A Course in Semantics[1]

[1] Terence Parsons has a semantics textbook available online at ucla.edu. With his permission, Roger Schwarzschild and Daniel Altshuler have been revising and modifying it. This is the Spring 2016 version.

Chapter 2: The Semantics of Symbolic Logic

In this chapter, we will introduce the language of symbolic logic. In so doing, we'll learn a bit about what semantic rules look like. We'll be using the symbolic logic in subsequent chapters as part of our grammar of English.

2.1 *Atomic Sentences and their Parts*

The simplest sentences of symbolic logic are called *atomic* sentences. Like the atoms of the physical universe, they themselves have smaller parts. These parts are of two kinds.

One kind of symbol is called an **individual constant**. These resemble proper names of English. A lexicon for this language pairs each individual constant with an individual. Unlike proper names of English, individual constants of symbolic logic are usually written with single, lower-case letters. Letters from near the end of the alphabet are reserved for another purpose, so the available individual constants are typically: 'a','b','c',...,'s','t'. (If more individual constants than these are needed, these are usually expanded by adding subscripts or primes: 'a_1','b_1','c_1',...,'s_1','t_1' or 'a′','a″','a‴', etc).

Another kind of symbol which is used to make an atomic sentence is called a **predicate**. An atomic sentence is formed by combining one or more individual constants with a predicate. Predicates are divided into classes, depending on the number of individual constants that they combine with. A **one-place predicate** combines with a single individual constant to make an atomic sentence; a **two-place predicate** combines with two individual constants to make an atomic sentence. We will not discuss predicates with more than two places. A lexicon for this language pairs each one-place predicate with a set, and each two-place predicate with a relation. Whether an atomic sentence is true or false is determined by these sets and relations, along with the things that the individual constants in the sentence stand for. (This will be explained in a moment.)

Predicates are usually symbolized by capital letters. In an atomic sentence containing a one-place predicate, the predicate precedes the individual constant. Suppose we have a lexicon *L* according to which 'C' stands for the set of clever individuals, and 'a' stands for Agatha. Then 'Ca' is a sentence of symbolic logic that corresponds to the English sentence '*Agatha is clever*'. Likewise, if according to *L*, 'H' stands for the set of all happy individuals, and 'f' stands for Frank, then 'Hf' is a sentence that corresponds to '*Frank is happy*'.

In an atomic sentence containing a two-place predicate, the predicate is followed by parentheses with two individual constants inside separated by a comma. If we have a lexicon according to which 'T' stands for the taller-than relation, 'm' stands for Mary, and 's' for Sue, then both 'T(m,s)' and 'T(s,m)' are sentences, the first corresponding to '*Mary is taller than Sue*' and the second to '*Sue is taller than Mary*'.

Before continuing, we should clarify a typographical aspect of the presentation so far. A lot of the text above appears cluttered with single-quotes (‘ ’). This has to do with the **use-mention distinction**. To understand what that is about, consider the two English sentences below:

(1) Boston is the capital of the Commonwealth of Massachusetts.
(2) ‘Boston’ starts with a consonant.

The sentence in (1) is about a city. The sentence in (2) is about a word. Correspondingly, the subject of (1) stands for the city and the subject of (2) stands for a word. The word ‘Boston’ is *used* in (1). The word ‘Boston’ is *mentioned* in (2) and this is indicated with single-quotes. Throughout this text, single-quotes indicate that a symbol, a word or a phrase is mentioned. Generally, when you see the mention-quotes, you can read them as “the phrase/word/symbol___”. The sentence in (2) is synonymous with:

(3) The word ‘Boston’ starts with a consonant

By contrast, the sentence in (1) is not synonymous with:

(4) The word ‘Boston’ is the capital of the Commonwealth of Massachusetts.

Often, when a whole sentence is mentioned, that is indicated with italics. (Reread the first 3 paragraphs of this section, paying attention to the use of mention-quotes).

EXERCISE A

One of the words in the sentence below is mentioned and not used. Rewrite the sentence using single-quotes to indicate that a word is mentioned.

Even even has an even number of letters.

✪ IMPORTANT ✪

There are two correct answers to this exercise, related to two different ways to pronounce the sentence. You only need to provide one answer.

✪ OPTIONAL ✪

Describe the difference in the pronunciation for the two correct answers. Say which pronunciation correlates with which answer. Suggest an explanation for the correlation.

Returning to the main theme, we direct our attention now to the interpretation of the symbolic language introduced above. If we have a lexicon *L* according to which ‘C’ stands for the set of clever individuals, and ‘a’ stands for Agatha, then it should ultimately follow from our rules that:

(5) ‘Ca’ is true with respect to L *iff* Agatha is clever.

This tells us what the truth conditions are for the sentence ‘Ca’. However, notice that (5) doesn’t tell us what the truth conditions are in terms of the meanings assigned to ‘C’ and to ‘a’. That’s not good if we want to arrive at a general rule that will apply to any atomic sentence formed from a one-place predicate and an individual constant. We’d like a statement like (5) but with the statement to the right of *iff* populated with L(C) and L(a), the meanings of ‘C’ and ‘a’ with respect to the lexicon L. As noted above, L(C) is the set of clever people, so we’d like to replace the statement ‘Agatha is clever’ with a statement about sets. This fact from set theory will be helpful:

(6) Agatha is clever *iff* Agatha is a member of the set of clever individuals.

Given the equivalence in (6), we can replace ‘Agatha is clever’ in (5) with the equivalent set theoretic statement to get:

(7) ‘Ca’ is true with respect to L *iff* Agatha is a member of the set of clever individuals.

Given what was said earlier about the lexicon L, we have the following equations:

(8) a. L(a) = Agatha.
b. L(C) = the set of clever individuals.

Given the equivalences in (8), we can replace ‘Agatha’ and ‘the set of clever individuals’ in (7) with the equivalent expressions that use L to get:

(9) ‘Ca’ is true with respect to L *iff* L(a) is a member of L(C).

And using our set-theory symbol ‘$\in$’ we have:

(10) ‘Ca’ is true with respect to L *iff* $L(\mathrm{a}) \in L(\mathrm{C})$.

As desired, we’ve arrived at a statement of the truth conditions of ‘Ca’ that makes reference to the meanings of ‘C’ and of ‘a’.

In moving from (5) through to (10), there were several places where we replaced one expression with an equivalent one in a larger context, leaving everything else the same. This type of inference will be important going forward so we’ll introduce an explicit rule for it. The rule makes use of the Greek letters ‘ϕ’ (phi) and ‘ψ’ (psi):

REPLACE [Rule of Inference]

If two statements ϕ and ψ are logically equivalent (ϕ *iff* ψ), then from a statement including ϕ, we can infer the statement that results from replacing ϕ with ψ.

If two expressions E_1 and E_2 name the same entity, then from a statement that includes E_1, we can infer the statement that results from replacing E_1 with E_2.

The first part of this rule of inference was used, for example, in going from (5) to (7) above. The second part was used twice in going from (7) to (9). As a result of our inferencing, we arrived at (10), repeated below:

(10) 'Ca' is true with respect to L *iff* $L(\mathrm{a}) \in L(\mathrm{C})$.

This statement achieves what we wanted: a statement of the truth conditions of 'Ca' in terms of the meanings of its parts.

