Note 9 with HW 9

We now look at another kind of information problem: observable but non-verifiable information in the following famous hold-up problem. When an event is not verifiable, it cannot be contracted. It is not possible to make the contract contingent on the event.

Hold-up Problem

Hold-up problem refers to the problem of under-investment when two parties have long term contract regarding their transactions. During the time of the contract, each side will have opportunities to make investments to increase the benefit from the contract, such as reducing cost of production, raising the quality of the product, or increase the revenue from sales. The investment from one side will benefit both the buyer and the seller, but the investor (could be the buyer or the seller) incurs the total cost, while only reaps part of the benefit from investment. This gives rise to lower investment and loss of efficiency.

Fisher Bodies and General Motors: In the 1920s, Fisher Bodies produced car bodies for General Motors. It invested in specialized equipments and tools to the special needs of General Motors. A contract signed in 1919 gave Fisher Bodies a ten-year exclusive contract to protect it from being held up by General Motors. This gave Fisher Bodies the possibility of raising prices outragenously. There is a cost-plus clause to protect General Moters from being help up by Fisher Bodies. Fisher Bodies manipulated by choosing a low captial intensity and located its plants far away from those of General Motors. Low capital density lowers the fixed cost of Fisher Bodies, but increased the higher marginal costs for General Motors and the higher marginal costs can be charged to General Motors by the cost-plus clause. Similarly, the more remote location saves the fixed cost of Fisher Bodies, and the higher transportation costs can be charged to General Motors. In other words, Fisher Bodies has no incentive to invest and increase productivity when its investment costs (higher capital intensity and closer location) cannot be covered by the contract. In the end, General Motors bought it in 1926. Investment and effort is harder to measure and difficult to have a effective contract based on the amount of investment by Fisher Bodies. Investment is not just the purchase of machinery or equipment. To make it work, many elements and efforts need to go with the investment decision which is not contractible. We say that it is observable but not verifiable.

Consider a contract between the supplier of an input (the seller) and a downstream manufacturer (the buyer). The seller must decide the quality of the inputs to supply. The seller can invest in adapting its plant to produce to buyer's specifications. Such investment improves the quality of the product, increases the value, and benefits the buyer. In addition, the buyer may also need to retool its manufacturing plant to accommodate the characteristics of the seller's design. Such investments also make the seller's product more valuable. We assume that each party's investment is observable by the other, but not verifiable. Therefore a contract cannot be made contingent on these investment decisions. In addition, there is substantial uncertainty in the buyer's product value, hence buyer's willingness to pay is uncertain. For simplicity, we assume that the buyer purchases either one unit or zero unit from the seller. The buyer and the seller can observe the demand for the buyer's product as events evolve, but a court cannot. Again these demand changes are observable by the parties of the contract but not by a court. In the case of General Motors, its promotion of products increases the sales and potentially benefits Fisher Bodies as Fisher Bodies can extract that additional surplus by charging more. Promotion schemes can be considered investments which are observable but not verifiable.

The events proceed in five steps. In the first period, the parties sign the contract. In the second period, the buyer and the seller makes investment decisions. In the third period, the uncertainty is resolved. In the fourth period, the parties have a chance to renegotiate. In the fifth, the parties decide whether to carry out the terms of the (renegotiated) contract or breach it.

The value of the product of the buyer is a sum of three elements $v + k_b + k_s$, The value of v is uncertain, and is uniformly distributed over $[0, 1]$. The number v represents the uncertain part of the consumer's willingness to pay for the product. The buyer and seller's investment-induced value is represented by k_b, k_s respectively, and the cost functions are given by $2k_s^2$, k_b^2 respectively. (In order to enhance the value by the amount k, it will cost $2k^2, k^2$ respectively). These numbers say that it is less costly for the buyer to contribute to the value of the buyer's product through investment. The buyer has no other costs in making the product (or only sunk costs), the only costs of the buyer and the seller are the price of the input paid to the seller and the cost of investments of each side.

The efficient investments are found by maximizing $k_b - k_b^2, k_s - 2k_s^2$ by choosing k_b, k_s jointly. We have the solution $k_b = \frac{1}{2}, k_s = \frac{1}{4}$. These are efficient investment levels and are used as benchmarks. Lower levels of investments are called under-investments, while higher levels are called over-investments.

