Andrew Zito

Brian Conolly

Ad Infinitum

10 October 2014

On the Continuity of Time

Time is one of the most foundational entities that we know of, so foundational that we often have trouble discussing its nature due to the difficulty of conceiving of anything separate from the principle of time. The nature of time, however, is a vital question of metaphysics. I am specifically interested in whether time is continuous – infinitely divisible – or discrete – having some smallest part that cannot be divided. I will first examine Aristotle's refutation of discrete time, and give my own analysis of his discussion. Next I will provide two arguments of my own, one in favor of continuous time and one against discrete time. Finally I will examine five arguments presented in Sorabji's *Time, Creation, and the Continuum* that are in favor of discrete time, and show that each of these is in error. Ultimately I will prove that discrete time is incoherent, and that time *must* be continuous.

Aristotle provides what I think is the most powerful refutation of discrete time. Suppose, he says, that we grant that continua are composed of indivisible parts. Like a point, these indivisibles would have no extension themselves. They would have to compose extension of time, space, or otherwise in one of three ways: continuously, contiguously, or successively. He gives these definitions: "…things being 'continuous' if their extremities are one, 'in contact' if their extremities are together, and 'in succession' if there is nothing of their own kind intermediate between them…" (VI.1) Here Aristotle is talking about space, and his indivisibles are points rather than instants. I will discuss the argument largely in the abstract, usually referring to indivisibles rather than points or instants (though occasionally using lines and points as a conveniently imaginable example). Afterwards I will apply this abstract to time specifically, to demonstrate that it is indeed compatible.

Before continuing, we must ensure that the definitions upon which Aristotle founds his discussion are consistent and exhaustive of all possibilities. Something continuous must indeed have no dividing extremities – its extremities must be one. We cannot say that there are any extremities within a continuous line. One might object that the line can be divided, and so it *does* have extremities within it. However, this assertion actually fits within the second possibility of contiguous. Here the line does indeed have extremities within it, because it has a finite number of parts which each share one extremity with the preceding part and one with the following part. (Remember that the infinite divisions possible in a continuum are *potential* extremities rather than *actual* extremities as in a contiguous entity. This distinction between potential and actual parts will be discussed more later.)

Our possibilities seem to be progressing from less separate to more separate; continuous indivisibles would have one extremity, and contiguous indivisibles would share each of their extremities. The only possibility remaining seems to be that they share *none* of their extremities, and this is what we call successive. Aristotle is right to say that successive indivisibles must be separated by something that is different from them in kind. In our example of a line, if it were composed of successive points that were separated by something that was the *same* as them in kind, they would either be separated by “line,” which would again have to be composed of points in some fashion, or separated by points, which would then in fact be contiguous.

These definitions established, let us examine the consequences on the idea of any extension being composed of indivisibles. Aristotle says:

> For the extremities of two points can neither be one (since of an indivisible there can be no extremity as distinct from some other part) nor together (since that which has no parts can have no extremity, the extremity and the thing of which it is the extremity being distinct). (VI.1)

Here Aristotle seems to say that indivisible points or instants have no extremities, because an extremity would be a thing separate from the entity itself. Therefore, they can't possibly be continuous because something is continuous only if "its extremities are one," and obviously even an infinite amount of entities without extremity cannot have even one extremity. They also can't be contiguous, because they have no extremities to be "in contact."

This solution seems unsatisfying. It would be more reasonable to suppose that indivisibles *do* have an extremity, but only one; perhaps you could say that they *are* an extremity. We might say that a line segment has two extremities – its endpoints. These endpoints then are in themselves single extremities. Taking this subtly different view does not obligate us to admit the possibility of continuous or contiguous indivisibles, however. As to continuity, since each indivisible has only one extremity, if they shared extremities "they" would actually be a single indivisible. Thus any extension of time or space would be impossible.

As to contiguity, Aristotle presents another argument against contiguous indivisibles that is compatible with the idea that indivisibles have or are a single extremity. Contiguous indivisibles would be those that are arranged in a timeline like blocks in a row, *touching* each other (so to speak) but not overlapping. Thus they would form an extension of time or space. The problem here stems from the above discussion of singular extremity, or to put it another way, lack of multiple parts. To be adjacent to two things on either side of it, any entity must have at least two parts – one to be adjacent to the entity preceding it and one to be adjacent to the entity following it. Instants of this kind would not be indivisible, because they could be divided into what we could call beginning and end. Aristotle says: "But since indivisibles have no parts, they must be in contact with one another as whole with whole." (VI.1) This brings us back to the same problem as above, that indivisibles sharing all extremities would not be separate.

