

# Innovation, monopolies and the poverty trap

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## Abstract

This paper considers an avenue by which the structure of industries in an economy can affect the development of new technologies through its general equilibrium impact on profits relative to wages. We show that a monopolistic structure in one industry, by increasing the share of profits in aggregate income, tends to increase the relative profitability of innovative activities elsewhere thereby leading to the creation of further monopoly rents which, in turn, feeds back into incentives for innovation thus causing a self-perpetuating cycle. This leads to the possibility of an economy exhibiting multiple steady states including a 'poverty trap' or situation of zero growth. We analyze the conditions under which multiple steady states exist and characterize the economy's behaviour out of the steady state. The role of government intervention, in the form of subsidies and direct provision of research is also examined.

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## 1. Introduction

This paper endogenizes market structure in a framework of growth through creative destruction in a simple and tractable way. In partial equilibrium analyses, i.e., at an industry level, it has long been understood that industrial structure matters in affecting incentives for innovation, as expounded upon by Schumpeter

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(1951) and surveyed in Kamien and Schwartz (1982). In contrast, here we explore a general equilibrium effect that industrial structure has on incentives for innovation and growth. An immediate implication of considering industrial structure endogenously is the possibility of multiple steady states, one of which is a growth trap or situation of zero growth. Multiple steady states arise here principally because of self-reinforcing effects between technological change and market structure, that is, because technological change affects market structure in a way which increases future technological change, thereby once again altering market structure and so on. An advantage of this explanation is that, instead of arising from spillovers or production non-convexities, or due to complementarities in demand or production as in previous models (i.e. through the exogenously posited characteristics of the technology), multiplicity here arises through the effect of technological change on market structure (i.e. from the process of technological change itself).

At an industry level, the impact of market structure on innovation has received much attention in partial equilibrium analyses (as expounded upon by Schumpeter (1951) and surveyed in Kamien and Schwartz (1982)). We, however, focus on the general equilibrium effect of industrial structure on innovation, through its impact on factor returns, and the subsequent effect, in turn, of innovation on industrial structure. The crucial factor we wish to highlight is that returns to a factor employed in research relative to production are lower when all sectors are competitive than when there exist sectors which are monopolistic. Thus a monopolistic industrial structure, i.e. one in which other industries tend to be dominated by monopolists rather than being competitive, raises relative returns to innovation in non-directly related industries and is, therefore, more conducive to technological progress and growth. Furthermore, since industrial structure is itself the outcome of past innovations (i.e. there are more likely to be monopolists in industries which have devoted resources to innovative activities than in industries which have not), current returns to innovation depend upon previous levels of innovation through the inherited industrial structure and can thus make innovation a self-sustaining process.

Recently there have been many models focusing on the process of growth through the purposeful development of new technology. In contrast to the previous literature in which aggregate productivity increased almost incidentally, often as a byproduct of investment in another activity (e.g. Arrow (1952); Romer (1986)) this generation of research focuses on the intended creation of new knowledge. Aghion and Howitt (1992) and Grossman and Helpman's (1991, Ch. 4) quality ladders model explore the Schumpeterian process of growth through innovation and creative destruction.<sup>1</sup> In these models, creative destruction takes the following form: innovation leads to the displacement of an incumbent monopolist by a

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<sup>1</sup> See also Shell (1957), Romer (1990) and Segerstrom et al. (1990)).

successful innovator and thus to the transfer of monopoly rents from the former to the latter. The lure of monopoly rents leads profit seeking agents to devote resources to innovative activities. As research activities are carried out in a decentralized manner, the size of potential rents is an important determinant of the intensity of research activity and therefore of growth. Importantly, these models implicitly start with incumbent monopolists already holding infinite patents on the use of their technology, so that when a new innovation occurs in an industry, one incumbent is simply replaced by another and the underlying industrial structure remains basically unchanged.

The point of these models is to capture, in a tractable way, the effects of knowledge spillovers and cascading technological progress in explaining how an economy can exhibit sustained growth. In contrast, this paper aims to understand the process leading to the point where these previous analyses take over. That is, by generalizing these previous approaches slightly and allowing some industries to not initially be dominated by monopolists, we firstly demonstrate that an economy otherwise capable of sustained technological progress and growth may be caught in a no growth steady state or 'Poverty Trap'. We determine, in terms of exogenous parameters, the conditions under which a poverty trap can emerge, and show the initial conditions under which an economy will be caught there. We then seek to understand the process of transition from stagnation to self-sustained growth, and the role of government therein, through an analysis of the model's transitional dynamics.

This paper is also related to another branch of the endogenous growth literature which has been concerned with explaining the diversity of observed cross country growth experiences through models exhibiting multiple steady states (Azariadis and Drazen, 1990; Durlauf, 1993; Matsuyama, 1991). Their analyses draw on a venerable tradition in development economics dating back to the theory of the Big Push of Rosenstein-Rodan (1943), the discussion of balanced growth of Nurkse (1953) and the emphasis on stages of growth and the take-off of Rostow (1956). This literature shared in common the belief that some form of complementarity between individuals (often investors) could explain underdevelopment as a low level equilibrium trap, and that the process of development therefore implied a drastic move of the economy from a low level equilibrium to a higher one. Although, retrospectively, the insights provided by the early contributors may not always appear theoretically well founded, they offered historical evidence of economies experiencing the sorts of dramatic changes which were consistent with their evocative explanations of complementarities and coordination problems. More recently, Murphy et al. (1989) provide a theoretical foundation for the possibility of poverty traps by explaining plausible mechanisms which give rise to these complementarities. In contrast, the endogenous growth literature on multiple steady states is not primarily concerned with the mechanism leading to complementarities, per se, but rather with developing the implications of such complementarities in an endogenous growth framework, or, as in Matsuyama (1991),

understanding the conditions under which coordination can affect a movement from a dominated steady state. Despite this being a model of endogenous growth, we think of our model as more closely related to the work of Murphy et al. (1989) in its focus on the mechanism giving rise to multiple steady states, though we also undertake some analysis of transition paths.

The terms ‘technological progress’ and ‘innovation’ used above have different interpretations depending on the context in which they are used. In the modern creative destruction literature, concerned with already developed economies, attention is focused on one particular aspect of innovative activities, namely, the creation of new or better products or inputs. Innovative activity there implies the use of resources in research and development with an aim to both generating new ideas (development of new products, technological improvements...) and seizing rents on them through exclusion (patents, confidentiality, contracts or licences). In the context of LDCs however, a different interpretation of innovation, emphasizing entrepreneurship rather than R&D, seems more relevant. It is not so much lack of R&D which appears to constrain LDCs to the use of less productive technology but rather insufficient entrepreneurial incentive to adopt new methods by borrowing from ideas already developed elsewhere. Thus, in our view, a theory of growth emphasizing the role of entrepreneurship may yield useful insights into the causes of low productivity in LDCs. In a later section, therefore, we emphasize the entrepreneurial interpretation of innovation, with a view to using the insights developed in the basic model to explain stagnation in LDCs.

Before developing the main model, Section 2 of the paper presents a stylised version in a static framework in order to provide some basic intuition for the general equilibrium effect causing the complementarity which motivates the paper. Section 3 then presents a dynamic model, based on Grossman and Helpman (1991), in which multiple steady states are shown to exist and determines the conditions under which they do. This section also examines the welfare properties of the steady states and describes the dynamics of the system between them. Section 4 discusses a possible role for government research support, either through subsidies or publicly provided research, in moving the economy away from Pareto dominated situations. In Section 5 the entrepreneurial interpretation of innovation is developed, with emphasis placed on the self-generating nature of the entrepreneurial process. A brief conclusion is provided in Section 6.

## 2. The static model

There is one final good denoted  $y$  which is produced competitively using a continuum of intermediate inputs distributed along a unit interval according to the following production function:

$$y = e^{\int_0^1 \ln x_i di}, \quad (1)$$

where  $x_i$  stands for the intermediate good  $i$ . The final good is taken as the numeraire:  $p_y = 1$ . There are  $L$  consumers with well-behaved preferences over the final good. Individuals are all identical and each is endowed with one unit of inelastically supplied labour.

