HOMEWORK - I

----- submit to your grader Jeonghwan Yun jeonghwy@usc.edu by Tuesday Feb. 14, 8am

[1] (8pts) a) (4pts) (*Path Independence for Rational Choice*) An agent is in a choice environment with X as the universal set of alternatives. From each menu $\mathbf{A} \subset \mathbf{X}$, he picks a single element $\mathbf{c(A)} \in \mathbf{A}$. Assume his choice is rational. His friend suggest him to apply the following procedure instead;

Divide A arbitrarily into two parts $\mathbf{A = A_1 \cup A_2}$, then pick the alternative $\mathbf{c(c(A_1) \cup c(A_2))}$.

(That is, he divides the original menu into two parts, he picks from each sub-menu his choice according to his original choice function –say **a** and **b** are the chosen alternatives respectively- and then he picks among **a** and **b** again according to the original choice function.)

Would this new two-step choice procedure always result in the same choice as in the original procedure?

b) (4pts) Suppose I am buying a used car. A used car is identified by two numbers (p, m), price (p) and mileage (m); both real numbers. All cars are different in both respects. For each menu of used cars, if there is a car that is (weakly) cheaper than $20k, I buy the cheapest car; if not I buy the car with the least mileage. Is my choice rational?

[2] (15pts) (***Value of Information***) You are betting for the winner in a game to be either team **A** or team **B**. There is no draw (deuce) in this game; exactly one team wins. The prior probability that A wins is **%40**. If you bet on team x and it turns out that team y wins; your payoff is **u(x,y)** in net dollars with

u(A , A) = \$20 , **u(B , B) = \$10** , **u(A , B) = -\$10** and **u(B , A) = -\$15**

Not betting gives you **u(not bet) = \$0.** You are an expected utility maximizer.

a) (3pts) Would you bet on a team? If yes, which team?

b) (6pts) Now you have an advisor who has some information about these teams' performances and advise you to bet on a team. He gives you a signal (a guess who is going to win) that is correct **%80** of the time;

if A is going to win, with **%80** probability he guesses that A is going to win (and guesses with %20 that team B is going to win), and

if B is going to win, with **%80** probability he guesses that B is going to win (and guesses with %20 that team A is going to win).

The advisor first gives you advice (a guess on the winning team) and you will then decide to bet or not, and if yes which team to bet on. If you are an expected utility maximizer, what is the maximum fee **f**, you would be willing to pay this advisor?

c) (6pts) Instead of %80, suppose the advisor is correct with probability **p ≥ %50.** What should **p** at least be so that the advisor's guess/advice is of any value to you? Suppose the advisor is an oracle, foreseeing the winner for sure. What is his advice worth to you in dollars?

[3] (4pts) Assume rationality is common knowledge in the two player game on the right. What is game theory's prediction for this game?

	X	Y	Z	T
A	4,7	6,5	9,4	7,4
B	6,9	3,8	8,7	5,7
C	3,5	2,6	1,5	9,2
D	1,12	2,4	5,12	2,3

[4] (8pts) (***All pay auction***) Assume **A** and **B** are bidding to win a **$100 prize**, by (simultaneously and independently) submitting a bid $**a ≥ 0** and $**b ≥ 0** respectively to an auctioneer. The higher bidder wins the **$100** but both pay their bids (the loser pays too!). A player's payoff is then the prize (if he wins) minus his bid.

Find, if any, all pure strategy NE of this game.

Assuming agents are risk neutral expected utility maximizers (they are maximizing the expected net win), find one mixed strategy symmetric equilibrium, where each player's strategy is to bid **x** with probability density **f(x)** (and the cdf is **F(x)**).

[5] (16pts) ***(Social Unrest)*** Suppose there is a continuum **[0,1]** of people in a country, each person is indexed with $\mathbf{i \in [0,1]}$. Each person can independently choose to either stay home (**H**) or protest (**P**) the government. Staying home gives 0 utility to each person. The utility of protesting is : $\mathbf{u(x, i) = 4x - 2 + \alpha i}$ where **x** is the proportion of the population that is protesting, and α is a given parameter. Hence **u** implicitly depends on what others are doing (home or at the protest). Is this a game of strategic substitutes or complements? For each of α**=1** and α**=3**,

a) (8pts) Find the set of pure NE,

b) (8pts) Find the set of rationalizable outcomes.

