MATH 425a ASSIGNMENT 4
FALL 2015 Prof. Alexander
Due Friday October 2.

Note this assignment is due after Midterm 1, but the material IS covered on the midterm.

Rudin Chapter 2 #12, 14, 16, 22 plus the problems (A)–(E) below:

(A)(i) Show that in any metric space, the closure of a neighborhood satisfies $\overline{N_r(x)} \subset \{y : d(x, y) \leq r\}$.
(ii) Find an example where the sets in (i) are not equal.

(B) Suppose E is infinite and all points of E are isolated. Show *directly from the definition* (of compact) that E is not compact.

(C)(i) Show that if L, M are compact, then $L \cup M$ is compact.
(ii) Show that if K is compact, D is closed, and $D \cap K^c$ is compact, then D is compact.

(D) Suppose $G_1 \subset G_2 \subset \dots$ are open in $\mathbb{R}$, and G_j^c is nonempty and bounded for all j. Show that $\cup_{j \geq 1} G_j \neq \mathbb{R}$.

(E) Identify which of the following sets are compact and which are not, with an explanation (not necessarily a full formal proof.)
(i) $\{1/k : k \in \mathbb{N}\} \cup \{0\}$
(ii) $\{(x, y) \in \mathbb{R}^2 : |xy| \leq 1\}$
(iii) $[2, 3] \cup [4, 5]$

HINTS:

(A)(ii) You can use $\mathbb{Z}$ as your metric space.

(C) For (i), use the definition of compactness. For (ii), use (i).

(D) This is closely related to one of the textbook's theorems about compact sets.

(14) What happens if you cover $(0, 1)$ with open intervals, all having left endpoints > 0? How can you define such a cover?

(16) For non-compactness, use Theorem 2.33.

(22) Let $\epsilon > 0$ and $x, y \in \mathbb{R}^k$. Fill in the blank: if the coordinates satisfy $|y_i - x_i| <$ ____ for all i, then $|y - x| < \epsilon$. Now use this together with the fact that $\mathbb{Q}$ is dense in $\mathbb{R}$.