MATH 425a ASSIGNMENT 2
FALL 2015 Prof. Alexander
Due Wednesday September 16.

Rudin Chapter 1 #12, 13, 17, Chapter 2 #3, 4 plus the problems (A) - (D) below:

(A) Let $A \subset \mathbb{C}$ and $\alpha = \sup\{|z| : z \in A\}$. Show that $\sup\{|z+1| : z \in A\} \leq \alpha + 1$.

(B) Let

$$\begin{aligned} A_3 &= \{(a,b,c) : a,b,c \in \mathbb{Z}\} \\ B_3 &= \{(a,b,c) \in A_3 : a \neq b \neq c\} \\ C_3 &= \{\text{all 3-element subsets of } \mathbb{Z}\}. \end{aligned}$$

Define $f : B_3 \to C_3$ by $f\big((a,b,c)\big) = \{a,b,c\}$.
(a) Why is B_3 countable?
(b) Is f 1-to-1? Onto? Explain.
(c) Show C_3 is countable.
(d) Show that $C = \{$ all finite subsets of $\mathbb{Z}\}$ is countable.

(C) Is the intersection of two uncountable sets necessarily uncountable? How about their union? (Prove, or disprove by giving an example. This is short!)

(D) Let us say that a sequence $\{z_n\}$ of integers *terminates* if for some $N, z_n = 0$ for all $n \geq N$. Thus for example $(1,3,0,3,0,0,0,...)$ and $(1,2,1,3,1,2,0,0,0,...)$ both terminate.
(a) Show that $A = \{$ all terminating sequences of 0's, 1's, 2's, and 3's $\}$ is countable.
(b) Show that $B = \{$ all terminating sequences of integers $\}$ is countable.

HINTS:

(Ch. 2 problems) Remember two useful facts from lecture (of course that's redundant, ALL facts from lecture are useful): (i) To show $|a| \leq b$, show $a \leq b$ and $-a \leq b$. (ii) To prove inequalities with norms, it is sometimes useful to square both sides, and use $|x|^2 = x \cdot x$.

(3,4) You may *assume problem 2* to do these. Then all you need is that (from lecture and problem 2) the rationals and algebraic numbers are countable, but the reals are not.

(D)(a) What happens if you fix the termination time? By this I mean the index of the last non-zero entry. For the two examples given in the problem, the termination times are 4 and 6.