Labour is used in production of intermediate inputs and in research activities. As labour is homogeneous across all sectors and activities, there is a unique wage rate denoted  $w$ .<sup>2</sup> Intermediate inputs are assumed to be produced according to a linear production technology with labour as the only input as follows:

$$x_i = A_i \ell_i, \quad (2)$$

where  $A_i$  denotes labour productivity and  $\ell_i$ , the amount of labour used in the production of intermediate good  $i$ . The level of labour productivity depends upon the arrival of an innovation in that sector. Innovations take the form of a new technology in which labour productivity is  $\gamma$  times higher than under the previous technology (with  $\gamma > 1$ ), so that the production function for the intermediate good  $i$  after innovation becomes

$$x_i = \gamma A_i \ell_i. \quad (3)$$

Following Grossman and Helpman (1991, p. 97) we assume constant returns to scale in research and a Poisson process governing the arrival of innovations so that if  $\ell_{Ri}$  units of labour are devoted to research in sector  $i$ , then the instantaneous probability of an innovation occurring in that sector is given by  $\delta \ell_{Ri}$ , where  $\delta$  is an exogenously fixed parameter exceeding zero. An advantage of this specification is that it abstracts from explicitly considering the structure of research firms. It is assumed that within each sector at most one innovation can occur.<sup>3</sup> A successful innovator becomes a monopolist in the production of that intermediate good using the new technology.

Since the final good sector is competitive, it follows from the final good production function that final good producers minimize costs by allocating expenditures evenly over all inputs. Thus, as the final good is the numeraire and it is a competitive sector, the unit cost of producing the final good is equal to 1:1 is therefore also equal to expenditure on each input  $i$  (since there is a unit interval of inputs). The monopolist's decision problem is therefore very simple; profit is

<sup>2</sup> Uniqueness of wage rates is a simplifying assumption only, the model could easily be extended to allow for heterogeneous labour (for example with private investment in human capital) and multiple equilibrium wage levels without affecting the results provided labour is substitutable between production and research.

<sup>3</sup> This is a strong assumption which is made for expositional convenience here. However in the dynamic version of the model, with continuous time, the analogous assumption is that innovations do not occur simultaneously.

maximized by limit pricing at the marginal cost of the firm's best competitor. Profits for a successful innovator in sector  $i$  are therefore denoted by

$$\begin{aligned}\Pi_i &= \left( \frac{1}{A_i} - \frac{1}{\gamma A_i} \right) w x_i = \left( \frac{1}{A_i} - \frac{1}{\gamma A_i} \right) w \frac{y}{p_i} = \left( \frac{1}{A_i} - \frac{1}{\gamma A_i} \right) w \frac{y A_i}{w} \\ &= \left( \frac{\gamma - 1}{\gamma} \right) y.\end{aligned}\quad (4)$$

We normalize  $L$  to 1, so that the labour-market clearing condition can be written as

$$\int_0^1 \ell_i di + \int_0^1 \ell_{Ri} di = 1. \quad (5)$$

### 2.1. The competitive economy equilibrium

Let us first consider a situation where all intermediate firms share a common technology and, therefore, produce all intermediate goods competitively. We proceed by first assuming that no research is undertaken in this economy, and then establish the conditions under which this assumption is valid. The production function of each firm is given by Eq. (2) and the price of any intermediate good is therefore given by  $w/A_i$ . From Eq. (1), the amount of each input used to produce output  $Y$  equals  $y/p_i$ , or equivalently  $A_i y/w$  which corresponds to  $y/w$  units of labour. Therefore, labour market clearing implies

$$1 = \int_0^1 \frac{y}{w} di \Rightarrow y = w. \quad (6)$$

Let us now consider the profits which would accrue to a successful innovator. From Eqs. (4) and (6), these are equal to  $((\gamma - 1)/\gamma)w$ . As long as the costs of undertaking research exceed the expected benefits, the above described equilibrium is the unique equilibrium of this economy. This is given by the following equation:

$$\left( \frac{\gamma - 1}{\gamma} \right) w \delta < w \quad \text{or} \quad \left( \frac{\gamma - 1}{\gamma} \right) \delta < 1. \quad (7)$$

If the above inequality does not hold, then any equilibrium must involve a positive amount of research.

### 2.2. The monopolistic economy equilibrium

We now consider the situation in which, within each industry, one firm only has access to the technology described in Eq. (2). All other firms have in common the following inferior technology:

$$x_i = \epsilon A_i \ell_i \quad (8)$$

where  $\epsilon < 1$ . Note that this case differs from the previous only in the relationship between firms *currently* producing with the technology and their competitors, new innovators, however, receive the same mark up in both cases. Thus, within each industry, the currently privileged firm limit prices its potential competitors out of the market by setting its price at their marginal cost of production which equals  $w/\epsilon A_i$ . Once again, competition in the final good market implies demand of  $y/p_i$ , which here corresponds to  $(y/w)\epsilon A_i$  units of each input. For each input, labour demanded therefore equals  $y\epsilon/w$ . Equilibrium in the labour market implies

$$1 = \int_0^1 \frac{\epsilon}{w} di \Rightarrow y = \frac{w}{\epsilon}. \quad (9)$$

A comparison of Eqs. (6) and (9) shows the main point of this section, namely that the ratio of  $w$  to  $y$  is lower when the economy initially has monopolies in each sector. Note, however that the total amount of output in the economy does not differ across the two situations; all labour is employed and the technology used in production of intermediate inputs is still given by Eq. (2). What differs is that the existence of monopolists in each sector reduces labour's share of total income and, since the real wage is lower, the cost of undertaking research is thereby reduced. Furthermore since the expected benefits of research depend on the size of sales and therefore income,  $y$ , they remain unaltered across situations. As a result, the condition for there to be no research in equilibrium, corresponding to that given by Eq. (7) above, is weaker when industries are monopolistic:

$$\left( \frac{\gamma - 1}{\gamma} \right) w\delta < \epsilon w \quad \text{or} \quad \left( \frac{\gamma - 1}{\gamma} \right) \delta < \epsilon. \quad (10)$$

A comparison of (10) with (7) shows that the existence of a monopolist in other intermediate sectors raises the incentives for research in any one sector. This arises even though the Cobb–Douglas specification of final good production ensures no direct complementarity between intermediate sectors, and, importantly, even though the mark-up rate a successful innovator would command is identical. It arises simply as a result of the general equilibrium impact of monopolies on relative returns. It therefore follows that there exists a range of values for  $\gamma$  such that condition (10) is not satisfied while condition (7) is. In other words, for such a range of parameters, a competitive industrial structure involves no research while, in a monopolistic structure, research will occur.<sup>4</sup> This holds even though, considering any one sector alone, returns to innovation in that sector do not depend upon whether the current best technology is used exclusively by an incumbent monopolist or available to all producers.

<sup>4</sup> Note that, in the above static framework, the no research outcome is socially optimal. We ignore this aspect of the issue here since, as will be clear later, this need not be true in a dynamic context.

### 3. The dynamic model

The above model showed that industrial structure affects incentives for research by influencing labour's share of total income, or alternatively, by altering the proportion of aggregate income accruing in the form of monopoly rents. However, properly considered, the process of research and innovation is inherently dynamic: current resources are invested in expectation of future profits. In this section we therefore develop the basic insight of the model presented above in an intertemporal framework similar to that of both Aghion and Howitt (1992) and Grossman and Helpman (1991). Though similar to Grossman and Helpman (1991) in particular, our approach differs in that we allow for endogenous change in industrial structure, that is for the possibility of industries which are, initially at least, not monopolistic. We also pursue a different method of solution and normalization to that used in Grossman and Helpman (1991) so as to best make clear the impact of changes in industrial structure on incentives for innovation and therefore growth. It will be shown that, in this situation, it is possible that the economy converges towards different steady states.

#### 3.1. The consumer's problem

There exist  $L$  infinitely lived agents with the following intertemporal preferences:

$$U_t = \int_t^{\infty} e^{-\rho(\tau-t)} \ln C(\tau) d\tau \quad (11)$$

where  $C(\tau)$  denotes the amount of final good consumed at time  $\tau$  and  $\rho$  is the rate of time preference. As in Section 2 the production function for final goods is given by

$$\ln C(\tau) = \int_0^1 \ln x_i(\tau) di, \quad (12)$$

where  $x_i(\tau)$  denotes the amount of industry  $i$ 's input used at time  $\tau$  in production of the final good. Let the final good be numeraire, then each individual's intertemporal budget constraint is given by

$$\int_t^{\infty} e^{-(R(\tau)-R(t))} C(\tau) d\tau \leq W(\tau) + \int_t^{\infty} e^{-(R(\tau)-R(t))} w(\tau) d\tau \quad (13)$$

with

$$R(\tau) = \int_0^{\tau} r(s) ds, \quad (14)$$

where  $R(\tau)$  represents the discount factor from time  $\tau$  to time 0 and  $W(\tau)$  denotes the net present value of the household's wealth at time  $\tau$  (see Eq. (2.7) in Grossman and Helpman (1991)). The left-hand side of Eq. (13) represents the net present value of expenditure and the right hand side, the net present value of wage



and asset income. In the aggregate, the right-hand side of the budget constraint consists of total wage income and net assets held by consumers.

