

DIFFERENTIATION

Let $y = f(x)$ be a given function, assume that Δy is the increment in y , corresponding to a small increment Δx in x . Then the ratio $\frac{\Delta y}{\Delta x}$ is known as the incremental ratio which gives average rate of change of y with x .

The instantaneous rate of change of y with respect to x or derivative of $w.r.t x$ is given

by

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}}$$

- 1st principle,
 $\Delta y = \frac{b}{x b}$

This method is known as "method" of 1st principle.

Q. Find the derivative of $y = x^n$ using the method of 1st principle.

A. $y = x^n$.

$$\Delta y = \text{increment in } y, \quad \Delta x = \text{increment in } x. \quad (f)$$

$$y + \Delta y = (x + \Delta x)^n$$

$$\frac{\Delta y}{\Delta x} = \frac{(x + \Delta x)^n - x^n}{\Delta x} = x^n \cdot \frac{1}{\Delta x} = x^n \frac{b}{x b} = (x^n) \frac{b}{x b}. \quad (g)$$

$$\Delta y = (x + \Delta x)^n - x^n.$$

$$= (x + \Delta x)^n - x^n = x^n \cdot \frac{1}{\Delta x} = (x^n) \frac{b}{x b}. \quad (h)$$

apply in 1st principle.

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

$$\frac{1}{\Delta x} = \left(\frac{1}{x}\right) \frac{b}{x b} \quad (e)$$

$$\Delta x \rightarrow 0 \Rightarrow x \Delta x \rightarrow 0 \quad (i)$$

$$x \Delta x = x \cdot x \Delta x \rightarrow 0 \quad (ii)$$

$$\therefore \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^n - x^n}{\Delta x}$$

as $\Delta x \rightarrow 0$, $x + \Delta x \rightarrow x$, and hence we get
that the above limit can be written as.

$$\text{and } \lim_{x + \Delta x \rightarrow x} \frac{(x + \Delta x)^n - x^n}{x + \Delta x - x} = nx^{n-1}. \quad (\text{where } n \text{ is rational no.})$$

similarly for $n < 0$

e.g.) $\frac{d(k)}{dx} = 0$, (k is constant). \Rightarrow understand $\text{def. } f'$.

2) $\frac{d(x)}{dx} = 1$ \Rightarrow constant no. 3 of $\frac{1}{x^3}$

3) $\frac{d(x^2)}{dx} = 2x$ $\boxed{\begin{array}{l} \text{DA rule} \\ x^2 \text{ here} \end{array}}$

4) $\frac{d(x^n)}{dx} = nx^{n-1}$. \Rightarrow constant of function def.

5) $\frac{d(x^{-2})}{dx} = -2x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$ \Rightarrow constant of function def.

6) $\frac{d(x^{-5})}{dx} = -5x^{-5-1} = -5x^{-6} = \frac{-5}{x^6}$ \Rightarrow constant of function def.

7) $\frac{d(x^n)}{dx} = \frac{-n}{x^{n+1}}$ \Rightarrow B. no. component = nA

8) $\frac{d(\sqrt{x})}{dx} = \frac{d(x^{1/2})}{dx} = \frac{1}{2} \cdot x^{1/2-1} = \frac{1}{2} \cdot x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

9) $\frac{d(x^{3/2})}{dx} = \frac{3}{2} \cdot x^{1/2} = \frac{3\sqrt{x}}{2}(x\Delta + x) =$

10) $\frac{d}{dx} \cos x = -\sin x$ \Rightarrow sign of x in $\cos x$

$$\frac{dx}{x^2} \text{ sign } = \frac{ab}{x^2}$$

11) $\frac{d}{dx} \sin x = \cos x$

properties of derivatives

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$x^n = \frac{d}{dx} x^n$$

$$\frac{d}{dx} x^n \cdot a = a \frac{d}{dx} x^n = a n x^{n-1}$$

a) $\frac{d}{dx} (k f(x)) = k \cdot \frac{d}{dx} (f(x))$.

$$\text{eg: } \frac{d}{dx} (5x^3) = 5 \cdot \frac{d}{dx} x^3 = 5 \cdot 3x^2 = 15x^2$$

$$\therefore \frac{d}{dx} (10x^2) = 10 \cdot \frac{d}{dx} (x^2) = 10 \cdot 2x = \frac{20}{x^3}$$

b) $\frac{d}{dx} (f(x) + g(x)) = \frac{d}{dx} (f(x)) + \frac{d}{dx} (g(x))$.

$$\text{eg: } \frac{d}{dx} (x^3 + \sqrt{x}) = \frac{d}{dx} x^3 + \frac{d}{dx} \sqrt{x} =$$

$$= 3x^2 + \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{d}{dx} (5x^2 + 2x^4) = \frac{d}{dx} (5x^2) + \frac{d}{dx} (2x^4) =$$

$$= 5 \cdot 2x + 2 \cdot 4x^3 =$$

$$= 10x + 8x^3$$

1) $y = 4x^5 + 10x^3$.

$$\therefore \frac{dy}{dx} = \frac{d}{dx} (4x^5) + \frac{d}{dx} (10x^3) =$$

$$\frac{x - 2A + x}{5} = 4 \cdot 5x^4 + 10 \cdot 3x^2 =$$

$$= 20x^4 + 30x^2$$

2) $y = 5x^2 + \frac{7}{x} + 3\sqrt{x}$

$$\frac{dy}{dx} = 5 \cdot 2x + 7 \cdot \frac{-1}{x^2} + 3 \cdot \frac{1}{2\sqrt{x}} =$$

$$= 10x + \frac{-7}{x^2} + \frac{3}{2\sqrt{x}}$$

$$3) y = 7x^{-4} - 5x^6$$

$$\frac{dy}{dx} = 7 \cdot \frac{-4}{x^5} - 5 \cdot 6 \cdot x^5$$

$$= -\frac{28}{x^5} - 30x^5$$

$$4) y = \frac{10}{x^3} + 7x^{3/2}$$

$$\frac{dy}{dx} = 10 \cdot \frac{1}{x^3} + 7 \cdot \frac{3}{2} x^{1/2}$$

$$= 10 \cdot \frac{-3}{x^4} + \frac{21\sqrt{x}}{2}$$

$$= -\frac{30}{x^4} + \frac{21\sqrt{x}}{2}$$

- Find $\frac{d}{dx}$ derivative of $\sin x$, using 1st principle.

$$A. y = \sin x$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

$$y + \Delta y = \sin(x + \Delta x)$$

$$\Delta y = \sin(x + \Delta x) - y$$

$$\Delta y = \sin(x + \Delta x) - \sin x$$

$$\sin A - \sin B = 2 \cdot \cos \frac{A+B}{2} \cdot \sin \frac{A-B}{2} = \frac{(\cos 01 + \cos \pi)}{2} \cdot \frac{(\sin x + \sin \Delta x)}{2}$$

$$\Delta y = 2 \cdot \cos \frac{x+\Delta x+x}{2} \cdot \sin \frac{\Delta x}{2}$$

$$\Delta y = 2 \cdot \cos x + \frac{\Delta x}{2} \cdot \sin \frac{\Delta x}{2}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{2 \cdot \cos x + \frac{\Delta x}{2} \cdot \sin \frac{\Delta x}{2}}{\Delta x}$$

$$= \frac{2 \cdot \cos x + \frac{1}{2} \cdot x + \frac{1}{2} \cdot \frac{\Delta x}{2} \cdot \sin \frac{\Delta x}{2}}{\Delta x}$$

$$= \frac{2 \cdot \cos x + \frac{1}{2} + x \cdot \frac{1}{2} \cdot \sin 01}{\Delta x} =$$

$\sin x$
 $\cos x$
 x

$$= \lim_{\Delta x \rightarrow 0} \frac{\cos x + \frac{\Delta x}{2} \cdot \sin \frac{\Delta x}{2}}{-\frac{\Delta x}{2}}$$

प्रथम नियम
परिवर्तन का विधि

$$= \lim_{\Delta x \rightarrow 0} \cos x + \frac{\Delta x}{2} \cdot \left[\lim_{\Delta x \rightarrow 0} \frac{\sin \frac{\Delta x}{2}}{\frac{\Delta x}{2}} \right]$$

(vii) b

$$= \frac{\cos x}{1}$$

ज्ञात करने का प्रकार्य

• Find the derivative of $\cos x$. by using 1st principle.

$$A. y = \cos x.$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}.$$

$$y + \Delta y = \cos(x + \Delta x) = \cos x + \frac{\partial \cos}{\partial x} x \Delta x$$

$$\Delta y = \cos(x + \Delta x) - y = \cos(x + \Delta x) - \cos x$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} (1 + \cos x + \frac{\partial x}{\partial}) = B$$

$$\Delta y = -2 \sin \frac{x + \Delta x + x}{2} \sin \frac{\Delta x + x - x}{2} = \frac{\partial}{\partial}$$

$$= -2 \sin \frac{2x + \Delta x}{2} \sin \frac{\Delta x}{2} =$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{-2 \sin x + \frac{\Delta x}{2} \cdot \sin \frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

सूत्र तात्पर्य

$$= \lim_{\Delta x \rightarrow 0} \frac{-\sin x + \frac{\Delta x}{2} \cdot \sin \frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\sin x + \frac{\Delta x}{2} \cdot \sin \frac{\Delta x}{2}}{\frac{\Delta x}{2}} = \frac{\frac{\partial}{\partial} \frac{\Delta x}{2}}{\frac{\Delta x}{2}} = \frac{\frac{\partial}{\partial} \frac{\sin \frac{\Delta x}{2}}{\Delta x / 2}}{\frac{\Delta x}{2}}$$

$$= -\sin x$$

product rule.

Let u, v are functions of x .

$$\boxed{\frac{d(uv)}{dx} = u \cdot \frac{dy}{dx} + v \cdot \frac{du}{dx}}$$

eg: $y = x^2 \sin x$

$$\frac{d}{dx} x^2 \sin x = x^2 \cdot \frac{d \sin x}{dx} + \sin x \cdot \frac{d x^2}{dx}$$

$$= x^2 \cdot \cos x + \sin x \cdot 2x$$

• $y = x^3 \cos x$

$$\begin{aligned} A. \frac{d}{dx} x^3 \cos x &= x^3 \cdot \frac{d \cos x}{dx} + \cos x \cdot \frac{d x^3}{dx} = x^3 \cdot (-\sin x) + \cos x \cdot 3x^2 \\ &= -x^3 \sin x + \cos x \cdot 3x^2 \end{aligned}$$

• $y = (x^3 + 2x + 1) \frac{\sin x}{\cos x}$

$$\begin{aligned} \frac{d}{dx} &= (x^3 + 2x + 1) \frac{d \sin x}{dx} + \sin x \frac{d}{dx} (x^3 + 2x + 1) \\ &= (x^3 + 2x + 1) \cos x + \sin x (3x^2 + 2) \end{aligned}$$

Quotient rule

$$\boxed{\frac{d(u/v)}{dx} = v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}$$

$$\frac{d(x^2 \sin x)}{dx} = \sin x \cdot 2x + x^2 \cos x$$

$$x \sin x =$$

- Find derivative of $\tan x$ using quotient rule.

A. $y = \tan x$

$$y = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x \cdot \frac{d \sin x}{dx} - \sin x \cdot \frac{d \cos x}{dx}}{(\cos x)^2}$$

$$= \frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{\cancel{\cos^2 x}/(1+\cancel{\sin^2 x})}{\cos^2 x} = 1$$

$$= \frac{\cancel{\cos^2 x} + \cancel{\sin^2 x}}{\cos^2 x} = 1 + \frac{\cancel{\sin^2 x}}{\cos^2 x} = \sec^2 x$$

$$= \frac{1}{\cos^2 x} = \underline{\underline{\sec^2 x}}$$

- $y = \cosec x$.

$$y = \frac{1}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x \cdot 0 - \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x \cdot \sin x} = \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x} = \frac{-\cot x}{\sin^2 x} = \cot x \cdot \frac{1}{\sin^2 x} = \cot x \cdot \frac{1}{1-\cos^2 x} = \cot x \cdot \frac{1}{(1-\cos x)(1+\cos x)} = \cot x \cdot \frac{1}{2\sin^2 x} = \cot x \cdot \frac{1}{2} \cdot \frac{1}{\sin^2 x} = \cot x \cdot \frac{1}{2} \cdot \frac{1}{x^2 b^2} = \cot x \cdot \frac{1}{2x^2 b^2}$$

$$= \frac{b}{x^2 b^2} \cdot (x^2 - 1) + \cosec x \cdot \frac{1}{\sin^2 x} = \frac{b}{x^2 b^2} \cdot (x^2 - 1) + \cosec x \cdot \frac{1}{1 - \cos^2 x} = \frac{b}{x^2 b^2} \cdot (x^2 - 1) + \cosec x \cdot \frac{1}{(1 - \cos x)(1 + \cos x)} = \frac{b}{x^2 b^2} \cdot (x^2 - 1) + \cosec x \cdot \frac{1}{2\sin^2 x} = \frac{b}{x^2 b^2} \cdot (x^2 - 1) + \cosec x \cdot \frac{1}{2} \cdot \frac{1}{\sin^2 x} = \frac{b}{x^2 b^2} \cdot (x^2 - 1) + \cosec x \cdot \frac{1}{2} \cdot \frac{1}{x^2 b^2} = \frac{b}{x^2 b^2} \cdot (x^2 - 1) + \cosec x \cdot \frac{1}{2x^2 b^2}$$

- $y = \sec x \cdot x^2$

$$y = \frac{1}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x \cdot 0 + \sin x}{\cos^2 x}$$

$$\sin x = \frac{\sin x}{\cos^2 x} = \sec x \cdot \tan x$$

$$y = \cot x$$

$$y = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x \cdot -\sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{1}{\sin^2 x} = \csc^2 x$$

