MATH 425a   MIDTERM EXAM 1
September 30, 2015
Prof. Alexander

Last Name: ______________________________

First Name: ______________________________

USC ID: ______________________________

Signature: ______________________________

Problem	Points	Score
1	21	
2	27	
3	31	
4	21	
Total	100	

Notes:

(1) Use the backs of the sheets if you need more room.

(2) If you can't do, say, part (a) of a problem, you can assume it and do parts (b), (c), etc.

(1)(21 points)(a) Prove Theorem 2.19: every neighborhood $N_r(p)$ is an open set.

(b) Direct from the definition: Is $\sqrt{2}$ an interior point of the set I of all irrational numbers, in the metric space $\mathbb{R}$? Why or why not? (No formal proof necessary–just a brief statement.)

(2)(27 points)(a) Define *open cover* (of a set E.)

(b) In $\mathbb{R}$, give an example of an open cover of $\mathbb{Z}$ with no finite subcover. You don't need a formal proof, but say how you know it has no finite subcover.

(c) Suppose F is an infinite set in a metric space, and every $x \in F$ has a neighborhood N_x containing no other point of F (besides x.) Show that F is not compact.

(3)(31 points) The *boundary* of a set E, denoted ∂E, is $\partial E = \overline{E} \cap \overline{E^c}$.

(a) Show that if every neighborhood of x contains a point of E and a point of E^c, then $x \in \partial E$. (These point of E and/or E^c might be x itself.)

(b) State the converse of part (a).

(c) *Slightly harder problem.* Prove the converse of part (a).

(4)(21 points) Suppose A is an infinite set, and there is a sequence $\alpha_1, \alpha_2, \ldots$ which contains each element of A at least once.

(a) Show that there is a bijection from A to a subset of $\mathbb{N} = \{1, 2, 3, \ldots\}$. HINT: Consider where an element of A first appears in the sequence $\{\alpha_i\}$.

(b) Show that A is countable.