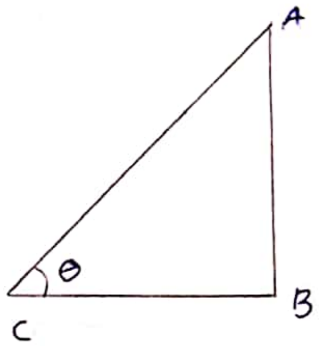
TRIGONOMETRY

15/7/2019

The word 'Trigonometry' is derived from 2 Greek words "Trigonon" which means triangle and "metron" means measurements.

Therefore, the greek word "Trigonometry" means the measurements of triangle.



1) $\sin\theta$ (sine of θ)

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{AC}$$

2) $\cos\theta$ (cosine of θ)

$$\cos\theta = \frac{\text{adjascent side}}{\text{hypotenuse}} = \frac{BC}{AC}$$

3) $\tan\theta$ (Tangent of θ)

$$\tan\theta = \frac{\text{opposite side}}{\text{adjascent side}} = \frac{AB}{BC}$$

4) $\operatorname{cosec}\theta$ (cosecant of θ)

$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{opossite side}}$$

$$= 1/\sin\theta = \frac{AC}{AB}$$

5) $\sec\theta$ (secant of θ)

$$\sec\theta = 1/\cos\theta = \frac{\text{hypotenuse}}{\text{adjascent side}} = \frac{AC}{BC}$$

6) $\cot\theta$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjascent side}}{\text{opposite side}} = \frac{BC}{AB}$$

Results :– (Strong θ)

1) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

ie $\sin\theta \times \operatorname{cosec}\theta = 1$

2) $\sec\theta = \frac{1}{\cos\theta}$ or $\cos\theta = \frac{1}{\sec\theta}$

ie, $\cos\theta \sec\theta = 1$

3) $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta = \frac{1}{\cot\theta}$

ie $\tan\theta \cot\theta = 1$

4) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$(\sin\theta)^2 = \sin^2\theta$

5) $\sin^2\theta + \cos^2\theta = 1$

$*$ $\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2$

$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$

$= \frac{AB^2 + BC^2}{AC^2}$

$= \frac{AC^2}{AC^2} = 1$

$*$ $\sin^2\theta = 1 - \cos^2\theta$

$\therefore \sin\theta = \pm\sqrt{1-\cos^2\theta}$

$*$ $\cos^2\theta = 1 - \sin^2\theta$

$\therefore \cos\theta = \pm\sqrt{1-\sin^2\theta}$

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6) $1 + \tan^2\theta = \sec^2\theta$

ie $\sec^2\theta - \tan^2\theta = 1$

7) $1+\cot^2\theta = \operatorname{cosec}^2\theta$

ie $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

acute angle $< 90°$

obtuse angle $> 90°$

$= 90°$ – Right angle

1) • If $\sin\theta = 1/2$, θ = is acute, find all other trigonometric functions.

given $\sin\theta = 1/2$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$
$$= \sqrt{1-(1/2)^2}$$
$$= \sqrt{1-1/4}$$
$$= \sqrt{\frac{4-1}{4}}$$
$$= \sqrt{3/4}$$
$$= \sqrt{3}/2$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$= \frac{1/2}{\sqrt{3}/2}$$
$$= 1/\cancel{2} \times \cancel{2}/\sqrt{3}$$
$$= 1/\sqrt{3}$$

$\sin\theta = 1/2$
$\cos\theta = \sqrt{3}/2$
$\tan\theta = 1/\sqrt{3}$
$\operatorname{cosec}\theta = 2$
$\sec\theta = 2/\sqrt{3}$
$\cot\theta = \sqrt{3}$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{1/2}$$
$$= 2$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$$

If $\cos\theta = 1/2$. find all other trigonometric functions.

If $\sin\theta = \frac{5}{13}$, find all other trigonometric ratios

given $\sin\theta = \frac{5}{13}$

~~cosec θ~~ -

$\cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-(5/13)^2}$

$= \sqrt{1-\frac{25}{169}}$

$= \sqrt{\frac{144}{169}}$

$= \frac{12}{13}$ //

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

$\sec\theta = \frac{13}{12}$

$\operatorname{cosec}\theta = \frac{13}{5}$

$\cot\theta = \frac{12}{5}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{5/13} = \frac{13}{5}$ //

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{12/13} = \frac{13}{12}$ //

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{\frac{12}{13}} \cdot \frac{5}{\cancel{13}} \times \frac{\cancel{13}}{12}$

$= \frac{5}{12}$ //

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{5/12} = 12/5$

4) If $\cot\theta = \frac{1}{3}$. find all other trigonometric ratios

given $\cot\theta = \frac{1}{3}$

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{1/3} = \underline{\underline{3}}$$

$$\cancel{\sec^2\theta - \tan^2\theta = 1}$$

$$\cancel{\tan^2\theta = 1 - \sec^2\theta}$$

$$\cancel{\operatorname{cosec}^2\theta = 1+\cot^2\theta}$$

$$\cancel{\operatorname{cosec}^2\theta = 1+(1/3)^2}$$

$$\operatorname{cosec}^2\theta = 1+\cot^2\theta$$

$$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+(1/3)^2}$$

$$= \sqrt{1+\frac{1}{9}}$$

$$= \sqrt{\frac{10}{9}}$$

$$= \sqrt{10}/3$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{3}{\sqrt{10}}$$

$$\cancel{\sec^2\theta = 1 - \sec^2\theta}$$

$$\sec^2\theta = 1+\tan^2\theta$$

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$= \sqrt{1+(3)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10} \,//$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{10}}$$

$\sin\theta = 3/\sqrt{10}$

$\cos\theta = 1/\sqrt{10}$

$\tan\theta = 3$

$\operatorname{cosec}\theta = \sqrt{10}/3$

$\sec\theta = \sqrt{10}$

$\cot\theta = 1/3$

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Method

$\cos\theta \times \tan\theta = \sin\theta$ —— (1)

$\sin\theta \times \cot\theta = \cancel{\sin\theta} \times \frac{\cos\theta}{\cancel{\sin\theta}}$

$= \cos\theta$ —— (2)

5) $\tan\theta = 2$, find al trigonometric functions.

given, $\tan\theta = 2$

$\cot\theta = \frac{1}{2}$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

~~cot~~

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}\theta = \sqrt{1 + \cot^2\theta}$

$= \sqrt{1 + (1/2)^2}$

$= \sqrt{1 + \frac{1}{4}}$

$= \sqrt{\frac{4+1}{4}} = \sqrt{5/4}$

$= \frac{\sqrt{5}}{2}$

$\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{2}{\sqrt{5}}$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$= \sqrt{1 - \left(\frac{2}{\sqrt{5}}\right)^2}$

$= \sqrt{1 - \frac{4}{5}} = \sqrt{\frac{5-4}{5}} = \sqrt{\frac{1}{5}}$

$= \sqrt{1}/\sqrt{5}$

$= \frac{1}{\sqrt{5}}$

$\sec\theta = \sqrt{5}$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

$\tan\theta = 2$

$\cot\theta = \frac{1}{2}$

$\mathrm{cosec}\,\theta = \sqrt{5}/2$

$\sec\theta = \sqrt{5}$

- If $\cos\theta = 1/2$ find all trigonometric functions.

given $\cos\theta = \frac{1}{2}$

$\sec\theta = 2$

~~cos θ second~~

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-(1/2)^2}$

$= \sqrt{1-\frac{1}{4}}$

$= \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \sqrt{3}/2$

$\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$

$= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{\cancel{2}} \times \frac{\cancel{2}}{1}$

$= \sqrt{3}$

$\cot\theta = \frac{1}{\sqrt{3}}$

$\sin\theta = \sqrt{3}/2$

$\cos\theta = 1/2$

$\tan\theta = \sqrt{3}$

$\operatorname{cosec}\theta = 2/\sqrt{3}$

$\sec\theta = 2$

$\cot\theta = \frac{1}{\sqrt{3}}$

- $\tan\theta = 3$ find all other trigonometric function
- If $\sin\theta = \frac{12}{13}$, find all other trigonometric functions.

Angle	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined.
$\cot\theta$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	1/infinity

not defined $\rightarrow \alpha$

Very large → Infinity

- find the value of $\tan^2 60^\circ + \tan^2 45^\circ$

$= (1)^2 + (\sqrt{3})^2$

$= 1 + 3$

$= \underline{\underline{4}}$

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• find $2\sin 30^\circ + \cos 60^\circ - \tan 45^\circ$

$2\sin 30^\circ + \cos 60^\circ - \tan 45^\circ$

$2 \times \frac{1}{2} + \frac{1}{2} - 1$

$\frac{2}{2} + \frac{1}{2} - 1$

$1 + \frac{1}{2} - 1$

$\frac{2+1}{2} - 1$

$\frac{3}{2} - 1$

$\frac{3-2}{2} = \frac{1}{2}$ //

• proove that $(1+\cos A)(1-\cos A) = \sin^2 A$

$\underbrace{(1+\cos A)}_{a+b}\ \underbrace{(1-\cos A)}_{a-b} = \sin^2 A$

$L.H.S = 1 - \cos^2 A$

$= \sin^2 A$

$= R.H.S$

• S.T $(\sin A + \cos A)^2 = 1 + 2\sin A \cos A$

$L.H.S = \sin^2 A + 2\sin A\cos A + \cos^2 A$ $\qquad (a+b)^2 = a^2 + 2ab + b^2$

$= 1 + 2\sin A \cos A$

$= R.H.S$

• P.T $\cos A \times \tan A \times \operatorname{cosec} A = 1$

$L.H.S = \cancel{\cos A} \times \frac{\cancel{\sin A}}{\cancel{\cos A}} \times \frac{\cancel{\sin A}}{\cancel{\cos A}} \cdot \frac{1}{\cancel{\sin A}}$

$= 1$

$= R.H.S$

- S.T $\operatorname{cosec}^2 x + \sec^2 x = \operatorname{cosec}^2 x \cdot \sec^2 x$

$$L.H.S = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x * \cos^2 x}$$

$$= \frac{1}{\sin^2 x \times \cos^2 x} = \frac{1}{\sin^2 x} \times \frac{1}{\cos^2 x}$$

$$= \operatorname{cosec}^2 x \cdot \sec^2 x$$

$= \underline{R.H.S}$

- S.T $\sin A \cdot \sec A \cdot \cot A = 1$

LHS $\sin\cancel{A} \frac{1}{\cancel{\cos A}} \frac{\cancel{\cos A}}{\sin\cancel{A}} = 1$

$= R.H.S$

- $\tan x + \cot x = \sec x \cdot \operatorname{cosec} x$

$$L.H.S = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x * \sin x}$$

$$= \frac{1}{\cos x \times \sin x}$$

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$= \sec x \cdot \operatorname{cosec} x$$

$= \underline{R.H.S}$

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$\ldots \times \cos 60 \times \sec 30 = \frac{2}{3}$

L.H.S $(\sqrt{2})^2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2$

$= 2 \times \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3}$

$= \frac{2}{3}$

- $\tan\theta = 3$, find all trigonometric function

given $\tan\theta = 3$

$\cot\theta = \frac{1}{3}$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec\theta = \sqrt{1+\tan^2\theta}$

$= \sqrt{1+\left(\frac{1}{3}\right)^2}$

$= \sqrt{1+\frac{1}{9}}$

$= \sqrt{\frac{9+1}{9}} = \sqrt{\frac{10}{9}}$

$= \frac{\sqrt{10}}{3}$

$\cos = \frac{1}{\sec\theta} = \frac{3}{\sqrt{10}}$

$\sin\theta = \frac{1}{\sqrt{10}}$

$\cos\theta = \frac{3}{\sqrt{10}}$

$\tan\theta = 3$

$\sec\theta = \frac{3}{\sqrt{10}}$

$\mathrm{cosec}\,\theta = \sqrt{10}$

$\cot\theta = \frac{1}{3}$

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{3}{\sqrt{10}}\right)^2$

$= 1 - \frac{9}{10}$

$= \frac{10-9}{10} = \frac{1}{10}$ //

$\sin\theta = \sqrt{1/10} =$ $\quad$ $\mathrm{cosec} = \frac{\sqrt{10}}{1}$

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S.T $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

$(a+b)^2 = a^2+2ab+b^2$

$$L.H.S = \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + 1^2 + 2\times1\times\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$$

$\cos^2\theta + \sin^2\theta =$

$$= \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\sin\theta} = \frac{2}{\sin\theta}$$

$= 2\operatorname{cosec}\theta$

$= RHS$

P.T $\frac{\cos\theta}{(1+\sin\theta)} + \frac{(1+\sin\theta)}{\cos\theta} = 2\sec\theta$

$$L.H.S = \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta}$$

$$= \frac{\cos^2\theta + 1^2 + 2\sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{2 + 2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{2\cancel{(1+\sin\theta)}}{\cancel{(1+\sin\theta)}\cos\theta} = \frac{2}{\cos\theta} = 2\times\frac{1}{\cos\theta}$$

$= 2\sec\theta$

P.T $\dfrac{\sec\theta}{\sec\theta-1} + \dfrac{\sec\theta}{\sec\theta+1} = 2\operatorname{cosec}^2\theta$

$$L.H.S = \frac{\sec\theta(\sec\theta+1)+\sec\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$$

$$= \frac{\sec^2\theta+\sec\theta+\sec^2\theta-\sec\theta}{(\sec\theta-1)(\sec\theta+1)}$$

$(a-b)(a+b) = a^2-b^2$

$$= \frac{\sec^2\theta+\cancel{\sec\theta}+\sec^2\theta-\cancel{\sec\theta}}{\sec^2\theta-1^2}$$

$$= \frac{2\sec^2\theta}{\sec^2\theta-1}$$

$\sec^2\theta-1=\tan^2\theta$

$$= \frac{2\sec^2\theta}{\tan^2\theta}$$

$$= \frac{2\times\frac{1}{\cancel{\cos^2\theta}}}{\frac{\sin^2\theta}{\cancel{\cos^2\theta}}}$$

$$= 2\times\frac{1}{\sin^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

$$= 2\times\operatorname{cosec}^2\theta$$

$= R.H.S$

• P.T $\dfrac{1+\sin A}{\cos A} = \dfrac{\cos A}{1-\sin A}$

$$L.H.S = \frac{1+\sin A}{\cos A} \times \frac{1-\sin A}{1-\sin A}$$

$$= \frac{1-\sin^2 A}{\cos(1-\sin A)}$$

$$= \frac{\cancel{\cos^2 A}^{\,1}}{\cancel{\cos A}(1-\sin A)}$$

$$= \frac{\cos A}{1-\sin A} = R.H.S$$

• S.T $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

$$L.H.S = \frac{\sin^2\theta + (1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(\sin\theta)}$$

$$= \frac{\sin^2\theta + 1^2 + \cos^2\theta}{(1-\cos\theta)\sin\theta}$$

$$= \frac{1+1}{(1-\cos\theta)\sin\theta}$$

$$= \frac{2}{(1-\cos\theta)\sin\theta}$$

$$= \frac{2}{\quad}$$

$$= \frac{\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta}{(1-\cos\theta)\sin\theta}$$

$$= \frac{2-2\cos\theta}{(1-\cos\theta)\sin\theta}$$

$$= \frac{2\cancel{(1-\cos\theta)}}{\cancel{(1-\cos\theta)}\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

$$= RHS$$

S.T $(\cot A+1)^2 + (\cot A-1)^2 = 2\operatorname{cosec}^2 A$

$L.H.S = \cot^2 A + 2\cancel{\cot A} + 1 + \cot^2 A - 2\cancel{\cot A} + 1$

$= 2\cot^2 A + 2$

$= 2(\cot^2 A + 1)$

$= 2\operatorname{cosec}^2 A$

$= \underline{R.H.S}$

P.T $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

$$L.H.S = \frac{\operatorname{cosec}\theta(\operatorname{cosec}\theta+1) + \operatorname{cosec}\theta(\operatorname{cosec}\theta-1)}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$$

$$= \frac{\operatorname{cosec}^2\theta + \cancel{\operatorname{cosec}\theta} + \operatorname{cosec}^2\theta - \cancel{\operatorname{cosec}\theta}}{\operatorname{cosec}^2\theta + 1}$$

$$= \frac{2\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta - 1}$$

$$= \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}$$

$$= 2 \times \frac{1}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}}{\cos^2\theta}$$

$$= \frac{2}{\cos^2\theta}$$

$= \underline{2\sec^2\theta}$

$= \underline{R.H.S}$

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1). P.T $\operatorname{cosec}^2 45^\circ \times \cos^2 60^\circ \times \sec^2 30^\circ = \frac{2}{3}$

