Note 11 with HW 11

A simple game repeated many times may yield different equilibrium behavior than the one played only once. More generally, dynamic games many exhibit many different behavior patterns in static games.

Competition issues or anti-trust issues such as tacit collusion, conscious parallelism and predatory pricing, are often examined in the context of dymaic or repeated games

The solution of dymaic or repeated games may involve backward induction.

In contract negotiations, there is a Statute of Frauds that conditions legal enforcement on the existence of a signed writing evidencing the agreement of the parties. The writing need only show that the parties have entered into a contract, and does not even require a price term.

A dynamic repeated game may be used to illustrate the need for such a legal rule.

Southwest Engineering v Martin Tractor illustrates this point.

Southwest Engineering was a general contractor that was installing runway lighting at an Air Force base. It needed to buy a stand-by generator and made inquiries at Martin Tractor. A series of negotiations eventually led to a lunch in an airport cafeteria. During the lunch, Martin's employee took out a piece of paper and listed the components of the generator as well as their prices. Southwest's employee took a copy of this piece of paper.

The deal fell through a few weeks later and Martin attempted to withdraw all verbal quotations. Southwest brought suit against Martin, alleging that they had reached an agreement during the lunch. Martin defended its position on the ground that it was not legally enforceable because the price quotation that its employee had written on the piece of paper was not sufficient to satisfy the Statute of Frauds.

The court ultimately rejected this argument.

Figure 5.1 represents such a dynamic negotiation in which each party may exit during negotiations and assert that a deal exists that is favorable to the party.

Backward induction then leads to the conclusion that the negotiation will never be commenced.

The Statute of Frauds solves the problem by making the exit strategy legally unenforceable, thus both sides have the incentive to reach a signed contract.

Although backward induction was used very often in analyzing dynamic or repeated games, and has a very intuitive appeal, it may lead to implausible outcomes in some games.

For instance in the installment sale game, the seller would like to ship the buyer goods in 100 equal installments. The buyer would promise to pay each installment after receiving it. Each shipment costs the seller \$1, and the buyer pays \$2 for each shipment which is worth \$3 to the buyer.

The seller and the buyer have no recourse against each other (due to high legal costs), if there is no performance (no shipment or payment).

This game tree is in Fig 5.2. The buyer may default after a shipment by not paying. If the seller worries about buyer default, it will not ship. If the buyer anticipates no further shipment, it is better not to pay.

By going through the backward induction, the seller would not ship in the beginning anyway even though there is benefits of trade.

The only equilibrium is no trade.

This conclusion is not plausible because each side has an incentive to continue the game, and keep the promise (there is long term benefits, but short-term opportunism).

In a dynamic or repeated game, each player may have an incentive to cooperate with the other player to achieve a mutually beneficial outcome, even though the one-period game is a prisoners' dilemma game.

For instance, firm competition may lead to lower prices benefitting the consumers but bad for the competitors. When firms play repeated games, they may jointly hold up the price (cooperate with each other in pricing strategies) to achieve a joint monopoly outcome.

In the 1940s, cigarette companies were accused of violations of antitrust laws in the pricing of cigarettes. The Supreme Court upheld the jury verdict against the tobacco companies even though there was not any explicit agreement or conspiracy to fix prices.

In later cases, the plaintiff must demonstrate more than parallel behavior in tacit collusion.

To illustrate tacit collusion, consider the following one-period game in pricing strategies.

	H	L
H	(10,10)	(0,16)
L	(16,0)	(5,5)

This is prisoners' dilemma game in which each firm has a dominant strategy to use the low price strategy (non-cooperative strategy). They both do better by adopting the high price strategy.

In the one-period game, the only Nash equilibrium is the (low, low).

The same is true when the game is repeated finitely many times.

We consider instead an infinitely repeated game (games with no ending periods). A game repeated infinitely many times is called a supergame.

The payoff in each payoff is discounted by a discount factor d.

In general, a strategy in a supergame is a function from the past plays into the next action (low or high), and the payoff is a discounted sum of the period payoffs.

In supergames, we often require equilibrium to be subgame perfect equilibrium.

Consider the following grim strategy: both firms adopt the high price strategy until some plays a low price strategy (out-of-equilibrium) which triggers (low,low) strategy in all future periods.

