

A Theory of Economic Development

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A THEORY OF ECONOMIC DEVELOPMENT

BY GUSTAV RANIS AND JOHN C. H. FEI*

This paper attempts to make a contribution towards the theory of growth by rigorously analyzing the transition process through which an underdeveloped economy hopes to move from a condition of stagnation to one of self-sustaining growth. Since the totality of economies bearing the "underdeveloped" label admittedly defies easy generalization, we shall be primarily concerned here with the labor-surplus, resource-poor variety in which the vast majority of the population is typically engaged in agriculture amidst widespread disguised unemployment and high rates of population growth. We hope to accomplish our task by drawing liberally on the stock of already accepted ideas and then proceeding to weave them into a general explanatory model of economic growth.

Our analysis begins with an economy's first departure from quasi-stagnation or the initiation of the so-called take-off process.¹ Rostow defines this as a period of two or three decades during which the economy transforms itself in such a way that economic growth becomes, subsequently, more or less automatic; its characteristics are a reduction of the rural proportion of the population, a doubling of savings rates and the first marked and continuous flowering of industry stimulated by the availability of surplus labor [11, pp. 25-32]. This well-known intuitive notion has been chosen as our point of departure. For our basic analytical tool-kit, however, we draw heavily on the work of Arthur Lewis.

In his celebrated articles Lewis [3] [4] presents a two-sector model and investigates the expansion of the capitalistic or industrial sector as it is nourished by supplies of cheap labor from the subsistence or agricultural

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¹ This is not to underestimate the importance of a prior preconditioning period (see [1] and [9]) when potentially expansionary institutional forces are being mobilized and render the system capable of a significantly positive response to a random stimulus.

sector.² Development consists of the re-allocation of surplus agricultural workers, whose contribution to output may have been zero or negligible, to industry where they become productive members of the labor force at a wage equal (or tied to) the institutional wage in agriculture. This process continues until the industrial labor supply curve begins to turn up.

Lewis, however, has failed to present a satisfactory analysis of the subsistence or agricultural sector. It seems clear that this sector must also grow if the mechanism he describes is not to grind to a premature halt. Pursuit of this notion of a required balance in growth then leads us to a logically consistent definition of the end of the take-off process.

Finally, the economy must be able to solve its Malthusian problem if the process of development along a balanced-growth path is to prove successful. Considerations of this nature have given rise to the so-called "critical minimum effort" theory [2], which deals with the size of the effort required to achieve a more-than-temporary departure from stagnation. We shall show, in the course of our analysis, that the concept of a critical minimum effort does not presuppose some absolute magnitude of effort but contains a built-in time dimension permitting the size of the effort to vary with the duration of the take-off process.

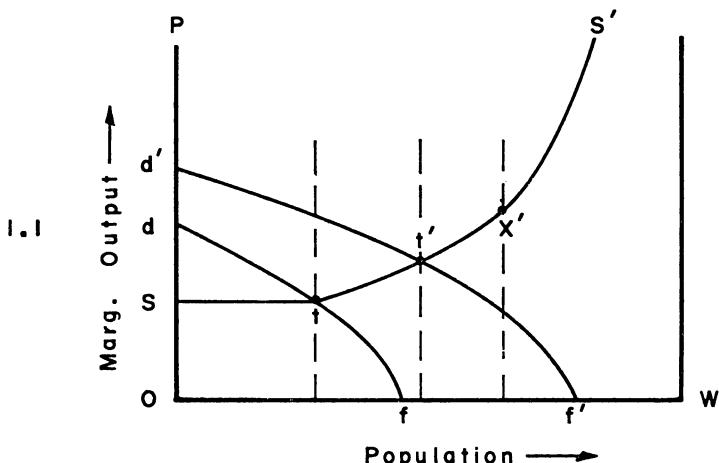
The contribution of this paper, then, is to construct a theory of economic growth of which the above ideas, rigorously formulated, constitute component parts. In Section I we present the basic structural assumptions of our model with emphasis on analysis of the role of the "neglected" agricultural sector. Section II generalizes the previously "static" analysis by admitting the possibility of a change of productivity in the agricultural sector. In Section III we introduce changes in industrial productivity and the notion of a "balanced growth criterion" by means of which the termination of the take-off process is formally defined. Section IV proceeds with a precise mathematical formulation of our theory which enables us to make certain quantitative conditional predictions as a first test of its empirical relevancy. Finally, in Section V, we integrate population growth as well as some other real-world complexities into our model and investigate the notion of the critical minimum effort in relation to the length of the take-off process.

I. *The Basic Assumptions*

Our formal explanatory model is presented with the help of Diagram 1. Diagram 1.1 depicts the industrial sector and Diagrams 1.2 and 1.3

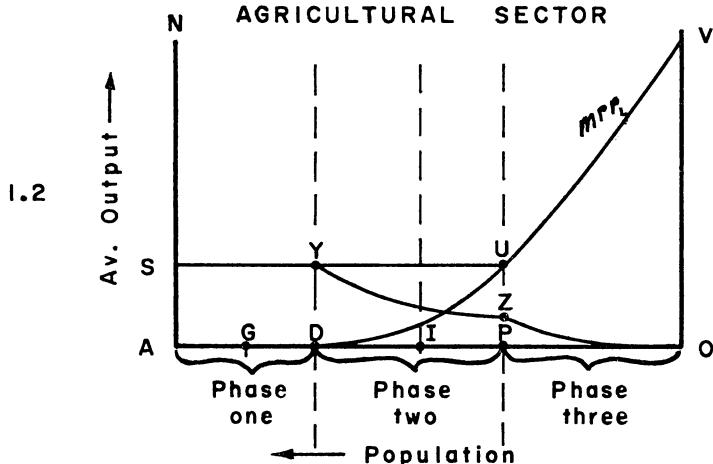
² We wish to underscore the absence of any necessary one-to-one relationship between the subsistence sector and agriculture, or between the capitalistic sector and industry in most less-developed economies. The existence of substantial islands of commercialized production in the primary sector and of sizable subsistence enclaves in the small-scale and service industries does not, however, bar Lewis, or us, from using this short-hand terminology.

INDUSTRIAL SECTOR



Population →

AGRICULTURAL SECTOR



Population ←

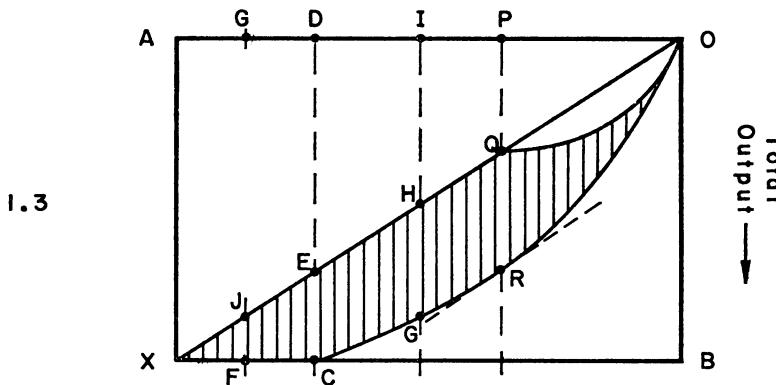


DIAGRAM I

the agricultural sector. The first is the familiar Lewis diagram measuring industrial labor on the horizontal axis OW and its marginal physical productivity (MPP) on the vertical axis OP . The demand curve for labor (i.e., the MPP curve dif), together with the supply curve of labor ($Stt'S'$), determines the employment of the industrial labor force (St). Since the marginal physical productivity curve depends on the size of the capital stock cooperating with the labor force, an increase in the capital stock leads to a shift of the MPP curve to the right, e.g. to $d't'f'$. Lewis' "unlimited" supply curve of labor is defined by the horizontal portion of the supply curve, i.e. St . When this supply curve turns up, unlimitedness comes to an end. Our first problem is to investigate the conditions of this turning point. This leads us to focus attention on the agricultural sector.

In Diagram 1.3 let the agricultural labor force be measured on the horizontal axis OA (reading from right to left), and let agricultural output be measured on the vertical axis OB (downward from O). The curve $ORCX$ describes the total physical productivity of labor (TPP) in the agricultural sector. This curve is assumed to have a concave portion ORC showing a gradually diminishing marginal productivity of agricultural labor and a horizontal portion XC where marginal product vanishes. The portion of any labor force in excess of OD may be considered redundant in that its withdrawal from agriculture would not affect agricultural output.

At the initial (or break-out) point let the entire labor force OA be committed to agriculture, producing a total agricultural output of AX . Let us assume that the agricultural output AX is totally consumed by the agricultural labor force OA . Then the real wage is equal to AX/OA or the slope of OX . The persistence of this wage level is sustained by institutional or nonmarket forces since under competitive assumptions the real wage would fall to zero, at equality with MPP. We shall call this the institutional wage.

Let point R on the total output curve be the point at which the MPP equals the institutional wage, i.e. the dotted tangential line at R is parallel to OX . We can then define AP as the disguisedly unemployed agricultural labor force since, beyond P , MPP is less than the institutional wage.³

Note that Diagrams 1.1, 1.2, and 1.3 are "lined up." Any point on the horizontal axis of Diagrams 1.1 to 1.3 represents a particular way in which the total population or labor force OA is distributed between the two sectors; for example, at point P (Diagrams 1.2 and 1.3) the

³ Redundancy is a technological phenomenon, i.e., determined by the production function. Disguised unemployment, on the other hand, depends upon the production function, the institutional wage, and the size of the agricultural population. In other words, it is an economic concept.

agricultural labor force is OP and the (already allocated) industrial labor force is AP . If, at the break-out point, the entire population, OA , is engaged in the agricultural sector, the allocation process during take-off can be represented by a series of points, A, G, D, I, P , etc., on OA , gradually moving towards O .⁴

The important concepts of disguised unemployment, redundant labor force and institutional wage can be more clearly depicted with the aid of Diagram 1.2, in which agricultural output per worker is measured on the vertical axis AN . Let $ADUV$ be the marginal physical productivity (MPP) curve of labor in the agricultural sector. Let the vertical distance AS equal the institutional wage (shown also as PU , equal to MPP of agricultural labor at U , lined up with P and R in Diagram 1.3). Three phases in the re-allocation process may now be distinguished: (1) Phase 1 is the range for which $MPP=0$, i.e., the total productivity curve in Diagram 1.3 is horizontal. This phase marks off the redundant labor force, AD . (2) Phase 2 is the range for which a positive MPP is less than the institutional wage. Phases 1 and 2 together mark off the existence of the disguisedly unemployed labor force, AP . (3) Phase 3 is the range for which MPP is greater than the institutional wage rate assumed to prevail at the break-out point.

