LIMITS

15/7/2019

* Quantities can be classified into 2;

→ constants
→ variable

* quantities which do not varry during a mathematical investication are known as constants

eg:- $\pi = 3.14\ldots$
e (exponential). 2.718
etc.

constants can be denoted using k, a, b, etc..

* quantities which may varry during a mathomatical investication are known as variable

eg:- atmospheric temperature
amound of rainfall on each day.
speed of a vehicle
Water level in a dam
angle b/w minute hand & second hand.

* Variables are usually denoted by x, y, z, t, etc.

• consider 2 variable x and y which are related by the rule $y = x^2+1$ then for each values of x we get a unique value for y as follows

x	0	1	2	3	-1	-2
y	1	2	5	10	2	5

$y = 0^2+1 = 1$ $y = 1^2+1 = 2$ $y = 2^2+1 = 5$, etc.....

such a rule i.e a rule which gives a unique value for y for each values of x is known as a function.

a relation $y=f(x)$ is said to be a function if it gives a unique value for y for each values of x.

$y=f(x)$
y is a function of x

2). $y = x^2+x+1$

x	0	1	2	3	-1	-2
y	1	3	7	13	1	3

$y = 0^2+0+1$
$= 1$

$y = 1^2+1+1$
$= 1+1+1$
$= 3$

$y = 2^2+2+1$
$= 4+2+1$
$= 7$

$y = 3^2+3+1$
$= 9+3+1$
$= 13$

$y = (-1)^2+(-1)+1$
$= 1-1+1$
$= 1$

$y = (-2)^2+-2+1$
$= 4-2+1$
$= 5-2$
$= 3$

* In the function $y=f(x)$, x is the independant varible, since it can take any values and y is the dependand variable since the value of y are determined by the values of x.

* In the above function $f(x)$, $f(3)=13$ i.e, the value of the function $f(x)$, when $x=3$ is 13

Ily $f(5) = 5^2+5+1 = 31$
$f(7) = 7^2+7+1 = 49+7+1$

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1) $f(x) = 2x^2 + 4x - 1$

find $f(0), f(1), f(-1), f(-3), f(4), f(0.5)$

$f(0) = 2\times 0^2 + 4\times 0 - 1$

$= 0 + 0 - 1$

$= -1$

$f(1) = 2\times 1^2 + 4\times 1 - 1$

$= 2 + 4 - 1$

$= 6 - 1$

$= 5$

$f(4) = 2\times(4)^2 + 4\times 4 - 1$

$= 2\times 16 + 16 - 1$

$= 32 + 16 - 1$

$= 48 - 1$

$= 47$

$f(0.5) = 2\times(0.5)^2 + 4\times 0.5 - 1$

$= 2\times .25 + .2.0 - 1$

$= .5 + 1 = 1.5$

$f(-3) = 2\times(-3)^2 + 4\times -3 - 1$

$= 2\times 9 + -12 - 1$

$= 18 - 12 - 1$

$= 18 - 13$

$= 5$

$f(-1) = 2\times(-1)^2 + 4\times -1 - 1$

$= 2 - 4 - 1$

$= 2 - 5$

$= -3$

$16\times 2 = 32$

$32 + 16 = 48$

2) $y = \sin x$

x	0	$\pi/4$	π	$3\pi/2$	2π
y	0	$1/\sqrt{2}$	0	-1	0

3) $y = 2\cos x$

x : 0 $\pi/6$ $\pi/4$ $\pi/2$ π

y :

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4) $f(x) = \sin 2x$

find $f(0)$, $f(\pi/4)$, $f(\pi/2)$, $f(\pi/8)$

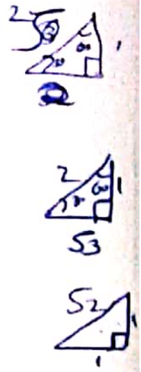
5) $f(x) = x^3 + 2x^2 + 1$

find $f(-1)$, $f(-2)$, $f(3)$

ANSWERS

3) $y = 2\cos x$

x :	0	$\pi/6$	$\pi/4$	$\pi/2$	π
y :	2	$\sqrt{3}$	$2/\sqrt{2}$	0	-2



$y = 2\cos x$
$= 2\cos 0$
$= 2 \times 1$
$= 2$

$y = 2\cos 30$
$= \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}}$
$= \sqrt{3}$

$y = 2\cos 45$
$= 2 \cdot \frac{1}{\sqrt{2}}$
$= 1$ [illegible]
$= \frac{2}{\sqrt{2}}$

$y = 2\cos 90$
$= 2 \times 0$
$= 0$

$y = 2\cos 180$
$= 2 \times -1$
$= -2$

4) $f(x) = \sin 2x$

$f(0) = \sin(2 \times 0)$
$= 0$

$f(\pi/4) = \sin \cancel{2} \times \pi/4$
$= \sin \pi/2$
$= \sin 90$
$= 1$

$f(\pi/2) = \sin \cancel{2} \times \pi/\cancel{2}$
$= \sin \pi$
$= 0$

$f(\pi/8) = \sin 2 \times \pi/\cancel{8}_4$
$= \sin \pi/4$
$= \sin 45$
$= 1/\sqrt{2}$

x	0	$\pi/4$	$\pi/2$	$\pi/8$
y	0	1	0	$1/\sqrt{2}$

5) $f(x) = x^3 + 2x^2 + 1$

$f(-1) = (-1)^3 + 2\times(-1)^2 + 1$

$= -1 + 2 + 1$

$= 2$

$f(-2) = (-2)^3 + 2\times(-2)^2 + 1$

$= -8 + 2\times 4 + 1$

$= \cancel{-8} + \cancel{8} + 1$

$= 1$

$3\times 3\times 3 = 9\times 3 = 2$

$f(3) = (3)^3 + 2\times(3)^2 + 1$

$= 27 + 2\times 9 + 1$

$= 27 + 18 + 1$

$= 46$

x	-1	-2	3
y	2	1	46

12/7/2019

LIMITS

- given a function $y=x+2$

as x tends to 1 the function $y=x+2$ tends to 3 as follow

x	0	0.5	0.8	0.9	0.995	$0.99999 \to 1$
y	2	2.5	2.8	2.9	2.995	$2.99999 \to 3$

then,

we say that; limit of the function $y=x+2$, as x tends to 1 is 3

or

$$\lim_{x \to 1} x+2 = 3$$

or

$$\lim_{x \to 1} y = 3$$

* By the Notation

$$\lim_{x \to a} f(x) = l$$

we mean that; ~~lim~~ As $x \to a$, $f(x) \to l$

Methods for finding limit;

a) limit by Substitution

eg:- i) $\lim_{x \to 2} 2x+3 = ?$

$= 2 \times 2 + 3$

$= 4+3$

$= 7$//

2) $\lim_{x \to 1} x^2 + 2x + 5 = 1^2 + 2 \times 1 + 5$

$= 1 + 2 + 5$

$= 8$

3) $\lim_{x \to 2} 2x^3 + 4x^2 + 1 = 2 \times 2^3 + 4 \times 2^2 + 1$

$= 2 \times 8 + 4 \times 4 + 1$

$= 16 + 16 + 1$

$= 33$

4) $\lim_{x \to -1} x^3 + 2x^2 - 9x + 1 = (-1)^3 + 2 \times (-1)^2 - 9 \times -1 + 1$

$= -1 + 2 + 9 + 1$

$= 12 - 1$

$= 11$

5) $\lim_{x \to 2} \frac{2x+3}{3x-1} = \frac{2 \times 2 + 3}{3 \times 2 - 1} = \frac{4+3}{6-1} = \frac{7}{5}$

6) $\lim_{x \to \pi/2} \sin x = \sin(\pi/2)$

$= 1$

7) $\lim_{x \to \pi/4} 3 \sin 2x = 3 \sin \cancel{2} \times \pi/\cancel{4}_2$

$= 3 \sin \pi/2$

$= 3 \times 1$

$= 3$

8) $\lim_{x \to \pi/4} \sin x + \cos x = \sin \pi/4 + \cos \pi/4$

$= 1/\sqrt{2} + 1/\sqrt{2}$

$= \frac{\sqrt{2} + \sqrt{2}}{\cancel{\sqrt{2}} \times \sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$= \sqrt{2}$

9) $\lim_{x \to 0} 2\cos x$

$= 2 \times 1$

$= 2$

b) Limid by factorization.

1) find dhe limit

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 3x + 2}$$

$\lim_{x \to 2}$

(To factorize $x^2 - 5x + 6$ we have to find 2 no's whose sum is -5 and pdt is $+6$, the 2 no's are -2 and -3, then the factors will be $(x-2)(x-3)$)

$x^2 - 5x + 6 = (x-2)(x-3)$

$-2 \times -3 = 6$

$-2 + -3 = -5$

$x^2 - 3x + 2 = (x-2)(x-1)$

$-2 + -1 = 3$

$-2 \times -1 = 2$

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 3x + 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x-1)}$$

$$= \lim_{x \to 2} \frac{x-3}{x-1} = \frac{2-3}{2-1} = -1$$

2) $\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 - 6x + 5}$

-1×-3

$-1 - 5 = -6$

$1 \times 5 = 5$

$$= \lim_{x \to 1} \frac{(x-1)(x-3)}{(x-1)(x-5)}$$

$x^2 - 4x + 3 = (x-1)(x-3)$

$x^2 - 6x + 5 = (x-1)(x-5)$

$$\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 - 6x + 5} = \lim_{x \to 1} \frac{\cancel{(x-1)}\,(x-3)}{\cancel{(x-1)}\,(x-5)}$$

$$= \lim_{x \to 1} \frac{(x-3)}{(x-5)}$$

$$= \frac{1-3}{1-5}$$

$$= \frac{-2}{-4} = \frac{1}{2}$$

3) $$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 + 2x - 8}$$

1×6

$(x-2)\,(x+3)$

$-2 \times 3 = 6$

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 + 2x - 8}$$

$$x^2 + x - 6 = (x-2)\,(x+3)$$

$$x^2 + 2x - 8 = (x+4)\,(x-2)$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}\,(x+3)}{(x+4)\,\cancel{(x-2)}}$$

$$\lim_{x \to 2} \frac{(x+3)}{(x+4)}$$

$$\frac{2+3}{2+4} = \frac{5}{6}$$

$$\lim_{x \to -2} \frac{x^2+5x+6}{x^2+x-2}$$

$$x^2+5x+6 = (x+3)(x+2)$$

$$x^2+x-2 = (x+2)(x-1)$$

$$\lim_{x \to -2} \frac{x^2+5x+6}{x^2+x-2} = \lim_{x \to -2} \frac{(x+3)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$$

$$= \lim_{x \to -2} \frac{x+3}{x-1}$$

$$= \frac{-2+3}{-2-1}$$

$$= \frac{1}{-3}$$

$$= -1/3$$

- $$\lim_{x \to -3} \frac{x^2+2x-3}{x^2+5x+6}$$

$$(x^2+2x-3) = (x+3)(x-1)$$

$$(x^2+5x+6) = (x+3)(x+2)$$

$$\lim_{x \to -3} \frac{x^2+2x-3}{x^2+5x+6} = \lim_{x \to -3} \frac{\cancel{(x+3)}(x-1)}{\cancel{(x+3)}(x+2)}$$

$$= \lim_{x \to -3} \frac{(x-1)}{x+2}$$

$$= \frac{-3-1}{-3+2}$$

$$= \frac{-4}{-1}$$

$$= \frac{4}{1}$$

$$= \underline{\underline{4}}$$

- $$\lim_{x \to 2/3} \frac{3x-2}{9x^2-4}$$

$$\lim_{x \to 2/3} \frac{3x-2}{9x^2-4}$$

$$= \frac{(3 \times 2/3) - 2}{9 \times \left(\frac{2}{3}\right)^2 - 4}$$

$$= \frac{\frac{6}{3} - 2}{\left(9 \times \frac{4}{9}\right) - 4}$$

$$= \frac{2-2}{4-4}$$

$$9x^2 - 2^2 = (3x)^2 - 2^2$$

$$= (3x+2)(3x-2)$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$= \lim_{x \to 2/3} \frac{\cancel{(3x-2)}}{(3x+2)\cancel{(3x-2)}}$$

$$= \lim_{x \to 2/3} \frac{1}{3x+2}$$

$$= \frac{1}{\cancel{3} \times 2/\cancel{3} + 2}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4} //$$

