In his *Treatise of Human Nature* Hume includes a section concerning the nature of space and time. This is intended as part of his larger point concerning the limits of rationality, but for our purposes it is interesting because he discusses the *infinite divisibility* of space and time. He presents one major argument and one secondary argument. I will explain both of these, and provide commentary on Hume's writing from an Aristotelean perspective. Additionally, James Franklin will be consulted towards the end, in the hopes of gleaning something more positive from Hume than arguments which, we shall see, are deeply flawed.

I am going to take Hume somewhat out of order. The sequence in which I choose to present his arguments is designed to flow better for the purposes of this paper. Frankly I think it also makes his reasoning more clear than his own presentation. First let us examine Hume's epistemology. He says that "[sense] impressions always take the precedency of [ideas], and that every idea, with which the imagination is furnished, first makes its appearance in a correspondent impression." (I.ii.iii) So for Hume, all our ideas originate from our senses. We perceive a piece of paper, and so we have the idea of a piece of paper. It is important to note that this strongly suggests that an idea is equal to a mental image (the image of a piece of paper, for example), and this conflation will play a role later on.

Hume will later argue that we cannot conceive of an infinitely divisible extension – that is, that our *idea* of extension is not infinitely divisible. Where does our idea of extension itself come from? As Hume says, "my senses convey to me only the impressions of coloured points" (I.ii.iii) The concept of extension is something separate from, and a good deal more abstract than, the images we receive from our eyes. According to Hume we obtain the idea of extension *from* perception through a process of abstraction. We see a vast variety of different colors and forms in our visual field, and we are then able to distill distance, or extension, as the factor which does not change. If we observe two books on opposite sides of a bookshelf and two birds flying overhead, we can notice that while the pairs have a great deal of differences, they share in common the quality of each being separated by distance. Rather than have separate ideas of extension for every possible situation in which two or more objects are separated, we generalize and abstract the idea of distance so that it can encompass a massive variety of situations and experiences. Hume puts it this way: "All abstract ideas are really nothing but particular ones… annexed to general terms, they are able to represent a vast variety" (I.ii.iii)

The same logic holds for temporal extension, which we perceive by means of the *succession* of spatial impressions. Hume maintains that someone asleep, or even deeply engrossed in a single line of thought, fails to perceive the passing of time. The sleeper or the daydreamer will have little or no sense of successive impressions. Without successive impressions, there is no way to know of a *before* or an *after*, a *present* or a *duration*. Hume says that "From these phenomena… we may conclude, that time cannot make its appearance to the mind, either alone, or attended with a steady unchangeable object" (I.ii.iii) It is interesting to note that in the cases of both temporal and special extension Hume believes that extension does not stand alone. It cannot be perceived separately from objects or matter.

This, then, is how we receive the idea of extension either spatial or temporal. Let us now consider the properties of this idea. Because the idea comes from the impression, we are also concerned with the properties of the impression. Both, says Hume, are indivisible. He asks us to imagine an object getting farther and farther away. He says that "the moment before it vanished the image or impression was perfectly indivisible… it is not for want of rays of light striking on our eyes… but because [distant bodies] are removed beyond that distance, at which their impressions were reduced to a minimum, and were incapable of any further diminution." (I.ii.i)

Again, this is intended to show that sense impressions are not infinitely divisible. Apart from lacking any actual argument and being a bare assertion, Hume's passage here also seems to have strange consequences. Does this mean, for example, that even an object thousands of light years away does not lack an impression on our senses, but rather that its impression is "reduced to a minimum" and "incapable of any further diminution"? This seems absurd. Surely Hume does not means to suggest that an object moving away from us *never* ceases to be represented as an impression in our sense experience.

At least in his discussion of the finite divisibility of ideas, Hume actually presents an argument. If we were to grant him the finite divisibility of sense impressions, it would provide further support for this position. That is, if ideas are derived from sense impressions, and if sense impressions are not infinitely divisible, then it would make sense that ideas are not infinitely divisible either. This interesting point aside, Hume's argument for the finite divisibility of ideas runs roughly as follows. The first part of the argument is that "the capacity of the mind is limited, and can never attain a full and adequate conception of infinity." (I.ii.i) Remember that earlier we saw that Hume conflates conception with mental imagery. Operating under this assumption it is easy to accept this first point. No one is capable of imagining an infinitely long line, neither all at once nor as successive parts. In the same way it exceeds human capabilities to imagine the infinite division of a line. It is interesting to ponder whether or not this is the only form of conception, and whether alternate types of ideas, perhaps ones that rely less upon imagery, might take out any wind this argument possesses. However, such considerations exceed, if not human capacity, then certainly the scope of this paper.