Standard dynamic optimization of the utility function, Eq. (12), subject to the intertemporal constraint, Eq. (13), yields

$$\frac{\dot{C}(\tau)}{C(\tau)} = -\rho + r(\tau), \quad (15)$$

which describes an agent's optimal consumption path.

### 3.2. The intermediate monopolist's problem

As in Section 2, in each sector, intermediate goods are produced linearly with labour as the only input. At time  $t$ , the production function for firm  $j$  in sector  $i$  is defined as

$$x_i^j = A_i^j(t) \ell_i^j \quad (16)$$

where  $A_i^j(t)$  denotes labour productivity of firm  $j$  in sector  $i$  at time  $t$ . Since production is linear and Eq. (12) implies unit elastic demand for intermediate goods, the firm with highest labour productivity within each sector, if unique, acts as a constrained monopolist and limit prices at the marginal cost of its closest competitor. Thus, if  $A_i^M(t)$  denotes the most productive firm's productivity in sector  $i$  and  $A_i^c(t)$  denotes the productivity of their closest rival, then instantaneous profits for the sector  $i$  monopolist are given by

$$\Pi_i(t) = \left( \frac{w(t)}{A_i^c(t)} - \frac{w(t)}{A_i^M(t)} \right) y(t) \frac{A_i^c(t)}{w(t)} \quad (17)$$

where  $y$  and  $w$  again denote output of the final good and wages respectively. Note that since there is no storage in the model, aggregate consumption always equals aggregate output, i.e.,  $C(t) = y(t)$ .

If the highest labour productivity technology is shared by more than one firm, it is assumed that these firms act as Bertrand competitors<sup>5</sup> and, by competition, all profits are dissipated with price equalling marginal costs of production, that is

$$p_i(t) = \frac{w(t)}{A_i(t)}. \quad (18)$$

Note that we interpret innovations as increases in the productivity of labour in intermediate production. There are many alternative interpretations which yield an identical structure. Grossman and Helpman (1991, Ch. 4) interpret innovations as

<sup>5</sup> Modelling interaction between firms with access to similar technology in some other way which does not imply the complete dissipation of profits will lower the magnitude of impact associated with industrial structure. However, the qualitative nature of the results presented here will be completely unaffected.

either raising the productivity of inputs used in production of the final good (as do Aghion and Howitt (1992)), or, assuming away intermediate industries, as increasing the quality of final goods measured by an index of utility.

### 3.3. Research

It is assumed that, if a firm successfully innovates, its labour productivity rises to a level  $\gamma$  times greater than that of the previous incumbent monopolist (where  $\gamma > 1$ ). Successful innovations within an industry are assumed not to arrive simultaneously.<sup>6</sup> Note that, as in Aghion and Howitt (1992) and Grossman and Helpman (1991), a research success yields spillovers: it raises the initial level of all future research to that of the successful innovator. Nevertheless, even though the knowledge embodied in new technology is completely public, the use of that technology in production is protected by a patent of infinite duration.

Within an industry, innovations arrive at the instantaneous rate  $\delta \ell_{Ri}(t)$  where  $\ell_{Ri}(t)$  denotes the total amount of labour devoted to research in industry  $i$  at time  $t$ . The parameter  $\delta$  denoting the productivity of labour in research, as in Section 2. A successful research firm obtains a flow of instantaneous profits as defined in Eq. (17), until the arrival of a superior technology reduces the value of its patent to zero. We can think of research firms as financing their investments by selling claims to the future stream of potential profits. As in both Aghion and Howitt (1992) and Grossman and Helpman (1991), incumbent monopolists do not undertake research because the net present value of a successful innovation is lower than for a potential entrant. Let  $V_i(t)$  denote the present value of this flow of profits in sector  $i$ . Then, as there is free entry into research, in equilibrium the marginal expected value of research cannot exceed its marginal costs:<sup>7</sup>

$$w(t) \geq \delta V_i(t) \text{ with equality if } \ell_{Ri}(t) > 0. \quad (19)$$

Since  $\gamma$  is identical across sectors and since there is equal expenditure on intermediate inputs in the production of the final good, profit to a monopolist within industry  $i$  is given by

$$\Pi_i(t) = \frac{\gamma - 1}{\gamma} y(t) \quad (20)$$

by substituting  $\gamma^{n_i(t)-1}$  for  $A_i^C$  and  $\gamma^{n_i(t)}$  for  $A_i^M$  in Eq. (17), where  $n_i(t)$  is the number of innovations that have taken place in industry  $i$  upto time  $t$ . As a result,

$$V_i(t) = V_j(t) = V(t) \quad \forall i, j. \quad (21)$$

<sup>6</sup> Or, since this is such a low probability event in a continuous time world, simply ignored as in Grossman and Helpman (1991).

<sup>7</sup> Since firms finance investment by the sale of equity shares in the future stream of potential profits, and since the arrival of innovations is governed by independent distributions across sectors, equity shareholders will diversify and receive riskless returns. Firms thus only consider the expected value of return to research, and not their variance, when deciding upon investment.

Moreover, since  $\delta$  is also identical across sectors, research effort will be distributed evenly:

$$\ell_{Ri}(t) = \ell_{Rj}(t) = \ell_R(t) \quad \forall i, j. \quad (22)$$

As a result, in equilibrium, Eq. (19) can be re-written as

$$w(t) \geq \delta \ell_R(t) V(t) \text{ with equality if } \ell_R(t) > 0. \quad (23)$$

Since the arrival of research success is independent across industries and since there exist a continuum of industries, the economy wide level of innovation success is completely determined by the level of research. Also, as capital markets are perfect, research investments can be completely diversified so that returns to research are non-stochastic. In equilibrium, the economy wide value of wealth simply equals the value of the aggregate claims on research firms.

### 3.4. Instantaneous equilibrium

Here we closely follow the procedure used in Section 2. By appropriate normalization we assume that  $L = 1$  and  $A_i(0) = 1$  for all  $i$ . Labour market clearing then implies that

$$1 = \int_0^1 \ell_{Ri}(t) di + \int_0^1 \ell_i(t) di, \quad (24)$$

$$1 = \ell_R(t) + \alpha(t) \frac{y(t)}{\gamma w(t)} + (1 - \alpha(t)) \frac{y(t)}{w(t)} \quad (25)$$

where  $\alpha(t)$  defines the number of sectors in which production is undertaken by a technologically dominant monopolist. This can be rearranged to yield

$$y(t) = \frac{(1 - \ell_R(t)) w(t) \gamma}{\alpha(t) + (1 - \alpha(t)) \gamma}. \quad (26)$$

It is worth noting that, as in the static model of Section 2, the relationship between  $y(t)$  and  $w(t)$  depends upon industrial structure. Eq. (26) shows that the general equilibrium consequence of an increase in the number of sectors dominated by a unique monopolist, an increase in  $\alpha$ , is a lowering of wages relative to output.

