q^2 - 20)\, q_i$

FOC: $\quad 180 - \frac{1}{2}Q^2 - Q \cdot q_i = 0$

$Q = 3q_i \quad \Rightarrow \quad 180 - \frac{9}{2}q_i^2 - 3q_i^2 = 0$

$\Rightarrow \quad q_i^2 = 24 \quad , \quad q_i \approx 4.90$

$P = 200 - \frac{1}{2} \cdot 9 q_i^2 = 200 - 108 = 92$

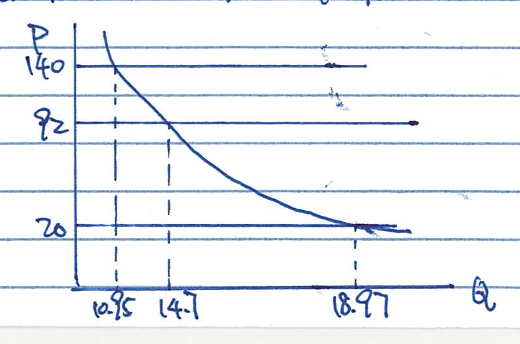
$\pi_1 = \pi_2 = \pi_3 = (92 - 20) \cdot 4.90 = 352.8$

② $\pi_m = (200 - \frac{1}{2}Q_m^2 - 20)\, Q_m$

FOC: $\quad 180 - \frac{1}{2}Q_m^2 - Q_m^2 = 0 \quad \Rightarrow \quad Q_m^2 = 120 \quad , \quad Q_m \approx 10.95$

$P = 200 - \frac{1}{2} \cdot 120 = 140$

$\pi_m = (140 - 20) \cdot 10.95 = 1314.53.$



③ $CS_c = \int_0^{14.7} (200 - \frac{1}{2}Q^2 - 92)\, dQ$

$DWL_c = \int_{14.7}^{18.97} (200 - \frac{1}{2}Q^2 - 20)\, dQ$

$CS_m = \int_0^{10.95} (200 - \frac{1}{2}Q^2 - 140)\, dQ$

$$DWL_m = \int_{10.95}^{18.97} \left(200 - \frac{1}{2}Q^2 - 20\right) dQ.$$

2\. (a) Budget constraint: $q_0 + P_1 q_1 + P_2 q_2 = I$

$$u(q_1, q_2) = (10-P_1)q_1 + (15-P_2)q_2 - \left(\frac{1}{2}q_1^2 + q_1 q_2 + q_2^2\right)$$

FOC: $[q_1]$ $\quad 10 - P_1 - \cancel{4}q_1 - q_2 = 0$

$[q_2]$ $\quad 15 - P_2 - q_1 - 2q_2 = 0$

$$\Rightarrow \begin{cases} P_1 = 10 - q_1 - q_2 \\ P_2 = 15 - q_1 - 2q_2 \end{cases}$$

$$\begin{cases} q_1 = 5 - 2P_1 + P_2 \\ q_2 = 5 - P_2 + P_1 \end{cases}$$

(b) These two goods are substitutes. An increase in the price of the other good causes an increase in the sales of this good.

(c) $\pi_1 = (P_1 - 1)(5 - 2P_1 + P_2)$ $\quad$ FOC: $5 - 2P_1 + P_2 - 2P_1 + 2 = 0$

$\pi_2 = (P_2 - 2)(5 - P_2 + P_1)$ $\quad$ $5 - P_2 + P_1 - P_2 + 2 = 0$

$$\Rightarrow \begin{cases} P_1 = 3,\ q_1 = 4,\ \pi_1 = 8 \\ P_2 = 5,\ q_2 = 3,\ \pi_2 = 9 \end{cases}$$

(d) $\pi_m = \pi_1 + \pi_2$

FOC: $[P_1]$ $\quad 5 + 2P_2 = 4P_1$

$[P_2]$ $\quad 6 + 2P_1 = 2P_2$

$$P_1 = 5.5, q_1 = 2.5, \pi_1 = 11.25$$

$$P_2 = 8.5, q_2 = 2, \pi_2 = 13$$

(e) Upon merger, consumer faces higher prices and lower quantities, therefore the welfare should be lower.