2). evaluate $4\tan^2 60^\circ - 3\tan^2 30^\circ + \tan^2 45^\circ$

3). S.T $\cos 60^\circ \times \cos 30^\circ - \sin 60^\circ \times \sin 30^\circ = 0$

4. P.T $\sin 60^\circ \times \cos 30^\circ + \cos 60^\circ \times \sin 30^\circ = 1$

1) $(\sqrt{2})^2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2$

$\cancel{2} \times \frac{1}{\cancel{4}_{2}} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$

2) $4 \times (\sqrt{3})^2 - 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + 1^2$

$= 4 \times 3 - \cancel{3} \times \frac{1}{\cancel{3}} + 1$

$= 12 - \cancel{1} + \cancel{1}$

$= 12$

3) $\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

4) $\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

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S.T $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

L.H.S $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}}$

dividing both numerator and denominator with $\sqrt{1-\sin\theta}$

$= \frac{\sqrt{1-\sin\theta} \times \sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta} \times \sqrt{1-\sin\theta}}$

$$= \frac{1-\sin\theta}{\sqrt{1+\sin\theta} \times \sqrt{1-\sin\theta}}$$

$(a+b)(a-b)$

$$= \frac{1-\sin\theta}{1-\sin^2\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

$$= \text{R.H.S}$$

- P.T $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

L.H.S $= (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$ $\sec\theta = \frac{1}{\cos\theta}$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)(\tan\theta + \cot\theta)$$

$$= \cos^2\theta$$

$$= \frac{\cos^2\theta}{\cancel{\sin\theta}} \times \frac{\sin^2\theta}{\cancel{\cos\theta}} \times \tan\theta + \cot\theta$$

$$= \cos\theta \times \sin\theta \times (\tan\theta + \cot\theta)$$

$$= \cos\theta \times \sin\theta \times \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \cos\theta \cancel{\times \sin\theta} \times \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \cancel{\sin\theta}}$$

$$= 1 = \text{R.H.S}$$

5.7 $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1+\sin\theta}{\cos\theta}$

$1 = \sec^2\theta - \tan^2\theta$

L.H.S $= \dfrac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$\dfrac{1}{\sec\theta}$

$= \dfrac{\tan\theta + \sec\theta - \left[(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$

$(a+b)(a-b)$

$= a^2 - b^2$

$= \dfrac{\sec\theta + \tan\theta\left[1 - (\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\sec\theta + \tan\theta)\cancel{(1 - \sec\theta + \tan\theta)}}{\cancel{\tan\theta - \sec\theta + 1}}$

$= \sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$ = R.H.S

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• S.T $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

$\cancel{\cos A\,(1-\tan}$

$$L.HS = \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$(a-b)(a-b)$

$$= \frac{\cos A}{\frac{\cos A-\sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$

$$= \frac{\cos A \times \cos A}{\cos A-\sin A} + \frac{\sin A \times \sin A}{\sin A - \cos A}$$

$\frac{a}{m} + \frac{b}{m}$

$$= \frac{\cos^2 A}{\cos A-\sin A} + \frac{\sin^2 A}{\sin A-\cos A}$$

$= \frac{a+b}{m}$

$$= \frac{\cos^2 A(\sin A-\cos A) + (\cos A-\sin A)\sin^2 A}{(\cos A-\sin A)(\sin A-\cos A)}$$

$$= \frac{\sin A\cos^2 A - \cos A\cos^2 A + \cos A\sin^2 A - \sin A\sin^2 A}{\cos A\sin A - \cos^2 A - \sin^2 A + \sin A\cos A}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} + \frac{\sin^2 A}{-(\cos A \mp \sin A)}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} + \frac{\sin^2 A}{-(\cos A-\sin A)}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} = \frac{(\cos A + \sin A)(\cancel{\cos A - \sin A})}{\cancel{\cos A - \sin A}}$$

$= \cancel{\cos A - \sin A}$

$= \cos A + \sin A$ $\quad a^2 - b^2 = (a+b)(a-b)$

$= \sin A + \cos A$

$= R.H.S$

Radian Measure

Radian is the angle, subtended @ the center of a circle by an arc, whose length is equal to Radius of the circle

1 radia is denoted by - 1^c

Relation b/w degree measure and radian measure

$$\pi^c = 180^\circ$$

. Find $\sin^3 \pi/2 \times \cos^2 \frac{\pi}{4} \times \tan \frac{\pi}{6}$

$= \sin^3 90^\circ \times \cos^2 45^\circ \times \tan 30^\circ$

$= 1 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)$

$= 1 \times \frac{1}{2} \times \frac{1}{\sqrt{3}}$

$= \frac{1}{2\sqrt{3}}$ //

• evaluate $4\sin^3\frac{\pi}{3} - 3\cos^2\frac{\pi}{6}$

$= 4\times \sin^3\frac{\pi}{3} - 3\times\cos^2\frac{\pi}{6}$

$= 4\times\left(\frac{\sqrt{3}}{2}\right)^3 - 3\times\left(\frac{\sqrt{3}}{2}\right)^2$

$= \; = 4\times\frac{3\sqrt{3}}{8} - 3\times\frac{3}{4}$

$= \frac{\cancel{12}^{3}\sqrt{3}}{\cancel{8}_{2}} - \frac{9}{4}$

$= \frac{3\sqrt{3}}{2} - \frac{9}{4}$

$= \frac{12\sqrt{3} - 18}{8}$

$= \frac{6\sqrt{3} - 9}{4}$

• evalate $\sin\frac{\pi}{2}\times\cos\frac{\pi}{3}\times\tan\frac{\pi}{4}$

$= 1\times\frac{1}{2}\times 1$

$= \frac{1}{2}$

• find $4\sin^3\frac{\pi}{3} - 3\sin\frac{\pi}{3}$

$4\times\left(\frac{\sqrt{3}}{2}\right)^3 - 3\times\frac{\sqrt{3}}{2}$

$= 4\times\frac{3}{8} - \frac{3\sqrt{3}}{2}$

$= \frac{12}{8} - \frac{3\sqrt{3}}{2}$ $\quad = \frac{24 - 24\sqrt{3}}{16}$ $\quad = \frac{\cancel{12}^{3}(2 - 2\sqrt{3})}{\cancel{16}_{4}}$

$= \frac{6-6\sqrt{3}}{4}$

$=$

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Signs of trigonometric functions in quadrants

II^{nd} quadrant: Sin, Cosec } +ve	I^{st} quadrant: All +ve
III^{rd} quadrant: Tan, Cot } +ve	(IV)th quadrant: Cos, Sec } +ve

important

- if $\sin\theta = \frac{3}{5}$, θ lies in the II^{nd} quadrant, find all other trigonometric functions.

— given $\sin\theta = \frac{3}{5}$

$$\operatorname{cosec}\theta = \frac{5}{3}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= \sqrt{1-\frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$\cos\theta$ −ve in II^{nd} quadrant

so

$$\cos\theta = -\frac{4}{5}$$

$$\sec\theta = -\frac{5}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/\cancel{5}}{-4/\cancel{5}} = -\frac{3}{4}$$

$$\cot\theta = -\frac{4}{3}$$

• If $\tan\theta = \frac{5}{12}$, θ lies in IIIrd quadrant

= given,

$\tan\theta = \frac{5}{12}$

$\cot\theta = \frac{12}{5}$

$\sec\theta = \sqrt{1+\tan^2\theta}$

$1+\tan\theta^2 = \sec\theta$

$= \sqrt{1+(\frac{5}{12})^2}$

$= \sqrt{1+\frac{25}{144}}$

$= \sqrt{\frac{144+25}{144}}$

$= \sqrt{\frac{169}{144}}$

$= \frac{13}{12}$, bt θ lies in IIIrd quadrant so

$\sec\theta = -\frac{13}{12}$

$\cos\theta = \frac{-12}{13}$

$169 - 144 = 025$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$\text{cosec}\,\theta = \frac{-13}{5}$

$= \sqrt{1-(\frac{-12}{13})^2}$

$= \sqrt{1-\frac{144}{169}}$

$= \sqrt{\frac{169-144}{169}}$

$= \sqrt{\frac{25}{169}}$

$= \frac{-5}{13}$

1) • If $\cos\theta = \frac{1}{2}$, θ lies in the IVth quadrant.

• If $\sin A = -\frac{4}{5}$, A lies in IIIrd quadrant.

given $\sin A = -\frac{4}{5}$

$\operatorname{cosec} A = -\frac{5}{4}$

$\cos A = \sqrt{1-\sin^2 A}$

$= \sqrt{1-\left(\frac{-4}{5}\right)^2}$

$= \sqrt{1-\frac{16}{25}}$

$= \sqrt{\frac{25-16}{25}}$

$\sqrt{9/25}$

$= \frac{-3}{5}$

$\sec A = -5/3$

$\tan A = \frac{\sin A}{\cos A} = \frac{-4/\cancel{5}}{-3/\cancel{5}}$

$= \frac{-4}{-3}$

$= 4/3$

$\cot A = 3/4$

2) • If $\tan A = -\frac{24}{7}$, A lies in 4th quadrant.

3) • If $\sin\theta = -\frac{24}{25}$, θ lies in 4th quadrant.

$\rightarrow \tan A = \frac{-24}{7}$

$\cot A = \frac{-7}{24}$

$\sec\theta = \sqrt{1+\tan^2\theta}$

$= \sqrt{1+\left(-\frac{24}{7}\right)^2}$

$= \sqrt{1+\frac{576}{49}}$

$= \sqrt{\frac{49+576}{49}}$

$= \sqrt{\frac{625}{49}}$

$= \frac{25}{7}$

$\cos A = \frac{7}{25}$

$\sin A = \sqrt{1-\cos^2 A}$

$= \sqrt{1-\left(\frac{7}{25}\right)^2}$

$= \sqrt{1-\frac{49}{625}}$

$= \sqrt{\frac{625-49}{625}}$

$= \sqrt{\frac{576}{625}} = \frac{24}{25}$

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{24/25}$

$= 25/24$

$$\begin{array}{r} 24 \times \\ 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array}$$

2) $\sin\theta = -\frac{24}{25}$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{24}{25}} = -\frac{25}{24}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\frac{24}{25}\right)^2}$$

$$= \sqrt{1-\frac{576}{625}}$$

$$= \sqrt{\frac{625-576}{625}}$$

$$= \sqrt{\frac{49}{625}}$$

$$= \frac{7}{25}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{7/25} = \frac{25}{7}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{24/25}{7/25} = \frac{-24}{7}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{24/7} = -7/24$$

- $\cos\theta = \frac{1}{2}$, θ lies in the 4^{th} quadrant

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1/2} = \underline{\underline{2}}$$

$$\sin\theta = \sqrt{1^2 - \cos^2\theta}$$

$$= \sqrt{1 - \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{4-1}{4}}$$

$$= \sqrt{3/4} = -\sqrt{3}/2$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\sqrt{3}/2} = \underline{\underline{-2/\sqrt{3}}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/4}{1/2} = \underline{\underline{-\sqrt{3}}}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{\sqrt{3}} //$$

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COMPOUND ANGLES

An angle which is made up of sum of difference of 2 or more angles is known as compound angles

eg:- $A+B$, $A-B$, $A+B+C$, $A+B-C$, ..

viimp **Addition formulae**

1) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

2) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

3) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

Substraction formulae

1) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

2) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

3) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

30/7/19

- If $\sin A = -3/5$, $\sin B = \frac{12}{13}$, A lies in III^{rd} quadrant

B lies in II^{nd} quadrant find i) $\sin(A+B)$
ii) $\cos(A-B)$

— given $\sin A = -3/5$

$\cos A = \sqrt{1-\sin^2 A}$

$= \sqrt{1-(-3/5)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{25-9}{25}}$

$= \sqrt{\frac{16}{25}}$

$= \frac{-4}{5}$ (A lies in III^{rd} quadrant)

$\sin B = \frac{12}{13}$

$\cos B = \sqrt{1-\sin^2 B}$

$= \sqrt{1-\left(\frac{12}{13}\right)^2}$

$= \sqrt{1-\frac{144}{169}}$

$= \sqrt{\frac{169-144}{169}}$

$= \sqrt{\frac{25}{169}}$

$= \frac{5}{13}$

$= \frac{-5}{13}$

169 −
144
025

$\sin(A+B) = \sin A \cos B + \cos A \sin B.$

$= -3/5 \times \frac{-5}{13} + \frac{-4}{5} \times \frac{12}{13}$

$= \frac{15}{65} + \frac{-48}{65}$

$= \frac{15}{65} - \frac{48}{65}$

$= \frac{-33}{65}$

either in III^{rd} or in IV^{th} quad.

13 ×
5
65

12 ×
4
48

15 − 4

48 −
15
33

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{-4}{5} \times \frac{-5}{13} + \frac{-3}{5} \times \frac{12}{13}$

$= \frac{-20}{65} - \frac{46}{65}$

$= \frac{\cancel{-66} - 16}{65}$

$\frac{12 \times 3}{46}$

$\frac{46 \; 20}{66}$

• If $\cos A = \frac{3}{5}$, $\tan B = \frac{5}{12}$, A & B are acute find

(i) $\sin(A-B)$

(ii) $\cos(A+B)$

→ given $\cos A = \frac{3}{5}$, $\tan B = \frac{5}{12}$

$\sin A = \sqrt{1-\cos^2 A}$

$= \sqrt{1-\left(\frac{3}{5}\right)^2}$

$= \sqrt{1-\frac{9}{25}}$

$= \sqrt{\frac{25-9}{25}}$

$= \sqrt{\frac{16}{25}}$

$= \frac{4}{5}$

~~Cosec A~~.

$\tan B = \frac{5}{12}$

~~$\tan B = \sqrt{1+}$~~

$\sec B = \sqrt{1+\tan^2 B}$

$= \sqrt{1+\left(\frac{5}{12}\right)^2}$

$= \sqrt{1+\frac{25}{144}}$

$$=\sqrt{\frac{144+25}{144}}$$

$$\begin{array}{r}144\\ 25\\ \hline 169\end{array}$$

$$=\sqrt{\frac{169}{144}}$$

$$=\frac{13}{12}$$

$$\cos B=\frac{12}{13}$$

$$\sin B=\sqrt{1-\cos^2 B}$$

$$=\sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$=\sqrt{1-\frac{144}{169}}$$

$$=\sqrt{\frac{169-144}{169}}$$

$$=\frac{5}{13}$$

$$\sin(A-B)=\sin A\cos B-\cos A\sin B.$$

$$=\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}$$

$$\begin{array}{r}12\times\\ 4\\ \hline 48\end{array}$$

$$=\frac{48}{65}-\frac{15}{65}$$

$$=\frac{33}{65}$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$=\frac{3}{5}\times\frac{12}{13}-\frac{4}{5}\times\frac{5}{13}$$

$$\begin{array}{r}12\times\\ 3\\ \hline 36\end{array}$$

$$=\frac{36}{65}-\frac{20}{65}$$

$$=\frac{16}{65}$$

A & B are acute so in 1^{st} quadrant.

