A Course in Semantics[1]

[1] Terence Parsons has a semantics textbook available online at ucla.edu. With his permission, Roger Schwarzschild and Daniel Altshuler have been revising and modifying it. This is the Spring 2016 version.

Chapter 1: Introduction

1.1 *Syntax and semantics*

When people are asked how they come to know the meanings of sentences in a language they learn in school or even in their native language, they will usually say something like the following:

> *Words have meanings, they are listed in the dictionary, and you get the meaning of a sentence by putting together the meanings of the words in that sentence.*

This idea has a very important consequence: whatever meanings are, they have to be combinable somehow and the combinations have to be meanings as well. This is true at the level of sentences but it's also true for other constituents. The meaning of the phrase '*near the house*' comes from combining the meanings of the three words that make it up.

While it is true that the meaning of a sentence is determined in part by the words in the sentence, as the quote above indicates, there is another important ingredient. To see that, compare these two example sentences:

(1) a. The triangle is above the square.
    b. The square is above the triangle.

(1)a and (1)b contain the same six words and yet they don't say the same thing. Clearly, the meaning difference arises from the fact that their word order is different. This suggests that there are **semantic rules** that produce meanings using as ingredients: (i) the meanings of words and (ii) the order in which they occur.

There are other ingredients as well. Examples of **syntactic ambiguity** are especially revealing. Before turning to syntactic ambiguity, we should say a few words about **ambiguity** in general.

Suppose Jack remarks to Jill:

(2) The soup is very hot.

That sentence is **ambiguous**. It can mean either (3) or (4):

(3) The temperature of the soup is very high.

(4) The soup is very spicy.

If Jill is uncertain about which meaning Jack intended when he uttered (2), she can use (3) and (4) to clarify. These sentences are possible **paraphrases** of Jack's remark. Furthermore, they are **unambiguous paraphrases** – each one captures one (and only one!) of the meanings of the sentence it is paraphrasing. It is because the paraphrases are unambiguous that they are useful for clarifying which meaning is intended. Notice that the word '*hot*' doesn't appear in either paraphrase. That's because the very source of the ambiguity in Jack's original remark is the word '*hot*', which has at least two meanings. In order to be unambiguous, the paraphrases replace this word. Since the source of the ambiguity is a word, we say that Jack's remark is **lexically ambiguous**.

The two paraphrases in (3)-(4) are distinct in a sense that will be important in this course, namely that there are possible situations in which one of the sentences is true and the other false. If Jack's soup is cold and very spicy, then (3) is false and (4) is true.

EXERCISE A Describe a situation in which (3) is true and (4) is false.

The two paraphrases in (3) and (4) represent different **readings** of (2). We say that in a situation where the soup is cold and very spicy, Jack's remark is false on the reading paraphrased in (3) and true on the reading paraphrased in (4). We will be using 'ambiguous' as a technical term. To say that a sentence is ambiguous is to say that it has at least two readings and that there are situations in which it is true on one reading and false on the other.

EXERCISE B

Provide a sentence of your own that is lexically ambiguous. For each reading of your sentence, provide an unambiguous paraphrase that captures that reading. Describe a situation in which your sentence is true on one reading and false on the other.

✪ IMPORTANT ✪

Make sure you do not include more information in your paraphrase than is in the original sentence.

- Example: 'the soup that he ordered is spicy' is not an adequate paraphrase of 'the soup is hot' which says nothing about ordering the soup.
- Make sure your situation is complete, so that the sentence is false/true on the relevant reading. It is not enough to give a description where the sentence *could* be false/true. For example, consider the situation in which I turned on the stove 15 minutes ago. In this situation, the sentence 'The soup is hot' *could* be true or false. Not enough information is provided to be sure (for example: was the soup ever put on top of the stove?).

Now consider the sentence below:

(5) Jim talked about the party in the church.

This sentence is ambiguous. One of the readings is captured in the unambiguous paraphrase below, in (6), and the other, in the unambiguous paraphrase in (7):

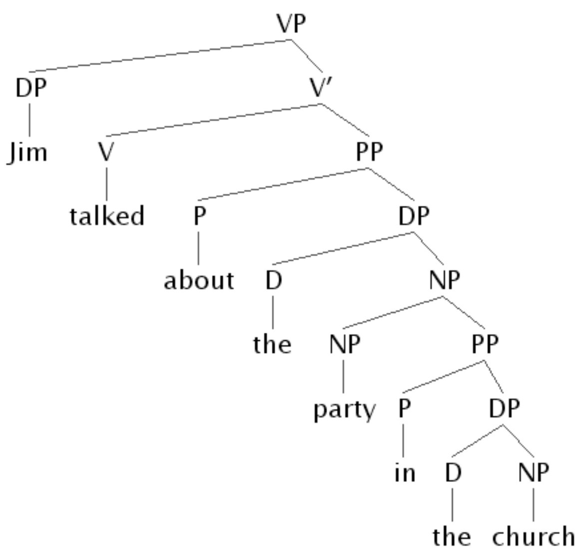
(6) Jim talked about the party, which took place in the church.
(7) Jim talked about the party, while he was in the church.

If the party in question took place on the beach and then Jim talked about it in the church, the sentence is false on the reading paraphrased in (6) and true on the reading paraphrased in (7).