This is a subgame perfect Nash equilibrium strategy if d is sufficiently high (close to 1).

The strategy of each player is a stationary strategy. The stationary structure allows us to compute payoffs in a simple way.

If both firms adopt high price for all periods (equilibrium strategy), in each period, each firm gets the payoff 10. The total discounted sum of payoffs of each firm is then 10/(1-d).

If a firm deviates (adopt low price strategy), it will get payoff 16 in the deviation period, but 5 in all future periods. Hence the total payoff from deviation is given by 16+5d/(1-d).

Deviation is not profitable if the payoff from deviation is smaller or 10/(1-d)>16+5d/(1-d).

The inequality gives us d>6/11. Thus if the discount factor is greater than 6/11, then the grim strategy is a subgame perfect Nash equilibrium for the supergame. In this equilibrium, the players enjoys the payoffs from cooperation in each period, and no firm wants to deviate from the equilibrium strategy.

The grim strategy is not the only subgame perfect Nash equilibrium of the supergame. For instance (low,low) every period is such an equilibrium as well.

There are many other interesting strategies in equilibrium in dynamic interactions.

Consider the following tit-for-tat strategy: a firm begins by charging a high price, and charge a low price only if the rival firm charges a low price in the preceding period. In such strategies, "punishment" for deviation lasts only one period.

When d is sufficiently close to 1, we can show that it is a Nash equilibrium (but not necessarily subgame perfect) of the supergame for both firms to adopt the tit-for-tat strategy.

If firm one deviates once by charging a low price in the first period, then return to high price next period, then the strategy plays of the two firms are: (low,high), (high,low), (low,high), (high,low), . . . , etc.

Firm one will get payoff 16 in odd periods, and 0 in even periods. The discounted sum of payoffs from deviation is 16/(1-dd). If firm one follows equilibrium strategies, it will get payoff 10 in each period, and the discounted sum of payoffs is 10/(1-d).

To prevent deviation, we need to have 10/(1-d)>16/(1-dd), or d>6/10.

Thus if d>6/10, then it is a Nash equilibrium. In equilibrium, each firm charges a high price.

Of course we should consider other deviations as well. What happens if firm one deviates for two periods and then follow the rules of tit-for-tat. The plays are: (L,H), (L,L), (L,L) forever with the payoff. It is easily seen that such deviation is not profitable. When firm one has three deviations right from the beginning, it is similar.

It is not subgame perfect because in the off-equilibrium path (penalty phase), the cycling strategies are not in equilibrium. Firm 2 can change to high price strategy and get a better payoff. This makes the punishment strategy non-credible. Firm 2 In this case will not have the incentive to carry out the punishment strategy.

To have a subgame perfect Nash equilibrium, the punishment strategy needs to be an equilibrium strategy as well.

To get a subgame perfect Nash equilibrium in tit-for-tat like strategies, we need to modify the strategies. A firm now starts with a high price strategy, and maintains the high price strategy if there was no deviation in the preceding period or both deviated in the preceding period. Otherwise, charge a low price. Off the equilibrium path, the plays become (low,low), (high,high),(high,high),... etc.

In fact, it is possible to use the grim punishment strategy to sustain any pattern of low and high prices. Whenever someone deviates from the pattern, then (low,low) is played forever. You can see that there are many different subgame perfect equilibria in repeated games.

Grim punishment strategy only makes sense when the deviation can be detected right away. If there is noise in the signal (for example, the actions of the other players are only observed with a noise), then it is not optimal to carry out the grim punishment strategy.

With noisy observation, the punishment strategy should be more moderate.

The Folk Theorem says that any payoff above (5,5) can be sustained as subgame perfect Nash equilibrium payoffs in the supergame as long as the discount rate is close to 1. This is true whether the actions are perfectly observable.

We say that repetition creates the possibility of cooperative (or collusive) behavior.

Tacit collusion is easier when firms can change price quickly (mail order firms cannot), or can observe the prices others charge. It is also easier if there are a small number of firms.

These models suggest that antitrust enforcers pay attention to how rapidly prices change, how information about prices is conveyed, and how many firms there are in the industry.

The court found that detailed disclosure of price information (by trade associations) was inconsistent with competition.