Having worked out how a particular sentence of this kind is interpreted, we'd like now to state a general rule that will apply no matter what the predicate or individual constant is. In order to state such a rule, we'll use the Greek letter 'π' (pi) to stand for any predicate, the Greek letter 'α' (alpha) to stand for any individual constant, and we'll use 'M' to stand for any lexicon of the language. Here then is the first rule of interpretation for our symbolic language:

ATOMIC-1 [Rule of Interpretation]

'$\pi\alpha$' is true with respect to M *iff* $M(\alpha) \in M(\pi)$

Above, we said that if according to L, 'H' stands for the set of all happy individuals, and 'f' stands for Frank, then 'Hf' is a sentence of the symbolic language that corresponds to the English sentence '*Frank is happy*'. Let's see how our new rule, ATOMIC-1, captures that. First we will **instantiate** the rule, by taking the predicate π to be 'H', the individual constant α to be 'f' and the lexicon will be L. That gives us:

(11) 'Hf' is true with respect to L *iff* $L(\mathrm{f}) \in L(\mathrm{H})$

Since $L(\mathrm{f})$ = Frank and $L(\mathrm{H})$ = the set of all happy individuals, by our inference rule REPLACE we get from (11) to:

(12) 'Hf' is true with respect to L *iff* Frank is a member of the set of all happy individuals.

From set theory we have:

(13) Frank is a member of the set of all happy individuals *iff* Frank is happy.

And so we can use REPLACE to get from (12) and (13) to:

(14) 'Hf' is true with respect to *L iff* Frank is happy.

This is the desired result. Assuming the lexicon *L,* 'Hf' corresponds to the sentence '*Frank is happy'*; they have the same truth conditions. Our rule works as intended.

ATOMIC-1 will apply to any sentence formed from any one-place predicate and any individual constant because it is stated in terms of **metavariables**, π and α. Metavariables are variables that are used in linguistic rules; they are variables that stand for variables. 'VP' and 'NP' are metavariables over expressions of the language. Phonologists use 'V' and 'C' as metavariables over vowels and consonants. By replacing the metavariables with actual expressions you get an instance of a rule. This is called **instantiation**. Here's a precise definition:

INSTANTIATION [Rule of Inference]

> From a rule stated in terms of metavariables, infer the result of substituting the metavariables with expressions of the right kind – taking care to substitute all occurrences of a given variable with the same expression.

INSTANTIATION is not a rule of our grammar. It is a general logical principle that allows us to deduce facts about our grammar. Note that we will continue to introduce rules of inference inside dashed boxes (as opposed to solid boxes, which are reserved for rules of interpretation, cf. ATOMIC-1).

SOMETHING TO THINK ABOUT

Notice that the rule REPLACE, discussed above, is also a rule of inference inside dashed boxes. Why do you think this is so?

The reasoning behind INSTANTIATION is very similar to the reasoning we employ in applying laws. Here's an illustration. Suppose there is a law that says:

(15) If a person owns a dog, the person has to get a license for the dog in the state where they reside.

That's a general rule. Suppose now that Toto and Barney are both dogs. The statements below are instantiations of the general rule in (15):

(16) If Dorothy owns Toto, then Dorothy has to get a license for Toto in the state where they reside.

(17) If Laura owns Barney, then Laura has to get a license for Barney in the state where they reside.

In (16), we instantiated with Dorothy/Toto and in (17) with Laura/Barney. In contrast to (16) and (17), (18) and (19) below would be incorrect instantiations:

(18) If Dorothy owns Toto, then Dorothy has to get a license for Barney in the state where they reside.

(19) If Dorothy owns Toto, then Toto has to get a license for Dorothy in the state where they reside.

The error made in (18) is that in one case we used 'Toto' for the dog and then later 'Barney' was used. The law is understood to apply to the same dog throughout. It's this intuition that is captured in INSTANT when it says: "substitute <u>all occurrences</u> of a given variable with the same expression".

The error made in (19) is that we used Toto to instantiate a person, when Toto is a dog; we used Dorothy to instantiate a dog, when she is a person. The law distinguishes people and dogs and the instantiations must respect that. It's this intuition that is captured in INSTANT when it says: "substituting the metavariables with expressions of the <u>right kind</u>".

Finally, suppose it's the case that Laura and her dog Barney reside in Texas. In that case, the state where they reside is Texas. So that means we can perform a replacement inference on (17) to get:

(20) If Laura owns Barney, then Laura has to get a license for Barney in Texas.

As just illustrated, these inferencing rules are not rules of our grammar. They are general logical principles that allow us to deduce facts about our grammar. Another common use of the rule REPLACE comes in editing. Suppose Jill plans to write a news report including the statement:

(21) If the weather is good, the White Sox will be beaten by the Tigers.

Jack doesn't like the use of the passive ('be beaten by') so he recommends that she change her statement to:

(22) If the weather is good, the Tigers will beat the White Sox.

Jack assumes that 'White Sox will be beaten by the Tigers' is equivalent to 'Tigers will beat the White Sox', and so he replaces one statement with the other. Notice that the remainder of the final statement 'if the weather is good,' is repeated untouched.

EXERCISE B

1. Which of the statements below are true with respect to a lexicon L_1, in which 'a' stands for Alan, 'b' stands for Betty, 'c' stands for Cathy, 'W' stands for the set of wealthy individuals, and 'D' for the set of doctors? Assume that Alan is a poor doctor, Betty is a wealthy doctor, and Cathy is not wealthy and is not a doctor.

i. Wa	iii. Wb	v. Dc
ii. Db	iv. Da	vi. Wc

2. Starting with the rule ATOMIC-1 write out the steps for determining the truth conditions for 'Wb' with respect to the lexicon *V*, where *V*(W) is the set of wealthy individuals and *V*(b) is Betty. At the end, you should have a statement with an *iff* in it and to the right of the *iff* should be a description of the world – no mention of *V,* 'b' or 'W'. [Hint: Follow the steps used in (11)-(14) above to interpret 'Hf'.]

3. The line below is the result of an incorrect application of the inference rule INSTANTIATE to the rule ATOMIC-1. What has gone wrong?

'Db' is true with respect to L_1 *iff* M(b) $\in$ M(D)

We now move on to sentences formed with two-place predicates and for that we need a new rule of interpretation. Since two-place predicates combine with two individual constants, we need a new metavariable for individual constants, β (beta). Here is the rule:

ATOMIC-2 [Rule of Interpretation]
'$\pi(\alpha,\beta)$' is true with respect to M *iff* M(α) bears the relation M(π) to M(β)

Let's now instantiate this rule for the sentence 'T(s,m)' and the lexicon *L*:

(23) ‘T(s,m)’ is true with respect to *L* *iff* *L*(s) bears the relation *L*(T) to *L*(m).

Let’s assume that *L*(m) is Mary, *L*(s) is Sue and *L*(T) is the taller-than-relation (α bears the taller-than-relation to β *iff* α is taller than β). Then, with a few applications of REPLACE we get:

(24) ‘T(s,m)’ is true with respect to *L* *iff* Sue is taller than Mary.

This coincides with the earlier claim that given the lexicon *L*, ‘T(s,m)’ corresponds to the English sentence ‘*Sue is taller than Mary*’.

