HOMEWORK 2

Econ 501: Macroeconomic Analysis and Policy

Spring 2016

1.

a) The Euler equation is $\frac{\beta\alpha A k_{t+1}^{\alpha-1}}{c_{t+1}} = \frac{1}{c_t}$

b) $k^* = (\beta\alpha A)^{\frac{1}{1-\alpha}}$, $y^* = Ak^{\alpha} = A(\beta\alpha A)^{\frac{\alpha}{1-\alpha}}$,

$c^* = y^* - k^* = A(\beta\alpha A)^{\frac{\alpha}{1-\alpha}} - (\beta\alpha A)^{\frac{1}{1-\alpha}}$

c) The steady state savings rate $s^* = \frac{y^* - c^*}{y^*} = \frac{(\beta\alpha A)^{\frac{1}{1-\alpha}}}{A(\beta\alpha A)^{\frac{\alpha}{1-\alpha}}}$

d) When α =1, the Euler equation becomes $\left(\frac{c_{t+1}}{c_t}\right) = \beta\alpha A$

In general, the growth rate of a variable x_t is $(x_{t+1}/x_t)-1$ (by definition) or $\log(x_{t+1}/x_t)$ (a convenient approximation), so the growth rate of consumption is: $\beta\alpha A - 1$ or $\log(\beta\alpha A)$

2.

a) Because of the production technology, there is a maximum possible level of capital, $k_{\max} = A^{\frac{1}{1-\alpha}}$. Since capital cannot be bigger than that number, consumption cannot be bigger than $Ak_{\max}$. Since $c_t < A^{\frac{1}{1-\alpha}}$, it must be that

$$\sum \beta^t c_t < \sum \beta^t A^{\frac{1}{1-\alpha}} < \frac{A^{\frac{1}{1-\alpha}}}{1-\beta}$$

In other words, any feasible consumption stream provides finite utility.

b) Let k* be the usual steady state capital stock i.e. $f'(k^*)=1/\beta$. Suppose you are deciding whether to consume a particular unit of output or save it for tomorrow. Today, it will give you 1 unit of utility. Tomorrow it will give you $\beta f'(k_{t+1})$ units of utility. If $\beta f'(k_{t+1})>1$, you will save 100% of output. In that case, $k_{t+1}=f(k_t)$, so saving 100% output is optimal if and only if $\beta f'(f(k_t))>1$ or equivalently, $f'(k_t)<k^*$. Similarly, if $\beta f'(k_{t+1})<1$, you will consume 100% of your output. If this policy is pursued, then we would have $k_{t+1}=0$ and $\beta f'(0)=\infty>1$. So, consuming 100% of optimal can never be optimal. If $\beta f'(k_{t+1})=1$ or equivalently, if $k_{t+1}=K^*$, you will be indifferent between saving and consuming. Any consumption allocation will be optimal.

This means the capital stock will follow the law of motion

$$k_{t+1} = Min\left\{f\left(k_t\right), k^*\right\} = Min\left\{k_t^\alpha, \left(\alpha\beta A\right)^{\frac{1}{1-\alpha}}\right\}$$

In other words, the planner will choose zero consumption until the steady state capital level is reached. If the planner can reach steady state starting at period s, then $c_{s-1}=f(k_{s-1})-k^*$ and $c_t=f(k^*)-k^*=c^*$for all t greater than equal to s.

These 2 problems help you see how the shape of the utility function u(.) affects the behavior of the model. I meantioned in class that log utility is a special case ($\theta=1$) of CRRA utility, and so is linear utility ($\theta=0$). The parameter θ measures a person's preference for smoothing consumption over time. The characteristics of the steady state are unrelated to θ. However, the speed at which the steady state is reached is closely related to θ. When θ is low (linear case), consumers will tolerate low early consumption to get to the steady state quickly. When θ is higher, consumer wil not tolerate low early consumtion, and will reach the steady state more slowly.

c) As I said before, the consumer will choose to save if $\beta f'(k)>1$ and spend if $\beta f'(k)<1$. When $\alpha=1$, the consumer will choose to save if $\beta A>1$ and spend if $\beta A<1$.

If $\beta A<1$, there is a solution to the planner's problem-consume everything right away ($c_0=ak_0$).

If $\beta A>1$, there is no solution to the planner's problem.

You don't have to prove this but if you wanted to, here is how to do it. Suppose that there is a solution $\{c_t\}$ which gives utility level U. We can prove that such a solution does not exist by finding a feasible allocation that improves on it. A proposed solution will have the characterstic that there exists some time T such that $c_T>0$. I propose a new soltuion $\{\hat{c}_t\}$

$$\hat{c}_t = \begin{cases} c_t \forall t \notin \{T, T+1\} \\ c_{T-1}, t = T \\ c_{T+1} + A, t = T+1 \end{cases}$$

The utility from this proposed solution is $\hat{U} = U + \beta^{T+1} A - \beta^T$. Since βA>1, $\hat{U} > U$. Therefore, U could not have been a solution.