We assume that the institutional wage AS prevails during phases 1 and 2 and a wage rate equal to MPP prevails in phase 3. Only when the disguisedly unemployed have been absorbed, i.e. in phase 3, does the marginal contribution of labor to output become as great as or greater than the institutional real wage. As a result, it is then to the advantage of the landlord to bid actively for labor; the agricultural sector can be said to have become commercialized as the institutional wage is abandoned and competitive market forces yield the commonly accepted equilibrium conditions. Under these assumptions the agricultural real wage in terms of agricultural goods is defined by the curve SUV in Diagram 1.2, consisting of a horizontal portion SU and a rising portion; UV . This curve may be called the supply-price curve of agricultural labor. It indicates for each level of real wage the amount of labor that may be released from the agricultural sector.

The transition into phase 3 constitutes a major landmark in the developmental process. With the completion of the transfer of the disguisedly unemployed, there will occur a switch, forced by circumstance, in employer behavior, i.e. the advent of a fully commercialized agricultural sector. This landmark may be defined as the end of the take-off process. We know no other way to establish a nonarbitrary criterion for an economy reaching the threshold of so-called self-sustaining growth.⁵

⁴ The present assumption of an unchanging population will later be relaxed.

⁵ Whether or not growth can ever really be "self-sustaining," in Rostow's phrase, is basi-

Returning now to Diagram 1.3, we see that, as agricultural workers are withdrawn, a surplus of agricultural goods begins to appear. That portion of total agricultural output in excess of the consumption requirements of the agricultural labor force at the institutional wage is defined as the total agricultural surplus (TAS). The amount of TAS can be seen to be a function of the amount of labor reallocated at each stage. For example, if agricultural workers to the extent of AG are withdrawn in phase 1 and re-allocated, JG is required to feed the remaining agricultural workers and a TAS of size JF results. The TAS at each point of allocation in phases 1 and 2 is represented by the vertical distance between the straight line OX and the total physical productivity curve $ORCX$. (For phase 3, due to the rise of the wage rate, TAS is somewhat less than this vertical distance and equals the vertical distance between the curve OQ and the total productivity curve).

TAS may be viewed as agricultural resources released to the market through the re-allocation of agricultural workers. Such resources can be siphoned off by means of the investment activities of the landlord class and/or government tax policy and can be utilized in support of the new industrial arrivals.⁶ The average agricultural surplus, or AAS, may now be defined as the total agricultural surplus available per head of allocated industrial workers.

The AAS curve is represented by curve $SYZO$ in Diagram 1.2. In phase 1 as TAS increases linearly with the allocation of the redundant labor force from A to D we can picture each allocated worker as carrying his own subsistence bundle along with him. The AAS curve for phase 1 thus coincides with the institutional wage curve SV . In phase 2, however since the MPP in agriculture of the now allocated workers was positive there will not be sufficient agricultural output to feed all the new industrial arrivals at the institutional wage level. Thus, while TAS is still rising, AAS begins to fall.⁷ It can, moreover, readily be seen that

cally not a problem amenable to the tools of traditional economic analysis. The role of saving rates and per capita income levels in setting it in motion remains undefined. All we are saying here is that, after the turning point, the real wage in agriculture is determined by impersonal competitive market forces, a qualitative transformation which constitutes a necessary (if not sufficient) condition for growth to become automatic and routinized. It is this point which Lewis [4, p. 26] seems to have in mind when he speaks of "two different stages of economic development with two different sets of results" and describes the second stage as a situation in which "all the factors of production are scarce [and] . . . wages are no longer constant as accumulation proceeds."

⁶ While it could easily be accommodated by the model, we neglect resource transfer costs as well as the possibility that it may be impossible to induce those left behind in agriculture to release the entire surplus.

⁷ The following analogy with individual-firm analysis may be drawn to show more clearly the relationship between the marginal, total and average concepts involved. We may think of the total agricultural output curve ($ORCX$) and the total agricultural consumption curve (OX) in Diagram 1.3 as analogous to the total revenue curve and the total cost curve, respectively. Then the gap between these curves is the total profit curve which is equivalent to our

during phase 3 AAS declines even more rapidly (and TAS also declines) as the now commercialized wage in agriculture becomes operative.

We may now consider the derivation of the Lewis turning point in the agricultural sector. Lewis himself [4, pp. 19–26] explains the turning point rather loosely as occurring when one of the following events puts an end to the horizontal supply curve of labor: (a) the worsening of the terms of trade for the industrial sector, and (b) the exhaustion of the labor surplus in the agricultural sector. But in our model any such explanation must take into account the basic determination of the entire industrial labor supply curve by the conditions postulated for the non-industrial sector.

The “worsening of the terms of trade” for the industrial sector occurs as the result of a relative shortage of agricultural commodities seeking exchange for industrial goods in the market. In our model, it will be recalled, this surplus is measured by total agricultural surplus (TAS) and, on a per-industrial-worker basis, average agricultural surplus (AAS). There is a tendency, then, for the industrial supply curve to turn up as phase 2 is entered because this is the time when there begins to appear a shortage of agricultural goods measured in AAS—causing a deterioration of the terms of trade of the industrial sector and a rise in the industrial real wage measured in terms of industrial goods. We thus see that the disappearance of the redundant labor force in the agricultural sector is a cause of the Lewis turning point.

The “exhaustion of the labor surplus” must be interpreted primarily as a market phenomenon rather than as a physical shortage of manpower; it is indicated by an increase in the real wage at the source of supply. If we assume that the real wage of the industrial worker is equal to the agricultural real wage,⁸ then there is a tendency for the industrial supply curve of labor ($S_{IT}'S'$ in Diagram 1.1) to turn upward when phase 3 is entered. With the disappearance of the disguisedly unemployed labor force and the commercialization of the agricultural sector, the agricultural real wage begins to rise (see Diagram 1.2). This leads to an increase in the industrial real wage level if the industrial employer

TAS curve. The total profit curve reaches a maximum when marginal cost equals marginal revenue. This occurs at point U in Diagram 1.2—because SU is the marginal cost curve and $ADUV$ is the marginal revenue curve. The AAS curve in Diagram 1.2 is equivalent to an “average profit curve.”

⁸ “Governed by” may be a more realistic description. Lewis [3, p. 150] points out that urbanization, transfer costs, etc. may require an industrial real wage at a constant (he believes approximately 30 per cent) margin or “hill” above the institutional wage in agriculture; while, for simplicity of exposition, our model initially maintains strict equality between the two wage rates, this assumption is later relaxed (Section V). In his second article [4], Lewis also refers to certain “exogenous factors,” including unionization and presumably other changes in the institutional milieu. Such a dynamically growing “hill” could also be accommodated by the model but has not been considered in this first approximation.

is to compete successfully with the landlord for the use of the, by now "limited," supply of labor.

Putting the two factors (a and b) together, we can say that as labor is re-allocated from the agricultural to the industrial sector, the industrial supply curve turns up (i.e. the Lewis turning point occurs), in the first instance (at t), due to a shortage of agricultural goods traceable to the disappearance of the redundant agricultural labor force; and that this upward trend in the industrial real wage is later accentuated (at X') by the upward movement of the agricultural real wage traceable to the complete disappearance of the disguisedly unemployed labor force and the commercialization of the agricultural sector.

To facilitate our later analysis, let us refer to the boundary between phases 1 and 2 (i.e., point Y in Diagram 1.2) as the "shortage point" signifying the beginning of shortages of agricultural goods as indicated by the fact that AAS falls below the minimum wage; let us also refer to the boundary between phases 2 and 3 as the "commercialization point" signifying the beginning of equality between marginal productivity and the real wage in agriculture. The Lewis turning point thus coincides with the shortage point and the upward movement of the industrial real wage is accentuated at the commercialization point.⁹

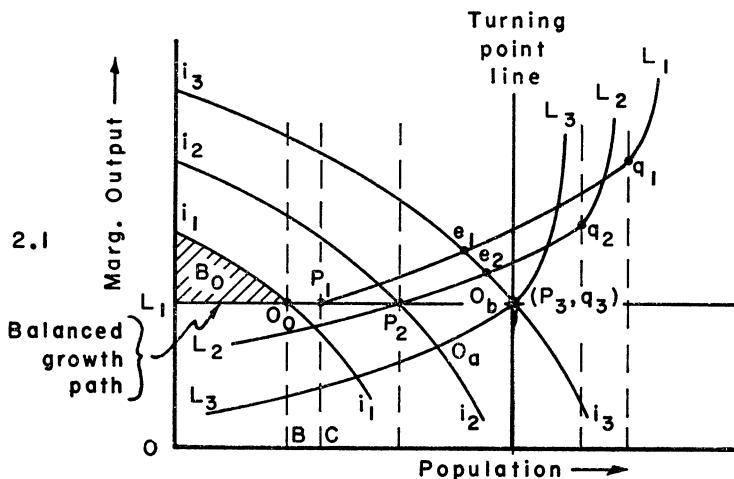
There are two factors which may lead to a postponement of the Lewis turning point: (1) increases in agricultural productivity, and (2) population growth. The fact that these two factors operate very differently—one, generally viewed as a blessing, by raising surplus agricultural output, the other, almost invariably considered a curse, by augmenting the supply of redundant labor, is intuitively obvious. We shall first examine the significance of an increase of agricultural productivity. The extension of our analysis to accommodate population growth will be undertaken later.

II. Changes in Agricultural Productivity

An increase in labor productivity in the agricultural sector can be described by an "upward" shift of the entire total physical productivity (TPP) curve of Diagram 1.3. Such productivity increases are depicted in Diagram 2.3 by a sequence of TPP curves marked I , II , $III \dots$ etc. among which the I -curve is the initial TPP curve (as in Diagram 1.3) and II , $III \dots$ represent the TPP curves after successive doses of agricultural investment. (For the present we assume no change in industrial productivity.)