Let’s try another example. Suppose we have a lexicon *G* and according to *G*, ‘d’ stands for the diamond in Fig. 1 below, and ‘c’ stands for the circle. Suppose that according to *G*, ‘A’ stands for the above-relation: x bears the above-relation to y *iff* x is above y.

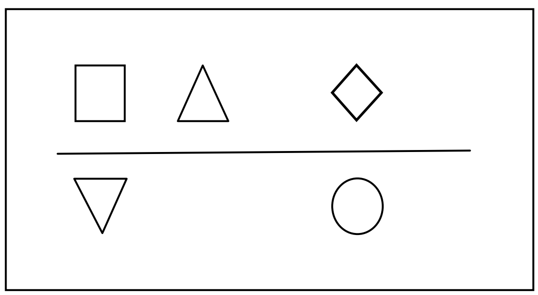


Figure 1

Using our ATOMIC-2 rule and INSTANTIATE, we have:

(25) ‘A(d,c)’ is true with respect to *G* *iff* *G*(d) bears the *G*(A) relation to *G*(c).
(26) ‘A(c,d)’ is true with respect to *G* *iff* *G*(c) bears the *G*(A) relation to *G*(d).

Given what we said above about the lexicon *G*, by REPLACE we get:

(27) ‘A(d,c)’ is true with respect to *G* *iff* the diamond is above the circle.
(28) ‘A(c,d)’ is true with respect to *G* *iff* the circle is above the diamond.

And now, given the facts in Fig. 1, the (29) follows with respect to *G*:

(29) ‘A(d,c)’ is true and ‘A(c,d)’ is false.

‘A(d,c)’ and ‘A(c,d)’ use all the same symbols but they are put together differently. As a result, they have different truth conditions as shown by the fact that we found a situation in which one is true and the other false. We have now a simple model for how syntax could play a role in determining truth conditions.

EXERCISE C

1. Which of the statements below are true with respect to a lexicon L_1 in which 'V' stands for the relation of loving, and 'T' for the relation of being taller than. Assume L_1(g) is George, L_1(m) is Mary and L_1(s) is Sam. Further assume that Sam loves himself and no one else; everyone else loves everybody (including Sam); Sam is taller than George, who is taller than Mary.

(i) V(s,m)	(ii) V(m,g)	(iii) T(s,m)
(iv) V(g,m)	(v) T(g,s)	(vi) T(m,g)

2. Instantiate ATOMIC-2 by replacing the metavariables '*M*', 'π', 'α' and 'β' with 'L_1', 'T', and 's' and 'm' respectively.

3. Starting with ATOMIC-2, write out the steps for determining the truth conditions for 'H(s,m)' with respect to the lexicon L_2, where L_2(H) is the hate relation, L_2(s) is Sam and L_2(m) is Mark. Make sure to state which rules you applied at each step. At the end, you should have a statement with an *iff* in it and to the right of the *iff* should be a description of the world – no mention of L_2, 's', 'm' or 'H'.

4. Above we showed that with respect to *G,* 'A(d,c)' is true and 'A(c,d)' is false in Fig. 1. Provide a diagram like the one in Fig. 1 depicting a situation in which 'A(d,c)' is false with respect to *G* and 'A(c,d)' is true with respect to *G*.

✪ HINT ✪

Use the derivation on the previous page as your guide for doing this exercise.

KEY IDEAS

In this section, we've introduced two rules of interpretation and two rules of inference:

ATOMIC-1 [Rule of Interpretation]

'πα' is true with respect to M *iff* M(α) ∈ M(π)

ATOMIC-2 [Rule of Interpretation]

'π(α,β)' is true with respect to M *iff* M(α) bears the relation M(π) to M(β)

REPLACE [Rule of Inference]

> If two statements ϕ and ψ are logically equivalent (ϕ *iff* ψ), then from a statement including ϕ, we can infer the statement that results from replacing ϕ with ψ.
>
> If two expressions E1 and E2 name the same entity, then from a statement that includes E1, we can infer the statement that results from replacing E1 with E2.

INSTANTIATE [Rule of Inference]

> From a rule stated in terms of metavariables, infer the result of substituting the metavariables with expressions of the right kind – taking care to substitute all occurrences of a given variable with the same expression.

- INSTANTIATE allows us to use ATOMIC-1 to arrive at statements such as:

(30) 'Bd' is true with respect to L *iff* L(d) $\in$ L(B).

- INSTANTIATE allows us to use ATOMIC-2 to arrive at statements such as:

(31) 'T(d,c)' is true with respect to L *iff* L(d) bears the relation L(T) to L(c).

- And REPLACE allows us to use facts from the lexicon to go from statements like (30) to statements like (32) and from statements like (31) to statements like (33).

(32) 'Bd' is true with respect to L *iff* David is bossy.
(33) 'T(d,c)' is true with respect to L *iff* David is taller than Corien.

The lexical facts needed for (32) and (33) are that L(d) is David, L(c) is Corien, L(B) is the set of bossy individuals and L(T) is the taller than relation.

NEW TERMINOLOGY

- individual constant
- predicate
- replace
- one-place predicate
- two-place predicate
- instantiate
- instantiation
- metavariables

2.2 *Connectives*

In the previous section, we introduced a syntax and semantics for atomic sentences such as 'Ca' and 'T(s,m)'. In this section, we'll introduce a syntax and semantics for **compound sentences**. A compound sentence has one or more smaller sentences as one of its parts. One way to form a compound sentence is to combine two sentences with the connective '&', pronounced as *and*, and to enclose them in parentheses. Here are some examples:

(34) a. (Ca & Kc)
b. (T(s,m) & Ca)
c. ((Ca & Kc) & Bd)

In order to spell out the syntax and semantics of compound sentences, we'll need metavariables over sentences. We'll use 'ϕ' and 'ψ'. So here's a general statement of the syntactic rule for '&':

(35) If ϕ and ψ are sentences, then '(ϕ & ψ)' is a sentence.

And here is the semantic rule that interprets sentences formed in this way:

AND
'(ϕ & ψ)' is true with respect to *M iff* 'ϕ' is true with respect to *M* and 'ψ' is true with respect to *M*

Let's use this rule to calculate the truth conditions of the sentence '(Ca & Hf)' assuming a lexicon *L* according to which 'C' stands for the set of clever individuals, 'a' stands for Agatha, 'H' stands for the set of all happy individuals, and 'f' stands for Frank.

First we instantiate the AND rule:

(36) '(Ca & Hf)' is true with respect to *L iff* 'Ca' is true with respect to *L* and 'Hf' is true with respect to *L*

The following statements come from our discussion earlier in section 1.1 of the truth conditions of 'Ca' and 'Hf' relative to this same lexicon *L*:

(37) 'Hf' is true with respect to *L iff* Frank is happy.
(38) 'Ca' is true with respect to *L iff* Agatha is clever.

So now we can use REPLACE twice in (36) to get:

(39) '(Ca & Hf)' is true with respect to *L iff* Agatha is clever and Frank is happy.

Assuming lexicon *L,* we deduce from this calculation that '(Ca & Hf)' has the same truth conditions as '*Agatha is clever and Frank is happy*' and therefore corresponds to this English sentence. Often (though not always – as we will see), sentences formed with '&' correspond to conjunctions with *'and'* in English.