Results

$$1) \frac{d}{dx} \sin x = \cos x + 1 = \frac{x' \cos x}{\sin x} + \frac{\sin x \cdot 0}{\cos^2 x} =$$

$$2) \frac{d}{dx} \cos x = -\sin x \quad x' \cos x = \frac{1}{\cos x}$$

$$3) \frac{d}{dx} \tan x = \sec^2 x$$

$$4) \frac{d}{dx} \sec x = \sec x \cdot \tan x$$

$$5) \frac{d}{dx} \csc x = -\csc x \cdot \cot x$$

$$6) \frac{d}{dx} \cot x = -\csc^2 x \quad x' \csc x = \frac{1}{\csc x} = \frac{1}{\sin x}$$

$$y = (x^2 + x + 3)(3x^4 - 5x^3)$$

$$\frac{dy}{dx} = (x^2 + x + 3) \frac{d}{dx} (3x^4 - 5x^3) + (3x^4 - 5x^3) \cdot \frac{d}{dx} (x^2 + x + 3)$$

$$= (x^2 + x + 3) \cdot (12x^3 - 15x^2) + (3x^4 - 5x^3) (2x + 1) = 0$$

$$= \cancel{x^2 + x + 3}$$

$$\frac{1}{\sin x + 0 \cdot \csc x} = \frac{1}{\sin x} = \frac{b}{ab}$$

$$\bullet y = (x^2 + 3\sqrt{x} + 1) \tan x$$

$$\frac{dy}{dx} = (x^2 + 3\sqrt{x} + 1) \cdot \frac{d}{dx} \tan x + \tan x \cdot \frac{d}{dx} (x^2 + 3\sqrt{x} + 1)$$

$$= (x^2 + 3\sqrt{x} + 1) \sec^2 x + \tan x \left(2x + \frac{3}{2\sqrt{x}} + 0 \right).$$

$$\bullet y = \sqrt{x} \cdot \operatorname{cosec} x$$

$$\frac{dy}{dx} = \sqrt{x} \cdot \frac{d}{dx} \operatorname{cosec} x + \operatorname{cosec} x \cdot \frac{d}{dx} \sqrt{x}$$

$$= \sqrt{x} \cdot -\operatorname{cosec} x \cdot \cot x + \operatorname{cosec} x \cdot \frac{1}{2\sqrt{x}}$$

$$\bullet y = \frac{8}{x^2} - 2 \cot x$$

$$\frac{dy}{dx} = 8 \cdot x^{-2} - 2 \cot x$$

$$= 8 \cdot \frac{x-2}{x^3} - 2 \operatorname{cosec}^2 x$$

$$= 2 \operatorname{cosec}^2 x - \frac{16}{x^3}$$

$$\bullet y = \frac{x^2 - 1}{x^2 + 1}$$

$$\frac{d}{dx} = (x^2 + 1) \cdot \frac{d}{dx} (x^2 - 1) - (x^2 - 1) \cdot \frac{d}{dx} (x^2 + 1)$$

$$(x^2 + 1)^2$$

$$= \frac{(x^2 + 1) \cdot 2x - (x^2 - 1) \cdot 2x}{(x^2 + 1)^2}$$

$$= \frac{2x(x^2 + 1 - (x^2 - 1))}{(x^2 + 1)^2} = \frac{2x(2)}{(x^2 + 1)^2}$$

$$= \frac{2x \cdot 2}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2}$$

$$= \frac{4x}{(x^2 + 1)^2} \cdot \operatorname{cosec} x + \frac{1}{x} \cdot \operatorname{cosec} x = \frac{4x}{x^4 + 2x^2 + 1} \cdot \operatorname{cosec} x + \frac{1}{x} \cdot \operatorname{cosec} x =$$

$$\bullet y = (x^2 + 2x + 3)(x^2 - 4).$$

$$y = \frac{1-x^2}{1+2x} \Rightarrow y = \frac{(1-x^2)(1+2x)}{1+2x} = 1-x^2 \Rightarrow (1+2x)^{-1} = \frac{1}{1+2x}$$

$$y = \frac{x^4+x^2+1}{x^2+2x+3} = x^2(1+2x) + \frac{1}{1+2x} \Rightarrow (1+2x)^{-2} = \frac{1}{(1+2x)^2}$$

Derivative of $\log x$ & e^x

$$1) \frac{d}{dx} \log x = \frac{1}{x}$$

$$2) \frac{d}{dx} e^x = e^x$$

$$\bullet \text{Find } \frac{d}{dx} ? \quad y = x^2 \log x.$$

$$\begin{aligned} \frac{dy}{dx} &= x^2 \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} x^2 \\ &= x^2 \cdot \frac{1}{x} + \log x \cdot 2x \\ &= x + \log x \cdot 2x \end{aligned}$$

$$\bullet y = x^3 \cdot e^x$$

$$\begin{aligned} \frac{dy}{dx} &= x^3 \cdot \frac{d}{dx} e^x + e^x \cdot \frac{d}{dx} x^3 \\ &= x^3 \cdot e^x + e^x \cdot 3x^2 \\ &= e^x \underbrace{\left(x^3 + 3x^2\right)}_{\frac{1}{(1+x)}} = \frac{(1+x) - 1}{(1+x)} x^2 = \end{aligned}$$

$$\bullet y = \cos x \cdot \log x$$

$$\frac{dy}{dx} = \cos x \cdot \frac{1}{x} + \log x \cdot \frac{-\sin x}{(1+x)} = \frac{\cos x}{x} - \frac{\sin x \log x}{(1+x)} =$$

$$= \frac{\cos x}{x} - \underline{\sin x \cdot \log x}$$

$$\frac{1+\sin^2 x}{\sin x + \cos x} = u$$

$$\begin{aligned} * y &= (x^2+x) \log x \\ &= (x^2+x) \cdot \frac{1}{x} + \log x (2x+1) \\ &= \underline{x^2+x} + \log x (2x+1) \\ &= \underline{x(x+1)} + \log x (2x+1) \\ &= \underline{x+1} + \log x (2x+1) \end{aligned}$$

$$* y = x^2 \sec x$$

$$= x^2 (\sec x \tan x) + \sec x \cdot 2x \underline{(x \sec x + x)}$$

$$\begin{aligned} * y &= (x^2 + e^x) \tan x \\ &= (x^2 + e^x) \cdot \sec^2 x + \tan x \cdot (2x + e^x) \end{aligned}$$

$$\begin{aligned} * y &= (x^2 + 2x + 3)(x^2 - 4) \\ &= (x^2 + 2x + 3) \cdot 2x + \underline{(x^2 - 4) \cdot (2x + 4)} \end{aligned}$$

$$\begin{aligned} * y &= \frac{1-x^2}{1+x^2} \\ &= \frac{(1+x^2)(0-2x) - (1-x^2) \cdot (0+2x)}{(1+x^2)^2} \\ &= \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} = \frac{-2x[1+x^2 + (x^2-1)]}{(1+x^2)^2} \\ &= \frac{-2x(2x^2)}{(1+x^2)^2} = \frac{-4x}{(1+x^2)^2} \end{aligned}$$

$$\bullet y = \frac{x^2+x+1}{x^2+2x+3}$$

$$\text{LHS} = \frac{\cos x}{x}$$

$$\Rightarrow \frac{(x^2+2x+3)(2x+1) - x^2+x+1 \cdot (2x+2)}{(x^2+2x+3)^2} =$$

$$= \frac{(1+x^2) \cancel{2x^2} + \cancel{x^2+3x+3}}{(1+x^2) \cancel{2x^2} + \cancel{x^2+3x+3}} =$$

$$\bullet \text{Find } \frac{dy}{dx} \text{ if } y = \frac{\cos x}{x+\sin x} (1+\cos x) \times \cancel{2x} + \frac{(1+\cos x)x}{x} =$$

$$\text{A. } \frac{dy}{dx} = \frac{(x+\sin x) \cancel{\frac{d}{dx} \cos x} - \cos x \cancel{\frac{d}{dx} (x+\sin x)} + x}{(x+\sin x)^2} =$$

$$= \frac{(x+\sin x) \cancel{\sin x} - \cos x (1+\cos x)}{(x+\sin x)^2} =$$

$$= \frac{-x \cdot \sin x - \sin^2 x - \cos x + \cos^2 x}{(x+\sin x)^2} = \text{B.}$$

$$= \frac{-x \sin x - \cos x - (\sin^2 x + \cos^2 x)}{(x+\sin x)^2} =$$

$$= \frac{-x \sin x - \cos x - 1}{(x+\sin x)^2} + x \cancel{\sin} \cdot (1+\cancel{\sin x}) =$$

$$\bullet y = \frac{x^3+2x^2+5}{x^3} \cdot (x-1) - (x^2-1) \cdot (x+1) =$$

$$\frac{dy}{dx} = \frac{x^3 \cdot \cancel{\frac{d}{dx} (x^3+2x^2+5)} - (x^3+2x^2+5) \cancel{\frac{d}{dx} x^3}}{(x+1)^2} =$$

$$= \frac{5x^2(3x^2-2) - (x^3+2x^2+5) \cdot 3x^2}{(x+1)^2} =$$

$$= \frac{15x^4 - 10x^2 - 3x^5 - 15x^4}{(x+1)^2} =$$

$$= \frac{-10x^2 - 3x^5}{(x+1)^2} =$$

$$\begin{aligned}
 &= \frac{3x^5 - 2x^3 - (x^3 - 2x + 5) \cdot 3x^2}{x^6} = \frac{3x^5 - 2x^3 - 3x^5 + 6x^3 - 15x^2}{x^6} = \frac{4x^3 - 15x^2}{x^6} = \frac{x^2(4x - 15)}{x^6} = \frac{4x - 15}{x^4} \\
 &= \frac{4x^3 - 15x^2}{x^6} \cdot \frac{1}{x^4} = \frac{4x - 15}{x^8} = \frac{4x - 15}{x^8} = 0 \quad (\text{d/m})
 \end{aligned}$$

$$\begin{aligned}
 \bullet y &= \frac{e^x}{\sqrt{x}} \quad \frac{(1-xe^{-x})}{(\sqrt{x}-xe^{-x})} = \frac{1+xe^{-x}}{(\sqrt{x}-xe^{-x})} = \frac{1}{x^{1/2}} = \frac{1}{x^{1/2}} \\
 \frac{dy}{dx} &= \frac{\sqrt{x} \cdot e^x - e^x \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = \frac{(\sqrt{x}e^x - \frac{1}{2\sqrt{x}})(\sqrt{x}e^x + \frac{1}{2\sqrt{x}})}{2\sqrt{x}\sqrt{x}} = \frac{(\sqrt{x}e^x - \frac{1}{2\sqrt{x}})(\sqrt{x}e^x + \frac{1}{2\sqrt{x}})}{2\sqrt{x}\sqrt{x}} = \frac{e^x(\frac{2x-1}{2\sqrt{x}})}{x} = \frac{e^x(\frac{2x-1}{2\sqrt{x}})}{x} = \frac{e^x(2x-1)}{2x^{3/2}}
 \end{aligned}$$

Function of a function rule

Let $y = f(u)$, where $u = g(x)$. Then there will be function rule.

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

$\frac{dy}{dx}$ = outer function $\cdot \frac{du}{dx}$ inner function

$$\frac{1}{(e+x^2)^2} = 0$$

$$\frac{1}{(e+x^2)\sqrt{x}} = 0$$

$$\text{eg: 1) } y = \sqrt{x^2 + 2x + 1} \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x^2 + 2x + 1}} \cdot (2x + 2) = \frac{2x + 2}{2\sqrt{x^2 + 2x + 1}}$$

$$2) y = \sin(x^2)$$

$$\frac{dy}{dx} = \cos(x^2) \cdot 2x = \frac{\cos(x^2) \cdot 2x}{x^2}$$

$$3) y = \frac{1}{x^2 + 2x + 1}$$

$$= \frac{-1}{(x^2 + 2x + 1)^2} \cdot 2x + 2 = \frac{-(2x + 2)}{(x^2 + 2x + 1)^2}$$

$$y = \sin(x^2 + 3x + 7) \quad \sin(u) = \cos u \cdot \frac{du}{dx}$$

$$= \cos(x^2 + 3x + 7) \cdot (2x + 3) \quad \left(\frac{1}{\sin u} - \frac{1}{\cos u}\right) x_3 - \frac{1}{\sin u} =$$

$$y = \log e^x \quad \log e^x = \log x = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{e^x} \cdot e^x = \frac{1}{\infty} \quad \frac{(1-xe^x)^{x_3}}{\infty} = x^{10}$$

$$y = e^{\sin x} \quad e^{\sin x} \Rightarrow e^x$$

$$\frac{dy}{dx} = e^{\sin x} \cdot \cos x \quad \frac{(1-xe^x)x_3}{e^{\sin x}} =$$

$$y = (3x^2 + 4x + 7)^{10}$$

$$= 10 \cdot (3x^2 + 4x + 7)^9 \cdot (6x + 4)$$

$$y = \frac{1}{(2x^2 + 3)^7} \quad \frac{1}{x^n} = x^{-n} \Rightarrow -7 \cdot x^{(-7)-1} = -7 \cdot x^{-8} = \frac{7}{x^8}$$

$$= \frac{1}{2\sqrt{(2x^2 + 3)^7}}$$

$$= \frac{-1}{(2x^2+3)^2} \cdot 4x$$

$$= \frac{-2x^3}{(2x^2+3)^3}$$

$$\bullet y = \frac{1}{(2x^2+3x+5)}$$

$$= \frac{-1}{(2x^2+3x+5)^2} (4x+3) = \frac{(4x+3)}{(2x^2+3x+5)^2} \cdot \frac{d}{dx}(1+x^2) = 0$$

$$\bullet y = \log(\sec x + \tan x)$$

$$= \frac{1}{\sec x + \tan x} \cdot \sec x \cdot \tan x + \sec^2 x$$

$$= \frac{1}{\sec x + \tan x} \cdot \sec x \cdot (\tan x + \sec x)$$

$$= \frac{\sec x}{\sec x + \tan x} \cdot \sec x \tan x + \sec x \tan x \cdot \sec x$$

$$\bullet y = \sin^2(5x), \quad \text{ज्येष्ठा ज्यानिका} + \text{ज्येष्ठा ज्यानिका} =$$

$$= (\sin(5x))^2, \quad (\text{ज्येष्ठा}) \text{ ज्येष्ठा} =$$

$$= 2(\sin(5x)) \cdot \cos(5x) \cdot 5$$

$$= 10 \sin x(5x) \cdot \cos(5x) \quad (\text{ज्येष्ठा}, 5 = 5)$$

$$= -5 \sin(5x) \cdot (5x) \cos(5x) + 5 \cos(5x) \cdot 5x \cdot (-5x)$$

$$\bullet y = \sqrt{\tan(2x+1)} \quad (3x) \text{ ज्येष्ठा} \cdot \sec x + \sec x \cdot \sec x =$$

$$= \frac{1}{2\sqrt{\tan(2x+1)}} \cdot \sec^2(2x+1) \cdot 2,$$

$$= \frac{2\sec^2(2x+1)}{2\sqrt{\tan(2x+1)}}$$

$$\bullet y = \log(\log x)$$

$$= \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$$

$$\bullet y = e^{(ax+b)}$$

$$= e^{(ax+b)} \cdot a$$

$$\bullet y = (x^2+1)^{10} \cdot \sec 5x$$

$$= (x^2+1)^{10} \cdot \frac{d}{dx} \sec 5x + \sec 5x \cdot \left(\frac{d}{dx} (x^2+1)^{10} \right) \text{sol} = 0$$

$$= (x^2+1)^{10} \cdot \sec 5x \cdot \tan(5x) + \sec 5x \cdot 10(x^2+1)^9 \cdot 2x$$

$$= 5(x^2+1)^9 \sec 5x \cdot \tan 5x + 20x(x^2+1)^9 \sec 5x$$

$$\bullet y = \sin 3x \cdot \sin 5x$$

$$= \sin 3x \cdot 5 \cos 5x + \sin 5x \cdot 3 \cos 3x$$

$$= 5 \sin 3x \cdot \cos 5x + 3 \sin 5x \cdot \cos 3x$$

$$= \cancel{\sin 3x \cdot \cos 5x} (2) \quad \cancel{\sin 5x \cdot \cos 3x} (2)$$

$$\bullet y = x^2 \cdot \cos(x^2)$$

$$= x^2 \cdot 2x \cdot -\sin(x^2) + \cos(x^2) \cdot 2x$$

$$= -2x^3 \cdot \sin(x^2) + 2x \cos(x^2)$$

$$\bullet y = x^2 \cdot \sec x$$

$$\begin{aligned}
 y &= \frac{\cot \pi x}{(x^3-1)^2} \\
 &= \frac{(x^3-1)^2 \cdot \frac{d}{dx} \cot \pi x - \cot \pi x \cdot \frac{d}{dx} (x^3-1)^2}{(x^3-1)^4} \\
 &= \frac{(x^3-1)^2 \cdot \csc^2(\pi x) \cdot \pi - \cot \pi x \cdot 2(x^3-1) \cdot 3x^2}{(x^3-1)^4} \\
 &= \frac{-\pi (x^3-1)^2 \csc^2(\pi x) - 6x^2 \cot \pi x \cdot \cot \pi x}{(x^3-1)^4} \\
 &= \frac{(x^3-1)^3 \left[-\pi (x^2-1) \csc^2(\pi x) - 6x^2 (\cot \pi x)^2 \right]}{(x^3-1)^4} \\
 &= \dots
 \end{aligned}$$