⁹ From a strictly logical standpoint the industrial supply curve of labor must be derived from the totality of conditions emerging from our analysis of the agricultural sector. The relevant conditions include (1) the agricultural real-wage curve, (2) the AAS curve, and (3) a consumer preference map specifying preferences for agricultural vs. industrial goods. Space limitations prevent us from rendering a rigorous derivation of the industrial real wage at each point through the terms-of-trade mechanism.

INDUSTRIAL SECTOR



AGRICULTURAL SECTOR

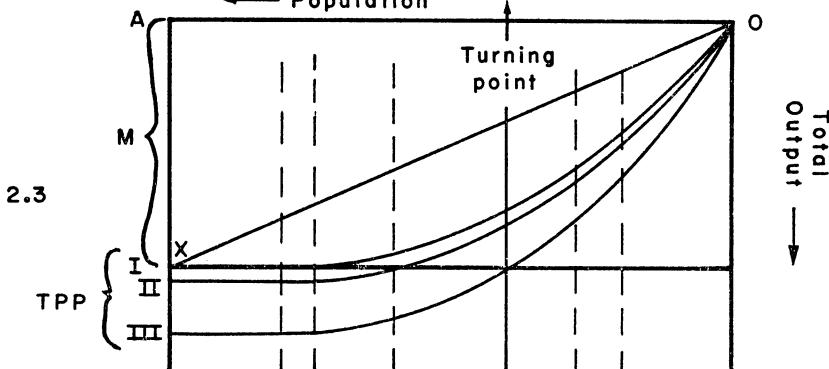
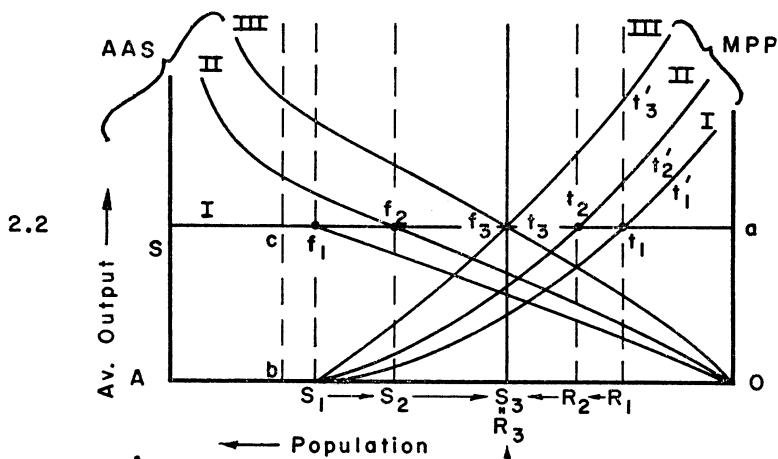


DIAGRAM 2

Let us make the assumption that as agricultural productivity increases the institutional wage remains unchanged, i.e. SA in Diagram 2.2 equals the slope of OX in Diagrams 1.3 and 2.3 as determined by the initial TPP curve.¹⁰ In Diagram 2.2 we may now plot the sequence of marginal physical productivity of labor curves marked $I, II, III \dots$ (all containing the flat portion AS_1) and the sequence of average agricultural surplus curves marked $I, II, III \dots$ corresponding to the total physical productivity curves $I, II, III \dots$ in Diagram 2.3. According to the method already indicated, we can now determine the three phases for each level of productivity, i.e., the sequence of shortage points, $S_1, S_2, S_3 \dots$ and the sequence of commercialization points, $R_1, R_2, R_3 \dots$. Reference to these points will facilitate our analysis of the effects of an increase in agricultural productivity on the supply-price curve of agricultural labor and on the AAS curve.

As depicted in Diagram 2.2, for every amount of labor employed in the agricultural sector, an increase in agricultural productivity also shifts the marginal physical productivity curve upward.¹¹ As a consequence, the agricultural labor supply price curve is transformed from St_1t_1' to St_2t_2' to $St_3t_3' \dots$ etc. with a shortening of its horizontal portion (i.e., phase 3 arrives earlier) as the sequence of commercialization points $R_1, R_2, R_3 \dots$ gradually shifts from right to left. On the other hand, the sequence of shortage points $S_1, S_2, S_3 \dots$ etc. gradually moves from left to right. This is due to the fact that, for each amount of labor allocated to the industrial sector, the AAS increases with the increase in total physical productivity; the amount of food consumed by agricultural labor remains unchanged, leaving more TAS (and hence AAS) for the industrial workers. Thus the effect of our increase in agricultural productivity is an upward shift of the AAS curve (to positions marked $II, III \dots$).

Sooner or later, the shortage point and the commercialization point coincide, the distance $S_1R_1, S_2R_2, S_3R_3 \dots$ vanishes and phase 2 is eliminated. In Diagram 2.2 such a point of coincidence is described by $R_3=S_3$. We shall call this point the turning point. There exists one level of agricultural productivity which, if achieved, will bring about this turning point. (In Diagram 2.3 this level of agricultural productivity is described by TPP curve III).

¹⁰ It is, of course, possible that the institutionally determined agricultural wage will be permitted to rise; but as the economy becomes increasingly capitalistic it seems highly doubtful that nonmarket forces in agriculture will be strengthened and thus prevent the closing of the artificial marginal productivity-wage gap. A second, and possibly more powerful, qualification arises from the fact that the institutional wage level in agriculture may be sufficiently close to caloric subsistence so that raising it may constitute a highly productive form of investment. We do not, however, consider this possibility in the context of the present model. Concerning the relative position of the industrial wage level see footnote 8.

¹¹ This is a reasonable assumption if the shift in TPP is proportional.

Let us now investigate the impact of an increase of agricultural productivity on the industrial supply curve L_1L_1 depicted in Diagram 2.1. On the one hand, the upward shift of the AAS curve will shift the industrial supply curve downward *before* the turning point. This is due to the fact that an increase of AAS will depress the terms of trade for the agricultural sector and, with the same institutional wage (in terms of agricultural goods) paid to the industrial workers, the industrial wage (in terms of industrial goods) must decline. On the other hand, the upward shift of the MPP curve which is accompanied by a higher real wage in the agricultural sector *after* the turning point raises the industrial supply curve after that point. Thus we see, for example, that the L_2L_2 curve crosses the L_1L_1 curve from below, indicating that ultimately the "terms-of-trade effect" (due to an increase of AAS) has been overcome by the "real-wage effect" (due to an increase of MPP). For purposes of this paper, we are, however, not very much concerned with phase 3 which lies beyond the turning point.

Let us now examine more closely the relative positions of the industrial supply curves before phase 3 is reached. Let the horizontal portion L_1P_1 of the initial industrial supply curve L_1L_1 be extended up to P_3 , the turning point, and let us call this horizontal line segment L_1P_3 the balanced-growth path (for reasons which will be fully explained in the next section). We may then claim that all the industrial supply curves between L_1L_1 (i.e. the initial one) and L_3L_3 (i.e., the one corresponding to the turning point) cross the balanced-growth path at the respective shortage points. This is due to the fact that at the shortage point for each case (e.g., point f_2 in Diagram 2.2 for the case of industrial supply curve L_2L_2 in Diagram 2.1) the subsistence wage rate and the AAS take on the same value as that prevailing in phase 1 before any increase in agricultural productivity has been recorded. Hence the same real wage, in terms of industrial goods, must prevail at the shortage point as prevailed previously. In short, before the turning point, the industrial labor supply curve lies above (below) the balanced growth path when the AAS curve lies below (above) the horizontal line S_a , causing a deterioration (improvement) of the industrial sector's terms of trade.

The economic significance of the equality between our turning point and the (final) shortage point is that, before the turning point, the economy moves along its balanced-growth path while exploiting (or making the best of) its under-employed agricultural labor force by means of increases in agricultural productivity. The economic significance of the equality between our turning point and the commercialization point is that, after the turning point, the industrial supply curve of labor finally rises as we enter a world in which the agricultural sector is no longer dominated by nonmarket institutional forces but assumes the characteristics of a commercialized capitalistic system.

III. *Changes in Industrial Productivity and Balanced Growth*

In addition to investment in the agricultural sector, the other major aspect of growth which must be considered is the simultaneous process of investment in the industrial sector. We know, moreover, that such activities in the two sectors do not constitute independent activities. For, from the output side, the two sectors must provide the marketing outlets for each other's products; and, from the input side, the industrial sector must provide the employment opportunities for the absorption of workers released by the agricultural sector. Consideration of this basic interdependence during the take-off process is really nothing else but consideration of the "balanced growth" problem, a key concept in the current development literature.¹² The purpose of this section is to formulate the problem of balanced growth rigorously and to investigate its significance in the context of our model.

Referring to Diagram 2.1 we see that during the take-off process the demand curve for labor, $i_1 i_1, i_2 i_2 \dots$, gradually shifts upward to the right as real capital is accumulated in the industrial sector. Simultaneously the investment activity proceeding in the agricultural sector shifts the supply curve of labor $L_1 L_1, L_2 L_2 \dots$ downward in the same direction. The central problem of balanced growth concerns the synchronization through time of the shifts of the two sequences of curves. At any moment of time during the take-off process, the question is how should the total investment fund be allocated to the two sectors to ensure that they are "harmonious" from the point of view of both the input and the output criteria.

The output criterion, i.e. provision of mutual market outlets, specifies that the allocation of investment funds must be such as to continuously sustain investment incentives in both sectors of the economy. In the context of our model, this means that the terms of trade between the two sectors should not deteriorate substantially against either sector. The input criterion, on the other hand, specifies that the allocation of the investment fund must be such as to enable the industrial sector to demand, at the constant industrial real wage consistent with the output criterion, the precise number of new workers now freed as a result of the investment activity in the agricultural sector. We shall now proceed to show that a balanced-growth path satisfying these conditions exists as an integral part of our model.

