USC, Fall 2016, Econ 513

Answer Key Midterm

Problem 1

a. The (own) price elasticity of the demand for that car model.

b. Cars differ in quality and it is likely that price and quality are positively related. Together with a positive effect of quality on demand, it is likely there is a positive omitted variable bias in the estimated price elasticity.

c. If $\ln p$ is log price and t the test score and q the unobserved quality than we require that (i)

$$\mathrm{E(y|\ln p, t, q) = E(y|\ln p, q)}$$

i.e. in the (infeasible) long regression with q included t has no effect on y and (ii)

$$\mathrm{E(q|\ln p, t) = E(q|t)}$$

i.e. given t quality and price are mean independent.

d. OLS with log price and test score as independent variables.

e. Let quality be equal to

$$q = \gamma_1 x_1 + \cdots \gamma_K x_K + \zeta$$

with ζ unobserved quality and $x_1, \ldots, x_K$ the observed attributes. In the infeasible relation

$$y = \beta_0 + \beta_1 \ln p + q + \varepsilon$$

the error is uncorrelated with $\ln p$ and q. Therefore if the price is determined independently of the attributes or only by the observable attributes and the observable and unobservable attributes are independent, then OLS gives consistent estimates.

f. It is likely that the price depends on the unobserved attribute(s).

Problem 2

a. The F-statistic is

$$F = \frac{(n-K)R^2}{K_2(1-R^2)}$$

and is equal to 8 with 3 and 36 degrees of freedom. The critical value is between 2.84 and 2.92 so that the null hypothesis is rejected.

b. Define $\gamma_k = \beta_k, k = 1, 3, 4$ and $\gamma_2 = \beta_2 + \beta_3$. Hence $\beta_2 = \gamma_2 - \gamma_3$ and substitution gives a linear model with right hand side variables $x_2, x_3 - x_2, x_4$. The coefficient of x_2 and its standard error give us the desired confidence interval.

c. The interval is $[.85 - 2.027 * .068, .85 - 2.027 * .068] = [0.712, 0.988]$.

d. The implicit assumption is that the n vector of random errors $\varepsilon \sim N(0, \sigma^2 I)$, i.e. normal, uncorrelated and homoskedastic.

e. If we maintain that the errors are uncorrelated and homoskedastic, but allow them to be non-normal, then the F-statistic above converges in distribution to a $\chi^2(K_2)/K_2$ distribution. Therefore the test statistic is equal to $3 * 8 = 24$. The 5% critical value is 7.81 so that we reject the null.

f. Take the test statistic

$$C = \frac{\hat{\beta}_2'((X'X)_2^{-1})^{-1}\hat{\beta}_2}{\hat{\sigma}^2}$$

that in large samples has a $\chi^2(K_2)$ distribution if the null is correct. For a bootstrap estimate $\hat{\beta}_{2b}$ (where we can use the parametric or non-parametric bootstrap to generate bootstrap samples) we compute (I assume the parametric bootstrap)

$$C_b = \frac{(\hat{\beta}_{2b} - \hat{\beta}_2)'((X'X)_2^{-1})^{-1}(\hat{\beta}_{2b} - \hat{\beta}_2)}{\hat{\sigma}^2}$$

The critical value is the 95% quantile of the empirical distribution of C_b with $b = 1, \ldots, B$. We compare C above with this quantile and reject the null if C is larger.