The second point of Hume's argument is that "whatever is capable of being divided in infinitum must consist of an infinite number of parts" (I.ii.i) Here Hume says that any infinitely divisible thing, for example an extension, must have an infinite number of parts. This is a far more fruitful ground for objections, or at least objections relating to the topic of infinity. Here resides a much larger debate concerning the nature of infinitesimals. Some believe, as Hume does, that to speak of infinite division necessitates the existence of an infinite number of parts. If we can divide a line infinitely, then it must have an infinite number (of infinitely small?) pieces. Others, such as Aristotle, claim that this is false. Aristotle created for us an extremely useful distinction between two types of infinity, which he calls "potential" and "actual." (Physics III.6) An actual infinity is an actually existing infinite quantity. It would be something like an infinitely long line, *or* a line with infinitely many parts (even one that was only finite in length). Aristotle says that actual infinities do not exist, and I think he would agree with Hume that we could never hope to conceive of one.

However, Aristotle would *not* agree that any infinitely divisible entity must have an infinite number of parts. Rather, he would say that such entities are *potentially* infinite. Potential infinities are characterized by processes, such as division or addition. An infinitely divisible entity does not *have* infinite parts; it is only *able* to have infinite parts. We can divide the entity as much as we like, and we will *never* reach a point at which it cannot be further divided. Yet at any point in the process of division, it has only a finite number of parts.

Aristotle's distinction will have consequences for Hume's conclusion. If our minds cannot conceive of an infinity of parts, and if an infinitely divisible extension must have an infinite number of parts, then, says Hume, it follows that we cannot conceive of an infinitely divisible extension. Our idea of extension, derived from our impression of extension (which Hume claimed was only finitely divisible), *cannot* be infinitely divisible. But, while we might accept the first premise – that our minds cannot conceive of an infinity – we are certainly not bound to accept the second. Rather we might be Aristoteleans and say that an infinitely divisible extension *does not* have an infinite number of parts, and that therefore there is nothing barring us from conceiving of such an extension.

At this point Hume's argument seems to be on precarious ground. However, one could always deny Aristotle's distinction, and argue that potential infinity is just another way of saying not infinite at all; that we might be able to conceive of a line that *can* be infinitely divided, but that since we can never *actually* infinitely divide it, it doesn't count. I think that this rather misses the point of Aristotle's distinction. However, if we exercise the principle of charity, we may continue Hume's argument and see what he has to say.

The crux of Hume's argument, and to my mind the most interesting part, is this: "Wherever ideas are adequate representations of objects, the relations, contradiction and agreements of the ideas are all applicable to the objects" (I.ɪɪ.ii) The principle described is that our ideas and the objects whose impressions they derive from share the same properties (apart from one being real and one being a mental construction, we must suppose). This principle is what allows Hume to make the jump from the finite divisibility of our *idea* of extension to the finite divisibility of *extension itself*. He says that "whatever appears impossible and contradictory upon the comparison of these ideas, must be really impossible and contradictory" (I.ɪɪ.ii) If we can't *conceive* of an infinitely divisible extension, then there cannot *be* an infinitely divisible extension.

I think that the principle upon which this argument is based is utterly fallacious. It is strange to me, too, that Hume – ever the empiricist – suddenly gives our minds ontological precedence over the real world. We saw earlier that he says all our ideas come from sense impressions; does this not imply the precedence of the real world? How can it be, then, that an argument can justifiably run from our ideas *back* to the world, and demonstrate the nature of something *real* based upon the nature of something *imaginary*? I think the fact that Hume does use such an argument shows that he believes our minds capable of reproducing perfectly the properties of the world. He is caught up in the theory that our minds can faithfully reproduce reality, despite his own discussion of examples in which this is not true (remember the example of the receding object; here our senses did not allow our minds to reproduce the outer world correctly).

Perhaps to further clarify the issue we could make a distinction. There are things which we cannot conceive of because they are *a priori* contradictory, such as a married bachelor. There are other things we cannot conceive of because they exceed the capacities of our imaginations (or our senses), such as nothingness, or the color of ultraviolet light. I would argue that infinities fall into the latter category. I think that Hume would be forced to agree, based upon his own reasoning – remember that he says that we cannot imagine an infinity because "the capacity of the mind is limited" (I.II.i) Ideas which are inconceivable due to *a priori* contradiction are, I would agree, impossible. There can be no such thing as a married bachelor because marriage violates the definition of bachelorhood. There can be no such thing as a bird which is simultaneously white all over and green all over, because being two different colors at the same time violates the properties of physical objects. Yet, I think the same may not be said of ideas which we cannot conceive due to limitations of the mind. For example, we know that voids exist, yet we cannot actually imagine emptiness – pure black or pure white are the closest the human imagination can come. Infinite divisibility, then, even if not conceivable, might still exist.