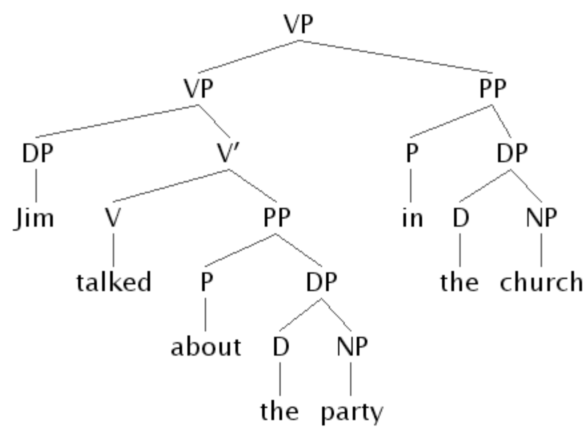
EXERCISE C

Describe a situation in which '*Jim talked about the party in the church.*' is true on the reading paraphrased in (6) and false on the reading paraphrased in (7).

The source of the ambiguity in (5) is not a particular word, but rather the way the words are put together. Below are two possible syntactic structures for the example sentence. The one on the right corresponds to the reading in (6) and the one on the left to the reading in (7).



party in the church



party talked about in the church

Notice that the source of the ambiguity is the attachment site of the PP *'in the church*'. In one case, it forms a **constituent** with the NP '*party*' and in the second, it forms a constituent with the VP headed by '*talked*'. A **syntactic ambiguity** is, thus, an ambiguity that arises when a string of words can be formed in two or more different ways and the results lead to differences in meaning. This leads us to the following hypothesis, called **compositionality**: that the meaning of a sentence is determined by the words used as well as by how they are composed. The study of how words are composed is called **syntax.** A central aim of this course will be to explain how syntactic differences like those depicted in the diagrams above (called *trees*) can lead to a difference in meaning.

EXERCISE D

Provide a sentence that is ambiguous due to the possibility of some constituent being attached in different places. For each reading of your sentence, provide an unambiguous paraphrase that captures that reading. Describe a situation in which your sentence is true on one reading and false on the other.

Here's a brief summary of discussion to this point:

CONCLUSIONS

- Meanings can be combined to form new meanings.
- The meaning of a complex expression is determined by the meanings of the words in that expression, as well as by its syntax (how it was put together).
- Syntactic input to meaning includes word order and constituent structure (groupings).

EVIDENCE

We looked at two kinds of evidence for the view that semantic rules make reference to syntactic structure in addition to word meaning.

- A group of words can be put together in two different orders to produce two sentences with different meanings (*The triangle is above the square* ≠ *The square is above the triangle*).
- A group of words can be put together in a particular order but with different constituent structures each with its own meaning (*Jim talked about the party in the church*).

NEW TERMINOLOGY

- semantics rules
- syntactic ambiguity
- ambiguity
- ambiguous
- paraphrases
- unambiguous paraphrases
- lexically ambiguous
- readings
- constituent
- compositionality
- syntax

APPLICATION

- ✓ A particular reading of an ambiguous sentence can be specified by using an unambiguous paraphrase.
- ✓ To show that two readings are distinct, we describe a situation in which one reading is true and the other false.

SOMETHING TO THINK ABOUT

Recall the quote we started with:

> *Words have meanings, they are listed in the dictionary, and you get the meaning of a sentence by putting together the meanings of the words in that sentence.*

Although this hypothesis leaves out the role that syntax plays in interpretation, it is generally correct that you get the meaning of a sentence by combining the meanings of the words making it up. In fact, this hypothesis implies that standard dictionary definitions may describe meanings but they can't be meanings. You do not get the meaning of a sentence by stringing together dictionary definitions. Try it some time!

Here is what we got using dictionary.com to 'interpret' the sentence '*a fish jumped*':

Not any particular or certain one of a class or group; any of various cold-blooded, aquatic vertebrates, having gills, commonly fins, and typically an elongated body covered with scales; to spring clear of the ground or other support by a sudden muscular effort; a suffix forming the past tense of weak verbs.

1.2 *Semantic Rules and Grammar*

As we have just seen, the meaning of a sentence is determined partly by the meanings of the words in the sentence and partly by the way in which the words are put together. This suggests that there are semantic rules which take as input: (a) the meanings of two or more expressions and (b) the syntactic structure in which they are combined. The semantic rules give as output the meaning of the combination. Our goal throughout this book is to specify what these rules are. As we progress, we will accumulate more and more such rules. Semantic rules together with syntactic rules will be called a **grammar**. The grammar generates a set of sentences paired with meanings.

Our rules are like Newton's Laws in physics or Mendel's Laws in genetics. They are precise descriptions of the subject matter. They are arrived at by generalizing based on what one takes to be representative examples. The rules or laws then make predictions about situations beyond those that motivated the rules and, by combining laws, further predictions are made.

These predictions can be tested. For semantic rules, that means comparing our intuitions about the meaning of a sentence with what the rules predict. If the predictions turn out to be incorrect, then the rules need to be revised. This is one way that progress is made.

NEW TERMINOLOGY

- grammar

APPLICATION

- ✓ The goal of semantic theory is to specify what the semantic rules are. These rules, along with syntactic rules, make up a grammar which generates a set of sentences paired with meanings.
- ✓ Grammars are tested by comparing the meanings they assign with native speakers' intuitions.

1.3 *Truth conditions*

Biology is a science concerned with the study of life. This doesn't mean that biologists begin their research by asking what life is. Instead they develop theories about phenomena that are related to life. Similarly, while semantics is the study of meaning, we will not start our investigation by directly asking what meaning is or what meanings are. Instead we will begin by focusing on a property of sentences that depends on their being meaningful, namely, that a sentence is something that can be TRUE or FALSE, unlike a vowel or a table or a neutrino, which aren't they kinds of things that can be assessed for truth and falsity.

Let's briefly explore the connection between truth and meaning. Whether a sentence is true or not, depends on what the facts are in the situation being described. For example, the sentence below, in (8), is true if the facts are as depicted in Fig. 3 below and it is false if the facts are as depicted in Fig. 4 below.

(8) The square is above the triangle.

