Chapter 7: Viscosity

Viscosity:

It is the property of a fluid which makes it tends to resist the relative motion between the different layers of a fluid.

Viscous force:

It is a shearing force present in fluids by the virtue of the velocity difference of different layers of the fluid. The faster moving layer tends to drag the slower moving layer and the slower moving layer tends to retard the faster moving layer due to viscous force.

$$F = \frac{\eta A(v_2 - v_1)}{d}$$

Coefficient of viscosity (η):

It is defined as the tangential force required per unit are to maintain a unit velocity gradient between two layers of a fluid.

$$\eta = \frac{F/A}{\frac{(v_2 - v_1)}{d}}$$

Where **F** is the tangential force acting on an area **A,** $\mathbf{v_1}$ and $\mathbf{v_2}$ are the velocity of the different layers, **d** is the thickness of the layer.

$\frac{(v_2 - v_1)}{d}$ is known as the velocity gradient.

SI unit of Coefficient of viscosity is **kg/m s** or **Ns/m**$^{-2}$

Cgs unit of Coefficient of viscosity is called **poise.**

Poiseuille's formula for the flow of a liquid through a capillary tube:

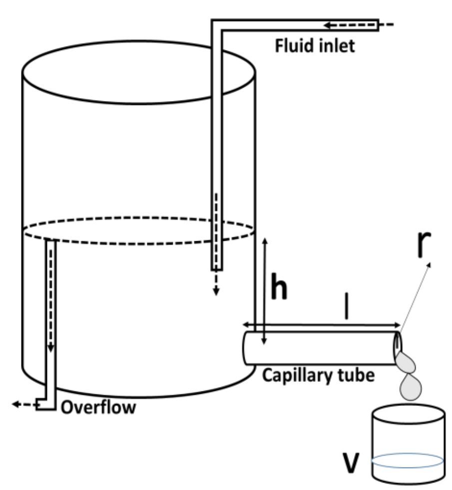
$$V = \frac{\pi P r^4}{8 l \eta}$$

Where **V** is the volume of liquid flown out through the capillary tube by applying an external pressure **P**.

r is the radius and **l** is the length of the capillary tube.

Determination of the coefficient of viscosity by Poiseuille's method:

The experimental setup is to be arranged as shown in the figure. The height of the fluid ***h*** is kept the same by arranging a fluid inlet and an overflow outlet pipe in the reservoir. A capillary tube of radius ***r*** and length ***l*** is attached on the reservoir and the volume of fluid ***V*** flown out through the capillary tube in each second is measured with a measuring cylinder.



From the Poiseuille's formula,

$$V = \frac{\pi P r^4}{8 l \eta} \rightarrow \eta = \frac{\pi P r^4}{8 l V}$$

Where **η** is the coefficient of viscosity, **r** is the radius and **l** is the length of the capillary tube, **V** is the volume flown out through the capillary tube per second and **P** is the pressure.

If **h** is the height and **d** is the density of the fluid, the pressure is given by $P = hdg$

by substituting

$$\eta = \frac{\pi h d g r^4}{8 l V}$$

The coefficient of viscosity of low viscous fluids can be measured by using the above equation.

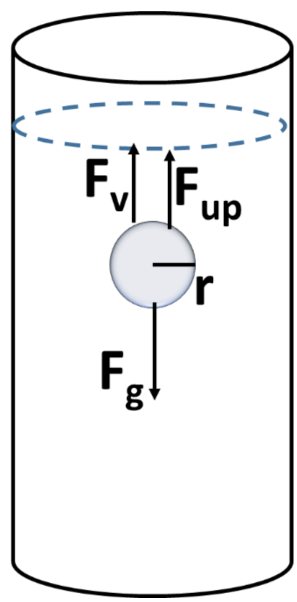
Stokes formula for highly viscous liquids:

When a sphere of mass m falls vertically downward through a highly viscous medium, the downward gravitational force $\mathbf{F_g}$ will be canceled by the up thrust $\mathbf{F_{up}}$ of the displaced liquid and the viscous force $\mathbf{F_v}$.

$$Ie F_g = F_{up} + F_v$$
$$where, F_g = mg$$

Mass of the sphere = Volume of the sphere x Density of the sphere

$$m = \frac{4}{3}\pi r^3 \rho$$
$$hence, F_g = \frac{4}{3}\pi r^3 \rho g$$
$$F_{up} = \frac{4}{3}\pi r^3 d$$



$F_v = 6\pi r \eta v$ (Stokes formula)

$As F_g = F_{up} + F_v$

Substituting the values,

$$\frac{4}{3}\pi r^3 \rho g = \left(\frac{4}{3}\pi r^3 d g + 6\pi r \eta v\right)$$
$$\frac{4}{3}\pi r^3 g(\rho - d) = 6\pi r \eta v$$
$$\eta = \frac{2r^3 g(\rho - d)}{9v}$$

Stokes method for determining the coefficient of viscosity of highly viscous liquids:

The fluid is to be taken in a long jar as shown in the figure, a metallic sphere of radius **r** and density $\rho$is allowed to fall vertically through the liquid. The sphere attains a constant velocity when the net force acting on the sphere become zero. The forces acting on the sphere is given by

$$F_g = F_{up} + F_v$$

By substituting the corresponding equations

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 d g + 6\pi r \eta v$$

By simplifying

$$\eta = \frac{2r^3 g(\rho - d)}{9v}$$

Where η is the coefficient of viscosity, **r** is the radius of the sphere, ρ is the density of the sphere, **g** is the acceleration due to gravity, **d** is the density of the fluid, and **v** is the terminal velocity of the sphere.

Temperature dependence of the coefficient of viscosity of gases:

$$\eta_t = \frac{\eta_0}{1 + at + bt^2} \rightarrow \eta_t \alpha \frac{1}{t}$$

Coefficient of viscosity of gases decreases as the temperature increases.

Temperature dependence of the coefficient of viscosity of liquids:

$$\eta_T = \frac{\eta_0 A\sqrt{T}}{1 + \frac{B}{T}} \rightarrow \eta_T \alpha T$$

Coefficient of viscosity of gases increases as the temperature increases.