We now derive an expression for the expected present value of the flow of profits to a monopolist. We first define the real wage rate at time  $t$  in terms of the state variables,  $\alpha(t)$  and  $n_i(t)$ , the number of innovations having occurred in industry  $i$  upto time  $t$ . From Eq. (12) and by substituting in the demand for each intermediate input,  $y(t)/p_i(t)$ , the amount of final good produced is given by

$$\ln y(t) = \int_0^1 \ln \frac{y(t)}{p_i(t)} di \quad (27)$$

which implies

$$0 = \int_0^1 \ln \frac{1}{p_i(t)} di = - \int_0^\alpha \ln \frac{w(t)}{\gamma^{n_i-1}} di - \int_\alpha^1 \ln \frac{w(t)}{\gamma^{n_i}} di, \quad (28)$$

and therefore,

$$\ln w(t) = (\ln \gamma) \int_0^{\alpha(t)} (n_i(t) - 1) di = (\ln \gamma) \int_0^t \delta \ell_R(\tau) d\tau - \alpha(t)(\ln \gamma). \quad (29)$$

This expression defines the wage rate in two equivalent ways. The middle expression is a slight manipulation of (28) where the integration over sectors 0 to  $\alpha(t)$  involves monopolistic sectors, while integration from  $\alpha(t)$  to 1 is over competitive sectors which disappear since no innovations have arrived in these sectors and the denominator equals 1. The last part of (29) makes use of the deterministic nature of innovation successes at the aggregate level and the symmetry of labour allocation to research. This implies that integrating across sectors with research successes to determine the aggregate number of successes is equivalent to integrating over the instantaneous probability of a research success arriving from 0 to the present date.<sup>8</sup>

The flow of instantaneous profits defined in (17) simplifies down to  $((\gamma - 1)/\gamma)y(t)$  which is then discounted by the interest rate and the probability of a new research success driving future profits to zero to determine the net present value of the flow of profits to a monopolist,  $V(t)$ , as

$$V(t) = \int_t^\infty e^{-(R(\tau) - R(t))} e^{\int_t^\tau -\delta \ell_R(s) ds} \frac{\gamma - 1}{\gamma} y(\tau) d\tau. \quad (30)$$

Thus, using (26) and  $w(t)$  from (29), we obtain

$$V(t) = \int_t^\infty e^{-(R(\tau) - R(t))} e^{\int_t^\tau -\delta \ell_R(s) ds} \frac{\gamma - 1}{\gamma} \frac{(1 - \ell_R(\tau))\gamma}{\alpha(\tau) + (1 - \alpha(\tau))\gamma} w(\tau) d\tau. \quad (31)$$

<sup>8</sup> There are a few technical issues glossed over here. Firstly, note that for the integral in the middle expression to be well defined we must be able to order sectors by their number of research successes, and, since these occur discretely, to integrate over the pieces. This is always possible since the arrangement of sectors along the continuum is arbitrary and can be adjusted to ensure integrability. A further issue is the integrability of  $\delta \ell_R(t)$  through time. This does not matter in steady states where  $\ell_R$  is fixed, nor on saddle paths where  $\ell_R$  will be shown to vary continuously, but is liable to be important when  $\ell_R$  changes discretely (as will be seen to occur when policy variables change). The integral is again only well defined up to the point of discontinuity so we must again proceed by integrating the pieces.

The first term within the first integral,  $e^{-(R(\tau)-R(t))}$ , is the discount rate, the second term,  $e^{\int_t^\tau -\delta \ell_R(s) ds}$ , is the probability of at least one innovation having occurred in the sector up to time  $\tau$ , i.e. the probability of being displaced before time  $\tau$ , the third,  $(\gamma - 1)/\gamma$ , is the monopolist's markup, the fourth,  $(1 - \ell_R(\tau))$ , is the amount of labour directly used in production, the fifth term,  $\gamma/[\alpha(\tau) + (1 - \alpha(\tau))\gamma]$ , the inverse of labour's factor share of income, and finally the term,  $w(\tau)$ , denotes the wage rate at  $\tau$ . An instantaneous equilibrium is then defined by a value of  $\ell_R(t)$  such that, after substituting in for  $V(t)$  from Eq. (31), the inequality in Eq. (23) is satisfied. It can be seen that expectations about future values matter both in affecting the expected life of the patent, through the second term and through their effect on future wages and aggregate income.

### 3.5. Steady states

In this framework, a steady state is defined as an allocation of labour, between research and production which does not change through time. We restrict attention to perfect foresight equilibria only. Here we will first establish the conditions under which there exists a steady state in which there is a positive amount of research, secondly we derive the conditions under which there is no labour allocated to research and no growth in the steady state, the poverty trap, and finally we show that there exist a range of parameter values such that the two steady states coexist.

#### 3.5.1. A steady state with positive research

In any steady state with positive  $\ell_R(t)$  it must be the case that  $\alpha = 1$ , i.e., the proportion of industries in which there exists a monopolist with a technological advantage equals one. This is because Eq. (20) shows that a leader's profit flow does not vary with the number of innovations in a sector and with constant returns to scale in research across sectors, research efforts are targeted evenly across sectors with equal profit flows. Thus with  $\ell_R(t) > 0$ , if  $\alpha < 1$  it will rise through time, and since Eqs. (23) and (31) show that  $\ell_R(t)$  increases with  $\alpha$ , this cannot be a steady state.

Define the steady state level of  $\ell_R(t)$ ,  $\bar{\ell}_R$ , which, as previously shown, will be equal across industries. Due to the arrival process governing innovations, the number of innovations that have taken place up to time  $t$  is given by

$$\int_0^t n_i(s) ds = \int_0^t \delta \ell_R(s) ds. \quad (32)$$

In the steady state, (26) becomes

$$y(t) = (1 - \bar{\ell}_R)w(t)\gamma. \quad (33)$$

Using Eqs. (29) and (32),  $w(t)$  can be expressed as follows:

$$w(t) = \frac{1}{\gamma} e^{(\ln \gamma) \int_0^t \delta \ell'_R(s) ds}. \quad (34)$$

The present value of holding a monopoly is then given by

$$V(t) = \int_t^\infty e^{-(R(\tau) - R(t))} e^{-\delta \ell'_R(\tau - t)} \frac{\gamma - 1}{\gamma} (1 - \bar{\ell}_R) \gamma \frac{1}{\gamma} e^{(\ln \gamma) \int_0^\tau \delta \ell'_R(s) ds} d\tau. \quad (35)$$

From Eqs. (33) and (34), it is clear that the wage rate and the output, at the stationary equilibrium, grow at a constant rate  $(\ln \gamma) \delta \ell'_R$ . As a result, from Eq. (14) and remembering that the numeraire is the final good, the stationary equilibrium interest rate is then given by  $\tilde{r} = \rho + (\ln \gamma) \delta \ell'_R$ . Since, by definition,  $R(s) = \tilde{r}s$ , integrating the expression in (35) and using Eq. (23) yields the following steady state value for  $\ell'_R$ :

$$\delta \frac{1 - \bar{\ell}_R}{\delta \bar{\ell}_R + \rho} (\gamma - 1) = 1. \quad (36)$$

It is easy to verify that any solution to (36), where it exists, is unique.<sup>9</sup> The existence of a solution to (36) between 0 and 1 depends upon the values of exogenous parameters  $\gamma$ ,  $\rho$  and  $\delta$ . We can re-arrange (36) to obtain an expression for the steady state allocation of labour in research  $\bar{\ell}_R = (\delta(\gamma - 1) - \rho) / \delta\gamma$ . It can be seen that it is always the case that  $\bar{\ell}_R < 1$  and  $\bar{\ell}_R$  will exceed zero, i.e., a steady state with growth will exist, provided

$$\delta(\gamma - 1) - \rho > 0. \quad (37)$$

A steady state with sustained growth is more likely to exist the higher is the productivity advantage of new products,  $\gamma$ , the higher the productivity of labour in research,  $\delta$ , and the lower the discounting of the future  $\rho$ , as one would expect.

### 3.5.2. A steady state with no research

In a steady state with no research, technology does not change and there is no growth in per capita incomes, this may be thought of as corresponding to a poverty trap. Without research in the steady state, the initial level of  $\alpha$ , which we denote  $\alpha_0$ , persists through time. Since there is no growth it must also be the case that

$$\frac{\dot{C}}{C} = \frac{\dot{y}}{y} = \frac{\dot{w}}{w} = 0. \quad (38)$$

<sup>9</sup> Note that by substituting  $\ell'_R/a$  for  $\delta \ell'_R$  and  $\delta$  for  $1/\gamma$  in Eq. (37) we obtain an expression which is identical to that obtained in Eq. (4.18) of Grossman and Helpman (1991).