As to successive indivisibles, Aristotle has this to say: "…things are in succession if there is nothing of their own kind intermediate between them, whereas that which is intermediate between points is always a line and that which is intermediate between moments is always a period of time." (VI.1) This is essentially what was discussed above when we examined the definition of successive. The only way around this objection would be to assume that there are "gaps" within an extension composed of indivisibles. It seems mysterious what these gaps would be composed of, what principle would necessitate their existence, and how they would form an extension. These difficulties become much clearer when we examine the specific extension of time or space.

This examination will first be applied to the previous two possibilities, to ensure that there is no special property of time which would allow instants to avoid the problems of indivisibles in general. An instant is a part of time that is a single extremity – as a line is bounded by endpoints a duration of time is bounded by indivisible instants. Thus there seems to be no way in which we could conceive of a continuous stream of indivisible instants without their becoming a single instant. Nothing about an instant suggests that a series of them would be able to have one extremity yet still form an extension. The same applies to contiguity, for there seems to be no way in which instants could be directly adjacent. Either they would share their one extremity and be continuous, and again a single instant, or they would share no extremities and be successive.

Here we return to where we left off with successive indivisibles. In order to form a duration of time, instants would have to be separated so that they did not form a single instant. They cannot share *any* extremity without sharing *all* extremities and becoming continuous and singular; therefore, they must share no extremities. At first this may seem intuitively attractive – time, perhaps, is like a flipbook, each instant following the other in a series of (extremely small) jumps or jerks. However, this possibility does not stand up to further consideration. A flipbook works because each page appears *after* the other one; the frames in a video work the same. These frames are contained *within* time. They are separated *by* time. But instants of time *are* time – if they are separated by time as well, we are right back where we started! The problem of successive indivisibles is no more resolvable in time than in the abstract. Either the instants would be separated by nothing, in which case they would in fact be adjacent (contiguous); or they would be separated by time, in which case that time too would be subject to the same problem of composition.

I would now like to offer an argument of my own for continuous time. It is a positive argument *for* my position, rather than against the opposition. We know that we can divide durations of time. A year can be divided into months, months into weeks, weeks into days, days into hours, hours into minutes, minutes into seconds, seconds into half seconds, and so on. There seems to be nothing essentially different about smaller durations of time that prevents them from being divided just like their larger counterparts. This analogy can be carried on indefinitely, unless some principle can be provided which stops the analogy from applying to some tiny duration of time. This argument is also meant in part as an answer to those who might say that

Aristotle is assuming continuous time, and that if we simply make the assumption that time is discrete his problems no longer make sense. Therefore, they might say, we are both just assuming our own definitions of time, and yours is just as arbitrary as mine. But, as I have shown, the assumption of continuous time is not arbitrary in any way, but rather based on a strong argument from analogy.

Another answer to anyone making the above objection would be to deny either that Aristotle's arguments assume continuous time, or that the counter-assumption of discrete time makes these problems go away. It is possible that one could reasonably object that successive instants need only be separated by time if we already assume continuous time; and that they are therefore not separated by *nothing* but rather by the principle of the discreteness of time, which we must assume. Again, this objection is weakened by the fact that such an assumption of discrete time seems arbitrary in a way that the assumption of continuous time is not. Moreover, even exercising the principle of charity and granting the objectors their conclusion, they are not freed from logical difficulties.

Before I present my final argument against discrete time, I think it is important to discuss further the nature of divisibility. I will distinguish two types of divisibility. First, there is what I will call conceptual divisibility – we can easily take any length of time and place it over two, even something as tiny as the famous planck time. This could also be thought of as mathematical divisibility. If there is a valid objection to this specific form of divisibility, I cannot think of it. However, there is a second type of divisibility that I think is more important to this discussion, because time exists outside of pure mathematical abstraction. This "physical divisibility" can best be thought of in just those terms – can time *actually* be divided infinitely? I think the best way to conceive of this is to consider the shortest duration of time over which we can (given the right instruments) measure motion. I do *not* think it is unreasonable to suppose that any duration of time in which motion is measurable can be further divided, in that we could measure the first and last halves of that motion (once again, given the right instruments). However, an indivisible portion of time would be one over which motion does not occur, like a page in a flipbook – a frozen image. It would still be conceptually or mathematically divisible, in that we could say "one indivisible quanta of time divided by two," but this would be a purely mathematical division which would not carry over to the actual nature of time.