Let the initial demand curve for industrial labor at the break-out point be indicated by $i_1 i_1$ and the initial supply curve by $L_1 L_1$ in Diagram 2.1, with OB units of labor already employed in the industrial

¹² See especially R. Nurkse [5] and [6, p. 192]: "Without [agricultural] reorganization the labor surplus in agriculture remains largely potential. On the other hand, reorganization may well prove impracticable without an active policy of absorbing the surplus manpower."

sector. (While it is realistic to assume that some industrial establishment already exists during the preconditioning period and is inherited at the beginning of the take-off process, it is also realistic to assume that the initial industrial labor force OB is very small.) At this level of employment the industrial sector is making a profit represented by the shaded area B_0 (Diagram 2.1) which may be taken to represent the economy's investment fund at this stage.¹⁸ This investment fund is to be allocated in part to the agricultural sector, thus raising agricultural productivity and shifting the industrial supply curve to the right, and in part to the industrial sector, thus raising the industrial capital stock and shifting the industrial demand curve to the right.

If the balanced growth criterion is to be satisfied, the new industrial demand curve, e.g., i_2i_2 , and the new industrial supply curve, e.g., L_2L_2 , must intersect at a point, e.g., P_2 , lying on the balanced-growth path (L_1P_3). Otherwise the stability-of-the-terms-of-trade condition is violated. At P_2 , where the balanced-growth criterion is met, the industrial sector will have absorbed O_0P_2 additional workers, which is the same number of workers which has been released by the agricultural sector (i.e., cf_2 in Diagram 2.2 equals O_0P_2 in Diagram 2.1).

Thus, as investment activity in both sectors proceeds through time, the balanced-growth path describes the actual growth path if the balanced-growth criterion is satisfied. It is, of course, likely that the actual growth path will deviate from the balanced-growth path in one direction or the other from time to time. Such a deviation, however, will call into play countervailing equilibrating forces which tend to bring it back to the balanced-growth path. The actual growth path is, in fact, likely to be oscillating around the balanced-growth path.

For example, if the actual growth path is above the balanced-growth path, say at e_2 in Diagram 2.1 (as would be the case if investment in the agricultural sector had shifted the industrial supply curve to L_2L_2 and investment in the industrial sector had shifted the industrial demand curve to i_3i_3), we have a case of overinvestment in the industrial sector. The shortage of food will result in a deterioration of the terms of trade of the industrial sector and will cause an increase in the industrial real wage. This will tend to discourage investment in the industrial, and tend to encourage investment in the agricultural sector, thus causing the actual growth path to turn back toward the balanced-growth path. Government policy may be assumed to work in the same direction if the price system proves inadequate. In this fashion, the economy,

¹⁸ If, for the sake of simplicity, capitalists' consumption can be ignored. It should be noted that the agricultural sector (Diagram 2.2) makes no contribution to the investment fund since the entire agricultural output (area $OaSA$) is just adequate to meet the consumption requirements of the agricultural workers (area $Obca$) and the consumption requirements of the industrial workers (area $AScb$).

proceeding along an actual growth path which coincides with or oscillates around the balanced-growth path, moves towards the turning point, P_3 , previously defined.¹⁴

IV. *Empirical Relevancy of the Basic Model*

In order to formulate our model more rigorously and render it amenable to statistical verification certain restrictive assumptions, not required for our previous qualitative analysis, must now be accepted. The first such assumption, that the marginal physical productivity of labor changes at a constant rate as employment in the agricultural sector varies, is concerned with the shape of the initial total physical productivity curve. This means that the initial MPP curve (I in Diagram 2.2) is composed of two straight-line segments: a horizontal segment, AS_1 , coinciding with the horizontal axis, and a segment S_1t_1' for the range of positive marginal physical productivity. The two segments are connected at point S_1 marking off the redundant agricultural labor force (AS_1 in Diagram 2.2). Under these assumptions, it can be shown (see the Appendix for all detailed derivations) that the initial TPP curve takes on the following form:

$$(1) \quad y = \begin{cases} M \left[-\left(\frac{x}{TL} \right)^2 + 2\left(\frac{x}{TL} \right) \right] & \text{for } x \leq TL \\ M & \text{for } x > TL \end{cases}$$

where the variables x and y , and the parameters M , T and L have the following economic and diagrammatic (Diagram 2.3) interpretation: (i) y =total agricultural output (measured downward from point O); (ii) x =labor force employed in the agricultural sector (measured to the left of point O); (iii) M =maximum agricultural output (the distance AI); (iv) L =size of the population at the break-out point (the distance OA); (v) T =the fraction of L which is nonredundant, i.e., TL is the nonredundant labor force (the distance OS_1 in Diagram 2.2) and $(1-T)L$ is the redundant labor force (the distance S_1A). The parameter T or nonredundancy coefficient may take on any nonnegative value. If T is less than 1, $(1-T)L$ is the redundant labor force at the break-out point. If T is greater than 1, $(T-1)L$ is the addition to the agricultural labor force L which would be tolerated before any portion of the agricultural labor force becomes redundant, i.e., of zero marginal physical

¹⁴ The "unlimited" portion of Lewis' supply curve of labor may thus be interpreted as an *ex post* supply curve defined as the locus of all points on our balanced-growth path under conditions of continuous increases in agricultural productivity. Neither we nor Lewis should, however, discount the possibility that the actual growth path may, in fact, be gently upward-sloping rather than horizontal. Such a growth path would imply gradually rising levels of the industrial real wage during the take-off period. (Also see footnote 8.)

productivity. The case of T less than 1 is depicted in Diagram 2.3.¹⁵

Our second restrictive assumption is that an increase in agricultural productivity shifts the entire TPP curve "upward" proportionally. In other words, the new TPP curve is obtained by multiplying the initial TPP curve by a constant k which will be called the productivity coefficient. As the productivity coefficient takes on successively larger values, a sequence of TPP curves (*II*, *III*, etc.) is generated, as depicted in Diagram 2.3.¹⁶

From the TPP curves we can easily derive expressions for the institutional wage, the marginal physical productivity (MPP) curves, and the average agricultural surplus (AAS) curves:

(2) $W = M/L$ (agricultural wage represented by distance *AS* in Diagram 2.2 or the slope of *OX* in Diagram 2.3)

$$(3) \quad y' = \frac{2kM}{(TL)^2} (-x + TL) \quad (\text{marginal physical productivity curve for the nonredundant agricultural labor force in Diagram 2.2; } 0 < T \leq 1)$$

$$(4) \quad AAS = \frac{ky - xW}{L - x} \quad (\text{average agricultural surplus curve})$$

These variables are functions of x (i.e. the agricultural labor force), with M , T and L as parameters and k the exogenous productivity coefficient.

A major objective of our model is to derive an expression for TALF, the turning-point agricultural labor force, represented by the distance *OS*₃ in Diagram 2.2. TALF is a fraction, V_t , of the total population L , i.e., $\text{TALF} = V_t L$. By solving for the turning-point value of k the following expression for V_t can be derived from (1) through (4):¹⁷

$$(5) \quad V_t = 1 + T - \sqrt{1 + T^2}.$$

This percentage of the population in agriculture at the turning point (V_t) depends only on T , the coefficient of nonredundancy. From the economic standpoint, this means that our model is independent of the size (i.e., the scale) of the economy (as described by the absolute population size, L , or the absolute amount of initial agricultural output, M).

¹⁵ There are those, e.g., Harry Oshima [8, p. 259], who believe that the MPP of agricultural labor in an underdeveloped area never really drops to zero. This position is represented by the second case, i.e., $T > 1$, for no one will probably deny that, with a fixed amount of land, there will be *some* size of agricultural population which is large enough to render MPP zero. While both cases are treated systematically in the appendix, for reasons of ease in exposition we only present the case for $0 < T \leq 1$ in the text. The conclusions for both cases are, however, incorporated in the body of the paper.

¹⁶ Notice that under these assumptions all the MPP curves contain the same horizontal segment *AS*₁.

¹⁷ As shown in the Appendix.

To subject our model to its first test of empirical relevancy, let us examine (Table 1) the values of V_t for a range of values for T (from .7 to 3) which represents, we think, a reasonable spectrum covering most countries. A small T , or a small nonredundancy coefficient, means that a country is initially unfavorably endowed with natural resources, i.e., a low land-labor ratio. Though precise estimates are scarce, most interested observers are agreed that the redundant labor force could be as high as 30 per cent in the densely populated regions of Asia, e.g., Pakistan, India, Ceylon. A nonredundancy coefficient of $T = .7$ thus represents the country with the most unfavorable initial resource endowment. At the other extreme of the spectrum lie certain Western countries, possibly Denmark, which have already completed their take-

TABLE 1

T	.7	.8	.9	1	1.1	1.2	1.3	1.4	2	3
V_t	.48	.52	.55	.58	.61	.64	.66	.68	.75	.80

off process. There is, of course, even less statistical knowledge of the nonredundancy coefficient for any such country at the relevant point in its history; we have picked a more or less arbitrary upper value of $T = 3$, although we are by no means committed to any such figure.¹⁸

For this reasonable range of values for T , the corresponding values for V_t extend from approximately 50 to 80 per cent. This means that at the end of the take-off process our model "predicts" that from 20 to 50 per cent of the total labor force must have been allocated to the industrial sector. Commonly held notions concerning these magnitudes suggest that our results also are reasonable.

From this table we can also see that the value for V_t increases as the value of T increases, a generally valid relation which can be easily established by taking the first derivative of (5). The economic interpretation of this relationship is straightforward: the larger the nonredundancy coefficient the more favorable (relatively) the initial resource endowment; and the more favorable this endowment the more likely that the economy will still be agriculture-oriented (as measured by a relatively large value of V_t) at the turning point. Conversely, the smaller the non-

¹⁸ Notice that V_t approaches 1 as T approaches infinity so that the value of V_t is not very sensitive to the change in T as T becomes larger. Hence we need not be overly concerned with the upper limit for the range of values postulated in Table 1. A large T , incidentally, should not be confused with the possibility that primary production, in say, Australia may always have been organized on a plantation basis, therefore never part of the "agricultural" sector as defined by us (footnote 2). As pointed out earlier, our model is not relevant where the entire economy is commercialized at the outset.

redundancy coefficient, the more unfavorable the initial resource endowment and the more likely that the economy will have to be industry-oriented (as measured by a relatively small value of V_t) by the time of completion of the take-off process.¹⁹ For the former (agriculture-oriented) case associated with some more advanced economies our theory then "predicts" a turning-point agricultural labor force upward of 65 per cent (for T greater than 1.2). For the latter (industry-oriented) case associated with the contemporary underdeveloped countries of Asia, our theory "predicts" a turning-point agricultural labor force downward of 55 per cent (for T smaller than .9). Evidently, if the take-off process is to be successfully completed the resource-poor countries, in which we are primarily interested here, will have to re-allocate a higher percentage of their total labor force to industry than did some of their better-endowed Western counterparts. And this already difficult task is further complicated by the fact that these countries are usually subject to severe population pressures at this stage. We now proceed to integrate this important facet of the developmental problem into our model.