To see another example, recall our earlier *G*, where *G*(d) = the diamond and *G*(c) = the circle and where according to *G*, 'A' stands for the above-relation: α bears the above relation to β *iff* α is above β. Relative to the lexicon *G,* the sentence '(A(d,c) & A(c,d))' corresponds to the English sentence '*The diamond is above the circle and the circle is above the diamond*'. That English sentence is false in Fig. 1, repeated below, because the circle is not above the diamond.

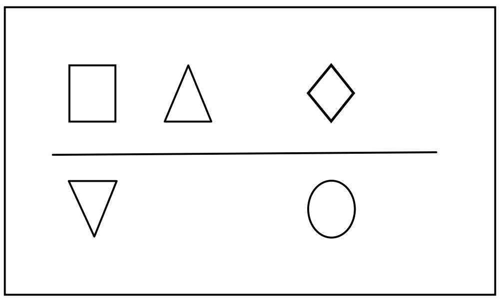


Figure 1

Likewise, the symbolic logic sentence '(A(d,c) & A(c,d))' is false relative to *G* given the facts depicted in Fig. 1 above. In general, as the rule AND states, for a conjunction to be true, <u>both</u> conjuncts have to be true.

SOMETHING TO THINK ABOUT

Could '(A(d,c) & A(c,d))' ever be true relative to *G*? Can you think of another lexicon where '(A(d,c) & A(c,d))' is true?

A second way to form a compound sentence is to put the symbol '¬' in front of a sentence. Here are some examples:

(40) a. ¬Ca
b. ¬T(s,m)
c. ¬(Ca & Kc)
d. ¬((Ca & Kc) & Bd)

The symbol '$\neg$' is pronounced '*it is not the case that*' or sometimes just '*not*'. Here's the rule for interpreting sentences formed with '$\neg$':

NOT '$\neg\phi$' is true with respect to *M* *iff* it is not the case that: 'ϕ' is true with respect to *M*

Let's use this rule to calculate the truth conditions of the sentence '$\neg$Ca' assuming a lexicon *L* according to which 'C' stands for the set of clever individuals and 'a' stands for Agatha.

First we instantiate the NOT rule:

(41) '$\neg$Ca' is true with respect to *L* *iff* it is not the case that: 'Ca' is true with respect to *L*

And now given that:

(42) 'Ca' is true with respect to *L* *iff* Agatha is clever.

By REPLACE we get:

(43) '$\neg$Ca' is true with respect to *L* *iff* it is not the case that: Agatha is clever.

According to (43), '$\neg$Ca' is true with respect to *L* if and only if it is not the case that Agatha is clever. So '$\neg$Ca' corresponds to the English sentence '*Agatha is not clever*'.

EXERCISE D

1. Write sentences of the symbolic logic that correspond to the English sentences below. Assume a lexicon L_1 in which ‘V’ stands for the relation of loving, and ‘T’ for the relation of being taller than. Assume further that <g, George> $\in L_1$, <m, Mary> $\in L_1$ and <s, Sam> $\in L_1$ (if this last notation using pairs is unfamiliar, see section 1.6 from the previous chapter).

   (i) George is not taller than Sam.
   (ii) Sam loves himself but Sam doesn’t love Mary.

2. Relative to the lexicon *G*, in the situation depicted by Fig. 1 on pg. 13 and repeated below, ‘A(c,d)’ is false and ‘A(d,c)’ is true (the diamond is above the circle). Using the rules AND and NOT, decide which of the sentences in (i)-(v) are true in the situation depicted by Fig. 1, relative to lexicon *G*.

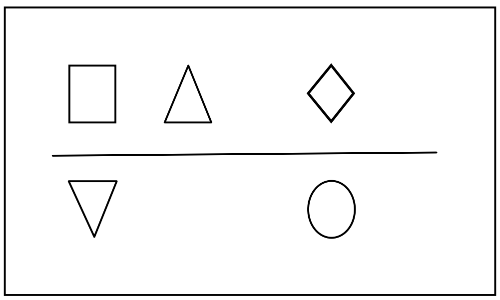


Figure 1

(i) ¬A(c,d) (ii) ¬A(d,c) (iii) (A(c,d) & A(d,c))
(iv) ¬(A(c,d) & A(d,c)) (v) (¬A(c,d) & A(d,c))

✪ IMPORTANT ✪
Make sure that you decide on the truth-values those sentences have in Fig. 1 based on the truth conditions assigned by the rules AND and NOT. They may or may not correspond to your intuitions about the corresponding English sentences. If they don’t correspond, you may add a comment about the discrepancy.

KEY IDEAS
In this section, we introduced two rules of interpretation.

> AND
> ‘(ϕ & ψ)’ is true with respect to *M iff*
> ‘ϕ’ is true with respect to *M* and ‘ψ’ is true with respect to *M*

NOT
'$\neg\phi$' is true with respect to M *iff* it is not the case that: 'ϕ' is true with respect to M

Instantiating these rules leads to statements about conjunctions and negations, such as:

(44) '(Ca & Hf)' is true with respect to L *iff* 'Ca' is true with respect to L and 'Hf' is true with respect to L

(45) '$\neg$Ca' is true with respect to L *iff* it is not the case that: 'Ca' is true with respect to L

The statements above contain atomic sentences following *'iff'*. This allows us to apply our rules for atomic sentences and then our rules of inference (as we did in the section 1.1). Using facts from the lexicon (L(a) is Agatha, L(f) is Frank, L(C) is the set of clever individuals, L(H) is the set of happy individuals), we can then go from statements like (44) to statements like (46) and from statements like (45) to statements like (47).

(46) '(Ca & Hf)' is true with respect to L *iff* Agatha is clever and Frank is happy.

(47) '$\neg$Ca' is true with respect to L *iff* it is not the case that: Agatha is clever.

NEW TERMINOLOGY
- compound sentence
- connective
- conjunctive
- negation

2.3 *Quantifiers*

The English examples below exemplify a kind of statement that we don't yet have in our symbolic language.

(48) a. No girl submitted a story about herself.
b. Every boy submitted a story about himself.
c. At least one adult submitted a story about themselves.

These sentences are taken from the description of a writing workshop attended by a group of adults and children. The sentences report on stories the instructor received at the end of the first week. Each statement reports on a **quantity**. The first says that the number of girls who submitted stories about themselves was 0. The second says that 100% of the boys submitted stories about themselves. And the last one says that the number of adults who submitted stories about themselves was greater than 0. These are quantificational statements. To include quantificational statements in our symbolic language, we’re going to need to add new symbols, along with their syntax and semantics. To get an idea of what needs to be added, we should consider some differences between the quantificational statements above and the sentence below:

(49) Jake submitted a story about himself.

The subject of (49), ‘*Jake*’, is a name. Like individual constants in the symbolic language, names refer to particular individuals. By contrast, the subjects of the quantificational statements don’t work semantically like names. There is no individual that is referred to by ‘*no girl*’ or by ‘*every boy*’. ‘*At least one adult*’ could be about a particular individual, in some sense, but it can’t be used to refer to that individual: as it moves from sentence to sentence it isn’t fixed to one person – think about the sentence ‘*At least one person sang and at least one person danced*’ as compared with ‘*Jake sang and Jake danced*’. To accomplish our task we’re going to need symbols called **quantifiers** whose semantics gives rise to truth conditions that depend on facts about several objects at once.