• given $\cos A = -\frac{12}{13}$, $\cot B = \frac{24}{7}$. A is in II^{nd} quadrant

B is in 1^{st} quadrant, find (i) $\sin(A+B)$

(ii) $\cos(A+B)$

→ given $\cos A = \frac{-12}{13}$

$$\sin A = \sqrt{1-\sin^2 A}$$

$$= \sqrt{1-\left(\frac{-12}{13}\right)^2}$$

$$= \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{169-144}{169}}$$

$$= \sqrt{\frac{25}{169}}$$

$$= \pm\frac{5}{13}$$

$\cot B = \frac{24}{7}$

$\tan B = \frac{7}{24}$

$$\sec B = \sqrt{1-\tan^2 B}$$

$$= \sqrt{1-\left(\frac{7}{24}\right)^2}$$

$$= \sqrt{1-\frac{49}{676}}$$

$$= \sqrt{\frac{676-49}{676}}$$

$\sin B = \frac{1}{\cos B} = \frac{7}{25}$

$\cot B = \sin B \times \cot B$

$$= \frac{\cancel{7}}{25} \times \frac{24}{\cancel{7}}$$

$$= \frac{24}{25}$$

24×24: 96, 48, 676

676 − 49 = 526

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{5}{13} \times \frac{24}{25} + -\frac{12}{13} \times \frac{7}{25}$

$= \frac{120}{325} + -\frac{84}{325}$

$= \frac{\cancel{71}}{\cancel{9k}} \quad \frac{36}{325}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{-12}{13} \times \frac{24}{25} - \frac{5}{13} \times \frac{7}{25}$

$= \frac{-288}{325} - \frac{35}{325}$

$= \frac{\cancel{-288}}{325}$

$13 \times 7 = 91$

$12 \times 7 = 84$

13×25: 65, 26, $= 325$

$125 - 54 = 71$

$24 \times 5 = 120$

24×12: 48, 24, $= 288$

$288 - 35 = 253$

- $\tan A = 2 \quad \tan B = 1$, A & B are acute angle find

(i) $\tan(A-B)$

(ii) $\cos(A-B)$

given $\tan A = 2$

$\tan B = 1$

(i) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{2-1}{1+(2 \times 1)}$

$= \frac{2-1}{1+2} = \frac{1}{3}$

(ii) $\cos(A-B)$

$\tan(A-B) = \frac{1}{3}$

$\sec(A-B) = \sqrt{1+\tan^2(A-B)}$

$= \sqrt{1-\left(\frac{1}{3}\right)^2}$

$= \sqrt{1+\frac{1}{9}}$

$= \sqrt{\frac{9+1}{9}}$

$= \sqrt{10/9}$

$= \frac{\sqrt{10}}{3}$

$\cos(A-B) = \frac{1}{\sec(A-B)} = \frac{3}{\sqrt{10}}$

- $\tan A = 3$, $\tan B = 1$ A&B are acute, find
  (i) $\tan(A-B)$
  (ii) $\cos(A-B)$

v.imp

- If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ A and B are acute angles S.T $A+B = 45°$

→ $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{3+2}{6}}{1-\frac{1}{6}}$

$$= \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

ie, $\tan(A+B) = 1$

$\therefore A+B = 45^\circ$

. If $\tan A = \frac{18}{17}$, $\tan B = \frac{1}{35}$, P.T $(A-B) = \frac{\pi}{4} = 45^\circ$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{\frac{18}{17} - \frac{1}{35}}{1+\frac{18}{17} \times \frac{1}{35}}$$

$$= \frac{\frac{630-17}{17 \times 35}}{1+\frac{18}{17 \times 35}}$$

$$= \frac{\frac{613}{595}}{1+\frac{18}{595}}$$

$$= \frac{\frac{613}{595}}{\frac{595+18}{595}}$$

$$= \frac{\frac{613}{595}}{\frac{613}{595}} = 1$$

$$\begin{array}{r} 623+ \\ 17 \\ \hline 640 \end{array}$$

$$\begin{array}{r} 18\times \\ 35 \\ \hline 190 \\ 54 \\ \hline 630 \end{array}$$

$$\begin{array}{r} 630- \\ 17 \\ \hline 623 \end{array}$$

$$\begin{array}{r} 35\times \\ 17 \\ \hline 245 \\ 35 \\ \hline 595 \end{array}$$

$$\begin{array}{r} 595+ \\ 18 \\ \hline 613 \end{array}$$

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$\tan(A-B) = 1$

$\therefore (A-B) = \pi/4$ $\quad$ $\pi/4 = 45°$

H.W. • $\tan A = 3$, $\tan B = 1$ $\quad$ A and B are acute, find ~~angles~~

(i) $\tan(A-B)$

(ii) $\cos(A-B)$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{3-1}{1+3\times 1}$$

$$= \frac{2}{1+3} = 2/4$$

$$= 1/2$$

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$\sec(A-B) = \sqrt{1+\tan^2(A-B}$$

$$\sec(A-B) = \sqrt{1+(1/2)^2}$$

$$= \sqrt{1+1/4}$$

$$= \sqrt{\frac{4+1}{4}}$$

$$= \sqrt{\frac{5}{4}} = \sqrt{5}/2$$

$$\cos(A-B) = \frac{1}{\sec(A-B)} = 1/\sqrt{5}/2 = \frac{2}{\sqrt{5}}$$

1

(d) (b) $\sin\left(A+\frac{\pi}{3}\right)+\sin\left(A-\frac{\pi}{3}\right)=\sin A$

$L.H.S = \left(\sin A\cos\frac{\pi}{3}+\cos A\sin\frac{\pi}{3}\right)+\left(\sin A\cos\frac{\pi}{3}-\cos A\sin\frac{\pi}{3}\right)$

$= \sin A\cdot\frac{1}{2}+\cancel{\cos A\cdot\frac{\sqrt{3}}{2}}+\sin A\cdot\frac{1}{2}-\cancel{\cos A\cdot\frac{\sqrt{3}}{2}}$

$=\cancel{2}\sin A\frac{1}{\cancel{2}}$

$=\sin A$

$=\underline{\underline{R.H.S}}$

• find the value of tan 75° & hence ST :

$\tan 75^\circ+\cot 75^\circ=4$

$\tan 75^\circ=\tan(\overset{A}{45}+\overset{B}{30})$

$=\dfrac{\tan 45+\tan 30}{1-\tan 45\tan 30}$

$=\dfrac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$

$=\dfrac{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}$

$=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$\cot 75^\circ=\dfrac{1}{\tan 75^\circ}=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$\therefore \tan 75^\circ+\cot 75^\circ=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}+\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$=\dfrac{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$=\dfrac{3+\cancel{2\sqrt{3}}+1+3-\cancel{2\sqrt{3}}+1}{3-1}=8/2=\underline{\underline{4}}$

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S.T $\tan 15° + \cot 15° = 4$

$\tan 15° = \tan(45° - 30)$

$= \dfrac{\tan 45° - \tan 30°}{1 + \tan 45 \tan 30}$

$= \dfrac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}}$

$= \dfrac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$\cot 15° = \dfrac{1}{\tan 15}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$\tan 15 + \cot 15° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} + \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}-1)^2 (\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \dfrac{3 - 2\sqrt{3} + 1 + 3 + 2\sqrt{3}...}{3-1}$

$= \dfrac{3+1+3+1}{2}$

$= 8/2 = 4$

• If $A+B = 45°$, S.T $(1+\tan A)(1+\tan B) = 2$

given $A+B = 45°$

$\tan(A+B) = \tan 45°$

$\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = 1$

$\tan A + \tan B = 1 - \tan A \tan B.$

ie $\tan A + \tan B + \tan A \tan B = 1$

adding one on both sides

$1 + \tan A + \tan B + \tan A \tan B = 2$

$(1+\tan A)(1+\tan B) = 2$

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• S.T $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$.

L.H.S $= \sin(A+B) \cdot \sin(A-B)$

$= (\sin A\cos B + \cos A \sin B)(\sin A\cos B - \cos A \sin B)$

this of the of the form $(a+b)(a-b)$

$= (\sin A\cos B)^2 - (\cos A \sin B)^2$

$= \sin^2 A\cos^2 B - \cos^2 A \sin^2 B$.

$\sin^2 A (1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \cancel{\sin^2 A \sin^2 B} - \sin^2 B + \cancel{\sin^2 A \sin^2 B}$

$= \underline{\underline{\sin^2 A - \sin^2 B}}$.

$= \underline{\underline{R.H.S}}$

• P.T $\sin(A+B)\sin(A-B) = \cos^2 B - \cos^2 A$

imp • S.T $\frac{\cos A - \sin A}{\cos A + \sin A} = \tan(45° - A)$

→ dividing with $\cos A$

L.H.S $= \dfrac{\frac{\cos A}{\cos A} - \frac{\sin A}{\cos A}}{\frac{\cos A}{\cos A} + \frac{\sin A}{\cos A}}$

$= \dfrac{1 - \tan A}{1 + \tan A}$

$= \dfrac{\tan 45° - \tan A}{1 + \tan 45° \tan A}$

(d3).

$= \tan(45° - A)$

$= \underline{\underline{R.H.S}}$

$\tan(45° - A) = \dfrac{\tan 45 - \tan A}{1 + \tan 45 \tan A}$

$= \dfrac{1 - \tan A}{1 + \tan A}$

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S.T $2\tan 10^\circ + \tan 40^\circ = \tan 50^\circ$ $\tan 10 =$

$\cancel{2\tan(50-40) + \tan 40^\circ}$

$\cancel{\left(\frac{\tan 50 \mp \tan 40}{1+\tan 50 \tan 40}\right) \tan 40^\circ} = \tan$

$2\tan(50-40) + \tan 40$

$$\tan 10 = \frac{\tan 50 - \tan 40}{1+\tan 50 \tan 40}$$

$$\tan 10 + \tan 40 = \frac{\tan 50}{1+\tan 50 \tan 40}$$

$\tan(90-\theta)$

$= \cot\theta$

$\tan 50^\circ = (\tan(90-40))$

$= \cot 40$

$\tan(90-50)$

$= \cot 40$

$$= \frac{\tan 5\theta}{1+(\tan(90-50)) \times \tan 40}$$

$\cot 40 \times \tan 40$

$= 1.$

$$\tan 10 + \tan 40 = \frac{\tan 50}{1+\underbrace{\cot 40 \times \tan 40}_{1}}$$

$2\tan 10 + \tan 40 = \tan 50$

- $\sin(A+B)\sin(A-B) = \cos^2 B - \cos^2 A$

$(a+b)(a-b)$

a^2-b^2 form

$\cancel{(\sin A \cos B)^2}$

$\sin(A+B)\ \sin(A-B)$

$= (\underbrace{\sin A \cos B}_{a} + \underbrace{\cos A \sin B}_{b})(\underbrace{\sin A \cos B}_{a} - \underbrace{\cos B \sin B}_{b})$

$= (\sin A \cos B)^2 - (\cos A \sin B)^2$

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$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$(1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)$

$=(\cos^2 B - \cancel{\cos^2 A \cos^2 B}) - (\cos^2 A - \cos^2 A \cos^2 B)$

$= \cos^2 B - \cos^2 A$

L.H.S = R.H.S

1). Find the values of $\sin 15^\circ$, $\cos 15^\circ$

2). If $\tan A = \frac{m}{2m+1}$, $\tan B = \frac{1}{m+1}$, S.T $A+B = 45^\circ$

1). $\sin 15^\circ = \sin(45-30)$

$\sin A = 45 = \frac{1}{\sqrt{2}}$ $\qquad \sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin B = 30 = \frac{1}{2}$

$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1-\frac{1}{2}} = \sqrt{\frac{1}{2}}$

$\cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$

$\therefore \sin 15 = \sin(45-30)$

$= \sin A \cos B - \cos A \sin B$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{(2\sqrt{2}\cdot\sqrt{3})-1}{(2\sqrt{2})^2} = \frac{2\sqrt{6}-1}{4\times 2} = \frac{\cancel{2}\sqrt{2}-1}{\cancel{8}4}$

$= \frac{\sqrt{2}-1}{\cancel{(2\times 2)}} = \frac{\sqrt{2}-1}{4}$

(backside 2 question)

2/8/2019

MULTIPLE ANGLES

For a given angle A, 2A, 3A, 4A, . . . are called Multiple angles

2A

1. $\boxed{\sin 2A = 2\sin A\cos A}$

ie $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

$\sin 4\theta = 2\sin\frac{4\theta}{2}\cos\frac{4\theta}{2}$

$= 2\sin 2\theta\cos 2\theta$

~~tan 2~~

$$\boxed{\sin 2A = \frac{2\tan A}{1+\tan^2 A}}$$

2. $$\boxed{\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 1 - 2\sin^2 A\\ &= 2\cos^2 A - 1\\ &= \frac{1-\tan^2 A}{1+\tan^2 A}\end{aligned}}$$

5 methods

3. $$\boxed{\tan 2A = \frac{2\tan A}{1-\tan^2 A}}$$

3A

1. $\boxed{\sin 3A = 3\sin A - 4\sin^3 A}$

2. $\boxed{\cos 3A = 4\cos^3 A - 3\cos A}$

3. $\boxed{\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}}$

• If $\sin A = 3/5$, A is acute find the values of $\sin 2A$, $\cos 2A$, $\sin 3A$ and $\cos 3A$.

— given $\sin A = \frac{3}{5}$

$$\cos A = \sqrt{1-\sin^2 A}$$

$$= \sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= \sqrt{1-\frac{9}{25}}$$

$$= \sqrt{\frac{25-9}{25}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\sin 2A = 2\sin A\cos A$$

$$= 2 \times \frac{3}{5} \times \frac{4}{5}$$

$$= \frac{24}{25}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$9 \times 3 = 27$

$$= 3 \times \frac{3}{5} - 4 \times \left(\frac{3}{5}\right)^3$$

$25 \times 5 = 125$

$$= \frac{9}{5} - 4 \times \frac{27}{125}$$

$$= \frac{9}{5} - \frac{108}{125} = \frac{225-108}{125} = \frac{117}{125}$$

$\cos 3A = 4\cos^3 A - 3\cos A$

$= 4\times\left(\frac{4}{5}\right)^3 - 3\times\frac{4}{5}$

$= \frac{256}{125} - \frac{12}{5}$

$= \frac{256-300}{125} = \frac{-44}{125}$

• if $\cos\theta = \frac{1}{2}$, θ is acute find $\sin 2\theta$, $\cos 2\theta$, $\sin 3\theta$, $\cos 3\theta$

given $\cos\theta = \frac{1}{2}$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \sqrt{1-\frac{1}{4}}$

$= \sqrt{\frac{4-1}{4}}$

$= \frac{\sqrt{3}}{2}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \cancel{2}\times\frac{\sqrt{3}}{\cancel{2}}\times\frac{1}{2}$

$= \frac{\sqrt{3}}{2}$

$\cos 2\theta = \cos^2 A - \sin^2 A$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{4}$

$= \frac{1-3}{4} = \frac{-2}{4} = -\frac{1}{2}$

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$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ 3

$= 3 \times \frac{\sqrt{3}}{2} - 4 \times \left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{2} - 4 \times \frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}}{2} - \frac{12\sqrt{3}}{8}$

$12 \times 2 = 24$

$= \frac{24\sqrt{3} - 24\sqrt{3}}{16}$

$= \underline{\underline{0}}$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$= 4 \times \left(\frac{1}{2}\right)^3 - 3 \times \frac{1}{2}$

$= 4 \times \frac{1}{8} - \frac{3}{2}$

$= \frac{4}{8} - \frac{3}{2}$

$= \frac{1}{2} - \frac{3}{2}$

$= \frac{-2}{2}$

$= -1$ //

- If $\tan B = 1$, B lies in IIIrd quaddrant, find the values of $\sin 2B$, $\cos 2B$ and $\tan 2B$.

