Date:

Assignment #. 02

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Matrix Determinant

It is a scalar value that is calculated from the elements of a scalar matrix. In matrix, the vertical lines are column and horizontal lines are rows. "n" order of determinant has "n" number of rows and columns.

Determinant of 2x2 matrix

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow (4) - (6) = -2$$

Determinant of 3x3 matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow 1[(5)(9)-(6)(8)] - 2[(4)(9)-(6)(7)] + 3[(4)(8)-(5)(7)]$$

$$= 1(45-48) - 2(36-42) + 3(32-35)$$

$$= 1(-3) - 2(-6) + 3(-3)$$

$$= -3 + 12 - 9$$

$$= 0$$

Date:

Properties of Determinant

change in value of determinant will not occur if row and columns are inter-changed but sign will be changed.

If two rows or columns in determinant have same value, determinant will be zero.

If any variable say "K" is multiplied by rows or columns then its value is also multiplied by "K".

If some or all elements of row or column are exposed as the sum of two or more terms, then determinant can be expressed in terms of two or more determinant.

1) Reflection property

value of determinant is unchanged by inter-changing rows and columns. eg

$$M = \begin{bmatrix} 1 & 5 & 6 \\ -3 & 2 & 8 \\ 7 & 0 & 4 \end{bmatrix} \rightarrow |M| = \begin{vmatrix} 1 & 5 & 6 \\ -3 & 2 & 8 \\ 7 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 7 \\ 5 & 2 & 0 \\ 6 & 8 & 4 \end{vmatrix}$$

case 01:

$$|M| = 1[(8-0)] - 5[(-12-56)] + 6[(0-14)]$$
$$= 8 + 340 - 84$$
$$= 264$$

case 2:

$$= 1(8-0) + 3(20-0) + 7(40-12)$$
$$= 8 + 60 + 196$$
$$= 264$$

Date:

2) Switching property

If any of two rows or columns are interchanged, sign of determinant is changed. e.g

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 5 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 1 & 2 \\ 3 & 0 & 4 \end{bmatrix}$$ interchanging R_2 & R_3

case # 01

$|M| = 1(0-4) - 2(20-6) + 3(3-0)$

$= -4 - 28 + 9$

$= -23$

Case # 02

$|M| = 1(4-0) - 2(20-6) + 3(0-3)$

$= 4 + 28 - 9$

$= 23$

3) All-zero property

If all the elements of any row or column are in matrix zero then its determinant is zero. e.g

$$M = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 0 & 0 \\ 6 & 7 & 8 \end{bmatrix}$$

$|M| = 0$

Date:

4) Repetion or Proportionality

If two rows or columns in matrix are same than determinant of that matrix will be zero. e.g

$$|M| = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix} \Rightarrow |M| = 0$$

5) Scalar multiplication

If elements of a row or column are multiplied by any non-zero constant then determinant also gets multiplied by same constant. e.g

$$\begin{vmatrix} 12 & 9 & 6 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 3(4) & 3(3) & 3(2) \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix} = 3 \begin{vmatrix} 4 & 3 & 2 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix}$$

Here $k = 3$, which is constant value.

6) Sum property

If elements of rows or column of a determinant are expressed as sum of two or more terms, then determinant can be expressed as a sum of two or more determinants. e.g

Date: ____________

e.g

$$\begin{vmatrix} a & b & c \\ a+3j & b+k & c+i \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 3j & k & i \\ x & y & z \end{vmatrix}$$

7) Invariance property

Suppose any scalar multiplied of corresponding elements of other two rows or columns are added to every element of any row or column of a determinant. In this case, value of determinant remains same.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+Kb_1 & a_2+Kb_2 & a_3+Kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

8) Factor property

If determinant Δ becomes zero when we insert $x = \alpha$ then $(x-\alpha)$ is factor of Δ

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \Rightarrow \Delta 1 = \begin{vmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{vmatrix}$$

Here x_{ij} denotes the cofactor of then element a_{ij} of a matrix.

Date:

9) Triangle property

If the elements in determinant is below and above the main diagonal are zero than determinant is the product of diagonal elements. e.g.,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}, \quad |A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{vmatrix}$$

So,

$$|A| = 0$$

same if

$$A = \begin{bmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow |A| = \begin{bmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$|A| = 0$$

10) Determinant of cofactor matrix

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \ \& \ \Delta 1 = \begin{vmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{vmatrix}$$

Here $\Delta 1$ represents the cofactors of the elements of a_{ij} in Δ.