Another important difference between the statements in (48) and (49) is the interpretation of the pronouns ‘*himself*’ and ‘*herself*’. In (49), the pronoun *himself* intuitively refers to Jake. It’s a story about Jake that’s being described. By contrast, the pronoun ‘*herself*’ in (48)a doesn’t refer to anyone in particular. In a sense, its reference varies according to which girl is under consideration. Suppose Karen and Rebecca were among the girls. Then the sentence says that Karen didn’t submit a story about Karen and it says that Rebecca didn’t submit a story about Rebecca. And so on, for each girl that we consider, we temporarily treat the pronoun as a name for that girl. The same type of procedure works for the other examples. For example, if Henry and Brett are among the boys, then (48)b tells us that Henry submitted a story about Henry and that Brett submitted a story about Brett.

To accomplish our task of adding quantificational statements to the symbolic language, we will need to have special symbols that work like ‘temporary names’. These symbols will be called **variables** and they will be lower-case letters from near the end of the alphabet: ‘x’, ‘y’, ‘z’. The semantic rule that interprets them will work in tandem with the quantifiers. Each time the quantifier considers a new object, we’ll add the variable to the lexicon with that object as its meaning. We’ll use the + notation to do that. For example, if we have a lexicon *L* and an object o, we’ll enlarge the lexicon to: *L*+<x,o>. Recall from Chapter 1, that *L*+<x,o> is a lexicon according to which ‘x’ stands for the object o. Summarizing now, what lies ahead is the introduction of two new quantifier symbols, as well as variables. Along with these new symbols, we’ll need syntactic rules to tell us how to form sentences with them and semantic rules that guide us to an interpretation of quantificational sentences.

As noted above, the variables are lower-case letters at the end of the alphabet. They have the same syntax as individual constants. They can therefore combine with one- and two-place predicates to form **formulas**:

(50) a. Cx
b. T(s,y)
c. (Cz & Hf)
d. T(x,y)

The metavariables ‘α’ and ‘β’ can now be instantiated with individual constants as well as with variables. That means our two ATOMIC rules apply to ‘Cx’ and ‘T(s,y)’ respectively. Finally, since variables are meant to function like temporary names, we’ll assume that they are not assigned a value by any of the lexicons that we consider. In other words, *L* does not by itself assign a meaning to ‘x’, its only when we form *L*+<x,o> that we get a lexicon that assigns a meaning to ‘x’. We’ll see how all that works in a moment, but first we need to introduce a quantifier, because the variables work in tandem with the quantifiers.

2.3.1 *Existential quantifier*

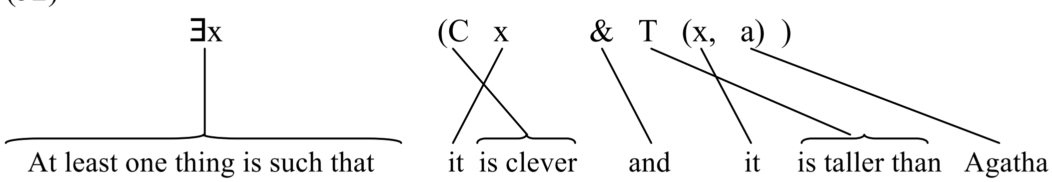
The first quantifier we’ll add is formed by putting the symbol ‘∃’ in front of a variable: ‘∃x’, ‘∃y’, ‘∃z’. This combination is called an **existential quantifier**, which is written with a backwards capital ‘e’ and is often read as “there is an x” or “there is a y” etc. Quantifiers precede formulas, as in the following examples:

(51) a. ∃xCx b. ∃y ¬T(y,a) c. ∃z(Cz & T(z,a))

It is usually possible to “read” a formula of symbolic logic into English in a stilted but accurate way. Just read ‘∃x’ as ‘at least one thing is such that’, and then use the pronoun ‘it’ wherever ‘x’ occurs, with the understanding that the antecedent for the pronoun ‘it’ is ‘at least one thing’. For example, assuming the lexicon *L,* in which ‘a’ stands for Agatha, ‘C’ stands for the set of clever people and ‘T’ stands for the taller-than relation, the formula in (51)c can be read: *At least one thing is such that it is clever and it is taller than Agatha.*

The parts go together like this:

(52)



And then that stilted English sentence roughly corresponds to: *Someone is clever and taller than Agatha.* Likewise assuming the lexicon *L,* (51)b translates as: *At least one object is such that: it is not the case that: it is taller than Agatha* which in turn can be put more colloquially as: *There is someone or something that is not taller than Agatha.*

In order to formulate a semantic rule for quantified formulas, we need a new metavariable, one that ranges over variables. We use *u* for that. Here is the rule:

> EXIST
> ‘$\exists u\phi$’ is true with respect to M *iff*
> There is at least one object o such that, ‘ϕ’ is true with respect to M+<*u*,o>

Let’s use that rule to see what truth conditions we get for the formula in (51)a relative to the lexicon *L*. First we instantiate the rule EXIST, using ‘x’ for ‘*u*’, ‘Cx’ for ‘ϕ’ and *L* for M.

(53) ‘$\exists$xCx’ is true with respect to *L iff*
There is at least one object o such that ‘Cx’ is true with respect to *L*+<x,o>

It is **very** important here to carefully follow the rule INSTANTIATE, making sure to replace **every** instance of *u* and ‘ϕ’ with the corresponding expression. Notice that ‘*u*’ occurs in the rule EXIST in two places and it was instantiated in two places in (53); same for ‘ϕ’.

The last part of (53) says:

(54) ‘Cx’ is true with respect to *L*+<x,o>

The rule ATOMIC-1, repeated below, applies to the statement in (54).

> ATOMIC-1
> ‘$\pi\alpha$’ is true with respect to *M iff* $M(\alpha) \in M(\pi)$

If we instantiate ATOMIC-1 using ‘C’ for π, ‘x’ for α and *L*+<x,o> for ‘*M*’, then from (54) we get:

(55) ‘Cx’ is true with respect to *L*+<x,o> *iff* *L*+<x,o>(x) $\in$ *L*+<x,o>(C).

Now, by the definition of the plus notation (cf. section 1.6 of Chapter 1), we get:

(56) a. L+<x,o>(x) = o
b. L+<x,o>(C) = L(C)

So now we can use REPLACE to get:

(57) 'Cx' is true with respect to L+<x,o> *iff* o ∈ L(C).

And now, (57) allows for a replacement in (53) as follows:

(58) '∃xCx' is true with respect to L *iff*
There is at least one object o, such that o ∈ L(C).

And now using that fact that L(C) = the set of all clever individuals, by REPLACE and set theory notation, we get:

(59) o ∈ L(C) *iff* o is clever

And then we REPLACE in (58) to get:

(60) '∃xCx' is true with respect to L *iff* there is at least one object o such that, o is clever.

(60) tells us what the truth conditions are for '∃xCx' relative to L. Those truth conditions are the same as the truth conditions for the English sentence: *At least one object is clever.* Given that only people and animals are clever, that statement is pretty close in meaning to the sentences *At least one person or animal is clever* and *Some person or animal is clever.* In general, formulas that begin with '∃' correspond to English sentences containing a **determiner phrase (DP)** of the form '*at least one…*'. Although there are meaning differences between '*a*', '*at least one*', and '*some*' – often a formula with '∃' is translated into English using '*a*' or '*some*'.