There is one further argument, separate from Hume's primary one set out above, which I think is of interest. Hume attributes this argument to "Monsieur Malezieu," and it begins with the premise that "existence in itself belongs only to unity, and is never applicable to number" (I.ɪɪ.ii) Essentially Malezieu is placing the existence of the whole ontologically prior to the existence of multiples or fractions. This is not so strange a doctrine as it may at first appear. As Hume says, "Twenty men may be said to exist; but it is only because one, two, three, four, &c. are existent" (I.ɪɪ.ii) Platonically speaking, it makes far more sense to speak of natural numbers (composed only of wholes) existing than decimal numbers. It is not hard to imagine that the concepts of one, two, or three have independent existence, while that same existence applies to a number such as 53.45 is far more ambiguous. Of course, the argument calls for something even stronger than this notion. Even multiple whole numbers are said not to exist; only one, a single whole, a "unity" has "existence in itself."

An infinitely divisible extension with infinite parts would not have any unities, any wholes, because each part would need to be infinitely small. Thus, say Malezieu and Hume, we cannot have such an extension. Again, Aristotle's potential infinity comes to the rescue. As Hume himself says, "That term of unity is merely a fictitious denomination, which the mind may apply to any quantity of objects it collects together" (I.ɪɪ.ii) During our process of division, we can apply the concept of a whole (or a unity) to the parts we have currently divided an extension into. Because the number of parts is only *potentially* infinite, we will never be forced to give up this application; the parts will always have some finite, non-zero length.

We've spend a great deal of time here discussing all the ways Hume went wrong. But is there anything we can learn from his discussions of the infinite divisibility of extension? James Franklin thinks that there is. First, he says, Hume was right to accuse the proponents of infinite divisibility of being unable to *prove* this property of extension. It is entirely possible for extension to be only finitely divisible, and there is not proof against the bare possibility. I have some reservations concerning Franklin and Hume's assertion here. Aristotle gives a refutation of the finite divisibility of extension that I think stands strong against all onslaught. However, since I have discussed this previously and in depth, I will refrain from doing so in this paper.

According to Franklin, there was much conflict concerning Hume's assertions about the divisibility of ideas. Remember that Hume asserts that ideas *are* divisible, but *only* finitely. Franklin says that "Later philosophical commentators have professed themselves unable to understand what a part of an idea is." (Franklin, Where Hume was right) I myself am unable to see where the difficulty lies. If we take idea to mean mental image, as Hum does, parts could be literally pieces of the image, divided like a jigsaw puzzle, or into components like color, line, etc., or otherwise. If an idea is something more abstract, such as a concept like "green," we can easily enough divide this idea into components such as "quality," "color," and "green" (as opposed to the other "colors"). But, Franklin presents evidence that others did not see things in this light, and if this is the case it is to Hume's credit that he put forward this view.

Finally, Franklin comments on Hume's "identifications of conceiving with imagining, and of imagining with forming a mental image" (Franklin, Where Hume was right) He uses the example of geometry to show that Hume was on the right track here – since in geometry "the modern evidence is that a spatial visualization capacity (read "imagination") is necessary in practice." (Franklin, Where Hume was right) This is a tad mystifying, since showing that Hume's conflation of conceiving and mental imagery is applicable to one branch of mathematics barely qualifies as a step towards showing that it is a generally valid principle. I would (and did) give Hume no credit here whatsoever, and simply accept that when he says "idea" he is talking about a mental image, understand his argument in those terms, and leave it at that.

While Hume's conclusion is, in my opinion, wrong, his arguments are subtle enough to prove interesting, and provide an excellent way to refine the pro-infinite-divisibility position. For example, I personally developed a distinction between two types of inconceivability which I had previously not considered. While I am not foolish enough to think that this is an original distinction, it is new to me and is demonstrative of the ways in which even the weakest of opposing arguments can strengthen our positions in the work of refutation. It is also important to remember that this entire discussion of his was merely a small part of a much larger argument concerning not infinite divisibility but the limits of reason. For Hume, tearing holes in the particulars of his arguments might not be such a big deal, if it leaves his larger point intact.

At many points throughout his argument Hume fails to take into account the distinction between actual and potential infinity. Even if one were to avoid this difficulty, his argument is fundamentally flawed – its essential principle, which ties together so closely reality and ideas, is wide open to attack. Yet this does not prevent us from learning from Hume's argument and using it to further our thinking. Rather than any new or exciting idea concerning infinity, the takeaway here has to do with the nature of argumentation and how we can advance our understanding by never dismissing a theory out of hand.

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