V. Population Growth and the Minimum Effort

Let us assume that, in the course of the take-off, the economy experiences a population increase of 100s per cent. Let the total population at the turning point be denoted by L_t . Then

$$(6) \quad L_t = (1 + s)L$$

where L is the size of the population at the break-out point. For such an increase in population the average agricultural surplus (AAS) function becomes:

$$(7) \quad AAS = \frac{ky - xW}{L(1 + s) - x}$$

When this equation is used in place of (4), we can derive the following expression:

$$(8) \quad V_t = 1 + \frac{T}{1 + s} - \sqrt{1 + \left(\frac{T}{1 + s}\right)^2}$$

¹⁹ Since, in our system, only the commercialized sector is in a position to earn profits and save, this conclusion is consistent with Lewis' prediction [4, p. 27] that "profit margins will be lowest in countries which reach their second stage [turning point] earliest and will be highest in countries where the second stage is longest delayed."

where V_t is the turning-point agricultural labor force (TALF) expressed as a fraction of L_t , i.e., $\text{TALF} = V_t L_t$. (In other words, when there is an increase in population, we use the total population L_t at the turning point, rather than that at the break-out point, as the basis for computing the TALF fraction).

Comparing (5) and (8), we see that our analysis in the last section, assuming no population growth, now reduces to a special case. Furthermore, as far as the impact on V_t is concerned, population increase is equivalent to a decrease in the value of the nonredundancy coefficient, T . This underlines the fact that both phenomena constitute a worsening of the economy's resource base. It follows that, for a given value of T , the larger the population increase (i.e. the larger s) the lower the value of V_t and hence the more industry-oriented the economy will have to be at the turning point.

The significance of expression (8) may now be more fully investigated. In Diagram 3, let time be measured on the horizontal and population on the vertical axis. Let the initial, or break-out population, L , be represented by the distance Ob_0 at the 0th year and the growth of population through time be described by the curve $b_0 bB$, which we shall call the population growth curve (PGC). Population growth will be treated as a known phenomenon exogenous to our model.

As population increases the industrial sector will obviously have to absorb more labor by the time the turning point is reached. In fact, the industrial sector will have to absorb not only more labor absolutely but a higher percentage of the enlarged total population. We may then ask the following hypothetical question: what will be the absolute size of the industrial labor force, L_{ti} , and of the agricultural labor force, L_{ta} , at the turning point, if the take-off process is to be completed in τ years? Let the total turning point population L_t , be represented by the distance $b\tau$. Since, for a given τ , the population growth curve gives us the values for both L and L_t , we can immediately determine the multiple factor $1+s(\tau)$ in (6). [Notice that s is now written as a function $s(\tau)$ of τ .] When the value for $1+s(\tau)$ is substituted in (8), we obtain:

$$(9) \quad V_t(\tau) = 1 + \frac{T}{1+s(\tau)} - \sqrt{\left(1 + \frac{T}{1+s(\tau)}\right)^2}$$

as the fraction of the total population in the agricultural sector at the turning point. It is now expressed as a function of τ , the specified length of time for the completion of the take-off process, treating T , the non-redundancy coefficient, as a parameter. From this equation, we can easily determine the absolute size of the turning-point industrial labor

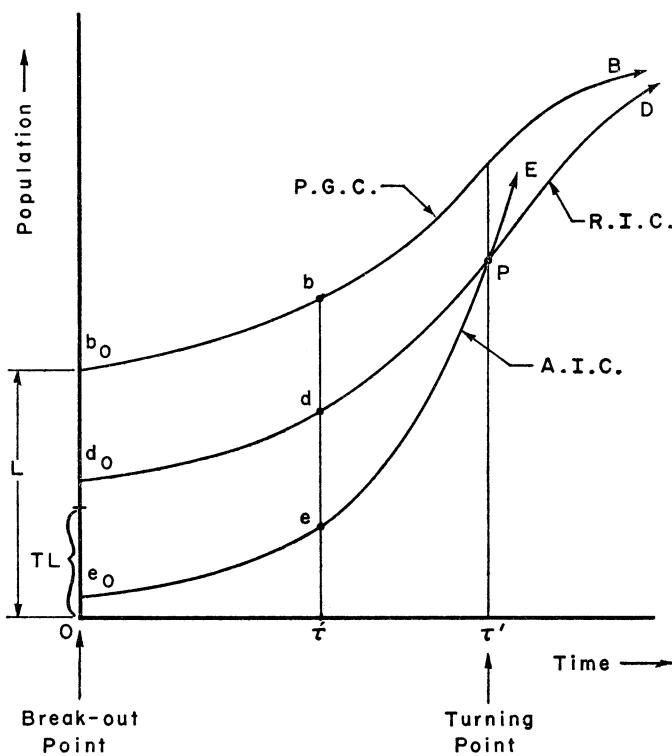


DIAGRAM 3

force L_{ti} and of the turning point agricultural labor force L_{ta} as a function of τ .

$$(10) \quad \begin{aligned} (a) \quad L_{ti} &= [1 - V_t(\tau)][1 + s(\tau)]L \\ (b) \quad L_{ta} &= V_t(\tau)[1 + s(\tau)]L \end{aligned}$$

where $V_t(\tau)$ is defined in (9).

The curve corresponding to (10a), i.e., d_0dD , is plotted in Diagram 3. We shall call this curve the required industrialization curve (RIC). [The vertical distance between RIC and PGC is represented by (10b)]. RIC marks off the absolute size of the population which must be absorbed by the industrial sector if the turning point is to occur at the time indicated on the horizontal axis. As we can see directly from equation (9), the value for $V_t(\tau)$ approaches 0 as τ increases. This means that RIC bends towards PGC as the time required for the take-off is lengthened. The economic significance of this phenomenon is that the

longer it takes to reach the turning point, i.e., the more time there is for the Malthusian devil to assert itself, the heavier the burden on the industrial sector in terms of the absorption of agricultural workers required. RIC indicates the total absorption requirements for each and every τ or length of the take-off process.

This important concept of a required industrialization curve may be interpreted in terms of a critical minimum effort thesis. It means that, for every value of τ , a certain minimum investment activity must be carried on in both the industrial and agricultural sectors during every year of the take-off process, from year 0 to year τ . For, as we have seen, investment in the industrial sector must be adequate to provide employment opportunities for the enlarged industrial labor force; and investment in the agricultural sector must be adequate to increase agricultural productivity sufficiently to feed the increased population in the face of a possibly reduced agricultural labor force. Thus, whether or not the take-off process can, in fact, be completed in τ years depends on whether or not the required effort is forthcoming in the intervening years.

To further clarify this point, let us now, in juxtaposition with the above-described required industrialization curve (RIC), postulate an actual industrialization curve (AIC) which shows the amount of labor actually absorbed by the industrial sector at each point in time. The equation for this curve may be written as

$$(11) \quad E_i = \phi(t)$$

where t measures time and E_i the actual size of the industrial labor force at time t . This curve is denoted by $e_0 e E$ in Diagram 3. At time τ , for example, out of the total labor force or population $b\tau$ the amount *actually* absorbed by the industrial sector equals $e\tau$. At the same time, as we have already seen, the amount of labor which *needs* to have been allocated to this sector is $d\tau$ if turning point is to occur at this time. Hence, in this case, it is impossible to achieve the turning point at time τ . It follows that the take-off process can be successfully completed if and only if AIC and RIC intersect, e.g., at point P , after τ' years.

The position of AIC then depends on the national effort, measured in terms of investment expenditures in both sectors, actually forthcoming in the course of the take-off process. With a larger national effort AIC rises more steeply and intersects RIC at an earlier date, i.e. a smaller τ . Conversely, with a smaller national effort AIC rises more slowly and intersects RIC at a later date; or, alternatively, it does not intersect it at all.

To investigate this problem, let us assume, for the sake of simplicity, that labor is actually being absorbed by industry at a constant annual rate i . AIC in (11) then takes on the following concrete form:

$$(13) \quad L_i = L(1 - V)e^{it}$$

where i , the rate of growth of the industrial labor force, may be taken as a summary index of the national effort.²⁰

In order to enable us to estimate the length of the take-off process with the help of our model, let us assume that the population grows at the constant rate r . PGC is then represented by

$$(14) \quad P = Le^{rt}$$

and, for this particular PGC, RIC in (10a) becomes

$$(15) \quad L_{ti} = [1 - V_t(\tau)]e^{rt}L$$

where, using (9), $V_t(\tau) = 1 + Te^{-rt} - \sqrt{1 + (Te^{-rt})^2}$

We know from our previous discussion that, if the take-off process is to be completed in τ years, the RIC and AIC must intersect at $t=\tau$. Thus the value of τ must satisfy the following equation [obtained by equating (13) and (15)]:

$$(16) \quad L(1 - V)e^{i\tau} = L[1 - V_t(\tau)]e^{r\tau}$$

signifying the intersection of the two curves. Equation (16) enables us to solve for i explicitly in terms of τ :

$$(17) \quad i = r + \frac{\ln (1/1 - V)}{\tau} + \frac{\ln [1 - V_t(\tau)]}{\tau}$$

We can therefore determine the minimum annual effort, as summarized by i , for any given value of τ . Conversely, if we know i we can determine τ , the duration of the take-off process.