Derivative of inverse trigonometric functions

$$\begin{aligned}
 y &= \frac{e^{2x} \log x}{x^2} \\
 f &= \frac{x^2 / 2 \cdot e^{2x}}{(x^2)^2} = \frac{e^{2x} - \log x \cdot 2x}{x^4} \\
 &= \dots
 \end{aligned}$$

$$(x)^{\frac{1}{2}} = 5^{\circ}$$

$$\frac{dx}{dx-1} = \frac{dx}{(x-1)^2} = \frac{1}{x-1} = \frac{1}{5^{\frac{1}{2}}} = \frac{1}{\sqrt{5}}$$

$$\bullet y = \frac{e^{2x} \cdot \log x}{x^2}$$

$$= x^2 \cdot \frac{d}{dx}(e^{2x} \cdot \log x) = \frac{e^{2x} \cdot \log x \cdot \frac{d}{dx}(\text{Cact}) + (1+x^2)}{(x^2)^2} \cdot \frac{d}{dx}(1+x^2)$$

$$= x^2 \cdot \frac{e^{2x} \cdot \log x \cdot 2 + (1+x^2) \cdot e^{2x} \cdot \frac{1}{x}}{(x^2)^2} \cdot \frac{d}{dx}(1+x^2)$$

$$= x^2 \cdot \left(e^{2x} \cdot \frac{1}{x} + \log x \cdot e^{2x} \cdot 2 \right) \cdot \frac{e^{2x} \cdot \log x \cdot 2x}{(x^2)^2} \cdot \frac{d}{dx}(1+x^2)$$

$$= \frac{x^2 \cdot \left(e^{2x} \cdot \frac{1}{x} + \log x \cdot e^{2x} \cdot 2 \right) \cdot (e^{2x} \cdot \log x \cdot 2x)}{x^4 \cdot (1+x^2)^2} \cdot \frac{d}{dx}(1+x^2)$$

$$= \frac{\left(x^2 \cdot \log x \cdot x^2 + (x^2)^2 \log x \cdot (1+x^2) \cdot 2x \right) \cdot (e^{2x} \cdot \log x \cdot 2x)}{x^4 \cdot (1+x^2)^2} \cdot \frac{d}{dx}(1+x^2)$$

Inverses of

Derivatives of inverse trigonometric functions

$$1) \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$2) \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$3) \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\bullet y = \sin^{-1}(x^2)$$

$$A. \frac{dy}{dx} = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

$$\bullet y = -\arcsin(\sqrt{x})$$

$$\frac{dy}{dx} = \frac{1}{1-x} \cdot \frac{1}{2\sqrt{x}} \cdot \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} \quad y = \log(\sin(x^2+a^2))$$

$$= \frac{1}{1+x(2\sqrt{x})} \quad y = e^{2x}$$

$$(x^2+a^2) \cdot \sin(x^2+a^2) + \frac{1}{x+1} (2x+a) y = \frac{\sin(\log x)}{x}$$

$$1) \text{ a) } y = \log(\sin(x^2+a^2))$$

$$= \frac{1}{\sin(x^2+a^2)} \cdot \cos(x^2+a^2) \cdot 2x \quad y = (1+x^2) \tan(x)$$

$$= \frac{2x \cdot \cos(x^2+a^2)}{\sin(x^2+a^2)} \quad y = \frac{e^{2x} + \tan x}{\sin x}$$

$$= 2x \tan(x^2+a^2)$$

$$2) y = e^{\cos x} \cdot \frac{\sin x}{\cos x} = (\sec x)^{-1} \cdot \frac{\sin x}{\cos x} = \frac{1}{\sqrt{1-x^2}}$$

$$= e^{\cos x} \cdot -\sin x \quad \cos^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$3) y = e^{2x} \cdot \cos(3x) \quad \frac{d}{dx} \cos^{-1} x = \frac{1}{1-x^2}$$

$$= e^{2x} \cdot \frac{d}{dx} \cos(3x) + \cos(3x) \cdot \frac{d}{dx} e^{2x}$$

$$= e^{2x} \cdot -7\sin(3x) \cdot 3 + \cos(3x) \cdot e^{2x} \cdot 2$$

$$= -2 \cdot e^{2x} \cdot \cos(3x) - 3 \cdot e^{2x} \sin(3x)$$

$$4) y = \frac{\sin(\log x)}{x}$$

$$= x \cdot \cos(\log x) \cdot \frac{1}{x} - \sin(\log x) \cdot \frac{dy}{dx} \quad !$$

$$= \frac{\cos(\log x) - \sin(\log x)}{x^2(1+x^2)}$$

5) $y = (1+x^2) \cdot \tan^{-1} x$

$$= (1+x^2) \frac{1}{1+x^2} + \tan^{-1} x \cdot (2x)$$

$$= 1 + \tan^{-1} x \cdot 2x$$

6) $y = \frac{e^{2x} \tan^{-1} 3x}{\sqrt{x}}$

$$\frac{d}{dx} = \sqrt{x} \cdot \frac{d}{dx} (e^{2x} \tan^{-1} 3x) - e^{2x} \tan^{-1} 3x \cdot \frac{d}{dx} \sqrt{x}$$

$$\frac{d}{dx} = x^{\frac{1}{2}} \cdot (x^{\frac{1}{2}})^{-\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{1}{x} \cdot x^{\frac{1}{2}} = \sqrt{x} \left[e^{2x} \cdot \frac{1}{1+(3x)^2} \cdot 3 + \tan^{-1} 3x \cdot e^{2x} \cdot 2 \right] - \frac{e^{2x} \tan^{-1} 3x}{2\sqrt{x}}$$

$$= \sqrt{x} \cdot \left[\frac{3e^{2x}}{1+9x^2} + 2e^{2x} \tan^{-1} 3x \right] - \frac{e^{2x} \tan^{-1} 3x}{2\sqrt{x}}$$

$$(x^{\frac{1}{2}})^{-\frac{1}{2}} \cdot (x^{\frac{1}{2}})^{-\frac{1}{2}} = \frac{1}{x}$$

$$\frac{(x^{\frac{1}{2}})^{-\frac{1}{2}}}{x} = \frac{1}{x}$$

$$\frac{1}{x} \cdot (x^{\frac{1}{2}})^{-\frac{1}{2}} - \frac{1}{x} \cdot (x^{\frac{1}{2}})^{-\frac{1}{2}} \cdot x =$$

$$\bullet y = \frac{\sin^3 \sqrt{x}}{x^3}$$

$$\frac{dy}{dx} = \frac{x^3 \cdot \frac{d}{dx} \sin^3 \sqrt{x} - \sin^3 \sqrt{x} \cdot \frac{d}{dx} x^3}{(x^3)^2}$$

$$= x^3 \cdot \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \sin^3 \sqrt{x} \cdot 3x^2 - \frac{\sin^3 \sqrt{x} \cdot 3x^2}{x^6}$$

$$= \frac{x^3}{2\sqrt{1-x}} - \frac{\sin^3 \sqrt{x} \cdot 3x^2}{x^6}$$

$$\bullet y = \frac{x \cdot \sec x}{3x+2}$$

$$= \frac{3x+2}{3x+2} \cdot \frac{d}{dx} x \sec x - x \sec x \cdot \frac{d}{dx} \frac{3x+2}{3x+2}$$

$$= (3x+2) \left[x \cdot \sec x \cdot \tan x + \sec x \right] - x \cdot \sec x \cdot 3$$

$$= (3x+2) \left[x \sec x \tan x + (3x+2) \sec x - 3x \sec x \right]$$

$$= (3x+2) \left[x \sec x \cdot \tan x + 3x \sec x + 2 \sec x - 3x \sec x \right]$$

$$= (3x+2) \left[x \sec x \cdot \tan x + 2 \sec x \right]$$

$$= (3x+2) \left[x \sec x \cdot \tan x + 2 \sec x \right]$$

$$\bullet y = (1-x^2) \sin^{-1}(x).$$

$$= (1-x^2) \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}(x) \cdot -2x$$

$$= \frac{1-x^2}{\sqrt{1-x^2}} - 2x \cdot \sin^{-1}(x)$$

$$\bullet y = ((x+1)(x+2))^{2x^2+1}.$$

$$= (x+1)(x+2) \cdot \frac{d}{dx} (2x^2+1) + (2x^2+1) \frac{d}{dx} (x+1)(x+2)$$

$$= (x+1)(x+2) \cdot 4x + (2x^2+1) \left((x+1) \cdot \frac{d}{dx} (x+2) + (x+2) \cdot \frac{d}{dx} (x+1) \right)$$

$$= (x+1)(x+2) \cdot 4x + (2x^2+1) [(x+1) \cdot 1 + (x+2) \cdot 1]$$

$$= 4x \cdot (x+1)(x+2) + (2x^2+1)(2x+3)$$

$$= 4x(x^2+3x+2) + 4x^3 + 6x^2 + 2x + 3$$

$$= 4x^3 + 12x^2 + 8x + 4x^3 + 6x^2 + 2x + 3$$

$$= 8x^3 + 18x^2 + 10x + 3$$

$$\bullet x \log x \cdot \sin x.$$

$$\bullet y = (x \log x) \sin x$$

$$= (x \log x) \cdot \frac{d}{dx} \sin x + \sin x \cdot \frac{d}{dx} (x \log x)$$

$$= x \log x \cdot \cos x + \sin x \cdot$$

$$= x \log x \cdot \cos x + \sin x \left[x \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} x \right]$$

$$= x \log x \cdot \cos x + \sin x \left[x \cdot \frac{1}{x} + \log x \cdot 1 \right]$$

$$= \sin x [1 + \log x].$$

$$\bullet y = x \cdot (x+2) \cdot (x+5).$$

$$\frac{dy}{dx} \cdot y^2 = \frac{\partial F}{\partial x} : C$$

Derivative of Implicit functions

$$\begin{aligned}
 \frac{dy}{dx} &= x(x+2) \cdot \frac{d}{dx}(x+5) + (x+5) \cdot \frac{d}{dx}x(x+2) \\
 &= x(x+2) \cancel{(+)} + (x+5) \cdot \frac{d}{dx}(x^2+2x) \\
 &= x(x+2) \cancel{(+)} + (x+5)(2x+2) \\
 &= \cancel{6x^2+12x} + (x+5)(2x+2) \\
 &= \cancel{6x^2+12x} + (x+5)2x + (x+5)2 \\
 &= \cancel{6x^2+12x} + 2x^2+10x+2x+10 \\
 &= \underline{2x^2+8x+10} \\
 &= x^2+2x+2x^2+12x+10 \\
 &= \underline{3x^2+14x+10}
 \end{aligned}$$

Derivative of Implicit function

A function of the form $f(x,y) = C$ is known as an implicit function.

$$\text{eg: } x^2+y^2=1, \quad \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$$

$$x^2+2xy+y^2 = (x+y)^2 \Rightarrow \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$$

$\frac{d}{dx} F(y) \Rightarrow \frac{d}{dy}(F(y)) \cdot \frac{dy}{dx} = \frac{\partial F}{\partial x}$
--

$$(x+y)(s+x) \cdot s = b$$

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15 C

$$\text{eg: } \frac{d}{dx} y^2 = 2y \cdot \frac{dy}{dx}$$

$$\frac{d}{dx} \sin y = \cos y \cdot \frac{dy}{dx}$$

$$(x+y) \frac{d}{dx} (y^2 + 2y) + (2y+2) \frac{dy}{dx} = \frac{b}{x+b}$$

$$\bullet \text{Find } \frac{d}{dx} \text{ if } x^2 + y^2 = 1. \quad (\text{---}) \cdot (s+x) x =$$

$$\text{A. } \cancel{\text{Diff: wrt } x} \quad (\text{---}) \cdot (s+x) x =$$

$$2x + 2y \cdot \frac{dy}{dx} + (xs + s^2) x =$$

$$-2x = 2y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\bullet x^2 + 2xy + y^2 = 2 \quad (s^2 + xs + s^2) + (xs + s^2) x =$$

$$2x + 2y \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 0 \quad + s^2 x =$$

$$2x + 2 \left[x \cdot \frac{dy}{dx} + y \cdot \frac{dy}{dx} \right] = 0 \quad + 2y \cdot \frac{dy}{dx} = 0.$$

$$\bullet \text{Divide by } 2 \frac{dy}{dx} \quad 2x + 2x \cdot \frac{dy}{dx} + 2y + 2y \cdot \frac{dy}{dx} = 0 \quad \text{about A}$$

$$2x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = -2x - 2y \quad \text{about B}$$

$$\frac{dy}{dx} (2x + 2y) = -(2x + 2y) \quad \text{about C}$$

$$\frac{dy}{dx} = -\frac{1}{2} \cdot \frac{b}{x+b} \cdot \frac{(s+b)}{b} \cdot \frac{b}{x+b}$$

$$x^3 + y^3 = 3axy \quad d,d,0 \quad 0 = xy + 3x^2y + 3xy^2$$

$$xy = c^2.$$

$$x \cdot \frac{dy}{dx} + y = 0 \quad \frac{xy - y^2}{x^2} + \frac{3x^2y + 3xy^2}{x^2} = 0$$

$$0 = \frac{dy}{dx} \cdot \frac{-y}{x^2} + \left(3y + \frac{3y^2}{x} \right) \quad \text{(cancel common terms)}$$

$$A. \quad 3x^2 + 3y^2 \frac{dy}{dx} = 3a \left(x \cdot \frac{dy}{dx} + y \right)$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 3a \cdot x \cdot \frac{dy}{dx} + 3ay$$

$$3x^2 + (3y - 3ax) \cdot \frac{dy}{dx} = 3ay$$

$$\frac{dy}{dx} = \frac{3ay - 3x^2}{3y - 3ax}$$

$$B. \quad \frac{dy}{dx} = \frac{(3y^2 - 3ax)}{(3y - 3ax)} \cdot \frac{xy}{xy}$$

(i.e. Prod) $\frac{d}{dx} \left(\frac{xy}{xy} \right) \text{ If } x^2 + y^2 + 2gx + 2fy + c = 0 \quad (g, f, \text{ & } c \text{ are constants})$

A. diff. wrt x.

$$ye + \frac{eb}{x^2} xe + \frac{eb}{x^2} ye + xe = xc \cdot y + \frac{eb}{x^2} ye \cdot x$$

$$2x + 2y \cdot \frac{dy}{dx} + 2g + 2f \frac{dy}{dx} + 0 = 0 \Rightarrow 0.$$

$$ye + \frac{eb}{x^2} xe + \frac{eb}{x^2} ye + xe = \frac{eb \cdot xc}{x^2} - \frac{eb}{x^2} \frac{dy}{dx}$$

$$2y \cdot \frac{dy}{dx} + 2f \cdot \frac{dy}{dx} = -2x - 2g$$

$$ye + \frac{eb}{x^2} xe + \frac{eb}{x^2} ye + xe = \frac{(xe - ye - eb)}{x^2}$$

$$(2y + 2f) \frac{dy}{dx} = -2(x + g).$$

$$\frac{ye + \frac{eb}{x^2} xe + \frac{eb}{x^2} ye + xe}{xe - ye - eb} = \frac{eb}{x^2}$$

$$\frac{2(y+f)}{xe - ye - eb} \frac{dy}{dx} = -\frac{eb}{x^2}(x+g)$$

$$\frac{dy}{dx} = \frac{-(x+g)}{y+f}$$

• $ax^2 + 2hxy + by^2 = 0$. (a, h, b are constant)

A. ~~$a \cdot 2x + 2h \cdot \frac{dy}{dx} + b \cdot 2y \cdot \frac{dy}{dx} = 0$~~

$$a \cdot 2x + 2h \left(x \frac{dy}{dx} + y \cdot 1 \right) + b \cdot 2y \cdot \frac{dy}{dx} = 0.$$

(Dividing with 2)

$$ax + hx \cdot \frac{dy}{dx} + hy + by \cdot \frac{dy}{dx} = 0 \quad \text{--- (1)}$$

($\cancel{hx \cdot \frac{dy}{dx}} + \cancel{by \cdot \frac{dy}{dx}}$) $\cancel{DE} = \cancel{xp}$

$$(hx + by) \frac{dy}{dx} = -ax - hy -$$

$\cancel{xp} + \cancel{hy}$

$$y^2 \cdot \frac{dy}{dx} = \frac{dy}{dx} \frac{(x - (ax + by))}{hx + by} + \frac{xp}{hx + by}$$

$$\frac{y^2 \cdot \frac{dy}{dx} - \frac{xp}{hx + by}}{y^2}$$

• Find $\frac{dy}{dx}$ $\frac{(xp - x^2y^2)}{y^2}$ $= x^3 + y^3 + 3xy$.