As a result,  $r = \rho$ . One can also express the equilibrium level of output in the steady state as follows:

$$y = \frac{\gamma}{\alpha_0 + (1 - \alpha_0)\gamma} w \quad (39)$$

Thus, the present value of a patent equals

$$V(t) = \int_t^\infty e^{-\rho(\tau-t)} \left( \frac{\gamma-1}{\gamma} \right) \frac{\gamma w}{\alpha_0 + (1 - \alpha_0)\gamma} d\tau. \quad (40)$$

Finally, there should be no incentive to allocate labour to research in the steady state. This holds as long as the inequality defined in Eq. (23) is satisfied, that is

$$\frac{\delta}{\rho} \frac{(\gamma-1)}{\alpha_0 + (1 - \alpha_0)\gamma} < 1. \quad (41)$$

For given  $\gamma$ , we define  $\alpha^c$  as the value of  $\alpha$  which solves (41) with equality, in the linear case above this becomes  $\alpha^c = -(\delta/\rho) + \gamma/(\gamma-1)$ . Note that, as expected,  $\alpha^c < 1$  if and only if (37) holds. Provided  $\alpha_0 < 1$ , it can be seen by comparing Eq. (36) to Eq. (41), that there always exist values of  $\gamma$  and  $\rho$  such that the economy could be caught in a poverty trap, even though it is also capable of experiencing sustained growth. For  $\alpha_0 = 0$  we can thus determine the broadest conditions under which both steady states exist. These are given by  $\gamma$  and  $\rho$  such that  $\delta(\gamma-1)/\rho\gamma < 1$  and  $\ell_R > 0$ . Thus, using (37) this yields the following range of parameter values for which both a steady state with sustained growth and a growth trap exist:

$$\rho < \delta(\gamma-1) < \rho\gamma. \quad (42)$$

If there were a unique steady state, any initial industrial structure and hence any initial ratio of aggregate income to wages would end up not mattering in the long run, provided the economy eventually attains its steady state. When both steady states exist, however, history matters, in the sense that the initial industrial structure may determine which of the steady states is eventually attained. But, if there exist multiple paths of transition between the steady states, sufficiently optimistic expectations may outweigh the effects of history and generate a ‘take-off’ out of the growth trap, as in Matsuyama (1991). Now, we therefore turn to an analysis of the model’s dynamics in order to determine the economy’s behaviour out of steady states.

### 3.6. Dynamics

The behaviour of the system out of steady states is defined by equations of motion for  $\alpha$  and  $\ell_R(t)$ . In this section we show that when in the neighbourhood of the steady state with growth, the economy converges, along a saddle path, to

that steady state. However, it will be seen that the equations of motion describing the economy's evolution are highly non-linear and thus do not admit the possibility of a qualitative global analysis. We therefore revert to simulation methods in order to describe the economy's transition between the steady states.

The equation of motion for  $\alpha(t)$  is simply given by the level of research in period  $t$ . The instantaneous arrival rate of innovation governed by the Poisson process is given by  $\delta\ell_R(t)$ , however  $\alpha(t)$  changes only when innovations arrive in the sectors which are not already monopolistic so that only a proportion  $1 - \alpha(t)$  of sectors experiencing a new innovation actually change industrial structure. Thus the instantaneous change in  $\alpha(t)$  is given by

$$\dot{\alpha}(t) = (1 - \alpha(t))\delta\ell_R(t), \quad (43)$$

thus  $\alpha(t)$  is unchanged if and only if either there is no research in period  $t$  or  $\alpha(t) = 1$ .

In the neighbourhood of the steady state with growth, or with any positive allocation of labour to research, Eq. (23) holds with equality, so that

$$\frac{w}{V} = \delta, \quad (44)$$

with time variables suppressed from here on. After some manipulation, the details of which are in Baland and Francois (1995), using Eqs. (15), (20), (26) and (43) we obtain an expression for  $\dot{\ell}_R$ :

$$\dot{\ell}_R = \left( \delta\ell_R \left( 1 + \frac{(\gamma - 1)(1 - \alpha)}{\alpha(1 - \gamma) + \gamma} \right) + \rho - \frac{(\gamma - 1)(1 - \ell_R)\delta}{\alpha + (1 - \alpha)\gamma} \right) (1 - \ell_R). \quad (45)$$

Fig. 1 depicts this equation for  $\dot{\ell}_R = 0$ , which is easily verified to be upward sloping in  $\ell_R, \alpha$  space and to cut the horizontal axis at  $\alpha = \alpha^c$ . The equation of motion for  $\alpha$  also shows that  $\dot{\alpha} = 0$  only where either  $\alpha = 1$  or  $\ell_R = 0$ . The diagram thus shows the models two possible types of steady state. If the initial value of  $\alpha$  lies between 0 and  $\alpha^c$ , incentives to invest in research are too low to induce research, so that in any instant the inequality in (23) holds strictly and  $\alpha$  remains at its initial value. However if the economy, starts with  $\alpha = 1$ ,  $\ell_R = \bar{\ell}_R$ , sustained research levels of  $\bar{\ell}_R$  are maintained each instant and (23) holds with equality.

We now show the existence of a saddle path, locally, to the steady state with sustained growth, i.e. where  $\ell_R = \bar{\ell}_R$ . By linearizing the system around the point where  $\alpha = 1$  we obtain

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\ell}_R \end{bmatrix} = \begin{bmatrix} -\delta\bar{\ell}_R & 0 \\ z & \delta\gamma \end{bmatrix} \begin{bmatrix} \alpha \\ \ell_R \end{bmatrix} + K, \quad (46)$$



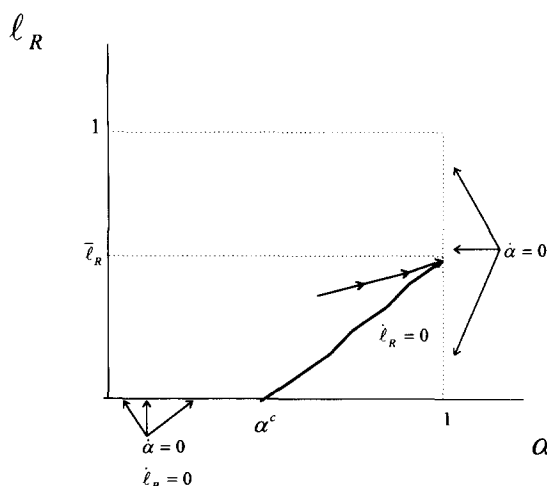


Fig. 1.

where  $K$  denotes residual terms and  $z$  is a constant. Since the product of the diagonal is always negative in the case of own effects and zero in the case of cross effects, the determinant of the square matrix is negative and there thus exists a saddle path to the steady state with sustained growth. The saddle path is depicted in Fig. 1. For a given value of  $\alpha$ , levels of research above the saddle path value imply an increase in labour allocated to research until eventually  $\ell_R = 1$ . However, in this case the instantaneous value of a leading firm's profits is infinite which violates the transversality condition in the consumer's optimization problem. On the contrary, too low a level of research implies that research eventually stops. However, with  $\alpha > \alpha^c$  and  $\ell_R = 0$ , this implies that the value of labour in research exceeds that in production which is inconsistent with the labour market clearing each instant. Thus in the neighbourhood of  $\alpha = 1$ ,  $\ell_R = \bar{\ell}_R$ , the economy moves along the saddle path until  $\alpha = 1$  and converges to a steady state with growth and constant allocation of labour to research.

Linearizing the model in the neighbourhood of the steady state with growth, unfortunately, yields no insight into the economy's performance a discrete distance away from that steady state. However the complexity of Eq. (45) in particular, implies that a qualitative analysis cannot be undertaken. We thus provide simulations of the economy's path when  $\alpha_0 \in (\alpha^c, 1)$ .