C) Algebraic Limit

a → any real no

n → any rational no

$$\boxed{\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1} \quad a \in R,\ n \in Q}$$

eg:- 1) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

$= \lim_{x \to 2} \frac{x^3 - 2^3}{x - 2} = 3 \times 2^{3-1} = 3 \times 2^2 = 3 \times 4$

$= 12$

$n = 3,\ a = 2$

2) $\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$

$= \lim_{x \to 3} \frac{x^4 - 3^4}{x - 3}$

$= \cancel{\lim_{x \to 3}} = 4 \times 3^{4-1}$

$= 4 \times 3^3$

$= 4 \times 27$

$= 108$

3) $\lim_{x \to 2} \frac{x^5 - 32}{x - 2}$

2^5

$2 \times 2 \times 2 \times 2 \times 2$

$4 \times 4 = 16 \times 2 = 32$

$= \lim_{x \to 2} \frac{x^5 - 2^5}{x - 2}$

$16 \times 5 = 80$

$= 5 \times 2^{5-1}$

$= 5 \times 2^4 = 5 \times 16 = 80$

• $\lim_{x \to 4} \dfrac{x^4 - 256}{x - 4}$

4^4

$4 \times 4 \times 4 \times 4$

16 ✓
16

$\lim_{x \to 4} \dfrac{x^4 - 4^4}{x - 4}$

$= 4 \times 4^{4-1}$

$= 4 \times 4^3$

$= 4 \times 16 \times 4$

$= 16 \times 16$

$= 256$

Q19 $\lim_{x \to -2} \dfrac{x^5 + 32}{x + 2}$

$\lim_{x \to a} \dfrac{x^n - a^n}{x - a}$

$\lim_{x \to -2} = \dfrac{x^5 - (-2)^5}{x + 2}$

$= \lim_{x \to -2} \dfrac{x^5 - (-2)^5}{x - -2}$

$n = 5$

$a = -2$

$= 5 \times (-2)^{5-1}$

$= 5 \times (-2)^4$

$= 5 \times 16$

$= 80$

• $\lim_{x \to -3} \dfrac{x^3+27}{x+3}$

$\lim_{x \to -3} \dfrac{x^3-(-3)^3}{x--3}$

$n = 3$

$a = -3$

$= n \times a^{n-1}$

$= 3 \times (-3)^{3-1}$

$= 3 \times (-3)^2$

$= 3 \times 9$

$= 27$

* $\sqrt{x} = x^{1/2}$

$x\sqrt{x} = x \, x^{1/2}$

$= x^{1+1/2}$

$= x^{3/2}$

$3\sqrt{3} = 3^{3/2}$

• $\lim_{x \to 2} \dfrac{x\sqrt{x} - 2\sqrt{2}}{x-2}$

$= \lim_{x \to 2} = \dfrac{x \, x^{1/2} - 2^1 . 2^{1/2}}{x-2}$

$= \lim_{x \to 2} = \dfrac{x^{3/2} - 2^{3/2}}{x-2}$

$n = 3/2$

$a = 2$

$= n \times a^{n-1} = 3/2 \times 2^{3/2-1} = 3/2 \times 2^{3/2-1}$

$= \frac{3}{2} \cdot 2^{1/2}$

$= \frac{3\sqrt{2}}{2}$

$= \frac{3\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\sqrt{2}}$

$= \frac{3}{\sqrt{2}}$

$\lim_{x \to 3} \frac{x\sqrt{x} - 3\sqrt{3}}{x-3}$

$= \lim_{x \to 3} \frac{x \cdot x^{1/2} - 3 \cdot 3^{1/2}}{x-3}$

$= \lim_{x \to 2} \frac{x^{3/2} - 3^{3/2}}{x-3}$

$= n \times a^{n-1}$

$= 3/2 \times 3^{3/2 - 1}$

$= 3/2 \times 3^{1/2}$

$= \frac{3\sqrt{3}}{2}$

$\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8}$

$\lim_{x \to 2} \frac{x^5 - 2^5}{x^3 - 2^3}$

$(a^m)^n = a^{m+n}$

dividing numerator and denominator by $(x-2)$
÷ both with $(x-2)$

$\lim_{x \to 2} \frac{x^5 - 2^5/(x-2)}{x^3 - 2^3/(x-2)}$

$$\frac{\lim_{x \to 2} \frac{x^5 - 2^5}{x-2}}{\lim_{x \to 2} \frac{x^3 - 2^3}{x-2}} = \frac{5 \cdot 2^{5-1}}{3 \cdot 2^{3-1}}$$

$$= \cancel{\frac{5 \cdot 2^{4}}{3 \cdot 2^{2}}} \qquad \frac{5 \cdot 2^{4\,2}}{3 \cdot 2^{x}}$$

$$= \cancel{5 \times 16}$$

$$= \frac{5 \cdot 2^2}{3 \cdot \cancel{2}}$$

$$= \frac{5 \cdot 4}{3} = \frac{20}{3}$$

- $$\lim_{x \to 3} \frac{x^4 - 81}{x^3 - 27}$$

$$\lim_{x \to 3} \frac{x^4 - 3^4}{x^3 - 3^3}$$

÷ numerator and denominator by $x-3$

$$= \lim_{x \to 3} \frac{x^4 - 3^4/(x-3)}{x^3 - 3^3/(x-3)}$$

$$= \frac{\lim_{x \to 3} \frac{x^4 - 3^4}{x-3}}{\lim_{x \to 3} \frac{x^3 - 3^3}{x-3}}$$

$$= \frac{4 \times 3^{4-1}}{3 \times 3^{3-1}} = \frac{4 \times 3^{\cancel{3}\,1}}{3 \times \cancel{3}^{\cancel{2}}}$$

$$= \frac{4 \times 3}{3} = 12/3 = 4$$

• $\lim_{x \to 5} \dfrac{x^4 - 625}{x^2 - 25}$

$= \lim_{x \to 5} \dfrac{x^4 - 5^4}{x^2 - 5^2}$

divide numerator and denominator by $(x-5)$

$= \lim_{x \to 5} \dfrac{x^4 - 5^4/(x-5)}{x^2 - 5^2/(x-5)}$

$= \dfrac{\lim_{x \to 5} \dfrac{x^4 - 5^4}{(x-5)}}{\lim_{x \to 5} \dfrac{x^2 - 5^2}{(x-5)}}$

$= \dfrac{4 \times 5^{4-1}}{2 \times 5^{2-1}}$

$= \dfrac{4 \times 5^{\cancel{3}\,2}}{2 \times \cancel{5}}$

$= \dfrac{4 \times 5^2}{2}$

$= \dfrac{\cancel{4}^{\,2} \times 25}{\cancel{2}}$

$= 2 \times 25$

$= 50$

imp

• $\lim_{x \to 0} \dfrac{\sqrt{1+x} - 1}{x}$

imp

- $$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$$

$$= \lim_{x\to 0} \frac{(1+x)^{1/2} - 1^{1/2}}{(1+x)-1}$$

~~1+x-1~~ $= x$

$1+x \to 1$ when $x=0$

$1+0 \to 1$

$x \to 0 \Rightarrow 1+x \to 1$

$$= \lim_{(1+x)\to 1} \frac{(1+x)^{1/2} - 1^{1/2}}{(1+x)-1}$$ which is of the for

$$\lim_{y\to a} \frac{y^n - a^n}{y-a} = n \times a^{n-1}$$ where $y = 1+x$, $n = 1/2$ and $a = 1$

$$= 1/2 \cdot 1^{1/2 - 1}$$

$$= 1/2 \cdot 1$$

$$= \underline{\underline{1/2}}$$

- $$\lim_{t\to 0} \frac{(t+2)^2 - 4}{t}$$

$$= \lim_{t\to 0} \frac{(t+2)^2 - 2^2}{(t+2)-2}$$

$t \to 0 \Rightarrow t+2 \to 2$

$$= \lim_{(t+2)\to 2} \frac{(t+2)^2 - 2^2}{(t+2)-2}$$ which is of the form

$$\lim_{y\to a} \frac{y^n - a^n}{y-a} = n \times a^{n-1}$$, where $y = t+2$ $n = 2$ $a = 2$

$= 2 \times 2^{2-1}$

$= 2 \times 2$

$= 4$

1). $\lim_{t \to 0} \frac{(t+3)^2 - 9}{t}$

2). $\lim_{x \to 3} \frac{x^5 - 243}{x^3 - 27}$

3). $\lim_{x \to 5} \frac{x\sqrt{x} - 5\sqrt{5}}{x - 5}$

1) $\lim_{t \to 0} \frac{(t+3)^2 - 9}{t}$

$= \lim_{t \to 0} \frac{(t+3)^2 - 3^2}{(t+3) - 3}$

$= t \to 0 \Rightarrow t+3 \to 3$

$= \lim_{(t+3) \to 3} \frac{(t+3)^2 - 3^2}{(t+3) - 3}$

$\lim_{y \to a} \frac{y^n - a^n}{y - a} = n \times a^{n-1}$, where $y = t+3$, $n = 2$, $a = 3$

$n \times a^{n-1} = 2 \times 3^{2-1}$

$= 2 \times 3^1$

$= 6$

2) $\lim_{x \to 3} \frac{x^5 - 243}{x^3 - 27}$

$\lim_{x \to 3} \frac{x^5 - 3^5}{x^3 - 3^3}$

$$= \lim_{x \to 3} \frac{x^5 - 3^5/(x-3)}{x^3 - 3^3/(x-3)}$$

$$= \lim_{x \to 3} \frac{\frac{x^5 - 3^5}{x-3}}{\frac{x^3 - 3^3}{x-3}} = \frac{5 \cdot 3^{5-1}}{3 \cdot 3^{3-1}} = \frac{5 \cdot 3^{\cancel{4}\,2}}{3 \cdot \cancel{3^2}} = \frac{5 \times 3^2}{3}$$

$$= \frac{5 \times 9}{3}$$

$$= \frac{45}{3}$$

$$= 15$$

3) $$\lim_{x \to 5} \frac{x\sqrt{x} - 5\sqrt{5}}{x-5}$$

we have $x\sqrt{x} = x \cdot x^{1/2}$

$= x^{1+1/2} = x^{3/2}$

$5\sqrt{5} = 5 \cdot 5^{1/2}$

$= 5^{1+1/2} = 5^{3/2}$

$$\lim_{x \to 5} \frac{x^{3/2} - 5^{3/2}}{x-5}$$

$n = 3/2$

$a = 5$

$= n \times a^{n-1}$

$= 3/2 \times 5^{3/2-1}$

$= 3/2 \times 5^{1/2}$

$$= \frac{3\sqrt{5}}{2}$$

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d) limit involving trigonometric functions.

$$\boxed{\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1}$$

eg:- 1) find $\lim_{\theta \to 0} \frac{\sin 2\theta}{\theta}$

$$= \lim_{\theta \to 0} \frac{2\sin 2\theta}{2\theta}$$

$$= 2 \times \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$= 2 \times 1$$

$$= 2$$

$\theta \to 0$
same to
$2\theta \to 0$
$2 \times 0 \to 0$

2) $\lim_{\theta \to 0} \frac{\sin 5\theta}{\theta}$

$$= \lim_{\theta \to 0} \frac{5 \cdot \sin 5\theta}{5\theta}$$

$$= 5 \times \lim_{\theta \to 0} \frac{\sin 5\theta}{5\theta}$$

$$= 5 \times 1 = 5$$

3) $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta}$

$$= \lim_{\theta \to 0} m \times \frac{\sin m\theta}{m\theta}$$

$$= m \times \lim_{\theta \to 0} \frac{\sin m\theta}{m\theta}$$

$$= m \times 1 = m$$

$$\boxed{\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = m.}$$

4) $\lim_{\theta \to 0} \frac{2\sin 4\theta}{5\theta}$

$$= \lim_{\theta \to 0} \frac{2\sin 4\theta}{5\theta}$$

$$= \frac{2}{5} \lim_{\theta \to 0} \frac{\sin 4\theta}{\theta} = \frac{2}{5} \cancel{\times 1}$$

$$= \frac{2}{5} \times \lim_{\theta \to 0} \frac{4\sin 4\theta}{4\theta} = \frac{2}{5} \times 4 \times \lim_{\theta \to 0} \frac{\sin 4\theta}{4\theta}$$

$$= \frac{8}{5} \times 1 = 8/5$$

5) $\lim_{\theta \to 0} \frac{3\sin 2\theta}{8\theta}$

$= 3 \lim_{\theta \to 0} \frac{\sin 2\theta}{8\theta}$

$= \frac{3}{8} \lim_{\theta \to 0} \frac{\sin 2\theta}{\theta}$

$= \frac{3}{8} \lim_{\theta \to 0} \frac{2\sin 2\theta}{2\theta}$

$= \frac{3}{8} \times 2 \lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta}$