(product rule)

A.

$$\frac{x^2 \cdot dy}{dx} y^2 + y^2 \cdot \frac{d}{dx}(x^2) = 3x^2 + 3y^2 \cdot \frac{dy}{dx} + 3 \left(x \cdot \frac{dy}{dx} + y \cdot 1 \right)$$

$$x^2 \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 2x = 3x^2 + 3y^2 \cdot \frac{dy}{dx} + 3x \cdot \frac{dy}{dx} + 3y$$

$$2x^2y \cdot \frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx} - 3x \cdot \frac{dy}{dx} = 3x^2 + 3y - 2xy^2$$

$$\frac{dy}{dx} (2x^2y - 3y^2 - 3x) = \frac{3x^2 + 3y - 2xy^2}{2x^2y + 3y^2 - 3x}$$

$$\frac{dy}{dx} = \frac{3x^2 + 3y - 2xy^2}{2x^2y + 3y^2 - 3x}$$

$$\frac{dy}{dx} = \frac{3x^2 + 3y - 2xy^2}{2x^2y + 3y^2 - 3x}$$

$$\frac{dy}{dx} = \frac{3x^2 + 3y - 2xy^2}{2x^2y + 3y^2 - 3x}$$

parametric Equations

An equ. of the form $x = f(t)$, $y = g(t)$ is known as a parametric equ. with parameter t .

$$\boxed{\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}} \quad \text{or} \quad \frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

- Find $\frac{dy}{dx}$ if $x = at^2$, $y = 2at$

A. $\frac{dy}{dt} = 2a \cdot \frac{dt}{dt} = 2a \cdot 1 = 2a.$

$$\frac{dx}{dt} = a \cdot 2t = 2at \cdot (a + x) =$$

$$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t(a+x)} = \frac{1}{ta}$$

- Find $\frac{dy}{dx}$, if $x = a \sin^3 t$, $y = b \cos^3 t$.

$$\frac{dy}{dt} = b \cdot 3 \cos^2 t \cdot -\sin t$$

$$= -3b \cos^2 t \cdot \sin t$$

$$\frac{dx}{dt} = a \cdot 3 \sin^2 t \cdot \cos t$$

$$= 3a \sin^2 t \cdot \cos t$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{-3b \cos^2 t \cdot \sin t}{3a \sin^2 t \cdot \cos t} \\ &= -\frac{b}{a} \cot t \end{aligned}$$

- 51 $\frac{dy}{dx} = \cot \frac{\theta}{2}$, if $x = a(\alpha - \sin \theta)$, $y = a(1 - \cos \theta)$

A. $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.



$$\frac{dy}{d\theta} = \frac{d}{d\theta}(1 - \cos \theta).$$

$$\Rightarrow a(\alpha + \sin \theta) = a \sin \theta,$$

$$\frac{dx}{d\theta} = \frac{d\alpha(\alpha - \sin \theta)}{d\theta} = \alpha(1 - \cos \theta),$$

$$\left(\frac{dy}{dx} \right) = \frac{a \sin \theta \cdot (s+1)}{a(1 - \cos \theta)} = \frac{\sin \theta \cdot (s+1)}{1 - \cos \theta}.$$

$$\sin \theta = \sin 2(\theta/2) = 2 \cdot \sin \theta/2 \cdot \cos \theta/2$$

$$\sin 2x = 2 \sin x \cos x \cdot (s+1)$$

$$1 - \cos \theta = 1 - \cos 2(\theta/2) = 2 \sin^2 \theta/2$$

$$1 - \cos 2x = 2 \sin^2 x$$

$$\frac{dy}{dx} = \frac{2 \sin \theta/2 \cdot (\cos \theta/2)}{2 \sin^2 \theta/2} = \frac{\cot \theta/2}{s+1}$$

* Find $\frac{dy}{dx}$ if $x = a \sec \theta$, $y = b \tan \theta$.

A. $\frac{dx}{d\theta} = \frac{d a \sec \theta}{d\theta} = a \sec \theta \cdot \tan \theta =$

$$\frac{dy}{d\theta} = \frac{d b \tan \theta}{d\theta} = b \sec^2 \theta$$

$$(x^2 - 1)^2 \frac{dy}{dx} = \frac{a \sec \theta + \tan \theta}{a b \sec^2 \theta} \quad \text{to,} \\ \frac{dy}{dx} = \frac{a \sec \theta + \tan \theta}{a b \sec^2 \theta} = \frac{b \sec \theta}{a \sec \theta} =$$

$$= \frac{b \sec \theta}{a + \tan \theta} =$$

$$\frac{(x^2 - 1) \cdot \frac{1}{\cos \theta}}{a \cdot \frac{\sin \theta}{\cos \theta}} \frac{\frac{dy}{dx}}{dt} = \frac{b}{a} \cosec \theta.$$

$$\text{Diseño} = (x^2 + 1) D \cosec \theta.$$

$$x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2} \quad \text{diseño} \frac{dy}{dx} = \frac{xy}{ab}$$

$$A. \frac{dx}{dt} = \frac{(1+t^2) \cdot \frac{d}{dt}(1-t^2) - (1-t^2) \cdot \frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= (1+t^2) \cdot 2t - (1-t^2) \cdot 2t = 0$$

$$\text{Diseño} = ((1+t^2)^2 - 1) = 1 - 1 = 0$$

$$= \frac{-2t(1+t^2 + 1-t^2)}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{(1+t^2) \cdot 2 - 2t \cdot 2t}{(1+t^2)^2} =$$

$$= \frac{2(1+t^2) - 4t^2}{(1+t^2)^2} = \frac{2 + 2t^2 - 4t^2}{(1+t^2)^2} = \frac{2 - 2t^2}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2} =$$

$$= \frac{b}{ab} = \frac{2(1-t^2)}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2} = \frac{b}{ab}$$

$$= \frac{2 - 2t^2}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)^2} \quad \text{cos}^2\theta = x \quad (6)$$

$$\frac{dy}{dx} = \frac{\frac{d}{dt}(2(1-t^2))}{\frac{d}{dt}(1+t^2)^2} = \frac{-4t}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2}$$

Q. 1) $x = 3\cos t - \cos^3 t$

$$y = 3\sin t - \sin^3 t = \frac{(3\cos t + 3\sin t)t}{(3\cos t - 3\sin t)t} = \frac{6t}{4t} = \frac{3}{2}$$

2) $x = e^t \cdot \cos t$

$$y = e^t \cdot \sin t$$

Orthocentroid Swastik

Q. 3) $\frac{dx}{dt} = 3\sin t - 3\cos^2 t \cdot (-\sin t)$ direction is curved

orthocentroid is straight because $\sin t + \cos t$ is straight

Now Q. 4) $\frac{dx}{dt} = 3\cos t - \cos^3 t$ is curved at $\frac{\pi}{2}$

$$\frac{dx}{dt} = 3\sin t - 3\cos^2 t \cdot \sin t = \frac{3\sin t}{\cos^2 t}$$

$$= 3(\cos t \sin t - \sin^3 t) = 3\cos t \sin t$$

$$= 3\cos t (\cos^2 t - 1) = 3\cos t \cdot -\frac{\sin^2 t}{\cos^2 t}$$

$$= -3\sin^3 t$$

$$\frac{dy}{dt} = 3\sin t - 3\sin^2 t \cdot \cos t = \frac{3\sin t}{\cos^2 t}$$

$$= 3\cos t (1 - \sin^2 t) = 3\cos^3 t$$

$$\frac{dy}{dx} = \frac{3\cos^3 t}{-3\sin^3 t} = \frac{-\cos^3 t \cdot \cos^3 t}{3\sin^3 t \cdot \sin^3 t} = \frac{\cot^3 t}{-\tan^3 t}$$

$$\therefore P = -\cot^3 t + \tan^3 t = -\frac{\cot^3 t}{\tan^3 t}$$

Q) $x = e^t \cos t$ (Given) $\frac{dy}{dx} = \frac{e^t \cos t - e^t \sin t}{(e^t + 1)^2}$

$$\frac{dy}{dx} = e^t \cos t + \sin t \cdot e^t = \frac{(e^t - 1)\sin t}{(e^t + 1)^2}$$

$$\frac{d^2y}{dt^2} = \frac{e^t(e^t \cos t + \sin t) - (e^t - 1)(e^t \cos t - \sin t)}{(e^t + 1)^3}$$

$$= e^t(e^t \cos t - \sin t)$$

$$\frac{dy}{dx} = \frac{e^t(e^t \cos t + \sin t)}{e^t(e^t \cos t - \sin t)} = \frac{\cos t + \sin t}{\cos t - \sin t}$$

Successive differentiation

Given a function $y = f(x)$, then $\frac{dy}{dx}$ is the 1st order derivative of y . The second order derivative $\frac{d^2y}{dx^2}$ is given by $\boxed{\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)}$ It is also

denote $\frac{dy}{dx} = y'$ and $\frac{d^2y}{dx^2} = y''$

- Find $\frac{d^2y}{dx^2}$ if $y = x^3 + 7x^2 + 5$

A) $\frac{dy}{dx} = (3x^2 + 14x + 14)$

$$\frac{d^2y}{dx^2} = 6x + 14$$

- Find y'' , $y = 2\sin x + 3\cos x$

A. $y' = 2\cos x - 3\sin x$

$$\begin{aligned} y'' &= -2\sin x - 3\cos x \\ &= -(2\sin x + 3\cos x) = -y \end{aligned}$$

$$\bullet y = \cos 3x + \sin 7x. \quad 0 = e^{-7x}(-7\sin 7x - 3\cos 3x) + e^{-3x}(-9\sin 3x + 7\cos 7x)$$

$$y' = -\sin 3x \cdot 3 + \cos 7x \cdot 7$$

$$= 7 \cdot \cos 7x - 3 \sin 3x$$

$$y'' = 7 \cdot -\sin 7x \cdot 7 - 3 \cdot \cos 3x \cdot 3$$

$$= -49 \cdot \sin 7x - 9 \cos 3x$$

$$= e^{-7x}(-49\sin 7x - 9\cos 3x) + e^{-3x}(9\cos 3x - 49\sin 7x)$$

$$\bullet y = e^{7x}$$

$$y' = e^{7x} \cdot 7$$

$$y'' = 7 \cdot e^{7x} \cdot 7$$

$$= 49 e^{7x}$$

$$e^{3x} + e^{7x} = A$$

$$e^{3x} + e^{7x} = B$$

$$-B + B =$$

$$(A + B) \Rightarrow (A + B) - (B - B) \Leftrightarrow 0 = A + B - B$$

$$\bullet y = e^{7x} \cdot \cos x.$$

$$y = e^{7x} - \sin x + \frac{\cos x}{e^{7x}} \cdot e^{7x} \Leftrightarrow$$

$$= -\cos x \cdot e^x - \sin x \cdot e^x$$

$$y'' = (\cos x \cdot e^x + e^x - \sin x \cdot e^x) - (\sin x \cdot e^x + e^x \cos x) \Leftrightarrow$$

$$= e^x \cos x + e^x (-\sin x) - \sin x \cdot e^x - e^x \cos x = 0$$

$$= -2e^x \sin x$$

$$\bullet y = x^2 \cdot \sin x$$

$$9 \cdot x^2 \cos x + 9 \cdot x^2 \sin x - 2x \cdot \sin x = A$$

$$y' = 2 \cdot x^2 \cdot \cos x + \sin x (2x \cos x - x^2 \sin x) \Leftrightarrow$$

$$y'' = (x^2 \cdot -\sin x + \cos x \cdot 2x) + 9(\sin x \cdot 2x + 2x^2 \cos x)$$

$$= 2x \cos x - x^2 \sin x + [2 \cdot \sin x + 2x \cos x]$$

$$= 4x \cos x + 2 \sin x (1 - x^2)$$

$$\bullet y = e^x + e^{-x} \text{ ST } y'' - y = 0$$

$$A: y' = e^x - e^{-x}$$

$$y'' = e^x + e^{-x}$$

$$\underline{y'' - y = 0}$$

$$\bullet y = e^{2x} + e^{3x}: \text{ST } y'' - 5y' + 6y = 0$$

$$A: y' = e^{2x} \cdot 2 + e^{3x} \cdot 3$$

$$y'' = 2e^{2x} \cdot 2 + 3e^{3x} \cdot 3$$

$$= 4e^{2x} + 9e^{3x}$$

$$y'' - 5y' + 6y = 0 \Rightarrow 4e^{2x} + 9e^{3x} - 5(e^{2x} \cdot 2 + e^{3x} \cdot 3) + 6(e^{2x} + e^{3x})$$

$$\Rightarrow 4e^{2x} + 9e^{3x} - 10e^{2x} - 15e^{3x} + 6e^{2x} + 6e^{3x}$$

$$(4e^{2x} + 9e^{3x}) - (10e^{2x} + 15e^{3x}) = 0$$

$$\bullet y = 2\cos px + 3\sin px, \text{ST } y'' + p^2y = 0 \quad (\text{where } p \text{ is constant})$$

~~the QM part is~~

$$y' = -2\sin px \cdot p + 3\cos px \cdot p$$

$$= p(-2\sin px + 3\cos px)$$

$$(y'') = p(-2\sin px \cdot p + 3\cos px \cdot p) \cdot (-2\sin px + 3\cos px) = 0$$

$$(y'') = p^2(-3\sin px \cdot p + 2\cos px \cdot p) \cdot (-2\sin px + 3\cos px) = 0$$

$$= p^2(2\cos px + 3\sin px) \cdot (-2\sin px + 3\cos px) = 0$$

$$+ \bar{P}^2 y = 0$$

$$\bullet y'' - p^2 y = y'' + p^2 y = 0 \quad \text{if } K \cos px + xP = Ax^2$$

$$\checkmark \bullet y = y = A \cos px + B \sin px, \text{ ST. } y'' \propto -y \text{ & } y'' \propto y.$$

