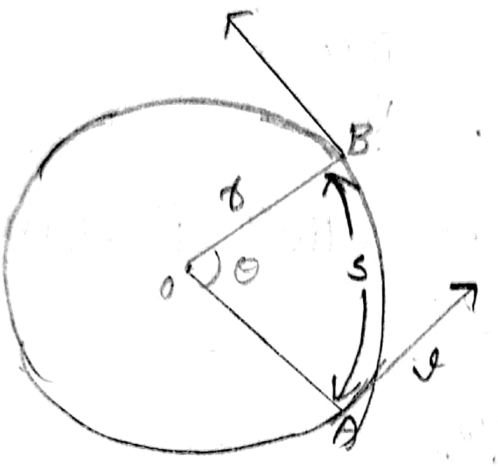
CIRCULAR MOTION



Most of the particle along the circumference of a circle with a constant speed is called 'uniform circular motion'.

From the figure a particle move from A to B with a distance 's'. It is a circular motion

Then its linear speed is $= \frac{\text{distance}}{\text{time}}$

$$v = \frac{s}{t}$$

From the figure, $\theta = \frac{\text{arc}}{\text{radius}}$.

$$\theta = \frac{s}{r}$$

Angular Velocity (ω)

Angular velocity of a particle is the rate at which the angular displacement is takes place.

ie, $\omega = \frac{\theta}{t}$

unit = rad/second.

Then the period $T = \frac{\text{distance}}{\text{velocity}}$

$$\boxed{T = \frac{2\pi r}{v}}$$

Reciprocal of ~~r~~ T is called frequency.

$$\boxed{f = \frac{v}{2\pi r}}$$

Q. What is the relation between ω & v.

ans. we have, $\omega = \theta/t$

$v = s/t$

$\theta = s/r$

from the above equations, we have

$s = \theta r$

$\therefore v = \frac{\theta}{t} r$

$$\boxed{v = \omega r}$$

or, $\boxed{\omega = \frac{v}{r}}$

In a complete revolution, $\theta = 2\pi$ radian.

~~for con~~ and the ~~at~~ time taken ~~is~~ T.

then $\omega = \frac{2\pi}{T}$

or linear velocity $V = \omega r$

$$V = \frac{2\pi}{T} r.$$

$$\therefore \omega = \frac{2\pi}{T}$$

$$\omega = 2\pi f.$$

Angular acceleration

It is the rate of change of angular velocity.

$$\alpha = \frac{\omega}{t}$$

let ω_1 & ω_2 be the initial and final angular velocities, within the time t. then,

$$\omega_1 = \frac{V_1}{r}$$

$$\omega_2 = \frac{V_2}{r}$$

where V_2 & V_1 are initial and final velocities.

$$\alpha = \frac{\frac{V_2}{r} - \frac{V_1}{r}}{t}$$

$$\alpha = \frac{V_2 - V_1}{tr}$$

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where, $\frac{V_2 - V_1}{t} = a$.

$$\boxed{\alpha = \frac{a}{r}}$$

$$\boxed{a = \alpha r}$$

unit = radian/s^2

Equations of Angular motion (circular motion)

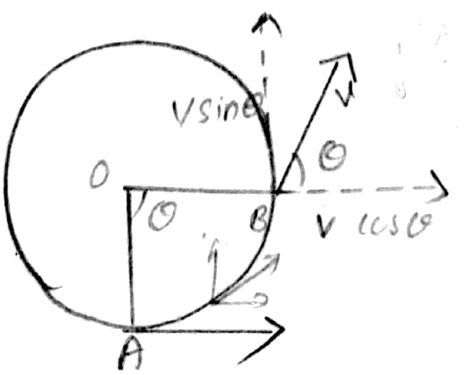
Let ω_2 & ω_1 be initial and final angular ~~distance~~ velocity and α be the angular acceleration. Let θ be the angular displacement in t seconds. Then the equations become

$v = u + at \rightarrow \omega_2 = \omega_1 + \alpha t$

$s = ut + \frac{1}{2}at^2 \rightarrow \theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$v^2 = u^2 + 2as \rightarrow \omega_2^2 = \omega_1^2 + 2\alpha\theta$

Centripetal acceleration.



When a particle moves ~~in~~ along a ~~elect~~ circular path, even if the motion is uniform, the change in direction causes acceleration. This acceleration is called centripetal acceleration. It is directed towards the center of the circle. If V is the velocity and r be the radius of circle, the centripetal acceleration is given by,

$$a = \frac{v^2}{r}$$

$$a = \frac{r^2\omega^2}{r} = r\omega^2$$

From figure θ is very small. At point B velocity has components V sinθ, parallel to the direction AO.

At point A there is no components of velocity along AO.

During time 't' particle moves from A to B so the change in velocity during the time t,

$$a = \frac{V\sin\theta - 0}{t} \quad V = V\sin\theta - 0.$$

∴ the centripetal acceleration,

$$a = \frac{V\sin\theta - 0}{t}$$

$$a = \frac{V\sin\theta}{t}$$

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Let θ be very small

$\therefore \quad a = \frac{v\theta}{t}$

where $\theta \rightarrow$ angular displacement.