— given $\tan B = 1$

$\sin 2B = \frac{2\tan B}{1 + \tan^2 B}$

$= \frac{2 \times 1}{1 + (1^2)}$

$= \frac{2}{1+1} = \frac{2}{2} = \underline{\underline{1}}$

$$\cos 2B = \frac{1-\tan^2 B}{1+\tan^2 B}$$

$$= \frac{1-1^2}{1+1^2}$$

$$= \frac{1-1}{1+1}$$

$$= \frac{0}{2} = 0$$

$$\tan 2B = \frac{2\tan B}{1-\tan^2 B}$$

$$= \frac{2\times 1}{1-1^2}$$

$$= \frac{2}{0} = \text{not defined.}$$

E.g. If $\tan\theta = 2$, find $\cos 2\theta$

given $\tan\theta = 2$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{1-2^2}{1+2^2}$$

$$= \frac{1-4}{1+4}$$

$$= \frac{-3}{5}$$

- If $\sin A = a$ Find $\sin 3A$

given $\sin A = a$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$= 3\times a - 4(a)^3$$

$= 3a - 4a^3$

$= \underline{\underline{a(3 - 4a^2)}}$

- If $\cos A = a$, find $\cos 3A$

given $\cos A = a$

$\cos 3A = 4\cos^3 A - 3\cos A$

$= 4a^3 - 3 \times a$

$= 4a^3 - 3a$

$= \underline{\underline{a(4a^2 - 3)}}$

- If $\tan A = 0.38$, find $\tan 2A$

given $\tan A = 0.38$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$= \dfrac{2 \times 0.38}{1 - (0.38)^2} = \dfrac{.76}{1 - .1444}$

$= \dfrac{.76}{0.8556} = \underline{\underline{.888}}$

- If $\sin A = 0.5$, A is acute find $\sin 2A$

given $\sin A = 0.5$

$\sin 2A = 2\sin A \cos A$

$= 2 \times 0.5 \times .866$

$= \underline{\underline{.866}}$

$\cos A = \sqrt{1 - \sin^2 A}$

$= \sqrt{1 - (0.5)^2}$

$= \sqrt{1 - 0.25}$

$= \sqrt{0.75}$

$= \underline{\underline{.866}}$

• S.T $\cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\rightarrow \cos^2 A - \sin^2 A = 1 - \sin^2 A - \sin^2 A = 1 - 2\sin^2 A$

$= 1 - 2(1 - \cos^2 A)$

$= 1 - 2 + 2\cos^2 A$

$= 2\cos^2 A - 1$

$\cos^2 A = 1 - \sin^2 A$

$-\sin^2 A - \sin^2 A$

$= -2\sin^2 A$

6/8/2019.

Imp

• S.T $\cos^4 A - \sin^4 A = \cos 2A$

$(a^2)^2 = a^4$

$\rightarrow$ L.H.S $= (\cos^2 A)^2 - (\sin^2 A)^2$

take $a = \cos^2 A$ $b = \sin^2 A$

$a^2 - b^2$

$= (\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$

$= (a+b)(a-b)$

$= 1(\cos^2 A - \sin^2 A)$

$\cos^2 A + \sin^2 A = 1$

$= \cos 2A$

$\cos 2A = \cos^2 A - \sin^2 A$

$=$ R.H.S

• S.T $\cos 4\theta = 1 - 8\sin^2\theta\cos^2\theta$

we have $\cos 2\theta = 1 - 2\sin^2\theta$

then $\cos 4\theta = 1 - 2\sin^2(2\theta)$

$= 1 - 2\sin 2\theta \cdot \sin 2\theta$

$= 1 - 2 \cdot 2\sin\theta \cdot \cos\theta \cdot 2\sin\theta\cos\theta$

$= 1 - 8\sin^2\theta\cos^2\theta$

$=$ R.H.S

vimp

- $$\boxed{\sin^2 A = \frac{1-\cos 2A}{2}}$$

$A \longrightarrow$ double $2A$

$$\Downarrow$$

$$\boxed{\begin{aligned} &\text{ie} \quad 1-\cos 2A = 2\sin^2 A \\ &\therefore 1-\cos A = 2\sin^2 A/2 \end{aligned}}$$

$$\boxed{\cos^2 A = \frac{1+\cos 2A}{2}}$$

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$$

$$= \frac{2\sin^2 \theta/2}{\theta^2}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} &\text{ie} \quad 1+\cos 2A = 2\cos^2 A \\ &\therefore 1+\cos A = 2\cos^2 A/2. \end{aligned}}$$

$$\boxed{\tan^2 A = \frac{\dfrac{1-\cos 2A}{\cancel{2}}}{\dfrac{1+\cos 2A}{\cancel{2}}} = \frac{1-\cos 2A}{1+\cos 2A}}$$

- If $\cos 2\theta = -\frac{3}{5}$, find $\tan\theta$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\therefore \tan\theta = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$= \sqrt{\frac{1-(-3/5)}{1+(-3/5)}}$$

$$= \sqrt{\frac{1+3/5}{1-3/5}}$$

$$= \sqrt{\frac{\frac{5+3}{5}}{\frac{5-3}{5}}} = \sqrt{\frac{\frac{8}{\cancel{5}}}{\frac{2}{\cancel{5}}}} = \sqrt{\frac{8}{2}} = \sqrt{4}$$

$$= 2$$

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imp
easy method.

S.T $\frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A} = 4\cos 2A$

sin3A value

$$L.H.S = \frac{\sin 3A \times \cos A + \cos 3A \times \sin A}{\sin A \cos A} =$$

$$= \frac{\sin(3A + A)}{\sin A \cos A}$$

$\sin(a+b) = \sin a \times \cos b + \cos a \sin b$

$\sin 2A = 2\sin A \cos A$

$$= \frac{\sin 4A}{\sin A \cos A}$$

$$= \frac{2(\sin 2A)\cos 2A}{\sin A \cos A}$$

$$= \frac{2 \times 2\sin \cancel{A \cos A} \times \cos 2A}{\sin \cancel{A \cos A}}$$

$$= \underline{4\cos 2A} = \underline{R.H.S}$$

OR

$$\frac{3\sin A \cancel{\cos A} - 4\sin^3 A}{\sin A} - \frac{4\cos^3 A - 3\cos A}{\cos A}$$

$= \quad \cos A$

• P.T $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = 2$

$$LHS = \frac{\sin \overset{a}{\widehat{3x}} \cos \overset{b}{\widehat{x}} - \sin \overset{b}{\widehat{x}} \cos \overset{a}{\widehat{3x}}}{\sin x \cos x}$$

$\sin(a-b)$

$$= \frac{\sin(3x-x)}{\sin x \cos x}$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$= \frac{\sin(2x)}{\sin x \cos x}$$

$$= \frac{2\cancel{\sin x \cos x}}{\cancel{\sin x \cos x}}$$

$= \underline{\underline{2}}$

$= \underline{\underline{R.H.S}}$

(i) P.T $\frac{\sin 2A}{1+\cos 2A} = \tan A$ and deduce the value of $\tan 15^\circ$

$$L.H.S = \frac{2\sin A \cos A}{1+\cos 2A}$$

$1+\cos 2A = 2\cos^2 A$

$$= \frac{\cancel{2}\sin A \cancel{\cos A}}{\cancel{2}\cos^{\cancel{2}} A}$$

$$= \frac{\sin A}{\cos A}$$

$= \tan A$

$= \underline{\underline{R.H.S}}$

By result:-

~~$\tan 15^\circ = $~~

put $a = 15^\circ$

$$\tan 15^\circ = \frac{\sin 2\times 15^\circ}{1+\cos 2\times 15^\circ}$$

$$= \frac{\sin 30^\circ}{1+\cos 30^\circ}$$

$$= \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{\frac{1}{\cancel{2}}}{\frac{2+\sqrt{3}}{\cancel{2}}} = \frac{1}{2+\sqrt{3}}$$

/7/2019

P.T $\frac{\cot A - \tan A}{\cot A + \tan A} = \cos 2A$

$$L.H.S = \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$$

$$= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cancel{\cos x}}}{\frac{\cos^2 A + \sin^2 A}{\sin A \cancel{\cos A}}}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$$

$$= \frac{\cos^2 A - \sin^2 A}{1}$$

$$= \cos 2A = R.H.S$$

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• P.T $\dfrac{\cos 3A + \cos A}{\sin 3A - \sin A} = \cot A$

ans $L.H.S = \dfrac{4\cos^3 A - 3\cos A + \cos A}{3\sin A - 4\sin^3 A - \sin A}$

$\left.\begin{array}{l}\sin 3A = 3 \\ \cos 3A\end{array}\right\}$ value [illegible]

$= \dfrac{4\cos^3 A - 2\cos A}{2\sin A - 4\sin^3 A}$

$= \dfrac{2(2\cos^3 A - \cos A)}{2(\sin A - 2\sin^3 A)} = \dfrac{\cos A(2\cos^2 A - 1)}{\sin(1 - 2\sin^2 A)}$

$= \dfrac{\cos A\,(\cancel{\cos 2A})}{\sin A\,(\cancel{\cos 2A})}$

$= \underline{\underline{\cot A}}$

[illegible] chapter [illegible]

OR,

1(d) • P.T $\frac{1+\cos 2A}{\sin 2A} = \cot A$ and deduce the value of $\cot 15^\circ$

out of syllabus

• Calculate the possible value of $\cos\theta$, if $4\cos 2\theta + 2\cos\theta + 3 = 0$

given $4\cos 2\theta + 2\cos\theta + 3 = 0$ $\qquad -1 \leq \cos\theta \leq 1$

ie $4(2\cos^2\theta - 1) + 2\cos\theta + 3 = 0$ $\qquad -1 \leq \sin\theta \leq 1$

$\tan\theta = $ infinity

ie $8\cos^2\theta - 4 + 2\cos\theta + 3 = 0$ $\qquad \cos 2\theta = 2\cos^2\theta - 1$

ie $8\cos^2\theta + 2\cos\theta - 1 = 0$

put $\cos\theta = x$

$$8x^2 + 2x - 1 = 0$$

quadratic eqn

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 8,\ b = 2,\ c = -1$$

$$= \frac{-2 \pm \sqrt{2^2 - 4 \times 8 \times -1}}{2 \times 8}$$

$$= \frac{-2 \pm \sqrt{4 + 32}}{16}$$

$$= \frac{-2 \pm \sqrt{36}}{16}$$

$$= \frac{-2 \pm 6}{16} = \frac{-2+6}{16} \text{ or } \frac{-2-6}{16}$$

$$= \frac{4}{16} \text{ or } \frac{-8}{16}$$

$$x = \frac{1}{4} \text{ or } \frac{-2}{4} \text{ or } -\frac{1}{2}$$

ie $x = \frac{1}{4}$ or $-\frac{1}{2}$

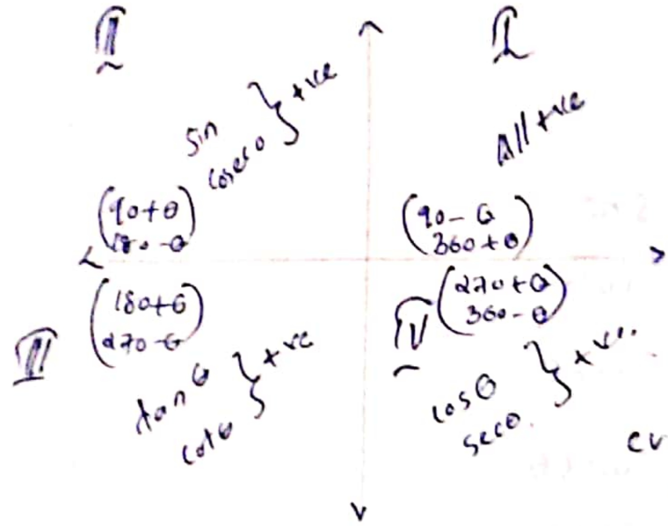
ie $\cos\theta = \frac{1}{4}$ or $-\frac{1}{2}$

$= .25$ or $-.5$

ie, 2 values apt for $\cos\theta$ becoz it in the range -1

- If $\tan\theta = 2$, find $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$
- If $\tan A = 3/4$ (A is acute) and $\sin B = 5/13$ (B lies in IInd qu
  find $\sin 2A$ and $\sin 2B$
- P.T $1 + \tan\theta \tan 2\theta$

Reduction formulae



8 angle
6 trigonometric functions
48 formulae

$90 \times 1 = 90$
$90 \times 2 = 180$
$90 \times 3 = 270$
$90 \times 4 = 36$

odd multiples (90×1, 90×3)

even multiples of 90 (90×2, 90×4)

* $\sin\theta \xleftrightarrow{\text{Co-function}} \cos\theta$
* $\sec\theta \xleftrightarrow{\text{Co-function}} \operatorname{cosec}\theta$
* $\tan\theta \xleftrightarrow{\text{Co-function}} \cot\theta$

$90-\theta$ (Reduction formulae)

1. $\sin(90 \mp \theta) = \cos\theta$
2. $\cos(90 \mp \theta) = \sin\theta$
3. $\tan(90 \mp \theta) = \cot\theta$
4. $\operatorname{cosec}(90 \mp \theta) = \sec\theta$
5. $\sec(90 \mp \theta) = \operatorname{cosec}\theta$
6. $\cot(90 \mp \theta) = \tan\theta$

(odd multiple 90 co function same. sign eg: $(90-\theta)$ in 1st quadrant All +ve)

$(90+\theta)$

1. $\sin(90+\theta) = \cos\theta$
2. $\cos(90+\theta) = -\sin\theta$
3. $\tan(90+\theta) = -\cot\theta$
4. $\operatorname{cosec}(90+\theta) = \sec\theta$

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5. $\sec(90+\theta) = -\operatorname{cosec}\theta$

6. $\cot(90+\theta) = -\tan\theta$

$180-\theta$

1- $\sin(180-\theta) = \sin\theta$

2- $\cos(180-\theta) = -\cos\theta$

3- $\tan(180-\theta) = -\tan\theta$

4- $\operatorname{cosec}(180-\theta) = \operatorname{cosec}\theta$

5- $\sec(180-\theta) = -\sec\theta$

6- $\cot(180-\theta) = -\cot\theta$

$180+\theta$

1- $\sin(180+\theta) = -\sin\theta$

2- $\cos(180+\theta) = -\cos\theta$

3- $\tan(180+\theta) = \tan\theta$

4- $\operatorname{cosec}(180+\theta) = -\operatorname{cosec}\theta$

5- $\sec(180+\theta) = -\sec\theta$

6- $\cot(180+\theta) = \cot\theta$

$270+\theta$

1- $\sin(270+\theta) = -\cos\theta$

2- $\cos(270+\theta) = \sin\theta$

3- $\tan(270+\theta) = \cancel{\tan\theta}\ -\cot\theta$

4- $\operatorname{cosec}(270+\theta) = -\sec\theta$

5- $\sec(270+\theta) = \operatorname{cosec}\theta$

6- $\cot(270+\theta) = -\tan\theta$

$270 - \theta$

1- $\sin(270 - \theta) = -\cos\theta$

2- $\cos(270 - \theta) = -\sin\theta$

3- $\tan(270 - \theta) = \cot\theta$

4- $\operatorname{cosec}(270 - \theta) = -\sec\theta$

5- $\sec(270 - \theta) = -\operatorname{cosec}\theta$

6- $\cot(270 - \theta) = \tan\theta$

$360 - \theta$

1- $\sin(360 - \theta) = -\sin\theta$

2- $\cos(360 - \theta) = \cos\theta$

3- $\tan(360 - \theta) = -\tan\theta$

4- $\operatorname{cosec}(360 - \theta) = -\operatorname{cosec}\theta$

5- $\sec(360 - \theta) = \sec\theta$

6- $\cot(360 - \theta) = -\cot\theta$

$360 + \theta$

1- $\sin(360 + \theta) = \sin\theta$

2- $\cos(360 + \theta) = \cos\theta$

3- $\tan(360 + \theta) = \tan\theta$

4- $\operatorname{cosec}(360 + \theta) = \operatorname{cosec}\theta$

5- $\sec(360 + \theta) = \sec\theta$

6- $\cot(360 + \theta) = \cot\theta$

• find the values of the following.