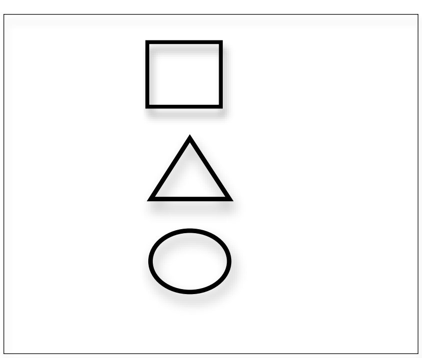


Fig. 3

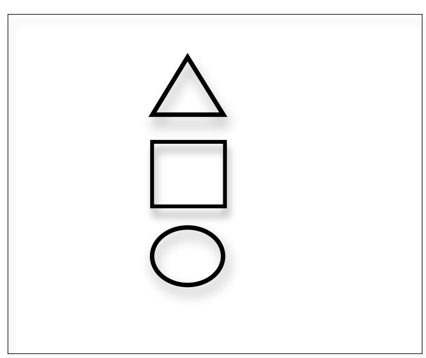


Fig. 4

In general, the conditions under which a sentence is true are called the sentence's **truth-conditions**. So we can say that the truth conditions for the sentence in (8) are met in Fig. 3 and are not met in Fig. 4. This may seem obvious to you, but that is because you are a speaker of English. Compare what you know about the English example sentence above, to what you know about the Q'eqchi' sentence in (9) below:

(9) Li oxxukuut wan rub'el li kaaxukuut

If you don't speak Q'eqchi', you don't know the meaning of that sentence, so you couldn't say if it is true or false in either of the two situations depicted in Figures 3 and 4 above. This illustrates a tight connection between knowing the meaning of a sentence and knowing its truth conditions. With the English example sentence in (8), you know its meaning and hence you know its truth-conditions; in the Q'eqchi' example sentence in (9), you don't know the meaning and you don't know its truth conditions.

While truth-conditions might not be the first thing that comes to mind when you think of meaning, it turns out to be extremely productive to focus on truth conditions in working out a grammar of the kind described in the previous subsection. So, our goal will be to develop syntactic and semantic rules that generate sentences paired with their truth conditions.

To get a sense of the advantage of focusing on truth conditions, think about the data that we need to collect to develop our grammar and to test its predictions. If you asked several English speakers to describe the meaning of the sentence '*The square is above the triangle*', you'd likely get a range of unwieldy answers. On the other hand, if you asked several English speakers if the sentence is true when the facts are as depicted in Figure 3 above, you would get the same answer across speakers and a clear answer: '*Yes*'.

In working with this idea of truth conditions, it is important to keep in mind the difference between <u>knowing the truth-conditions of a sentence</u> and <u>knowing whether or not a sentence is true</u>. Consider the following example:

(10) The Barker Library is on fire.

I don't know whether or not (10) is true. What I'm missing are facts about the world. If I went over to the library I could ascertain whether it is true or not. In so doing, I would <u>not</u> be learning more English. Rather, I would be using my knowledge of the truth conditions, and depending on whether those conditions are met or not, I would determine that the sentence is true or not.

The true-ness or false-ness of a sentences (henceforth: TRUE and FALSE) are called **truth-values**. And we have just been discussing the idea that the truth-value of a sentence is determined by its truth conditions plus whatever facts obtain in the world. The following formula summarizes this idea:

(11) Truth conditions + facts about the world → truth value

This formula applies to sentences since sentences have truth conditions and truth-values. When we confront other expressions, non-sentential ones, with facts in the world we don't get a truth-value, we get different types of entities. Here are some examples. '*Barack Obama*' is an expression of the language – it has phonological properties and syntactic properties. And there is some non-linguistic entity, namely an **individual**, that that expression stands for. This individual is the man who was elected president of the U.S. in 2008. The noun '*horse*' stands for not one entity, but a **set** of entities – the set of all horses. The preposition '*above*' stands for a **relation** between entities. The square bears that relation to the triangle in the situation depicted below:

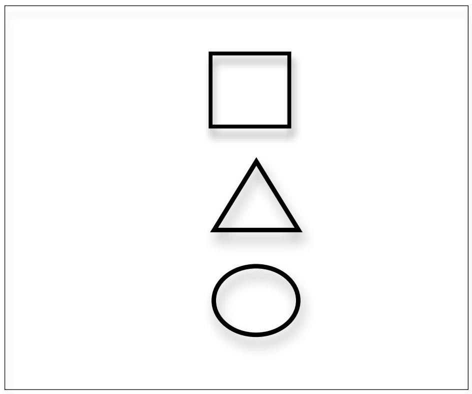


Fig 5: Illustrating the above relation

And the triangle bears that relation to the circle but not to the square. The noun '*brother*' stands for a different relation. George Bush bears that relation to Jeb Bush but he doesn't bear that relation to Barack Obama.

The entities that expressions stand for—a man, a set of horses, a relation—these are called **extensions**. The extension of an expression is determined by its meaning along with facts about the world. For a large part of natural language, the truth-value of a sentence is determined by the extensions of the words in that sentence and the syntax of the sentence. We will use that idea as a starting point for our study of the semantics of English. For much of this course, we will be developing semantic rules that will take us from word extensions, via syntactic structure, to truth-values for sentences. We will judge our progress in terms of how closely the system we develop tracks the intuitions speakers have about the truth of a sentence in different situations.

Up to this point, we've established that semantic rules need to take syntactic structure as input and they need to be capable of producing meanings out of combinations of other meanings. But what do such rules look like? It turns out that logicians have addressed this question in some detail and so in the next chapter, we'll be taking a look at how logicians define and interpret symbolic languages.