First assume that there is no contract between the parties. Without any contract between the parties, the price of the input will be determined by the bargaining among the buyer and the seller after the investments have been made, and the uncertainty is resolved. We assume that the buyer and the seller divide the gains from trade equally between them (in general the sharing of trade surplus depends on the bargaining powers of the buyer and the seller). The gains from trade (after the investments have been made) is $v + k_b + k_s$, therefore, both anticipate the price to be $\frac{1}{2}(v + k_b + k_s)$ when the realized uncertainty is v and the investment levels are k_b, k_s respectively. When the seller makes decision about the investment levels, it chooses k_s to maximize $\frac{1}{2}(v+k_b+k_s)-2k_s^2$. The optimal k_s is this case is $k_s = \frac{1}{8}$. Similarly, at the time of investment, the buyer chooses k_b to maximize $\frac{1}{2}(v + k_b + k_s) - k_b^2$. The optimal k_b is $k_b = \frac{1}{4}$. These separate optimal investments without contract is lower than the efficient levels. Therefore we conclude that there is under-investment by both the buyer and the seller.

Now we consider possible improvements through a contract. If the investments and v were verifiable, then we can make a contractual price $\frac{1}{2}v + k_s$ of the input (price reflecting the cost of investments). With this anticipated price, the seller profit becomes $\frac{1}{2}v + k_s - 2k_s^2$, and the buyer profit is $v + k_b + k_s - \frac{1}{2}v - k_s - k_b^2 = \frac{1}{2}v + k_b - k_b^2$. When they maximize the profit, both the buyer and the seller will make efficient investments, and we get efficient outcome. There would not be a hold-up problem. However this is not possible because of non-verifiability.

Contracts can be written based on price to pay or quantity to buy (which is assumed to be 0 or 1). Consider the fixed quantity ($q = 1$), and fixed price p contract. Since the seller does not derive any benefit from her investment costs, as she always get the same price, hence she has no incentive to invest. Whatever investment made only benefits the buyer, and costs the seller.

Consider a fixed quantity ($q = 1$) but variable price contract. By variable price we mean that the price is not specified in the beginning, but subject to negotiation laters. There are many possible negotiation processes we can consider, but we only consider two possibilities: either the seller makes a take-it-or-leave-it offer, or the buyer makes a take-it-or-leave-it offer to the other side. If the seller makes offers, the price will be equal to the consumers' willingness to pay $v + k_b + k_s$, and the seller has full incentive to invest, but the buyer has no incentive to invest, as she gets $0 - k_b^2$ profit, and the best investment amount is 0. If the buyer makes offers, the price will be 0 (marginal cost). The buyer captures all gain from trade), and the seller has no incentive to invest.

Another better alternative is for the parties to enter into a fixed price variable quantity contract. The buyer can choose either to buy one unit at a fixed price p, or not buy any at all and pay no damages. When the uncertainty is resolved, p may be below $v+k_b+k_s$. In this event, the buyer will buy one unit and pay p. In the event p is above $v+k_b+k_s$, the buyer does not want to place any order. The seller will be willing to lower the price to $v+k_b+k_s$ in the renegotiation (with the seller getting all the trade surplus). The seller has incentive to invest in such events.

To look at the incentive to invest, consider first the buyer. The buyer gross profit is

$$E(v+k_b+k_s-p|v+k_b+k_s>p)=\int_{p-k_b-k_s}^{1}(v+k_b+k_s-p)dv=\frac{1}{2}(1-p+k_b+k_s)^2$$

after incurring cost k_b^2. Hence the the buyer chooses k_b to maximize $\frac{1}{2}(1-p+k_b+k_s)^2-k_b^2$, and we have the first order condition

