

## Mechanism Design with Interdependent Values

In the preceding chapters we considered the relative performance of three common formats—the first-price, second-price, and English auctions—in a setting with interdependent values and affiliated signals. Here, as in Chapter 5, we study abstract selling mechanisms with a view to comparing the performance of the common auction forms to that of the “ideal” method of selling a single object. Again our search is for both optimal and efficient mechanisms, but in a more general informational setting. In this setting, it is convenient to tackle the efficiency question first, so we begin by ascertaining circumstances under which an efficient mechanism exists. We then look for optimal, or revenue maximizing, mechanisms.

As a preliminary observation, note that the revelation principle from Chapter 5 applies equally well to the setting of interdependent values and affiliated signals. Proposition 5.1 continues to hold in this general setting without amendment: Given a mechanism and an equilibrium for that mechanism, there exists a direct mechanism in which (1) it is an equilibrium for each buyer to report his signal truthfully, and (2) the outcomes are the same as in the original mechanism. A direct mechanism asks buyers to report their private information—in this case, their signals—and replicates the equilibrium outcomes of the original mechanism.

As before, denote by  $\mathcal{X}_i$  the set of signals that buyer  $i$  can receive and let  $\mathcal{X} = \times_j \mathcal{X}_j$ . Let  $\Delta$  denote the set of probability distributions over the set of buyers  $\mathcal{N}$ . The revelation principle allows us to restrict attention to mechanisms of the form  $(\mathbf{Q}, \mathbf{M})$  consisting of a pair of functions  $\mathbf{Q}: \mathcal{X} \rightarrow \Delta$  and  $\mathbf{M}: \mathcal{X} \rightarrow \mathbb{R}^N$ , where  $Q_i(\mathbf{x})$  is the probability that  $i$  will get the object and  $M_i(\mathbf{x})$  is the payment that  $i$  is asked to make.

## 10.1 EFFICIENT MECHANISMS

With private values, the second-price auction allocates efficiently but as we have seen, the question is more delicate once values are interdependent. In particular, with interdependent values none of the common auction forms is efficient in general. The previous chapter was devoted to deriving a sufficient condition for the English auction to have an efficient equilibrium—the average crossing condition—but question of whether there exists a mechanism that allocates efficiently even when this condition does not hold remains to be explored.

Recall from the Chapter 9 that the valuations  $\mathbf{v}$  are said to satisfy the *single crossing condition* if for all  $j$  and  $i \neq j$ ,

$$\frac{\partial v_j}{\partial x_j}(\mathbf{x}) > \frac{\partial v_i}{\partial x_j}(\mathbf{x})$$

at every  $\mathbf{x}$  such that  $v_i(\mathbf{x})$  and  $v_j(\mathbf{x})$  are equal and maximal over all buyers.

We have already seen that the English auction need not allocate efficiently if the single crossing condition is not satisfied. A simple example illustrates that the single crossing condition cannot be dispensed with even if we consider general, abstract mechanisms.

**Example 10.1.** *If the single crossing condition does not hold, then there may be no mechanism that allocates the object efficiently. Suppose*

$$\begin{aligned} v_1(x_1, x_2) &= x_1 \\ v_2(x_1, x_2) &= (x_1)^2 \end{aligned}$$

Suppose that buyer 1's signal  $X_1$  lies in  $[0, 2]$ . Notice that buyer 2's signal does not affect the value of either buyer, so there is no loss in supposing that it is a constant. The valuations do not satisfy the single crossing condition, since  $v_1(1, x_2) = v_2(1, x_2)$ , but

$$\frac{\partial v_1}{\partial x_1}(1, x_2) < \frac{\partial v_2}{\partial x_1}(1, x_2)$$

Clearly,  $v_1(x_1, x_2) > v_2(x_1, x_2)$  if and only if  $x_1 < 1$ , so it is efficient to allocate the object to buyer 1 when his signal is low and to buyer 2 when it is high.