Uniform delivered pricing, a practice in which all buyers pay a single price independent of transportation costs, can make it easier for firms to observe rivals' prices.

Most-favored nation clauses reduce the firm's incentive to cut prices because it bears the additional cost of the rebates to previous customers.

The use of airline computer reservation systems facilitates collusion, but it may offer many benefits to consumers.

Predatory pricing is the practice of a large incumbent firm to deliberately cut prices below cost for a time until the small firms are driven out of business, at which time it will return to charging the monopoly price.

One of the best-known predation cases involves the American Tobacco Company. The Supreme Court found that AT would enter a market of a much smaller firm, lower its prices dramatically to persuade the small firm to merge with it, and then shut that firm down.

A simple one period game of entry deterrence is in Fig 5.4. In this one-shot game, predation is costly, and the unique equilibrium is accommodation.

If the game is repeated indefinitely, and the entrant is the same for each period, then the Folk Theorem says that it is possible to have predatory pricing equilibrium.

Another approach is to introduce an aggressive incumbent in Fig. 5.5. This incumbent has no interest in accommodation (negative payoff from accommodation). Most incumbents are rational (in Fig 5.4) rather than aggressive (in Fig 5.5), but the entrant cannot distinguish the two.

When the entrant believes that the probability of the rational incumbent is less than 5/9, it is better to stay out.

Depending on the initial belief of the entrant, the rational incumbent may behave in such a way that she establishes a reputation for "toughness" (by mimicking the actions of the non-rational one) and changes the belief of the entrant so that there is no entry.

If the incumbent accommodates in previous period, then it must be rational. However, if it predates, the firm may still be rational but imitate the aggressive firm. (Fig 5.6)

A rational incumbent may decide to predates in the initial period (to establish reputation) so that entrant will not enter in later periods.

Homework 11

Due April 19

1. Consider the following normal form game of the two firms engaging in price competition. Each firm can choose either high or low price strategy. The payoffs are given by

	H	L
H	(15,15)	(0,30)
	(30,0)	(5,5)

Consider the supergame with a discount factor δ.

(a) What is the condition for the Grim strategy to be a Nash equilibrium?

(b) Determine the range of δ to have a Nash equilibrium with Grim strategy for each firm.

(c) Why is this equilibrium a subgame perfect Nash equilibrium?

2. An investor has a chance to undertake a new project. The expected value of the project is \$115 (It yields \$84 with probability 0.2, and \$122.75 with probability 0.8). The investor needs to borrow \$100 for capital investment.

Two creditors lead money (\$50 each) to the investor. The competitive interest rate is 2%. Assume that creditors worry about default. Each creditor can decide on its own whether to monitor the project. The monitoring costs are \$4, \$6 for creditor 1,2 respectively. If only one creditor monitors, and the project fails, that creditor can take its money back from the investor (plus interest payments), and the other will bear the cost of failure. If both monitor, and the project fails, the assets are distributed pro rata. If neither monitors, pro rata shares also apply.

(a) What is the interest payment when the project succeeds in order to meet the competitive interest rate of 2%?

(b) Compute the payoffs in the normal form game table.

(c) Is no monitoring still a dominant strategy equilibrium?

3. Now assume that the risk of project failure is prob. 0.5 rather than prob. 0.2 in problem 2. We modify the value of project success, failure as \$146, \$84 respectively so the project still has expected value \$115. The costs of monitoring will be added to the interest payment so that a firm nets 2% competitive interest rate in equilibrium.

(a) Compute the table of the normal form game.

(b) Check if It is a dominant strategy for both firms to monitor in this case.

(c) Check if the equilibrium outcome is efficient.

4. In problem 3, assume that creditor 1 now has a right to be paid before creditor 2. This right is set up with a collateral. The cost of setting up the secured loan is assumed to be zero.

(a) Compute the table of the normal form game with secured loan.

(b) Check if it is dominant strategy for both firms not to monitor.

5. In problem 1, assume that the two firms decide to follow the following patterns of pricing behavior: (H,L), (L,H) in the first two periods then followed by (H,H) forever. Any time there is a deviation, they will follow a grim punishment strategy after the deviation. Show that this is a subgame perfect Nash equilibrium for δ which is sufficiently close to 1. Determine the exact interval of δ for which the result holds.