$\theta = \omega t$

$$a = \frac{v\omega t}{t}$$

$$a = v\omega$$

~~whe~~ we have $\omega = v/r$

$$a = \frac{v v}{r}$$

$$a = \frac{v^2}{r}$$

$$a = \frac{r^2\omega^2}{r}$$

$$a = \omega^2 r.$$

centripetal force

To get a centripetal acceleration a force must act on the particle. This force is called centripetal force

$$\text{centripetal force} = m\frac{v^2}{r}$$

$$= m r\omega^2$$

No work is done by the centripetal force. Because the centripetal force is ⊥r to the direction of velocity and hence to the direction of displacement.

Examples of centripetal force.

18/12/19. 1) The tension in the string ~~forwar~~ provides the centripetal force for a mass 'm' attached to a string which is whirled around

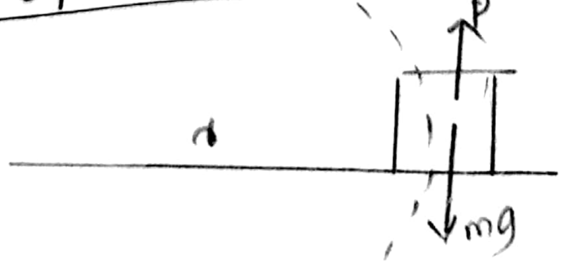
2) The friction between wheel and road supplies centripetal force for a car turning ~~roun~~ a horizontal curve.

3) If a train is turning around a horizontal curve, the flanges of the wheel press on the rail. The lateral thrust exerted by the rail on the flangs provides the centripetal force.

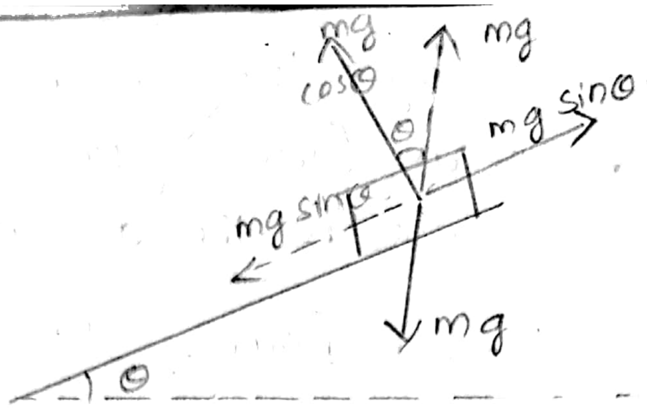
4) In the case of planetary motion gravitation attraction supplies the centripetal force.

5. In the case of motion of eln around the nucleus the coulombs attraction provides centripetal force.

Banking of Road & Rails.



No banking.



A vehicle may take a safe turn without depending on the frictional force, if there must be some agency to supply the necessary centripetal force. For this the outer portion of the curved path is raised slightly above the inner. This process is called banking

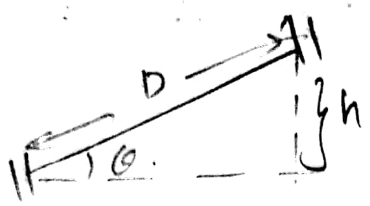
If the curved tracks are banked in this way a component of the normal reaction will contribute to the centripetal force and lateral thrust can be avoided.

From the figure the angle between the elevated surface and horizontal is called "angle of banking". ie, $\tan\theta = \frac{v^2}{rg}$.

$v \rightarrow$ optimum speed.

$r \rightarrow$ radius of curvature.

Banking of rail.



In the case of curved railway track outer rail is at a higher level than inner one to avoid lateral thrust between the wheel and rail.

From the figure,

$$\sin\theta = \frac{h}{D}$$

here θ is
But the angle of banking 'θ' is very small.

so, $\sin\theta = \tan\theta = \frac{v^2}{rg}$.

$$\frac{h}{D} = \frac{v^2}{rg}$$

$$h = \frac{v^2 D}{rg}.$$

h → super elevation.

When a cyclist is riding through a curved path, the cyclist, instingctively ~~deals~~ leaves turns towards the center of the curve.
If θ is the inclination of the plane of the cycle with vertical, $\tan\theta = \frac{v^2}{rg}$.

1/1/2020

If the velocity 'v' of the cycle is very large and radius is small, the ~~liting~~ leining angle θ becomes very large.

Hence the risk of falling is greate. So the cyclist must ride slowly in sharp curves.

Q. A string can sustain a maximum tension of 100 N without breaking. A mass of 1 kg is attached to the end of the string. 1 m ~~to~~ long and is rotated in a horizontal plan. Find out the maximum no. of revolution possi per second.

Here centripetal force is given by $\frac{mv^2}{r}$ that is given by tension along the string.

ie, $\frac{mv^2}{r} = 100\ N$

$\therefore$ $m = 1\ kg$

$r = 1\ m$

$$v^2 = \frac{100 \times r}{m}$$

$$= \frac{100 \times 1}{1}$$

$$= 100$$

$$v = 10\ m/s$$

$\omega = \frac{v}{r} = \frac{10}{1} = 10$ rad/second.

$\omega = \frac{2\pi}{T}$

$\frac{1}{T} = \frac{\omega}{2\pi}$ = no. of revolution / second

$= \frac{10}{2\pi}$

$= \underline{\underline{1.59}}\ s^{-1} = 1.6$