3\. (a) Let the wholesale price be P_W, the retail price be P_R

$\pi_R = (P_R - c_r - P_W) \cdot Q$

$= (10 - Q - 1 - P_W)Q$

FOC: $10 - Q - \underset{9}{1} - P_W - Q = 0$

$\Rightarrow Q = \frac{9}{2} - \frac{1}{2}P_W$

$\pi_M = (P_W - c_m)Q = \frac{1}{2}(P_W - 2)(9 - P_W)$

FOC: $P_W = \frac{11}{2}$

$\Rightarrow Q = \frac{7}{4}$, $P_R = 10 - \frac{7}{4} = \frac{33}{4}$, $CS = \frac{1}{2}\left(\frac{7}{4}\right)^2 = \frac{49}{32}$

(b) $\pi_I = (P_I - c)Q = (P_I - 3)Q = (10 - Q - 3)Q$

FOC: $10 - Q - 3 - Q = 0$

$\Rightarrow Q = \frac{7}{2}$

$\Rightarrow P_I = 10 - \frac{7}{2} = \frac{13}{2}$

$CS = \frac{1}{2}\cdot\left(\frac{7}{2}\right)^2 = \frac{49}{8}$

(c) Consumer surplus is lower when there exists a downstream retailer. The reason is that with a retailer, both the manufacturer and the retailer need to extract some profit margin from the consumer $\Rightarrow$ double marginalization.

(d) Constant elasticity demand: $Q = Ap^{-2}$, $A > 0$ constant.

① $\pi_R = (P_R - P_W - 1)Q = (A^{\frac{1}{2}}Q^{-\frac{1}{2}} - P_W - 1)Q$

FOC: $A^{\frac{1}{2}}Q^{-\frac{1}{2}} - P_W - 1 - \frac{1}{2}A^{\frac{1}{2}}Q^{-\frac{3}{2}}\cdot Q = 0$

$\frac{1}{2}A^{\frac{1}{2}}Q^{-\frac{1}{2}} = P_W + 1$

$Q = \frac{1}{4}A(P_W+1)^{-2}$

$\pi_M = (P_W - 2)Q = \frac{1}{4}A(P_W-2)(P_W+1)^{-2}$

FOC: $\frac{1}{4}A(P_W+1)^{-2} + \frac{1}{4}A(P_W-2)(-2)(P_W+1)^{-3} = 0$

$\Rightarrow \frac{1}{4}A(P_W+1) - \frac{1}{2}A(P_W-2) = 0$

$\Rightarrow P_W = 5$

$\Rightarrow Q = \frac{1}{4}A\cdot 6^{-2} = \frac{1}{144}A$

$P_R = A^{\frac{1}{2}}Q^{-\frac{1}{2}} = 144^2$

② $\pi_I = (P_I - 3)Q = (A^{\frac{1}{2}}Q^{-\frac{1}{2}} - 3)Q$

FOC: $A^{\frac{1}{2}}Q^{-\frac{1}{2}} - 3 - \frac{1}{2}A^{\frac{1}{2}}Q^{-\frac{1}{2}} = 0$

$\Rightarrow Q = \frac{1}{36}A$.

$\Rightarrow P_I = A^{\frac{1}{2}}Q^{-\frac{1}{2}} = 36^2$.

4. (a) $\pi_1 = (P_1 - 4)(10 - 2P_1 + P_2)$

$\pi_2 = (P_2 - 4)(10 - 2P_2 + P_1)$

FOC: $\begin{cases} 10 - 2P_1 + P_2 - 2P_1 + 8 = 0 \\ 10 - 2P_2 + P_1 - 2P_2 + 8 = 0 \end{cases}$

$\Rightarrow \begin{cases} P_1^c = 6,\ q_1^c = 4,\ \pi_1^c = 8 \\ P_2^c = 6,\ q_2^c = 4,\ \pi_2^c = 8 \end{cases}$

$\pi_m = (P_1 - 4)(10 - 2P_1 + P_2) + (P_2 - 4)(10 - 2P_2 + P_1)$

FOC: $10 - 2P_1 + P_2 - 2P_1 + 8\ \cancel{-2P_2+8=0}\ + P_2 - 4 = 0$

$\Rightarrow P_1^m = P_2^m = 7,\ q_1^m = q_2^m = 3,\ \pi_1^m = \pi_2^m = 9.$

(b) Suppose Firm 1 deviates.

$\pi_1 = (P_1 - 4)(10 - 2P_1 + 7)$.

