Unit 2: Motion in One Dimension

Distance: It is the total path length traveled by an object, it is a scalar quantity. It will never be negative.
Displacement: It is the shortest distance between the initial position and final position of an object. It is a vector quantity.
Vector quantity: It is defined as the physical quantity which require both direction and magnitude for its specification. Eg force, velocity, displacement, acceleration...
Scalar quantity: The physical quantity which does not have a direction is called scalar quantity.
Speed: It is the rate of change of distance. It a scalar quantity. Its SI unit is m/s and dimensional formula is LT^{-1}.
Uniform speed: It is the motion of object which travels equal distance in equal interval of time.
Instantaneous speed: It represents the speed of an object at an instant.
Velocity: It is the rate of change of displacement. It is a vector quantity. Its SI unit is m/s and dimensional formula is LT^{-1}. As being a vector quantity, its variation can be in direction or magnitude or both.

$velocity(v) = \frac{x_2 - x_1}{t_2 - t_1}$ Where X_2 and X_1 are the final and initial positions respectively.

$instantaneousspeed = \frac{ds}{dt}$ Where s is the displacement.

Acceleration: it is defined as the rate of change of velocity. It is vector quantity. Its SI unit is m/s^2. Its dimensional formula is LT^{-2}.

$acceeleration(a) = \frac{v-u}{t}$ Where v and u are final and initial velocities respectively.

Equations of motion for uniformly accelerated bodies

1. $a = \frac{v-u}{t}$
2. $v = u + at$
3. $v^2 = u^2 + 2as$
4. $s = ut + \frac{1}{2}at^2$
5. $s = \frac{u+v}{2}t$

Equations of motion under free fall.

1. $v = u + gt$
2. $s = ut + \frac{1}{2}gt^2$
3. $v^2 = u^2 + 2gs$

Equations of motion for objects moving against gravity

1. $v = u - gt$
2. $s = ut - \frac{1}{2}gt^2$
3. $v^2 = u^2 - 2gs$
4. $maximaumheightS_{max} = \frac{u^2}{2g}$

Derivation for the distance traveled by an object during the n^{th} second of its motion

A uniformly accelerated body traverse unequal distance in equal interval of time.
To calculate the distance travelled (S_n) in n^{th} second, we have to subtract the total distance travelled (S_2) by the object in $(n-1)^{th}$ second from the distance travelled (S_1)by the object in the nth second of its motion.

$S_n = S_1 - S_2$

$$S_1 = un + \frac{a}{2}n^2$$

$$S_2 = u(n-1) + \frac{a}{2}(n-1)^2$$

$$S_2 = un - u + \frac{a}{2}(n^2 - 2n + 1)$$

$$S_2 = un - u + \left(\frac{a}{2}n^2 - an + \frac{a}{2}\right)$$

$$S_1 - S_2 = un + \frac{a}{2}n^2 - \left[un - u + \left(\frac{a}{2}n^2 - an + \frac{a}{2}\right)\right]$$

$$S_1 - S_2 = un + \frac{a}{2}n^2 - un + u - \frac{a}{2}n^2 + an - \frac{a}{2}$$

$$S_1 - S_2 = u + an - \frac{a}{2}$$

$$S_1 - S_2 = u + a\left(n - \frac{1}{2}\right)$$

Derivation for proving the *time of ascent is equal to the time of descent* and for the maximum height for a body projected upward

let t_1 be the time of ascent (time taken by a body to reach at the maximum height for a body projected upward)

Time of ascent (t_1)

from the equations of motion

$$v = u + at$$

$0 = u - gt$| v =0 at the maximum height

$$t_1 = \frac{u}{g} \rightarrow (1)$$

Maximum height (h)

from the equations of motion

$$v^2 = u^2 + 2as$$

$$0 = u^2 - 2gh|$$

At the maximum height, S = h; v = 0.

$$h = \frac{u^2}{(2g)} \rightarrow (2)$$

Time of descent (t_2)

from the equations of motion

$$s = ut + \frac{1}{2}at^2$$

$$h = \frac{1}{2}gt_2^2$$

by substituting the value of h in (2)

$$\frac{u^2}{(2g)} = \frac{1}{2}gt_2^2$$

$$t_2^2 = \frac{u^2}{g^2}$$

$$t_2 = \frac{u}{g} \rightarrow (3)$$

by comparing (1) and (3)

$t_1 = t_2$|The time of ascent = The time of descent