$= \frac{3}{8} \times 2 \times 1$

$= \frac{6}{8} = 3/4$ //

6) $\lim_{\theta \to 0} \frac{2\sin\theta + \sin 3\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{2\sin\theta}{\theta} + \lim_{\theta \to 0} \frac{\sin 3\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{2\times 2\sin\theta}{2\theta} + \lim_{\theta \to 0} \frac{3\times\sin 3\theta}{3\theta}$

$= 2\times 1 + 3\times 1$

$= 2+3$

$= 5$ //

$\boxed{\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}}$

7) $\lim_{\theta \to 0} \frac{3\sin 4\theta + 5\sin 7\theta}{8\theta}$

$= \lim_{\theta \to 0} \frac{3\sin 4\theta}{8\theta} + \lim_{\theta \to 0} \frac{5\sin 7\theta}{8\theta}$

$= 3/8 \lim_{\theta \to 0} \frac{4\times\sin 4\theta}{4\theta} + \frac{5}{8} \lim_{\theta \to 0} \frac{7\times\sin 7\theta}{7\theta}$

$= 3/8 \times 4 \times 1 + 5/8 \times 7 \times 1$

$= \frac{12}{8} + \frac{35}{8}$

$= \frac{6}{4} + \frac{35}{8}$

$= 3/2 + \frac{35}{8} = \frac{24+70}{16}$

$= \frac{94}{16} = 47/8$

• S.T $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

ie $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = \lim_{\theta \to 0} \frac{\frac{\sin\theta}{\cos\theta}}{\theta}$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta \times \cos\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{1}{\cos\theta}$$

$$= 1 \times \lim_{\theta \to 0} \frac{1}{\cos\theta}$$

$$= 1 \times \frac{1}{\cos 0} = 1 \times \frac{1}{\cos 0} = 1 \times 1/1 = 1$$

- $$\lim_{\theta \to 0} \frac{\sin 3\theta}{\sin 2\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin 3\theta/\theta}{\sin 2\theta/\theta}$$

$$= \frac{\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta}}{\lim_{\theta \to 0} \frac{\sin 2\theta}{\theta}}$$

$$= \frac{\lim_{\theta \to 0} \frac{3\sin 3\theta}{3\theta}}{\lim_{\theta \to 0} \frac{2\sin 2\theta}{2\theta}}$$

$$= \frac{3 \times 1}{2 \times 1} = 3/2$$

- $\lim_{\theta \to 0} \frac{\sin m\theta}{\sin n\theta}$, where m, n as constant

$$\lim_{\theta \to 0} \frac{\sin m\theta}{\sin n\theta} = \frac{\lim_{\theta \to 0} \frac{\sin m\theta}{\theta}}{\lim_{\theta \to 0} \frac{\sin n\theta}{\theta}} = \frac{m \times 1}{n \times 1} = m/n$$

- $$\lim_{\theta \to 0} \frac{\sin 2\theta \cdot \cos\theta}{\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin 2\theta}{\theta} \times \lim_{\theta \to 0} \cos\theta$$

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$\lim_{\theta \to} = 2 \times 1 \times \cos 0$

$= 2 \times 1$

$= 2$

$\lim_{\theta \to 0} \frac{\sin(\theta \times \theta)}{\theta \times \theta}$

$\lim_{\theta \to 0} \frac{\sin(}{(}$

- $\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta^2}$

$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \times \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$

$= 1 \times 1 = 1$

$1 - \cos 2\theta = 2\sin^2\theta$

$1 + \cos 2\theta = 2\cos^2\theta$

$\cos^2\theta$

- $\lim_{\theta \to 0} \frac{1 - \cos 2\theta}{\theta^2}$

$= \lim_{\theta \to 0} \frac{2\sin^2\theta}{\theta^2}$

$$\boxed{\begin{aligned} 1 - \cos 2\theta &= 2\sin^2\theta \\ 1 + \cos 2\theta &= 2\cos^2\theta \end{aligned}}$$

we know that $\lim_{\theta \to 0} \frac{\sin^2\theta}{\theta^2} = 1$

$= 2 \times 1 = 2$

$\cos x = \sin(\pi/2 + x)$

$\cos x = \sin(\pi/2 - x)$

$\sin(n\pi/2 \pm x)$

$= \pm \cos x$ (if n is odd)

$= \pm \sin x$, if n is even

- $\lim_{x \to \pi/2} \frac{\cos x}{\pi/2 - x}$

$= \lim_{x \to \pi/2} \frac{\sin(\pi/2 - x)}{(\pi/2 - x)}$

as $x \to \pi/2$, $\pi/2 - x = 0$

$\pi/2 - x \to 0$

$= \lim_{(\pi/2 - x) \to 0} \frac{\sin(\pi/2 - x)}{(\pi/2 - x)}$

$= 1$

• $\lim_{x \to -\pi/2} \dfrac{\cos x}{\pi/2 + x}$

we know that $\cos x = \sin(\pi/2 + x)$

$$= \lim_{x \to -\pi/2} \frac{\sin(\pi/2 + x)}{(\pi/2 + x)}$$

$$= \lim_{(\pi/2 + x) \to 0} \frac{\sin(\pi/2 + x)}{(\pi/2 + x)}$$

$x \to \pi/2$

$\pi/2 + x \to 0$

$= 1$

• e) Limit involving infinity.

$$\boxed{\lim_{x \to \infty} \frac{1}{x} = 0}$$

||ly we get

$$\lim_{x \to \infty} \frac{1}{x^2} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^3} = 0$$

$$\lim_{x \to \infty} \frac{1}{5x} = 0$$

1) $\lim_{x \to \infty} 2 + \dfrac{3}{x}$

$$= \lim_{x \to \infty} 2 + 3 \lim_{x \to \infty} \frac{1}{x}$$

$$= 2 + 3 \times 0$$

$$= 2$$

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· find $\lim_{x \to \alpha} 2 + \frac{3}{x} - \frac{5}{x^2}$

· $\lim_{x \to \alpha} 2 + \lim_{x \to \alpha} \frac{3}{x} - \lim_{x \to \alpha} \frac{5}{x^2}$

$$= 2 + 3 \lim_{x \to \alpha} \frac{1}{x} - 5 \times \lim_{x \to \alpha} \frac{1}{x^2}$$

$$= 2 + 3 \times 0 - 5 \times 0 = 2$$

· find $\lim_{x \to \alpha} \frac{3x^2 + 5x - 7}{2x^2 - 3x + 2}$

$$= \lim_{x \to \alpha} \frac{\cancel{x^2}\left(3 + \frac{5}{x} - \frac{7}{x^2}\right)}{\cancel{x^2}\left(2 + \frac{3}{x} + \frac{2}{x^2}\right)}$$

$$= \frac{\lim_{x \to \alpha} 3 + \frac{5}{x} - \frac{7}{x^2}}{\lim_{x \to \alpha} 2 - \frac{3}{x} + \frac{2}{x^2}} = \frac{3 + 0 - 0}{2 - 0 + 0} = \frac{3}{2}$$

· $\lim_{x \to \alpha} \frac{7x^2 - 5x + 1}{3x^2 + 5}$

$$\frac{\lim_{x \to \alpha} \cancel{x^2}\left(7 - \frac{5}{x} + \frac{1}{x^2}\right)}{\lim_{x \to \alpha} \cancel{x^2}\left(3 + \frac{5}{x^2}\right)}$$

$$= \frac{\lim_{x \to \alpha} 7 - \frac{5}{x} + \frac{1}{x^2}}{\lim_{x \to \alpha} 3 \cdot \frac{5}{x^2}}$$

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$= 7/3$ //

- $$\lim_{x \to \alpha} \frac{2 + 3x - 7x^2}{5 + x^2}$$

$$\frac{\lim_{x \to \alpha} \cancel{x^2}\left(-7 + \frac{3\cancel{x}}{\cancel{x}} + \frac{2}{x^2}\right)}{\lim_{x \to \alpha} \cancel{x^2}\left(1 + \frac{5}{x^2}\right)}$$

$$= \frac{\lim_{x \to \alpha} -7 + \frac{3}{x} + \frac{2}{x^2}}{\lim_{x \to \alpha} 1 + \frac{5}{x^2}}$$

$$= -7/1$$

$$= -7 //$$

- $$\lim_{x \to \alpha} \frac{2x^2 + 3x - 4}{x^3 - 4x^2 + 1}$$

$$\cancel{\frac{\lim_{x \to \alpha} x^3\left(\frac{2}{x} + \frac{3}{x} - \frac{4}{x^2}\right)}{\lim_{x \to \alpha} x^3\left(2x + \frac{4}{x^2}\right)}}$$

$$= \frac{\lim_{x \to \alpha} \cancel{x^3}\left(\frac{2}{\cancel{x}} + \frac{3}{x^2} - \frac{4}{x^3}\right)}{\lim_{x \to \alpha} \cancel{x^3}\left(1 - \frac{4}{x} + \frac{1}{x^3}\right)}$$

$$= \frac{\lim_{x \to \alpha} \frac{2}{x} + \frac{3}{x^2} - \frac{4}{x^3}}{\lim_{x \to \alpha} 1 - \frac{4}{x} + \frac{1}{x^3}} = \frac{0 + 0 - 0}{1} = \frac{0}{1} = 0 //$$

- $\lim\limits_{x \to \alpha} \dfrac{2x^3+4x-1}{3x^2+5}$

$$= \frac{\lim\limits_{x\to\alpha} \cancel{x^3}\left(2+\frac{4}{x^2}-\frac{1}{x^3}\right)}{\lim\limits_{x\to\alpha} \cancel{x^3}\left(\frac{3}{x}+\frac{5}{x^3}\right)}$$

$$= \frac{\lim\limits_{x\to\alpha} 2+\frac{4}{x^2}-\frac{1}{x^3}}{\lim\limits_{x\to\alpha} \frac{3}{x}+\frac{5}{x^3}}$$

$$= \frac{2+0-0}{0+0} = \frac{2}{0} = \text{infinity}$$

$= \alpha$

$\lim\limits_{x\to\alpha}$ Denominat[illegible]

degree ↑

answer $= 0$

$\frac{n}{d}$

Just opposite answer [illegible]

- $\lim\limits_{x\to\alpha} \dfrac{\sqrt{x^2+2x+3}}{2x-3}$

$$= \lim_{x\to\alpha} \cancel{\sqrt{x^2}} \left(1+\frac{2}{x}+3\right.$$

$$= \lim_{x\to\alpha} \frac{\sqrt{x^2\left(1+\frac{2}{x}+\frac{3}{x^2}\right)}}{x\left(2-3/x\right)}$$

$$= \lim_{x\to\alpha} \frac{\cancel{x}\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}}{\cancel{x}\left(2-3/x\right)}$$

$$= \frac{\lim\limits_{x\to\alpha}\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}}{\lim\limits_{x\to\alpha} 2-3/x}$$

$$= \frac{\sqrt{1+0+0}}{2-0}$$

$\sqrt{x^2} = x$

[illegible]

$\sqrt{x^2}$ [illegible]

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$= \frac{\sqrt{1}}{2} = \frac{1}{2}$ //