²⁰ And $1-V$ is the fraction of the initial population engaged in industry. The significance of i as a national effort index is, of course, by no means a simple matter. A larger i means a faster annual rate of labor absorption by the industrial sector; but this, it should be recalled, necessitates both a higher rate of investment in the agricultural sector, to feed the growing population (in the face of a possible absolute diminution of the agricultural labor force), and a higher rate of investment in the industrial sector, to absorb the newly freed agricultural workers—with allocations between the two sectors obeying our balanced-growth criterion. The national effort behind i is thus a function of the absolute size of the investment fund which can be made available in each year during the course of the take-off process and a function of the efficiency of its use in the two sectors. For the industrial sector, for example, if we assume that only capital-widening takes place, then i also indicates the required annual rate of investment. With respect to the investment requirements of the agricultural sector, the rate of increase of agricultural output must be at least equal to that of the total population and the required annual rate of increase in agricultural productivity can be uniquely determined. Admittedly the real measure of sacrifice lies in the rate of accumulation of profits. But linking this with the rate of industrialization and the rate of change of agricultural productivity which lie behind i requires precise knowledge of the relative efficiency of investment and the impact of technological change on the two sectors. An elaboration of this aspect of the dynamic balanced-growth problem is currently under investigation by the authors but would take us beyond the confines of the present paper.

Before subjecting this result to further conditional testing it remains to generalize our model to bring it an important step closer to reality. In addition to the consumption requirements of both industrial and agricultural workers at the institutional wage there may well be other claims on (or markets for) agricultural output. Specifically, the industrial sector may require raw materials and the industrial worker may require a wage premium over the institutional wage level in agriculture. We may classify these demands as proportional to the industrial labor force, $L(1+s)-x$, with d as the factor of proportionality.²¹ Other, hitherto neglected, markets include landlord consumption requirements and export demand for agricultural products.²² We may classify these demands (for want of a better hypothesis) as proportional to the growth of total agricultural output, with $1-\theta$ as the factor of proportionality. When we subtract these additional items from total agricultural output, we obtain

$$(18) \quad AAS = ky - xW - dW[L(1+s) - x] - (1-\theta)ky$$

in place of (7). With these complications incorporated in our model, and with a given PGC, the following expression can be derived (see appendix) for V_t , the turning-point agricultural labor force as a fraction of the total turning-point population:

$$(19) \quad V_t(\tau) = \frac{1}{(\theta + 2d)(1 + s(\tau))} \left[(\theta + d)T + [1 + s(\tau)](1 + d) - \sqrt{[dT - (1 + s(\tau))(1 + d)]^2 + (\theta + 2d)T^2\theta} \right].$$

V_t is thus a function of parameters T , θ , d and s and we see that our previous formulation in (8) becomes a special case of (19) if we let $d=s=1-\theta=0$. Furthermore, it can be shown that as T increases or as s decreases the value of V_t increases, which is the identical conclusion reached for the simple case.

The analysis of the duration of the take-off process can then once again be summarized by (17) above, but with $V_t(\tau)$ now defined by (19) instead of (15). Using (17), we may now obtain varying values for i with varying values for τ , r , T , V , θ and d . The results are presented in Table 2.²³ They permit us to determine the minimum annual effort

²¹ For computational convenience d can be measured in terms of institutional wage units, W . For example, suppose this "additional" support of industrial workers takes the form of raw materials plus wage premiums to the amount of \$2 per worker and the institutional wage is \$4 per worker; then d equals .5.

²² The authors are currently investigating the fuller open-economy implications of the model.

²³ Parameters V , θ and d have been estimated from relevant empirical data, principally for Japan in the late nineteenth century. The estimate for V (= .8) is based on [7], θ (= .9) on

for any given value of τ . Conversely, if we know the average annual effort, i , which can be elicited we can derive τ , the duration of the take-off process.²⁴

TABLE 2—AVERAGE ANNUAL MINIMUM EFFORT (i) ($\theta=.9$, $d=1.3$, $V=.8$) (per cent)

(T=.7)					(T=.9)				
τ (years)	5	20	35	50	τ (years)	5	20	35	50
r					r				
1.0	8.15	3.05	2.29	1.99	1.0	6.00	2.58	2.07	1.85
2.0	9.52	4.34	3.51	3.16	2.0	7.47	3.96	3.36	3.08
2.5	10.19	4.96	4.10	3.71	2.5	8.20	4.62	3.97	3.66
3.0	10.86	5.56	4.67	4.26	3.0	8.92	5.26	4.57	4.21
3.5	11.53	6.16	5.22	4.79	3.5	9.63	5.89	5.14	4.76

For application to the heavily labor-surplus areas of Asia, e.g. India and Pakistan, the left-hand side of Table 2 is perhaps more relevant. With annual population growth estimated in the vicinity of 2.5 per cent the annual minimum effort in terms of the annual growth of the industrial sector must be more than 10 per cent if take-off is to be achieved within a five-year period. If the country, more realistically, sets a take-off completion goal of 20 or 50 years, the industrial sector must grow at only 4.96 or 3.71 per cent, respectively. Moreover, if population-control programs now under way are successful in bringing the population growth rate down to, say, 1 per cent, the equivalent burden on the economy in terms of minimum effort would be further lowered to 3.05 and 1.99 per cent, respectively. For the case of a Latin American or African country where we can afford to be somewhat more optimistic with respect to the initial resource endowment, the right-hand side of Table 2 may have more relevance.

With the help of Diagram 4, the results of this section may be briefly summarized in the form of a number of comparative static "theorems." Evidently, the larger the annual effort, the shorter the take-off process. This simply confirms the well-known advantage of economies capable

[10], and d (1.3) largely on the results of a recent unpublished input-output study for 1953-54 by the Indian Statistical Institute, Calcutta. Independent estimates for T are admittedly more difficult to come by. We have used two of the more frequently made "guesstimates" covering the range of the plausible (.7 and .9). Well-behaved r is permitted to vary from 1 to 3.5 and τ from 5 to 50.

²⁴ The authors are currently engaged in further testing the empirical validity of the model and thus its predictive value, by examining the extent to which it provides a consistent explanatory framework in the case of countries whose take-off has already been completed. Theoretical performance indices thrown up by the model can, for example, be compared with actual performance indices for given economies over given periods. While an elaboration of this effort takes us beyond the intended scope of the present paper, it may not be inappropriate to report encouraging first results, dealing with the case of Japan.

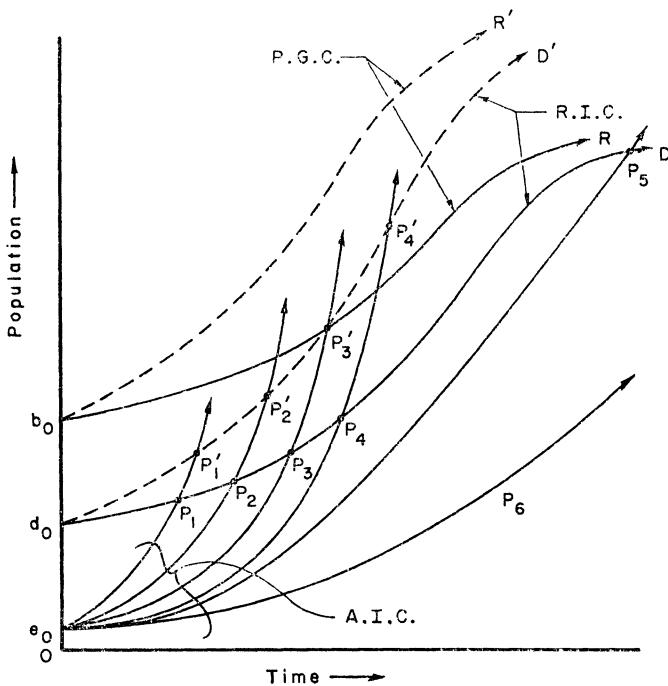


DIAGRAM 4

and willing to submit to temporary austerity for the sake of a larger investment fund. For the same initial industrial population, Oe_0 , in Diagram 4, and the same population growth curve, b_0R , hence the same required industrialization curve, d_0D , a series of actual industrialization curves, e_0P_1, e_0P_2, \dots may be shown. Clearly, the greater the actual annual effort put forth, the more steeply rising AIC and the earlier (i.e. at a lower τ) the occurrence of the turning point marked by the intersection of RIC and AIC (at P_1, P_2, \dots).

The faster the rate of population growth, the longer will be the duration of the take-off process or the more likely that take-off becomes impossible of achievement. With the help of Diagram 4 we can easily trace the effect of an increase in r , shifting PGC from b_0R to, say, b_0R' . Consequently RIC also shifts up (from d_0D to d_0D') and the turning point is delayed as its intersection with the relevant AIC now occurs at P'_1, P'_2, \dots . For the case of AIC e_0P_5 , on the other hand, which intersected the old RIC at P_5 , the turning point may now have become impossible of achievement. The same annual effort is now insufficient to cope with the challenge of an accelerated growth in numbers.

If the minimum effort which can be elicited from the economy is not

sufficiently large, successful take-off may prove impossible altogether. This observation simply confirms the notion that some economies may be unable to reach the turning point, no matter how long they are willing to wait, because their resource endowment or their motivations are inadequate. This situation is represented in Diagram 4 by AIC $e_0 P_6$ which, it will be noted, does not intersect RIC at any point regardless of the time period permitted. It is perhaps only in this sense that we may speak of a unique critical minimum effort as that minimum annual rate of growth of the industrial labor force which just leads to tangency with the relevant RIC. If i falls below this critical minimum, the value for τ will be infinitely large. For such a country, since the turning point does not occur and the take-off process is not successfully completed, we may say, without violating common sense, that the process has never really begun. The economy is really experiencing only a temporary departure from stagnation and is, in fact, still in its pre-conditioning stage.

As we can see from equation (17), the take-off can occur only if $i > r$; if $i = r$ or $i < r$, no matter how large a τ is permitted, take-off becomes impossible. If r increases, usually due to a fall in mortality, the economy must either bring it back down again, through a lowering of fertility by means of a planned parenthood program, or must increase its national development effort, i , by further tightening its belt. It should thus be emphasized that the concept of a critical minimum effort cannot have an independent life but must be defined in terms of a given rate of population growth as well as a given target date for completion of the take-off process. A "big push" is required not to achieve a once-and-for-all departure from stagnation but to provide a sustained effort over time relative to the strength of the Malthusian pressures at hand and the growth aspirations of a given society. Using the by now familiar analogy, it is not sufficient for a plane to achieve an initial velocity permitting it to escape the earth's gravitational pull; it must be able to carry enough fuel to enable it to get over the surrounding mountains and reach its destination at a speed dictated by the ambitiousness of the pilot.

