Answers for homework 4:

1 (a) If GF uses high test, dl will use low test when it sells to GF, as it reduces cost, and gives the firm profit \$5. The diary firm dh will use low test too, and get the same profit \$5.

(b) Since dl only sells to GF with probability 1/2, and dh always sells, the conditional probability when it sells is 2/3 from dh, and 1/3 from dh.

(c) The profit from high test is always \$1 whether it is from dh or dl. The profit from low test is 2/3*3=2. Hence it is optimal to use the low test.

(d) The high test is not optimal because it is more likely that the milk is from dh.

2. When GF chooses high test, dl is willing to sell as it is optimal for dl to choose low test which gives the firm \$5 in profit equal to the profit from processing its own milk. The equality of profit also means that dl sells to GF with probability 0.5. As to dh, it always sells and chooses low test. Therefore, the updated belief about the firm when they sells to GF is $\frac{p}{0.5(1-p)+p}$ from dh, and $\frac{0.5(1-p)}{0.5(1-p)+p}$ from dl. Therefore when GF chooses high test, the profit is always \$1. When it chooses low test, it has profit only when the milk comes from dh, and the expected profit is $\frac{3p}{0.5(1-p)+p}$.

For the high test to be optimal for GF, it has to be true that

$$\frac{3p}{0.5(1-p)+p} < 1,$$

or

$$p < 0.2.$$

3. (a) under the out-equilibrium belief, GF gets payoff 3 from low test, and 3 from high test. Hence GF optimal response is low test. Knowing this, when the diary firm chooses to sell to GF, the optimal strategy is high test, yielding payoff 2 which is lower than processing its own milk and gets 10. Therefore it is a perfect Bayesian equilibrium for the diary firm to process its own milk. (b) Using the refinement idea, we know that selling to GF and adopting high test can only yield 2 which is lower than 10. Thus diary firm high test is a dominated strategy. The refinement idea says that we should assign o probability to this strategy. Hence the out-of-equilibrium belief of GF is that diary firm will choose low test. GF will then choose high test. This implies that the diary firm will sell to GF with the payoff 15 rather than processing its own milk with payoff 10. Hence the unique perfect Bayesian equilibrium is: the diary firm sells to GF, and choose low test, while GF chooses high test.

4. (a) If the seller finds a bad furnace and reports the true information, he gets \$180 from the house. However if he hides the information, the buyer does not necessarily conclude that it is a bad one, because it is possible that it may be a good one. Therefore, the house will be worth more than \$180. Hence it is better that he hides information.

(b) The probability that a low cost seller acquires information and finds a good furnace is p*q. This will be reported. When there is no report, either the low cost seller finds bad furnace, or it is a high cost seller who does not have information. The buyer cannot distinguish between the two cases. The total probability is 1-pq. The probability that it is a good furnace even though there is no report is q(1-p). The probability that it is a bad furnace when there is no report is 1-pq-q(1-p)=1-q. Therefore, conditional on no report, the probability that it is bad furnace is r=(1-q)/(1-pq) and the probability that it is a good furnace is 1-r=q(1-p)/(1-pq).

(c) Acquiring information in this case leads to $200 with probability q. With probability 1-q, the seller gets 200r+180(1-r). Therefore with information, he gets

$$Va = 200q + (1-q)(200(1-r) + 180r) = \frac{40q - 20q^2 - 200pq + 180}{1-pq}.$$

When the low cost seller does not acquire information, (the buyer does not know this) there is no information and no report. The buyer is still willing to pay

$$Vn = 200(1-r) + 180r = \frac{20}{1-pq}(q - 10pq + 9)$$

The value of information is then

$$Va - Vn = 200q + (1-q)(200(1-r) + 180r) - 200(1-r) - 180r = 20qr = \frac{20q(1-q)}{1-pq}.$$

5. (a) If the seller finds a bad furnace and reports the true information, he gets $180 from the house. If he hides the information, the buyer can still conclude that it is a bad one, because of unraveling. Therefore, the house will be worth $180 in this case as well. Hence it is not better that he hides information.

(b) If there is no report, the buyer thinks the house has a bad furnace.

(c) The probability that a low cost (or high cost) seller acquires information and finds a good furnace q. In this case, the house is worth 200 when reported. If he finds a bad furnace, it is worth $180 whether it is reported or hidden. Hence the seller gets 200q+180(1-q) with information. If the seller does not acquire information, and has nothing to report, the buyer still thinks that the furnace is bad. The seller gets $180 in this case. Therefore the value of information is the difference 200q+180(1-q)-180=20q in this case.