Midterm, March 9, 2010, Answer key

Problem 1 (10 points each)

a. The relation is

$$y_i = \beta_1 + \beta_2 x_i + \beta_3 x_i^2 + \varepsilon_i$$

The estimate of the average marginal utility is

$$\hat{\beta}_2 + 2\hat{\beta}_3 \overline{x}$$

with $\overline{x}$ the sample mean of labor input.

b. It is likely that labor and capital inputs are correlated for two reasons (i) a scale effect, i.e. large firms have high levels of both labor and capital inputs, (ii) a substitution effect, i.e. for given size firms that employ relatively more labor inputs may employ relatively fewer capital inputs. The first will most likely dominate so that labor inputs is positively related the omitted variable that itself has a positive effect. The omitted variable bias is likely to be positive, but I have given full credit to everyone who thinks that the other effect is larger.

c. Not likely a proxy because capital inputs are not mean independent of labor inputs given the wage, but see my suggested answer to d.

d. One could argue either that the wage is a proxy or that it is an instrument, although the case for the latter is much stronger. Any argument should make sense economically. It could be argued that the wage is a proxy by an appeal to the marginal productivity of labor being equal to the wage rate. This requires that capital is an input that can be adjusted flexibly. However the same equality violates the second requirement for a proxy that capital input is mean independent of labor input given the wage. For the wage rate to be a valid instrument it should not have a direct effect on output and also

be mean independent of capital inputs. If capital is a fixed input (or hard to adjust in the short run) and the wage is not set by the firm, but the firm hires labor on the market (or one of many markets if labor inputs are heterogeneous) where it is a small player and hence considers the wage rate as given, then the equality of the wage and the marginal product of labor determines the firms demand for labor inputs. Hence the condition that the instrument has an effect on the endogenous variable holds. Because the wage is not set by the firm, it is likely to be independent of the capital input of the firm. The wage only affects the output through its effect on labor inputs. Of course if capital inputs are not fixed then they may depend on the wage rate, so that the wage is not a valid instrument. I have given full credit to everyone who argued either case (proxy or instrument) in a fully consistent manner.

e. The IV estimator with wage as instrument will give a consistent estimator. Consistency means that the IV estimator converges in probability to the population regression coefficient.

f. The model is

$$y_{it} = \beta_1 x_{it} + \alpha_i + \varepsilon_{it}$$

with α_i the contribution of capital inputs (and other time constant variables) to output.

g. Because labor and capital inputs are likely to be correlated, we prefer an FE type estimator. We can either use the FD estimator or the FE estimator.

h. The FE estimator requires that $\mathrm{E}[\varepsilon_{\mathrm{it}}|\mathrm{x}_{\mathrm{i1}}, \ldots, \mathrm{x}_{\mathrm{iT}}] = 0$ while the FD estimator is consistent if $\mathrm{E}[\varepsilon_{\mathrm{it}}|\mathrm{x}_{\mathrm{i,t-1}}, \mathrm{x}_{\mathrm{it}}, \mathrm{x}_{\mathrm{i,t+1}}] = 0$.

Problem 2 (20 points each) The model is

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i$$

a. The new average is $\hat{\beta}_1 + \hat{\beta}_2 1.2\overline{x}_2 + \hat{\beta}_3(\overline{x}_3 + .04$, so that the change for the average household is $.2\hat{\beta}_2\overline{x}_2 + .04\hat{\beta}_3$.

b. Define $\gamma_1 = \beta_1$, $\gamma_2 = .2\beta_2\overline{x}_2 + .04\beta_3$, $\gamma_3 = \beta_3$. Hence

$$\beta_2 = \frac{\gamma_2 - .04\gamma_3}{.2\overline{x}_2} = \frac{5}{\overline{x}_2}\gamma_2 - \frac{.2}{\overline{x}_2}\gamma_3$$

so that

$$y_i = \gamma_2 + 5\frac{x_{i2}}{\overline{x}_2}\gamma_2 + \left(x_{i3} - .2\frac{x_{i2}}{\overline{x}_2}\right)\gamma_3 + \varepsilon_i$$

c. With 5000 observations we use the normal approximation, so that the confidence interval is [-20.55-1.96*3.75, -20.55+ 1.96*3.75]= [-27.9, 13.20].