• $\lim_{x \to 3\pi/2} \frac{\cos x}{3\pi/2 - x}$

IIIrd quad - only ? +ve

$\sin(3\pi/2 - x) = -\cos x$

ie $\cos x = -\sin\left(\frac{3\pi}{2} - x\right)$

$\lim_{x \to 3\pi/2} \frac{-\sin(3\pi/2 - x)}{(3\pi/2 - x)}$

minuze is a constant

$= - \lim_{x \to 3\pi/2} \frac{\sin(3\pi/2 - x)}{3\pi/2 - x}$

$x \to 3\pi/2$, $3\pi/2 - x \to 0$

$= - \lim_{3\pi/2 - x \to 0} \frac{\sin(3\pi/2 - x)}{(3\pi/2 - x)}$ .... 1 //

$= -1$ //

1) • $\lim_{x \to \alpha} \frac{2x^2 + 4 - 1}{x^3 + 2x - 5}$

2) • $\lim_{x \to \alpha} \frac{5x^2 + 1}{2x^2 - x + 2}$

$= \frac{\cancel{\lim_{x \to \alpha}} \; x^2\left(2 + \frac{4}{x^2} - \frac{1}{x^2}\right)}{\lim_{x \to \alpha} \; x^2 \left(\cancel{\alpha}\right.}$

• $\lim_{x \to \alpha} \dfrac{2x^2+4-1}{x^3+2x-5}$

$$= \frac{\lim_{x \to \alpha} \cancel{x^3}\left(\frac{2}{x} + \frac{4}{x^3} - \frac{1}{x^3}\right)}{\lim_{x \to \alpha} \cancel{x^3}\left(1+\frac{2}{x^2} - \frac{5}{x^3}\right)}$$

$$= \frac{\lim_{x \to \alpha} \frac{2}{x} + \frac{4}{x^3} - \frac{1}{x^3}}{\lim_{x \to \alpha} 1+\frac{2}{x^2} - \frac{5}{x^3}}$$

$\dfrac{2x^{\cancel{3}2}}{\cancel{x}}$

$$= \frac{0+0+0}{1+0+0} = \frac{0}{1} = \underline{\underline{0}}$$

• $\lim_{x \to \alpha} \dfrac{5x^2+1}{2x^2-x+2}$

$\dfrac{x}{x^3}$

$= \cancel{\lim_{x \to \alpha} x^3\left(\frac{2}{x^2} + 0 - \frac{1}{x}\right.}$

$$= \lim_{x \to \alpha} \frac{\cancel{x^2}(5+1/x)}{\cancel{x^2}(2-1/x+2/x^2)}$$

$$= \frac{\lim_{x \to \alpha} 5+\frac{1}{x}}{\lim_{x \to \alpha} 2-1/x+\frac{2}{x^2}} = \frac{5+0}{2-0+0} = 5/2$$

24/7/2019

DIFFERENTIATION

given a function $y = f(x)$, let Δy be the increment in 'y' corresponding to an increment Δx in x

Then, the ratio $\frac{\Delta y}{\Delta x}$ is known as incremental ratio and it gives the average rate of change of y with respect to x. The instantaneous rate of change of y with respect to x can be found using $\frac{dy}{dx}$

$\frac{dy}{dx}$ ie, derivative of y with respect to x.

$\frac{dy}{dx}$ is given by the following formula

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$$

The process of finding, derivative of a given function is known as differenciation

Method of first Principle

given a function $y = f(x)$, let 'Δy' be the incremend in 'y', corresponding to an increment Δx in x, then, we have

$$y + \Delta y = f(x + \Delta x)$$

ie $$\Delta y = f(x + \Delta x) - y$$

$$\Rightarrow \Delta y = f(x + \Delta x) - f(x)$$

we know that $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$$\boxed{\text{ie, } \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

The process of finding derivative of a given func. using the above formula is known as method of 1st principle.

- find derivative of $y = x^n$ find using the method of 1st principle.

<u>proof :-</u>

$y = x^n$

$f(x) = x^n$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

$f(x+\Delta x) = (x+\Delta x)^n$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

16/7/2019.

$$\lim_{y \to a} \frac{y^n - a^n}{y-a} = n \times a^{n-1}$$

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{(x+\Delta x) - x}$$

AS $\Delta x \to 0$, $x + \Delta x \to x$

$$\lim_{x+\Delta x \to x} \frac{(x+\Delta x)^n - x^n}{(x+\Delta x) - x} = n \times x^{n-1}$$

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

Where 'n' is rational numbers.

1) $\frac{d}{dx} x = 1x^0 = 1$

2) $\frac{d}{dx} x^2 = 2x$

3) $\frac{d}{dx} x^3 = 3x^2$

db 4) $\frac{d}{dx} x^{20} = 20x^{19}$

5) $\frac{d}{dx} k = 0$

$\frac{d}{dx} 3 = 0$

6) $\frac{d}{dx} \frac{1}{x} = \frac{d}{dx} x^{-1} = -1 \cdot x^{-1-1}$

$= -1 x^{-2}$

$= \frac{-1}{x^2}$

7) $\frac{d}{dx} \frac{1}{x^3} = \frac{d}{dx} x^{-3} = -3 \cdot x^{-3-1}$

$= -3x^{-4}$

$= -3 \frac{1}{x^4}$

db 8) $\frac{d}{dx} \frac{1}{x^n} = \frac{-n}{x^{n+1}}$

9) $\frac{d}{dx} \sqrt{x}$

$\frac{d}{dx} x^{1/2} = 1/2 \cdot x^{1/2 - 1}$

$= 1/2 \, x^{1/2}$

$= \frac{1}{2\sqrt{x}}$ //

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Result :-

1) $\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

2) $\frac{d}{dx}(f(x)-g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

3) $\frac{d}{dx}k\,f(x) = k\frac{d}{dx}f(x)$

→ find $\frac{d}{dx}(x^3+x^5)$

$= \frac{d}{dx}x^3 + \frac{d}{dx}x^5$

$= 3 \cdot x^2 + 5x^4$

→ $\frac{d}{dx}3x^7+5x$

$= \frac{d}{dx}3x^7 + \frac{d}{dx}5x$

$= 3\frac{d}{dx}x^7 + 5\frac{d}{dx}x$

$= 3\times 7x^6 + 5\times 1$

$= 21x^6 + 5$

→ $\frac{d}{dx}5x^{10}+5$

$= \frac{d}{dx}5x^{10} + \frac{d}{dx}5$

$= 5\frac{d}{dx}x^{10} + \frac{d}{dx}5$

$5\times 10x^9 + 0 = 50x^9$

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$\rightarrow \frac{d}{dx} \; 2\sqrt{x} + 3/x$

$$\frac{d}{dx} 2\sqrt{x} + \frac{d}{dx} \frac{3}{x}$$

$$= 2 \frac{d}{dx} \sqrt{x} + 3 \frac{d}{dx} \frac{1}{x}$$

$$= \cancel{2} \frac{1}{\cancel{2}\sqrt{x}} + 3 \times -\frac{1}{x^2}$$

$$= \frac{1}{\sqrt{x}} - 3 \times \frac{1}{x^2}$$

$$= \frac{1}{\sqrt{x}} - \frac{3}{x^2}$$

$\rightarrow$ find $\frac{dy}{dx}$ if $y = 7x^5 - 3x^2 - \frac{2}{x^2}$

$$= \frac{d}{dx}\left(7x^5 - 3x^2 - \frac{2}{x^2}\right)$$

$$= \frac{d}{dx} 7x^5 - \frac{d}{dx} 3x^2 - \frac{d}{dx} 2/x^2$$

$$= 7 \frac{d}{dx} x^5 - 3 \frac{d}{dx} x^2 - 2 \frac{d}{dx} \frac{1}{x^2}$$

$$= 7 \cdot 5x^4 - 3 \cdot 2x - 2 \cdot -2 \cdot \frac{1}{x^3}$$

$$= 35x^4 - 6x + \frac{4}{x^3}$$

→ find $\frac{d}{dx}$, $y = 5x^7 - \frac{3}{x} + 7$

$$= \frac{d}{dx} 5x^7 - \frac{d}{dx} \frac{3}{x} + \frac{d}{dx} 7$$

$$= 5 \frac{d}{dx} x^7 - 3 \frac{d}{dx} \frac{1}{x} + \frac{d}{dx} 7$$

$$= 5 \cdot 7x^6 - 3 \cdot \frac{-1}{x^2} + 0$$

$$= 35x^6 + \frac{3}{x^2}$$

→ $y = 20x^6 + \frac{4}{x} + 2\sqrt{x}$

$$= \frac{d}{dx}\left(20x^6 + \frac{4}{x} + 2\sqrt{x}\right)$$

$$= \frac{d}{dx} 20x^6 + \frac{d}{dx} \frac{4}{x} + \frac{d}{dx} 2\sqrt{x}$$

$$= 20 \frac{d}{dx} x^6 + 4 \frac{d}{dx} \frac{1}{x} + 2 \frac{d}{dx} \sqrt{x}$$

$$= 20 \times 6x^5 + 4 \frac{-1}{x^2} + \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x}}$$

$$= 120 \cdot x^5 - \frac{4}{x^2} + \frac{1}{\sqrt{x}}$$

$\rightarrow \quad y = 4x^{3/2} + 5x^7 + 10$

$$\frac{d}{dx}\left(4x^{3/2} + 5x^7 + 10\right)$$

$$\frac{d}{dx} 4x^{3/2} + \frac{d}{dx} 5x^7 + \frac{d}{dx} 10$$

$$= 4 \frac{d}{dx} x^{3/2} + 5 \frac{d}{dx} x^7 + \frac{d}{dx} 10$$

$$= \cancel{4} \cdot \frac{3}{2} x^{3/2 - 1} + 5 \cdot 7 x^6 + 0$$

$$= \cancel{4}\; 2 \cdot 3 x^{1/2} + 35x^6$$

$$= \underline{\underline{6\sqrt{x} + 35x^6}}$$

$$\frac{d}{dx}(k) = 0$$

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$$

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imp 17/12/2019 Find the derivative of $y = \sin x$ using the method of 1^{st} principle.

$$y = \sin x$$

$$f(x) = \sin x$$

$$\frac{dy}{dx} = ?$$

By the method of 1^{st} principle

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = \sin(x + \Delta x) \cancel{- \sin x}$$

$$f(x+\Delta x) - f(x) = \overbrace{\sin(x + \Delta x)}^{A} - \overbrace{\sin x}^{B}$$

$$\left(\sin A - \sin B = 2\cos\frac{A+B}{2} \sin\frac{A-B}{2}\right)$$

$\cancel{2\cos\frac{A+B}{2} = \sin A - \sin B}$

$2\cos\frac{A+B}{2} \sin\frac{A-B}{2}$

$= \sin A - \sin B$

$$= 2\cos\left(\frac{x + \Delta x + x}{2}\right) \cdot \sin\left(\frac{x + \Delta x - x}{2}\right)$$

$$= 2\cos\left(\frac{2x + \Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)$$

$$= 2\cos\left(\frac{\cancel{2}x}{\cancel{2}} + \frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)$$

$$= 2\cos\left(x + \frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)$$

Now $$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{2\cos\left(x + \frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \cos\left(x + \frac{\Delta x}{2}\right) \cdot \frac{\sin \frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

which is of the form $\frac{\sin\theta}{\theta}$

$$= \lim_{\Delta x \to 0} \cos\left(x + \frac{\Delta x}{2}\right) \cdot \lim_{\Delta x \to 0} \frac{\sin \frac{\Delta x}{2}}{\Delta x/2}$$

$$= \cos x + 0 \times 1$$

$$= \cos x \times 1$$

$$= \underline{\underline{\cos x}}$$

✓ find the derivative of $y = \cos x$, using the method 1st principle.

