ECON 504 Spring 2017 — Yilmaz KOCER — 1

FINAL EXAM ANSWER KEY

(1) a)- Each student chooses e to maximize $-e^2 \Rightarrow$ FOC $-2e=0$ $e=0$ //
Payoffs are $-0^2=0$ for each student.

b)- If both chooses $x=y=0 \Rightarrow u_A = 0 = u_B$; they get the same payoffs as before.
No, they won't ; given $y=0$ $u_A = -x^2+2x$ FOC $\rightarrow -2x+2=0$ $x=1$ would be chosen instead of $x=0$. Each student would prefer to "overstudy" his classmate.

c)- If (x,y) is a pure NE x maximizes $-x^2+x(y+2)-y^2$ given y and
y " $-y^2+y(x+2)-x^2$ given x.

$\Rightarrow$ FOC_x $-2x+y+2=0$ $\underline{2x-y=2}$ FOC_y $-2y+x+2=0$ $\underline{2y-x=2}$
$\Rightarrow$ $x=y=2$ is the unique pure NE
They study more under the curve. However $u_A(2,2) = -2^2+2(2+2)-2^2 = 0 = u_B(2,2)$ again !

(2)

①

	L	R
U	2,2	−10,x
D	y,0	0,0

②

	L	R
U	8,4	0,0
D	0,0	4,8

Notice that any SPE should have a NE played in the second (last) round. (U,L) & (D,R) are the pure NE in the second round.

a)- player 1 : play U in round 1 ; and play U in round 2 if (U,L) played in round 1 and play D otherwise
player 2 : play L " ; and play L " if (U,L) played " " R "
Second round play is NE as ; if (U,L) played in round 1, (U,L) will be played in round 2.
(U,L) not " , (D,R) "
hence noone can deviate in the second round.
In the first round: if player 1 deviates ; $y+4 = 5+4 < 2+8 = 10$ player 1 gets in equilibrium.
if player 2 deviates : $x+3 = -2+8 = 6 \leq 2+4 = 6$ — player 2 gets in equilibrium.
(Notice that in eqm, (U,L),(U,L) is played players get 10 and 6 respectively in total.

b)- (U, L) can be played in eqm only by playing one of the two NE in the second period and such that if one deviates from (U,L); the other NE will be played, to discipline the agents not to deviate in the first round.

That is if no matter what happens in round 1, the same NE were played in round 2; then, to have SPE, the round 1 play (U,L) has to be NE itself in round 1, which it isn't. (both players would want to deviate in round 1, if it were the case, that is)

Suppose after (U, L) 1)- (U, L) is the NE agreed to be played & if someone deviates (D, R) NE were to be played in round 2; then

player 2 deviates $\Rightarrow$ $y + 8 = 4+8 = 12 > 2 + 4$ eqm payoff for player 2.

2)- (D,R) is the agreed upon NE to be played, and after a deviation (U,L) were to be played, then this time player 1 would want to deviate;

player 1 deviates $\Rightarrow$ $x+8 = 4+8 = 12 > 2+4 \rightarrow$ eqm payoff player 1

③ a)- Assume the other bidder is using $s(\theta_j) = k\,\theta_j^2$ $\; j \neq i$ and as player i type θ_i,
your bid b should maximize $EU_i = pr(win)\,(\theta_i - b) + (1 - pr(win))(-b)$

when you lose, you still pay your

$$EU_i = pr(win)\cdot\theta_i - b = pr(s(\theta_j) < b)\;\theta_i - b$$

$$= pr\left(k\,\theta_j^2 \le b\right)\cdot\theta_i - b = \left(\sqrt{\frac{b}{k}}\right)\cdot\theta_i - b$$ remember there is only 1 opponent.

$\Rightarrow \theta_j \le \sqrt{\frac{b}{k}}$

$$\text{FOC} \Rightarrow \frac{1}{2}\left(\frac{b}{k}\right)^{-\frac{1}{2}}\cdot\frac{1}{k}\,\theta_i - 1 = 0$$

$$\frac{\theta_i}{2k} = \left(\frac{b}{k}\right)^{\frac{1}{2}} \Rightarrow b = \frac{1}{4k}\,\theta_i^2$$

We're looking for a symmetric BNE, $\quad b = \underline{k}\,\theta_i^2 \quad k = \frac{1}{4k} \quad \boxed{k = \frac{1}{2}}$

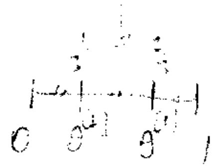
b)- For all $\theta_i \in [0,1]$ $\quad \frac{1}{2}\theta_i^2 < \frac{1}{2}\theta_i \longrightarrow$ optimal bid for the standard first price auction; hence they bid less!