EXERCISE E

1. Review the steps taken in the text to arrive at truth conditions using the rule EXIST and using the rule AND. Calculate the truth conditions assigned to the sentence '∃x(Cx & Hx)', assuming the lexicon *L* according to which 'C' is the set of clever individuals and 'H' is the set of happy individuals. You should begin by instantiating the rule EXIST and work from there. Each step you take should be justified either by a rule of inference (REPLACE or INSTANTIATE) or by a rule of interpretation (AND, ATOMIC-1) or by some fact about set-theory or notation. Show all steps in your calculation.

2. Suppose we have a lexicon *G* and according to *G*, 'd' stands for a particular diamond, 'c' stands for a particular circle and 'A' stands for the above-relation: x bears the above relation to y *iff* x is above y. Calculate the truth conditions for the formula '∃zA(z,d)' with respect to the lexicon *G*. For this exercise, **do not turn in the intermediate steps in your calculation – just the final line**. That should be a statement that looks like (60) above.

3. Consider the statements in (a) and (b):

   (a) There is at least one object o such that: o is round and o touches the diamond.
   (b) There is at least one object o such that: o touches the diamond.

   Draw a diagram in which (a) is true and (b) is false. If that is not possible write: '(a) entails (b)'. Next, draw a diagram in which (b) is true and (a) is false. If that is not possible write: '(b) entails (a)'.

4. The next few tasks make reference to a lexicon which we call G_1. According to G_1:

   - 'd' stands for the diamond referred to in the statement (a) and (b) in task 3.
   - 'b' stands for some particular geometrical figure.
   - 'S' stands for the set of all squares.
   - 'R' stands for {x | x is round}.
   - 'T' stands for the touch-relation: x bears the touch relation to y *iff* x touches y.
   - 'A' stands for the above-relation: x bears the above relation to y *iff* x is above y.

   Write a sentence of the symbolic language that adequately translates statement (a) from task 3, assuming the lexicon G_1. Then write a sentence of the symbolic language that adequately translates statement (b) from task 3, assuming the lexicon G_1.

5. Write a sentence of the symbolic language that is true with respect to G_1 (described in task 4) if and only if (c) below is true:

   (c) There is something round that is touching a square.

6. Draw a diagram in which the following sentence of the symbolic language is true with respect to G_1 (described in task 4): ∃x(Rx & ¬(S(b) & T(x,b))

7. Assume the lexicon G_1 described in task 4. Suppose we have some object, let's call it o′. What facts must be true of o′ in order for the formula below to be true with respect to G_1+<*y*, o′>?

   ∃x(A(y,x) & T(x,d) & Rx)

2.3.2 *Universal quantifier*

Our next symbol is '∀' which has the same syntax as '∃'. '∀' combines with a variable to form a **universal quantifier**. Examples include '∀x', '∀y' and '∀z'. The symbol '∀' is an upside down letter 'A' and '∀x' is often read "for all x". Similarly, '∀z(Cz & Hz)' is read "for all z, Cz and Hz" and '∀x ¬Cx' is read "for all x, not Cx". The rule for interpreting universal quantifiers is as follows:

> UNIVERSAL
> '∀*u*ϕ' is true with respect to M *iff*
> Every object o is such that 'ϕ' is true with respect to $M+<u,o>$

To see what truth conditions are assigned to the formula '∀zRz' with respect to lexicon *L,* we instantiate **UNIVERSAL** with 'z' for '*u*', 'Rz' for 'ϕ' and *L* for M to get:

(61) '∀zRz' is true with respect to *L iff*
Every object o is such that 'Rz' is true with respect to $L+<z,o>$

By our rule ATOMIC-1 we have:

(62) 'Rz' is true with respect to $L+<z,o>$ *iff* $L+<z,o>(z) \in L+<z,o>(R)$

Given how the plus notation works we have:

(63) a. $L+<z,o>(z) = o$
b. $L+<z,o>(R) = L(R)$

So using REPLACE, from (62) we get:

(64) 'Rz' is true with respect to $L+<z,o>$ *iff* $o \in L(R)$

And that in turn can be used to go from (61) to (65) below via REPLACE:

(65) '∀zRz' is true with respect to *L iff* every object o is such that $o \in L(R)$

If *L* assigns to 'R' the set of round objects, it now follows that:

(66) '∀zRz' is true with respect to *L iff* every object o is such that it is round.

And so, assuming lexicon *L*, '∀zRz' could be translated into English as: '*Every object is round*' or perhaps as '*Everything is round*'.

Now let's assume that according to *L*, the two-place predicate 'A' stands for the above-relation (x bears the above-relation to y *iff* x is above y) and that *L*(s) is the sky. In that case, using our inferencing rules, along with the interpretation rules UNIVERSAL, NOT, and ATOMIC-2, we arrive at the following conclusion (check for yourself whether this is correct):

(67) ‘$\forall x\ \neg A(x,s)$’ is true with respect to *L iff* every object o is such that it is not above the sky.

So with respect to *L*, ‘$\forall x\ \neg A(x,s)$’ could be translated into stilted English as: *Everything is such that it is not above the sky.* Assuming that whatever is not above the sky is below it, we could translate ‘$\forall x \neg A(x,s)$’ into more colloquial English as: *Everything is below the sky*. ‘$\forall x\ \neg A(x,s)$’ could also be translated as *Nothing is above the sky.* You can usually mechanically arrive at the meaning of a universal formula by following the same steps as with existential, just using ‘everything is such that’ for the universal:

$\forall x$ (C x & T (x, a))

Every thing is such that it is clever and it is taller than Agatha.

EXERCISE F

1. Consider the following two statements:

   (i) Every object is such that it is not the case that it is above the line.
   (ii) It is not the case that every object is such that it is above the line.

   Draw a diagram (like the one in Fig. 1, page 13) with geometric objects and with a line in the middle in which (i) is false and (ii) is true.

2. Assume a lexicon *G* according to which ‘n’ stands for the line in your diagram in the previous question and ‘A’ stands for the above-relation. Provide a sentence of symbolic logic that with respect to *G* has the truth conditions of (i) above and provide a second formula that has the truth conditions of (ii) above. [Hint: for (i), review the discussion in the text of the truth conditions of ‘$\forall x\ \neg A(x,s)$’.]

2.3.3 *Free Variables*

We began our discussion of quantifiers by introducing variables, special symbols that were meant to work like ‘temporary names’. In the course of applying the semantic rules for quantifiers, variables are added to the lexicon using the plus notation. Here’s our definition for plus notation:

M+<…. , …> notation

If M is a set of pairs that does not include <*a,b*>, then **M+<*a,b*>** is a set which consists of the pairs in M plus the pair <*a,b*>.

It follows from this, that variables are not in the lexicon to begin with. That is the sense in which they are temporary names.