FOC: $P_1 = \frac{25}{4}$

$\pi_1^D = (\frac{25}{4} - 4)(10 - \frac{25}{2} + 7) = \frac{9}{4} \cdot \frac{9}{2} = \frac{81}{8}$

(c) For the collusion to sustain, we need:

$\frac{81}{8} + 8\delta + 8\delta^2 + \cdots \leq 9 + 9\delta + 9\delta^2 + \cdots$

$\Rightarrow \frac{81}{8} + 8 \cdot (\frac{\delta}{1-\delta}) \leq 9(\frac{1}{1-\delta})$

$\Rightarrow \frac{81}{8}(1-\delta) + 8\delta \leq 9$

$\Rightarrow \frac{9}{8} \leq \frac{17}{8}\delta$

$\Rightarrow \delta \geq \frac{9}{17}$

(d) ① $\pi_1 = (P_1 - 3)(10 - 2P_1 + P_2)$ FOC: $\begin{cases} 10 - 2P_1 + P_2 - 2P_1 + 6 = 0 \\ 10 - 2P_2 + P_1 - 2P_2 + 8 = 0 \end{cases}$

$\pi_2 = (P_2 - 4)(10 - 2P_2 + P_1)$

$\Rightarrow \begin{cases} P_1 = \frac{82}{15}, & q_1 = \frac{74}{15}, & \pi_1 = \frac{37}{15} \cdot \frac{74}{15} \approx 12.17 \\ P_2 = \frac{88}{15}, & q_2 = \frac{56}{15}, & \pi_2 = \frac{28}{15} \cdot \frac{56}{15} \approx 6.97 \end{cases}$

$\pi_m = (P_1 - 3)(10 - 2P_1 + P_2) + (P_2 - 4)(10 - 2P_2 + P_1)$

FOC: $\begin{cases} 16 - 4P_1 + P_2 + P_2 - 4 = 0 \\ 18 - 4P_2 + P_1 + P_1 - 3 = 0 \end{cases}$

$\Rightarrow \begin{cases} P_1 = 7.5, & q_1 = 2, & \pi_1 = 9 \\ P_2 = 7, & q_2 = 3.5, & \pi_2 = 10.5 \end{cases}$

Notice that a transfer from Firm 2 to Firm 1 is needed.

② Let the amount of transfer be t, $t \geq 12.17 - 9 = 3.17$, $t \leq 10.5 - 6.97 = 3.53$

- Firm 1 deviates:

$\pi_1 = (P_1 - 3)(10 - 2P_1 + 7)$, $P_1 = \frac{23}{4}$

$\pi_1^D = (\frac{23}{4} - 3)(10 - \frac{23}{2} + 7) = \frac{11}{4} \cdot \frac{11}{2} = \frac{121}{8} = 15.125$

- Firm 2 deviates:

$\pi_2 = (P_2 - 4)(10 - 2P_2 + 7.5)$, $P_2 = \frac{51}{8} = 6.375$

$\pi_2^D = (\frac{51}{8} - 4)(10 - \frac{51}{4} + \frac{15}{2}) = \frac{19}{4} \cdot \frac{19}{8} = \frac{361}{32} \approx 11.281$

~~For the collusion to sustain:~~

~~- Firm 1 not deviate:~~

$\cancel{15.125 + (9+t)\delta + (9+t)\delta^2 + \cdots \leq}$

For the collusion to sustain:

- Firm 1 not deviate

$$15.125 + 12.17\delta + 12.17\delta^2 + \cdots \leq (9+t) + (9+t)\delta + (9+t)\delta^2 + \cdots$$

$$12.17\left(\frac{1}{1-\delta}\right) + 2.96 \leq (9+t)\left(\frac{1}{1-\delta}\right)$$

$$\delta_1 \geq \frac{6.13-t}{2.96} = \delta_1^*$$

- Firm 2 not deviate:

$$11.281 + 6.97\delta + 6.97\delta^2 + \cdots \leq (10.5-t) + (10.5-t)\delta + \cdots$$

$$6.97\left(\frac{1}{1-\delta}\right) + 4.311 \leq (10.5-t)\left(\frac{1}{1-\delta}\right)$$

$$\delta_2 \geq \frac{0.781+t}{4.311} = \delta_2^*$$

If firm 1 & 2 have a common discount factor δ:

$$\delta \geq \max\{\delta_1^*, \delta_2^*\}$$