Suppose there is a mechanism that is efficient and has the payment rule  $M_1 : [0, 2] \rightarrow \mathbb{R}$  for buyer 1. Since buyer 2 has no private information that is relevant, her signal is assumed to be a constant, so buyer 1's payment can only depend on his own reported signal.

Now if  $y_1 < 1 < z_1$ , then efficiency and incentive compatibility together require that when his true signal is  $z_1$ ,

$$0 - M_1(z_1) \geq z_1 - M_1(y_1)$$

and likewise, when his true signal is  $y_1$ ,

$$y_1 - M_1(y_1) \geq 0 - M_1(z_1)$$

Together these imply that  $y_1 \geq z_1$ , which is a contradiction. ▲

In general, suppose that there exists an efficient mechanism with an *ex post* equilibrium. Then by a version of the revelation principle there exists an efficient direct mechanism in which truth-telling is an *ex post* equilibrium. We will now argue that the valuation functions must satisfy the single crossing condition. Consider the signals of all buyers other than  $i$ ,  $\mathbf{x}_{-i}$ . If regardless of his signal  $x_i$ , buyer  $i$  either always wins or always loses, then the single crossing condition holds vacuously for  $x_i$ . Otherwise, we will say that buyer  $i$  is *pivotal* at  $\mathbf{x}_{-i}$  if there exist signals  $y_i$  and  $z_i$  such that  $v_i(y_i, \mathbf{x}_{-i}) > \max_{j \neq i} v_j(y_i, \mathbf{x}_{-i})$  and  $v_i(z_i, \mathbf{x}_{-i}) < \max_{j \neq i} v_j(z_i, \mathbf{x}_{-i})$ . In other words, when the others' signals are  $\mathbf{x}_{-i}$ ,  $i$ 's signal is crucial in determining whether or not it is efficient for him to get the object. Incentive compatibility requires that when his signal is  $y_i$ , it is optimal for  $i$  to report  $y_i$  rather than  $z_i$ , so that

$$v_i(y_i, \mathbf{x}_{-i}) - M_i(y_i, \mathbf{x}_{-i}) \geq -M_i(z_i, \mathbf{x}_{-i})$$

Likewise, when his signal is  $z_i$ , it is optimal to report  $z_i$  rather than  $y_i$ , so that

$$-M_i(z_i, \mathbf{x}_{-i}) \geq v_i(z_i, \mathbf{x}_{-i}) - M_i(y_i, \mathbf{x}_{-i})$$

Combining the two conditions results in

$$v_i(y_i, \mathbf{x}_{-i}) \geq M_i(y_i, \mathbf{x}_{-i}) - M_i(z_i, \mathbf{x}_{-i}) \geq v_i(z_i, \mathbf{x}_{-i})$$

so a necessary condition for incentive compatibility is

$$v_i(y_i, \mathbf{x}_{-i}) \geq v_i(z_i, \mathbf{x}_{-i}) \tag{10.1}$$

that is, buyer  $i$ 's value when he wins the object must be at least as high as when he does not. Put another way, keeping others' signals fixed, an increase in buyer  $i$ 's value that results from a change in his own signal cannot cause him to lose if he were winning earlier. Thus, *ex post* incentive compatibility implies that the mechanism must be *monotonic* in values. But by assumption buyer  $i$ 's value is an increasing function of his own signal, so we conclude that an increase in buyer  $i$ 's signal cannot cause him to lose if he were winning earlier.

Efficiency now requires that if  $i$  has the highest value and wins the object, he should still have the highest value to win the object, if his signal increases.

This in turn requires that at any  $x_i$  such that  $v_i(x_i, \mathbf{x}_{-i}) = v_j(x_i, \mathbf{x}_{-i})$ , we must have

$$\frac{\partial v_i}{\partial x_i}(x_i, \mathbf{x}_{-i}) > \frac{\partial v_j}{\partial x_i}(x_i, \mathbf{x}_{-i})$$

Thus, the single crossing condition is necessary for efficiency.

