MATH 425a MIDTERM EXAM 2
November 6, 2015
Prof. Alexander

Last Name: ______________________________

First Name: ______________________________

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Signature: ______________________________

Problem	Points	Score
1	25	
2	20	
3	32	
4	28	
Total	100	

Notes:

(1) Use the backs of the sheets if you need more room.

(2) If you can't do, say, part (a) of a problem, you can assume it and do parts (b), (c), etc.

(3) Points total 105 but 100 is maximum. This means the first 5 points you lose don't count against you.

(1)(25 points)(a) State what it means for a series $\sum_n a_n$ to converge.

(b) Prove the following part of Theorem 4.8: for $f : X \to Y$, if f is continuous then $f^{-1}(V)$ is open in X for every open set V in Y.

(2)(20 points) These two questions are "yes/no with justification." This means formal proof is not required, just say enough to show you understand the reason.

(a) Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(x) = 0$ for all rational numbers x. Is it necessarily true that $f(x) = 0$ for all $x \in \mathbb{R}$?

(b) Suppose the Ratio Test is used to show that the series $\sum_n a_n$ converges to some finite s. Is it possible that some rearrangement $\sum_n a'_n$ converges to a value other than s?

(3)(32 points) Establish convergence or divergence of the series (a), (b) and (c):

(a) $\sum \dfrac{n}{n^3 - 2}$

(b) $\sum 2^{-\sqrt{\log_2 n}}$ (Note $\log_2 n$ means base 2 logarithm.)

(c) $\sum (-1)^n \dfrac{n}{n+1}$

(d) Find the radius of convergence: $\sum n^2 \left(\frac{2}{n}\right)^n z^n$.

(e) Suppose $a_n > 0$ and $a_n/n \to 0$ as $n \to \infty$. Show that $\sum 1/a_n$ diverges. HINT: Comparison test.

(4)(28 points) Suppose $g : (0, 1] \to \mathbb{R}$ is uniformly continuous, and $p_n > 0$ with $p_n \to 0$.

(a) Show that $\{g(p_n)\}$ is a Cauchy sequence. HINT: All you need are the relevant definitions.

(b) If also $t_n > 0$ with $t_n \to 0$, show that $|g(p_n) - g(t_n)| \to 0$.

(c) Show that $\lim_{x\to 0} g(x)$ exists. HINT: Recall that by Theorem 4.2, this limit exists and is equal to q, if and only if $g(t_n) \to q$ for all sequences $t_n \to 0$ in $(0, 1]$.

REMINDER LIST–TESTS FOR CONVERGENCE

1. Cauchy criterion ($\{s_n\}$ must be a Cauchy sequence, where s_n is the nth partial sum)
2. Comparison test
3. Cauchy condensation test
4. Root test
5. Ratio test
6. Alternating series test
7. "Two bounds" method