Suppose now that *L* is a particular lexicon. Given what we just observed, *L*(*z*) is not defined. It's not defined for the same reason that it makes no sense to look up the definition of a Q'eqchi' word in a German-French dictionary (recall Chapter 1, Section 6, pg. 18). And given that *L*(*z*) is not defined, the following statement couldn't be true:

(68) $L(\mathrm{z}) \in L(\mathrm{R})$

and so given our rule ATOMIC-1, 'Rz' could never be true. And since the truth of a conjunction depends on the truth of the conjuncts, '(Rz & Ca)' also could never be true. We might go even further and argue that if *L*(z) is not defined, then the statement in (68) isn't defined either, and so 'Rz' shouldn't be true or false nor should '(Rz & Ca)'.

But now we have a puzzle. Q'eqchi' words don't have interpretations in English and therefore <u>any</u> sentence that is half English and half Q'eqchi' also lacks an interpretation in English. We just said that 'Rz' doesn't have an interpretation relative to *L*, so how is it that '∀zRz' has perfectly well defined truth conditions with respect to *L*? The answer lies in the semantic rule for the quantifier '∀z'. It changes the lexicon so that 'Rz' gets interpreted not relative to *L* but relative to a lexicon that in fact does include 'z'. In this case, we say that the quantifier **binds** the variable.

A variable that is not bound by a quantifier is called a **free variable.** The formula '∀xRz' is well-formed, in other words, its syntax is fine. We might say it is grammatical. However, it contains the free variable 'z' and because of that '∀xRz' is undefined with respect to *L*, for the same reason that 'Rz' is. We began this chapter by talking about how to form sentences of the symbolic logic. Technically, a sentence is a formula that contains no free variables.

EXERCISE G

Jack gets on a crowded city bus and finds himself standing next to a man and a woman engaged in conversation. The man says: "She was married to Norman Baldy for several years". The woman reacts with shock and horror. This arouses Jack's interest. He's curious to know the meaning of the man's utterance. He knows it concerns a marriage, but Jack doesn't know who "she" refers to or who Norman Baldy is. He quickly whips out his phone and looks up Norman Baldy on the web.

(a) Why would an internet search not similarly help Jack identify who the man was referring to when he said "she"?

(b) What could Jack do to find out who "she" is? What is the general rule that would determine who the man referred to with "she"?

(c) Will the rule you provided in (b) help to identify who "she" refers to in the sentence: *Every woman in Jack's club owns a dog that she adores.* If so, how will it apply? If not, why not?

(d) Write a short paragraph comparing the interpretation of pronouns in English with the interpretation of variables in the symbolic logic.

2.3.4 *Generalized Quantifiers*

In the previous two subsections we discussed the semantics of quantifiers such as '∀x' and '∃y', which correspond roughly to the English DPs 'everything' and 'something'. Most English DP's are more complicated, however, having forms such as 'every giraffe', 'some elephant', 'the tall horses', 'few striped zebras', 'every purple cow that has no tail', and so on. The symbolic analogues of these DP's are called **generalized quantifiers**. A generalized quantifier combines with a formula to make something that behaves like the quantifiers we have already been studying. In English, we combine a word like 'every' with a noun like 'giraffe' to form 'every giraffe', which goes into sentences in the same places as 'everything'; in logical notation we combine a generalized quantifier word '$\mathbf{every}_x$' with a formula 'Gx' ("x is a giraffe") to form the generalized quantifier '$\mathbf{every}_x\{Gx\}$'. The generalized quantifier then combines with a formula, just like simple quantifiers such as '∀x'. Compare these translations (assuming the obvious meanings for 'S', 'G', 'R' and 'B'):

(69)	*Everything is spotted.*	$\forall x\ Sx$
(70)	*Every giraffe is spotted.*	$\mathbf{every}_x\ \{Gx\}\ Sx$
(71)	*Something is round and spotted.*	$\exists x(Rx \ \&\ Sx)$
(72)	*Some ball is round and spotted.*	$\mathbf{some}_x\{Bx\}\ (Rx \ \&\ Sx)$

Just as a determiner of English combines with an NP to form a DP, the symbolic analogue of a determiner combines with a **restrictor**, a formula enclosed in curly brackets '{ }', to make a generalized quantifier. The generalized quantifier then combines with another formula, which is called its **scope.** Additional examples with the syntactic structure of the English DP's indicated are:

(73) *Every purple giraffe is spotted*
$[_{DP}[_{D}$ *Every*$][_{NP}$ *purple giraffe*$]]$ *is spotted* $\mathbf{every}_x\{Px \ \&\ Gx\}\ Sx$

(74) *The giraffe is spotted*
$[_{DP}[_{D}$ *The*$]\ [_{NP}$ *giraffe*$]]$ *is spotted* $\mathbf{the}_x\{Gx\}\ Sx$

(75) *Few zebras are spotted*
$[_{DP}[_{D}$ *Few*$]\ [_{NP}$ *zebras*$]]$ *are spotted* $\mathbf{few}_x\{Zx\}\ Sx$

EXERCISE H

1. Write a logical formula that corresponds to each sentence below, using a generalized quantifier.

(i) Every pig grunted. (ii) Mary met some gardener.
(iii) The black horse liked Mary. (iv) Every man knows Sue

2. Consider the sentence *Every man knows some woman*. According to intuitions of native speakers, this sentence is ambiguous; it has two truth-conditionally distinct readings. To guide your intuitions, consider these following paraphrases of these readings:

Reading 1: *Every man is such that he knows some woman or other* (doesn't have to be the same one).
Reading 2: *There is some particular woman* (e.g. Sue) *such that every man knows her*.

This means that there should be two distinct logical formals corresponding to *Every man knows some woman* (one corresponding to Reading 1 and another for Reading 2). Provide two such formulas.

✪ IMPORTANT ✪
Keep in mind that the syntax of the symbolic logic only allows quantifiers to precede a formula

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Here is the semantic rule that allows us to interpret sentences that have a generalized quantifier built from '**every**':

EVERY
'$\mathbf{every}_u\{\phi\}\ \psi$' is true with respect to *M iff*
For every object o such that 'ϕ' is true with respect to M+<u,o>, 'ψ' is also true with respect to M+<u,o>

Suppose we have a lexicon *L* according to which 'P' stands for the set of all pigs, and 'H' stands for the set of all happy things. Then we can instantiate the EVERY rule as follows:

(76) '$\mathbf{every}_x$ {Px} Hx' is true with respect to *L iff* for every object o such that 'Px' is true with respect to *L*+<x,o>, 'Hx' is also true with respect to *L*+<x,o>.

And using our inference rules and facts about set theory and the plus notation, we get from (76) to:

(77) '$\mathbf{every}_x$ {Px} Hx' is true with respect to *L iff*
For every object o such that o is a pig, o is happy.

And so, with respect to *L*, an adequate English translation for '**every**$_x${Px} Hx' would be: *Every pig is happy.*

We can now extend our language by adding the quantifiers 'no', 'the', 'atlst1' and 'some' all with the same syntax as 'every' and by adding the semantic rules on the next page.