EXERCISE E

According to this chapter, sentences can be true or false; they have truth conditions. Does this hold for all sentences? Attempt to provide one or more examples of grammatical sentences whose form indicates that they are not capable of being true or false.

✪ IMPORTANT ✪

We're looking for sentences that cannot be true or false – not sentences whose truth value nobody knows because we lack the relevant facts.

EXERCISE F

Reconsider the Q'eqchi' sentence provided in (9) – repeated below – and Figures 3 and 4, also repeated below. It turns out that the truth conditions of (9) are met in Fig. 3 and they are not met in Fig. 4. The sentence in (12) below, however, has the opposite truth conditions.

(9) Li oxxukuut wan rub'el li kaaxukuut
(12) Li kaaxukuut wan rub'el li oxxukuut

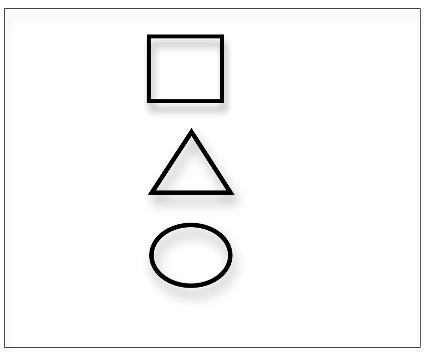


Fig. 3

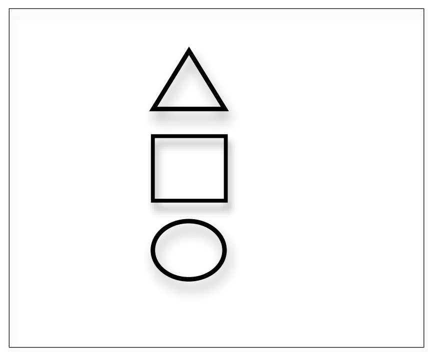


Fig. 4

The truth conditions of (12) are met in Fig. 4 but not in Fig. 3. Come up with a hypothesis about the meanings of the words making up those sentences. In particular, you should:

(i) Make a list of the words used in the Q'eqchi' sentences. Next to each word, provide its English translation according to your hypothesis.

(ii) Based on your translations, provide the extensions for the words '*kaaxukuut*' and '*oxxukuut*'.

✪ PRACTICE ✪

Make a guess about the meanings of the morphemes making up the words '*kaaxukuut*' and '*oxxukuut*'.

KEY IDEAS

- In the previous subsection, we claimed that a grammar generates a set of sentences paired with meanings.
- In developing a grammar, we focus on meaning related notions: *truth-conditions*, *truth-values, extensions.*
- Truth-conditions, together with whatever facts obtain in the world, determine the truth-value of a sentence.
- The meaning of a word, together with whatever facts obtain in the world, determine its extension.
- The goal of semantic theory is to develop semantic rules that will take us from word extensions, via syntactic structure, to truth-values for sentences.
- We will judge our progress in terms of how closely the system we develop tracks the intuitions speakers have about the truth of a sentence in different situations.

NEW TERMINOLOGY

- truth-conditions
- truth-values
- extensions
- individual
- set
- relation

1.4 *Entailment and synonymy*

Consider the following pair of sentences:

(13) On July 4th, Jack clumsily assembled a metal bookcase.
(14) Jack assembled a bookcase on July 4th.

There are various possible situations in which (13) would be true. It may be that Jack assembled the bookcase at home, at his friend's house or in a store. He might have done so while listening to music or while talking to a friend. But no matter what situation makes (13) true, it will also make (14) true. In this case we say that (13) **entails** (14). Notice that the reverse does not hold. There are situations that would make (14) true that would not make (13) true – for example if Jack assembled a wooden bookcase on July 4th and that was all he assembled. In other words, (14) doesn't entail (13). Compare that to the following pair of sentences:

(15) Jack sold a car to Jill.
(16) Jill bought a car from Jack.

It is not possible for there to be a situation in which (15) is true and (16) is false. And it is not possible for there to be a situation in which (16) is true and (15) is false. In other words, (15) and (16) entail each other. In this case, we say that they are **synonymous.**

Entailment and synonymy are relations that hold between sentences in virtue of their truth conditions. They therefore offer another source of data to test the grammars we will be constructing. Speakers have the intuition that (13) entails (14). If our grammar accurately captures our intuitions, then for any situation where the grammar assigns TRUE to (13), it will also assign TRUE to (14). And if the grammar doesn't do that, it will need to be revised.

EXERCISE G

For each pair of sentences in (17)-(19) below, try to find a situation in which the first member of the pair is true and the second false and then try to find a situation in which the second is true and the first false. In some cases, this won't be possible because of entailment or synonymy relations holding between the two sentences. Report your results as follows: for each pair, describe the situation that makes (a) true and (b) false and describe the situation that makes (b) true and (a) false. If it's not possible, state so. Then, for each pair, state whether (a) entails (b), (b) entails (a), both or neither. If (a) and (b) are synonymous state so.

(17) a. Jack is older than Jill.
b. Jill is younger than Jack.

(18) a. Jack lives in Texas.
b. Jill lives in Texas.

(19) a. Jack owns a submarine.
b. Jack owns a yellow submarine.

NEW TERMINOLOGY

- synonymy
- entailment
    - If one sentence entails another then any situation that makes the first true, also makes the second true.

APPLICATION

- ✓ A grammar that assigns truth conditions to sentences can be tested against speaker intuitions about entailment
- ✓ To show that one sentence does **not** entail another, you need to describe a situation in which the first is true and the second is false.

