Answers to HW 9:

1. (a)&(b) With the anticipated price, the seller profit becomes $\frac{1}{2}v+k_s-2k_s^2$, and the buyer profit is $v+k_b+k_s-\frac{1}{2}v-k_s-k_b^2=\frac{1}{2}v+k_b-k_b^2$. When they maximize the profit, both the buyer and the seller will make efficient investments, and we get efficient outcome. There would not be a hold-up problem. (c) However this is not possible when there is non-verifiability. The pricing strategy of the contract requires the knowledge of v, k_b, k_s. Without this knowledge, the contract price cannot be enforced when there is a breach. (d) In the non-verifiable case, p is chosen to maximize the total surplus produced. The total social surplus is given by

$$S = Ev + k_b - k_b^2 + k_s - 2k_s^2.$$

Given p, the equilibrium investment levels are solved in the note:

$$k_b = \frac{5}{6} - \frac{2}{3}p, k_s = \frac{p}{3} - \frac{1}{6}.$$

Substitute into the social surplus, we get

$$S = \frac{1}{2} + (\frac{5}{6} - \frac{2}{3}p) - (\frac{5}{6} - \frac{2}{3}p)^2 + (\frac{p}{3} - \frac{1}{6}) - 2(\frac{p}{3} - \frac{1}{6})^2 = -\frac{2}{3}p^2 + p + \frac{5}{12}.$$

The social surplus is maximized at the solution of the first-order condition

$$-\frac{4}{3}p + 1 = 0$$

or $p = \frac{3}{4}$.

2. (a) By carrying out the default option, the buyer payoff is $q^0 v_h - P^0$. If the buyer instead proposes the trade (q, P), the offer should make the seller indifferent between this trade and the default trade, so that we must have $P = P^0 - q^0 c_h + q c_h$. Hence the buyer payoff from the offer (q, P) is $q v_h - P = q v_h - (P^0 - q^0 c_h + q c_h)$
$= q(v_h - c_h) - P^0 + q^0 c_h$. Since $v_h - c_h < 0$, this payoff is the highest when $q = 0$. In equilibrium, the offer must be $q^* = 0, P^* = q^0 c_h - P^0$. The equilibrium payoff is $q^0 c_h - P^0 > q^0 v_h - P^0$. Hence the buyer does better by offering the equilibrium trade 0 rather than the inefficient trade q^0.

(b) By carrying out the default option, the buyer payoff is $q^0 v_h - P^0$. If the buyer instead proposes the trade (q, P), the offer should make the seller indifferent between this trade and the default trade, so that we must have $P = P^0 - q^0 c_l + q c_l$. Hence the buyer payoff from the offer (q, P) is $q v_h - P = q v_h - (P^0 - q^0 c_l + q c_l)$
$= q(v_h - c_l) - P^0 + q^0 c_l$. Since $v_h - c_l > 0$, this payoff is the highest when $q = 0$. In equilibrium, the offer must be $q^* = 1, P^* = P^0 - q^0 c_l + c_l$. The equilibrium payoff is $v_h - c_l - P^0 + q^0 c_l$. To show this is higher than the payoff from the default trade, we have $v_h - c_l - P^0 + q^0 c_l - (q^0 v_h - P^0) = (1 - q^0)(v_h - c_l) > 0$.

Hence the buyer does better by offering the equilibrium trade 1 rather than the inefficient trade q^0.

(c) By carrying out the default option, the buyer payoff is $q^0 v_l - P^0$. If the buyer instead proposes the trade (q, P), the offer should make the seller indifferent between this trade and the default trade, so that we must have $P = P^0 - q^0 c_h + q c_h$. Hence the buyer payoff from the offer (q, P) is $qv_l - P = qv_l - (P^0 - q^0 c_h + q c_h)$

$= q(v_l - c_h) - P^0 + q^0 c_h$. Since $v_l - c_h < 0$, this payoff is the highest when $q = 0$. In equilibrium, the offer must be $q^* = 0, P^* = q^0 c_h - P^0$. The equilibrium payoff is $q^0 c_h - P^0 > q^0 v_l - P^0$. Hence the buyer does better by offering the equilibrium trade 0 rather than the inefficient trade q^0.