What we have thus attempted in this paper is to construct an explanatory model of the less developed economy's transition from stagnation to self-sustaining growth. In the course of this attempt a number of familiar notions current in the literature on development have been stated in a rigorous way and assimilated into what we consider to be a meaningful pattern. A reformulation of the assumptions underlying the Lewis unlimited supply curve of labor enabled us to define the take-off process in a nonarbitrary fashion and, with the help of a balanced growth concept for the short run and a refurbished minimum effort

thesis for the long run, to elaborate the conditions for its successful completion.

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APPENDIX

By JOHN C. H. FEI AND GUSTAV RANIS

I. Total Product and Marginal Product Functions

In diagrams A.1 and A.2 let point O be the origin and let the agricultural population, x , be measured on the horizontal axis to the left of O . The TPP (total physical productivity) function, $f(x)$, and the MPP (marginal physical productivity) function, $f'(x)$, are measured on the vertical axis; downward for $f(x)$ and upward for $f'(x)$.

Let the initial agricultural population be L (located at point S in both diagrams A.1 and A.2) and let the total output for $x=L$ be M (located at point S'). Let the nonredundant labor force in each case be TL (i.e., located at point P). The definition of the nonredundant labor force is $f'(x)=0$ for $x \geq TL$.

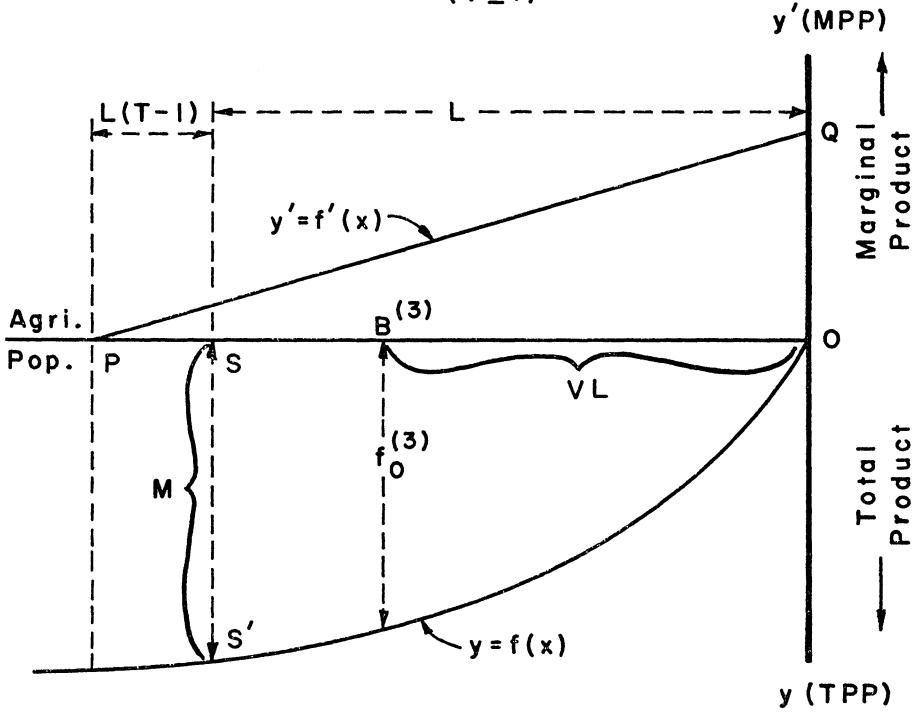
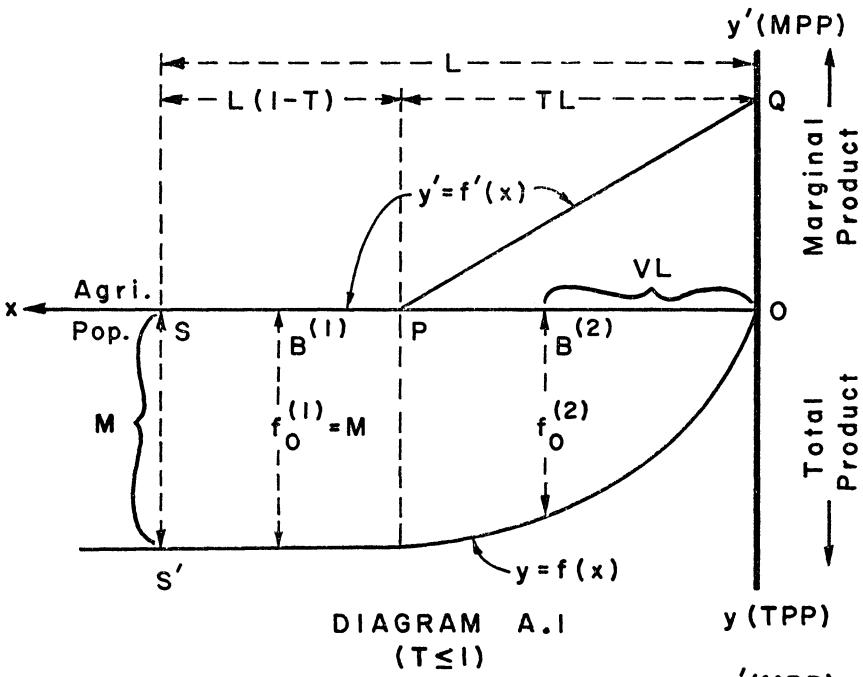
In deriving the TPP function, two cases must be distinguished, namely, $0 < T \leq 1$ (diagram A.1) and $T > 1$ (diagram A.2). The first case means that a part of L , to be more precise, $(1-T)L$, is already redundant. The second case means that the existing supply of land *could* have tolerated a further increase [to the amount of $(T-1)L$] of population beyond the initial population, L , before any portion of the population would become redundant. Assuming that $f''(x)=0$ (i.e., the MPP function is a straight line), the TPP function, $f(x)$, must satisfy the following conditions for the two cases just distinguished:

- (1) (a) $f''(x)=0$ (MPP curve is formed of straight lines)
 (b) $f'(x)=0$ (MPP curve is a horizontal line beyond point P) for
 $x \geq TL$
 (c) $f(0)=0$ (TPP curve starts from the origin)
 (d) $\begin{cases} f(x) = M & \text{for the case } 0 < T \leq 1 \text{ (diagram A.1)} \\ f(L) = M & \text{for the case } T > 1 \text{ (diagram A.2)} \end{cases}$

It is easy to prove that the TPP function, $f(x)$, will take on the following forms if all the conditions in (1) are to be satisfied:

$$(2) \quad \begin{aligned} (a) \quad y &= \begin{cases} M[-(x/TL)^2 + 2(x/TL)] & \text{for } x \leq TL \\ M & \text{for the case } T \leq 1 \text{ (diagram A.1)} \\ M & \text{for } x > TL \end{cases} \\ (b) \quad y &= [M/(2T - 1)][-(x/L)^2 + 2T(x/L)] \quad \text{for } x \leq TL^{25} \\ &\qquad\qquad\qquad \text{for the case } T > 1 \text{ (diagram A.2)} \end{aligned}$$

² TPP is constant for $x \geq TL$. However, since the agricultural population will decline rather than increase during the take-off process, we shall not be concerned with this portion of the TPP function in our analysis.



An increase of agricultural productivity is defined as an upward proportional shift of the entire TPP curve. This can be stated as:

$$(3) \quad (a) \quad y = f^{(1)}(k, x) = \begin{cases} kM[-(x/TL)^2 + 2(x/TL)] & \text{for } x \leq TL \\ & \text{for the case } T \leq 1 \text{ (diagram A.1)} \\ kM & \text{for } x > TL \end{cases}$$

$$(b) \quad y = f^{(3)}(k, x) = [kM/(2T - 1)][-(x/L)^2 + 2T(x/L)]$$

for the case $T > 1$ (diagram A.2)

In other words, after an increase of agricultural productivity has taken place, the new TPP function is a k -multiple of the functions in (2). The constant $k \geq 1$ will be referred to as the "productivity coefficient" because it measures the degree of increase of agricultural productivity. (The choice of notation $f^{(3)}(k, x)$ in (3b) is to facilitate later exposition).

From (2) and (3), the MPP functions can be derived:

$$(4) \quad (a) \quad y' = \begin{cases} [2kM/(TL)^2][-x + TL] & \text{for } x \leq TL \\ & \text{for the case } T \leq 1 \text{ (diagram A.1)} \\ 0 & \text{for } x > TL \end{cases}$$

$$(b) \quad y' = [2kM/(2T - 1)L^2][-x + TL]$$

for the case $T > 1$ (diagram A.2)

II. Total Output and the Institutional Wage at the Break-Out Point

At the break-out point, a part of the initial population, L , may have already been allocated to the industrial sector. Let the agricultural population at the break-out point be VL where $0 \leq V \leq 1$ is the fraction of L in agriculture at that time. The break-out point is indicated by the points $B^{(i)}$ ($i = 1, 2, 3$) in diagrams A.1 and A.2. These notations are chosen to distinguish three possible cases:

Case one: $V \geq T$ for $T \leq 1$ (represented by point $B^{(1)}$ in diagram A.1)

Case two: $V < T$ for $T \leq 1$ (represented by point $B^{(2)}$ in diagram A.1)

Case three: $V < T$ for $T > 1$ (represented by point $B^{(3)}$ in diagram A.2)

These cases will be indexed by $i = 1, 2, 3$ throughout this appendix. (For case one, the MPP = 0; for cases two and three the MPP is positive.)