I refrained from making this distinction previously because I felt that it was not relevant to the discussion at the time. Furthermore, this final argument of mine shows that the idea of an indivisible duration of time is impossible, and so the second type of divisibility does not threaten the earlier arguments. That is, one could not reasonably object that instants could be contiguous by having a duration and being mathematically divisible yet physically indivisible – so that they could have multiple conceptual (or mathematical) parts that would touch the preceding and following instants, but still be indivisible in the physical or real sense. This is flawed because conceptual divisibility is not enough to justify contiguity, since the instants of time are touching in the real world, not just in pure mathematical abstraction – so, in fact, they *must* be *physically* indivisible. More to my point, this objection does not hold water because indivisible durations are impossible, which I will now show.

We measure time by movement. Without movement, time would not exist in any meaningful way; and without time, there could be no movement. Imagine a universe composed of empty space with a single atom bouncing back and forth between two plates. If this atom stopped moving (completely, even at the subatomic level) there would be no way to measure time. Every moment would be *exactly* the same; and in this way, all moments would be identical, all moments would be one, and there would be no meaningful time flow. Those who might imagine themselves standing in such a universe, and say that they would still be able to perceive the passage of time, make the mistake of ignoring the “movement” of their own minds. This movement is better described as change, since we could hypothetically have a non-physical observer with an incorporeal mind. But that mind would be changing, adding memories each second – perhaps “Wow this is boring.” In the same way, if time stopped, no motion would be possible, for motion is defined as something existing at one point at one instant and then at a following instant being in a different place. If there are no progressing instants, then there can be no change in position and no motion. There can be no change at all, even of the incorporeal mind type, because such change too is predicated on the idea of being in one state at one time and in another state at a later time.

Having established this dependency between time and motion or change, we can now show that the idea of an indivisible duration is incoherent. A duration is some measurable amount of time, a second or half a second or a quarter of a second and so on. If time requires motion, then *any* duration, no matter how small, must be able to contain motion. As was stated earlier, if a duration has motion, it must be divisible – for we can divide it into the beginning of the motion and the end of the motion. Therefore, the idea of an indivisible duration makes no sense. Either it must be indivisible and have no duration (and thus be zero seconds long), or be divisible and have a duration.

Now imagine that we are trying to find the smallest unit of time. We have a hypothetical instrument than can measure any duration of time, no matter how short. Keep in mind that such an instrument, like any time measurement device, would rely on motion of some sort. Based upon the previous arguments made, we must conclude that such an instrument would never find the shortest duration of time that can no longer be divided, since any duration can be divided again by measuring the parts of its motion. Again we are lead to the conclusion that the only indivisible part of time is an instant, with no duration – and we have shown that indivisible instants without duration cannot constitute an extension of time.

Now I will examine some arguments in favor of discrete time, specifically the five discussed by Sorabji on page 343 of Time, Creation, and the Continuum. These five arguments are attributed to an unknown member of Aristotle's school. They are, like Aristotle's discussion, focused upon special continua, or at least abstract continua, rather than time; but also like Aristotle's discussion, they apply equally well to any continua you wish to consider. I will quote Sorabji's quotation of the arguments and follow each one with my own analysis.

> Are there atomic lines, and in general in all quantities is there something which lacks parts, as some people say?
>
> (1) For if much and large and their opposites, few and small, exist alike, but what has almost infinite divisions is not few but much, clearly the few and the little will have a finite number of divisions. But if the divisions are finite, there must be a magnitude which lacks parts, so that there will be something without parts in everything, since everything contains few and little.

This objection is based on the fallacy that infinite divisibility is the same as *actually* having infinite parts. Aristotle's well-known distinction between potential and actual infinites serves us well here. Infinite divisibility is a *potential* infinite. Nothing can ever be actually divided an infinite number of times, in the same way that we cannot count to infinity. Let us imagine two lengths, one shorter and one longer. The objection would have us believe that if we allowed both to have infinite divisions, they would not be differentiated by length. The shorter must have fewer divisions in order to be shorter. Again, this is a fallacy of equating divisions (or potential parts) with actual parts. If we were to claim that both had infinite *actualized* parts, this objection would be accurate. However, this is not the case. Imagine that we divided both the short and the long lengths by some number, say four hundred. One four hundredth of the shorter length would be shorter than one four hundredth of the longer length. Moreover, they would both have only a finite number of actualized parts. No matter how large a number you divide them by, the parts of the shorter line will always be shorter than the parts of the longer line. In the same way, if we divided each line into parts of the same length, the shorter *would* have fewer parts than the longer – but this does not suggest the existence of indivisible lengths, since we can always divide them again into even smaller parts. This is how they are differentiated.

