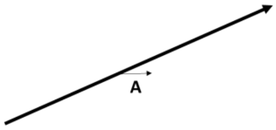
Unit:4 Vectors and statistics

Representation of a vector:

A vector can be represented geometrically by a straight line with an arrowhead. The length of the line segment indicates its magnitude and the arrow head specifies the direction. Vector is symbolically represented by a letter with an arrow on the top.



Modulus of the vector: The magnitude of a vector is called its modulus.

Collinear vector: Vectors that are lying on the same line is called collinear vector.

Coplanar vectors: Vectors that are lying in the same plane is called coplanar vectors.

Equal vectors: Vectors that are equal in both magnitude and direction are called equal vectors.

Negative vectors: Vectors that are equal in magnitude and opposite in direction are called negative vectors.

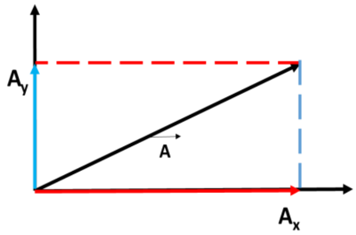
Zero vector: It is the vector with zero magnitude.

Like and unlike vectors:

Parallel vectors are called like forces and antiparallel vectors are known as unlike vectors.

Resolution of a vector:

The process of splitting up a vector in to any two different direction is called the resolution of a vector. A vector resolved into two mutually perpendicular direction is called ***rectangular component of the vector.***



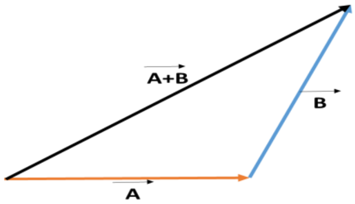
A_x=A cosθ

A_y=Asinθ

<u>Vector addition:</u>

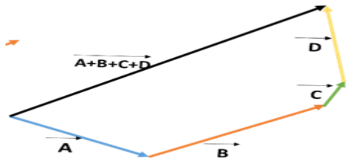
Triangular law of vector addition:

If two vectors are represented both in magnitude and direction by the two sides of triangle taken in order, their resultant or the vector sum is represented by the third side of the triangle taken in reverse order.



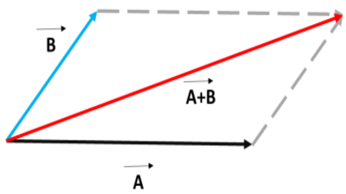
Polygon law of vectors:

If a number of vectors can be represented both in magnitude and direction by the sides of a polygon taken in order, the vector sum is represented by the vector drawn from the tail of the first vector to the head of the last vector.



Parallelogram law of vector addition:

If two vectors acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from the point, their resultant vector is represented both in magnitude and direction by the diagonal of the parallelogram passing through the same point.



Concurrent forces:

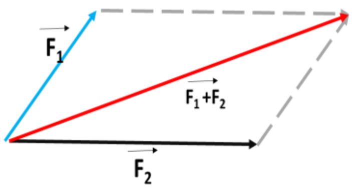
It is the forces whose lines of action pass through a common point.

Like and unlike forces:

Parallel forces are called like forces and antiparallel forces are known as unlike forces.

Law of parallelogram of forces:

If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from the point, their resultant is



represented both in magnitude and direction by the diagonal of the parallelogram passing through the same point.

Analytical method to find the resultant of two vectors:

From the figure

OA=BC=P

OB=AC=Q

OC=R

Where P and Q are the vectors and R is its resultant vector.