The seller revenue in the all pay auction is;

$$rev_{\text{all pay}} = E(b_1 + b_2) = 2E(b_1) = 2\,E\left(\frac{1}{2}\theta_i^2\right) = \int_0^1 \theta^2\cdot \underset{\text{pdf}}{1}\cdot d\theta = \Big|_0^1 \frac{\theta^3}{3} = \boxed{\frac{1}{3}}$$

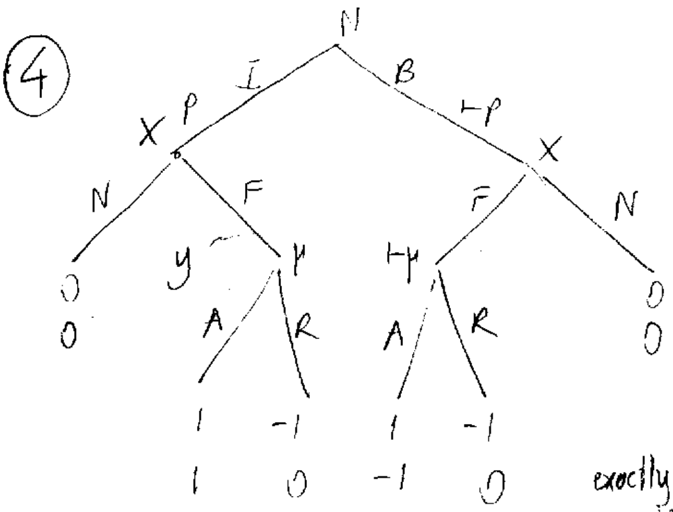
$$rev_{\text{first price}} = E\left(\frac{n-1}{n}\,\theta_i^{(1)}\right) = E\left(\frac{2-1}{2}\,\theta^{(1)}\right) = \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}$$ equal revenue!

highest θ_i among the bidders



Seller makes the same expected revenue!
Revenue equivalence theorem applies!

(4)



a)- Notice that for a BNE not to be a PBE, some information sets should not be visited with positive probability.

Otherwise, if under the strategies all info. sets are visited; a BNE qualifies as PBE (prop 15.1) exactly as a NE would pass as a SPE under the same condition.

Hence $X \Rightarrow (N\ N)$ should be the strategy for X. For this to be optimal for X, y should choose R. For y to choose R he should believe X is interesting with probability μ such that Accept $\Rightarrow \mu \cdot 1 + (1-\mu)(-1) < 0 \Leftarrow$ reject should hold; hence any $\mu < \frac{1}{2}$ would do. Notice that as this info set is not visited under the strategies, y can believe anything and he is not constrained by nature's probability p in a PBE.

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b) NO! If y chooses A; both types would choose F in a BNE and
y " R " N ".

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(5) a). $u_A(a, b) = ab - a^2$ FOC for a maximizing this; $\frac{\partial u_A}{\partial a} = 0 = b - 2a$ $a^* = \frac{b}{2}$

Hence as $b \leq 1$ $a^* \leq \frac{1}{2}$; $a > \frac{1}{2}$ can never be a best response! $\Rightarrow a \leq \frac{1}{2}$

Similarly for b; $b \leq \frac{1}{2}$ should hold. But then, in the 2nd iteration;

$a^* = \frac{b}{2} \leq \frac{\frac{1}{2}}{2} = \frac{1}{4}$ (a should be in $[0, \frac{1}{4}]$). Similarly $b \in [0, \frac{1}{4}]$.

3rd iteration; $a = \frac{b}{2} \leq \frac{\frac{1}{4}}{2} = \frac{1}{8}$, ... $a^k, b^k \in [0, 2^{-k}]$ at the k^{th} iteration;

$\Rightarrow$ after infinite steps only $a = b = 0$ is rationalizable!

b).

	A	B	C
A	1,0	0,1	0,0
B	0,1	1,0	0,0
C	0,0	0,0	2,2

matching pennies

unique NE!

Notice that A & B (surely C too) are rationalizable for each player too, as they are a Best response to each other ($A \leftrightarrow A$, $B \leftrightarrow B$ for player 1; $A \leftrightarrow B$, $B \leftrightarrow A$ for player 2.)