$$1-p+k_b+k_s-2k_b=0$$

or

$$k_b-k_s=1-p. \tag{1}$$

Similarly, the seller gross profit is

$$E\min(v+k_b+k_s,p)=\int_{0}^{p-k_b-k_s}(v+k_b+k_s)dv+p(1-p+k_b+k_s) \tag{2}$$

$$=\frac{1}{2}p^2-\frac{1}{2}(k_b+k_s)^2+p(1-p+k_b+k_s)$$

after incurring the cost $2k_s^2$. The seller chooses k_s to maximize

$$\frac{1}{2}p^2-\frac{1}{2}(k_b+k_s)^2+p(1-p+k_b+k_s)-2k_s^2$$

The first order condition is

$$-k_b-k_s+p-4k_s=0$$

or

$$k_b + 5k_s = p \tag{3}$$

Combining the two equations (1),(3), we get the equilibrium investment levels for a given p:

$$k_b = \frac{5}{6} - \frac{2}{3}p, k_s = \frac{p}{3} - \frac{1}{6}.$$

Which p will prevail depends on the bargaining power. Both parties may also find it optimal to agree on the price p that maximizes the total social surplus created together with a transfer payment between them. The price that maximizes the social surplus is $p = \frac{3}{4}$ (calculations not shown, left as an exercise). With this price, we get $k_b = \frac{1}{3}, k_s = \frac{1}{12}$. It can be shown that the associated social surplus is greater than the case with no contract. Thus the fixed price variable quantity contract reduces the inefficiency of the hold-up problem. However, the optinal incentives in this case are not provided.

Efficient Incentives with Default Options

In the following we provide an example of how the inefficiency of the hold-up problem can be completely eliminated by contract with renegotiations. This example differs from above in the following elements: (1) The quantity of trade can be any number between 0 and 1. (2) There is demand uncertainty as well as cost uncertainty, but the uncertainty is in a discrete form. (3) There is a default quantity and price pair which are fixed before investments are chosen. (4) the buyer get all the bargaining power in the renegotiation.

Two contracting parties, a prospective buyer and a prospective seller, can enter a trading relationship selling and buying quantity level $q \in [0, 1]$ at a (total) price P. The profits of the players depends on the buyer's valuation v (per unit) and seller's cost c (per unit). These v, c are uncertain at the time of contracting, and can be influenced by the specific investments made by each party. The buyer's valuation can be either high v_h or low v_l. The Buyer's investment j costs the buyer $\varphi(j)$ and results in $\text{Prob}(v_h) = b(j)$. We assume that $b(j)$ is concave and increasing in j. Therefore higher investment leads to higher probability of high valuation. The seller's costs are either high c_h or low c_l. The seller's investment i costs the seller $\phi(i)$ and results in $\text{Prob}(c_l) = a(i)$. The function $a(i)$ is also increasing and concave. Therefore higher investment leads to higher probability of low cost. We assume that these investments are sunk costs whatever the ex-post level of trade (even if there is no trade ex-post). The buyer's ex-post payoff can be written as

$$vq - P - \varphi(j)$$

and the seller's ex-post payoff is

$$P - cq - \phi(i)$$

The timing is as follows: First the parties contract; secondly, they simultaneously choose their investment levels i and j. Thirdly, they both observe the state of nature $\theta = (v, c)$; finally they execute the contracts or renegotiate. We assume that investment levels i, j and v, c are not verifiable or contractible. By contrast, quantities q and price P are contractible or verifiable. Both parties are assumed to be risk neutral. We also assume for simplicity that

$$c_h > v_h > c_l > v_l$$

This means that there is no profitable trade unless the buyer's valuation is high, and the seller's cost is low. Under these assumptions, the ex-post efficient trade is $q = 1$ if $\theta = (v_h, c_l)$ and 0 otherwise. The ex-ante efficient investments are determined by the maximization of

$$\max[a(i)b(j)(v_h - c_l) - \phi(i) - \varphi(j)$$

by choosing i, j. The first-order conditions for maximization give us the optimal investment levels i^*, j^* satisfying

$$a'(i^*)b(j^*)(v_h - c_l) = \phi'(i^*)$$

$$a(i^*)b'(j^*)(v_h - c_l) = \varphi'(j^*)$$

The holdup problem is expressed by the statement that there will be underinvestments in equilibrium if there is spot contract ex-post (no ex-ante contract), and after θ is realized, the gains from trade are evenly divided between the two parties. No one has the incentive to invest sufficiently because some of the increased surplus from investment will be taken away by the other side in the bargaining for spot contracts.

