## Spring 2017

Get acquainted Quiz: The purpose of this quiz is to find out your knowledge about basic game theory concepts, and your economic background so that the lectures will be delivered later at the proper level. It has nothing to do with grades. Work only on questions 1-5 unless you are comfortable with the starred questions after them.

## Name:

There is a principal who is considering hiring an agent. The principal cannot get any income or profit without the agent working for her. The agent has an outside option if he doesn't work for the principal. The outside option depends on the productivity of the agent. If the productivity is represented by  $\theta$ , the outside option (or the highest expected income from other firms' offers) is  $\mu(\theta) = \frac{4}{3}\theta$ . The productivity also affects the income generated by the agent if the agent works for the principal. The total generated income is  $2\theta$  if the productivity is  $\theta$ . This income belongs to the principal, and the agent only gets the wage income not profit income. The parameter  $\theta$  is uniformly distributed over the interval [1,2].

The productivity of the agent is known only to the agent. but unknown to the principal. When the productivity of the agent is unknown to the principal, the principal does not know whether the wage offer w will be accepted or not. It will only be accepted if the wage offer is above the agent's outside option (which is also unknown to the principal). Once the offer is accepted, the principal's income is also uncertain, as it is  $2\theta - w$  depending on the productivity of the worker.

Note that the more able agent can get higher offers from outside. We assume that the agent will accept the wage offer if it exceeds his outside offer. Given the probability distribution of the productivity, if the accepting agent has productivity  $\theta$  in the interval  $[1,\theta_0]$ , the probability of the agent accepting the wage is  $\theta_0 - 1$ . The threshold  $\theta_0$  is determined by setting the wage equal to the outside option.

Since the agent has no profit share, there is no incentive design in the wage structure. The agent is less likely to accept a given wage if he is more productive. Hence given a fixed wage, the productivity of the accepting agent will be increasing in the wage offer.

## Questions:

- 1. If the agent productivity is  $\theta$ , and the offer is w, what is the condition that the offer is accepted? (Write an inequality in  $\theta$  and w)
- 2. If the principal offers the wage  $w = \frac{4}{3}$ , argue that the offer is accepted with probability 0.
- 3. If the principal offers the wage  $w = \frac{8}{3}$ , argue that the offer is accepted with probability 1.
  - 4. Given  $w, \frac{4}{3} \le w \le \frac{8}{3}$ , what is the range of  $\theta$  so that the offer is accepted?
  - 5. Given  $w, \frac{4}{3} \le w \le \frac{8}{3}$ , what is the probability that the offer is accepted?
- \*6. Given  $w, \frac{4}{3} \le w \le \frac{8}{3}$ , and assume that the offer is accepted, what is the expected profit of the principal?
- \*7. The expected profit of the principal (whether or not the offer is accepted or not) is the probability of acceptance (answer from question 5) times the profit after the offer is

accepted (answer from equation 6). Write down the expected profit as a function of w.

\* 8. Assume that the first order condition is sufficient for the optimal solution, find out the optimal wage offer by maximizing the function in question 7.