Unit 3: Force and motion

State of rest: If the position of an object is not changing with time with respect to its surroundings, then it is in the state of rest.
State of motion: If the position of an object is changing with time with respect to its surroundings, then it is at the state of motion.
First law of motion: Every body continues in its state of rest or state of uniform motion along a straight line unless it is compelled by an external force to change that state.
Inertia: It is the inability of an object to change its original state.
Inertia of rest: It is the inability of an object to change its state of rest to the state of motion, e.g.; A person standing in a bus falls backward as the bus starts to move forward.
Inertia of motion: It is the inability of an object to change its state of motion to the state of rest, e.g. A person standing in a moving bus falls forward as the bus stops its motion.
Force: It is the physical quantity that changes or tends t change the state of a system. It is a vector quantity, its SI unit is newton (N) and the dimensional formula is $[M^1L^1T^{-2}]$.
1 newton = 1 kgm/s^2.
Momentum: It is defined as the product of mass and velocity. It is a vector quantity with the direction along the direction of velocity. Its SI unit is kgm/s and the dimensional formula is $[M^1L^1T^{-1}]$.
p= mv
Second law of motion: The rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the same direction of force.
Proof for the second law of motion:

$$initialmomentum\ \ p_i = mu$$

$$finalmomentum\ \ p_f = mv$$

$$chngeofmomenntum\ \ \delta p = mv - mu$$

$$rateofchngeofmomenntum \frac{dp}{dt} = \frac{p_f - p_i}{t}$$

$$\frac{dp}{dt} = \frac{mv - mu}{t} = m\frac{(v-u)}{t}$$

$$\frac{dp}{dt} = ma = F$$

Third law of motion:
To every action, there is an equal and opposite reaction; it exists as action-reaction pairs.
Force of action= - Force of reaction
Law of conservation of momentum:
The law of conservation of momentum states that when two or more bodies collide, the sum of the momenta before their impact is equal to the sum of the momenta after the collision.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Derivation for proving the law of conservation of momentum
If the mass m_2 exert a force on the mass m_1, then
Initial momentum of the mass m_1; $p_{1i} = m_1u_1$
Final momentum of the mass m_1;$p_{1f} = m_1v_1$
Rate of change of momentum of the mass m_1 =>$\frac{dp_1}{dt} = \frac{m_1v_1 - m_1u_1}{t} = F_{reaction}$
Initial momentum of the mass m_2; $p_{2i} = m_2u_2$
Final momentum of the mass m_2; $p_{2f} = m_2v_2$

Rate of change of momentum of the mass m2 => $\frac{dp_2}{dt} = \frac{m_2v_2 - m_2u_2}{t} = F_{action}$

According to the 3rd law of motion

Force of action= - Force of reaction

$$F_{action} = -F_{reaction}$$

$$\frac{m_2v_2 - m_2u_2}{t} = -\left(\frac{m_1v_1 - m_1u_1}{t}\right)$$

$$m_2v_2 - m_2u_2 = -(m_1v_1 - m_1u_1)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Recoil of gun

The backward movement of a gun while firing a bullet is called the recoil of a gun. It is an example of the law of conservation of momentum.

Derivation for the recoil velocity of a gun:

Let the M and m are the masses of gun and bullet and V and v are the velocities of the gun and the bullet respectively,

As the $V = 0, v = 0$, the initial momentum $p_i = 0$

According to the conservation of momentum; $p_i = p_f$

Hence $MV + mv = 0$

Recoil velocity of gun $V = -\left(\frac{mv}{M}\right)$

Force required to stop the gun:

Initial velocity of the gun (the recoil velocity) $= u = -\left(\frac{mv}{M}\right)$

Final velocity = 0

Distance moved before stopping the gun by the shoulder = s

Substituting the values in the formula $v^2 = u^2 - 2as$(- ve sign indicate the retardation)

$$0 = \left(\frac{mv}{M}\right)^2 - 2as$$

$$a = \frac{m^2v^2}{2sM^2}$$

$$Force = mass x acceleration$$

$$F = Mx\frac{m^2v^2}{2sM^2}$$

$$F = \frac{m^2v^2}{2sM}$$

Rocket propulsion:

The propulsion of rocket is an example of the law of conservation of momentum. The combusted fuels are ejected out at a very high velocity as the form of gases gives a thrust on the rocket to move upward. Its motion is also an example of a system with varying mass as the combusted fuels and casing are removed during the propulsion. It gives a considerable increase in the velocity.

Impulse (I):

It is the product of the force and the duration time it exerts. It is equivalent to the change in momentum. I = F t

Derivation for proving the impulse is equal to the change in momentum

$$I = Ft$$

$$F = ma = \frac{m(v-u)}{t}$$

$$I = Ft = tx\frac{m(v-u)}{t}$$

$$I = m(v-u) = mv - mu$$

Work:
It is defined as the product of force and the component of displacement along the direction o force.
$W = F.s$ Or $W = Fscos\theta$. Its SI unit is jule and the dimensional formula is $[M^1L^2T^{-2}]$.
One joule: 1 joule is the work done by a force of 1N when its point of application is displaced through one meter in the direction of force.
In CGS system, erg is the unit of work, 1 erg = 10^{-7} joule
Power: It is the rate of doing work.
Power $P = \frac{W}{t}$
But $W = F.s$
Hence $P = \frac{F.s}{t}$ but $\frac{s}{t} = v$
Hence power $P = Fv$
Energy: It is the capacity of doing work. SI unit of energy is joule. 1 J = 1 kgm^2/s^2.
Kinetic energy: It is the energy possessed by a body by the virtue of its motion. It can be found by using the formula $KE = \frac{1}{2}mv^2$
Potential energy: It is the energy possessed by a body by the virtue of its position or atomic configuration. It can be found by using the formula $PE = mgh$(h represent the height of the object from the ground)
Relation between the kinetic energy and the momentum:

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2}mv^2\frac{m}{m}$$

$$KE = \frac{1}{2m}m^2v^2$$

$$KE = \frac{p^2}{2m}$$

$$p = \sqrt{2mKE}$$