We now show that the single crossing condition is also sufficient to guarantee efficiency. If it is satisfied, then a generalization of the Vickrey-Clarke-Groves (VCG) mechanism, introduced in Chapter 5, to the interdependent values environment accomplishes the task.

### THE GENERALIZED VCG MECHANISM

Consider the following direct mechanism. Each buyer is asked to report his or her signal. The object is then awarded efficiently relative to these reports—it is awarded to the buyer whose value is the highest when evaluated at the reported signals. Formally,

$$Q_i^*(\mathbf{x}) = \begin{cases} 1 & \text{if } v_i(\mathbf{x}) > \max_{j \neq i} v_j(\mathbf{x}) \\ 0 & \text{if } v_i(\mathbf{x}) < \max_{j \neq i} v_j(\mathbf{x}) \end{cases} \quad (10.2)$$

and if more than one buyer has the highest value, the object is awarded to each of these buyers with equal probability. The buyer who gets the object pays an amount

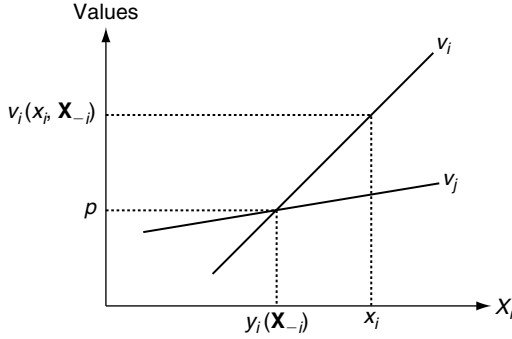
$$M_i^*(\mathbf{x}) = v_i(y_i(\mathbf{x}_{-i}), \mathbf{x}_{-i}), \quad (10.3)$$

where

$$y_i(\mathbf{x}_{-i}) = \inf \left\{ z_i : v_i(z_i, \mathbf{x}_{-i}) \geq \max_{j \neq i} v_j(z_i, \mathbf{x}_{-i}) \right\}$$

is the smallest signal such that given the reports  $\mathbf{x}_{-i}$  of the other buyers, it would still be efficient for buyer  $i$  to get the object. A buyer who does not obtain the object does not pay anything. The workings of the mechanism are illustrated in Figure 10.1, which depicts the values of buyers  $i$  and  $j$  as functions of  $i$ 's signal (the reported signals of the others,  $\mathbf{x}_{-i}$ , are held fixed). At the signal  $x_i$ , buyer  $i$  has the highest value. In particular, it exceeds that of buyer  $j$ . The signal  $y_i(\mathbf{x}_{-i}) < x_i$  is the smallest signal such that his value is at least as large as that of another buyer—in this case, buyer  $j$ —and buyer  $i$  is asked to pay an amount  $p = v_i(y_i(\mathbf{x}_{-i}), \mathbf{x}_{-i})$ .

The generalized VCG mechanism adapts the workings of a second-price auction with private values to the interdependent values setting. Notice that if the buyer—say,  $i$ —who obtains the object were asked to pay the second-highest value at the reported signals, say  $v_j(x_i, \mathbf{x}_{-i})$ , then he would have the



**FIGURE 10.1** The generalized VCG mechanism.

incentive to report a lower signal in order to lower the price paid. The generalized VCG mechanism restores the incentive to tell the truth by asking the winning buyer to pay  $v_j(y_i(\mathbf{x}_{-i}), \mathbf{x}_{-i})$  instead of  $v_j(x_i, \mathbf{x}_{-i})$ . The key point is that, as in a second-price auction with private values, the reports of a buyer influence whether or not he obtains the object but do not influence the price paid if indeed he does so.