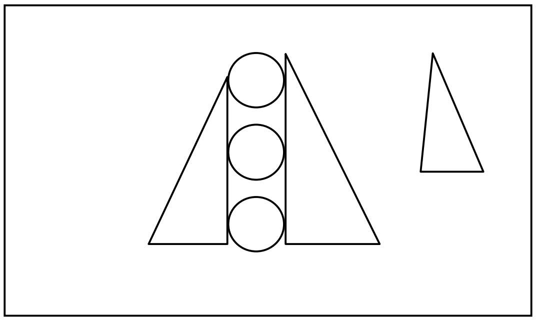
PRACTICE

- ➢ Below you will find a series of exercises giving you practice with entailment and paraphrase. Many of the examples involve quantifiers, a topic we will take up in chapter 2. These exercises give you the opportunity to develop your intuitions beforehand.

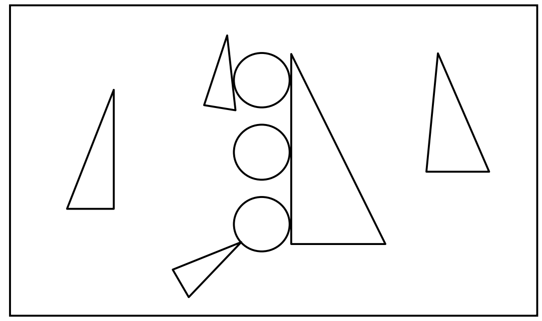
EXERCISE H

There are two situations depicted below, s_1 and s_2. The circles in those diagrams are touching triangles. Choose the situation in which it is true that:

Exactly one triangle is touching every circle.



s_1



s_2

EXERCISE I

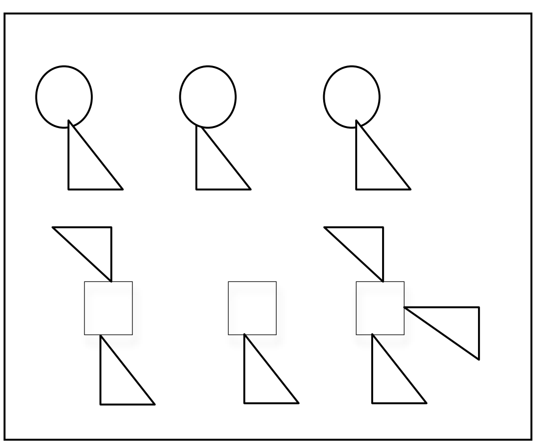
In which, if any, of the two situations s_1 and s_2 depicted in EXERCISE H above, is the following sentence true:

At least one triangle is touching no circle.

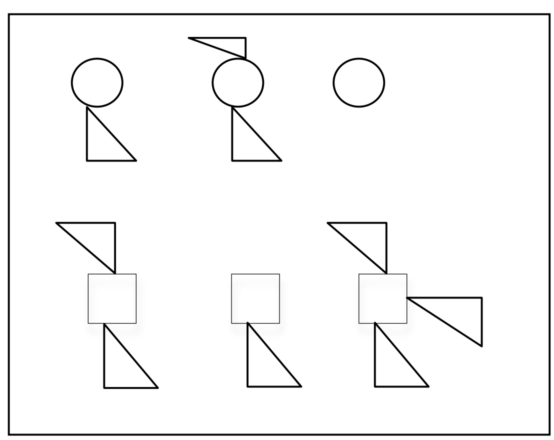
EXERCISE J

There are two situations depicted below, s_3 and s_4. In which situation is the sentence below true:

Between 1 and 3 triangles are touching every square, but exactly one triangle is touching every circle.



s_3



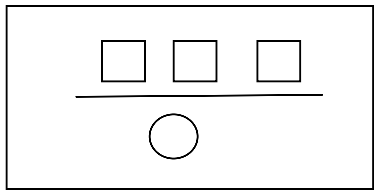
s_4

EXERCISE K

In which, if any, of the two situations s_3 and s_4 depicted in EXERCISE J above, is the following sentence true: *At least one triangle is touching no circle.*

EXERCISE L

In the diagram below, there is a line with three squares and a circle. The diagram depicts a situation in which the following is true: *Every square is above the line.*



i. If possible, draw a diagram depicting a situation in which (a) below is true and (b) below is false. If it is not possible, write "(a) entails (b)".

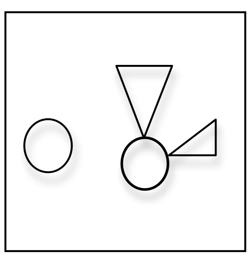
(a) *It is not the case that every square is above the line.*

(b) *Every square is such that it is not above the line.*

ii. If possible, draw a diagram depicting a situation in which (b) above is true and (a) above is false. If it is not possible, write "(b) entails (a)".

EXERCISE M

In the diagram below, there are two circles and two triangles. The diagram depicts a situation in which the following is true: *A circle is touching two triangles.*



i. Draw a diagram depicting a situation in which (a) below is true and (b) below is false. If it is not possible, write "(a) entails (b)".

(a) *There is exactly one triangle such that it is touching every circle.*
(b) *For every circle, there is exactly one triangle that is touching it.*

ii. Draw a diagram depicting a situation in which (b) above is true and (a) above is false. If it is not possible, write "(b) entails (a)".

Note: You can use one of the diagrams s_1-s_4 from previous exercises, if it fits the requirements.

EXERCISE N

Consider these two quotes:

(SCAN) "… a House subcommittee on Wednesday lambasted body scanners and pat-downs used by the Transportation Security Administration. Earlier in the meeting, some members of the panel said the scans were ineffective because every passenger was not examined. Former Department of Homeland Security Assistant Secretary for Policy Stewart Baker defended the randomization, saying that knowing a scan was possible acted as a deterrent for would-be terrorists."

(MEDICAL) "He couldn't figure out why there were so many complaints about the medical care on the ship. He assumed that some passengers may not have been examined after the outbreak, but surely most of them were. Then he went back to the records and he was horrified to learn that every passenger was not examined."

