USC, Spring 2012

Economics 513, Final, Answer key

Problem 1 Let D be the indicator that is 1 if the firm replaced its CEO at the start of period 3 (and 0 otherwise). Further Y_{dt} is the profits in year t with $d = 1$ if the firm has a new CEO in year t and $d = 0$ if not. The observed profits in year t is Y_t with

$$Y_3 = D \cdot Y_{13} + (1 - D) \cdot Y_{03} \qquad Y_2 = Y_{02} \qquad Y_1 = Y_{01}$$

See lecture 11. We have a sample of size N, i.e. we observe $Y_{1i}, Y_{2i}, Y_{3i}, D_i, i = 1, \dots, N$. The number of observations with $D = 0$ is N_0 and that with $D = 1$ is N_1.

(i)(10) The estimator estimates

$$\beta = E(Y_3|D = 1) - E(Y_3|D = 0)$$

The regression model is

$$Y_3 = \alpha + \beta D + \varepsilon$$

and the OLS estimator of β is the CEO replacement effect. Homoskedasticity means here that $Var(\varepsilon|D = 1) = Var(\varepsilon|D = 0) = \sigma^2$. The variance of the OLS estimator under homoskedasticity is

$$Var(\hat{\beta}) = \frac{\sigma_\varepsilon^2}{\sum_{i=1}^N (D_i - \overline{D})^2} = \frac{\sigma^2}{N\overline{D}(1 - \overline{D})} = \sigma^2 \frac{N_0 + N_1}{N_0 N_1}$$

with the final expression in lecture 11. If $\sigma_0^2 = Var(\varepsilon|D = 0)$ and $\sigma_1^2 = Var(\varepsilon|D = 1)$, then

$$Var(\hat{\beta}) = \frac{\sum_{i=1}^N D_i (D_i - \overline{D})^2}{\left(\sum_{i=1}^N (D_i - \overline{D})^2\right)^2} \sigma_1^2 + \frac{\sum_{i=1}^N (1 - D_i)(D_i - \overline{D})^2}{\left(\sum_{i=1}^N (D_i - \overline{D})^2\right)^2} \sigma_0^2 =$$

$$\frac{\sigma_1^2}{N\overline{D}} + \frac{\sigma_0^2}{N(1 - \overline{D})} = \frac{\sigma_1^2}{N_1} + \frac{\sigma_0^2}{N_0}$$

with the final expression in lecture 11.

(ii)(10) The assumption is that $Y_{03}, Y_{13} \perp D$ (independence) or $E(Y_{13}|D = 1) = E(Y_{13}|D = 0) = E(Y_{13})$ and $E(Y_{03}|D = 1) = E(Y_{03}|D = 0) = E(Y_{03})$ (mean independence). In the second year $Y_2 = Y_{02}$ so that we can compute from the second year profits $E(Y_{02}|D = 1) = E(Y_2|D = 1)$ and $E(Y_{02}|D = 0) = E(Y_2|D = 0)$. If Y_{02} is not mean independent of D then it is unlikely that Y_{03} is mean independent of D.

(iii)(10) The assumption we can make is the one that supports the dif-in-dif estimator

$$E(Y_{03} - Y_{02}|D = 1) = E(Y_{03} - Y_{02}|D = 0)$$

or

$$Y_{03} - Y_{02} \perp D$$

The dif-in-dif estimator estimates the ATET $E(Y_{13} - Y_{03}|D = 1)$.

(iv)(10) Define the indicator T that is 1 in period 3 and 0 in period 2. The regression model is

$$Y = \alpha + \gamma_1 D + \gamma_2 T + \beta D \cdot T + \varepsilon$$

and the OLS estimator of β is the estimate of the CEO replacement effect. Note that Y now is the dependent variable in either period 2 or 3. If we assume a homoskedastic error

$$\mathrm{Var}(\varepsilon | D = d, T = t) = \sigma^2$$

for $d = 0, 1$ and $t = 0, 1$, then

$$\mathrm{Var}(\hat{\beta}) = \left(\frac{1}{N_{11}} + \frac{1}{N_{10}} + \frac{1}{N_{01}} + \frac{1}{N_{00}} \right) \sigma^2$$

with e.g. N_{11} the number of firms in period 3 that replaced their CEO at the beginning of period 3. If all firms are observed in periods 2 and 3 then $N_{11} = N_{10}$ etc. If the error is heteroskedastic

$$\mathrm{Var}(\varepsilon | D = d, T = t) = \sigma^2_{dt}$$

then

$$\mathrm{Var}(\hat{\beta}) = \frac{\sigma^2_{11}}{N_{11}} + \frac{\sigma^2_{10}}{N_{10}} + \frac{\sigma^2_{01}}{N_{01}} + \frac{\sigma^2_{00}}{N_{00}}$$

(v)(10) No, because a negative change in profits $Y_{03} - Y_{02}$ is more likely to lead to CEO replacement $D = 1$ then a positive change in profits. Therefore $E(Y_{03} - Y_{02} | D = 1) < E(Y_{03} - Y_{02} | D = 0)$ so that the dif-in-dif estimator is biased.

(vi)(10) Because $Y_1 = Y_{01}$ and $Y_2 = Y_{02}$ we can check whether $E(Y_2 - Y_1 | D = 1) < E(Y_2 - Y_1 | D = 0)$. If this is the case then the assumption in (iii) is unlikely to hold.

(vii)(10) We can assume that

$$E(Y_{03} - Y_{02} | Y_1, Y_2, D = 1) = E(Y_{03} - Y_{02} | Y_1, Y_2, D = 0)$$

or

$$Y_{03} - Y_{02} \perp D | Y_1, Y_2$$

(viii)(10) The simplest approach would be to estimate

$$\Delta Y_3 = \alpha + \gamma_1 Y_2 + \gamma_2 Y_1 + \beta D + \varepsilon$$

The OLS estimator of β is the CEO replacement effect.

Problem 2 D is the selection indicator and Y_0, Y_1 are the non-treated and treated outcomes. The observed outcome is $Y = D \cdot Y_1 + (1 - D) \cdot Y_0$.

(i)(20) $Y_0, Y_1 \perp D$.

(ii)(20) Let T be the intervention indicator with $T \neq D$. Now we need $T \perp Y_0, Y_1$.

(iii)(20) The model is

$$Y = \alpha + \beta T + \varepsilon$$

with the assumption $E(\varepsilon | T) = 0$.

(iv)(20) We can use the IV estimator with D as instrumental variable. Because of random selection $D \perp \varepsilon$. We can regress T on D and use the predicted value of this regression as independent variable instead of T. The 2SLS estimator can be expressed as

$$\hat{\beta} = \frac{\overline{Y}_1 - \overline{Y}_0}{\overline{T}_1 - \overline{T}_0}$$

with

$$\overline{Y}_1 = \frac{1}{N_1} \sum_{i=1}^{N} D_i Y_i \quad \overline{Y}_0 = \frac{1}{N_0} \sum_{i=1}^{N} (1 - D_i) Y_i$$

with N_0, N_1 the number of observations with $D = 0$ and $D = 1$ respectively. We define $\overline{T}_0, \overline{T}_1$ in the same way.