In choosing parameter values from the admissible range we are somewhat hampered by the model's abstraction. In particular, as a referee points out, without capital accumulation, it is difficult to generate plausibly high growth levels without assuming an unreasonably high mark-up rate. We thus conduct and include results from both high mark-up/growth simulations, low ones and inter-

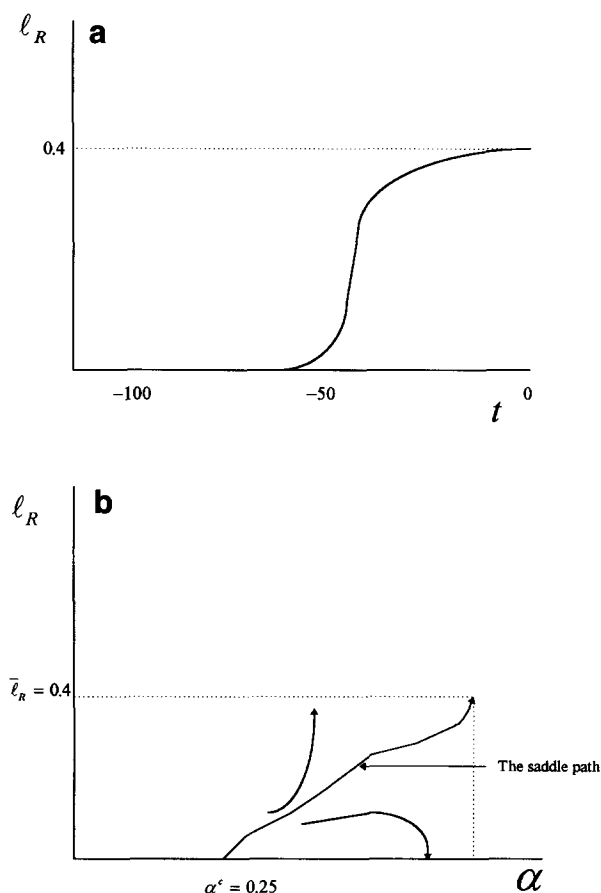


Fig. 2. Middle values of  $\rho$  and  $\gamma$ .  $\gamma = 3$ ,  $\rho = 0.8$ ,  $\bar{\ell}_R = 0.4$ ,  $\alpha^c = 0.25$ . Time path of labour in research from vicinity of growth trap to steady state with growth.

mediate values, with their corresponding mark-up rates and growth rates. Importantly, the qualitative characteristics of the analysis are identical in all three cases and also in all other simulations which we have conducted from the admissible range. We simulate for the simplest possible research technology, i.e.,  $\delta = 1$ , choosing values of  $\gamma$  and  $\rho$  to satisfy Eq. (42) evaluated with  $\alpha_0 = 0$ , i.e. to ensure that both steady states exist. The three  $\gamma$  and  $\rho$  pairs included are: ( $\gamma = 30$ ,  $\rho = 0.99$ ), ( $\gamma = 3$ ,  $\rho = 0.8$ ) and ( $\gamma = 1.02$ ,  $\rho = 0.0197$ ). The mark-up and instantaneous growth rates for each of the three cases are:  $\gamma = 30$ , mark-up = 0.9667, growth = 3.175, for  $\gamma = 3$ , mark-up = 0.6667, growth = 0.43 and if  $\gamma = 1.02$ , mark-up = 0.0197 and growth = 0.0000058. We present detailed analysis for the case of  $\gamma = 3$ ,  $\rho = 0.8$ , which implies  $\alpha^c = 0.25$  and  $\bar{\ell}_R = 0.4$ . Fig. 2b shows a

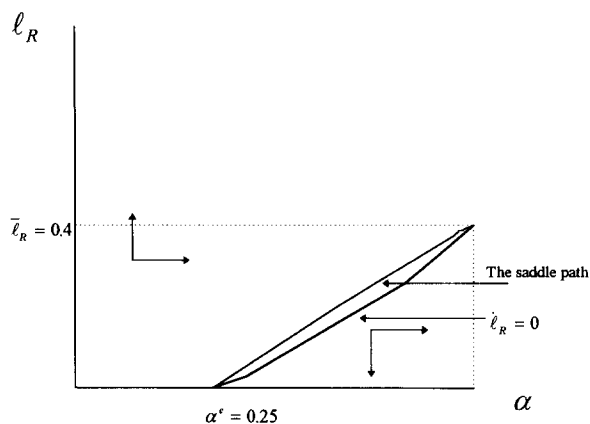


Fig. 3. The phase diagram.  $\gamma = 3$ ,  $\rho = 0.8$ ,  $\bar{l}_R = 0.4$ ,  $\alpha^c = 0.25$ .

plot of the simulated path running between  $(\alpha = \alpha^c = 0.25, \ell_R = 0)$  to  $(\alpha = 1, \ell_R = \bar{\ell}_R = 0.4)$ <sup>10</sup>. The simulations show that for  $\alpha, \ell_R$  pairs above the path,  $\ell_R$  explodes upwards until it equals one. By identical reasoning to that above, such paths are not admissible. The simulations also show that for values of  $\ell_R$  below the saddle path, the economy follows a trajectory in which research goes to zero, however with  $\alpha > \alpha^c$  this is again inadmissible. Thus the path obtained, for starting values of  $\alpha > \alpha^c$ , sketches the economy's convergence to the steady state with sustained growth and is the unique saddle path since all others lead to an explosion in the value of  $\ell_R$  to inadmissible values. Fig. 2a shows the monotonically increasing time path of  $\ell_R$  starting from a point on the saddle path arbitrarily close to  $\alpha^c$ . For values of  $\alpha$  less than  $\alpha^c$  the economy is in a no growth steady state, Eq. (23) holds with a strict inequality, and thus remains there. Fig. 3 thus extends the plot in Fig. 2b and depicts the complete behaviour of the system in  $\alpha, \ell_R$  space. To the left of  $\alpha^c$  simulations show that there does not exist a saddle path from the no growth steady state to the one with sustained growth, all paths for positive  $\ell_R$  lead to an explosion in the value of  $\ell_R$  until it equals one. The economy is therefore stuck in a poverty trap and can only be moved from there by an underlying change in industrial structure, i.e., an increase in  $\alpha$  to a point greater than  $\alpha^c$ , since there does not exist a path to  $(\ell_R, \alpha = 1)$  from  $\alpha < \alpha^c$ . Notice the essential difference between this result and the results obtained in Matsuyama (1991). Here the uniqueness of instantaneous labour market equilibrium implies that optimistic expectations are not sufficient to raise the economy out of the no growth steady state. The reason for the difference can

<sup>10</sup> Simulations were cross-checked using two different simulation programs the details of which are available from the authors on request.

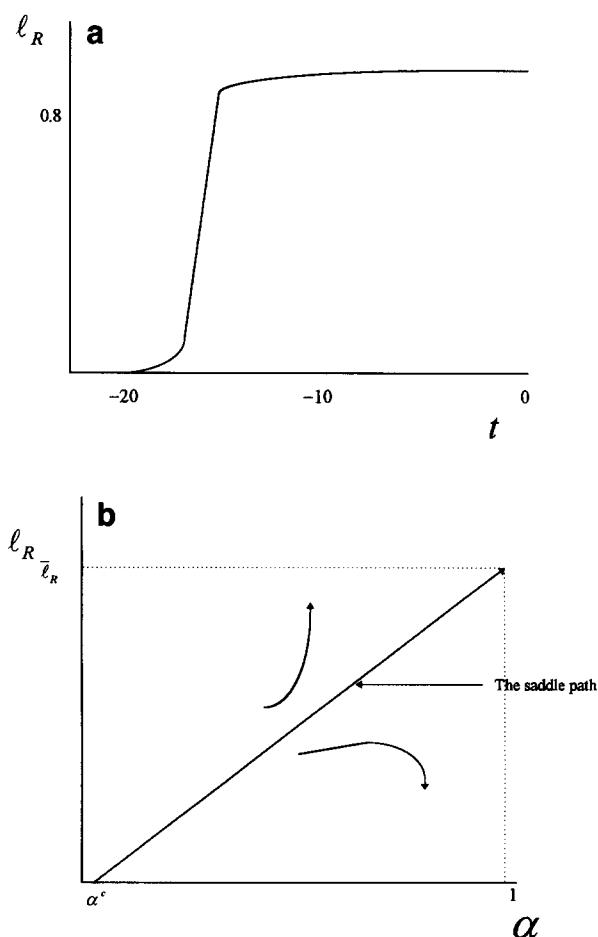


Fig. 4. High values of  $\rho$  and  $\gamma$ .  $\gamma = 30$ ,  $\rho = 0.99$ ,  $\bar{\ell}_R = 0.9336$ ,  $\alpha^c = 0.024381$ . Time path of labour in research from vicinity of growth trap to steady state with growth.

be readily seen by differentiating the relative wage obtained in research versus production,  $V(t)/w(t)$ , with respect to  $\ell_R(t)$ . This is always strictly less than zero, implying that labour devoted to research is never a strategic complement with contemporaneous labour devoted to research. Thus, unlike Matsuyama (1991) where such a complementarity can exist, optimistic expectations here, leading to a rush of research activity, serve only to lower returns to innovation thus implying that the expectations can never be fulfilled.

At points to the right of  $\alpha^c$  the analysis above applies and the economy must be on a saddle path converging to the steady state with sustained growth. Figs. 4 and 5 are plots derived from the same process of simulation for parameter values in both the low and high  $(\gamma, \rho)$  cases, which yields similar results to the first one.