The sentence '*every passenger was not examined*' occurs in both quotes, but with different readings. **Provide two unambiguous paraphrases**, one for each reading. The quotes above are labeled (SCAN) and (MEDICAL). The paraphrase that corresponds to the first quote should be labeled (SCAN) and the paraphrase that corresponds to the second quote should be labeled (MEDICAL).

Here are some tips for creating the paraphrases that capture the readings. Your paraphrases should capture the meaning of the sentence and nothing more. So it should describe passengers and examining but not other information contained in the discourse – such ships or scanning.

Your paraphrase should be unambiguous. The point of the paraphrase is to single out a particular reading. Here are ways to do that:

i. Paraphrase complex DPs using the such-that construction. For example:

Jack saw every cat ⇝ *Every cat is such that*: *Jack saw it*.

No cat scratched Jack ⇝ *There is no cat such that*: *it scratched Jack.*

ii. Paraphrase negation using 'it is not the case that':

Jack didn't see Jill ⇝ *It is not the case that Jack saw Jill.*

1.5 *Set theory*

Semantic rules are often written using the language of elementary set theory. We already saw above that the extension of the noun '*horse*' is the set of all horses. It will be helpful, therefore, to say a few words about sets before we dive into semantics.

A set has **members**. Boston is a member of the set of all cities in the United States. The number 5 is a member of the set of numbers greater than 2. There is no constraint on what can be a member of a set. And elements of a set do not have to have anything in common. There is a set whose members are the number 5, the sun and all the horses in New Zealand. The members of a set are also called **elements** of the set.

The word '*class*' is sometimes used synonymously with 'set', although within modern set theory these terms are not synonymous.

A set is completely defined by its members. That means that if you remove an element from a set, you've got a different set. And if you add an element, again you have a different set. In this respect, sets are different than groups of people. We might say that the committee that passed resolution A is the same committee that passed resolution B, regardless of the fact that the composition of the committee changed between resolutions. Or I might say my family lived in Paterson in 1990 and in Jefferson in 1995. Even though the family grew between 1990 and 1995, it is the same family. But the set consisting of the members of my family in 1990 is a different set from the one that consists of the members of my family in 1995.

The symbol '$\in$' is used to mean "is a member of". So, if E is the set of all even numbers, (20) below is true and (21) is false.

(20) $4 \in E$ (true)
(21) $5 \in E$ (false)

The symbol '$\notin$' is used to mean "is not a member of". So, if E is the set of all even numbers, (22) below is false and (23) is true.

(22) $4 \notin E$ (false)
(23) $5 \notin E$ (true)

There are several ways to describe a set. One way is to just list the elements. For that we can use **list notation** in which the elements are enclosed in curly brackets: $\{3, 5, 7\}$. Since a set is completely defined by its members, it doesn't matter what order they come in, in list notation: $\{3, 5, 7\} = \{5, 7, 3\}$. This is intuitive. Whether I list the names of the students in our class in alphabetical order or in reverse alphabetical order, it is the same set of students that I've listed.

Generally lower case letters are used to name non-sets and upper case letters are used for sets:

(24) $B = \{a, b, d, m\}$ $K = \{a, d\}$ $C = \{d\}$

Notice that the set C in (24) consists of just one element. That's called a **singleton set**. This is one place where the technical term 'set' and the colloquial word 'set' part ways.

A set can have many elements. In fact, there are sets with an infinite number of elements. When sets are large, it becomes unwieldy or impossible to name the set using list notation. In that case, we make use of another option known as **predicate notation** also sometimes referred to as **definition by condition**. This is done by using curly brackets and a vertical line or a colon after which a condition on membership in the set is stated. Here is an example:

(25) $\{x \mid x \text{ lives in Boston}\}$

The set in (25) names the set of all individuals who live in Boston. You read that notation as "the set of all x, such that x lives in Boston". The following statement is true:

(26) Jack is a member of {x | x lives in Boston} *if and only if* Jack lives in Boston.

And similarly, the following statements are all true:

(27) Jill is a member of {y | Rihanna knows y} *if and only if* Rihanna knows Jill.
(28) $a \in \{x \mid$ Jack is taller than $x\}$ *if and only if* Jack is taller than a
(29) $5 \in \{z \mid 1+z = 9\}$ *if and only if* $1+5 = 9$
(30) $b \in \{x \mid$ Karen likes x and x is short$\}$ *if and only if* Karen likes b and b is short.

Notice that the statements in (26)-(30) all use the expression 'if and only if'. That expression is common in definitions and often is abbreviated as 'iff'. 'if and only if' is stronger than 'if' by itself. Here's an example to show that. Suppose Randolph gets angry easily. One of the things that ticks him off is when people leave the garage door open. On our way home, I might say: '*If the garage door is open, Randolph will be pissed.*' If we come home to find Randolph in a rage and the garage door closed, we might try to figure out what has pissed him off this time. We would not conclude necessarily that the above statement was false. On the other hand, you would conclude that I had been mistaken if I had said: *'If the garage door is open and only if the garage door is open will Randolph be pissed off.*'

EXERCISE O
i. Fill in the blank below in a way that doesn't use any set theory notation. That is, only use colloquial English:

Carol $\in$ {y | y is hungry or y is tired} *if and only if*__________________________________

ii. Use predicate notation to name the set of numbers that are greater than 2 and less than 69.

iii. Which of the following statements is true and which is false?