**Proposition 10.1.** *Suppose that the valuations  $\mathbf{v}$  satisfy the single crossing condition. Then truth-telling is an efficient ex post equilibrium of the generalized VCG mechanism  $(\mathbf{Q}^*, \mathbf{M}^*)$ .*

*Proof.* Suppose that when all buyers report their signals truthfully, it is efficient for buyer  $i$  to be awarded the object with positive probability. In other words, buyer  $i$ 's value is the highest when evaluated at the reported signals so that

$$v_i(x_i, \mathbf{x}_{-i}) \geq \max_{j \neq i} v_j(x_i, \mathbf{x}_{-i})$$

Buyer  $i$  pays an amount  $v_i(y_i(\mathbf{x}_{-i}), \mathbf{x}_{-i})$ , which is no greater than the true value of the object, so he makes a nonnegative surplus. If buyer  $i$  reports a  $z_i$  such that  $z_i > y_i(\mathbf{x}_{-i})$ , then by the single crossing condition,  $v_i(z_i, \mathbf{x}_{-i}) > \max_{j \neq i} v_j(z_i, \mathbf{x}_{-i})$ , so he would still obtain the object and pay the same amount as if he had reported  $x_i$ . Thus, reporting a  $z_i > y_i(\mathbf{x}_{-i})$  makes no difference to the outcome. On the other hand, if he reports a  $z_i \leq y_i(\mathbf{x}_{-i})$ , then again by the single crossing condition his surplus is zero. Thus, no  $z_i \neq x_i$  can be a profitable deviation in the circumstances that it is efficient for  $i$  to win the object.

Now suppose that when all buyers report their signals truthfully, it is efficient for a buyer other than  $i$  to win the object; that is

$$v_i(x_i, \mathbf{x}_{-i}) < \max_{j \neq i} v_j(x_i, \mathbf{x}_{-i})$$

and so buyer  $i$ 's payoff is zero. This means that  $x_i < y_i(\mathbf{x}_{-i})$  and for him to win, the single crossing condition ensures that buyer  $i$  would have to report a  $z_i \geq y_i(\mathbf{x}_{-i}) > x_i$ . In that case, he would pay an amount

$$M_i^*(z_i, \mathbf{x}_{-i}) = v_i(y_i(\mathbf{x}_{-i}), \mathbf{x}_{-i}) > v_i(\mathbf{x})$$

and so this would not be profitable either. ■

In the case of private values, the generalized VCG mechanism reduces to the ordinary second-price auction, and in that case, of course, truth-telling is a dominant strategy. Also, when there are only *two* buyers the generalized VCG mechanism is the direct mechanism that corresponds to the efficient equilibrium of the English auction identified in Proposition 9.1: The allocations and payments in the two are the same.

Finally, note that when values are interdependent, the generalized VCG mechanism is *not* “detail free” in the sense discussed in Chapter 5: It is not an auction. The mechanism designer is assumed to have knowledge of the valuation functions  $v_i$ , and the mechanism is then able to elicit information regarding the signals  $x_i$  that is privately held by the buyers. Moreover, buyers with different valuation functions are treated differently so that the mechanism is not anonymous.

## 10.2 OPTIMAL MECHANISMS

In the independent private values model considered in Chapter 5, we saw that even in the optimal mechanism each buyer was able to appropriate positive informational rents. Put another way, the incomplete nature of the information available to the seller—his knowledge consisted only of the underlying distributions and not the actual realized values—meant that he was unable to extract all the surplus from the buyers. Actually, as we will see in this section, the key feature is not that the values are private but rather that they are independently distributed. This means that each buyer has private information that is exclusive in a strong sense. Not only does no one else know his value, but no one else knows anything that could provide even statistical information about it. The informational rents accruing to the buyers come solely from this strong exclusivity of information.

In this section we argue that if buyers' information is correlated—and so, statistically dependent—they are unable to garner any informational rents whatsoever. The surprising conclusion is that the slightest degree of correlation in information among the buyers allows the seller to extract *all* the surplus.