$\sin 120^\circ$

$\cancel{\sin(90+30)}$

$\sin 120^\circ = \sin(90+30)$

this of the form $\sin(90+\theta) = \cos\theta$

$= \cos 30$

$= \sqrt{3}/2$

• $\cos 120^\circ$

$\cos 120^\circ = \cos(90+30^\circ)$

$= -\sin 30^\circ$

$= -1/2$

• $\tan 150^\circ = \tan(180-30)$

$= \cancel{\tan\ \cot 30}\ -\tan 30$

$= -1/\sqrt{3}$

• $\tan 135^\circ = \tan(90+45)$ 180+

$= -\cot 45$

$= -1$

• $\operatorname{cosec} 210 = \operatorname{cosec}(180+30)$

$= -\operatorname{cosec} 30$

$= -2$

- $\sec 225^\circ = \sec(180 + 45)$

$= -\sec 45^\circ$

$= \underline{\underline{-\sqrt{2}}}$

- $\cot 240^\circ = \cot(270 - 30)$

$= \tan 30$

$= \underline{\underline{1/\sqrt{3}}}$

- $\sin 300 = \sin(360 - 60)$

$= -\sin 60$

$= \underline{\underline{-\sqrt{3}/2}}$

- $\sin 390 = \sin(360 + 30)$

$= \sin 30$

$= \underline{1/2}$

- $\sin 330^\circ = \sin(360 - 30)$

$= -\sin 30$

$= \underline{\underline{-1/2}}$

- $\cos 300^\circ = \cos(360 - 60)$

$= \cos 60$

$= 1/2$

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Results :-

1) $\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$

2) $\cos(-\theta) = \cos\theta$

3) $\tan(-\theta) = -\tan\theta$

4) $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

5) $\sec(-\theta) = \sec\theta$

6) $\cot(-\theta) = -\cot\theta$

• $\cos(-390) = \cos 390^\circ$

$= \cos(360+30)$

$= +\cos 30$

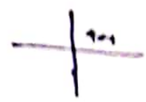
$= \sqrt{3}/2$

• PT $\sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ = 1$

$\sin(120) = \sin(90+30)$

$= \cos 30$

$= \sqrt{3}/2$



$\cos 330 = \cos(360-30)$

$= \cos 30$

$= \sqrt{3}/2$

$\sin 330^\circ = \sin(360-30)$

$= -\sin 30$

$= -1/2$

$\cos 240 = \cos(180+60)$

$= -\cos 60$

$= -1/2$

$\frac{\sqrt{3}}{2} * \frac{\sqrt{3}}{2} + -\frac{1}{2} \times -\frac{1}{2}$

$\frac{1}{2} + \frac{1}{2}$

$\cancel{2\sqrt{3}/2}\ \frac{3}{4} + \frac{1}{4}$

$= \frac{2+2}{4}$

$= \frac{4}{4} = 1$ //

$= \frac{4}{4}$

$= \frac{1}{2}$

• P.T $\cos 210^\circ \times \cos 150^\circ + \sin 330^\circ \times \sin 150^\circ = \frac{1}{2}$.

$\cos(270 - 60) = -\sin 60$

$= -\frac{\sqrt{3}}{2}$

$\cos 150^\circ = \cos(180 - 30)$

$= -\cos 30$

$= -\frac{\sqrt{3}}{2}$

$\sin 330^\circ = \sin(360 - 30)$

$= -\sin 30$

$= -\frac{1}{2}$

$\sin 150 = \sin(180 - 30)$

$= \sin(30)$

$= \frac{1}{2}$

L.HS $= -\frac{\sqrt{3}}{2} \times -\frac{\sqrt{3}}{2} + -\frac{1}{2} * \frac{1}{2}$

$= \frac{+3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ = R.H.S

$= \cancel{\frac{-4}{4} = 1}$ //

- S.T $\tan 225^\circ \times \cot 405^\circ + \tan 675^\circ \times \cot 675^\circ$

$\tan 225 = \tan(270 - 45)$

$= \cot 45$

$= 1$

$\cot 405 = \cot(360 + 45)$

$= \cot 45$

$= 1$

~~tan 675°~~ =

$\tan\theta \times \cot\theta = 1$

ie $\tan 675^\circ \times \cot 675^\circ = 1$

$L.H.S = 1 \times 1 + 1$

$= 1 + 1$

$= \underline{\underline{2}}$

$180 + 45 = 225$

$270 - 45 = 225$

- S.T $\sin 780^\circ \times \sin 480^\circ + \cos 120^\circ \sin 30^\circ = \frac{1}{2}$

$\sin(780^\circ) = \sin(\overset{\text{neglect}}{2} \times 360 + 60)$

$= \sin 60$

$= \underline{\frac{\sqrt{3}}{2}}$

$\sin(360 + \theta)$

$= \sin\theta$

~~Sin θ x θ~~

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675

$\sin 480° = \sin(360+120)$ $\quad \sin(360+\theta)$

$= \sin 120$

$= \sin(90+30)$ lies in IInd quad.

$= \cos 30$

$= \frac{\sqrt{3}}{2}$

$\cos 120 = \cos(\cancel{180}-90+30)$

$= -\sin 30$

$= -\frac{1}{2}$

$\sin 30 = \frac{1}{2}$

$L.H.S = \frac{\sqrt{3}}{2} * \frac{\sqrt{3}}{2} + -\frac{1}{2} * \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

$360+$
90
$450+$
30
480
IInd quad.

- simplify $\dfrac{\cos(90+\theta) \times \sec(-\theta) \tan(180-\theta)}{\sec(360-\theta) \times \sin(180+\theta) \times \cot(90+\theta)}$

$\therefore$ given expression

$= \dfrac{-\sin\theta \times \cancel{\sec\theta} \times -\tan\theta}{\cancel{\sec\theta} \times -\sin\theta \times \tan\theta}$

$= \dfrac{-1 \times 1 \times -1}{1 \times -1 \times 1}$

$= \dfrac{1}{-1} = -1$

$\cos(90+\theta) = -\sin\theta$

$\sec(-\theta) = \sec\theta$

$\tan(180-\theta) = -\tan\theta$

$\sec(360-\theta) = \sec\theta$

$\sin(180+\theta) = -\sin\theta$

$\cot(90+\theta) = \tan\theta$

imp

- P.T $\dfrac{\cos(90+A)\ \sec(360+A)\ \tan(180-A)}{\sec(A-720)\ \sin(540+A)\ \cot(A-90)}$

$\cos(90+A) = -\sin A$

$\sec(360+A) = \sec A$

$\tan(180-A) = -\tan A$

$\sec(A-720) = \sec\left(-\overbrace{(720-A)}^{\theta}\right)$

$(720-A) \rightarrow$ take as θ

$\sec(-\theta) = \sec\theta$

ie $\sec\left(-(720-A)\right) = \sec(720-A)$

$= \sec(2\times360-A)$

$= \underline{\underline{\sec A}}$

$\sin(540+A) = \sin\left((360+180)+A\right)$

$180+A = \theta$

$= \sin\left(360+[180+A]\right)$

$\sin(360+\theta) = \sin\theta$

ie $= \sin(180+A)$

$\sin(180+\theta) = -\sin\theta$ $= \underline{\underline{-\sin A}}$

$\cot(A-90) = \cot\left[-(90-A)\right]$

$\cot(-\theta) = -\cot\theta$

ie $\cot(90-A) = -\tan A$

$$= \frac{-\sin A \times \sec A \times -\tan A}{\sec A \times -\sin A \times -\tan A}$$

$$= \frac{-1 \times -1}{1 \times -1 \times -1} = \frac{1}{1} = 1$$

H.W

- $\sin 420 \times \cos 390 + \sin(-300) \times \cos 330 = 3/2$

$\sin(-300) = -\sin(300)$

$= -\sin(360 + -60)$ $\quad -\,-\sin 60 = +\sin 60.$

$= \sin 60$

$= \sqrt{3}/2$

$\sin 420 = \sin(360 + 60)$ $\quad 420 - 360 = 60$

$= \sin 60$

$= \sqrt{3}/2$

$\cos 390 = \cos(360 + 30)$

$= \cos 30$

$= \sqrt{3}/2$

$\cos 330 = \cos(360 - 30)$

$= \cos 30$

$= \sqrt{3}/2$

$$\sqrt{3}/2 \times \sqrt{3}/2 + \sqrt{3}/2 \times \frac{\sqrt{3}}{2}$$

$$\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2}$$

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• $$\frac{\sin(180+A) \times \cos(90-A) \times \tan(270+A)}{\sec(540-A) \times \cos(360+A)} = \frac{-\sin A}{\cos A}$$

$90+\theta$, $180-\theta$ | $90-\theta$, $360+\theta$
$180+\theta$, $270-\theta$ | $270+\theta$, $360-\theta$

→ $\sin(180+A) = -\sin A$

$\cos(90-A) = \sin A$

$\tan(270+A) = -\cot A$

$\sec(540-A) = \sec(360+(180-A))$

$= \sec(180-A)$

$= -\sec A$

$360+180=540$

$\sec(360+(180-A))$ also can be written in $\sec(180+(360-A))$

$\cos(360+A) = \cos A$

$$\therefore \text{L.H.S} = \frac{-\sin A \times \sin A \times -\cot A}{-\sec A \times \cos A}$$

$$= \frac{-\sin A \times \cancel{\sin A} \times -\frac{\cos A}{\cancel{\sin A}}}{\cancel{-\frac{1}{\cos A} \times \cos A}\ -1}$$

$\sec\theta \times \cos\theta = 1$

$$= \frac{+\sin A \cos A}{-1} = \underline{\underline{-\sin A \cos A}}$$

$= \underline{\underline{\text{R.H.S}}}$

H.W evaluate $\cos 57° + \sin 510 - \sin 330 \cos 390$

Remark :—

n) 1. function $(n\,90 \pm \theta) = \pm$ function θ, if n is even

2. function $(n90 \pm \theta) = \pm$ cofunction θ, if n is odd.

for – sign depends on the quadrant in which the angle lies.

11/12/2019.

PRODUCT FORMULAE AND CONVERSE

Product formulae.

1. $\sin C + \sin D$ where C and D are angles

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

3. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

4. $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

- ~~find~~ express $\sin 5\theta + \sin 7\theta$ as a product.

$\sin 5\theta + \sin 7\theta$

we know, $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$C = 5\theta$

$D = 7\theta$

$$\sin 5\theta + \sin 7\theta = 2\sin\left(\frac{5\theta+7\theta}{2}\right)\cos\left(\frac{5\theta-7\theta}{2}\right)$$

$$= 2\sin\left(\frac{\cancel{12}^{6}\theta}{\cancel{2}}\right)\cos\left(\frac{-\cancel{2}\theta}{\cancel{2}}\right)$$

$$= 2\sin 6\theta \cos(-\theta)$$

$$= 2\sin 6\theta \cos\theta$$

$\cos(-\theta)$

$= \cos\theta$

• $\cos 7\theta - \cos 3\theta$

this is of the for $\cos C - \cos D$

$C = 7\theta$

$D = 3\theta$

$$\cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$= -2\sin\left(\frac{7\theta+3\theta}{2}\right) \sin\left(\frac{7\theta-3\theta}{2}\right)$$

$$= -2\sin\left(\frac{10\theta}{2}\right) \sin\left(\frac{4\theta}{2}\right)$$

$$= -2\sin 5\theta \quad \sin 2\theta$$

imp

• S.T $\cos 5^\circ - \sin 25^\circ = \sin 35^\circ$

→ To proove $\sin 35 + \sin 25 = \cos 5$

$$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

[illegible] ഒരു side-ൽ 2 same function

$$L.H.S = 2 \sin\left(\frac{35+25}{2}\right) \cos\left(\frac{35-25}{2}\right)$$

$$= 2\sin\left(\frac{60}{2}\right) \cos\left(\frac{10}{2}\right)$$

$$= 2\sin 30 \cos 5$$

$$= 2 \times \frac{1}{2} \times \cos 5^\circ$$

$$= \cos 5^\circ$$

$$= R.H.S$$

• ~~P T S~~

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P.T $\sin 33° + \cos 63° = \cos 3$

To prove $\sin 33° = \cos 3 - \cos 63°$

$$L.H.S = \cancel{\cos 3 - \cos 63} \quad -2\sin\left(\frac{3+63}{2}\right)\sin\left(\frac{3-63}{2}\right)$$

$$= -2\sin\left(\frac{66}{2}\right)\sin\left(\frac{-60}{2}\right)$$

$$= -2\sin 33 \times \sin(-30)$$ $\sin(-\theta) = -\sin\theta$

$$= -2\sin 33 \times -\sin 30$$

$$= +\cancel{2}\sin 33 \times \frac{1}{\cancel{2}}$$

$$= \sin 33°$$

$$= R.H.S$$

v.imp

- P.T $\sin 50° - \sin 70° + \cos 80° = 0$

$$L.H.S = (\sin 50 - \sin 70) + \cos 80° +$$

$$= 2\cos\left(\frac{50+70}{2}\right)\sin\left(\frac{50-70}{2}\right) + \cos 80$$

$$= 2\cos 60\ \sin(-10) + \cos 80°$$

$$= 2\cos 60 \times -\sin 10 + \cos 80°$$

$$= -2\cos 60\ \sin 10 + \cos 80°$$

$$= -\cancel{2} \times \frac{1}{\cancel{2}} \times \sin 10 + \cos 80$$

$$= -\sin 10 + \cos 80$$ apply rdn formula to change it into sin 10

$$= -\sin 10 + \cos(90-10)$$

$$= \cancel{-\sin 10} + \cancel{\sin 10}$$

$$= 0 = R.H.S$$

$= \cos 80$
$= \cos(90-10)$
$= \sin 10$

v.imp

imp

- S.T $\cos 65° + \cos 55° + \cos 175° = 0$

$L.H.S = (\cos 65° + \cos 55°) + \cos 175°$

$= 2\cos\left(\frac{65+55}{2}\right)\cos\left(\frac{65-55}{2}\right) + \cos 175°$