$c \in \{a,b,c\}$ $d \notin \{a,b,c\}$ Boston $\in$ {z | z is a city and z is located in China}

NEW TERMINOLOGY
- member
- element
- list notation
- singleton set
- predicate notation
- definition by condition

1.6 *Lexicons*

Throughout much of the course, we will be studying how meanings of expressions of various sizes are combined to form meanings of larger expressions. But to get the ball rolling, our grammar will have to include word meanings and we'll need a way to make reference to the meaning of a particular word. To make this precise, we'll introduce a formal device that works like a dictionary. We'll call that device a **lexicon** and then we can introduce mathematical notation for "finding a meaning in the dictionary" and for "adding words to a dictionary".

Let's begin by reflecting a bit on how dictionaries commonly work. Below are a few entries from an online dictionary of English:

> **red** adjective \ˈred\
> : having the color of blood
>
> **read** verb \ˈrēd\
> : to look at and understand the meaning of letters, words, symbols, etc
>
> **reed** noun \ˈrēd\
> : a tall, thin grass that grows in wet areas

Each word, identified by its spelling, syntactic category and pronunciation, is paired with a description of its meaning. In the simplest case then, a dictionary is a set of entries, each entry being a pair of an expression and a meaning description.

Below are some entries from a Q'eqchi'-English dictionary[2]:

ch'och'	land	choxa	sky	pek	stone or rock
ch'ina ha'	spring	ha'	water	po	moon
ch'och'	soil	k'iche'	forest	saq'e	sun
chahim	star	nima'	river or stream	sulul	mud
che'	wood	ochoch pek	cave	tzuul	mountain or hill
chi re ha'	shore	palaw	sea		

Fig. 6: Q'eqchi'-English dictionary

Once again, the dictionary is a set of pairs, each pair consisting of a word that is defined and its definition. In this case, the defined words are Q'eqchi' and the definitions are expressions of English. To look a word up in a dictionary, is to find the item that is paired with that word in an

[2] http://wold.clld.org/vocabulary/34

entry in the dictionary. In the list above, 'sulul' is paired with 'mud', so according to that dictionary 'sulul' means mud.

To formalize these ideas, we introduce the mathematical notion of an **ordered pair**. An ordered pair <*a,b*> consists of two elements, *a* is the first element of the pair and *b* is the second. The pair <*a,b*> is distinct from the pair <*b,a*>. This contrasts with sets where order plays no role; $\{a,b\} = \{b,a\}$.

Let QE be the set of ordered pairs whose first element is a Q'eqchi' word from the chart above and whose second element is the English expression in the adjacent column. The following statements are true:

<*sulul*, mud> ∈ QE

<*palaw*, sea> ∈ QE

<*sulul*, sea> ∉ QE

<mud, *sulul*> ∉ QE

To look a word up in the dictionary is to locate that word and then to check what it is paired with. The following statement introduces the notation 'M(...)' corresponding to that activity:

M(...) notation

If M is a set of pairs and α is a first element of exactly one of the pairs in M, then **M(α)** is the object that is paired with α in M.

Here are some true statements:

QE(*sulul*) = mud.

QE(*palaw*) = sea

Above, we looked at some entries from an online dictionary of English. If we let Dict name the set of pairs of words and definitions in that dictionary, then the following is true:

Dict(*reed*) = a tall, thin grass that grows in wet areas

It follows from the 'M(…)' notation, that if M(x) = y, then <x,y> ∈ M.

EXERCISE P

1. Using Fig. 6 above, fill in the blanks below so that the resulting statement is true:

   (i) QE(*ha'*) = __________.

   (ii) QE(*choxa*) = __________.

   (iii) QE(__________) = moon.

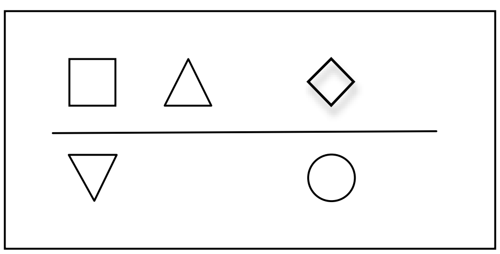
2. If α and β are words that are defined in the online dictionary mentioned above, under what circumstances would the following be true: Dict(α) = Dict(β).

EXERCISE Q

1. Let $G = \{\langle a, \square\rangle, \langle b, \lozenge\rangle, \langle c, \mathrm{O}\rangle, \langle d, \nabla\rangle, \langle e, \Delta\rangle\}$

Assume the facts depicted below in the situation labeled s_6. For each of the statements below, create a true statement about s_6 by inserting in the blank an expression of the form 'G(α)' where α is *a ,b, c, d* or *e.*

i. G(*a*) is above ________ ii. G(*a*) is right next to ______ iii. G(*b*) is above _____



s_6

2. Let L be a set of pairs defined as follows:

$$L = \{\langle a, 1\rangle, \langle b, 2\rangle, \langle P, \{1,2,3\}\rangle, \langle B, \{2,3,4\}\rangle\}.$$

Fill in the blanks below so that the resulting statement is true:

i. L(*a*) = ________ ii. L(*b*) = _________ iii. L(*B*) = _____________

Which of the following statements are true:

i. $L(a) \in L(P)$ ii. $L(a) \in L(B)$ iii. $L(b) \in L(P)$ iv. $L(b) \in L(B)$

So far we considered two kinds of dictionaries. In the first, words are paired with descriptions of their meanings. In the second, words in one language are paired with words from another language. Given the goals of the course, our focus will be on a lexicon in which an expression of the language is paired with an extension. For example, the name '*Barack Obama*' would be paired with the man elected US President in 2008 and the noun '*horse*' would be paired with the set of all horses.

During a person's life, especially the earlier part, they construct a mental lexicon which grows with time. New entries are added to the mental lexicon to create a new, expanded lexicon. To formalize that idea, we'll introduce notation, 'M+<….,…>', for adding a new pair to a set of pairs:

M+<….,…> notation

If M is a set of pairs that does not include <*a,b*>, then **M+<*a,b*>** is a set which consists of the pairs in M plus the pair <*a,b*>.