This quite remarkable result is easiest to derive in a context in which each buyer's signal is a *discrete* random variable. This constitutes a departure from

the informational structure used until now: Each buyer's information was in the form of a continuous random variable distributed over some interval  $[0, \omega_i]$ —but simplifies the analysis greatly and makes the point most clearly. Specifically, in this section we assume that each buyer's signal  $X_i$  is drawn at random from a finite set

$$\mathcal{X}_i = \{0, \Delta, 2\Delta, \dots, (t_i - 1)\Delta\}$$

with  $t_i$  possible signals. All other features of the model remain unaltered, in essence. Specifically, buyers' values are determined by the joint signal via the valuation functions  $v_i : \mathcal{X} \rightarrow \mathbb{R}_+$  satisfying  $v_i(\mathbf{0}) = 0$ . Other assumptions regarding the valuation functions are translated in a natural manner into their discrete counterparts. We suppose that the  $v_i$  are nondecreasing:  $v_i(x_j + \Delta, \mathbf{x}_{-j}) \geq v_i(x_j, \mathbf{x}_{-j})$  with a strict inequality if  $i = j$ . The discrete version of the *single crossing condition* is as follows: for all  $i$  and  $j \neq i$ ,

$$v_i(x_i, \mathbf{x}_{-i}) \geq v_j(x_i, \mathbf{x}_{-i}) \Rightarrow v_i(x_i + \Delta, \mathbf{x}_{-i}) \geq v_j(x_i + \Delta, \mathbf{x}_{-i}) \quad (10.4)$$

and if the former is a strict inequality, then so is the latter.

#### FULL SURPLUS EXTRACTION

In the preceding section we saw that, provided that the single crossing condition was satisfied, truth-telling was an efficient *ex post* equilibrium of the generalized VCG mechanism. Although we derived this result in a context where signals were continuously distributed, the same is true when they are discrete variables. Thus, as long as the (discrete) single crossing condition (10.4) is satisfied, the generalized VCG mechanism is efficient in the current context as well. Recall also that the efficiency properties of the generalized VCG mechanism did not depend on the distribution of signals but only on the valuation functions  $v_i$ . The optimal, revenue maximizing, mechanism when values are interdependent and buyers' information—their signals—are statistically related is a modification of the generalized VCG mechanism. It shares many important features with the generalized VCG mechanism; it also allocates efficiently and truth-telling is an *ex post* equilibrium. Unlike the generalized VCG mechanism, however, the optimal mechanism depends critically on the distribution of signals. This last feature allows it to extract *all* the surplus from buyers so that their expected payoffs are exactly zero. From the perspective of the seller, this is clearly the best possible outcome as he is able to, in effect, act as a perfectly price-discriminating monopolist.

Let  $\Pi$  denote the joint probability distribution of buyers' signals:  $\Pi(\mathbf{x})$  is the probability that  $\mathbf{X} = \mathbf{x}$ . (This is just the discrete analog of the joint density function,  $f$ , in the case of continuously distributed signals.)

Let  $\Pi_i$  be a matrix with  $t_i$  rows and  $\times_{j \neq i} t_j$  columns whose elements are the conditional probabilities  $\pi(\mathbf{x}_{-i} | x_i)$ . Each row of  $\Pi_i$  corresponds to a signal  $x_i$  of buyer  $i$ , whereas each column corresponds to a vector of signals  $\mathbf{x}_{-i}$  of the other buyers. The entry  $\pi(\mathbf{x}_{-i} | x_i)$  then represents the *beliefs* of buyer  $i$  regarding the signals of the other buyers conditional on his own information. Hence, we will refer to  $\Pi_i$  as the matrix of beliefs of buyer  $i$ . If the signals are independent, buyer  $i$ 's own signal provides no information about the signals of the other buyers. As a result, with independent signals the rows of  $\Pi_i$  are identical and hence  $\Pi_i$  is of rank one. When signals are correlated,  $i$ 's signal provides information about the signals of the others and the rows of  $\Pi_i$  are typically different.