$A \cdot y \propto y$; if and only if $y'' \propto -cy$, (K is constant)

$$y' = -A \cdot \sin px \cdot p + B \cdot \cos px \cdot p$$

$$= p(-B \cos px + A \sin px)^{(x+p)} + b x^2 - v^2 x$$

$$y'' = p(-B \sin px \cdot p - A \cos px \cdot p) + x^2 v^2 =$$

$$(x^2 v^2 + p^2 p)(-B \sin px - A \cos px).$$

$$= -p^2(B \sin px + A \cos px) + x^2 v^2 =$$

$$= -p^2 y.$$

$$\frac{y''}{y} = -p^2 \Rightarrow \text{is a constant}$$

$$\therefore \underline{y'' \propto y}$$

$$C = C(x^2 + 1) + p^2 x^2 - v^2 x^2 \cdot 2 \cdot 2 \cdot x^2 \cos^2 x = C$$

$$\bullet y = x^2 \sin x, \text{ PT } x^2 y'' - 4x y' + (x^2 + 6)y = 0.$$

$$\text{A? } y' = x^2 \cdot \cos x + \sin x \cdot 2x$$

$$y'' = (x^2 \cdot \sin x + \cos x \cdot 2x) + (\sin x \cdot 2 + 2x \cdot \cos x)$$

$$= 2x \cos x - x^2 \sin x + (2 \sin x + 2x \cdot \cos x)$$

$$= 4x \cos x + \sin x (2 + 2x)$$

$$x^2 y'' = x^2 (4x \cos x + \sin x (2 + 2x)) = 0$$

$$= 4x^3 \cos x + \sin x (2x^2 - x^4) = 0$$

$$4xy' = 4x(x^2 \cos x + 2x^2 \sin x). \quad \text{Eq 1}$$

$$\therefore y'' = 4x^3 \cos x + 8x^2 \sin x - 2x^2 \cos x. \quad \text{Eq 2}$$

$$(x^2+6)y = (x^2+6) \cdot x^2 \sin x + \text{some terms} \cdot A$$

$$= x^4 \sin x + 6x^2 \sin x - 2x^2 \cos x + q \cdot x^2 \sin x \cdot A = 1$$

$$x^2y'' - 4xy' + (6+x^2)y \Rightarrow \text{Eq 1} - \text{Eq 2} + \text{Eq 3} = 1$$

$$= [4x^3 \cos x + 6x^2 \sin x (2x^3 - x^4)] - [4x^3 \cos x + 8x^2 \sin x] + (x^2 \sin x [6x^2 \sin x + 6x^2 \sin x])$$

$$= -x^4 \sin x + 2x^4 \sin x + 4x^3 \cos x - 4x^3 \cos x - 2x^2 \sin x - 8x^2 \sin x + 6x^2 \sin x$$

$\Rightarrow 0$

$$\text{Hence } \lambda \in \frac{V}{U}$$

H.W.

$$\bullet y = x^2 \cos x, \text{ S.T } x^2y'' - 4xy' + (x^2+6)y = 0.$$

$$\therefore 0 = y(2+4x) + x^2 \cos x \cdot 2 \cdot 2x + x^2 \cos x \cdot 2x = x^2 \cos x \cdot 4x + x^2 \cos x \cdot 2x = 6x^3 \cos x$$

$$\bullet y = a \cos(\log x) + b \sin(\log x), \text{ S.T } x^2y'' + xy' + y = 0$$

$$y' = -a \sin(\log x) \cdot \frac{1}{x} + b \cos(\log x) \cdot \frac{1}{x} \quad \text{Eq 1}$$

$$y'' = \frac{1}{x^2} (b \cos(\log x) - a \sin(\log x)) \quad \text{Eq 2}$$

~~multiply by x^2~~

$$xy' = b \cos(\log x) - a \sin(\log x)$$

$$\text{Diff. w.r.t } x \quad \left(x^2 \cos x + x^2 \sin x \right) \cdot x^2 = xy^2$$

$$x \cdot y'' + y' = \frac{1}{x} b \cos(\log x) - a \sin x (\log x) - \frac{1}{x}$$

Multiply by x

$$x^2 y'' + xy' = b \cos(\log x) - a \sin x (\log x)$$

$$= -(b \cos(\log x)) + (a \sin(\log x))$$

$$= -y.$$

$$\underline{x^2 y'' + xy'} - y = 0.$$

$$y = x^2 \cos x, \text{ s.t } x^2 y'' - 4xy' + (x^2 + 6)y = 0,$$

$$y = x^2 \cdot \cancel{\cos x} \quad \therefore 0 = \cancel{x^2} - b(x-1) \quad \text{and} \quad x^2 \cos x = b$$

$$y' = x^2 \cdot -\sin x + \cos x \cdot 2x \quad \cdot \frac{1}{x-1} = b$$

$$y'' = 2x \cdot \cos x - x^2 \cdot \sin x \quad \text{(by product rule)} \quad \therefore b = \frac{1}{x-1} \cdot \cancel{x^2} \cdot \sin x$$

$$y'' = 2(x \sin x + \cos x) - (2x \sin x + x^2 \cos x) \quad \text{(by product rule)} \quad \therefore b = (x)(x-1)$$

$$= 2\cos x - 2x \sin x - 2x \sin x - x^2 \cos x \quad \therefore b = (x)(x-1) \cdot \frac{b}{x^2}$$

$$= (2+x^2) \cdot \cos x - 4x \cdot \sin x$$

$$x^2 y'' = x^2 [(2+x^2) \cos x - 4x \sin x] \quad \text{(from above)}$$

$$= \dots = x^2 - \frac{b}{(x-1)} + \frac{b}{(x-1)} \cdot x^2$$

(by division)

$$0 = bx - b(x-1)$$

$$x \cdot A'' + A' = \frac{1}{2} \cos(\text{mid}) - \sin(\text{mid})$$

differentiate wrt x

$$x^2 A'' + 2x A' = -\cos(\text{mid}) - \sin(\text{mid})$$

$$(x^2 A'' + 2x A') = -(\cos(\text{mid}) + \sin(\text{mid}))$$

$\therefore A'' + 2A' =$

$$A = x - x^2 + x^3$$

~~$y = x(\sin x) + x^2 \cos x$~~

~~$y = \sin x + x^2 \cos x$~~

$$\text{or } y = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} \cdot y' = 1, \text{ squaring all}$$

$$(1-x^2)(y')^2 = 1, \text{ now diff. wrt.}$$

$$\frac{d}{dx}(1-x^2)(y')^2 = \frac{d}{dx} 1$$

$$\text{or } 2(1-x^2) \cdot 2y' y'' + (y')^2 \cdot (-2x) = 0.$$

$$2y'(1-x^2)y'' + (y')^2 \cdot -2x = 0.$$

(dividing by $2y'$)

$$(1-x^2)y'' - xy' = 0$$

• $y = \sqrt{\sin x + y}$, differentiate to find slope

$$y' = \frac{1}{2\sqrt{\sin x + y}} \left(\cos x + \frac{dy}{dx} \right).$$

$$\frac{y'}{2\sqrt{\sin x + y}} = \cos x + \frac{dy}{dx}$$

$$\frac{1}{2\sqrt{\sin x + y}} \cdot y' = \cos x + \frac{dy}{dx}$$

$$\Rightarrow 2y \cdot y' = \cos x + \frac{dy}{dx}$$

$$2y \cdot y' - y' = \cos x$$

$$(2y - 1)y' = \cos x$$

$$\frac{(2y - 1)y'}{2y - 1} = \frac{\cos x}{2y - 1}$$

• $e^y = \sin(x+y)$.

diff. wrt. x

$$e^y \cdot \frac{dy}{dx} = \cos(x+y) \cdot \left(1 + \frac{dy}{dx} \right)$$

$$e^y \frac{dy}{dx} = \cos(x+y) \cdot 1 + \cos(x+y) \cdot \frac{dy}{dx}$$

$$e^y \frac{dy}{dx} - \cos(x+y) \frac{dy}{dx} = \cos(x+y)$$

$$\frac{dy}{dx} (e^y - \cos(x+y)) = \cos(x+y)$$

$$\frac{\frac{dy}{dx}}{e^y - \cos(x+y)} = \frac{\cos(x+y)}{e^y - \cos(x+y)}$$

$$(e^y - \cos(x+y))$$

Applications of Differentiation

Chap. 1 Tangents & Normals

slope of a straight line is given by $m = \tan \alpha$ where α is the angle made by the straight line with x -axis.

Given a curve $y = f(x)$, let $P(x_0, y_0)$ be a given point on this curve then the slope of tangent at $P(x_0, y_0)$ is given by $\left(\frac{dy}{dx}\right)_{(x_0, y_0)}$

$$m = \left(\frac{dy}{dx}\right)_{(x_0, y_0)}$$

- Find the Slope of tangent to the curve $y = 3x^2 + 4x - 5$ at $(1, 2)$

A) Slope of tangent at $(1, 2) = \left(\frac{dy}{dx}\right)_{(1, 2)} = \frac{dy}{dx} = 6x + 4$

$$\frac{dy}{dx} = 6x + 4 \Rightarrow (y+x) \cos = \frac{dy}{dx} = \frac{6x+4}{x}$$

$$m = \underline{(y+x) \cos} = \underline{(6x+4) \cos} = \frac{6x+4}{x}$$

- $y^2 = 4ax$ at $(a, 2a)$, Step?

A. $\frac{dy}{dx} = \frac{(y+x) \cos}{x} = \frac{2a}{x}$

$$\left(\frac{dy}{dx}\right)_{(a, 2a)} = \underline{\underline{\frac{2a}{x}}}$$

$$\frac{dy}{dx} \text{?}$$

Diff. w.r.t x

$$2y \cdot \frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

$$\left(\frac{dy}{dx} \right)_{(a, 2a)} = \frac{2a}{2a} = 1$$

- Find the slope of curve $x^2 + y^2 = 25$ at $(3, 4)$.
- Slope of tangent at $(3, 4) = \left(\frac{dy}{dx} \right)_{(3, 4)}$.

$$x^2 + y^2 = 25$$

Diff. w.r.t x

$$2x + 2y \cdot y' = 0.$$

$$y' = -\frac{2x}{2y} = -\frac{x}{y}$$

$$\left(\frac{dy}{dx} \right)_{(3, 4)} = -\frac{3}{4}$$

- Find slope of tangent : $x = 2t$, $y = \frac{2}{t}$, at $t = 3$.

Slope at $(t=3) \Rightarrow \left(\frac{dy}{dx} \right)_{(t=3)}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2}{t^2}}{2} = \frac{1}{t^2} = \frac{1}{9}$$

$$\left(\frac{dy}{dx} \right)_{(t=3)} = -\frac{1}{9}$$

• $y = \tan x$ at $x = \pi/3$.

∴ $\frac{dy}{dx} = \frac{1}{1+x^2}$.

$$\left(\frac{dy}{dx}\right)_{(x=\pi/3)} = \frac{1}{1+(\pi/3)^2} = \frac{1}{1+\frac{\pi^2}{9}} = \frac{9}{9+\pi^2}$$

• $y = \tan x$

$$\frac{dy}{dx} = \sec^2 x$$

∴ $\left(\frac{dy}{dx}\right)_{(x=\pi/3)} = \sec^2(\pi/3) = \frac{1}{\cos^2(\pi/3)} = \frac{1}{(\frac{1}{2})^2} = 4$ //

Normal

Slope of normal to the curve $y = f(x)$ at $P(x_0, y_0)$.

$$= -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_0, y_0)}}$$

$$P(x_0, y_0) = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_0, y_0)}}$$

- Find the slope of normal to curve $y = \tan x$ at $x = \pi/3$

∴ we know that $\left(\frac{dy}{dx}\right)_{(x=\pi/3)} = 4$ insert to get $\frac{dy}{dx} = \sec^2 x$
 \therefore slope $= -\frac{1}{4}$ //

- Find slope of tangent & normal to the curve $y^2 = 4x$ at $(4, 4)$

$$A. \frac{dy}{dx} \Rightarrow 2y \cdot \frac{dy}{dx} = 4. \quad \text{at } P(4,2) \Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{4}{2 \cdot 2} = 1$$

$\frac{dy}{dx}$ at $(4,2)$ is 1 . Hence tangent to curve at $(4,2)$.

$$\frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y} = \frac{-1}{\frac{1}{2}} = -2$$

$$\left(\frac{dy}{dx}\right)_{(4,2)} = \frac{2}{4} = \frac{1}{2}$$

$$P(4,2) \Rightarrow \frac{-1}{\frac{1}{2}} = -2 \text{ or } -2 = \frac{1}{2}y \Rightarrow y = -4$$

$$\Rightarrow \frac{-1-y_1}{x_1} = \frac{-4}{2} = -2$$

gradient of normal to tangent to curve at $(4,2)$.

$(-2)^{-1} = \frac{1}{2}$

Equ: of Tangent & Normal

Gives a curve, $y = f(x)$ and let $P(x_1, y_1)$ be a point on this curve. Then equ: of tangent.

$$P(x_1, y_1) = y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1).$$

$$\text{Equ: of normal at } P(x_1, y_1) = y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} (x - x_1).$$

- Find equ: of tangent & normal to curve.

$$y = 3x^2 + 4x - 5 \text{ at } (1,4)$$

$$A. (x_1, y_1) = (1, 4)$$

$$\text{Equ: of tangent at } (1,4) \Rightarrow y - 4 = \left(\frac{dy}{dx}\right)_{(1,4)} (x - 1)$$

$$\frac{dy}{dx} = 6x + 4$$

$$\left(\frac{dy}{dx}\right)_{(1,4)} = 6x+4 = 10.$$

* Equ: of tangent : $y-4 = 10(x-1)$.

$$\Rightarrow y-4 = 10x-10 \\ \Rightarrow 10x-y-6=0$$

Equ: of normal $= y-4 = -\frac{1}{10}(x-1)$

$$\Rightarrow 10(y-4) = -(x-1) \\ \Rightarrow 10y-40+x-1=0 \\ \Rightarrow x+10y-41=0$$

- Find the eqn: of tangent and normal to the curve $y^2 = 4ax$ at $(a, 2a)$

A. $y^2 = 4ax$

Normal \Rightarrow tangent to imp

$$2y \cdot \frac{dy}{dx} = 4a \quad \text{tangent to curve given to curve}$$

$$\frac{dy}{dx} = \frac{2a}{y} \quad \text{tangent to curve given to curve}$$

$$(x-a) \cdot \frac{dy}{dx} = 2a \quad (x-a) = 2a \quad (x, y) = (a, 2a)$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

$$(x-a) \cdot \frac{dy}{dx} = 2a \quad (x-a) = 2a \quad (x, y) = (a, 2a)$$

$$\left(\frac{dy}{dx}\right)_{(a,2a)} = \frac{2a}{2a} = 1$$

Equ: of tangent $= y-2a = 1(x-a)$

$$\Rightarrow x-a-y+2a=0$$

$$\Rightarrow x+a-y=0$$

$(x-a)(y-2a)$ Equ: of normal $= y-2a = -1(x-a)$

$$\Rightarrow y-2a+x-a=0$$

$$\Rightarrow x+y-3a=0$$

$y = \cos x$ at $x = \pi/6$, find the tangent & normal.

A. Equ: of tangent at (x_1, y_1) .

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x,y)} (x - x_1), \text{ where } y = \cos x \text{ at } x = \pi/6$$

$$x_1 = \pi/6, y_1 = \cos \pi/6 = \sqrt{3}/2$$

$$\text{Equ: of tangent is } y - \frac{\sqrt{3}}{2} = \left(\frac{dy}{dx}\right)_{(x=\pi/6)} (x - \pi/6)$$

$$\therefore (1-x)^{-\frac{1}{2}} = \frac{dy}{dx} = -\sin x \quad (1-x)^{\frac{1}{2}} = \sqrt{x-1}$$

$$1+x^2 = (1-x)^2 \quad \Rightarrow \quad 1+x^2 = 1-2x+x^2$$

$$1+2x = \left(\frac{dy}{dx}\right)_{(x=\pi/6)} = -\sin \pi/6 = -\frac{1}{2}$$

$$\text{Equ: of tangent} = y - \frac{\sqrt{3}}{2} = -\frac{1}{2}(x - \pi/6)$$

Multiply with 2 to make it easier

$$\Rightarrow 2y - \sqrt{3} = -1(x - \pi/6)$$

$$\Rightarrow 2y - \sqrt{3} + x - \pi/6 = 0 \quad \text{both sides add}$$

$$\Rightarrow x + 2y - \sqrt{3} - \pi/6 = 0$$

Equ: of normal

$$y - \frac{\sqrt{3}}{2} = 2(x - \pi/6)$$

$$\Rightarrow y - \frac{\sqrt{3}}{2} = 2x - \frac{\pi}{3}$$

$$\Rightarrow 2x - y - \frac{\pi}{3} + \frac{\sqrt{3}}{2} = 0$$

- Equ. of tangent & normal to the curve $y = 3x^2 - 1$ at $(1, 2)$.