$y = \cos x$

$f(x) = \cos x$

$\frac{dy}{dx} = ?$

By the method of 1st principle

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$f(x + \Delta x) = \cos(x + \Delta x)$

$\cos A - \cos B = -2\sin\frac{A+B}{2} \cdot \sin\frac{A-B}{2}$

$f(x + \Delta x) - f(x) = \cos(x + \Delta x) - \cos x$

$\left(\cos A - \cos B = -2\sin\frac{A+B}{2} \cdot \sin\frac{A-B}{2}\right)$

$$= -2\sin\left(\frac{x + \Delta x + x}{2}\right) \cdot \sin\left(\frac{x + \Delta x - x}{2}\right)$$

$$= -2\sin\left(\frac{2x + \Delta x}{2}\right) \sin\left(\frac{\Delta x}{2}\right)$$

$$= -2\sin\left(x + \frac{\Delta x}{2}\right) \sin\left(\frac{\Delta x}{2}\right)$$

Now $$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{-2\sin\left(x + \frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)}{\Delta x}.$$

$$= \lim_{\Delta x \to 0} \frac{-\sin\left(x+\frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)}{\Delta x/2}$$

$$= \lim_{\Delta x \to 0} -\sin\left(\frac{x+\Delta x}{2}\right) \cdot \lim_{\Delta x \to 0} \frac{\sin \frac{\Delta x}{2}}{\Delta x/2}$$

$$= -\sin x \times 1$$

$$= \underline{\underline{-\sin x}}$$

- $\frac{dy}{dx} =$ find $\frac{dy}{dx}$ if $y = \underline{2\sin x + 3\cos x}$

$f(x) = 2\sin x + 3\cos x$

$$\frac{dy}{dx} = 2 \frac{d}{dx} \sin x + 3 \cdot \frac{d}{dx} \cos x$$

$$= 2 \times \cos x + 3 \times -\sin x$$

$$= 2\cos x + -3\sin x$$

$$= \underline{\underline{2\cos x}} - 3\sin x$$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \sin x = \cos x$

- $y = x^2 - \cos x$

$$\frac{dy}{dx} = \frac{d}{dx} x^2 - \frac{d}{dx} \cos x$$

$$= 2x - -\sin x$$

$$= \underline{2x + \sin x}$$

$\frac{d}{dx} x^2 = 2x$

• $y = \sqrt{x} + \sin x - \cos x$

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{x} + \frac{d}{dx}\sin x - \frac{d}{dx}\cos x$$

$$= \frac{1}{2}\sqrt{x} + \cos x - -\sin x$$

$$= \frac{1}{2\sqrt{x}} + \cos x + \sin x$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

upto here

• product rule of differenciation.

given 2 functions $f(x)$ and $g(x)$ then,

$$\boxed{\frac{d}{dx} f(x).g(x) = f(x).\frac{d}{dx} g(x) + g(x).\frac{d}{dx} f(x)}$$

eg:- $\frac{d}{dx} x^2 \sin x = x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2$

$$= x^2 \cos x + \sin x \, 2x$$

$$= x^2 \cos x + 2x \sin x$$

• find $\frac{dy}{dx}$ if $y = x^3 \cos x$

$$\frac{dy}{dx} = f(y) = x^3 \cos x$$

$$\frac{d}{dx} x^3 \cos x = x^3 \frac{d}{dx} \cos x + \cos x \frac{d}{dx} x^3$$

$$= x^3 \times -\sin x + \cos x \; 3x^2$$

$$= -x^3 \sin x + \cos x \, 3x^2$$

$$= 3x^2 \cos x \; \cancel{} - x^3 \sin x$$

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- $y = 4x^5 \cos x$

$$\frac{d}{dx} 4x^5 \cos x = 4x^5 \frac{d}{dx} \cos x + \cos x \frac{d}{dx} 4x^5$$

$$= 4x^5 \times -\sin x + \cos x \times 4 \cdot 5x^4$$

$$= -4x^5 \sin x + \cos x \times 20\, x^4$$

$$\frac{d}{dx} 4x^5$$

$$= 5 \cancel{4} x^{4\ 5-1}$$

$$= 5x^4$$

$$4 \frac{d}{dx} x^5$$

$$4 \times 5x^4$$

- $y = \sqrt{x} \sin x$

$$\frac{d}{dx} \sqrt{x} \sin x = \sqrt{x} \frac{d}{dx} \sin x + \sin x \frac{d}{dx} \sqrt{x}$$

$$= \sqrt{x} \cos x + \sin x \frac{1}{2\sqrt{x}}$$

$$= \sqrt{x} \cos x + \frac{\sin x}{2\sqrt{x}}$$

- $y = \cos x \cdot \sin x$

$$\frac{d}{dx} \cos x \cdot \sin x = \cos x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} \cos x$$

$$= \cos x \times \cos x + \sin x \times -\sin x$$

$$= \cos^2 x + -\sin^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

- $y = (x^2 + 2x + 3)(x+1)$

$$\frac{d}{dx}(x^2+2x+3)(x+1) = x^2+2x+3 \frac{d}{dx}(x+1) + x+1 \frac{d}{dx}$$

$$(x^2 + 2x + 3)$$

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$=(x^2+2x+3)(1+0) + x+1\ (2x^1+2+0)$

$=(x^2+2x+3) + (x+1)\ (2x+2+0)$

$= \cancel{x^2+2x+3+(x+1)(2x+5)}$

$= \cancel{x^2+2x+3+2x^2+5x+2x+5}$

$= \cancel{x^2+2x+3+2x^2+7x+5}$

$= \cancel{3x^2+9x}+8$

$= (x^2+2x+3) (1+0) + (x+1)\ (2x+2)$

$= x^2+2x+3 + 2x^2+2x+2x+2$

$= \underline{\underline{3x^2+6x+5}}$

find $\frac{dy}{dx}$, if $y=(x+1)(x+2)$

$\frac{d}{dx}(x+1)(x+2) = \frac{d}{dx}\ (x+1)\frac{d}{dx}(x+2) + (x+2)\frac{d}{dx}(x+$

$=(x+1) \times 1 + x+2 \times 1$

$= x+1 + x+2$

$= \underline{\underline{2x+3}}$

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Quotient rule of Differentiation.

let u and v be the given functions of x then,

$$\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.}$$, where $v \neq 0$.

- Find $\frac{dy}{dx}$ if $y = \frac{x^3}{\sin x}$

$$\frac{d}{dx}\left(\frac{x^3}{\sin x}\right) = \frac{\sin x \cdot \frac{d}{dx}x^3 - x^3\frac{d}{dx}\sin x}{(\sin x)^2}$$

$$= \frac{\sin x \cdot 3x^2 - x^3\cos x}{\sin^2 x}$$

$$= \frac{3x^2\sin x - x^3\cos x}{\sin^2 x}$$

- Find $\frac{dy}{dx}$, if $y = \frac{x^2}{\cos x}$

$$\frac{d}{dx}\left(\frac{x^2}{\cos x}\right) = \frac{\cos x \cdot \frac{d}{dx}x^2 - x^2 \cdot \frac{d}{dx}\cos x}{(\cos x)^2}$$

$$= \frac{\cos x \cdot 2x - x^2 \times -\sin x}{\cos^2 x}$$

$$= \frac{2x\cos x + x^2\sin x}{\cos^2 x}$$

info. $y = \tan x$, find derivative using quotient rule

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}\cos x}{(\cos x)^2}$$

$$= \frac{\cos x \times \cos x - \sin x \times -\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

• find derivative of $\cot x$ using quotient rule

$$\cot x = \frac{\cos x}{\sin x}$$

$$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{\sin x \cdot \frac{d}{dx}\cos x - \cos x \cdot \frac{d}{dx}\sin x}{(\sin x)^2}$$

$$= \frac{\sin x \times -\sin x - \cos x \times \cos x}{\sin^2 x}.$$

$$= \frac{-\sin^2 x - \cos x \cancel{\sin x} \cos x}{\sin^2 x}$$

$$= \cancel{\cos x \sin x} \quad \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x)}$$

$$= \frac{-1}{\sin^2 x}$$

$$= \underline{\underline{-\csc^2 x}}$$

- find derivative of $\operatorname{cosec} x$ using quotient rule.

$$\frac{d}{dx}(\operatorname{cosec} x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= \frac{\sin x \cdot \frac{d}{dx} 1 - 1 \times \frac{d}{dx} \sin x}{(\sin x)^2}$$

$$= \frac{\sin x \times 0 - 1 \times \cos x}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x}$$

$$= \frac{-1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= \underline{\underline{-\operatorname{cosec} x \cot x}}$$

- $y = \sec x$, quotient

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{\cos x \cdot \frac{d}{dx} 1 - 1 \times \frac{d}{dx} \cos x}{(\cos x)^2}$$

$$= \frac{\cos x \times 0 - 1 \times -\sin x}{\cos^2 x}$$

$$= \frac{+\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \underline{\underline{\tan x \cdot \sec x}}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$$

$$\frac{d}{dx} \sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x.$$

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find $\frac{dy}{dx}$, if $y = \frac{x^3}{\tan x}$

$$\frac{d}{dx}\left(\frac{x^3}{\tan x}\right) = \frac{\tan x \cdot \frac{d}{dx} x^3 - x^3 \frac{d}{dx}\tan x}{(\tan x)^2}$$

$$= \frac{\tan x \cdot 3x^2 - x^3 \cdot \sec^2 x}{\tan^2 x}$$

$$= \frac{3x^2 \tan x - x^3 \cdot \sec^2 x}{\tan^2 x}$$

- $y = \frac{\sqrt{x}}{\sin x}$

$$\frac{d}{dx}\left(\frac{\sqrt{x}}{\sin x}\right) = \frac{\sin x \cdot \frac{d}{dx}(\sqrt{x}) - \sqrt{x} \cdot \frac{d}{dx}\sin x}{(\sin x)^2}$$

$$= \frac{\sin x \cdot \frac{1}{2\sqrt{x}} - \sqrt{x} \times \cos x}{\sin^2 x}$$

$$= \frac{\frac{\sin x}{2\sqrt{x}} - \sqrt{x}\cos x}{\sin^2 x}$$

- $y = \frac{1-x^2}{1+x^2}$

$$\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) = \frac{(1+x^2) \times \frac{d}{dx}(1-x^2) - (1-x^2) \times \frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)' \times -2x - (1-x^2)' \times 2x}{(1+x^2)^2}$$

$$= \frac{2x\left(-(1+x^2) - (1-x^2)\right)}{(1+x^2)^2}$$

$$= \frac{2x\left(-1 - \cancel{x^2} - 1 + \cancel{x^2}\right)}{(1+x^2)^2}$$

$$= \frac{2x(-2)}{(1+x^2)^2} = \frac{-4x}{(1+x^2)^2}$$

- find $\frac{dy}{dx}$ $\quad y = \frac{x+\sqrt{x}}{1+\sqrt{x}}$ $\quad$ s.t $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$$\frac{d}{dx}\left(\frac{x+\sqrt{x}}{1+\sqrt{x}}\right) = \frac{(1+\sqrt{x})\frac{d}{dx}(x+\sqrt{x}) - x+\sqrt{x}\,\frac{d}{dx}(1+\sqrt{x})}{(1+\sqrt{x})^2}$$

$$= \frac{(1+\sqrt{x})\left(1+\frac{1}{2\sqrt{x}}\right) - x+\sqrt{x} \times 0 + \frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2}$$

$$= \frac{(1+\sqrt{x})\left(1+\frac{1}{2\sqrt{x}}\right) - x+\sqrt{x} \cdot \frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2}$$

$$= \frac{\cancel{\dfrac{2\sqrt{x}+1}{2\sqrt{x}}}}{1+\cancel{\sqrt{x}}}$$

$$= \frac{\cancel{(1+\sqrt{x})}\left(1+\dfrac{1}{2\sqrt{x}} - \cancel{\sqrt{x}}\cdot\dfrac{1}{2\cancel{\sqrt{x}}}\right)}{(1+\sqrt{x})^{\cancel{2}}}$$

$$= \frac{1+\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2}}{1+\sqrt{x}}$$

$$= \frac{\dfrac{1}{2}+\dfrac{1}{2\sqrt{x}}}{1+\sqrt{x}}$$

$$= \frac{\cancel{2\sqrt{x}+2}}{\cancel{2(2\sqrt{x})}} = \frac{\dfrac{2\sqrt{x}+2}{2\times 2\sqrt{x}}}{1+\sqrt{x}}$$

$$= \frac{\dfrac{2\sqrt{x}+2}{4\sqrt{x}}}{1+\sqrt{x}}$$

$$\frac{\cancel{2}(\sqrt{x}+1)}{2\cancel{2}(\sqrt{x})}$$

$$= \beta \quad \frac{\dfrac{\sqrt{x}+1}{2\sqrt{x}}}{1+\sqrt{x}}$$

$$= \frac{\cancel{\sqrt{x}+1}}{2\sqrt{x}} \times \frac{1}{\cancel{1+\sqrt{x}}} = \frac{1}{2\sqrt{x}}$$