The main result of this section is as follows:

**Proposition 10.2.** *Suppose that signals are discrete and the valuations  $\mathbf{v}$  satisfy the single crossing condition. If for every  $i$ , the matrix of beliefs  $\Pi_i$  is of full rank, then there exists a mechanism in which truth-telling is an efficient ex post equilibrium in which the expected payoff of every buyer is exactly zero.*

*Proof.* First, consider the generalized VCG mechanism  $(\mathbf{Q}^*, \mathbf{M}^*)$  defined in (10.2) and (10.3). Define

$$U_i^*(x_i) = \sum_{\mathbf{x}_{-i}} \pi(\mathbf{x}_{-i} | x_i) [Q_i^*(\mathbf{x}) v_i(\mathbf{x}) - M_i^*(\mathbf{x})]$$

to be the expected payoff of buyer  $i$  with signal  $x_i$  in the truth-telling equilibrium of the generalized VCG mechanism. Let  $\mathbf{u}_i^*$  denote the  $t_i$  sized column vector  $(U_i^*(x_i))_{x_i \in \mathcal{X}_i}$ .

Since the matrix  $\Pi_i$  is of full row rank  $t_i$ , there exists a column vector  $\mathbf{c}_i = (c_i(\mathbf{x}_{-i}))_{\mathbf{x}_{-i} \in \mathcal{X}_{-i}}$  of size  $\times_{j \neq i} t_j$  such that

$$\Pi_i \mathbf{c}_i = \mathbf{u}_i^*$$

Equivalently, for all  $x_i$ ,

$$\sum_{\mathbf{x}_{-i}} \pi(\mathbf{x}_{-i} | x_i) c_i(\mathbf{x}_{-i}) = U_i^*(x_i)$$

Consider the Crémer-McLean (CM) mechanism  $(\mathbf{Q}^*, \mathbf{M}^C)$  defined by

$$M_i^C(\mathbf{x}) = M_i^*(\mathbf{x}) + c_i(\mathbf{x}_{-i})$$

Now observe that truth-telling is also an *ex post* equilibrium of the CM mechanism. This is because the allocation rule  $\mathbf{Q}^*$  is the same as in the generalized VCG mechanism and the payment rule  $M_i^C$  for buyer  $i$  differs from  $M_i^*$



by an amount that does not depend on his own report. In this equilibrium, the expected payoff of buyer  $i$  with signal  $x_i$  is

$$U_i^C(x_i) = \sum_{\mathbf{x}_{-i}} \pi(\mathbf{x}_{-i} | x_i) \left[ Q_i^*(\mathbf{x}) v_i(\mathbf{x}) - M_i^C(\mathbf{x}) \right] = 0$$

by construction. ■

The previous result is quite remarkable in that it shows that with the slightest degree of correlation among the signals, the seller can prevent the buyers from sharing any of the surplus resulting from the sale. Some remarks are in order.

First, if there are private values but these are correlated, then the payment in an optimal mechanism is just the payment in a second-price auction plus the terms  $c_i(\mathbf{x}_{-i})$ . In that case, truth-telling is a dominant strategy in the optimal auction as well.

Second, the optimal mechanism  $(\mathbf{Q}^*, \mathbf{M}^C)$  has two separate components: the generalized VCG mechanism and the additional payments determined by the function  $c_i$ . The latter constitute a “lottery”  $c_i(\mathbf{x}_{-i})$  that buyer  $i$  faces and the outcomes of this lottery—the amounts he is asked to pay—are determined by the reports of the other buyers. How buyer  $i$  evaluates this lottery depends on his own signal since, given the statistical dependence among signals, for different realizations of  $X_i$  the expected payment implicit in the lottery is different. The lottery is, as it were, an entry fee that allows buyer  $i$  to participate in the workings of the generalized VCG mechanism and in expectation the buyer is just indifferent between entering and not.

Third, for some realizations of all the signals, a buyer’s payoffs may be negative. He may end up paying something to the seller even if he does not get the object—and so, unlike the common auction formats, the mechanism is not *ex post* individually rational. Of course, by construction, the mechanism is *interim* individually rational: For every realization of his own signal, a buyer’s expected payoff is exactly zero.

