Answers for HW2:

1. (a) The offer will be accepted if and only if $2\theta * \frac{2}{3} + p \geq \frac{4}{3}\theta$, or $p \geq 0$. That is, any offer of non-negative payment to agent 2 is accepted. The probability of acceptance is 1.

(b) From (a), we know that any charge will be refused. Therefore only the probability of acceptance is 0 if $f > 0$. When $f = 0$, it is accepted with probability 1.

(c) Since the offer is accepted whenever $p \geq 0$, the optimal p is 0.

(d) The optimal profit for agent 1 is the expected value of $\frac{2}{3}\theta$ or $\frac{a+b}{3}$.

(e) The optimal profit in the original sharing arrangement is $\frac{1}{2}(a + \frac{b}{3}) = \frac{1}{2}a + \frac{1}{6}b$. Since the difference of profits is $\frac{1}{6}(b - a) > 0$, the profit in (d) is higher.

(f) The intuition you get higher profit in (d) is that agent 1 extracts all the surplus from agent 2, and while agent 2 keeps some surplus in the original sharing contract.

2. (a) In the first step, agent 2 accepts the offer if and only if $\theta + p \geq \mu(\theta) = \frac{3}{4}\theta$, or $\theta \geq -4p$. Clearly p needs to be negative, and we let $f = -p$ represent the fee that needs to be paid by agent 2. So agent 2 accepts the offer if and only if $\theta \geq 4f$. The probability of the offer being accepted is $\frac{b-4f}{b-a}$.

In the second step, the profit of agent 1 after the offer is accepted is $\frac{b+4f}{2} + f = \frac{b+6f}{2}$.

In the third step, the expected profit of agent 1 is $\frac{b-4f}{b-a}\frac{b+6f}{2}$.

To find the optimal fee, we take the first-order condition with respect to f, and gets

$$2b - 48f = 0,$$

or $f = \frac{b}{24} = \frac{1}{24}$. The optimal $p = -\frac{1}{24}$.

(b) The optimal p is negative.

(c) The equilibrium profit for agent 1 is

$$\frac{b - 4f}{b - a}\frac{b + 6f}{2} = (1 - \frac{1}{6})\frac{1.25}{2} = \frac{25}{48} = 0.520\,83$$

(d) yes, higher option leads to lower f or higher p.

3. (a) The conditional probability of winning without removal when the original choice was wrong, and you switch is $\frac{1}{5}$.

(b) The probability of winning is $\frac{5}{6} * \frac{1}{5} = \frac{1}{6}$.

(c) The conditional probability of winning without removal when the original choice was wrong, and you switch is $\frac{1}{4}$.

(d) The probability of winning is $\frac{5}{6} * \frac{1}{4} = \frac{5}{24}$.

(e) You should switch because it improves your chance of winning from $\frac{1}{6}$ to $\frac{5}{24}$.

(f) If you don't switch, the probability of winning is $\frac{1}{n}$. If you switch, the probability of winning is $\frac{n-1}{n} * \frac{1}{n-2} = \frac{1}{n} * \frac{n-1}{n-2} > \frac{1}{n}$ because $n - 1 > n - 2$. Hence it is always better to switch.