- $$\frac{d}{dx}\,\frac{x+\sqrt{x}}{1+\sqrt{x}} = \frac{d}{dx}\,\frac{\sqrt{x}\,(\cancel{\sqrt{x}+1})}{(\cancel{1+\sqrt{x}})}$$

$$= \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

- $$y = \frac{x^2+x+1}{x^2+2x+3} \qquad \frac{dy}{dx} = ?$$

$$\rightarrow \quad \frac{d}{dx}\left(\frac{x^2+x+1}{x^2+2x+3}\right) = \frac{(x^2+2x+3)\frac{d}{dx}(x^2+x+1) - (x^2+x+1)\frac{d}{dx}(x^2+2x+3)}{(x^2+2x+3)^2}$$

$$= \frac{(x^2+2x+3)(2x+1+0) - (x^2+x+1)(2x+2+0)}{(x^2+2x+3)^2}$$

$\frac{d}{dx}2x$

2×1

$$= \frac{(x^2+2x+3)(2x+1) - (x^2+x+1)(2x+2)}{(x^2+2x+3)^2}$$

$$= \frac{(2x^3+x^2+4x^2+2x+6x+3) - (2x^3+2x^2+2x^2+\cancel{2x}+\cancel{2x}-2)}{(x^2+2x+3)^2}$$

$2x\,r$

$$\frac{dy}{dx} = \frac{(2x^3+5x^2+8x+3) - (2x^3+4x^2+\cancel{8x}-2)}{(x^2+2x+3)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{2x^3}+5x^2+8x+3-\cancel{2x^3}-4x^2}{(x^2+2x+3)^2}$$

$$\frac{db}{} = \frac{x^2+8x+5}{(x^2+2x+3)^2}$$

$$= \frac{x^2+4x+1}{(x^2+2x+3)^2}$$

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$$y = \frac{\cos x}{x + \sin x}$$

$$\frac{d}{dx}\left(\frac{\cos x}{x+\sin x}\right)$$

$$= \frac{(x+\sin x) \times \frac{d}{dx}\cos x - \cos x \times \frac{d}{dx}(x+\sin x)}{(x+\sin x)^2}$$

$$= \frac{(x+\sin x) \times -\sin x - \cos x \times (1+\cos x)}{(x+\sin x)^2}$$

$$= \cancel{(x+\sin x)}\sin x$$

$$= \cancel{\frac{-x\sin x \times \sin x - \cos x + \cos^2 x}{(x+\sin x)^2}}$$

$$= \cancel{\frac{-x\sin^2 x - \cos x - \cos^2 x}{(x+\sin x)^2}}$$

$$= \cancel{\frac{-(x\sin^2 x + 2\cos x)}{(x+\sin x)^2}}$$

$$-x \sin x \times -$$

$$= \frac{-x\sin x - \sin^2 x - \cos x - \cos^2 x}{(x+\sin x)^2}$$

$$= \frac{-(x\sin x + \cos x + \sin^2 x + \cos^2 x)}{(x+\sin x)^2}$$

$$= \frac{(x \sin x + \cos x + 1)}{(x + \sin x)^2}$$

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• $y = \frac{2x^2-3}{x^2+2x+2}$ find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left(\frac{2x^2-3}{x^2+2x+2}\right) = \frac{(x^2+2x+2)\frac{d}{dx}(2x^2-3) - (2x^2-3)\frac{d}{dx}(x^2+2x+2)}{(x^2+2x+2)^2}$$

$$= \frac{(x^2+2x+2)(2\times 2\cancel{x})\cancel{x^2} - (2x^2-3)(2x+2)}{(x^2+2x+2)^2}$$

$$= \frac{(x^2+2x+2)(4x) - (4x^3+4x^2-6x-6)}{(x^2+2x+2)^2}$$

$$= \frac{\cancel{4x^3} + 8x^2 + 8x - \cancel{4x^3} - 4x^2 + 6x + 6}{(x^2+2x+2)^2}$$

$$= \frac{\cancel{12x^2 + 8x - 4x^3 - 4x^2 + 6x + 6}}{(\cancel{x^2+2x+2})^2}$$

$$= \cancel{12x^2 - 8x^2}$$

$$= \frac{4x^2 + 14x + 6}{(x^2+2x+2)^2}$$

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1) $y = \frac{\cot x}{5x}$

2) $y = \frac{x^2+2}{\cos x}$

3) $y = \frac{x^2-3x-2}{2x^2-3}$

1) $y = \frac{\cot x}{5x}$

$$\frac{d}{dx}\left(\frac{\cot x}{5x}\right) = \frac{5x\frac{d}{dx}(\cot x) - \cot x\frac{d}{dx}5x}{(5x)^2}$$

$$= \frac{5x \times -\operatorname{cosec}^2 x - \cot x \times \frac{1}{25x}}{(5x)^2}$$

$$= \frac{-5x \operatorname{cosec}^2 x - \frac{\cot x}{25x}}{x}$$

$$= \frac{-5x \operatorname{cosec}^2 x - \cot x}{x\,25x}$$

2) $y = \frac{x^2+2}{\cos x}$

$$\frac{d}{dx}\left(\frac{x^2+2}{\cos x}\right) = \frac{\cos x\frac{d}{dx}(x^2+2) - (x^2+2)\frac{d}{dx}\cos x}{(\cos x)^2}$$

$$= \frac{\cos x \times 2x - (x^2+2) \times -\sin x}{\cos^2 x}$$

$$= \frac{\cos x \, 2x - (-x^2 \sin x - 2\sin x)}{\cos^2 x}$$

$$= \frac{\cos x \, 2x + x^2 \sin x + 2\sin x}{\cos^2 x}$$

3) $y = \dfrac{x^2 - 3x - 2}{2x^2 - 3}$

$$\frac{d}{dx}\left(\frac{x^2-3x-2}{2x^2-3}\right)$$

$$= \frac{(2x^2-3) \times \frac{d}{dx}(x^2-3x-2) - (x^2-3x-2)\frac{d}{dx}(2x^2-3)}{(2x^2-3)^2}$$

$$= \frac{(2x^2-3) \times (2x-3) - (x^2-3x-2)(2 \times 2x)}{(2x^2-3)^2}$$

=

$\cancel{\frac{d}{dx}(e^x \cdot \log x)}$ derivative of $e^x, \log x$.

$\cancel{\frac{d}{dx}(e^x \log}$

• logarithm function.

~~loga~~

$$\boxed{\begin{array}{c} \log_a y = x \\ \Rightarrow a^x = y \end{array}}$$

eg:- 1) $\log_{10} 100 = 2$

2) $\log_{10} 1000 = 3$

3) $\log_2 8 = 3$

4) $\log_3 27 = 3$

• Find x ?

1) $\log_{10} x = -1$

$= \log_{10} (10^{-1}) = -1$

$= \log_{10} 1/10 = -1, \quad = \log_{10}(.1) = -1$

2) $\log_{\sqrt{2}} 4 = x$

$x = 4$

$\log_{\sqrt{2}} 4 = 4$

3) $\log_x 64 = 2$

$\log_8 64 = 2$

$\log_{10} 1000 = 3 \quad 10^3 = 1000$

$\log_{10} x = -1 \quad 10^{-1}$

$10^{-1} = x$

$x = 1/10 = .1$

$\log_{\sqrt{2}} 4 = x$

$= (\sqrt{2})^x$

$= 4$

$= (\sqrt{2})^4$

$= \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$

$= 2 \times 2$

$= \underline{4}$

$\log_x 64 = 2$

$= (x)^2$

$= 64$

ie $(8)^2 = 64$

$x^2 = 64$

$x = 8$

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- A logarithm with base 'e' is known as natural logarithm.
- $\log x$ and e^x are inverse functions.

$$\boxed{\begin{aligned} \text{ie } \log e^x &= x \\ e^{\log x} &= x \end{aligned}}$$

$\Rightarrow (e)^x = e^x$

Derivative of e^x $\log x$

$$\boxed{\begin{aligned} \frac{d}{dx}(e^x) &= e^x \\ \frac{d}{dx}(\log x) &= \frac{1}{x} \end{aligned}}$$

- find $\frac{dy}{dx}$, if $y = x^2 \log x$

$$\frac{d}{dx}(x^2 \log x) = \cancel{\frac{d}{dx}(x^2) + \frac{d}{dx}(\log x)}$$

$$= x^2 \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^2)$$

$$= x^2 \times \frac{1}{x} + \log x \times 2x$$

$$= \frac{x^2}{x} + 2x \log x$$

$$= \underline{x + 2x \log x}$$

- $y = e^x \log x$

$$\frac{d}{dx}(e^x \log x) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)$$

$$= e^x \times \frac{1}{x} + \log x \times e^x$$

$$= \frac{e^x}{x} + e^x \log x$$

$$= \underline{e^x\left(\frac{1}{x} + \log x\right)}$$

- $y = 5x \log x$

$$\frac{d}{dx}(5x \log x) = 5x \times \frac{d}{dx}(\log x) + \log x \times \frac{d}{dx}(5x)$$

$$= 5x \times \frac{1}{x} + \log x \times \frac{1}{25x}$$

$$= \frac{5x}{x} + \frac{\log x}{25x}$$

$$= \frac{1}{5x} + \frac{\log x}{25x}$$

$$= \frac{1}{5x}\left(1 + \frac{\log x}{2}\right)$$

$$\frac{5x}{x}$$

$$= \frac{\cancel{x}}{\cancel{x} \times 5x}$$

- $y = e^x \sin x$

$$\frac{d}{dx}(e^x \sin x) = e^x \times \frac{d}{dx} \sin x + \sin x \times \frac{d}{dx} e^x$$

$$= e^x \times \cos x + \sin x \times e^x$$

$$= e^x \cos x + e^x \sin x$$

$$= e^x (\cos x + \sin x)$$

- $y = \frac{\log x}{x}$

$$\frac{d}{dx}\left(\frac{\log x}{x}\right) = \frac{x \times \frac{d}{dx}(\log x) - \log x \times \frac{d}{dx}(x)}{x^2}$$

$$= \frac{x \times \frac{1}{x} - \log x \times 1}{x^2}$$

$$= \frac{\frac{\cancel{x}}{\cancel{x}} - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

- $y = \frac{e^x}{5x}$

$$= \frac{5x \times \frac{d}{dx} e^x - e^x \times \frac{d}{dx} \sqrt{x}}{(\sqrt{x})^2}$$

$$= \frac{\sqrt{x} \times e^x - e^x \frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{\sqrt{x}\, e^x - \frac{e^x}{2\sqrt{x}}}{x}$$

$$= \frac{e^x \left(\sqrt{x} - \frac{1}{2\sqrt{x}}\right)}{x}$$

$$= \frac{e^x \left(\frac{2x - 1}{2\sqrt{x}}\right)}{x}$$

$$= \frac{e^x (2x-1)}{2x\sqrt{x}}$$

$\sqrt{x} \times 2\sqrt{x}$

$\frac{2x-1}{2\sqrt{x}}$

$x\left(\frac{x-1}{\sqrt{x}}\right)$

$2x$

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- Derivatives of trigonometric functions.