[6] (20pts) (***Hotelling Competition***) Consumers are uniformly distributed along a boardwalk that is 1 mile long. They all like ice cream the same and dislike walking the same. Prices are regulated and equal for every vendor. The cost of producing ice creams is zero. If more than one vendor is at the same location, they split the business evenly (similarly, if two vendors are at the same distance, the consumer goes to each of them with the same probability). Assume that at the regulated prices the maximum distance that a consumer is willing to walk is 1 mile.

a) (4pts) Consider a game in which two ice-cream vendors pick their locations simultaneously. Write down the utility function of each vendor. Find the pure strategy NE of the game.

b) (6pts) Find the pure strategy NE of the game when three vendors choose locations simultaneously and the maximum distance that a consumer is willing to walk is **1/2** mile.

c) (6pts) There exists a maximum distance **x** that consumers are willing to walk such that a pure strategy Nash Equilibrium with 3 vendors exists. Find it. What happens when the maximum distance is greater than **x**?

d) (4pts) Interpret the results very briefly.

[7] (21pts) Give an example of a two-player game with finite actions (2 or 3 actions for each player should be enough) for each of the following scenarios or show that the scenario is impossible, it cannot happen;

a) (3pts) For a player, there is a pure strategy that is not strictly dominated by any pure strategy but is strictly dominated by a mixed (probabilistic/random/stochastic) strategy.

b) (3pts) For a player, there is a pure strategy that is not strictly dominated by any pure strategy against all pure beliefs about opponents' play (we assume opponents play pure strategies; $\mathbf{s_{-i} \in S_{-i}}$) **,** but is strictly dominated by a pure strategy when we allow probabilistic beliefs (we allow $\boldsymbol{\sigma_{-i} \in \Delta(S_{-i})}$).

c) (3pts) Neither of the two strategies $\mathbf{s_1}$ and $\mathbf{s'_1}$ for player 1 are strictly dominated; yet the mixed strategy $\boldsymbol{\sigma_1 = \frac{1}{2}\, s_1 + \frac{1}{2}\, s'_1}$, is strictly dominated.

d) (3pts) If $\mathbf{s_1 \in S_1}$ is a rationalizable strategy for player 1 and $\mathbf{s_2}$ is a best response to $\mathbf{s_1}$, is $\mathbf{s_2}$ always a rationalizable strategy for player 2?

e) (3pts) If $\mathbf{s_1} \in \mathbf{S_1}$ is a rationalizable strategy for player 1 and it is a best response against $\mathbf{s_2}$, is $\mathbf{s_2}$ always a rationalizable strategy for player 2?

f) (3pts) For a player, there is a mixed strategy (not pure; there are at least two strategies in the support that is) that is strictly dominant.

g) (3pts) For one player, there is a strategy which is not a best response to any pure strategy of the opponent, but is a best response to a mixed strategy of the opponent.

[8] (8pts) Give an example for the following scenarios, or show that it is impossible (a two player 2x2 or 2x3 game should be sufficient to find an example if it is possible).

a) (4pts) *(Is NE payoffs monotonic in game payoffs?)* Assume game G has a unique pure NE. We increase player 1's payoffs strictly for each strategy profile (for each outcome box in the game) to arrive at another game G', again with a unique pure NE (possibly different than the NE of game G); but such that player 1 is worse off in the new NE (of the new game) compared to the old game.

b) (4pts) *(Is NE payoffs monotonic in available strategies?)* Assume game G has a unique pure NE. We make some of player 1's strategies unavailable, to arrive at another game G', again with a unique pure NE (possibly different than the NE of game G); but such that player 1 is better off in the new NE (of the new game) compared to the old game's unique NE.

--------- GOOD LUCK-----