NO

'**no**$_u${φ} ψ' is true with respect to *M iff*

There is no object o such that 'φ' is true with respect to *M*+<*u*,o> and 'ψ' is also true with respect to *M*+<*u*,o>

THE

'**the**$_u${φ} ψ' is true with respect to *M iff*

There is exactly one object o such that 'φ' is true with respect to *M*+<*u*,o>, and 'ψ' is also true with respect to *M*+<*u*,o>

SOME

'**some**$_u$ {φ} ψ' is true with respect to *M iff*

For some object o such that 'φ' is true with respect to *M*+<*u*,o>, 'ψ' is also true with respect to *M*+<*u*,o>

ATLST1

'**atlst1**$_u$ {φ} ψ' is true with respect to *M iff*

There is at least one object o such that 'φ' is true with respect to *M*+<*u*,o> and 'ψ' is also true with respect to *M*+<*u*,o>

EXERCISE I

1. Calculate the truth conditions for the following sentence relative to a lexicon *L* according to which 'G' stands for the set of grey things, 'P' the set of all pigs, and 'S' the set of things that sing: $\mathbf{no}_y$\{Gy & Py\} Sy. Remember to show all steps in your derivation and to cite all rules that you used in each step.

2. Assume a lexicon G_1 according to which:
   - 'S' stands for the set of all squares.
   - 'R' stands for: {x : x is round}.
   - 'D' stands for the set of all diamond-shaped figures.
   - 'T' stands for the touch-relation: x bears the touch relation to y *iff* x touches y.
   - 'A' stands for the above-relation: x bears the above relation to y *iff* x is above y.

   Draw a diagram in which the symbolic language sentence below is true with respect to G_1:

$$\mathbf{the}_y\{Dy\}\ \exists x\ (Rx\ \&\ \exists z(T(x,z)\ \&\ Sz\ \&\ A(z,y))).$$

3. Suppose we have a generalized-quantifier forming symbol '**ex1**' whose semantic rule along with other rules guarantees that:

   '$\mathbf{ex1}_x$\{Px\}Hx' is true with respect to *L iff* there is exactly one object o such that o is a pig and o is happy.

assuming that 'P' stands for the set of all pigs, and 'H' stands for the set of all happy things, according to *L*. Write the semantic rule that needs to be added to allow us to interpret sentences formed with **ex1**.

4. Assume a lexicon *K* in which 'T' stands for the set of all triangles and 'C' stands for the set of all circles. Use the quantifiers 'ex1' and 'every' to write a sentence of symbolic logic that is true relative to *K* if and only if:

(a) *There is exactly one triangle such that it is touching every circle.*

Now, use the quantifiers 'ex1' and 'every' to write a sentence of symbolic logic that is true relative to *K* if and only if:

(b) *For every circle, there is exactly one triangle that is touching it.*

2.4 *Summary*

What we have described here is a simple language with a syntax and a semantics. The set of symbols constitute the morphemes of the language; its vocabulary. Along the way, we said how these symbols combine. These are the rules of syntax. And finally, we gave semantic rules for interpreting this language. Below, the rules are collected together giving us a definition of this language. This type of language is often called a **predicate logic**.

This language illustrates how syntax can affect interpretation. In section 2.1, we saw that 'A(d,c)' and 'A(c,d)' have different truth conditions relative to a given lexicon. And in one of the exercises, we saw that the order of negation and a quantifier could lead to different truth conditions.

DEFINITION OF OUR SYMBOLIC LOGIC LANGUAGE

Metavariables

π	predicates (1-place and 2-place)
α, β	variables and individual constants
ϕ, ψ	sentences / formulas
u	variables
M	lexicons

Syntactic Rules

If π is a 1-place predicate, and α is an individual constant or a variable, then '$\pi\alpha$' is a formula.
If π is a 2-place predicate and α is an individual constant or a variable and so is β, then '$\pi(\alpha,\beta)$' is a formula.
If ϕ is a formula, then '$\neg\phi$' is a formula.
If ϕ and ψ are formulas, then '$(\phi \,\&\, \psi)$' is a formula.
If ϕ is a formula and u is a variable, then '$\exists u\phi$' and '$\forall u\phi$' are formulas.
If ϕ and ψ are formulas and u is a variable, then '$\mathbf{every}_u\{\phi\}\ \psi$', '$\mathbf{no}_u\ \{\phi\}\ \psi$' , '$\mathbf{some}_u\ \{\phi\}\ \psi$', '$\mathbf{atlst1}_u\ \{\phi\}\ \psi$' and '$\mathbf{the}_u\ \{\phi\}\ \psi$' are formulas.

Semantic Rules

ATOMIC-1
'$\pi\alpha$' is true with respect to M *iff* $M(\alpha) \in \mathrm{M}(\pi)$

ATOMIC-2
'$\pi(\alpha,\beta)$' is true with respect to M *iff* $\mathrm{M}(\alpha)$ bears the relation $M(\pi)$ to $M(\beta)$

AND
'$(\phi \ \& \ \psi)$' is true with respect to *M* *iff* 'ϕ' is true with respect to *M* and 'ψ' is true with respect to *M*

NOT
'$\neg\phi$' is true with respect to *M* *iff* It is not the case that: 'ϕ' is true with respect to *M*

EXIST
'$\exists u\phi$' is true with respect to *M* *iff*
There is at least one object o such that, 'ϕ' is true with respect to M+<*u*,o>

UNIVERSAL
'$\forall u\phi$' is true with respect to *M* *iff*
Every object o is such that 'ϕ' is true with respect to M+<*u*,o>

EVERY
'$\mathbf{every}_u\{\phi\}\ \psi$' is true with respect to *M* *iff*

For every object o such that 'ϕ' is true with respect to M+<*u*,o> , 'ψ' is also true with respect to M+<*u*,o>

NO
'$\mathbf{no}_u\ \{\phi\}\ \psi$' is true with respect to *M* *iff*

There is no object o such that 'ϕ' is true with respect to M+<*u*,o> and 'ψ' is also true with respect to M+<*u*,o>

THE
'$\mathbf{the}_u\{\phi\}\ \psi$' is true with respect to M *iff*

There is exactly one object o such that 'ϕ' is true with respect to M+<*u*,o>, and 'ψ' is also true with respect to M+<*u*,o>

SOME
'$\mathbf{some}_u\ \{\phi\}\ \psi$' is true with respect to *M* *iff*

For some object o such that 'ϕ' is true with respect to M+<*u*,o>, 'ψ' is also true with respect to M+<*u*,o>

ATLST1

'**atlst1**$_u$ {ϕ} ψ' is true with respect to M *iff*

There is at least one object o such that 'ϕ' is true with respect to M+<u,o> and 'ψ' is also true with respect to M+<u,o>

That completes the definition of the language. In addition, we've introduced two rules of inference. They are repeated below. These rules are not part of the language; they only guide us in reasoning logically about the rules in the language. In the same way that the laws of our nation do not include logical principles, but logical principles are used to apply those laws to actual situations.

REPLACE [Rule of Inference]

If two statements ϕ and ψ are logically equivalent (ϕ *iff* ψ), then from a statement including ϕ, we can infer the statement that results from replacing ϕ with ψ.

If two expressions E1 and E2 name the same entity, then from a statement that includes E1, we can infer the statement that results from replacing E1 with E2.

INSTANTIATE [Rule of Inference]

From a rule stated in terms of metavariables, infer the result of substituting the metavariables with expressions of the right kind – taking care to substitute all occurrences of a given variable with the same expression.

NEW TERMINOLOGY

- quantifiers
- variables, free variables
- formulas
- determiner phrase (DP)
- existential quantifier
- universal quantifier
- generalized quantifier
- restrictor
- scope
- predicate logic