$65 + 55 = 120°$

$= 2\cos 60 \cos 5 + \cos 175°$

$65 - 55 = 10$

$= 2 \times \frac{1}{2} \cos 5 + \cos 175°$

$= \cos 5° + \cos 175°$ or apply formulae $\cos C + \cos D$ ~~cosC+D~~

$= \cos 5° + \cos(180° - 5)$

$= \cos 5° \; \cancel{\cos 5°} - \cos 5°$

$= \underline{\underline{0}} = \underline{\underline{R.H.S}}$

$\sin 30 \longleftrightarrow \cos 60$

$\cos 72 \longleftrightarrow \sin 18$

$\cot 50 \longleftrightarrow \tan 40.$

- P.T $\cos 80 + \cos 40 - \cos 20 = 0$

$L.H.S = (\cos 80 + \cos 40) - \cos 20$

~~$= \cos 80 + (2\sin(40$~~

$= 2\cos\left(\frac{80+40}{2}\right)\cos\left(\frac{80-40}{2}\right) - \cos 20$

$= 2\cos(60)\cos(20) - \cos 20$

$= 2 \times \frac{1}{2}\cos 20 - \cos 20$

$= \cos 20 - \cos 20$

$= \underline{\underline{0}} = R.H.S$

V. imp

- P.T $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 4\cos\theta \cos 2\theta \sin 4\theta$

easy = • P.T

L.H.S $= (\sin 7\theta + \sin\theta) + (\sin 5\theta + \sin 3\theta)$

$= \left[2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)\right] + \left[2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)\right]$

$= 2\sin 4\theta \cos 3\theta + 2\sin 4\theta \cos\theta$

$= 2\sin 4\theta (\cos 3\theta + \cos\theta)$

$\cos 3\theta + \cos\theta$

$\cos C + \cos C$

$= 2\sin 4\theta \left(2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right)$

$= 2\sin 4\theta \; 2\cos 2\theta \; \cos\theta$

$= \underline{\underline{4\cos\theta \cos 2\theta \sin 4\theta}} = R.H.S$

S.T $\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right) = 0$

$$L.H.S = \left(\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right)\right) + \left[\cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right)\right]$$

$$= \left[2\cos\left(\frac{\frac{\pi}{8}+\frac{7\pi}{8}}{2}\right)\cos\left(\frac{\frac{7\pi}{8}-\frac{\pi}{8}}{2}\right)\right] + \left[2\cos\left(\frac{\frac{5\pi}{8}+\frac{3\pi}{8}}{2}\right)\cos\left(\frac{\frac{5\pi}{8}-\frac{3\pi}{8}}{2}\right)\right]$$

$$= 2\cos\frac{\pi}{2}\cos\frac{6\pi}{8} + 2\cos\frac{4\pi}{8}\cos\frac{\pi}{8}$$

$$= 2\cos\frac{\pi}{2}\cos\frac{3\pi}{8} + 2\cos\frac{\pi}{2}\cos\frac{\pi}{8}$$

$$= 0 + 0$$

$$= \underline{0} = R.H.S$$

$\cos\pi/2 = 0$

$\frac{6}{8} = \frac{3}{4}$, $\frac{3/4}{2}$, $\frac{4}{4}$, $\frac{6\pi}{16}$, $\frac{4}{8}\pi = \frac{1\pi}{2}$

• P.T $\frac{\cos 3A - \cos A}{\sin A - \sin 3A} = \tan 2A$

$$L.H.S = \frac{-2\sin\left(\frac{3A+A}{2}\right)\sin\left(\frac{3A-A}{2}\right)}{2\cos\left(\frac{A+3A}{2}\right)\sin\left(\frac{A-3A}{2}\right)}$$

$\sin(-A) = -\sin A$

$$= \frac{-2\sin 2A \times \sin A}{2\cos 2A\ \sin A}$$

$$= \frac{-\sin 2A \times \sin A}{\cos 2A \times -\sin A}$$

$$= -\left(\frac{\sin 2A\ \cancel{\sin A}}{\cos 2A\ \cancel{\sin A}}\right)$$

$$= \frac{\sin 2A}{\cos 2A} = \tan 2A = \underline{R.H.S}$$

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• P.T $\dfrac{\sin 4A + \sin 2A}{\cos 4A + \cos 2A} = \tan 3A$

$$L.H.S = \frac{2\sin\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right)}{2\cos\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{6A}{2}\right)\cos\left(\frac{2A}{2}\right)}{2\cos\left(\frac{6A}{2}\right)\cos\left(\frac{2A}{2}\right)}$$

$$= \frac{\cancel{2}\sin 3A \ \cancel{\cos A}}{\cancel{2}\cos 3A \ \cancel{\cos A}}$$

$$= \frac{\sin 3A}{\cos 3A}$$

$$= \underline{\tan 3A} = R.H.S$$

• S.T $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$

$$L.H.S = \frac{\cancel{2}\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cancel{2}\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} = \frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} \ \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

$$= \underline{R.H.S}$$

(H.W) imp

• P.T $\cos 3A + \cos 5A + \cos 9A + \cos 17A = 4\cos 4A \times \cos 6A \times \cos 7A$

Imp ?
S.T $\sin 20 \sin 40 \sin 80 = \frac{\sqrt{3}}{8}$

24/8/19

$L.H.S = (\sin 20 \sin 40) \sin 80$

$LHS = \frac{1}{2}(2\sin 20 \sin 40) \sin 80$

$= \frac{1}{2}(\cos 20 - \cos 60) \sin 80$

$= \frac{1}{2}\cos 20 \sin 80 - \frac{1}{2}\cos 60 \sin 80$

$= \frac{1}{2}\cos 20 \sin 80 - \frac{1}{2} \times \frac{1}{2} \sin 80$

$\cos 60 = 1/2$

$= \frac{1}{2}\cos 20 \sin 80 - \frac{1}{4}\sin 80$

$\sin(-60) = \sin 60$

$= \frac{1}{2\times 2}(2\cos 20 \sin 80) - \frac{1}{4}\sin 80$

$= \frac{1}{4}(\sin 100 - \sin(-60)) - \frac{1}{4}\sin 80$

$= \frac{1}{4}(\sin 100 - -\sin 60) - \frac{1}{4}\sin 80$

$= \frac{1}{4}(\sin 100 + \sin 60) - \frac{1}{4}\sin 80$

$= \frac{1}{4}\sin 100 + \frac{1}{4}\sin 60 - \frac{1}{4}\sin 80$

$= \frac{1}{4}\sin 100 + \frac{\sqrt{3}}{8} - \frac{1}{4}\sin 80$

$\sin 60 = \frac{\sqrt{3}}{2}$

$= \frac{1}{4}\sin 100 + \frac{\sqrt{3}}{8} - \frac{1}{4}\sin(180-100)$

$= \cancel{\frac{1}{4}\sin 100} + \frac{\sqrt{3}}{8} - \cancel{\frac{1}{4}\sin(100)}$

$= \frac{\sqrt{3}}{8}$

$= \underline{R.H.S}$

S.T $\cos 20 \cdot \cos 40 \cdot \cos 80 = \frac{1}{8}$

$L.H.S = (\cos 20 \cdot \cos 40) \cos 80$

$L.H.S = \frac{1}{2}(2\cos 20 \cos 40) \cos 80$

$= \frac{1}{2}(\cos(20+40) + \cos(20-40)) \cos 80$

$= \frac{1}{2}(\cos 60 + \cos(-20)) \cos 80$

$= \frac{1}{2}(\cos 60 + \cos 20) \cos 80$

$= \cancel{\frac{1}{2}(\cos 80) \cos 80}$

$= \frac{1}{2} \cos 60 \cos 80 + \frac{1}{2} \cos 20 \cos 80$

$= \frac{1}{2} \times \frac{1}{2} \times \cos 80 + \frac{1}{2} \cos 20 \times \cos 80$

$= \frac{1}{4} \cos 80 + \frac{1}{2} \cos 20 \cos 80.$

$= \frac{1}{2 \times 2}(2\cos 20 \cos 80) + \frac{1}{4} \cos 80$

$= \frac{1}{4}(\cos 100 + \cos(-60)) + \frac{1}{4} \cos 80$

$= \frac{1}{4}(\cos 100 + \cos 60) + \frac{1}{4} \cos 80$

$= \frac{1}{4} \cos 100 + \frac{1}{4} \cos 60 + \frac{1}{4} \cos 80$

$= \frac{1}{4} \cos 100 + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \cos 80$

$= \frac{1}{4} \cos 100 + \frac{1}{8} + \frac{1}{4} \cos 80$

$= \frac{1}{4}(\cos(180-80)) + \frac{1}{8} + \frac{1}{4} \cos\theta$

$= \frac{1}{4} \times -\cos\theta + \frac{1}{8} + \frac{1}{4} \cos\theta$

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$$= -\cancel{\frac{1}{4}\cos\theta} + \frac{1}{8} + \cancel{\frac{1}{4}\cos\theta}$$

$$= \frac{1}{8}$$

$= \underline{\underline{R.H.S}}$

PROPERTIES OF TRIANGLES

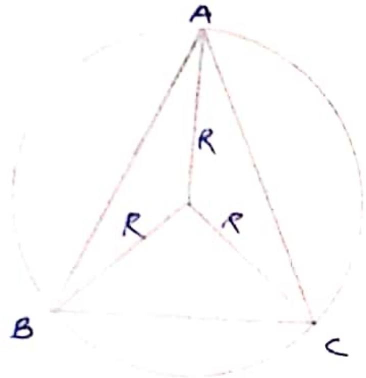
HOT
V.V

1 - Sine Rule (Sin law)

(In any Δ)

In any triangle ABC , $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$,

where R is the circumradius of the triangle ABC



ie, $\frac{a}{\sin A} = 2R$ $\quad \therefore \boxed{a = 2R\sin A}$

$\frac{b}{\sin B} = 2R$ $\quad \therefore \boxed{b = 2R\sin B}$

$\frac{c}{\sin C} = 2R$ $\quad \therefore \boxed{c = 2R\sin C}$

2. Cosine Rule (Cosine law)

In any triangle ABC,

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \quad (1) \\ b^2 &= a^2 + c^2 - 2ac\cos B \quad (2) \\ c^2 &= a^2 + b^2 - 2ab\cos C \quad (3) \end{aligned}}$$

From(1) $\Rightarrow$ $\boxed{2bc\cos A = b^2 + c^2 - a^2}$

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

From (2) $\Rightarrow$ $\boxed{2ac\cos B = a^2 + c^2 - b^2}$

$$\boxed{\cos B = \frac{a^2 + c^2 - b^2}{2ac}}$$

From (3) $\Rightarrow$ $2ab\cos C = a^2+b^2-c^2$

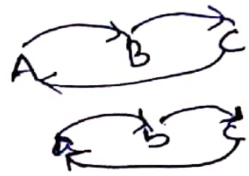
$$\boxed{\cos C = \frac{a^2+b^2-c^2}{2ab}}$$

projection formula

An any triangle ABC, a =

$$\boxed{\begin{aligned} a &= b\cos C + c\cos B \\ b &= a\cos C + c\cos A \\ c &= a\cos B + b\cos A \end{aligned}}$$

Area of a triangle ABC ;



$$\boxed{\begin{aligned} \text{Area}, \Delta &= \frac{1}{2} bc \sin A \\ &= \frac{1}{2} ca \sin B \\ &= \frac{1}{2} ab \sin C \end{aligned}}$$

$$\boxed{\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}}$$

27/8/2019

- In a triangle ABC, if $a = 6cm$, $b = 8cm$ and $\sin B = 3/5$ find $\sin A$

given,

$a = 6cm$
$b = 8cm$
$\sin B = 3/5$
$\sin A = ?$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{6}{\sin A} = \frac{8cm}{3/5}$$

$$6 \times \frac{3}{5} = \sin A \times 8$$

$$\frac{18}{5} = \sin A \times 8$$

$$\frac{18}{8 \times 5} = \sin A$$

$$\frac{18}{40} = \sin A$$

$$\tfrac{9}{20}\ \ \cancel{4.5}0 = \sin A$$

$4.5 \times 4 = 18.0$

$18 \div 4 = 4.5$ (16, 20)

$4.5 \times 4 = 18.0$

• In a triangle ABC, $\sin A = 4/5$, $\sin B = 1/4$ $a = 24cm$ find b,

~~SinA~~ given $\sin A = 4/5$

$\sin B = 1/4$

$a = 24$

$b = ?$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$b = \frac{a \sin B}{\sin A}$$

$$= \frac{24 \times 1/4}{4/5}$$

$$= \frac{\frac{24}{4}}{4/5}$$

$$= \frac{6}{4/5} = \frac{6 \times 5}{4}$$

$$4 \overline{)30} \quad 7.5$$

$$28$$

$$20$$

$$b = \frac{30}{4} = \cancel{7.5cm} \; 15/2 \, cm.$$

• In a triangle ABC $A = 30°$ $C = 45°$ $a = 2cm$ find $c = ?$

given $A = 30°$

$C = 45°$

$a = 2cm$

$c = ?$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a \sin C}{\sin A} = \frac{2 \times \sin 45}{\sin 30}$$

$$= \cancel{\frac{90}{30}} = \cancel{3}$$

$$= \cancel{3cm}$$

$$\frac{2 \times \frac{1}{\sqrt{2}}}{1/2}$$

$$= \frac{\sqrt{2}}{1/2}$$

$$= 2\sqrt{2} \, cm$$

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- Find the area of the triangle for which $a = 3cm$ $b = 7cm$, $c = 8cm$

given, $a = 3cm$

$b = 7cm$

$c = 8cm$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$S = \frac{a+b+c}{2}$$

$$= \frac{3+7+8}{2} = \frac{18}{2} = 9$$

$$\begin{array}{r} 18 \\ \times 6 \\ \hline 108 \end{array}$$

ie $\Delta = \sqrt{9(9-3)(9-7)(9-8)}$

$$= \sqrt{9(6)(2)(1)}$$

$$= \sqrt{9 \times 6 \times 2}$$

$$= \sqrt{18 \times 6}$$

$$= \sqrt{108}\ cm^2$$

- Find the area of a triangle having $a = 4cm$, $b = 2cm$ $C = 30^\circ$

$\frac{1}{2}ab\sin C$

~~Ase~~ given $a = 4cm$

$b = 2cm$

$C = 30^\circ$

$$\Delta = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 4 \times 2 \times \sin 30^\circ$$

$$= \frac{1}{\cancel{2}} \times \cancel{4}^{2} \times \cancel{2} \times \frac{1}{\cancel{2}}$$

$$= 2cm^2$$

HW • Find the area of the triangle ABC, having,

$a = 4\text{ cm},\ b = 5\text{ cm},\ c = 3\text{ cm}$

• Find the area of the triangle ABC, having $b = 3$, $c = 2\text{ cm}$ $A = 30°$

• In a triangle ABC, $a = 2\text{ cm}$, $c = 4\text{ cm}$ $C = 30°$ find $\sin A$

• In a triangle ABC. $A = 45°$, $B = 60°$, $a = 5\text{ cm}$ find $b = ?$

v.imp

⊙ In a triangle ABC, S.T $2(bc\cos A + ca\cos B + ab\cos C) =$
$a^2+b^2+c^2$

$\rightarrow$ L.H.S $= 2bc\cos A + 2ca\cos B + 2ab\cos C$

$= (b^2+c^2-a^2) + (a^2+c^2-b^2) + (a^2+b^2-c^2)$

$= \cancel{b^2}+\cancel{c^2}-\cancel{a^2}+\cancel{a^2}+c^2-\cancel{b^2}+a^2+b^2-\cancel{c^2}$