(d) By carrying out the default option, the buyer payoff is $q^0 v_l - P^0$. If the buyer instead proposes the trade (q, P), the offer should make the seller indifferent between this trade and the default trade, so that we must have $P = P^0 - q^0 c_l + q c_l$. Hence the buyer payoff from the offer (q, P) is $qv_l - P = qv_l - (P^0 - q^0 c_l + q c_l)$

$= q(v_l - c_l) - P^0 + q^0 c_l$. Since $v_l - c_l < 0$, this payoff is the highest when $q = 0$. In equilibrium, the offer must be $q^* = 0, P^* = q^0 c_l - P^0$. The equilibrium payoff is $q^0 c_l - P^0 > q^0 v_l - P^0$. Hence the buyer does better by offering the equilibrium trade 0 rather than the inefficient trade q^0.

3. The total surplus for the investment levels i, j is

$$2.5 i^{0.5} j^{0.5} - i - 2j^2$$

The optimal i^*, j^* satisfy the first order conditions for the maximization of the above surplus. The first order conditions are

$$1.25 i^{-0.5} j^{0.5} - 1 = 0$$

$$1.25 i^{0.5} j^{-0.5} - 4j = 0$$

$$\frac{j}{i} = \frac{16}{25}, \frac{i}{j^3} = \frac{16^2}{25}$$

The product of the two equations gives us

$$\frac{1}{j^2} = \frac{4^6}{25^2}$$

or

$$j = \frac{25}{64} = 0.39$$

and

$$i = \frac{25}{16} j = \frac{25}{16}\frac{25}{64} = 0.61$$

Thus we have

$$i^* = 0.61, j^* = 0.39$$

(b) The equation for q^0 is

$$q^0(c_h - c_l)a'(i^*) = \phi'(i^*)$$

or

$$4.5q^0 * 0.5(0.61)^{-0.5} = 1$$

and we have

$$q^0 = \frac{(0.61)^{0.5}}{4.5 * 0.5} = 0.347$$

This is the default level of trade in the optimal contract.

(c) In the optimal contract, the payoff for the seller with investment level i is

$$\begin{aligned} & P^0 - q^0(i^{0.5}c_l + (1 - i^{0.5})c_h) - i \\ = \quad & P^0 - 0.347(i^{0.5} * 5.5 + (1 - i^{0.5}) * 10) - i \end{aligned}$$

When we take the first order condition to determine the optimal investment level for the seller, we get

$$0.347 * (5.5 * 0.5i^{-0.5} - 5i^{-0.5}) + 1 = 0$$

or

$$0.347 * 2.25 = i^{0.5}$$

and we have

$$i = (0.347 * 2.25)^2 = 0.61$$

which is the same as the efficient investment level i^*. Hence the seller has the incentive to invest the efficient amount.

(d) The (ex-post) payoff to the seller from the default option is

$$\begin{aligned} & P^0 - q^0(i^{0.5}c_l + (1 - i^{0.5})c_h) - i \\ = \quad & P^0 - 0.347(0.61^{0.5} * 5.5 + (1 - 0.61^{0.5}) * 10) - 0.61 \\ = \quad & P^0 - 2.86 \end{aligned}$$

hence P is chosen so that

$$P - (0.61^{0.5} * 5.5 + (1 - 0.61^{0.5}) * 10) - 0.61 = P^0 - 2.86$$

or

$$P - 7.1 = P^0 - 2.86$$

$$P = P^0 + 4.24$$

hence the equilibrium price of trade is higher than the default price of trade by the amount 4.24.

(e) The total surplus created from the investments is

$$0.61^{0.5}0.39^{0.5} * 2.5 = 1.22$$

and the net surplus is

$$1.22 - 0.61 - 2 * 0.39^2 = 0.31$$

Hence we have

$$P^0 - 2.86 = 0.155$$

or

$$P^0 = 3.06$$