Denote the total agricultural output at the break-out point by $f_0^{(i)}$ ($i = 1, 2, 3$). The values for $f_0^{(i)}$ can be computed from (3):

$$(5) \quad (a) \quad f_0^{(1)} = f^{(1)}(k = 1, x = VL) = M \quad (\text{for } i = 1 \text{ in diagram A.1})$$

$$(b) \quad f_0^{(2)} = f^{(1)}(k = 1, x = VL) = [MV/T^2][-V + 2T]$$

(for $i = 2$ in diagram A.1)

$$(c) \quad f_0^{(3)} = f^{(3)}(k = 1, x = VL) = [MV(2T - 1)][-V + 2T]$$

(for $i = 3$ in diagram A.2)

Let the institutional wage rate be denoted by $W^{(i)}$ ($i = 1, 2, 3$). The value

of $W^{(i)}$ is determined by the requirement that the total agricultural output $f_0^{(i)}$ at the break-out point be just adequate to provide for:

- (1) consumption of the agricultural population (VL) at the wage rate $W^{(i)}$;
- (2) consumption of the industrial labor force $[(1 - V)L]$ at the wage rate, $W^{(i)}$;
- (3) consumption of the landlord and other uses proportional to total agricultural output assumed to be a fraction, $1 - \theta$, of $f_0^{(i)}$;
- (4) demand for agricultural product as industrial raw materials and other requirements assumed to be proportional to the industrial labor force $[(1 - V)L]$.²⁶

This can be written as:

$$(6) \quad f_0^{(i)} = W^{(i)}VL + W^{(i)}(1 - V)L + (1 - \theta)f_0^{(i)} + dW^{(i)}(1 - V)L$$

(The parameter d is the "input coefficient," i.e., the amount of agricultural product used as industrial raw materials per industrial worker employed in the industrial sector, and is measured in terms of wage units.)

From (6) the institutional wage can be determined as:

$$(7) \quad W^{(i)} = (\theta/RL)f_0^{(i)} \quad \text{for } i = 1, 2, 3$$

where

$$(8) \quad R = 1 + d - dV.$$

An explicit expression of $W^{(i)}$, defined in terms of the parameters M , L , θ , V , T , d (so far introduced in our system), can be obtained from (5) and (7):

- (a) $W^{(1)} = \theta M/RL$
- (b) $W^{(2)} = \theta MV(-V + 2T)/RLT^2$
- (c) $W^{(3)} = \theta MV(-V + 2T)/RL(2T - 1)$

III. *Balanced Agricultural Labor Force (BALF) and Commercialized Agricultural Labor Force (CALF)*

Let the size of the agricultural labor force at the shortage (commercialization) point (see Section I) be called balanced (commercialized) agricultural labor force, i.e., BALF (CALF). We want to determine BALF and CALF as a function of k . Suppose there is a 100s per cent increase of total population in the agricultural sector after the break-out point. The total population increases from L to $(1+s)L$. If x is the size of the agricultural population, the industrial population is $L(1+s) - x$. For this distribution of the total population between the two sectors, the demand for agricultural

²⁶ Please note that in the text we initially introduce a simplified version of our model for which (3) and (4) are not taken into consideration, i.e., d is assumed to be equal to 0 and θ equal to 1. Under these circumstances all other results in this appendix, which reflect the complete model, could be appropriately simplified. In like fashion, of course, we initially abstract from population growth in the text, i.e., we assume $s=0$. The interested reader may verify the results presented in the text by letting $s=d=(1-\theta)=0$.

output contains the following components [when the same behavioristic assumptions, (1)–(4), identified in the last section, obtain]:

- (10) (a) landlord consumption $A^{(i)}(x) = (1 - \theta)f^{(i)}(k, x)$
- (b) farm labor consumption $B^{(i)}(x) = W^{(i)}x$
- (c) industrial raw materials $C^{(i)}(x) = dW^{(i)}[L(1 + s) - x]$
- (d) industrial labor consumption $D^{(i)}(x) = W^{(i)}[L(1 + s) - x]$

where $W^{(i)}$ is defined in (7) or (9) and $f^{(i)}(k, x)$ is defined in (3). [Notice that, for $i = 1, 2$, $f^{(i)}(k, x)$ are both defined by (2a)]. Since the supply of agricultural output equals demand, then

$$(11) \quad f^{(i)}(k, x) = A^{(i)}(x) + B^{(i)}(x) + C^{(i)}(x) + D^{(i)}(x)$$

The solution for x in (11) is the balanced agricultural labor force (BALF). This is just an alternative way of saying that BALF is determined by the requirement that the average agricultural surplus (AAS) should equal the institutional wage $W^{(i)}$ which is the expression we have used in the text. TAS (total agricultural surplus) and AAS (average agricultural surplus) are defined as:

$$\begin{aligned} \text{TAS} &= f^{(i)}(k, x) - A^{(i)}(x) - B^{(i)}(x) - C^{(i)}(x) \\ \text{AAS} &= \text{TAS}/[L(1 + s) - x] \end{aligned}$$

Equation (11) can also be obtained by equating AAS with $W^{(i)}$.

To solve for x in (11), let U^* be BALF expressed as a fraction of the total population, i.e., $\text{BALF} = U^*L(1 + s)$. Substituting:

$$(12) \quad x = U^*L(1 + s)$$

in (11) above, the value of U^* can be determined. In other words, it is the fraction, U^* , rather than the absolute amount of BALF, that will be determined. Substituting (7) and (12) in (11), we have:

$$(13) \quad f^{(i)}(k, x)/f_0^{(i)}(1 + s)R^*/R$$

where

$$(14) \quad R^* = 1 + d - dU^*$$

and where the left-hand side can be computed from (3), (5) and (12). Equation (13) then becomes:

- (a) $(kR/T^2)[-(U^*(1 + s))^2 + 2U^*T(1 + s)] - (1 + s)(1 + d) + dU^*(1 + s) = 0 \quad \text{for } i = 1$
- (15) $(b) (ZR/T^2)[-(U^*(1 + s))^2 + 2U^*T(1 + s)] - (1 + s)(1 + d) + dU^*(1 + s) = 0, \quad \text{for } i = 2, 3$

where, for (15b),

$$(16) \quad Z = kT^2/V(-V + 2T)$$

These equations define U^* as a function of k , the productivity coefficient. Letting this function, in its explicit form, be denoted by $U^* = U^*(k)$, and noticing that Z takes the place of k in (15b), we have, after simplification,

$$(17) \quad \begin{aligned} (a) \quad U^* &= U^*(k) = \frac{T}{2kR(1+s)} [2kR + dT \\ &\quad - \sqrt{(2kR + dT)^2 - 4kR(1+s)(1+d)}], \quad \text{for } i = 1 \\ (b) \quad U^* &= U^*(Z), \quad \text{for } i = 2, 3 \end{aligned}$$

To compute the commercialized agricultural labor force (CALF), first set MPP [in (4)] equal to the institutional wage $W^{(i)}$ in (9):

$$(18) \quad \begin{aligned} (a) \quad W^{(1)} &= [2kM/(TL)^2](-x + TL), \quad \text{for } i = 1 \\ (b) \quad W^{(2)} &= [2kM/(TL)^2](-x + TL), \quad \text{for } i = 2 \\ (c) \quad W^{(3)} &= [2kM/(2T - 1)L^2](-x + TL), \quad \text{for } i = 3 \end{aligned}$$

The solution of x in (18) gives us the CALF. Denoting CALF by $V^*(1+s)L$, (i.e., V^* is the fraction of total population which is CALF), the expression:

$$(19) \quad x = V^*(1+s)L$$

can be substituted in (18) in order to solve for V^* as a function $V^*(k)$ of k . After substituting (19) and (9) in (18), we derive:

$$(20) \quad \begin{aligned} (a) \quad V^* &= V^*(k) = [T/(1+s)](1 - T\theta/2kR) \quad \text{for } i = 1 \\ (b) \quad V^* &= V^*(Z) \quad \text{for } i = 2, 3 \end{aligned}$$

where Z in (20b) is defined as in (16).

IV. Turning-Point Productivity Coefficient ($k^{(i)}$) and Turning-Point Agricultural Labor Force (TALF)

The turning point productivity coefficient $k^{(i)}$ is that level of productivity coefficient which equates $U^*(k)$, (17), and $V^*(k)$, (20). Solving for k by setting $U^*(k) = V^*(k)$, we have:

$$(21) \quad \begin{aligned} (a) \quad k^{(i)} &= (1/2R)[(1+s)(1+d) - dT \\ &\quad + \sqrt{(dT - (1+s)(1+d))^2 + (\theta + 2d)T^2\theta}], \quad \text{for } i = 1 \\ (b) \quad k^{(i)} &= [V(-V + 2T)/T^2]k^{(1)} \quad \text{for } i = 2, 3 \end{aligned}$$

expressing $k^{(i)}$ as a function of the parameters, V , s , d , T , θ . The turning-point agricultural labor force, V_t , (TALF) is the BALF (=CALF) when the productivity coefficient, k , takes on the turning point value (21). Thus substituting (21) in (20), we have:

$$V_t = A + B - \sqrt{(A + B)^2 - 2AB\left(\frac{2d + \theta}{d + \theta}\right)} \quad \text{where}$$

$$(22) \quad \begin{aligned} A &= \frac{1+d}{\theta+2d} \\ B &= \frac{Q(d+\theta)}{\theta+2d} \\ Q &= \frac{T}{1+s} \end{aligned}$$

The value of V_t , which is the same for all three cases ($i=1, 2, 3$), is seen to be a function of the parameters, d, T, s, θ . It should also be noted that in (22), the parameters L (the initial population) and M (the initial total agricultural output) are not involved. The economic significance of this fact is that the absolute size, i.e. the scale, of the economy, measured in terms of L and/or M , is irrelevant to the arguments of this paper.

From (22) we see that V_t is nonnegative. Furthermore, it can be shown that $V_t \leq 1$ if the following condition is satisfied:

$$(23) \quad T \leq \frac{(1-\theta/2)(1+s)}{(1-\theta)}$$

This, as we have pointed out in the course of the empirical discussion of our model in the text, permits all reasonable values of T to be postulated to yield $0 \leq V_t \leq 1$. Assuming (23) is satisfied, it can easily be shown that

$$\frac{\partial V_t}{\partial T} \geq 0 \quad \text{and} \quad \frac{\partial V_t}{\partial s} \leq 0.$$