$= c^2+a^2+b^2$

$= a^2+b^2+c^2$

$=$ R.HS

also question can be given like this

$\Sigma a(b^2+c^2).\cos A$

$* \ 2bc\cos A = b^2+c^2-a^2$

$* \ 2ca\cos B = a^2+c^2-b^2$

$* \ 2ab\cos C = a^2+b^2-c^2$

v.imp

⊙ $a(b^2+c^2)\cos A + b(c^2+a^2)\cos B + c(a^2+b^2)\cos C = 3abc.$

L.H.S $= ab^2\cos A + ac^2\cos A + bc^2\cos B + ba^2\cos B + ca^2\cos C + cb^2\cos C$

$= ab(\underbrace{b\cos A + a\cos B}_{c}) + ac(\underbrace{c\cos A + a\cos C}_{b}) + bc(\underbrace{c\cos B + b\cos C}_{a})$

$= abc + acb + bca$

$= abc + abc + abc$

$= 3(abc) =$ R.H.S

- S.T in any triangle ABC $a(b\cos C - c\cos B) = b^2 - c^2$

$= a(b\cos C - c\cos B)$

$b = 3cm$

L.H.S $= ab\cos C - ac\cos B$

we know

$2ab\cos C = a^2+b^2-c^2$

$ab\cos C = a^2+b^2-c^2$

$2ac\cos B = (a^2+c^2-b^2)$

$$= \frac{a^2+b^2-c^2}{2} - \left(\frac{a^2+c^2-b^2}{2}\right)$$

$$= \frac{a^2+b^2-c^2}{2} - \frac{a^2-c^2+b^2}{2}$$

$\frac{a^2}{2} - \frac{a^2}{2} =$ cancell

b^2

$$= \frac{\cancel{a^2}}{\cancel{2}} + \frac{b^2}{2} - \frac{c^2}{2} - \frac{\cancel{a^2}}{\cancel{2}} - \frac{c^2}{2} + \frac{b^2}{2}$$

$$= \frac{\cancel{2}b^2}{\cancel{2}} - \frac{\cancel{2}c^2}{\cancel{2}}$$

$$= b^2 - c^2$$

= R.H.S

- In a triangle ABC, S.T $abc = 4R\Delta$

$\frac{a}{\sin A} = 2R$

$a = 2R\sin A$

$\therefore \sin A = \frac{a}{2R}$

$\sin B = \frac{b}{2R}$

$\sin C = \frac{c}{R}$

$$\Delta = \frac{1}{2}bc\sin A$$

$$= \frac{1}{2}bc \times \frac{a}{2R}$$

ie $\Delta = \frac{abc}{4R}$

ie $abc = 4R\Delta$

= RHS

$\cos C$

$B + b\cos C)$

a

• S.T $R(a^2+b^2+c^2) = abc(\cot A + \cot B + \cot C)$

$$R.H.S = abc\cot A + abc\cot B + abc\cot C$$

$$= abc\frac{\cos A}{\sin A} + abc\frac{\cos B}{\sin B} + abc\frac{\cos C}{\sin C}$$

$$= \frac{a}{\sin A}bc\cos A + \frac{b}{\sin B}ac\cos B + \frac{c}{\sin C}ab\cos C$$

$$= 2R\,bc\cos A + 2R\,ac\cos B + 2R\,ab\cos C$$

$$= R(2bc\cos A + 2ac\cos B + 2ab\cos C)$$

$$= R(\cancel{b^2}+c^2\cancel{-a^2} + \cancel{a^2}+\cancel{c^2}\cancel{-b^2} + a^2+b^2\cancel{-c^2})$$

$2bc\cos A = b^2+c^2-a^2$

$2ac\cos B = a^2+c^2-b^2$

$2ab\cos C = a^2+b^2-c^2$

$$= R(c^2+a^2+b^2)$$

$$= R(a^2+b^2+c^2)$$

$$= \underline{\underline{L.H.S}}$$

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29/8/2019 In the ΔABC, $a = \sqrt{3}+1$, $c = 2$, $B = 30°$ find b using cosine formulae

$a = \sqrt{3}+1$, $c = 2$, $B = 30°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$= (\sqrt{3}+1)^2 + 2^2 - 2 \times (\sqrt{3}+1) \times 2 \cos 30°$

$= 3 + 2\sqrt{3} + 1 + 4 - \cancel{4}^{2}(\sqrt{3}+1) \times \frac{\sqrt{3}}{\cancel{2}}$

$= 8 + 2\sqrt{3} - 2 \times 3 - 2\sqrt{3}$

$= 8 + \cancel{2\sqrt{3}} - 6 - \cancel{2\sqrt{3}}$

$= 2$

$b = \sqrt{2}$

S.T in a ΔABC, $(a+b) \sin \frac{C}{2} = c \cdot \cos \frac{A-B}{2}$

$(a+b) \sin \frac{C}{2} = C \cdot \cos \frac{A-B}{2}$

$\frac{a+b}{c} = ?$

$\sin\theta = 2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$

$a = 2R \sin A$
$b = 2R \sin B$
$c = 2R \sin C$.

$\frac{a+b}{c} = \frac{2R \sin A + 2R \sin B}{2R \sin C}$

$= \frac{\cancel{2R} (\sin A + \sin B)}{\cancel{2R} (\sin C)}$

$= \frac{\sin A + \sin B}{\sin C}$

$= \frac{\cancel{2} \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)}{\cancel{2} \sin\frac{C}{2} \cos\frac{C}{2}}$

we know $A+B+C = 180$
$A+B = 180 - C$

$A+B+C = 180$
$A+B = 180 - C$
$\therefore \frac{A+B}{2} = \frac{180-C}{2}$
$\frac{A+B}{2} = 90 - \frac{C}{2}$

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ie $= \dfrac{2\sin\left(90-\frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}\ \cos\frac{C}{2}}$

$\sin(90-\theta) = \cos\theta$

$= \dfrac{\cancel{\cos\left(\frac{C}{2}\right)}\cdot\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}\cdot\cancel{\cos\frac{C}{2}}}$

ie $\dfrac{a+b}{c} = \dfrac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$

crossmultiply

$(a+b)\sin\frac{C}{2} = c\cos\left(\frac{A-B}{2}\right)$

In any triangle, ABC $(b+c)\sin\frac{A}{2} = a\cos\left(\frac{B-C}{2}\right)$

$\dfrac{b+c}{a} = \dfrac{2R\sin B + 2R\sin C}{2R\sin A}$

$= \dfrac{\cancel{2R}(\sin B+\sin C)}{\cancel{2R}(\sin A)}$

$= \dfrac{\sin B+\sin C}{\sin A}$

$= \dfrac{\cancel{2}\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)}{\cancel{2}\sin\frac{A}{2}\ \cos\frac{A}{2}}$

$A+B+C=180$

$B+C = 180-A$

$\dfrac{B+C}{2} = \dfrac{180}{2} - \dfrac{A}{2}$

$= 90-\dfrac{A}{2}$

$= \dfrac{\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}\ \cos\frac{A}{2}}$

ie $= \dfrac{\sin\left(90-\frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)}{\sin A/2\cdot\cos A/2}$

$$= \frac{\cancel{\cos A/2} \cdot \cos\left(\frac{B-C}{2}\right)}{\sin A/2 \cdot \cancel{\cos A/2}}$$

$$\frac{b+c}{a} = \frac{\cos\left(\frac{B-C}{2}\right)}{\sin A/2}$$

$$\therefore (b+c) \sin A/2 = a \cdot \cos\left(\frac{B-C}{2}\right)$$

$$(a-b) \cos \frac{C}{2} = c \cdot \sin \frac{A-B}{2}$$

$$\frac{a+b}{c} = \frac{2R\sin A \mp 2R\sin B}{2R\sin C}$$

$$= \frac{\cancel{2R}(\sin A \mp \sin B)}{\cancel{2R}(\sin C)}$$

$$= \frac{\sin A \mp \sin B}{\sin C}$$

$$= \frac{2\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)}{2\sin\frac{C}{2} \cos\frac{C}{2}}$$

$\cos(90-\theta) = \sin\theta$

$A+B+C = 180$

$A+B = 180 - C$

$\frac{A+B}{2} = 90 - \frac{C}{2}$

$$= \frac{\cancel{2}\cos\left(90-\frac{C}{2}\right) \sin\left(\frac{A-B}{2}\right)}{\cancel{2}\sin\frac{C}{2} \cos\frac{C}{2}}$$

$\frac{a+b}{c}$

$$= \frac{\cancel{2}\cancel{\sin\frac{C}{2}} \sin\left(\frac{A-B}{2}\right)}{\cancel{2}\cancel{\sin\frac{C}{2}} \cos\frac{C}{2}}$$

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$$\frac{a+b}{c} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$$

$$\therefore \quad (a+b)\cos\frac{C}{2} = c \cdot \sin\left(\frac{A-B}{2}\right)$$

- S.T in a ΔABC $\quad \sum a(\sin B - \sin C) = 0$

$L.H.S = \sum a(\sin B - \sin C)$

$$= \sum a\left(\frac{b}{2R} - \frac{c}{2R}\right)$$

$$= \frac{1}{2R} \sum a(b-c)$$

$$= \frac{1}{2R}\left[a(b-c) + b(c-a) + c(a-b)\right]$$

$$= \frac{1}{2R}\left(\cancel{ab} - \cancel{ac} + \cancel{bc} - \cancel{ba} + \cancel{ca} - \cancel{cb}\right)$$

$$= \frac{1}{2R} \times 0$$

$$= 0$$

$= R.H.S$

* $\frac{b}{\sin B} = 2R$

$\therefore \sin B = \frac{b}{2R}$

* $\frac{c}{\sin C} = 2R$

$\therefore \sin C = \frac{c}{2R}$

* R is constant it has no cyclic change. angles and sides have cyclic change.

Σ remove ~~[illegible]~~

first ~~[illegible]~~ term again

- In a ΔABC, If $A = 60^\circ$. S.T $a^2 + bc = b^2 + c^2$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = b^2 + c^2 - 2bc \times \frac{1}{2}$$

$$a^2 = b^2 + c^2 - bc$$

$$a^2 + bc = b^2 + c^2$$

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3/8/17 addn formula (chapterless)

Ⓞ Express $\sqrt{3}\cos x + \sin x$ in the form $R\sin(x+\alpha)$ where α is acute

(α acute/obtuse is not imp)

$3x+2y = ax+by$
then
$a=3$
$b=2$

→ ~~equati~~

let ~~$\sqrt{3}\cos$~~

$\sqrt{3}\cos x + \sin x = R\sin(x+\alpha)$

$= R(\sin x\cos\alpha + \cos x\sin\alpha)$

$\sqrt{3}\cos x + \sin x = R\sin x\cos\alpha + R\cos x\sin\alpha$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

equating co-efficients of like terms on both sides.

$\therefore \sqrt{3} = R\sin\alpha$ —(1)

$1 = R\cos\alpha$ —(2)

squaring and adding (1)&(2)

$(\sqrt{3})^2 + 1 = (R\sin\alpha)^2 + 1$

$3+1 = R^2\sin^2\alpha + 1$

$4 = R^2\sin^2\alpha + 1$

$1^2 + 1 = (R\cos\alpha)^2 + 1$

$2 = R^2\cos^2\alpha + 1$

$4 = R^2(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1}) = R^2$

~~4or~~ $(\sqrt{3})^2 = (R\sin\alpha)^2$

$(1)^2 = (R\cos\alpha)^2$

$$3 = R^2 \sin^2\alpha \; +$$
$$1 = R^2 \cos^2\alpha$$

$$4 = R^2 \sin^2\alpha + R^2\cos^2\alpha$$
$$4 = R^2 \underbrace{(\sin^2\alpha + \cos^2\alpha)}_{1}$$

$$4 = R^2 \times 1$$
$$4 = R^2$$
$$\underline{\underline{R = 2}}$$

put $R = 2$ in eqn (1)

$$\sqrt{3} = 2\sin\alpha$$

ie $\sin\alpha = \frac{\sqrt{3}}{2}$

$$\therefore \alpha = \underline{\underline{60^\circ}}$$

$\sin(a+b)$
$= \sin a \cos b$
$\cos a$

$$\therefore \sqrt{3}\cos x + \sin x = 2\sin(x + 60^\circ)$$
$$= \cancel{2}\sin x \times \frac{1}{\cancel{2}} + \cancel{2}\cos x \times \frac{\sqrt{3}}{\cancel{2}}$$
$$= \sin x + \sqrt{3}\cos x$$
$$= \underline{\underline{\sqrt{3}\cos x + \sin x}}$$

- Express $4\cos x + 3\sin x$ in the form $k\sin(x+\alpha)$, α i. acute.

→ let $4\cos x + 3\sin x = k\sin(x+\alpha)$
$$= k\sin x\cos\alpha + k\cos x\sin\alpha.$$

equating co-effs....

$$4 = k\sin\alpha \quad \text{——(1)}$$
$$3 = k\cos\alpha \quad \text{——(2)}$$

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squaring and adding (1)&(2)

$$16 = k^2\sin^2\alpha \; +$$
$$9 = k^2\cos^2\alpha$$

$$25 = k^2(\sin^2\alpha + \cos^2\alpha)$$

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$25 = k^2 \times 1$

$k = \sqrt{25}$

$= \underline{\underline{5}}$

put $k = 5$ in (1)

$4 = k \sin\alpha$

$4 = 5 \sin\alpha$

$\sin\alpha = \frac{4}{5}$

$\alpha = \sin^{-1}\left(\frac{4}{5}\right)$

$=$

$\therefore 4\cos x + 3\sin x = 5 \sin\left(x + \sin^{-1}\left(\frac{4}{5}\right)\right)$

- An AC ~~current~~ (alternating current) B 'i' is given by.

$i = 3\sin\omega t + 4\cos\omega t.$

→ express (This this in the form $i = I\sin(\omega t + \alpha)$

where α is acute.

let $i = I\sin(\omega t + \alpha)$

$3\sin\omega t + 4\cos\omega t = I \sin\omega t\cos\alpha + \sin\alpha\cos\omega t$

equating co-eff. like terms

$3 = I\cos\alpha$ —— (1)

$4 = I\sin\alpha$, —— (2)

---

squaring and adding (1) & (2)

$9 = I^2\cos^2\alpha$

$16 = I^2\sin^2\alpha$

$25 = I^2(\cos^2\alpha + \sin^2\alpha)$

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$25 = I^2 (1)$

$25 = I^2$

$I = 5$

sub $I = 5$ in (2)

$4 = I \sin\alpha$

$4 = 5 \sin\alpha$

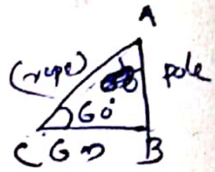
$\sin\alpha = \frac{4}{5}$

$\alpha = \sin^{-1}\left(\frac{4}{5}\right)$

$\therefore$ $3\sin\omega t + 4\cos\omega t = 5\sin\left(\omega t + \sin^{-1}\left(\frac{4}{5}\right)\right)$

HEIGHT & DISTANCE

- A rope is streched from the top of a vertical pole to a point 6 m from the foot of the pole. The rope makes an angle of 60° with the horizontal, find the height of the pole.



→ From the figure,

$$\tan 60^\circ = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\frac{\text{opposite}}{\text{adjacent}}$$

$$\sqrt{3} = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{6}$$

$$AB = 6\sqrt{3}$$

$$\begin{array}{r} 1.732 \times \\ 6 \\ \hline 10.392 \end{array}$$

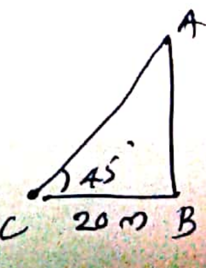
ht. of pole $= \underline{6\sqrt{3}}$

$= 6 \times 1.732$

$= 10.392$

$= \underline{10.39 m}$

- The rope supporting a flag post is fixed to the ground, 20 m away from the post making an angle of elevation 45° of the ground. find the length of the rope.



