HOMEWORK 0: Solutions

Econ 501: Macroeconomic Analysis and Policy

Spring 2016

1. The function is twice-differentiable, because it is a polynomial. We have $f'(x) = -2x/3 + 8$ and $f''(x) = -2/3 < 0$ for all x, so f is strictly concave.

2. We have $f'(x) = \alpha A x^{\alpha-1}$ and $f''(x) = \alpha(\alpha - 1)Ax^{\alpha-2}$. For any value of β we have $x^{\beta} \geq 0$ for all $x \geq 0$, so for f to be nondecreasing and concave we need $\alpha \geq 0$ and $\alpha(\alpha - 1) \leq 0$, or equivalently $0 \leq \alpha \leq 1$.

3. The function f is twice-differentiable for $x > 0$. We have $f'(x) = (1/4)x^{-3/4}$ and $f''(x) = -(3/16)x^{-7/4} < 0$ for all x, so f is concave for $x > 0$. It is continuous, so it is concave for all $x \geq 0$. The firm's profit, $p f(x) - wx$, is thus the sum of two concave functions, and is hence concave.

4. Because f'_x is homogeneous of degree 0, we have

   $x f''_{xx}(x, y) + y f''_{xy}(x, y) = 0$, so that $f''_{xy}(x, y) = -(x/y) f''_{xx}(x, y)$.

   a. Not homogeneous: Suppose, to the contrary, that there exists some value of k such that $(tx)^2 + (tx)^3 = t^k(x^2 + x^3)$ for all t and all x. Then, in particular, $4x^2 + 8x^3 = 2^k(x^2 + x^3)$ for all x (taking $t = 2$), and hence $6 = 2^k$ (taking $x = 1$), and $20/3 = 2^k$ (taking $x = 2$). These two conditions are inconsistent, so the function is not homogeneous of any degree.

   b. Homogeneous of degree np: $(g(tx_1, ..., tx_n))^p = (t^n g(x_1, ..., x_n)))^p = t^{np}(g(x_1, ..., x_n))^p$.

5.

   a) If a < w Homogeneous of degree 2: $2(tx)^2 + (tx)(ty) = t^2(2x^2 + xy)$.

   b) Not homogeneous: Suppose, to the contrary, that there exists some value of k such that $(tx)^2 + (tx)^3 = t^k(x^2 + x^3)$ for all t and all x. Then, in particular, $4x^2 + 8x^3 = 2^k(x^2 + x^3)$ for all x (taking t = 2), and hence $6 = 2^k$ (taking x = 1), and $20/3 = 2^k$ (taking x = 2). These two conditions are inconsistent, so the function is not homogeneous of any degree.

c. Homogeneous of degree np: $(g(tx_1, ..., tx_n))p = (t^n g(x_1, ..., x_n)))^p = t^{np}(g(x_1, ..., x_n))^p$.