A. Equ. of tangent $y - y_1 = \frac{dy}{dx} \Big|_{(1,2)} (x - x_1)$

$$\left(\frac{dy}{dx}\right) = 6x \quad \text{At } x=1, \frac{dy}{dx} = 6.$$

$$\left(\frac{dy}{dx}\right)_{(1,2)} = 6 \Rightarrow y - 2 = 6(x - 1)$$

$$\Rightarrow y - 2 = 6(x - 1) \quad y - 2 = \frac{1}{6}(x - 1).$$

$$\Rightarrow y - 2 = 6x - 6. \quad 6(y - 2) = -x + 1$$

$$\Rightarrow y - 2 + 6 - 6x = 0 \quad 6y - 12 = -x + 1$$

$$\Rightarrow y - 6x + 4 = 0. \quad \underline{\underline{6y - 13 + x = 0}}$$

~~$\frac{1}{6} = \frac{1}{6}x - \frac{1}{6}$~~ = tangent to N.P

- For what values of x is the tangent to curve $y = x^3 - 2x^2 + x + 1$ parallel to x -axis?

we have find to the values of x where

$$\frac{dy}{dx} = 0.$$

$$\frac{dy}{dx} = 3x^2 - 4x + 1 = 0$$

$$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 4x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \times 3 \times 1}}{2 \times 3}$$

$$\therefore x = \frac{4 \pm \sqrt{16 - 12}}{6} = \frac{4 \pm 2}{6} = \frac{2}{3}, 2$$

- Find the values of x at which tangent to curve

$$y = \frac{x}{x^2+1} \parallel \text{to } x\text{-axis}$$

A. $y = \frac{x}{x^2+1}$

$$\frac{dy}{dx} = \frac{(x^2+1) \cdot 1 - x \cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$\frac{dy}{dx} = 0.$$

$$\frac{1-x^2}{(1+x^2)^2} = 0 \Rightarrow 1-x^2=0, \quad x^2=1$$

$$x = \pm 1$$

∴ $x = \pm 1$ to ans

- Find the values of x where tangent to curve

$$y = \frac{x^2+1}{(1-x)^2}$$

b) \parallel to y -axis

A. a) $\frac{dy}{dx} = \frac{(1-x)^2 \cdot 1 - x \cdot (2(1-x)(-1))}{(1-x)^2 x^2}$

$$\begin{aligned} &= \frac{(1-x)^2 + 2x - 2x^2}{(1-x)^4} \quad \text{cancel } (1-x) \\ &= \frac{1-2x+2x^2}{(1-x)^4} \quad \text{cancel } (1-x) \\ &= \frac{1-2x+2x^2}{(1-x)^3} = \frac{1+x}{(1-x)^3} \end{aligned}$$

$$\begin{aligned} &= \frac{1+x}{(1-x)^3} = 0 \Rightarrow 1+x=0 \\ &\qquad \qquad \qquad \underline{\underline{x=-1}} \end{aligned}$$

b) we have find value of x , where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{1+x}{(1-x)^3} = 0$$

$$(1-x)^3 = 0 \Rightarrow \underline{x=1}$$

$$x=1 \text{ then } f(x) = x^3 - 1 \cdot (1+x) = \frac{6b}{Lb}$$

$$(L+2)^3 - (L+1)^3 = 6(L+1)^2 - 6(L+2)^2 = 6(1+5x) \dots$$

$$6(1+5x) = 6(1+10x+25) \dots$$

$$6 = 60x + 150 \dots$$

$$0 = 5x + 25 \dots$$

$$x = -5 \dots$$

$$\text{Area of Circle} = \pi r^2$$

$$\text{Volume of Cube} = a^3$$

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of Sphere} = 4\pi r^2$$

$$\text{Surface area of Hollow cylinder} = 2\pi rh$$

$$S = \pi r^2 + 2\pi rh$$

$$V = \pi r^2 h$$

$$S = 4\pi r^2$$

$$\text{Surface area of Balloon} = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

$$\text{Surface area of Hollow cylinder} = 2\pi rh$$

$$V = \pi r^2 h$$

$$S = 4\pi r^2$$

� 2nd year I) RATES & MOTION.

rate of change of function wrt x. $S + J\partial + S\partial + E = 2$
 Let $y = f(x)$ be a given function, then $\frac{dy}{dx}$ is represent
 rate of change of y. wrt x. $S + J\partial + S\partial + E = 2$

For eg: Consider the area of circle of radius 'r'
 which is given by, $a = \pi r^2$

Then the rate of change of area wrt radius

$$\frac{da}{dr} = \pi \cdot 2r = 2\pi r. + + J\partial + S\partial =$$

eg: Volume with of cube with edge = a

$$V = a^3.$$

$$\frac{dv}{da} = 3a^2.$$

$$= 3a^2 = \frac{d}{dr} \text{ rad w}$$

eg: Volume of sphere radius 'r'

$$V = \frac{4}{3}\pi r^3. \text{ rad w} = . + + S \times \partial + (S) \times S =$$

$$\frac{dv}{dr} = \frac{4}{3}\pi r^2. \text{ rad w} = 2 + S \times \partial = 2 + J\partial = D$$

$$= 4\pi r^2$$

� 2nd year I) ~~contd~~ shifting to technology *

curve, differentiation & motion but $S + J\partial = 2$

Let s be the displacement of particle in time 't'. Then

$$\text{velocity}, v = \frac{ds}{dt} \quad (\frac{S}{J} + J\partial) \frac{b}{Jb} = \frac{eb}{Jb} = v$$

$$\text{acceleration}, a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \frac{Jb}{Jb} = \frac{eb}{Jb} =$$

- Displacement of a particle in time t is given by
 $s = t^3 + 3t^2 + 4t + 2$, find velocity & acceleration when
 const. $A = 100$ m/s² starts from rest at $t = 0$

A. $s = t^3 + 3t^2 + 4t + 2$.

Find $v = \frac{ds}{dt}$ when $s = 0$ & $t = 0$.

where $v = \frac{d}{dt}(t^3 + 3t^2 + 4t + 2)$

$$= 3t^2 + 6t + 4, v = 3t^2 + 6t + 4 = \frac{dv}{dt} = \frac{ab}{s^b}$$

$a = \frac{d^2s}{dt^2} = \frac{d}{dt}(3t^2 + 6t + 4)$

$$\Rightarrow 6t + 6.$$

$$v = \frac{ab}{s^b}$$

when $t = 2$ sec

$v = 3t^2 + 6t + 4$

$$= 3 \times (2)^2 + 6 \times 2 + 4, v = 28 \text{ m/s}$$

$$a = 6t + 6 = 6 \times 2 + 6 = \frac{18}{2} = \frac{ab}{s^b}$$

- Displacement of a particle in time t is given by
 $s = 2t + \frac{3}{t}$ Find velocity & acceleration, when
 const. $A = 100$ m/s² starts from rest at $t = 2$ sec

A. $v = \frac{ds}{dt} = \frac{d}{dt}(2t + \frac{3}{t})$

$$= 2 + 3 \cdot \frac{1}{t^2} = 2 + \frac{3}{t^2}$$

$$a = \frac{d^2 s}{dt^2} = \frac{d}{dt} \left(2 - \frac{3}{t^3} \right) = 0 + 3 \cdot \frac{6}{t^4} = \frac{18}{t^4}$$

when $t = 3 \text{ sec}$.

$$v = 2 - \frac{3}{9} = \frac{15}{9} \text{ m/s} = \frac{5}{3} \text{ m/s}$$

$$a = \frac{6}{27} = \frac{2}{9} \text{ m/s}^2$$

- A particle moves such that the displacement from a fixed point O is given by

$$s = 5 \cos nt + 4 \sin nt$$

where $n = \text{constant}$; acceleration varies as displacement.

Q. we have $s = 5 \cos nt + 4 \sin nt$, where $k = \text{constant}$

$$\frac{ds}{dt} = 5 \cos nt + 4 \sin nt \cdot n$$

$$= n(4 \cos nt + 5 \sin nt)$$

$$\frac{d^2 s}{dt^2} = n(4 \sin nt \cdot n + 5 \cos nt \cdot n)$$

$$= n^2(5 \cos nt + 4 \sin nt)$$

$$z = s \cdot d + t \cdot P = r^2$$

$$\therefore r^2 = \text{constant}$$

$$a = k \cdot s \quad z = R$$

$$\therefore a \propto s$$

- displacement $s = ae^{nt} + be^{-nt}$, S.T acceleration varies

as it's displacement

$$\frac{ds}{dt} = \frac{ae^{nt} - be^{-nt}}{t} = 0$$

A. we S.T $a = s$

$$a = k.s.$$

$$\frac{ds}{dt} = a \cdot e^{nt} \cdot n + b \cdot e^{-nt} \cdot (-n) = \frac{-ns - s}{t} = V$$

$$\frac{d^2s}{dt^2} = n(a \cdot e^{nt} + b \cdot e^{-nt}) \cdot \frac{n}{t^2} = \frac{a}{t^2} = n$$

$$\frac{d^2s}{dt^2} = n(a \cdot e^{nt} + b \cdot e^{-nt})$$

$$= n^2(a \cdot e^{nt} + b \cdot e^{-nt})$$

$$= n^2 \cdot s \text{ (since } a = s)$$

A. we $s = t^n$ natural $\Rightarrow n^2 \cdot s$: t constant = a constant

for $n^2 \cdot s$ ~~constant~~

- displacement $s = ae^t + be^{-t}$ S.T $a = s$

A. we S.T $a = s$.

$$\frac{ds}{dt} = a \cdot e^t + b \cdot (-e^{-t}) = \frac{sb}{t}$$

$$(a \cdot e^t + b \cdot (-e^{-t}))n = \frac{sb}{t^2}$$

$$\frac{d^2s}{dt^2} = a \cdot e^t + b \cdot e^{-t} = 1$$

$$= a \cdot e^t + b \cdot e^{-t} = s$$

$$\therefore a = s \quad 2.s = 0$$

2.s = 0

A dist. traveled by a particle in time s_1 at t is given by

$$s = t^3 - 6t^2 + 8t - 4.$$

i) Find the time 't' when $a = 12 \text{ cm/s}^2$ (Ans 12 cm/s^2)

ii) Find velocity at this time. $v = \frac{ds}{dt}$ (Ans $\frac{28}{3} \text{ cm/s}$)

$$A. s = t^3 - 6t^2 + 8t - 4 \quad (i)$$

$$\frac{ds}{dt} = 3t^2 - 12t + 8$$

$$\frac{d^2s}{dt^2} = 6t - 12 \quad (ii)$$

i) when $a = 12 \text{ cm/s}^2$

$$6t - 12 = 12 \quad \Rightarrow \quad t = \frac{24}{6} = 4 \text{ sec}$$

$$6t = 24 \quad \Rightarrow \quad t = 4 \text{ sec}$$

$$t = 4 \text{ sec} \quad (Ans)$$

Ans $v = \frac{ds}{dt} = 3t^2 - 12t + 8 \quad (\text{when } t=4)$

$$= 3 \times 4^2 - 12 \times 4 + 8 \quad (\text{zero, zero, zero})$$

\therefore uniform motion with $\underline{\underline{s = 8 \text{ cm/s}}}$ comes to rest at $t=4$

* The displacement of a particle from a mean position 'o' in time 't' given by $s = 2t^3 - 9t^2 + 12t + 6$

i) Find 't' when $a = 0$.

ii) also find velocity at this time

$$A. s = 2t^3 - 9t^2 + 12t + 6$$

$$\frac{ds}{dt} = 6t^2 - 18t + 12 \quad (i)$$

$$\text{Given } \frac{d^2s}{dt^2} = 12t - 18, \text{ if object is at rest at } t=0 \text{ then } s = 0, v = 0$$

$$+ - 18 + 3t^2 - 18 = 0$$

(when $a=0$, then $s = 0$ now it's 2nd diff b/w s)

$$\text{i) } \frac{d^2s}{dt^2} = 0 \Rightarrow 12t - 18 = 0, \text{ velocity b/w } (i)$$

$$\Rightarrow 12t = 18 \Rightarrow t = \frac{18}{12} = \frac{3}{2} \text{ sec}$$

$$t = \frac{3}{2} \text{ sec}$$

$$s = 18t - \frac{18}{2} t^2 = \frac{18}{2} t - \frac{18}{2} t^2$$

$$\text{ii) Velocity } \frac{ds}{dt} \text{ at } t = \frac{3}{2} \text{ sec}$$

$s_1 = 0$ m/s (i)

$$\frac{ds}{dt} = 6t^2 - 18t + 12$$

$$= 6 \times \left(\frac{3}{2}\right)^2 - 18 \times \frac{3}{2} + 12$$

$$= \frac{27}{2} - 27 + 12 = \frac{-3}{2} \text{ m/s}$$

• work done by a moving body is given by $W = \frac{1}{2} m v^2$

units over an interval of t seconds.

• Find power of force creating this motion. when $t = 2$ sec,

$$A. \omega = at = 3$$

now $\omega = \frac{d\theta}{dt}$

$$P = \frac{W}{t} \text{ (we know)}$$

$P = \frac{d\omega}{dt}$ & since in '0' motion

$$\therefore \frac{d\omega}{dt} = 2 + \frac{3}{t^2} \text{ rad/s} (i)$$

$$= 2 + \frac{3}{4} \text{ rad/s} (ii)$$

$$\text{when } t = 2 \text{ sec} \Rightarrow 2 + \frac{3}{(2)^2} = 2 + \frac{3}{4} = 2.75 \text{ rad/s}$$

$$(2.75) \text{ unit } \frac{\text{rad}}{\text{s}}$$

Related rates \rightarrow or $\frac{dA}{dt} = \frac{dr}{dt} \cdot 2\pi r$ also write

If 2 variables are related then their rate of change also related. we can find the rate of change of one variable, if the rate of change of the other variable is given. for ex. if $r = 2t$, then $\frac{dr}{dt} = 2$ so if $r = 2t$, then $\frac{dr}{dt} = 2$
 eg: The radius of a circle increasing at a rate of 2 cm per second. Find the rate at which the area is increasing when the radius is 6 cm.

$$A = \pi r^2 \quad \text{or } \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} = \frac{\pi b^2}{t^2}$$

$$\frac{dA}{dt} = \frac{2\pi r \cdot dr}{dt} = \frac{dA}{dt}$$

$$\text{given } r = 6.$$

$$\frac{dr}{dt} = 2 \cdot \text{bpo} \cdot 0.5 = \frac{\pi b}{t^2}$$

$$\Rightarrow \frac{dA}{dt} = 2\pi \cdot 6 \cdot 2 \frac{\pi b}{t^2} = 0.5 \quad \text{cm}^2/\text{sec}$$

$$= 24\pi \text{ cm}^2/\text{sec} \quad \frac{cm^2}{\pi \cdot t^2} = \frac{rb^2}{t^2}$$

A circular disc expanding when heated at a rate of $12 \text{ cm}^2/\text{sec}$, find the rate at which the area is increasing when the radius is 4 cm.

A. $A = \pi r^2$ area of a circle

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

given $\frac{dA}{dt} = 12 \text{ cm}^2/\text{sec}$

and $r = 4 \text{ cm}$

also given $\frac{dr}{dt} = ?$

given $\frac{dA}{dt} = 12 = 2\pi \cdot 4 \cdot \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{12}{8\pi} = \frac{3}{2\pi/1}$

- gas is escaping from a spherical balloon at a rate of 20 cc/sec . Find the radius at which the radius is decreasing when radius is 4 cm , and at which rate.

