

Quiz

1. Suppose the market demand for a product is given by $P = 200 - Q$. Each firm's cost function is $C(q_i) = 20q_i$, where $i = 1, 2, 3$.
 - (a) Suppose the three firms compete in a Cournot fashion, compute each firm's output, profit, and the market price.
 - (b) Suppose that the three firms collude or merge to become one firm (monopoly) and the cost function does not change. Find their joint profit-maximizing (monopoly) price, output, and profit.
 - (c) Compute and compare consumers' surplus and deadweight losses of both cases.
 - (d) Compute the Herfindahl-Hirschman index of this market (in the case of competition). Without any calculation, answer the following question: suppose there is a technology innovation that decreases the marginal cost of each firm *by the same amount*, how would the HHI index change?
 - (e) (*Optional*) Answer the above questions if the demand is given by $P = 200 - \frac{1}{2}Q^2$.

2. Consider a representative consumer with the following utility function over three goods:

$$U(q_0, q_1, q_2) = q_0 + 10q_1 + 15q_2 - \left(\frac{1}{2}q_1^2 + q_1q_2 + q_2^2\right)$$

- (a) Derive the consumer's demand for goods 1 and 2 (both direct and inverse demand systems).
- (b) Are these two goods complements or substitutes? Why?
- (c) Suppose two firms produce these two goods at constant marginal cost $c_1 = 1$ and $c_2 = 2$, respectively. Compute the Bertrand-Nash equilibrium when the firms compete by setting prices simultaneously (prices, quantities, and profits).
- (d) Suppose the two firms merge to become a monopolist over the two products (or to maximize their joint profits). Compute the profit-maximizing prices, quantities, and profits. What do you conclude about the merger from the comparison to your results in (c)?
- (e) What can you say about the consumer welfare between the two cases?

3. A monopolist manufacturer produces a product at constant marginal cost $c_m = 2$. There is one downstream retailer who orders this product from the manufacturer and sell it to the consumers. In addition to the wholesale price, the retailer faces a constant marginal cost $c_r = 1$. The inverse demand function is given by: $P = 10 - Q$.
- Suppose the manufacturer determines the wholesale price first, and the retailer determines the retail price and quantity based on the wholesale price. Compute the wholesale price, retail price, quantity and consumer surplus in equilibrium.
 - Suppose a vertical integration occurs and the integrated firm faces a constant marginal cost $c = c_m + c_r$. Compute the price, quantity and consumer surplus.
 - What can you conclude from the comparison between (a) and (b)? Why is there a difference?
 - (Optional) Answer the above questions if the demand has a constant elasticity of 2.
4. Suppose the two firms play the Bertrand game an infinite number of periods. The demand system is given by:

$$q_1 = 10 - 2p_1 + p_2, MC_1 = 4$$

$$q_2 = 10 - 2p_2 + p_1, MC_2 = 4$$

In each period, the two firms may collude and play as a monopolist to maximize their joint profit. If a firm was to deviate from the collusive strategy, it will take the other firm's collusive price as given and maximize its own profit. If any of the firms deviates, the cooperation ends and both firms play the Nash competitive strategy forever. The two firms have the same discount factors δ .

- Compute the Nash competitive and collusive price and profit for each firm.
- If one of the firms deviates from the collusive strategy based on its belief that the other firm will cooperate, compute its price and profit.
- What is the condition for the collusion to sustain in this market?
- (Optional) Answer the above questions if $MC_1 = 3$.