From the triangle ADC

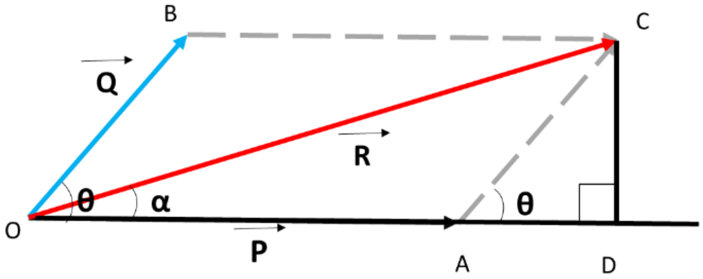
cos θ = AD/AC

AD = AC cos θ = Qcos θ

Similarly

sin θ = CD/AC

CD = AC sin θ = Qsin θ



From the triangle ODC

By applying Pythagoras theorem,

$OC^2=OD^2+CD^2$

i.e. $OC^2 = (OA+AD)^2+CD^2$

By expanding

$OC^2 = OA^2+2OA.AD +AD^2+CD^2$

By applying Pythagoras theorem in **triangle ADC**,

$AC^2=AD^2+CD^2$

Hence $OC^2 = OA^2+2OA.AD +AC^2$

By substituting the values of OC =R, OA = P, AC= Q,AD =Qcos θ,

$R^2 = P^2+2P.Q\cos\theta+Q^2$

$R = \sqrt{P^2 + Q^2 + 2P.Qcos\theta}$

The above equation represents the magnitude of resultant vector.

Direction of the resultant vector(α):

tan α = CD/OD

tan α = CD/(OA+AD)

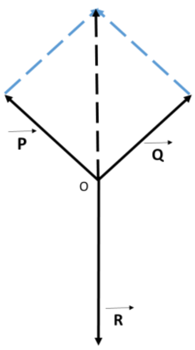
By substituting the values

$$tan\alpha = \frac{Qsin\theta}{P + Qcos\theta}$$

$$\alpha = tan^{-1}\left(\frac{Qsin\theta}{P + Qcos\theta}\right)$$

Different cases:

1. when $\theta = 0^0$, cosθ= 1, then R=P+Q
2. when $\theta =180^0$, cosθ= -1, then R=P-Q
3. when $\theta = 90^0$, cosθ= 0, then $R = \sqrt{P^2 + Q^2}$

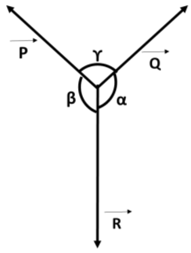


Equilibrant:

Resultant of any forces can be balanced by another vector of same magnitude in the opposite direction. This force is called the equilibrant force. The equilibrant force can keep the body in equilibrium.

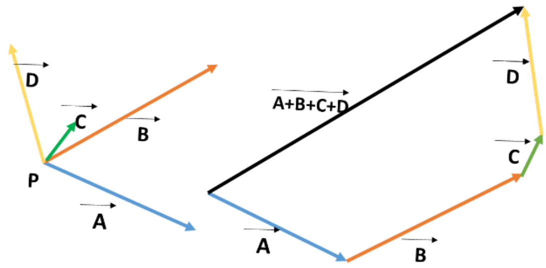
Law of triangle of forces:
If tree forces acting at point are represented in magnitude and direction by the sides of a triangle taken in order, they will be inn equilibrium.

Lami's theorem:
If three forces acting on a body keep the body in equilibrium, each force is proportional to the sine of the angle between the other two.

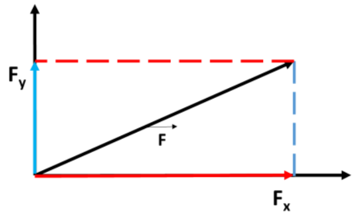


$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$

Polygon law of forces:
If a number of forces acting on a particle can be represented in magnitude and direction by the sides of a polygon taken in order, the forces will be in equilibrium.



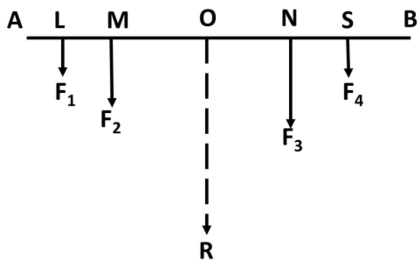
Rectangular component of a force:
The process of splitting up a force in to any two different direction is called the resolution of a vector. A force resolved into two mutually perpendicular direction is called ***rectangular component of the force.***



F_x=F cosθ
F_y=F sinθ

Moment of a force:
The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force. Clock wise moments are taken as positive and anticlockwise moment are considered as negative.