A. Volume of spherical

$$\text{balloon}, V = \frac{4}{3}\pi r^3 \quad r^2 = A \Rightarrow A = \pi r^2$$

$$\begin{aligned} \frac{dV}{dt} &= \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} \\ &= \frac{4\pi r^2}{3} \cdot \frac{dr}{dt} = \frac{4\pi}{3} r^2 \cdot \frac{dr}{dt} \end{aligned}$$

given $\frac{dV}{dt} = -20$ and $V = \frac{4\pi}{3} r^3$

$$\Rightarrow -20 = 4\pi \cdot 4^2 \cdot \frac{dr}{dt} \cdot 3 \cdot \pi G = \frac{4\pi}{3} \cdot 4^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{-5}{16\pi}$$

Hence the radius is decreasing at the rate of $\frac{5}{16\pi} \text{ cm/sec}$

- A spherical balloon inflated by pumping 15 cc/sec find the rate at which the radius is increasing when the radius is 3 cm .

A. i) $V = \frac{4}{3}\pi r^3$ given surface to vol ratio A = $\frac{r^2}{V}$

$$\text{Surface} = \frac{\partial V}{\partial t} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} \text{ Given } \frac{dr}{dt} = 10 \text{ cm/min.} \text{ Surface area} = 4\pi r^2 \frac{dr}{dt}$$

Given $\frac{dV}{dt} = 15 \text{ cc/sec}$ and $r^2 \cdot 3\pi = A$

$$\Rightarrow 15 = 4\pi \cdot 3^2 \cdot \frac{dr}{dt} \quad \frac{rb \cdot rs \cdot \pi}{3b} = \frac{rb}{3b}$$

$$\text{If } \frac{dr}{dt} = \frac{15}{4\pi \cdot 9} = \text{ given. use } \frac{rb}{3b}$$

$$= \frac{15}{36\pi} \quad 6 \times 11 \times 8 \cdot \pi \cdot \frac{rb}{3b} = \frac{rb}{3b}$$

$$= \frac{15}{36\pi} \quad \text{surface area} = \frac{rb}{3b}$$

at end of 29 sec surface area of balloon is doubled
• Air is pumped into a spherical rubber bladder
of radius 3. If the radius increases at uniform
rate of 1 cm/min. Find the rate at which
the volume is increased at the end of 3 minutes.

A. $V = \frac{4}{3}\pi r^3 \quad \frac{rb \cdot rs \cdot \pi}{3b} = \frac{rb}{3b}$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} =$$

$$= 4\pi r^2 \cdot \frac{dr}{dt} \quad \text{now } r = 3$$

$$\frac{dr}{dt} = 1 \text{ cm/min.} \quad \text{initial radius } \frac{rb}{3b} \text{ bnf of}$$

$$\text{radius after } 3 \text{ min.} = 3 + (3 \times 1) = 6 \quad \frac{rb}{3b}$$

$$\frac{dV}{dt} = \frac{rb}{3b} \cdot 4\pi \cdot 6^2 \cdot \frac{rb}{3b} = 4\pi \times 36 = 144\pi \text{ cm}^3/\text{min}$$

- A circular plate of radius 3 inches expands when heated at the rate of $\frac{1}{2}$ inch/sec. Find the rate at which area of the plate increasing at the end of 4 sec.

$$A = \pi r^2 \cdot \text{area} = \frac{\pi r^2}{\frac{dt}{dr}} \cdot \frac{dr}{dt} = \frac{\pi r^2}{\frac{dt}{dr}} \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} \cdot \frac{rb}{\frac{dt}{dr}} \cdot \frac{dt}{dr} = 21 \quad \leftarrow$$

$$\text{after 4 sec. radius} = 3 + (\frac{1}{2} \times 4) = 5 \text{ in.}$$

$$\frac{dr}{dt} = 2$$

$$\frac{dr}{dt} = \frac{2}{\frac{dt}{dr}} \cdot \frac{dt}{dr} = \frac{2}{\pi \cdot 5} = \frac{2}{5\pi}$$

$$= \frac{4}{5\pi} \text{ in/sec}$$

- ~~* * *~~ A balloon is spherical in shape gas is escaping from it at the rate of 10 cc/sec. How fast is the surface area shrinking when the radius is 15 cm, given to 3 decimal places.

A surface area $S = 4\pi r^2$ at a given time $t = 15 \text{ cm.}$

$$\frac{ds}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt} \cdot \frac{dt}{dr} = V \cdot A$$

$$= 8\pi r \cdot \frac{dr}{dt} \cdot \frac{dt}{dr} = -10 \cdot \frac{10}{15} = -\frac{20}{3}$$

$$r = 15 \text{ cm.}$$

To find $\frac{dv}{dt}$ - Given that $\frac{dr}{dt} = -10$.

$$\frac{dv}{dt} = \frac{4}{3}\pi r^3 \cdot \frac{dr}{dt} = \frac{4}{3}\pi \cdot 15^3 \cdot -10 = -\frac{10}{3}\pi \cdot 3375 = -3750\pi$$

$$\text{solution } \frac{dv}{dt} = 2\pi r \cdot \frac{dr}{dt} = \frac{4}{3}\pi r^2 \cdot \frac{dr}{dt} = \frac{4}{3}\pi \cdot 15^2 \cdot -10 = -3000\pi$$

$$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow -10 = 4\pi \cdot 15^2 \cdot \frac{dr}{dt} \quad \text{soft salt coming out is const}$$

$$\Rightarrow \frac{dr}{dt} = \frac{-10}{4\pi \cdot 15^2} \cdot \frac{1}{(r-15)} \quad r=30 \text{ taking out the denominator}$$

From ①,

$$\frac{ds}{dt} = 8\pi r^2 \cdot \frac{-10}{4\pi \cdot 15^2} \cdot \frac{1}{(r-15)}$$

$$= \frac{-20}{15} = \frac{4}{3} \cdot \text{station coming out to}$$

$$S-E = S + 1 + xS - Sx = E \quad (1)$$

- Gas is escaped from aspherical balloon at the rate of 20 cc/sec. Find the rate at which the surface area shrinking) when the radius is 10 cm.

$$\frac{ds}{dt} = \frac{4}{3}\pi r^2 \cdot \frac{dr}{dt}$$

$$A = 4\pi r^2, \quad dA/dt = 8\pi r \cdot \frac{dr}{dt} = \frac{4}{3}\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{ds}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

$$r = 10 \text{ cm.}, \quad S = S + xS - Sx = E$$

$$\text{To find } \frac{dv}{dt}, \text{ given that } \frac{ds}{dt} = \frac{-20}{15} = \frac{4}{3} \cdot \frac{10}{10-15} = \frac{4}{3} \cdot \frac{10}{-5} = -4$$

$$V = \frac{4}{3}\pi r^3, \quad E + (S)S - (S).E = \frac{4}{3}\pi r^3 \cdot \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} < 14 = E + S - S =$$

$$\Rightarrow -20 = 4\pi \cdot 10^2 \cdot \frac{dr}{dt} \cdot S - (S).E = \frac{4}{3}\pi r^3 \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-20}{4\pi \cdot 10^2} = \frac{1}{\pi} \cdot \frac{-20}{100} = \frac{-2}{\pi} = -\frac{2}{\pi}$$

$$\text{from ①} \Rightarrow \frac{ds}{dt} = 8\pi r^2 \cdot \frac{-20}{4\pi \cdot 10^2} = \frac{-40}{100} = -4$$

MAXIMA & MINIMA

Given a function $y = f(x)$, the function is increasing at the point $x=a$, if $\left(\frac{dy}{dx}\right)_{(x=a)} > 0$. The function is decreasing at the point $x=b$.

$$\left(\frac{dy}{dx}\right)_{(x=b)} < 0. \quad \text{①most}$$

e.g: check whether the function is increase/decrease at the given points.

i) $y = 4x^2 - 2x + 1$ at $x=3, -2$.

After diff to modded lowe angles chift basq ades si hra.

$\text{diff} \frac{dy}{dx} = 8x - 2$ to straight bnf . \Rightarrow $x > \frac{1}{4}$ of 90

$\left(\frac{dy}{dx}\right)_{(x=3)} = 8 \cdot 3 - 2 = 22 > 0$ (Function is increasing)

$\left(\frac{dy}{dx}\right)_{(x=-2)} = 8 \cdot (-2) - 2 = -18 < 0$ (Function is decreasing)

$$\frac{rb \cdot \pi \theta}{\pi b} = \frac{rb}{\pi b}$$

• $y = x^3 - 6x^2 + 5x + 2$ at $x = -2, 2$. $\text{mod} \theta = r$

A. $\frac{dy}{dx} = 3x^2 - 12x + 5$ to diff to moded. $\frac{rb}{\pi b}$ bnf of $\frac{5}{-12}$

$\left(\frac{dy}{dx}\right)_{(x=-2)} = 3 \cdot (-2)^2 - 12(-2) + 5. \quad \text{er } \frac{\pi \theta}{\pi b} = V$
 $= 12 + 24 + 5 = 41 > 0$ (Increasing)

$\left(\frac{dy}{dx}\right)_{(x=2)} = 3 \cdot (2)^2 - 12 \cdot (2) + 5 = 01 \cdot \pi \theta = 05 - (-7)$
 $= 12 - 24 + 5 = -7 < 0$ (Decreasing).

$\frac{rb}{\pi b} = \frac{\theta - \pi}{\pi b} = \frac{05 - (-7)}{\pi b} = \frac{12}{\pi b} \text{ or } \frac{12}{\pi b} \text{ ①most}$

- Find the range of values of x for which the function $y = 2x^2 - 3x + 4$ is increasing.

A. we have to find the range of values of x for which $\frac{dy}{dx} > 0$.

$$\frac{dy}{dx} = 2x^2 - 3 \Rightarrow \text{when all signs match with } 2x^2 > 0.$$

$$2x > 3.$$

$$x > \underline{\underline{\frac{3}{2}}}$$

i.e. the function is increasing at $x > \underline{\underline{\frac{3}{2}}}$
 (long $\frac{dy}{dx} = 2x^2 - 3$)

- Find the range of values of x for which $y = 4x^2 - 3x + 10$ is decreasing.

$$A. y = 4x^2 - 3x + 10.$$

$$\frac{dy}{dx} = 8x - 3.$$

now to find the range of x for which $8x - 3 < 0$.
 we have to find $(\frac{dy}{dx}) < 0$.

$$8x - 3 < 0.$$

$$8x < 3.$$

$$x < \underline{\underline{\frac{3}{8}}}.$$

i.e. function decreasing at $x < \underline{\underline{\frac{3}{8}}}$

$$\underline{\underline{x}}$$

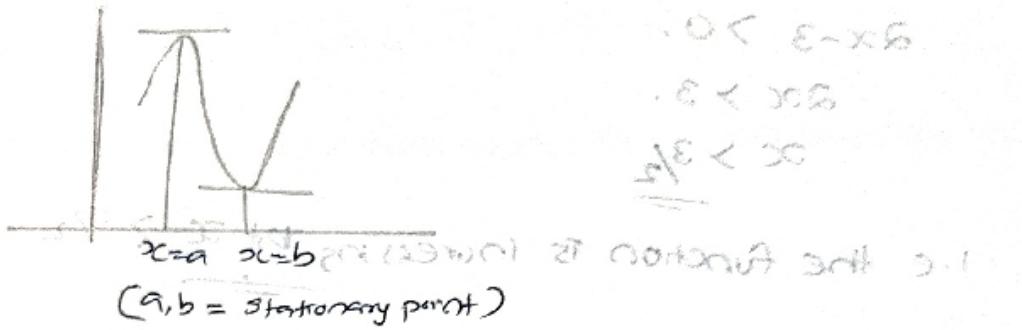
$$5 + x^2 + 3x^2 - 3x^2 + 5$$

$$5 + x^2 - 3x^2 = \underline{\underline{\frac{5b}{3b}}} \quad A$$

Stationary points / turning points

given a function $y = f(x)$, A point $x=a$ is a stationary point of $y=f(x)$, if the function neither increases nor decreases at that point.

A stationary point is also called "turning point" since the function changes its nature at that point.



* $x=a$ is stationary point of $y=f(x)$, if and only if.

$$\left(\frac{dy}{dx}\right)_{(x=a)} = 0.$$

Now

$$0 < \Delta x - x_b < \epsilon$$

- Find the stationary points of following functions.

$$1) y = x^2 - 4x + 10.$$

$$\frac{dy}{dx} = 2x - 4.$$

$$\frac{dy}{dx} = 0. \Rightarrow 2x - 4 = 0.$$

$$\Rightarrow 2x = 4$$

$$\underline{\underline{x = 2}}$$

$$2) y = 2x^3 - 9x^2 + 12x + 2$$

$$\frac{dy}{dx} = 6x^2 - 18x + 12$$

$$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 18x + 12 = 0 \text{ given } x \in \mathbb{R}$$

$$6x^2 - 18x = -12 \quad \begin{matrix} \text{add } +12 \\ \text{pract } = 2 \end{matrix}$$

$$\Rightarrow 6(x^2 - 3x + 2) = 0 \quad \begin{matrix} \text{pract } = 6 \\ \text{pract } = 2 \end{matrix}$$

$$\Rightarrow (x^2 - 3x + 2) = 0 \quad \begin{matrix} \text{letting } \text{pract } = 0 \\ \text{pract } = 2 \end{matrix}$$

$$\Rightarrow (x-1)(x-2) = 0 \quad \begin{matrix} \text{as } x^2 - 3x + 2 = 0 \\ \text{pract } = 2 \end{matrix}$$

$$\Rightarrow x = 1, 2, \quad 0 = (x-1)(x-2)$$

$$x=3 \rightarrow \infty$$

$$\bullet y = x^3 - 3x^2 - 9x + 5 \quad \text{find } \text{pract } = 2 \text{ and } 3$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9 \quad \begin{matrix} \text{pract } = 3x^2 - 6x - 9 \\ \text{pract } = 9 \end{matrix}$$

$$(\frac{dy}{dx}) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0 \quad \begin{matrix} \text{min. point} \\ \text{pract } = 3x^2 - 6x - 9 \end{matrix}$$

$$\Rightarrow x^2 - 2x - 3 = 0 \quad \begin{matrix} \text{pract } = 3x^2 - 6x - 9 \\ \text{pract } = 9 \end{matrix}$$

$$\Rightarrow x = 3, -1 \quad \begin{matrix} \text{pract } = 3x^2 - 6x - 9 \\ \text{pract } = 9 \end{matrix}$$

$$\therefore x = 3 \text{ or } x = -1$$

Maximum & minimum point

Given a function $y = f(x)$, Then $x = a$ is the max. point if $f'(a) = 0$ and $f''(a) < 0$.

or if $f''(a) > 0$ then $x = a$ is the min. point.

$$1) \frac{dy}{dx} = 0, 01 + 5x + 2x^2 - 2x^2 - 5 = 0 \quad \begin{matrix} \text{pract } = 0 \\ \frac{dy}{dx} = 0 \end{matrix}$$

$$2) \frac{d^2y}{dx^2} < 0 \quad \begin{matrix} \text{pract } = 0 \\ \frac{d^2y}{dx^2} < 0 \end{matrix}$$

$$\text{at } x = a$$

a point $x = b$ is the minimum point of $y = f(x)$.

$$1) \frac{dy}{dx} = 0$$

$$2) \frac{d^2y}{dx^2} > 0$$

Find the max. value of function $y = 2x^3 - 3x^2 - 36x + 10$.

