Note 1, Econ 537

The Principal Agent Model:

There is a principal who is considering hiring an agent. The principal cannot get any income or profit without the agent working for her. The agent has an outside option if he doesn't work for the principal. The outside option depends on the productivity of the agent. If the productivity is represented by $\theta,$ the outside option (or the highest expected income from other firms' offers) is $\mu(\theta) = \frac{4}{3}\theta.$ The productivity also affects the income generated by the agent if the agent works for the principal. The total generated income is 2θ if the productivity is $\theta.$ This income belongs to the principal, and the agent only gets the wage income not profit income. The parameter θ is uniformly distributed over the interval $[a,b].$

The productivity of the agent is known only to the agent. but unknown to the principal. Note that if the principal knows the productivity $\theta,$ then the best wage offer (from the principal's perspective) is $\mu(\theta) = \frac{4}{3}\theta.$ This is the minimum wage the principal has to offer to induce the agent to work for her. With this offer, the principal gets 2θ minus the wage $w = \frac{4}{3}\theta.$ Hence the principal's net profit is $\frac{2}{3}\theta.$ The agent gets $\frac{4}{3}\theta.$ This is often called the complete information contract. The agent only gets income from the outside option. Any additional surplus is extracted by the principal.

When the productivity of the agent is unknown to the principal, the principal does not know whether the wage offer w will be accepted or not. It will only be accepted if the wage offer is above the agent's outside option (which is also unknown to the principal). Once the offer is accepted, the principal's income is also uncertain, as it is $2\theta - w$ depending on the productivity of the worker. To analyze the optimal wage offer (from the principal's perspective), we take the following steps:

First we decide the probability that the offer will be accepted. If wage offer is $w,$ the agent accepts the offer and work for the principal if $w \geq \mu(\theta) = \frac{4}{3}\theta.$ Therefore the offer is accepted if and only if $\theta \leq \frac{3}{4}w.$ Note that the principal only gets the lower part of the productivity level, and will not get an agent with productivity above $\frac{3}{4}w.$ The possibility of offer rejection (by high productivity agents) means that the principal is losing profit potential. The principal may not want to attract the agent with the highest productivity $b,$ because she fears that the agent may turn out to have low productivity, and in this case, she is paying too high a wage. If $\frac{3}{4}w \in [a,b],$ then the probability of getting the offer accepted is $\frac{\frac{3}{4}w-a}{b-a}.$ If $\frac{3}{4}w$ is below $a,$ then the offer is not accepted. If $\frac{3}{4}w$ is above $b,$ it is accepted for sure. There is no need to consider wage offer such that $\frac{3}{4}w$ is outside the range $[a,b].$ If it is below, then it is rejected anyway, and no profit is generated. If it is above, then you can lower the wage cost by offering $w = \frac{4}{3}b,$ and get it accepted with a higher profit for the principal.

The second step is to figure out what is the profit for the principal if the offer is accepted. When it is accepted, then the income generated by the agent is uniformly distributed between $2a$ and $\frac{3}{2}w.$ A simple formula for the expected income is $\frac{2a+\frac{3}{2}w}{2} = a + \frac{3}{4}w.$ The expected profit of the principal is that income subtracted by the wage cost or $a + \frac{3}{4}w - w = a - \frac{1}{4}w.$

The third step is to figure out the expected profit. This is just the product of the probability of acceptance and the profit after acceptance. Hence the expected profit is

$$\frac{\frac{3}{4}w - a}{b - a}(a - \frac{1}{4}w).$$

Note that when the principal raises the wage $w,$ the probability of getting accepted is higher, but the profit after acceptance is lower. This is the trade-offer in determining the optimal offer $w.$

The last step is to find out the optimal offer w by maximizing the expected profit. This profit function is quadratic, and in general it is easy to figure out what is the optimal offer by solving the wage offer w which maximizes the profit function. Note that corner solutions are possible. When we take the first-order condition

$$\frac{d}{dw}\left[\frac{\frac{3}{4}w-a}{b-a}(a-\frac{1}{4}w)\right]=\frac{1}{b-a}[a-\frac{3}{8}w]=0,$$

and solve for $w,$ we get $w=\frac{8}{3}a.$ Note that w should be in the range $[\frac{4}{3}a,\frac{4}{3}b].$ The only situation in which this may occur is when $w=\frac{8}{3}a>\frac{4}{3}b.$ When this is the case, the optimal offer is to have $w=\frac{4}{3}b.$ At this wage, the offer is accepted for sure. This is a corner solution (not the one determined by the first-order condition). In this solution, there is always acceptance of the contract (probability of acceptance is one).

The payoff of the agent is $w=\frac{8}{3}a.$ If the agent accepts the offer, then $\theta\leq\frac{3}{4}w=2a,$ or $a\geq 0.5\theta,$ as we have shown earlier. Hence the payoff of the agent when there is acceptance of the offer is $\frac{8}{3}a\geq\frac{4}{3}\theta$ with strict inequality when $\theta<\frac{3}{4}w.$ In other words, the agent always get a payoff better than his outside option, and the principal is not able to extract the surplus entirely. Thus in the case of incomplete information, the principal suffers for two reasons: (1) her offer may be too low, and rejected, even though the two can share the surplus from working together. (2) Even if the offer is accepted, the principal is not able to pay the lowest possible wage (meaning $\mu(\theta)$), and yield some surplus to the agent.

Contractual Failure

One thing we notice is that when $a=0,$ the optimal offer is $w=0.$ This implies the offer is rejected with probability one. No agent wants to take up the offer because the offer is too low, and the outside option is preferred. This is called contractual failure. When the principal lowers the wage, less productive agents accept the offer, and the lower income generated out weighs the wage saving. So the principal ends up reducing the wage to 0. If the principal tries to raise the wage to attract more productive agents, the higher income generated is not sufficient to pay for the higher wage, because she cannot get the most productive agents to accept the offer.

Efficient Outcome Under Incomplete Information

When there is incomplete information, it is possible that the outcome is inefficient as in the case of contractual failure. However, there are situations in which contractual offer is accepted with probability one despite the lack of information about productivity by the principal. In this case, we say that we have efficient outcome even when there is incomplete information. In the example above, the corner solution means efficient contractual outcome.

Questions to ask:

How does outside option affect the optimal wage offer?
How does productivity affect the optimal wage offer?
Under what conditions (on productivity and outside options) do we have efficient outcome?

Homework 1

Due on Jan 18.

Based on your readings above, do all problems in the Get Acquainted Quiz posted online.