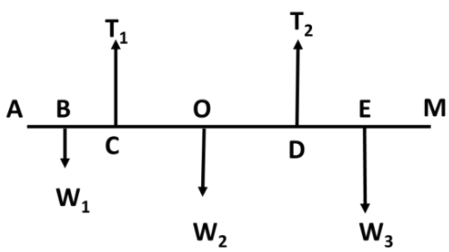
Moment of a resultant:
If a number of forces are acting on a rigid body; the algebraic sum of their moment in any point in the plane is equal to their moment of their resultant about the same point.



Consider the parallel forces F_1, F_2, F_3andF_4 are acting on the body AB, at the points L,M, N and S respectively.
The algebraic sum of moments = F_1.Al + F_2.AM+ F_3.AN+F_4.AS
The resultant force R = F_1+F_2+F_3+F_4
Hence the sum of moments can be modified as R.AO
i.e. F_1.Al + F_2.AM+ F_3.AN+F_4.AS = R.AO

Coplanar parallel forces: condition for equilibrium:
1. Algebraic sum of their moments of forces about any point in their plane should be zero
2. Algebraic sum of the forces acting on the body should be zero.

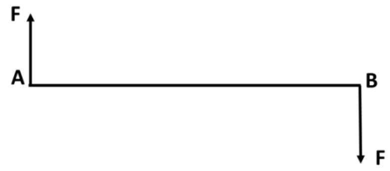
Illustration
Consider the equilibrium of a rod AM which acted by parallel forces in downward direction W_1, W_2, and W_3 and T_1, and T_2 in upward direction. As it is in equilibrium,



upward forces = downward forces
W_1+ W_2+ W_3 = T_1+ T_2
Similarly Sum of clockwise moment = Sum of anticlockwise moment
W_1.AB+ W_2.AO+ W_3.AE = T1.AC+ T_2.AD

Couple:

Two equal unlike forces acting on a body whose line of action is not the same constitutes a couple.



Arm of the couple:

The perpendicular distance between the lines of action of forces in a couple is known as the arm of the couple. In the figure AB is the arm of the couple.

Characteristics of a couple:

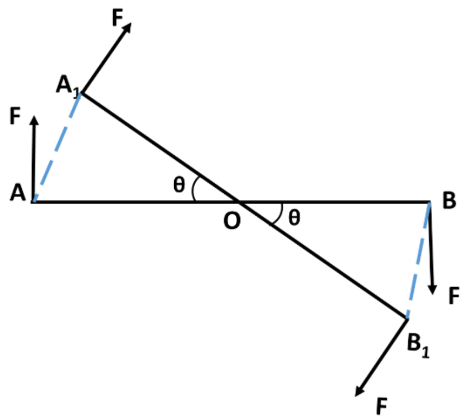
1. Since the algebraic sum of the forces constituting a couple is zero, the effect of the couple is to produce pure rotation without translation.
2. The algebraic sum of the moments of the forces constituting a couple about any point in their plane is a constant and equal to the moment of the couple.
3. Two couples acting in one plane up on a rigid body balance each other if their moments are equal and opposite.

<u>**Work done by a coupe:**</u>

Consider the forces F and F acting on a body AB.

The moment of the couple C = F.AB

Let the couple rotate an angle through an angle $\boldsymbol{\theta}$ about any point O leads to the movement of the point A to A_1 and B to B_1



As the work done W = F.S

Work done by the couple $W= F.AA_1+F.BB_1 \rightarrow 1$

As the angle = arc / radius

$\theta = AA_1 / AO = BB_1 / BO$

$AA_1 = AO\theta$

$BB_1 = BO\theta$

Substituting in the equation 1

$W = F.AO\theta + F.BO\theta$

$W = F (AO+BO)\, \theta$

$W = AB\, \theta$

$\mathbf{W= C\, \theta}$

Work is independent about the point of rotation.

For a complete revolution, $\theta = 2\pi$,

Work done to complete one revolution $= 2\, \pi C$

If the body perform N revolutions in a second,

Power (work done in unit time) $P = 2\, \pi NC$