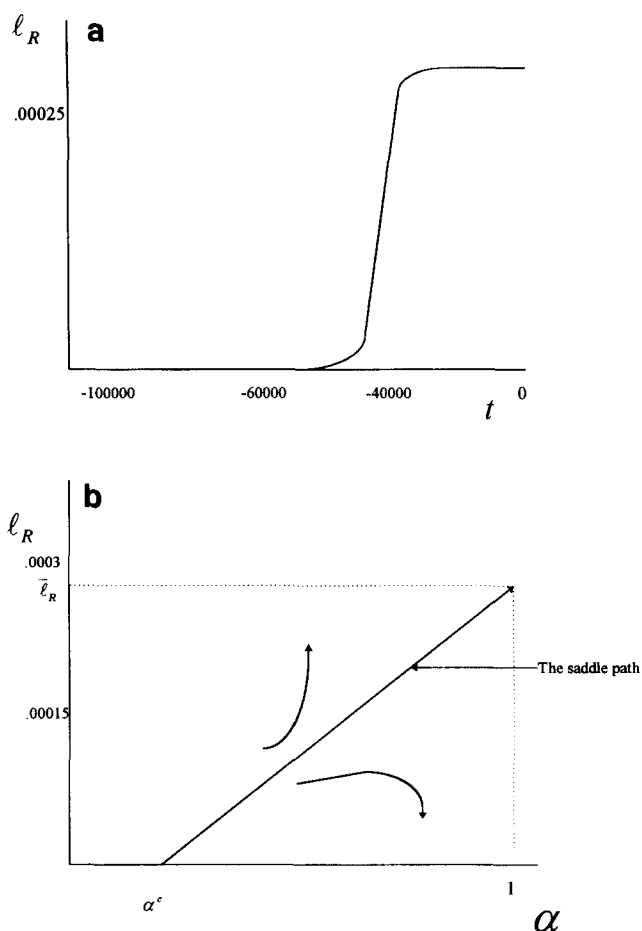


Fig. 5. Low values of  $\rho$  and  $\gamma$ .  $\gamma = 1.02$ ,  $\rho = 0.0197$ ,  $\bar{\ell}_R = 0.000294$ ,  $\alpha^c = 0.2385$ . Time path of labour in research from vicinity of growth trap to steady state with growth.

From the simulations we conclude that when both steady states exist, for values of  $\alpha_0 < \alpha^c$  the economy stagnates with no research and no growth. Small changes in  $\alpha$  will not move it from this situation unless it is close to  $\alpha^c$ . However for an economy starting with  $\alpha > \alpha^c$  there is monotonic convergence to the steady state with sustained growth.

### 3.7. Properties of the optimal path

The social optimum is calculated by maximizing an agent's intertemporal utility function by choice of  $\ell_{Rt}(t)$ , the amount of labour devoted to research in each

industry at time  $t$ . Due to the Cobb–Douglas production function, labour devoted to research or production will be allocated evenly across industries. Thus we re-specify variables in terms of the aggregate allocations of labour to research and production,  $\ell_R$  and  $\ell_i$ . Define  $n(t) = \int_0^1 \delta \ell_R(\tau) d\tau$ , then  $\dot{n}(t) = \delta \ell_R(\tau)$ . Thus, since there is no storage and instantaneous consumption equals total output of the final good, this is produced using the Cobb–Douglas technology from intermediate goods so that  $\ln C(t) = \int_0^1 \ln x_i(t) di = \int_0^1 \ln \gamma^{n_i(t)} \ell_i(t) di$ . Exploiting the symmetry of the problem  $n_i(t) = n(t)$  so that the expression becomes  $n(t)(\ln \gamma) + \ln(1 - \ell_R(t))$ . Using the aggregate labour constraint,  $\ell_R + \ell_i = 1$ , allows us to write the present value Hamiltonian as

$$H = \ln(1 - \ell_R(t)) + (\ln \gamma)n(\tau) + \theta(t)\delta \ell_R(t), \quad (47)$$

where  $\theta(t)$  is the co-state variable. Dynamic optimization yields

$$\ell_R^* = 1 - \frac{\rho}{\delta \ln \gamma}, \quad (48)$$

where  $\ell_R^*$  denotes the solution. A comparison of  $\ell_R^*$  with  $\bar{\ell}_R$  from Eq. (36) shows that the optimal level of investment may diverge from that which occurs in a decentralized economy experiencing growth. In particular, when the following inequality holds, the socially optimal level of research exceeds that which would be undertaken in the steady state with growth:

$$\frac{\delta}{\rho} > \frac{\gamma}{\ln \gamma} - 1. \quad (49)$$

Furthermore, using (36), when  $\ln(\gamma) < \rho/\delta < \gamma - 1$  it is optimal for the economy to have zero growth but a steady state with growth exists. Conversely, using (41), when  $\ln(\gamma) > \rho/\delta > \gamma - 1/\gamma$  optimal growth is positive yet a growth trap exists.

#### 4. Government subsidies and research

The potential for divergence between the socially optimal level of research and that which obtains in either steady state suggests the possibility of a role for government intervention. In this section, we consider the impact of two distinct forms of government intervention in the development of new technologies: government subsidies (or taxes) to private research and publicly provided research.

##### 4.1. Pareto improving government subsidies

In this section we show that an economy in a zero growth steady state *can* be moved to a path with sustained growth by a Pareto improving government subsidy to research. For the purpose of this example we consider the case where Eq. (42) is satisfied (both steady states exist) and Eq. (49) also holds (the steady state with

growth involves less than socially optimal research). We also assume that  $\alpha_0$  is less than  $\alpha^c$  by an arbitrarily small amount so that

$$1 = \frac{1}{\rho} \left( \frac{\gamma - 1}{\alpha_0 + (1 - \alpha_0)\gamma} \right) + \epsilon, \quad (50)$$

where  $\epsilon$  is arbitrarily small and strictly positive, so that the economy is currently in the zero growth steady state. Our previous example where,  $\delta = 1$ ,  $\rho = 0.8$ ,  $\gamma = 3$ , satisfies these conditions for  $\alpha_0$  arbitrarily below 0.25. The government now sets  $S > \epsilon$  by a small amount at time  $t$ ,<sup>11</sup> implying that (41) is no longer satisfied. Since  $\epsilon$  is arbitrarily small, once investment has been undertaken in all industries, the increase in  $\alpha$ , as described by (43), increases relative returns to innovation in subsequent periods so that some research is now always worthwhile.<sup>12</sup> The economy now moves onto the saddle path, depicted for simulation values in Fig. 2b, i.e., the rational expectations equilibrium in the labour market involves a positive amount of research. Research now increases monotonically in all periods after the subsidy and the economy converges towards the steady state with growth.

Since the control problem of maximizing social welfare involves a concave objective function with a convex constraint set, all  $\ell_R$  such that  $0 < \ell_R \leq \ell_R^*$  will be socially preferred to  $\ell_R = 0$ . Also, since (49) holds ( $\ell_R < \ell_R^*$ ) and convergence to  $\ell_R$  along the saddle path is monotonic, we know that all subsequent values of  $\ell_R$  will lie between  $\ell_R$  and  $\ell_R^*$ , thus, in all future instants, labour market equilibrium will yield a socially preferred level of research. Finally, since by construction  $s$  is arbitrarily small, its current period cost can be ignored and it follows that the subsidy constitutes a Pareto improvement for this economy.

In general, it is possible to specify the optimal level of subsidy for moving the economy from the low growth steady state, however considering discrete subsidy levels adds the complication of having to explicitly consider the government's method of finance and its welfare impact. Note finally that (49) is sufficient though not necessary for such subsidies to be Pareto improving, it may still be the case that moving from the zero growth steady state is Pareto preferred even if it eventually implies that the economy converges to a level of research which is greater than optimal.

<sup>11</sup> It can be assumed that the government finances this subsidy by a once off, lump sum tax on all individuals. By specifying  $\epsilon$ , and therefore  $s$ , as arbitrarily small, we are able to ignore the implications of this tax on individual decisions.

<sup>12</sup> If  $\epsilon$  were larger, then a once off subsidy may not raise  $\alpha$  sufficiently to move the economy from the zero growth steady state. In that case, a subsidy sustained over an interval may be required, it is, however, always possible to effect a move from the zero state by this method provided (42) is satisfied.