The underinvestment problem can be solved by using ex-ante contracts that allow for default options. The default option we will use is a trade quantity q^0 satisfying

$$q^0(c_h - c_l)a'(i^*) = \phi'(i^*)$$

The contractual mechanism is the following: There is a default price P^0 specified in the contract. The parties then play the following game. After θ

is realized, the buyer makes an offer (q, P) to the seller. The seller can either accept the offer (and the trade takes place at those terms) or rejects it. When the offer is rejected, trade takes place according to the default option (q^0, P^0). The price P^0 is determined by the initial bargaining strengths of the two sides.

The buyer has full bargaining power in the game played in the contract. The mechanism implements the first best. She will offer to trade the ex-post efficient quantity while leaving the seller indifferent between the trade and the default-option payoff. This is how we get the ex-post efficiency. The investment efficiency holds for the seller. This is because the seller anticipates obtaining the default-option payoff, and chooses the investment level i to maximize

$$P^0 - a(i)c_l q^0 - (1 - a(i))c_h q^0 - \phi(i)$$

and the first order condition is satisfied by i^* due to the way q^0 is chosen. This means that the seller will make the optimal amount of investment i^*. The seller has incentive to invest despite having no bargaining power in the game. The incentive comes from the default option.

Note that the final payoff of the seller depends only on the seller's investment and is independent of the buyer's investment. This also means that the buyer is the residual claimant on her investment. Therefore the buyer also has the incentive to invest the optimal amount j^*. Note that when $q^0 \in (0, 1)$, the actual trade is the efficient one (either 0 or 1), as the buyer will offer alternative trade to replace the default option. This is because it is always better for the buyer to do so.

For a numerical example, let the investment cost functions be

$$\phi(i) = i, \varphi(j) = j^2$$

and assume that $c_h = 10, v_h = 8, c_l = 6, v_l = 4$. The way the investment levels affect the valuation and the cost are described by the following probability functions

$$a(i) = i^{0.5}, b(j) = j^{0.5}$$

(1) How to determine the ex-ante investment levels i^*, j^*?

The total surplus from the investments is the expected surplus from trade minus the costs of investments. The total surplus can be written as

$$i^{0.5}j^{0.5}(v_h - c_l) - \phi(i) - \varphi(j) = 2i^{0.5}j^{0.5} - i - j^2$$

When we maximize this total surplus function, the first order conditions are

$$i^{-0.5}j^{0.5} - 1 = 0, i^{0.5}j^{-0.5} - 2j = 0$$

The first equation gives us $i = j$, and the second equation gives us $j = 0.5$. Hence the optimal investment levels are $i^* = j^* = 0.5$. The total surplus is obtained by substituting these values into the expression for the total surplus, and we get the total surplus

$$2 * 0.5 - 0.5 - 0.25 = 0.25$$

(2) How to determine the default trade level?
The equation for determining the default trade level is

$$q^0(c_h - c_l)a'(i^*) = \phi'(i^*)$$

or

$$q^0 4 * 0.5i^{-0.5} - 1 = 0$$

hence the default trade level is

$$q^0 = 0.5i^{0.5} = 0.353\,55$$

(3) How to verify the incentive of the seller?
The seller is assured of the trade amount q^0. When she made the investment choice ex-ante, she chooses i to maximize the expected profit

$$P^0 - q^0(i^{0.5}c_l + (1 - i^{0.5})c_h) - i$$

The first order condition gives us

$$q^0(c_h - c_l)0.5i^{-0.5} - 1 = 2q^0 i^{-0.5} - 1 = 0$$

therefore the optimal choice of i satisfies

$$q^0 = 0.5i^{0.5}$$

which is the same condition satisfied by the ex-ante efficient investment i^*. Therefore, her optimal choice is also the efficient choice. The seller thus has the incentive to invest the efficient amount $i^* = 0.5$

(4) How to determine the equilibrium proposal for trade by the buyer after the state of nature is observed?