> (2) Again, if there is an Ideal Form of the Line, and if an Ideal Form is first among the things which share its name, and if parts are prior in nature to the whole, then the Line Itself [i.e. the Ideal Form of the Line] will be indivisible. And the same applies to the Square and the Triangle, and the other Figures, and in general to Surface Itself and to Body. For [otherwise] there will be things [i.e. parts] which are prior to these [Ideal Forms].

This objection is based upon the same fallacy as the last one. Wholes may be composed of their parts, but at least in the case of a continuum, these parts are *not* prior to the whole. They are *derived* from the whole by a process of division. Again, the objection confuses divisible parts and *actual* parts. Moreover, it claims that Form of Line is indivisible – and this seems a difficult assertion to support, given the obvious existence of line segments, as well as the fact that allowing this conclusion would obviate both discrete and continuous extensions of time and space.

I will skip the third argument, as does Sorabji, because it is essentially a repetition of the previous one and falls prey to the same flaws.

> (4) Again, by Zeno's argument, there must be a magnitude which lacks parts, if it is impossible to touch an infinity one by one in a finite time, and if anything that moves must first reach half way, and if there is always a half of that which does not lack parts. And if anything that travels along a line does indeed touch an infinity in a finite time, and if a faster moving thing completes a longer journey in a given time, and if the movement of thought is the fastest, then thought too will touch an infinity one by one in a finite time. Thus if thought's touching things one by one constitutes counting, it is possible to count an infinity in a finite time. If that is impossible, there must be such a thing as an atomic line.

The objection is correct to assert that we must not accept that an infinity can be counted in a finite time. It is confused, however, in the same way as the previous ones. To traverse a continuum of space, time, thought, or otherwise, is *not* to touch an infinity of parts. The importance of this distinction cannot be underestimated, and so despite the fact that it has been stated twice already I will state it again: *actual* parts are not the same as *potential* parts derived from division. Again, continua are not composed of infinite *actual* parts – for this indeed would be absurd. They are composed of infinite *potential* parts derived from division. Motion is *not* an act of division, and so there is no need to traverse an infinity of parts.

> (5) Again, there must be such a thing as an atomic line, so they say, because of what is said by the mathematicians themselves, if those lines are commensurable which are measured by the same measure, and if all lines which are measured are commensurable. For there must be some length which will measure them all, and this must be indivisible.

> For if it is divisible, then its parts will have something that measures them, since the parts are commensurable with the whole. Thus a half portion of some measure will be a double portion. But since this is impossible, there must be an indivisible measure.

Sorabji refutes this argument by pointing out that, in fact, not all measurements are commensurable. "The number pi… is not equal to 22/7 or $3\frac{1}{7}$… No fraction expresses it correctly. To put the point another way, diameter and circumference have no common measure… There cannot then be some standard *atomic* size of which both diameter and circumference are multiples." (345) This refutation seems sufficient, and my knowledge of mathematics is insufficient to decide whether or not the objection is still valid if we pretend that all measurements *are* commensurable. Fortunately this is not the case.

Let us now summarize all the reasons we have to believe that time is continuous rather than discrete. First there is Aristotle's argument that discrete time is incoherent because indivisibles cannot be continuous, contiguous, nor successive. Second there is the argument by analogy from larger to smaller durations of time. This refutes the objection that it is acceptable to assert an arbitrary separating principle for successive indivisibles because asserting the lack of such a principle (continuity of time) is just as arbitrary. Third there is the argument that it makes no sense to speak of indivisible durations, thus refuting any attempt to get around Aristotle's argument by creating indivisibles with multiple extremities. Finally there is the examination of multiple arguments in favor of discrete time, which served primarily to illuminate the error made by the opposition in overlooking the important distinction between *actual* parts and *potential* parts derived from division. Thus it is shown that maintaining a theory of discrete time is logically incoherent. I would like to conclude by saying that, if modern physics were able to empirically discover that time is quantized – although as I said I cannot imagine how any instrument would be able to measure this – then I would need to reconsider my position. However, that possibility is outside the realm of metaphysical discussion, and so until such a time as we have any empirical evidence of discrete time, we must wrestle with the problem on grounds of logic alone – where, as I have said, continuous time is the victor.