HOMEWORK 3

Econ 501: Macroeconomic Analysis and Policy

Spring 2016

1. Suppose that we have an overlapping generations model in which agents have Leontief preferences over consumption

$$U_t = Min\ \{c_{1t}, c_{2t+1}\}$$

Production is Cobb-Douglas, with α <0.5

a) Define the consumer's problem, the firm's problem, and equilibrium for this economy.
b) Find the savings rate of the young worker as a function of R_{t+1}. Is savings increasing or decreasing in the interest rate?
c) Find a difference equation expressing k_{t+1} as a function of k_t.
d) Find the steady state capital stock.
e) If the Utility was defined as $U_t = \log c_{1t} + \beta \log c_{2t+1}$, Solve for the worker's savings rate, the golden rule level of the capital stock as well as the steady-state savings rate needed to maintain the golden rule level of the capital stock. Under what conditions on the model parameters will the savings rate exceed the golden rule level?

2. Consider a two-period endowment economy. Suppose there are two consumers, consumer 1 and consumer 2. They have their own preferences:

$$U_1(c_1, c_1') = \log(c_1) + \beta \log(c_1')$$
$$U_2(c_2, c_2') = \min\{c_2, \beta c_2'\}$$

where c_i and c_i' indicate current and future consumption for consumer i = 1; 2 and b = 0.95 is a discount factor. They can lend and borrow from each other at a real interest rate r > 0 in the unit of consumption, but there is no bank or government. Suppose two consumers' income process is given as $\{y_1, y_1'\} = \{0,55\}$ and $\{y_2, y_2'\} = \{45,0\}$ respectively.

a) Find optimal conditions for each consumer's consumption. Also find goods market clearing conditions.

b) Using above conditions, find an optimal current and future consumption $\{c_i^*, c_i'^*\}$ and lending/borrowing $\{s_1^*, s_2^*\}$ for each consumer, i=1,2. Also find a market clearing interest rate r. Who is going to be a lender today and how much does she lends?
c) Is this allocation Pareto optimal? Discuss.