ECON 504 Game Theory
Spring 2017

MIDTERM 1 ANSWER KEY

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[1] a)- With no signal he guesses A, being %60 right $\boxed{EU_{\text{no signal}} = 0.6}$

After any signal(s), you'll guess A iff posterior belief ≥ 0.5

1 signal: First check what to do after signals;

After A signal posterior belief: $Pr(A \mid \text{signal } A) = \dfrac{\%60\ \%80}{\%60\ \%80 + \%40\ \%20} = \dfrac{6}{7} > \dfrac{1}{2}$ $\Rightarrow$ guess A

B signal " : $Pr(A \mid \text{signal } B) = \dfrac{\%60\ \%20}{\%60\ \%20 + \%40\ \%80} = \dfrac{3}{11} < \dfrac{1}{2}$ $\Rightarrow$ guess B indeed

$EU_1 = \%60(\%80 \cdot 1 + \%20 \cdot 0) + \%40(\%20 \cdot 0 + \%80 \cdot 1)$

$= 0.8$ marginal value of 1st signal $= 0.8 - 0.6 = \boxed{0.2}$

b)- 2 signals: Given above analysis: after AA signal $\rightarrow$ guess A

BB signal $\rightarrow$ guess B

and as signals are symmetric, AB or BA signal $\rightarrow$ guess A

AB signals cancels each other out & you're left with the prior $\rightarrow$ hence guess A

$EU_2 = \%60(\%80\%80 \cdot 1 + \%20\%20 \cdot 0 + \%80\%20 \cdot 1 + \%20\%80 \cdot 1)$

$+ \%40(\%80\%80 \cdot 1 + \%20\%20 \cdot 0 + \%80\%20 \cdot 0 + \%20\%80 \cdot 0) = 0.832$

marginal value of 2nd signal is $\boxed{0.032}$

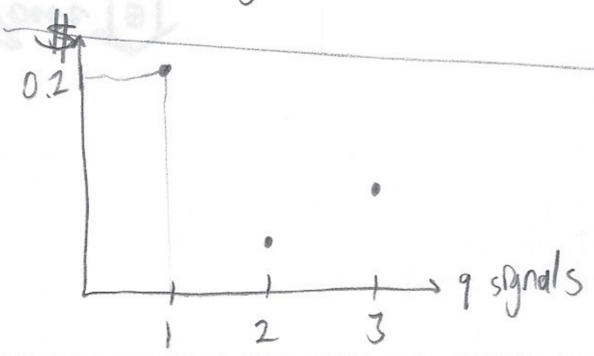
c) 3 signals Given above analysis, you'd guess the majority of the signals (at least 2 out of 3) You'd be wrong iff either 2 or all 3 signals are wrong;

$EU_3 = pr(\text{at least 2 signals are correct}) \cdot 1 = (0.8)^3 + \binom{3}{1}(0.8)^2(0.2) = 0.512 + 0.384$

$= 0.896$

marginal value of 3rd signal is $0.896 - 0.832 = \boxed{0.064}$

d)-

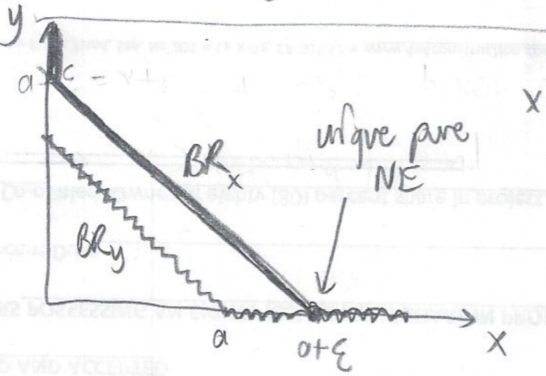


No value of info. not diminishing
(this is due to a discreteness effect;
for ex 2 signals, AB ends up with no info.
at all – similarly for BA –)

[2] a) $\frac{\partial}{\partial x} u_x(x,y) = \frac{d}{dx}\left(-\left(x+y-(a+\varepsilon)\right)^2\right) = -2\left(x+y-(a+\varepsilon)\right) = 0 \quad x = a+\varepsilon-y$

$$BR_x(y) = \begin{cases} a+\varepsilon-y & \text{if } y \le a+\varepsilon \\ 0 & \text{o.w} \end{cases} \quad \text{similarly} \quad BR_y(x) = \begin{cases} a-x & x \le a \\ 0 & \text{o.w} \end{cases}$$

b)

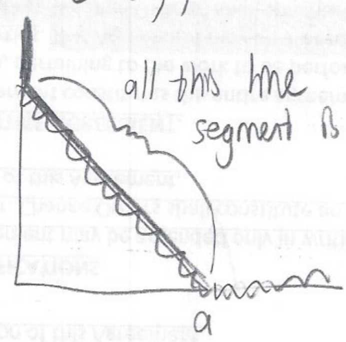


$x = a+\varepsilon \quad y = 0$

Heather does all the work!