The inverse sin function $\sin^{-1}(x)$ is defines as follows;

$$y = \sin^{-1} x \iff x = \sin y$$

Ily we can define the inverse of the each of trigonometric functions. $\iff$ implies and implied by.

eg:- $\sin(\pi/2) = 1 \iff \sin^{-1}(1) = \pi/2$

$\cos(\pi/3) = \frac{1}{2} \iff \cos^{-1}\left(\frac{1}{2}\right) = \pi/3$

$\sin^{-1}(0) = 0$

$\tan^{-1}(1) = \pi/4$ or 45°

$\tan^{-1}(\infty) = \pi/2$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

- find $\frac{dy}{dx}$, If $y = x^2 \sin^{-1}x$

$$\frac{d}{dx}(x^2\sin^{-1}x) = x^2\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}x^2$$

$$= x^2 \times \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \; 2x$$

$$= \frac{x^2}{\sqrt{1-x^2}} + 2x\sin^{-1}x$$

- $y = (1-x^2)^2\cos^{-1}x$

$$\frac{d}{dx}((\cancel{1}-x^2)(\cos^{-1}x)) = (1-x^2)\frac{d}{dx}\cos^{-1}x + \cos^{-1}x\frac{d}{dx}(1-x^2)$$

$$= (1-x^2) \times \frac{-1}{\sqrt{1-x^2}} + \cos^{-1}x \times -2x$$

$$= \cancel{\frac{-1+x^2)}{\sqrt{1-x^2}} - 2x} + \cos^{-1}x$$

$$= -\sqrt{1-x^2} - 2x\cos^{-1}x$$

$-1(1-x^2)x$

$-1+x^2$

- $y = \dfrac{\tan^{-1}x}{1+x^2}$

$$\frac{d}{dx}\left(\frac{\tan^{-1}x}{1+x^2}\right) = \frac{1+x^2 \times \frac{d}{dx}(\tan^{-1}x) - \tan^{-1}(x)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2) \times \frac{1}{1+x^2} - \tan^{-1}x \times 2x}{(1+x^2)^2}$$

$$= \frac{\frac{\cancel{1+x^2}}{\cancel{1+x^2}} - \tan^{-1}x\, 2x}{(1+x^2)^2}$$

$$= \frac{1-2x\tan^{-1}x}{(1+x^2)^2}$$

- $y = \dfrac{\sin^{-1}x}{x}$

$$\frac{d}{dx}\left(\frac{\sin^{-1}x}{x}\right) = \frac{x \times \frac{d}{dx}(\sin^{-1}x) - \sin^{-1}x \times \frac{d}{dx}x}{x^2}$$

$$= \frac{x \times \frac{1}{\sqrt{1-x^2}} - \sin^{-1}x \times 1}{x^2}$$

$$= \frac{\frac{x}{\sqrt{1-x^2}} - \sin^{-1}x}{x^2}$$

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$y = x^2 e^x \sin^{-1}x$

$$\frac{d}{dx} x^2 e^x \sin^{-1}x = x^2 e^x \frac{d}{dx}\sin^{-1}x + \sin^{-1}(x)\frac{d}{dx}x^2 e^x$$

$$= x^2 e^x \times \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot x^2 e^x + e^x\, 2x$$

$$= \frac{x^2 e^x}{\sqrt{1-x^2}} + \sin^{-1}x\left(x^2 e^x + e^x\, 2x\right)$$

$$= \frac{x^2 e^x}{\sqrt{1-x^2}} + e^x\left(x^2+2x\right)\sin^{-1}x$$

$\frac{d}{dx}x^2 e^x$

$= \cancel{2x}$

$= x^2\frac{d}{dx}e^x + e^x\frac{d}{dx}x^2$

$= x^2 e^x + e^x\, 2x$

$y = (x+1)(x+2)(x+2)$

$$\frac{d}{dx}(x+1)(x+2)(x+3)$$

$$\frac{d}{d} = (x+1)(x+2)\frac{d}{dx}(x+3) + (x+3)\frac{d}{dx}(x+1)(x+2)$$

$$= (x+1)(x+2)\times 1 + (x+3)(x+1)+(x+2)$$

$= \cancel{x+2x}$

$\cancel{= (x+1)(x+2)\left(1+(x+3)\right)}$

$$= (x+1)(x+2) + (x+3)(2x+3)$$

$$= x^2+3x+2+2x^2+3x+6x+9$$

$$= 3x^2+12x+11$$

$\frac{d}{dx}(x+1)(x+2)$

$= (x+1)\frac{d}{dx}(x+2) +$

$(x+2)\frac{d}{dx}(x+1)$

$= (x+1)\,1 + (x+2)\,1$

$= (x+1)+(x+2)$

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- $y = \dfrac{x\sin^{-1}x}{1-x^2}$

$$\frac{d}{dx}\left(\frac{x\sin^{-1}x}{1-x^2}\right) = \frac{(1-x^2)\frac{d}{dx}(x\sin^{-1}x) - x\sin^{-1}x\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$2x$

$$= \frac{(1-x^2)\cdot\frac{d}{dx}(x\sin^{-1}x) - x\sin^{-1}x \times -2x}{\cancel{\sqrt{1-x^2}}}$$

$$= \frac{\cancel{(1-x^2)} \times \frac{x}{\sqrt{1-x^2}} \times \sin^{-1}x + x\sin^{-1}x\, 2x}{(1-x^2)^2}$$

$$= \frac{\frac{\sqrt{1-x^2}\; x\sin^{-1}x}{\cancel{\sqrt{1-x^2}}} + x\sin^{-1}x\, 2x}{(1-x^2)^2}$$

$$= \frac{\cancel{x\sin^{-1}x\left(\frac{\sqrt{1+x^2}}{\sqrt{1-x^2}} + 2x\right)}}{(1-x^2)^2}$$

$$\frac{d}{dx}x\sin^{-1}x$$

$$= x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}x$$

$$= x \times \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x$$

$$= \frac{x}{\sqrt{1-x^2}} \times \sin^{-1}x$$

$$= \frac{x\sqrt{1-x^2} + (1-x^2)\sin^{-1}x + 2x^2\sin^{-1}x}{(1-x^2)^2}$$

$$= \frac{x\sqrt{1-x^2} + \sin^{-1}x - x^2\sin^{-1}x + 2x^2\sin^{-1}x}{(1-x^2)^2}$$

$$= \frac{x\sqrt{1-x^2} + \sin^{-1}x + x^2\sin^{-1}x}{(1-x^2)^2}$$

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DIFFERENTIATION - 2.

FUNCTION OF A FUNCTION rule

Given a function $y = f(u)$, where $u = g(x)$, then y is called a composite function of 'x'. To find $\frac{dy}{dx}$ we use function of a function rule , which is given by

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}.$$

which is equivalent to differen-tiating the outer function first which is multiplied by the derivative of inner function.

eg:- 1) $y = \sin(x^2)$

$$\frac{dy}{dx} = \cos(x^2) \cdot 2x$$

$$= \underline{\underline{2x \cos x^2}}$$

2) $y = \cos(\log x)$

$$\frac{dy}{dx} = -\sin(\log x) \frac{1}{x}$$

$$= \underline{\underline{\frac{-\sin(\log x)}{x}}}$$

$5x = \frac{1}{25x}$

3) $y = \sqrt{\sin x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin x}} \times \cos x = \underline{\underline{\frac{\cos x}{2\sqrt{\sin x}}}}$$

- $y = (x^2+5x+1)^{10}$

$$\frac{dy}{dx} = 10x^9 (x^2+5x+1)^9 (2x+5)$$

$$= 10 \times (2x+5)(x^2+5x+1)^9$$

$x^{10} \Rightarrow$ derivative $= 10x^9$

- $y = \sqrt{3x^2 - 5x + 10}$

$$= \frac{1}{2\sqrt{3x^2-5x+10}} \times (3\times 2x - 5)$$

$$= \frac{6x-5}{2\sqrt{3x^2-5x+10}}$$

- $y = \sin^2 x$

$\sin^2 x = (\sin x)^2$

$$\frac{dy}{dx} = 2\sin x \cdot \cos x$$

$$= 2\sin x \cos x \quad \text{or} \quad \sin 2x$$

- $y = \frac{1}{(x^2+10)}$

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

$$\frac{dy}{dx} = \frac{-1}{(x^2+10)^2} \times 2x$$

$$= \frac{-2x}{(x^2+10)^2}$$

- $y = \sec(\log x)$

$$\frac{dy}{dx} = \sec(\log x)\tan(\log x) \times \frac{1}{x}$$

$$= \frac{\sec(\log x)\tan(\log x)}{x}$$

- $y = \log(\sec x)$

$$= \frac{1}{\cancel{\sec x}} \cdot \cancel{\sec x} \tan x$$

$$= \underline{\tan x}$$

- P.T $\frac{dy}{dx} = \sec x$, if $y = \log(\sec x + \tan x)$

$$\frac{dy}{dx} = \frac{1}{(\sec x + \tan x)} \times \sec x \tan x + \sec^2 x$$

$$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

$$= \frac{\cancel{\sec x \tan x (1 +}}{\cancel{\sec^2 x}}$$

$$= \frac{\cancel{\sec x}(\tan x + \sec x)}{\cancel{\sec x}(1 + \tan x)}$$

$=$

$$= \frac{\sec x \, (\cancel{\tan x + \sec x})}{(\cancel{\sec x + \tan x})}$$

$$= \underline{\sec x}$$

- $y = \sin 2x$

$$\frac{dy}{dx} = \cos 2x \cdot 2$$

$$= 2\cos 2x$$

- $y = \tan 5x$

$$\frac{dy}{dx} = \sec^2 5x \cdot 5$$

$$= 5\sec^2 5x$$

In general

$$\frac{d}{dx} \sin kx = k \cos kx$$

$$\frac{d}{dx} \cot kx = k \times - \operatorname{cosec}^2 kx$$

$$\frac{d}{dx} e^{kx} = e^{kx} \cdot k = k e^{kx}$$

$$\frac{d}{dx} \log kx = \frac{1}{kx} k = \frac{\cancel{k}}{\cancel{k}x} = \frac{1}{x}$$

- $y = \sin^2(2x)$

$$\cancel{\frac{dy}{dx} = 2\sin(2x)\cos(2x)} \cdot 2$$

$$y = (\sin(2x))^2$$

$$\frac{dy}{dx} = 2\sin(2x)\cos(2x) \cdot 2$$

$$= 4\sin 2x \cos 2x$$

$$= 2(2\sin 2x \cdot \cos(2x))$$

$$= 2\sin 4x$$

• $y = e^{\log x^2}$ $\qquad e \to \log \to x^2$

$$= e^{\log x^2} \cdot \frac{1}{x^2} \cdot 2x$$

$$= \frac{2\cancel{x} e^{\log x^2}}{x^{\cancel{2}}}$$

$$= \frac{2e^{\log x^2}}{x}$$

• $y = \sin^{-1}(x^2)$

$$= \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$= \frac{1}{\sqrt{1-x^4}} \cdot 2x$$

• $y = \cos^{-1}(\sqrt{x})$

$$= \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$$

$\sqrt{x} \times \sqrt{x} = x$

$$= \frac{-1}{\sqrt{1-x} \cdot 2\sqrt{x}}$$

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{a} \times \sqrt{a} = a$

$$= \frac{-1}{2\sqrt{x(1-x)}}$$

• $y = \log(x^2 + 2x + 1)$

$$\frac{dy}{dx} = \frac{1}{(x^2+2x+1)} \cdot 2x+2$$

$$= \frac{2x+2}{x^2+2x+1} \quad = \frac{\cancel{2x}(\cancel{1+2})}{2x(} \quad = \frac{2(\cancel{x+1})}{(x+1)^2}$$

$$= \frac{2}{(x+1)}$$

⊙ $y = x^2 \sec 5x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx} \sec 5x + \sec 5x \frac{d}{dx} x^2$$

$$= x^2 . \sec 5x \tan 5x \cdot 5x + \sec 5x . 2x$$

$$x^2 \sec(5x) \tan(5x) . 5 + \sec 5x . 2x$$

$$= \cancel{\sec 5x (x^2 \tan 5x + 2x)}$$

$$= 5x^2 \sec 5x . \tan 5x + 2x \sec 5x$$

$y = e^{2x} . \cos 3x$

$$= . e^{2x} \frac{d}{dx} \cos 3x + \cos 3x \times \frac{d}{dx} e^{2x}$$

$$= e^{2x} \times 3x - \sin x + \cos 3x \times e^{2x} . 2$$

$$= e - 3 \sin x \, e^{2x} + e^{2x} \cos 3x . 2$$

$$= \cancel{e^{2x} \cos 3x - 3 \sin x e^{2x}}$$

$$= \cancel{e^{2x}} (\cos 3x - 3 \sin x)$$

$$= -. 3 e^{2x} \sin$$

$$= e^{2x} \times - \sin 3x . 3 + \cos 3x . e^{x} . \cancel{2}.$$

$$= - 3e^{2x} \sin 3x + 2 e^{2x} . \cos 3x.$$

$$= e^{2x} (-3 \sin 3x + 2 \cos 3x)$$

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imp • $y = (x^2+1)^{10} \sec 5x$

$$\frac{dy}{dx} = (x^2+1)^{10} \frac{d}{dx} \sec 5x + \sec 5x \frac{d}{dx} (x^2+1)^{10}$$

$$= (x^2+1)^{10} \cdot \sec 5x \cdot \tan 5x \cdot 5 + \sec 5x \, 10(x^2 \cdot \ \cdot 2x$$

$$= 5(x^2+1)^{10} \cdot \sec 5x \tan 5x + 20x(x^2+1)^9 \sec 5x$$

imp • $y = \dfrac{\cot 11x}{(x^3-1)^2}$

By Q.R.