→ from the figure

$$\cancel{\tan}\cos 45^\circ = \frac{BC}{AC}$$

$$\cos 45^\circ = \frac{20}{AC}$$

$A\cos 45° = 20$

$\cos 45° = \sqrt{2}$

$A = \cos 45° \times 20$

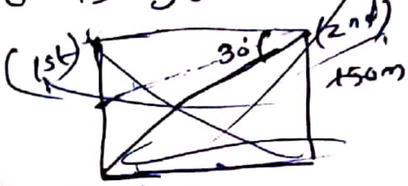
$= \frac{\sqrt{2}}{2} \times 20$

$= \frac{20\sqrt{2}}{2}$

$= 10$

$= 28.28\,m$

• The horizontal distance between 2 towers is 60m and the angle of depression of the 1st tower as seen from the second, which is in 150 m height is 30°



→ from the figure $AC = BE = 60\,m$

$AB = CE$

$CE = CD - DE$

$= 150 - DE$

From Δ DBE

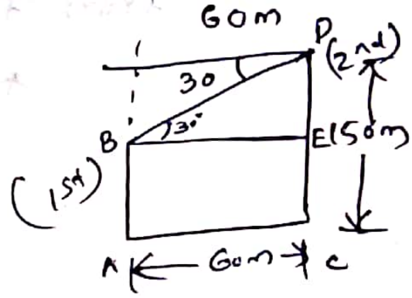
$\tan 30° = \frac{DE}{BE}$

$\frac{1}{\sqrt{3}} = \frac{DE}{60}$

$60 = \sqrt{3}\,DE$

$\frac{60}{\sqrt{3}} = DE$

$34.64 = DE$



$\therefore$ ht of the 1st tower, $AB = 150 - \frac{60}{\sqrt{3}}$

$= 150 - 34.64$

$= 115.358$

$= 115.36\,m$

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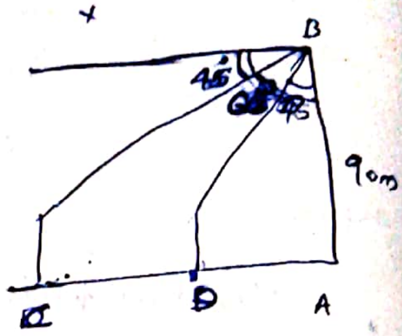
• From the top of a light house 90m height, the angles of depressions of 2 boats, on the sea level are 45° and 60°. find the distance b/w the boats.

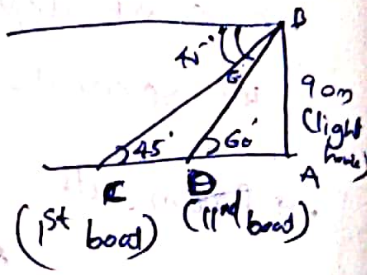


→ From the figure

$$\tan 45° = \frac{AB}{AC}$$

$$1 = \frac{90}{AC}$$

$$\underline{\underline{AC = 90m}}$$



• ~~from fi~~

$$\tan 60° = \frac{AB}{AD}$$

$$\sqrt{3} = \frac{90m}{AD}$$

$$AD = \frac{90}{\sqrt{3}} m$$

∴ Distance b/w 2 boats = CD

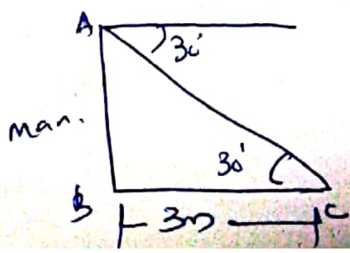
$$= AC - AD$$

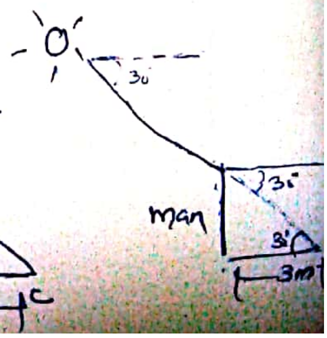
$$= 90 - \frac{90}{\sqrt{3}}$$

$$= 90\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$= \underline{38.04m}$$

• A man sees his shadow is 3m long (casts) when the sun's altitude is 30°. find the height of the man.





From figure;

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{3}$$

$$AB\sqrt{3} = 3$$

$$AB = \frac{3}{\sqrt{3}}$$

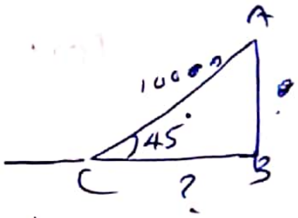
$$= \frac{\sqrt{3} \times \cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$$

$$= \sqrt{3}$$

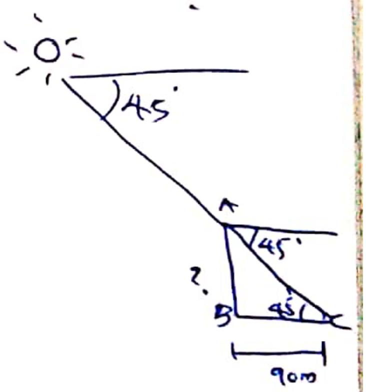
H.W

1. A tower casts a shadow 90m long when the sun is altitude is 45° find the height of the tower.

2). An aeroplane starts from a place and flies 1000m along a strt line, at 45° to the horizontal. find the horizontal distance discribed. (travelled)



1)



$$\tan 45^\circ = \frac{AB}{AC}$$

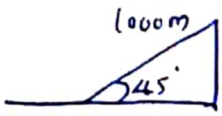
$$1 = \frac{AB}{90}$$

$$AB = 90m.$$

2)

$$\cancel{\tan 45^\circ} = \cos 45^\circ = \frac{BC}{AC}$$

$$\sqrt{2} = \frac{BC}{1000}$$



$$BC = \sqrt{2} \times 1000$$

$$= 1000\sqrt{2}\ m$$

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SOLUTION OF TRIANGLES

The 3 sides and the 3 angles of a triangle are called the elements of a triangle.

Case 1

imp

→ Three sides are given

Solve the ΔABC given $a = 4cm$ $b = 5cm$ $c = 7cm$

A - Given $a = 4cm$

$b = 5cm$

$c = 7cm$

$\angle A = ?$ $\angle B = ?$ $\angle C = ?$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{25+49-16}{2\times5\times7}$$

$$= 58/70 = 0.8286$$

$\cos A = 0.8286$

$A = \cos^{-1}(0.8286)$

$= 34.0448$

$\angle A = 34°.3' = \underline{34°3'}$

$A = \cos^{-1}(0.8286)$

$1° = 60'$

0.448×60

$= 2.6$

$= 3$ (rounded figure)

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \cancel{\frac{9+16-4}{2\times3\times4} = \frac{25-4}{6\times4} = \frac{21}{24} = 0.875}$$

$$\text{Ans} = \frac{4^2+7^2-5^2}{2\times4\times7} = \frac{16+49-25}{8\times7}$$

$$\cos B = \frac{40}{56} = 0.7143$$

$$B = \cos^{-1}(0.7143)$$

$$B = \cos^{-1}(0.7143)$$

$$= 44.4141$$

$$\angle B = 44^\circ\ 25' = \underline{\underline{44^\circ\ 25'}}$$

[illegible]

16/9/12. Monday

Solve $\triangle ABC$, $a=\sqrt{2}$, $b=\sqrt{3}$, $c=\sqrt{5}$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(\sqrt{3})^2+(\sqrt{5})^2-(\sqrt{2})^2}{2\times\sqrt{3}\times\sqrt{5}}$$

$$= \frac{3+5-2}{2\times\sqrt{15}} \qquad = \frac{3}{\sqrt{15}} = 0.7745$$

$$= \frac{8-3}{2\sqrt{35}}$$

$$= \frac{5}{2\sqrt{15}}$$

$$A = \cos^{-1}(0.7745)$$

$$= 39^\circ 14' \text{ (from logarithm)}$$

$$.2402 \times 60 = 14.412\ldots$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

BODMAS.

$$= \frac{(\sqrt{2})^2+(\sqrt{5})^2-(\sqrt{3})^2}{2\times\sqrt{2}\times\sqrt{5}}$$

$$= \frac{2+5-3}{2\times\sqrt{10}}$$

$$= \frac{7-3}{2\sqrt{10}} = \frac{\cancel{4}\,2}{\cancel{2}\sqrt{10}}$$

$$= \frac{2}{\sqrt{10}} = 0.6324$$

$$\therefore B = \cos^{-1}(0.6324)$$

$\therefore \quad C = 180 - (A+B)$

$= 180 - (39^\circ 14' + 50^\circ 46')$

$= 180 - 90$

$= 90^\circ$

$c = 12.5$

Case 2 :-

2 sides and the included angle are given.

tangents law (Napier's formula)

In any triangle ABC,

A · B · C

$$\boxed{\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}}$$

• Solve the triangle ABC given $a = 87$ cm, $b = 53$ cm $C = 70^\circ$

Case 2:-

$\cot 35 = \cot(90-55)$
$= \tan 55$

$a = 87$ cm, $b = 53$, $C = 70$

here $a > b$ so $a - b$

cot cofunction tan

So $\cot 35 = \tan 55$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\begin{array}{r} 87 - \\ 53 \\ \hline 34 \end{array}$$

$$\tan\frac{A-B}{2} = \frac{87-53}{87+53}\cot\left(\frac{70^\circ}{2}\right)$$

$$= \frac{34}{140}\cot 35^\circ$$

$\frac{1}{\tan 35^\circ} = \cot 35^\circ$

$= 1.428$

$$= \frac{34}{140}\tan 55^\circ$$

$$= \frac{34 \times 1.4281}{140} = \frac{48.5554}{140}$$

$$\begin{array}{r} .1265 \times \\ 60 \\ \hline 8' \end{array}$$

$$\tan\frac{A-B}{2} = \underline{\underline{0.3468}}$$

7.59

$$\therefore \frac{A-B}{2} = \tan^{-1}(0.3468)$$

$$= \underline{\underline{19.1265}}$$

$\therefore A-B = 38^\circ 16'$

$A+B = 180 - C$

$= 180^\circ - 70^\circ$ given, $C = 70^\circ$

$= \underline{\underline{110^\circ}}$

$$\begin{array}{r} 74 \\ 2\overline{)148} \\ 148 \\ \hline 0 \end{array}$$

$A - B = 38^\circ 16'$ +

$A + B = 110^\circ$

$2A = 148^\circ 16'$

$A = \dfrac{148^\circ 16'}{2}$

$$\begin{array}{r} 8' \\ 52 \\ \hline \end{array}$$

$= \underline{74^\circ 8'}$

$$\begin{array}{r} 74^\circ 8' \\ \hline 110^\circ \end{array}$$

$119^\circ 60'$

$A+B = 110$

$B = 110 - A$

$= 110 - 74^\circ 8'$

$= \underline{\underline{35^\circ 52'}}$

~~$c^2 = a^2 + b^2 + 2ab \cos$~~

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{57}{\sin 74^\circ 8'} = \dfrac{c}{\sin 70^\circ}$

siv

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 87^2 + 53^2 - 2 \times 87 \times 53 \cos 70^\circ$

$= 7569 + 2809 - 9222 \cos 70$

$= 10378 \quad - 3154.109$

$= \underline{\underline{7223.891}}$

fence = a+b+c

$c = \underline{\underline{84.993 \text{ cm.}}}$

$= \cancel{84}$

Given $a = 5cm$,

• Solve the ΔABC gi~~ven a = 5cm, c = 5cm, B = 30°~~

$c = 8cm \quad B = 30^\circ$

$\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot B/2$

$= \frac{8-5}{8+5} \cot 30/2$

$= \frac{3}{13} \cot 15$

$\tan \frac{C-A}{2} = 0.2307 \times 3.732$

$\tan \frac{C-A}{2} = 0.8608$

$\frac{C-A}{2} = \tan^{-1}(0.8608)$

0.7218×60
$= 43.3$
$= 44$

$= 40^\circ 44'$

$C-A = 40^\circ 44' \times 2$

$= 80^\circ 88'$

$C-A = 81^\circ \cdot 28'$ — 1

$C+A = 180 - B$

$C+A = 180 - 30^\circ$

$C+A = 150$ — 2

$2C = 231^\circ 28'$

$c = \frac{230}{2}$ $\frac{28+60}{2}$

$= 115^\circ 44'$

$A = 34^\circ 16'$

$b^2 = a^2 + c^2 - 2ac \cos 13$

$= 25 + 64 - 2 \times 5 \times 8 \times \cos 30$

$= 89 - 80 \sqrt{3}/2$

$= 89 - 4 \cdot \sqrt{3}$

$= 89 - 69.282$

$= 19.718$

$b = 4.44$ cm

Given $a = 3$ cm, $b = 7$ cm, $C = 38^\circ$

$\tan \frac{B-A}{2} = \frac{b-a}{b+a} \cot c/2$

$= \frac{7-3}{7+3} \cot 38/2$

$= \frac{4}{10} \cot 71$

$\tan \frac{B-A}{2} = 4/10 \times 2.904$

$\tan \frac{B-A}{2} = 1.616$

$\frac{B-A}{2} = 49.275 = 49^\circ 17'$

$B - A = 98^\circ 34'$ — (1)

$B + C = 180$

$B + A = 180 - c$

$B + A = 180 - 38$

$B + A = 142^\circ$ — (2)

$2B = 246^\circ 34'$

$B = 120^\circ 34'$

$A = 142^\circ - 120^\circ 34'$

$\underline{A = 21^\circ 26'}$

$c^2 = 3^2 + 7^2 - 3 \times 7 \times 2 \cos 38$

$c^2 = 9 + 49 - 3 \times 14 \cos 38$

$c^2 = 58 - 42 \times 0.788$

$c^2 = 24.904$

$\underline{C = 4.990\text{cm}}$

→ $a = 87\text{cm}$, $b = 53\text{cm}$, $C = 110^\circ$

$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot C/2$

$\tan\left(\frac{A-B}{2}\right) = \frac{87-53}{87+53} \cot 110/2$

$= 34/140 \quad \cot 55$

$= \frac{34}{140} \quad \frac{1}{\tan 55}$

$\tan \frac{A-B}{2} = \frac{34}{140} \times 0.700$

$= 0.17$

$\frac{A-B}{2} = 9.648$

$A - B = 9^\circ 38' \times 2$

$A - B = 19^\circ 16'$

$A+B = 70$

$2A = 89°16'$

$A = \frac{88}{2} \quad \frac{16+60}{2}$

$= 44°38'$

$B = 70 - 44°38'$

$= 25°22'$

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 87^2 + 53^2 - 2 \times 87 \times 5.3 \times \cos 110$

$= 7569 + 2809 - 9222 \times 0.342$

$= 10378 - 3153.924$

$c^2 = 7224.076$

$c = 84.995 cm$

Case 3:-

- 2 angles and one side aregiven

- given $A = 35°$ $B = 68°$ $c = 25cm$

→ ~~Given~~ $A = 35°$ $B = 68°$ $c = 25cm$

$C = 180 - (A+B)$

$= 180 - (35+68)$

$= 180 - (103)$

$= 77°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 35°} = \frac{25}{\sin 77°}$, $a = \frac{c \times \sin A}{\sin C} = \frac{25 \times \sin 35°}{\sin 77°}$

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