So now the following statement is true:

{<1,2>, <3,4>, <5,6>}+<7,8> = {<1,2>, <3,4>, <5,6>, <7,8>}

ak'ach is a Q'eqchi' word that is not defined in the Q'eqchi'-English dictionary entries listed above. This means that the expression 'QE(*ak'ach*)' is undefined. It turns out that *ak'ach* means turkey. We can add that word to our dictionary, to create a new, expanded dictionary. We refer to the expanded dictionary as: QE+<*ak'ach*, turkey>. And the following are true statements:

QE+<*ak'ach*, turkey>(*ak'ach*) = turkey.

QE+<*ak'ach*, turkey>(*sulul*) = mud.

The second statement says that the definition of *sulul* in the expanded dictionary is mud, just as it was before the expansion. This process of expansion can be repeated. If we discover that the word *mes* means cat, we can add this to get an even larger dictionary and the following are true:

QE+<*ak'ach*, turkey>+<*mes,* cat>(*mes*) = cat.

QE+<*ak'ach*, turkey>+<*mes,* cat>(*ak'ach*) = turkey.

QE+<*ak'ach*, turkey>+<*mes,* cat>(*sulul*) = mud.

These statements say what the meanings are of the words *mes, ak'ach* and *sulul* in the expanded dictionary.

EXERCISE R

1. Suppose L is a set of pairs whose first elements are: *a, b, c, P.* And the pairings are as follows: *a* is paired with Abraham Lincoln (the 16th president of the U.S.), *b* is paired with the novelist Charlotte Brontë, *c* is paired with Marie Curie (the co-discoverer of radium), and *P* is paired with the set of all U.S. presidents. The pairing with *P* can also be described this way:

<*P,* {x | x is a U.S. president}> ∈ L

Which of the following are true statements:

i. L(*a*) ∈ L(*P*) ii. L(*b*) ∈ L(*P*) iii. L(*c*) ∈ L(*P*)

Now we add the pair whose first element is *d* and whose second element is Dwight David Eisenhower (the 34th president of the U.S.). Which of the following are true statements:

iv. L+<*d,*Eisenhower>(*a*) ∈ L(*P*) v. L+<*d,*Eisenhower>(*c*) ∈ L(*P*)

vi. L+<*d,*Eisenhower>(*b*) ∈ L(*P*) vii. L+<*d,*Eisenhower>(*d*) ∈ L(*P*)

2. Let T = {<1, $>, <2, #>}.

a. Use list notation to name the set T+<3,@>.

b. Use list notation to name the set T+<3,@>+<4,%>.

c. Put a single symbol in each blank below to make the statement true. If the expression to the left of the equal sign is undefined, then just write 'undefined' in the blank.

i. T(3) = ________ ii. T+<3,@>(3) = ________ iii. T+<3,@>+<4,%>(3) = ________

Here's a question that is addressed in research on bilingualism: Does a bilingual have two mental dictionaries to recognize the words in each language, or a single combined dictionary? We can extend our plus notation to allow us a simple specification of what a combined dictionary is.

If M and N are sets of pairs having no pairs in common, then **M+N** is a set that consists of all the pairs in M along with all the pairs in N.

For example, if:

$L = \{<a, 1>, <b, 2>, <P, \{1,2,3\}>, <B, \{2,3,4\}>\}.$

$C = \{<c, 9>, <d, 0>\}$

then:

$L{+}C = \{<a, 1>, <b, 2>, <P, \{1,2,3\}>, <B, \{2,3,4\}>, <c, 9>, <d, 0>\}$

So now, if Eng is the mental lexicon of a monolingual English speaker and Qeq is the mental lexicon of a monolingual Q'eqchi' speaker, then Eng+Qeq could be the combined lexicon possessed by a bilingual English-Q'eqchi' speaker.

EXERCISE S

1. Which one of the following expressions is **undefined**:

   i. Eng+Qeq(*sulul*) ii. Qeq(*sulul*) iii. Eng(*sulul*).

2. Suppose L is a set of pairs whose first elements are: *a, b, c, P*. And the pairings are as follows: *a* is paired with Abraham Lincoln (the 16th president of the U.S.), *b* is paired with the novelist Charlotte Brontë, *c* is paired with Marie Curie (the co-discoverer of radium), and *P* is paired with the set of all U.S. presidents. And suppose we have another set of pairs C whose first elements are 1, 2, 3, *S*, where 1 is paired with the physicist Albert Einstein, 2 is paired with L(*b*), 3 is paired with L(*c*) and *S* is paired with the set of all scientists. Which of the following are true?

   i. $L{+}C(1) \in L{+}C(P)$ ii. $L{+}C(c) \in L{+}C(S)$
   iii. $L{+}C(3) \in L{+}C(S)$ iv. $L{+}C(b) = L{+}C(2)$

KEY IDEAS

- In some dictionaries, a word is paired with a description of its meaning(s).
- In some dictionaries, a word is paired with a word from another language.
- In this class, we'll focus on a lexicon in which an expression of the language is paired with an extension.

DEFINITIONS

M(...) notation
If M is a set of pairs and α is a first element of exactly one of the pairs in M, then **M(α)** is the object that is paired with α in M.

M+<….,…> notation
If M is a set of pairs that does not include <*a,b*>, then **M+<*a,b*>** is a set which consists of the pairs in M plus the pair <*a,b*>.

If M and N are sets of pairs having no pairs in common, then **M+N** is a set that consists of all the pairs in M along with all the pairs in N

NEW TERMINOLOGY

- lexicon
- ordered pair
- undefined