$$A. \quad y = 2x^3 - 3x^2 - 36x + 10$$

$$\frac{dy}{dx} = 6x^2 - 6x - 36$$

where $\frac{dy}{dx} = 0$, (Find stationary points)

$$\Rightarrow 6x^2 - 6x - 36 = 0$$

$$6(x^2 - x - 6) = 0 \quad \therefore x = 3, -2$$

$$\therefore x = 3, -2$$

The stationary points are $3, -2$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(6x^2 - 6x - 36)$$

$$= 12x - 6$$

where $\frac{d^2y}{dx^2} < 0$. (at max. point)

$$\text{when } x = 3, \frac{d^2y}{dx^2} = 12 \cdot 3 - 6 = 30 > 0.$$

$$\text{when } x = -2, \frac{d^2y}{dx^2} = 12 \cdot (-2) - 6 = -30 < 0.$$

$\therefore x = -2$ is the max. point

$$\text{Max. value at } y = 2 \cdot (-2)^3 - (3(-2))^2 - 36(-2) + 10$$

$$= 2 \cdot -16 - 12 + 72 + 10. \quad \therefore \frac{eb}{xb} (1)$$

$$= -28 + 72 + 10. = 44 + 10 \quad \therefore \frac{eb}{xb} (2)$$

$$= \underline{\underline{54}} \quad \therefore \frac{eb}{xb} (3)$$

$(x-2)^2 = 0$ to find minimum at $x = 2$ by taking $\frac{dy}{dx}$

$$0 = \frac{eb}{xb} (1)$$

$$0 < \frac{eb}{xb} (2)$$

Deflection of a beam is given by ($y = 2x^3 - 9x^2 + 12x$)

find the max. deflection at end of beam

$$y = 2x^3 - 9x^2 + 12x$$

$$\frac{dy}{dx} = 6x^2 - 18x + 12$$

Find stationary point

$$\frac{dy}{dx} = 0.$$

$$\Rightarrow 6x^2 - 18x + 12 = 0$$

$$\Rightarrow 6(x^2 - 3x + 2) = 0.$$

$$(x^2 - 3x + 2) = 0 \quad \text{Factorize soft 2)} \quad \underline{x-2} \quad \underline{x-1}$$

$$x = 2, 1 \Rightarrow \text{stationary points } P((x=0), 0) \frac{dy}{dx} = \frac{m^3 b}{30 b}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(6x^2 - 18x + 12)$$

$$= 12x - 18$$

$$\text{at } x=1; \frac{d^2y}{dx^2} = 12(1) - 18 = -6 < 0 \quad \text{so } x=1 \text{ is max deflection}$$

$$\text{at } x=2 \quad \frac{d^2y}{dx^2} = 12(2) - 18 = 6 > 0 \quad \text{so } x=2 \text{ is min deflection}$$

$\therefore x=1$ is the max. point so $m = 1$ to solve $m^3 b$

$$\text{Max. deflection} = 2(1)^3 - 9(1)^2 + 12(1)$$

$$= 2 - 9 + 12 = -7 + 12 = 5$$

✓ Bending moment of a rod of 10 m long and weighing 40 kg & resting at its ends at a dist. of 20 cm from one end is given by $M = 2(10x - x^2)$. find the max. bending moment

✓ Bending moment of a rod of 10 m long and weighing 40 kg & resting at its ends at a dist. of 20 cm from one end is given by $M = 2(10x - x^2)$. find the max. bending moment

$M = 2(10x - x^2)$ Deflection is increased up to deflection

Here we have to find the max. value of M soft beam
finding stationary points.

$$\frac{dm}{dx} = 2(10 - 2x).$$

$$\frac{dm}{dx} = 0.$$

$$2(10 - 2x) = 0.$$

$$10 - 2x = 0$$

$$+2x = +10$$

$x = 5$ (it is the stationary point)

$$\frac{d^2m}{dx^2} = \frac{d}{dx}(2(10 - 2x))$$
 monotonic $\in 1, 2 = \underline{x}$

$$= 2(0 - 2) = -4$$

$$S1 + xS1 - \frac{1}{2}x^2 = 8$$

$$S1 + xS1 - \frac{1}{2}x^2 = \frac{8b}{20}$$

monotonic b \rightarrow

$$D = \frac{8b}{20}$$

$$D = S1 + xS1 - \frac{1}{2}x^2 \leftarrow$$

$$D = (S + xS - \frac{1}{2}x^2)^2 \leftarrow$$

$x = 5$ (it is the stationary point) $+ xS - \frac{1}{2}x^2$

$$S1 + xS1 - \frac{1}{2}x^2 \frac{b}{20} = \frac{8b}{20}$$

$$81 - xS1 =$$

when $x = 5$; $\frac{d^2m}{dx^2} = -4 < 0$.

$$0 > 0^- = 81 - 5S1 = \frac{8b}{20}; 1 = \infty \rightarrow 10$$

$\therefore x = 5$ is the max. point $81 - 5S1 = \frac{8b}{20} \rightarrow x = 10$

Max. Value of $M = 2(10x - x^2)$ at $x = 10$

$$= 2(50 - 25) = 2 \times 25$$

$$(1) S1 + \frac{1}{2}(1)^2 - \frac{1}{2}(1)S = 200 \times 10 \times 25$$

$$= 50 \text{ kg m}$$

$$B = S + \frac{1}{2} + \sqrt{\frac{1}{4} + S - 1} =$$

beam

Deflection of a wave is given $y = 4x^3 + 9x^2 - 12x + 2$
find the Max. deflection. \rightarrow to maximum deflection.

max. deflection \rightarrow $(x - x0)^2 = 0$ \rightarrow $x = x0$ to order 2. $y = 0$

max. deflection $(x - x0)^2 = 0$ to give a true zero

maximum deflection

then gradient is constant

$$RS = \frac{h}{5} = \frac{AB}{5}$$

$$O = \frac{AB}{3b}$$

$$OS = RS - \frac{1}{3}$$

$$RS = \frac{h}{4}$$

$$AB = x$$

$$OS = \frac{A^2B}{3D}, \text{ if } x = \text{ const. } \therefore S = \frac{A^2b}{3D}$$

for a given area of A $\frac{1}{b}$ = const.

$\therefore A = \text{const.} \times b$ or A is $\propto b$

\checkmark PT a rectangular of fixed perimeter has its max. area when it becomes a square. P

A. x : length of rectangle in cm
y : breadth

$$A = xy \quad \text{--- ①}$$

$2x + 2y = l$ \Rightarrow $2y = l - 2x$ \Rightarrow $y = \frac{l}{2} - x$

$\therefore A = x \left(\frac{l}{2} - x \right)$ \Rightarrow $A = \frac{l}{2}x - x^2$ \therefore A is minimum when $x = \frac{l}{4}$

Sub ① \Rightarrow $y = \frac{l}{4}$ \therefore A is maximum when $x = \frac{l}{4}$

$$\begin{aligned} A &= x \left(\frac{l}{2} - x \right) \\ &= \frac{l}{2}x - x^2 \\ &\quad dA/dx + dx/dx = 0 \text{ for max.} \\ &\quad 0 = \frac{l}{2} - 2x \Rightarrow x = \frac{l}{4} \\ \text{we have to find value of } x, A \text{ is max.} \end{aligned}$$

Finding stationary point

$$\frac{dA}{dx} = \frac{l}{2} - 2x$$

$$\frac{dA}{dx} = 0$$

$$\frac{l}{2} - 2x = 0$$

$$\frac{l}{2} = 2x$$

$$x = \frac{l}{4}$$

x in cm^2 is maximum.

$$\frac{d^2A}{dx^2} = -2, \text{ when } x = \frac{l}{4}, \frac{d^2A}{dx^2} = -2 < 0.$$

when $x = \frac{l}{4}$ is the max. point

i.e. Area A is max. when $x = \frac{l}{4}$.

when $x = \frac{l}{4}$, $y = \frac{l}{2} - \frac{l}{4} = \frac{l}{4}$ which is TQ

i.e. A is max. when $x = y = \frac{l}{4}$.

when rectangular is square $x = A = \text{breadth} = b$



The hollow of cylinder can 100 cc of water.

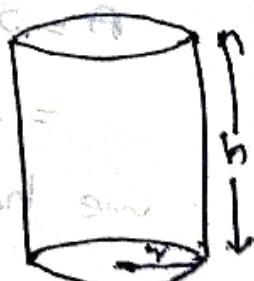
is to be made so that the area of the metal used is minimum. P.T the radius which will give minimum area is $3\sqrt{\frac{100}{\pi}}$ cm. \Rightarrow ① due to

A.

$$\text{Surface area} = \pi r^2 + 2\pi rh \quad (x = l) \rightarrow ①$$

$$\text{Given } V = \pi r^2 h = 100.$$

$$h = \frac{100}{\pi r^2}$$



Sub ① \Rightarrow object from rotating shaft to outer shell:

$$S = \pi r^2 + 2\pi r + \frac{200}{\pi r^2}, \text{ is easy to calculate}$$

$$S' = 2\pi r + \frac{200}{r^2} \text{ outer shell surface is constant}$$



We have to find the value of r , S is min.

To solve with help of first derivative

Finding stationary points

$$\frac{ds}{dr} = 2\pi r - \frac{200}{r^2} \quad d^2r = V$$

$$\frac{ds}{dr} = 0 \quad \Rightarrow \quad 2\pi r - \frac{200}{r^2} = 0$$

$$\Rightarrow \quad \Rightarrow (2\pi r) \frac{200}{r^2} = V \in \text{① due}$$

$$2\pi r^3 - \frac{200}{r^2} =$$

$$r^3 = \frac{100}{\pi}$$

or a point of stationary

$$r = 3\sqrt{\frac{100}{\pi b}}. \quad (\text{It is stationary point})$$

$$\frac{d^2s}{dr^2} = 2\pi - \frac{400}{r^3} \in 0 = \frac{Vb}{r^3}$$

$$2\pi r^2 = 2\pi r^2 - \frac{400}{r^3}$$

$$\frac{Vb}{r^3} = \frac{400}{r^3} \quad \text{when } r = 3\sqrt{\frac{100}{\pi}}, \quad \therefore r^3 = \frac{100}{\pi}$$

$$\text{calculate it. } \frac{d^2s}{dr^2} = 2\pi + \frac{400}{100/\pi} = \frac{\sqrt{b}}{r^3} - \frac{400}{\pi^2} = \frac{\sqrt{b}}{r^3}$$

$$= 2\pi + 4\pi$$

> 0 or concave

$$= 6\pi > 0$$

$$\frac{\sqrt{b}}{r^3} - \frac{400}{\pi^2} = \frac{\sqrt{b}}{r^3}$$

\therefore when $r = 3\sqrt{\frac{100}{\pi}}$ the S is minimum

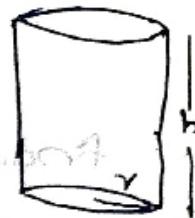
$$= \frac{\sqrt{b}}{r^3} - \frac{400}{\pi^2}$$

$$= \frac{\sqrt{b}}{r^3} - \frac{400}{\pi^2}$$

1. The sum of the diameter and length of an open cylindrical vessel is 40 cm . P.T. = max. Volume obtained when the radius = length.

A. Volume of cylinder $\Rightarrow V = \pi r^2 h$.

Here we have to find the value of r and h , and volume is max.



$$V = \pi r^2 h \quad \text{--- (1)}$$

Given that $2r + h = 40$

$$\Rightarrow h = 40 - 2r$$

$$\begin{aligned} \text{Sub (1)} \Rightarrow V &= \pi r^2 (40 - 2r) \\ &= 40\pi r^2 - 2\pi r^3. \end{aligned}$$

Stationary points finding are

$$\frac{dV}{dr} = 80\pi r - 6\pi r^2$$

$$\frac{dV}{dr} = 0 \Rightarrow 80\pi r - 6\pi r^2 = 0$$

$$\frac{80\pi}{6} r = 6\pi r^2$$

$$\frac{80}{6} = r \Rightarrow \frac{80}{6} = r \Rightarrow r = \frac{80}{6} = \frac{40}{3}$$

$$\frac{d^2V}{dr^2} = 80\pi - 12\pi r \quad \frac{80\pi}{6} > 0 \Rightarrow \frac{40}{3} \text{ is stationary}$$

when $r = \frac{40}{3}$ $\Delta A + \Delta C =$

$$\frac{d^2V}{dr^2} = 80\pi - 12\pi \cdot \frac{40}{3} =$$

$$= 80\pi - 160\pi = -80\pi < 0$$

$$= -80\pi < 0$$

\therefore i.e. V max. when $r = \frac{40}{3}$

when $r = \frac{40}{3}$

$$h = 40 - 2 \cdot \frac{40}{3}$$
$$= \underline{\underline{\frac{40}{3}}}$$

$\therefore V$ is max. at $r=h = \frac{40}{3}$

- Find 2 numbers whose sum is 14 and sum of squares is minimum.

whose sum is 14 too to obtain fixed no. A.

A. $x+y = 20$ to get y . mst $x+y = 20$
 $x = y$.

whose sum $= x^2 + y^2$ but bcoz $x+y = 20$
then $x+y = 20$ both x & y shift to side term

$x+y = 14$ x & y shift to another term

In ① $\Rightarrow s = x^2 + (14-x)^2$
shift to side shift x to

$\frac{ds}{dx} = 2x + 2(4-x) \cdot (-1)$

finding point of stationarity. suppose
 $\frac{ds}{dx} = 0$

$$\frac{ds}{dx} = 2x + 2(4-x) \cdot (-1) = 0$$
$$= 2x - 2(4-x) + 2x$$

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$2x + 2(4-x) = 0$ $\Rightarrow 2x + 8 - 2x = 0$ $\Rightarrow x = 4$

$$x \left(\frac{d^2s}{dx^2} = 2 \right) \Rightarrow 2 = 2$$

$$x(2x+2x-12) = 2$$

$$\frac{d^2s}{dx^2}$$

$$x(4x-12) = 2$$

$$x(4x-12) = 2 \Rightarrow 4x^2 - 12x - 2 = 0$$

$$x = 7, \therefore y = 14 - 7 = \underline{\underline{7}}$$

To find how far it can be search maximum so how

maximization of volume

- A open box is made out of a square sheet of side 18 cm. by cutting off equal squares at each corners and turning up this sides what size of the square should be cuts in order that volume of the box is maximum.

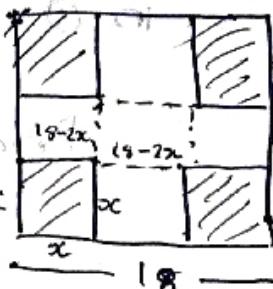
A.

Let x be the side of the square which are cut off

Then volume of box

$$(18-x)^2 \cdot x + x^2 = abx$$

$$V = (18-2x)(18-2x)x$$



Now we have to find that the value of x , V is max.

$$V = (324 - 36x - 36x^2 + 4x^3)x$$

$$= (324 - 72x + 4x^2)x$$

$$= 4x^3 - 72x^2 + 324x$$

$$= 4x^3 - 72x^2 + 324x$$

Finding the stationary points.

$$\frac{dv}{dx} = 12x^2 - 144x + 324$$
$$= 12(x^2 - 12x + 27).$$

$$\frac{dv}{dx} = 0 \Rightarrow 12(x^2 - 12x + 27) = 0$$
$$\Rightarrow x^2 - 12x + 27 = 0 \rightarrow \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow x = 3, 9 \text{ (one the st. points)}$$

$x=9$ is not possible \because box \therefore (total length = 18)

$$\Rightarrow x = 3.$$

$$\frac{d^2v}{dx^2} = 24x - 144.$$

when $x = 3$.

$$\frac{d^2v}{dx^2} = 24 \times 3 - 144,$$
$$= 72 - 144$$
$$= \underline{-72} < 0.$$

$$\begin{aligned} 24 \times \\ \frac{3}{72} \\ \frac{144}{72} \end{aligned}$$

\therefore Volume of the box is max. at $x = 3$.

Volume of box will be max. If the remove square of side 3.