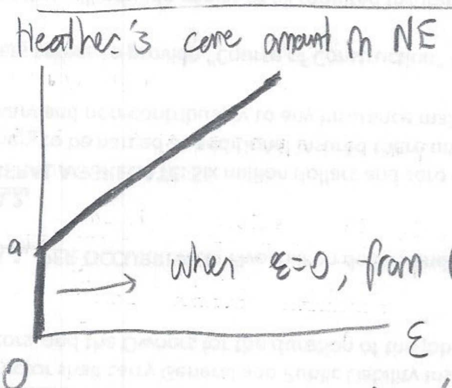
Yilmaz gives 0 care!

c) If $\varepsilon = 0$, in the above graph $BR_x \Leftrightarrow BR_y$ coincide



$NE = \{(x,y) \geq (0,0)$ such that $x+y=a\}$

d)



when $\varepsilon = 0$, from (c) any $x \in [0,a]$ would be a part of NE.

This is not a function! (at 0, it takes infinitely many values).

At $\varepsilon = 0$, the correspondence "shrinks" for any $\varepsilon > 0$ in the neighborhood.

Hence NE is upper semicontinuous but not lower-semicontinuous.

[3] a)

		K^p	L^q	M^{1-p-q}
a	A	3,1	0,0	1,3
$1-a$	B	0,0	2,2	0,0

(First Note that M weakly dominates K, hence if A is used with positive pr. by player 1 ; K cannot be chosen.)

Note that if player 1 plays a pure str. (doesn't mix); player 2 doesn't mix either as best response.

If player 2 plays a pure str., similarly player 1 has a pure best response.

So, in mixed NE both players have to mix.

Then by the first fact, K cannot be played $p=0$ $\left(a+(1-a)\cdot 0 \le 3a+(1-a)\cdot 0 \quad \forall a>0 \;\therefore\; K \text{ is worse than } M\right)$

For pl. 1 to be indifferent btw A & B; $1(1-q) = 2q \quad q=\frac{1}{3}$

Hence player 2 should play $\left(0,\frac{1}{3},\frac{2}{3}\right)$

For pl. 2 to be indifferent btw L & M $\quad 2(1-a) = 3a \quad a=\frac{2}{5}$

unique NE $= \left(\left(\frac{2}{5},\frac{3}{5}\right),\left(0,\frac{1}{3},\frac{2}{3}\right)\right)$

(b) 1st round of elimination: $d < c$ (strictly dominated) then $y < x$

2nd: $a, b < c \quad w, z < x$

$\Rightarrow (x, c)$ uniquely survives IESDS

the only rationalizable outcome

[3

[4] ① Nobody can be ticketed in a pure NE, as they could safely choose 70 and not get ticketed, and be better off.

② Suppose not everybody is driving with the same speed. Then there is at least two speeds being used. Anybody in the slow speed group can switch to a higher speed (and not get ticketed ①) and be better off.

③ Everybody driving the same speed, for each $v \in [70, 100]$ is a NE. And there's no other pure NE. The eqm. set doesn't depend on n.

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[5] a) $\begin{array}{|c|c|} \hline a,b & c,d \\ \hline e,f & g,h \\ \hline \end{array}$ Suppose top left is str. dom. eqm. $a>e$ $c>g$ $b>d$ $f>h$ should hold

Probability $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

For any one of the 4 boxes, total prob $= 4 \times \frac{1}{16} = \boxed{\frac{1}{4}}$

(note that all 4 events are mutually exclusive)

b) For each of the a^N outcomes to be str. dominant eqm, for one player's choice should be str. dominant $\Rightarrow pr = \left(\frac{1}{a}\right)^{a^{N-1}}$

hence for the outcome to be a str. dom. eqm. $= \left(\left(\frac{1}{a}\right)^{a^{N-1}}\right)^N$

total prob over a^N outcomes $= \boxed{a^N \cdot \left(\frac{1}{a}\right)^{a^{N-1} N}}$

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[6] Assume other player is playing a x^* cutoff strategy that is drawing a new number when the first draw $\leq x^*$

For x^* to be an optimal cutoff for me, I should be indifferent between redrawing or not when I get exactly x^* in my first draw.

I drew x^* in the first draw;

i). If I keep my number (not redraw);

$$EU = (1-x^*)\cdot 0 + x^*\cdot x^* = (x^*)^2$$

opponent draws $\geq x^*$ or not in his first draw

I lose if I stick to my first number

opponent draws another number as he drew $\leq x^*$ first time, and draws another number $\leq x^*$ for me to win.

ii). If I redraw;

$$EU = (1-x^*)(1-x^*)\frac{1}{2} + x^*\cdot\frac{1}{2} = \frac{(1-x^*)^2+x}{2}$$

opponent's first draw being $\leq x^*$ or not

in case my second draw $\geq x^*$ and only then with $\frac{1}{2}$ prob., I win.

opponent's first draw was $\leq x^*$. Now we are both drawing a second time, $\frac{1}{2}$ probability I win.

Now for x^* to be a symmetric NE;

$$x^2 = \frac{(1-x)^2+x}{2} \Rightarrow 2x^2 - x^2 + 2x - 1 = x$$

$$x^2 + x - 1 = 0$$

$$x = \frac{-b\mp\sqrt{\Delta}}{2a} \Rightarrow x = \frac{-1+\sqrt{1-(-4)}}{2} = \frac{\sqrt{5}-1}{2} = 0.62.$$

(note that if I draw 0.5 first time, I draw again, for example.)