#### 4.2. Publicly provided research

In addition to granting research subsidies, the government also has the option of increasing innovation and growth by undertaking research of its own. In most models this is qualitatively equivalent in effect to directly subsidizing research. However, the implications of this research effort for growth in our framework depend critically upon the way in which the government uses it. In the case where the government acts as a profit maximizing monopolist, such research can be beneficial if it draws the economy out of the zero growth trap.<sup>13</sup>

It will have this positive effect if the research allows the proportion of industries with monopolists to increase past the critical level of  $\alpha$ ,  $\alpha^c$ . However, starting from a steady state with growth, the effect of public research is markedly different if, instead of appropriating the returns to that research, the government makes it publicly available. There are two distinct effects here. Firstly, a displacement effect: government research crowds out private research. More importantly, a disincentive effect: since access to government research is public, the proportion of competitive industries in the economy rises ( $\alpha$  falls) and therefore wages rise relative to private returns from research. As a result, the aggregate level of research falls, lowering the rate of growth (as long as the level of public research is less than the initial steady state level). Furthermore, if government research is large enough to reduce the proportion of monopolistic sectors below the critical level  $\alpha^c$ , private research disappears, i.e. the economy can move to the zero growth steady state.

### 5. Entrepreneurship: A reinterpretation for LDCs

As mentioned earlier, establishing the existence of multiple steady states has been motivated, in the previous literature, by the desire to provide an explanation for observed differences in cross-country growth experiences. Thus the stagnation of some LDCs has been interpreted as corresponding to their being trapped in a low level steady state. However, it is hard to ascribe poor growth performances in LDCs to insufficient investment in R&D and the want of new knowledge, as would be suggested by the present model. The results established above may therefore seem of little relevance to this issue.

However, a broader interpretation of the model can provide some insights. The main thrust of the model is that the process of innovation in one sector provides a spillover to potential future innovators (though not contemporary innovators) by increasing the relative profitability of research activities. In the context of LDCs, it

<sup>13</sup> Note that, if the economy is currently in a steady state with growth, such research simply displaces private research with no aggregate effect.



is likely to be the implementation of new and superior production methods, rather than their invention, which provides the engine of growth. In this case, the important actors are entrepreneurs taking the risk of departing from existing or traditional production methods, rather than researchers. If, however, such entrepreneurship does not require unique or specialized skills, then we can simply interpret  $\ell_R$  as the amount of labour devoted to entrepreneurial activity (instead of R&D), with  $(1 - \delta\ell_R)$  then representing the risk of a new venture failing. By employing a superior technology, the successful entrepreneur displaces traditional producers from her sector by limit pricing at their marginal cost and receiving a markup on each unit sold until she, in turn, is displaced by a new entrant. Note that there is still a knowledge creation effect of such entrepreneurial activity: assuming that some part of the method of production used by the entrepreneur is observable, it will be useful to new entrants in that sector seeking to improve upon the first entrepreneur's methods. Moreover, as it is not protected by patent, it can also be readily imitated. However, as long as such imitation carries a positive probability of failure or a positive cost (no matter how small), it will not be undertaken in a perfect equilibrium since the ensuing Bertrand competition between imitator and incumbent yields zero profit.

Under this interpretation, the basic results developed in the previous sections carry over. Two steady states may exist. In the no growth steady state, all resources are used in traditional production and, since production techniques are commonly known, there are zero profits. Disequilibrating entrepreneurial activity displaces traditional producers to other sectors, thereby lowering relative returns to traditional production there. This displacement effect makes entrepreneurship there relatively more worthwhile. Entrepreneurial activity in one segment of the economy can thus launch a surge of such activities throughout all traditional sectors and move the economy towards a path of self-sustained growth.

The main difficulty with the interpretation expounded above is that it assumes entrepreneurship can be undertaken by anyone whereas, more realistically, entrepreneurship requires particular skills which may not be widespread. We now show that a simple modification of the existing model can accommodate a non-uniform distribution of entrepreneurial skills without altering the basic results. Define another factor of production,  $h$ , used in all intermediate sectors, so that the linear production function of Eq. (16) is replaced by

$$x_i^j = A_i^j(t) f(h_i^j, \ell_i^j), \quad (16')$$

with  $f(h_i^j, \ell_i^j)$  homogeneous of degree one in both inputs and  $h_i^j$  denotes firm  $j$ 's input of  $h$  in industry  $i$ .  $h$  can be used either in production or research with a possible interpretation being as a unit of human capital.<sup>14</sup> Unlike  $\ell$ , we assume

<sup>14</sup> A more complete explanation would describe the process of human capital formation instead of treating it as fixed. However, allowing for this in the present model yields no changes since relative returns to  $\ell$  and  $h$  are the same in all equilibria.

that  $h$  need not be distributed uniformly. Profit maximization implies the following cost function:  $c_i^j(t) = C(w(t), r(t), x_i^j(t), A_i^j(t))$ , and, using the homogeneity of the production function, average cost is equal to  $c(w(t), r(t))/A_i^j(t)$ . This will imply that  $c(w(t), r(t))$  actually replaces the  $w(t)$  term everywhere in the previous section. As a result, Eq. (17) can now be written as

$$\Pi_i(t) = \left( \frac{c(w(t), r(t))}{A_i^c(t)} - \frac{c(w(t), r(t))}{A_i^M(t)} \right) y(t) - \frac{A_i^c(t)}{c(w(t), r(t))}. \quad (17')$$

Eq. (18) similarly becomes

$$p_i(t) = \frac{c(w(t), r(t))}{A_i(t)}. \quad (18')$$

We model entrepreneurial activity analogously with research in the previous section, so that  $\delta e(t)$  denotes the sector wide probability of an entrepreneurial success. We assume that the technology used to produce entrepreneurial effort is analogous to the one used in the productive sectors, so that  $e(t) = f(h_e(t), \ell_e(t))$ , with  $f(\dots)$  identical to that appearing in Eq. (16'). This simplifies calculation without affecting the qualitative results of our analysis, and, in fact, yields a slightly modified expression for Eq. (36):

$$\delta \frac{1 - \tilde{e}}{\delta \tilde{e} + \rho} (\gamma - 1) = 1 \quad (36')$$

with

$$\tilde{e} = f(\tilde{h}_e, \tilde{\ell}_e)$$

and Eq. (41) remains unchanged. Thus it can be seen that the distribution of  $h$  does not affect the existence of two steady states.

The aim of this section was to show that the model, when suitably interpreted, could provide a realistic description of sectoral linkages giving rise to multiple steady states. In particular, having factors which are not uniformly distributed, such as entrepreneurship, does not affect the configuration of steady states. What really matters is that the factor (or combination of factors) used in entrepreneurship is also used in traditional production and that this factor is mobile. Under such conditions, successful entrepreneurship in sector  $i$  induces a reallocation of factors previously used in sector  $i$  to other sectors, thereby lowering returns to those factors in traditional production relative to their returns from entrepreneurship. The existence of two steady states again suggests that the status quo in traditional production may be self sustaining in economies not having experienced entrepreneurship, even though, once some critical proportion of sectors have been modernized by successful entrepreneurs, traditional production is no longer attractive since returns to the remaining traditional producers are now so low that they find entrepreneurial risks worth bearing.

## 6. Conclusion

Schumpeter (1951) argued that monopolies were an important element in the process of capitalist innovation and growth. His principal concern, and that of most subsequent literature, has been with the incentive effect provided by monopoly rents. In this paper we considered another avenue through which the existence of monopolies can affect the development of new technologies by investigating the general equilibrium effect of industrial structure on relative prices.

The main result of our analysis is that a monopolistic structure, by increasing the share of profits in aggregate income, tends to increase the profitability of innovative activities. An important implication is that in a multi-sector model of growth through creative destruction there may exist more than one steady state, with the initial industrial structure determining the state eventually attained: thus, the more competitive the initial industrial structure, the less likely that the positive growth steady state is attained. In particular, in LDCs, an interpretation is that the avenue through which this occurs is entrepreneurial activity. This suggests that, though perhaps difficult to set in motion, there are forces at work, even in the absence of direct externalities, which make entrepreneurship, once started, self-sustaining. This allows for the possibility of Pareto improving government intervention. However, though government subsidies to private research are shown to be beneficial, in contrast, government development of new technologies, if made publicly available, may be detrimental to growth.

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