Fourth, while the conditions of the result require only that the matrix of beliefs be of full rank, when buyers’ signals are “almost independent,” the lottery  $c_i(\mathbf{x}_{-i})$  may involve, with small probabilities, very large payments. In this case, it becomes increasingly untenable to maintain the assumption that buyers remain risk-neutral over the range of payoffs they may encounter while participating in a mechanism.

Finally, we draw attention to the fact that while the results of the preceding section regarding efficient mechanisms did not depend on the distribution of signals—all the complications there arose solely from the interdependence of values—the results of this section depend only on the correlation between the signals of the buyers—they are unaffected by the interdependence of the values.

## PROBLEMS

All of the problems below concern the following environment. Suppose that there are two potential buyers for one indivisible object. Each buyer's private value  $X_i$  for the object is drawn at random from the set  $\mathcal{X} = \{10, 20\}$ . Buyers' values are jointly distributed as follows:

$$\Pr[10, 10] = \Pr[20, 20] = 0.2$$

$$\Pr[10, 20] = \Pr[20, 10] = 0.3,$$

where  $\Pr[x_1, x_2]$  denotes  $\Pr[X_1 = x_1, X_2 = x_2]$ .

- 10.1.** (Generalized VCG mechanism) What is the generalized VCG mechanism  $(Q^*, M^*)$  for the environment specified above? Determine the expected revenue from this mechanism.
- 10.2.** Consider the following mechanism. If both buyers report values  $z_1 = z_2 = 20$ , then pick a buyer randomly with probability  $\frac{1}{2}$ —say this is buyer  $i$ —and give him the object for a price of  $M_i = 20$ . The other buyer  $j$  pays nothing. If both report values  $z_1 = z_2 = 10$ , again pick a buyer randomly with probability  $\frac{1}{2}$ —say,  $i$ —and give him the object for a price of  $M_i = 19$ . The other buyer—say,  $j$ —pays  $M_j = 9$  without, of course, getting the object. If one buyer reports  $z_i = 20$  and the other  $x_j = 10$ , then give the object to buyer  $i$  for a price  $M_i = 20$ . The other buyer  $j$  receives a transfer of 6—that is,  $M_j = -6$ .
  - a.** Show that the mechanism described above is incentive compatible and individually rational.
  - b.** What is the expected revenue in the truthful equilibrium of this mechanism?
  - c.** Does the mechanism have other (nontruthful) equilibria?
- 10.3.** (Cr mer-McLean mechanism) What is the Cr mer-McLean mechanism  $(Q^*, M^C)$  for the environment just specified?
- 10.4.** (Non-negativity payoff constraint) For the environment just specified, does there exist a mechanism that (a) is incentive compatible and individually rational; (b) gives each buyer a nonnegative payoff for every realization of the values; and (c) extracts all the surplus from the buyers?

## CHAPTER NOTES

The possibility that the seller may be able to extract all the surplus when signals are correlated was raised by Myerson (1981) in the context of an example. The developments of this chapter are based, in large part, on the subsequent work of Cr mer and McLean (1985, 1988). The generalized VCG mechanism and the single crossing condition were introduced there. The second paper also provides a slightly weaker condition on the matrix of beliefs  $\Pi_i$  that ensures full surplus

extraction. More recent work on the generalized VCG mechanism is contained in Ausubel (1999).

In the context of a pure common value auction, McAfee, McMillan, and Reny (1989) relax the assumption that buyers' signals are discrete. With continuously distributed signals, almost all the surplus can be extracted: For any  $\varepsilon > 0$ , there exists a mechanism such that no buyer's surplus exceeds  $\varepsilon$ . They also show that this is the best possible result in general—in an example, no mechanism can leave the buyers with a surplus of exactly zero. McAfee and Reny (1992) extend this result to other mechanism design settings. In particular, they show that with correlated signals and interdependent values, efficient trade between a privately informed seller and a privately informed buyer is possible. Thus, the impossibility of efficient bilateral trade, derived in Chapter 5, does not hold once signals are correlated.