The buyer will propose no trade when the valuation is low or when the cost is high. Note that we assume that these are observable by both players. In the state (v_h, c_l), the buyer proposes to trade the efficient level $q = 1$ at a price P which is chosen to make the seller indifferent between the equilibrium trade and the default option. The (ex-post) payoff to the seller from the default option is

$$P^0 - q^0(0.5^{0.5}c_l + (1 - 0.5^{0.5})c_h) - 0.5 = P^0 - 3.0355$$

hence P is chosen so that

$$P - (0.5^{0.5}c_l + (1 - 0.5^{0.5})c_h) - 0.5 = P^0 - 3.0355$$

or

$$P - 7.6716 = P^0 - 3.0355$$

$$P = P^0 + 4.6361$$

Therefore the equilibrium trade in this case is at a higher amount and price than the default option. The seller is indifferent between the two choices. When the state of nature is different from (v_h, c_l), the seller will reject the no trade proposal and choose the default option instead. Therefore the seller's payoff is independent of the realized state of nature.

(5) How to determine the default price level P^0?

Note that when the default P^0, q^0 is chosen, and q^0 determined optimally, the seller's optimal investment is determined (and equal to the efficient level i^*). The payoff to the seller is also determined and is shown above as

$$P^0 - 3.0355$$

therefore the choice of P^0 only affects the surplus accrued to the seller. The choice of P^0 is therefore determined by the bargaining strengths of both sides before the signing of the contract (note that after agreeing to the contract, the seller no longer has the bargaining power, and has to rely on the default option to protect her interests). If the seller is able to negotiate in a way so that she has half share of the total surplus, then we have

$$P^0 - 3.0355 = 0.125$$

and

$$P^0 = 3.1605$$

(6) Why does the buyer have proper incentives to invest?

This is because the default price P^0 and therefore the surplus to the seller is independent of the investment decision of the buyer. The buyer is the residual claimant of his investment. Any additional surplus created by his investment belongs to him fully. No underinvestment will occur in this case.

Assignment 9
Due April 3

1. In the first example of the hold-up problem, suppose that the investments $k_{b,}k_s$ and v were verifiable, then there will be no problem with hold-up. To see this, let the two firms sign a contract specifying the price of the input to be $\frac{1}{2}v + k_s$.

(a) Show that the seller will make efficient investment.

(b) Show that the buyer will also make the efficient investment.

(c) When the investments and v are not verifiable, explain why the contractual solution does not work.

(d) In the non-verifiable case, we say that fixed price (p) flexible quantity contracts can be used to improve contract efficiency. How would you choose p to achieve the most efficient outcome? Show it to be $p = 3/4$.

2. In the hold-up example with default option of the note, the buyer and the seller initially sign a contract with a default option (q^0, P^0), with $0 < q^0 < 1$. As time progresses, assume that the parties observe $\theta = (v, c)$. The buyer may make an offer to the seller (q, P) instead of carrying out the contract trade option.

(a) Assume that the observed value is $\theta = (v_h, c_h)$. Find the equilibrium trade (q^*, P^*), and shows that trade must be efficient in equilibrium.

(b) Assume that the observed value is $\theta = (v_h, c_l)$. Find the equilibrium trade (q^*, P^*), and shows that trade must be efficient in equilibrium.

(c) Assume that the observed value is $\theta = (v_l, c_h)$. Find the equilibrium trade (q^*, P^*), and shows that trade must be efficient in equilibrium.

(d) Assume that the observed value is $\theta = (v_l, c_l)$. Find the equilibrium trade (q^*, P^*), and shows that trade must be efficient in equilibrium.

3. In the hold-up example with default options, we make the following changes. Assume that the investment costs are given by

$$\phi(i) = i, \varphi(j) = 2j^2$$

and $c_h = 10, v_h = 8, c_l = 5.5, v_l = 4$. We keep the same assumptions on $a(i), b(j)$:

$$a(i) = i^{0.5}, b(j) = j^{0.5}.$$

(a) Determine the ex-ante investment levels i^*, j^*.

(b) Determine the default level of trade in the optimal contract solving the underinvestment problem.

(c) Compute the optimal level of investment for the seller under the contract with the default trade and show that it is the same as i^*.

(d) Find the equilibrium price of trade proposed by the buyer when the state of nature turns out to be efficient for trade. In this problem, we assume that the default price of trade is known to be P^0.

(e) Assume that both sides end up sharing the net surplus equally. What is the default price of trade in this case?