$$\frac{dy}{dx} = \frac{(x^3-1)^2 \frac{d}{dx} \cot 11x - \cot 11x \times \frac{d}{dx} (x^3-1)^2}{((x^3-1)^2)^2}$$

$$= \frac{(x^3-1)^2 \times -\mathrm{cosec}^2 11x \cdot 11 - \cot 11x \cdot 2(x^3-1) \cdot 3x^2}{\cancel{((x^3-1)^2)^2} \quad (x^3-1)^4}$$

$$= \cancel{(x^3-1)^2 \times \mathrm{cosec}^2 11x \times 11} -$$

cancelling (x^3-1),

$$= \frac{-11 (x^3-1) \mathrm{cosec}^2 11x - 6x^2 \cot 11x}{(x^3-1)^3}$$

H.W 1). $y = \frac{\sin(\log x)}{x}$

2). $y = \frac{1}{\sec\sqrt{x}}$

3). $y = e^{x} \log(\sin x)$

1) $y = \frac{\sin(\log x)}{x}$

$$= \frac{x \frac{d}{dx} \sin(\log x) - \sin(\log x) \frac{xd}{dx} x}{x^2}$$

$$= \frac{\cancel{x} \times \cos\log x \times \frac{1}{\cancel{x}} - \sin(\log x)}{x^2}$$

$$\frac{dy}{dx} = \frac{\cos\log x - \sin\log x}{x^2}$$

2) $y = \frac{1}{\sec(\sqrt{x})}$

$$= \frac{-1}{(\sec\sqrt{x})^2} \cdot \sec\sqrt{x}\tan\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\sec^2\sqrt{x} \cdot \sec\sqrt{x} \tan\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{\cancel{-\sec^2\sqrt{x}} \tan\sqrt{x}}{\sqrt{x} \cancel{\sec^2\sqrt{x}}}$$

$$= \frac{\tan\sqrt{x}}{\sqrt{x}}$$

3) $y = e^x \log \sin x$

$$\frac{dy}{dx} = e^x \times \frac{1}{\sin x} \times \cos x + \log \sin x \, e^x$$

$$= e^x \cot x + e^x \log \sin x$$

• $y = \dfrac{\sqrt{x^2+1}}{\sec 5x}$

$$= \frac{\sec 5x \times \frac{d}{dx}\sqrt{x^2+1} - \sqrt{x^2+1} \times \frac{d}{dx}\sec 5x}{(\sec 5x)^2}$$

$$= \frac{\sec 5x \times \frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x - \sqrt{x^2+1} \times 5 \sec 5x \tan 5x}{\sec^2 5x}$$

cancelling $\sec 5x$.

$$= \frac{\sec\cancel{5x}\left(\frac{x}{\sqrt{x^2+1}} - 5\sqrt{x^2+1}\tan 5x\right)}{\cancel{\sec^2} 5x}$$

$$= \frac{\frac{x}{\sqrt{x^2+1}} - 5\sqrt{x^2+1}\tan 5x}{\sec 5x}$$

implicit x explic

x- opposite

27/8/2019

Derivatives of implicit functions:-

A function of the form $y=f(x)$ is called an explicit function eg:- $y=\sin x$

$y=e^x$

$y=x^2+2x+3$

etc

If a function is given in the form $f(x,y)=0$, then it is called an implicit function eg:- 1) $x^2+y^2=1$

2) $\sin(x+y)=2$

3) $e^{x+y}-4xy=1$

etc.

To find $\frac{dy}{dx}$ in this we use the following rules:-

$$\frac{d}{dx} f(y) = \frac{d}{dy} f(y) \times \frac{dy}{dx}.$$

ie, differentiate $f(y)$ with respect to y first, which is multiplied by the term $\frac{dy}{dx}$

eg:- 1) $\frac{d}{dx} \cos y = -\sin y \times \frac{dy}{dx}$

2) $\frac{d}{dx} e^y = e^y \frac{dy}{dx}$

3) $\frac{d}{dx} \log y = \frac{1}{y} \frac{dy}{dx}$

- find $\frac{dy}{dx}$, if $x^2+y^2=1$

$r=1$
center $(0,0)$
[illegible] point-[illegible]
tangent[illegible]
slope

diff. w.r.to x.

$$\frac{d}{dx} x^2+y^2 = 2x + 2y \cdot \frac{dy}{dx} = 0$$

~~diff~~.

$$2x+2y \cdot \frac{dy}{dx} = 0$$

$$2y \cdot \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{+\cancel{2}y}$$

$$= -x/y$$

- $xy = c^2$

$\frac{dy}{dx} = ?$

$xy = c^2$

diff. w.r. to x.

[illegible] $= \cancel{00}$

~~dy~~ $\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$

PR apply

$$= x \times \frac{d}{dx} y + y \times \frac{d}{dx} x = \frac{d}{dx} c^2$$

$$= x \frac{dy}{dx} + y \times 1 = 0$$

$$\Rightarrow x \frac{dy}{dx} = -y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

- $x^2 + xy + y^2 = 0$

$x^2 + xy + y^2 = 0$

differenticiate with respect to x.

$$= \frac{d}{dx} x^2 + \frac{d}{dx} xy + \frac{d}{dx} y^2 = 0$$

$$= 2x + \cancel{\frac{dy}{x}} \; \cancel{+xy} \; x \frac{dy}{dx} + y + 2y \times \frac{dy}{dx}$$

$$= 2x + 2y \times \frac{dy}{dx} \cancel{\frac{xy}{x}} + x \frac{dy}{dx} + y$$

$$\Rightarrow x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - y$$

$$\Rightarrow \frac{dy}{dx}(x+2y) = -(2x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$$

imp ① $x^2+y^2+2gx+2fy+c=0$, where g, f, c are constant

= differentiate w.r.to x.

$$\frac{d}{dx}x^2+\frac{d}{dx}y^2+\frac{d}{dx}2gx+\frac{d}{dx}2fy+\frac{d}{dx}c=0$$

$$= 2x + 2y\cdot\frac{dy}{dx} + 2g\times 1 + 2f\frac{dy}{dx} + 0 = 0$$

$$= 2x + 2y\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$$

$$= 2y\frac{dy}{dx} + 2f\frac{dy}{dx} = -2x-2g$$

$$\frac{dy}{dx}(2y+2f) = -2x-2g$$

$$= -(2x+2g)$$

$$\frac{dy}{dx} = \frac{-(2x+2g)}{(2y+2f)}$$

$$= \frac{2(-x+g)}{2(y+f)}$$

$$= \frac{-(x+g)}{(y+f)}$$

y and y aaina derivate then $\frac{dy}{dx}$

x m only one derivative

$-2x-2g$

$= -(2x+2g)$

$= \frac{-\cancel{2}(x+g)}{\cancel{2}(y+f)}$

$= \frac{-(x+g)}{(y+f)}$

• $x^{2/3} + y^{2/3} = a^{2/3}$

diff. w.r.to x

$$\frac{d}{dx} x^{2/3} + \frac{d}{dx} y^{2/3} = \frac{d}{dx} a^{2/3}$$

$$\frac{2}{3} x^{2/3-1} + \frac{2}{3} y^{2/3-1} \times \frac{dy}{dx} = \cancel{\frac{2}{3} a^{2/3-1}} \; 0$$

$$= \frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = \frac{2}{3} a^{-1/3}$$

$\frac{2}{3} - \frac{1}{1}$

$$\frac{2}{3} y^{-1/3} \frac{dy}{dx} = \frac{2}{3} a^{-1/3} - \frac{2}{3} x^{-1/3}$$

$\frac{2-3}{3}$

$= \frac{-1}{3}$

$$\frac{dy}{dx} = \frac{\frac{2}{3} a^{-1/3} - \frac{2}{3} x^{-1/3}}{\frac{2}{3} y^{-1/3}}$$

$\frac{2}{3} - 1$

$\frac{2-3}{3}$

$= \frac{-1}{3}$

$$= \frac{\cancel{\frac{2}{3}} (a^{-1/3} - x^{-1/3})}{\cancel{\frac{2}{3}} (y^{-1/3})}$$

a constant so

$$= \frac{a^{-1/3} - x^{-1/3}}{y^{-1/3}}$$

$\frac{2}{3} - 1$

$\frac{2-3}{3} = \frac{-1}{3}$

$$\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = 0$$

$$= \frac{a - x^{-1/3}}{y^{-1/3}}$$

$$\frac{2}{3} y^{-1/3} \frac{dy}{dx} = -\frac{2}{3} x^{-1/3}$$

$$= -\left(\frac{x}{y}\right)^{-1/3} = \frac{-y^{1/3}}{x^{1/3}}$$

$$= -\left(\frac{y}{x}\right)^{1/3}$$

$$\frac{dy}{dx} = \frac{-\frac{2x^{-1/3}}{3}}{\frac{2y^{-1/3}}{3}}$$

$$\frac{dy}{dx} = \frac{-\cancel{2} x^{-1/3}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2} y^{-1/3}}$$

$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}}$$

$$\frac{dy}{dx} \left(\frac{-x}{y}\right)^{-1/3}$$

$$-(y/x)^{1/3}$$

29/8/19 $x^2 y^2 = x^3 + y^3 + 3xy$

find $\frac{dy}{dx} = ?$

P.R Diff. w.r.t. x

$$\frac{dy}{dx} = x^2 \frac{d}{dx} y^2 + y^2 \cdot \frac{d}{dx} x^2 = \frac{d}{dx} x^3 + \frac{d}{dx} y^3 + 3\frac{d}{dx}$$

$$+3\left(x\frac{dy}{dx} + y\frac{d}{dx}x\right)$$

$$= x^2 \times 2y\frac{dy}{dx} + y^2 \cdot 2x = 3x^2 + 3y^2\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right)$$

$$\Rightarrow 2x^2 y\frac{dy}{dx} + 2xy^2 = 3x^2 + 3y^2\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y$$

$$\Rightarrow 2x^2 y\frac{dy}{dx} - 3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} = 3x^2 + 3y - 2xy^2$$

$$\frac{dy}{dx}(2x^2 y - 3y^2 - 3x) = 3x^2 + 3y - 2xy^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 + 3y - 2xy^2}{2x^2 y - 3y^2 - 3x}$$

$\cancel{x^3} y^2$

- $y^4 = (x^2+3)^3$

Diff. w.r.t. x

$y^4 = 4xy^{4-1} = 4xy^3$

$$\frac{d}{dx}(y^4) = \frac{d}{dx}(x^2+3)^3$$

$$4y^3 \cdot \frac{dy}{dx} = 3(x^2+3)^2 \times 2x + 0$$

$$4y^3 \cdot \frac{dy}{dx} = 3(x^2+3)^2 \times 2x.$$

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$$\frac{dy}{dx} = \frac{3(x^2+3)^2 \cdot 2x}{4y^3}$$

$$\frac{dy}{dx} = \frac{\cancel{6}^{3} x \,(x^2+3^2)^2}{\cancel{4}^{2} y^3}$$

$$\frac{dy}{dx} = \frac{3x(x^2+3)^2}{2y^3}$$

$$e^y = \sin(x+y)$$

$$\frac{d}{dx}(e^y) = \frac{d}{dx}\sin(x+y)$$

$$e^y \cdot \frac{dy}{dx} = \cos(x+y)\frac{d}{dx}(x+y)$$

$$e^y \cdot \frac{dy}{dx} = \cos(x+y)\left(1+\frac{dy}{dx}\right)$$

$$e^y \frac{dy}{dx} = \cos(x+y) + \cos(x+y)\frac{dy}{dx}$$

$$e^y \frac{dy}{dx} - \cos(x+y)\frac{dy}{dx} = \cos(2x+y)$$

$$\frac{dy}{dx}\left(e^y - \cos(x+y)\right) = \cos(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(x+y)}{e^y - \cos(x+y)}$$

Parametric equations

A function of the form $x = f(t)$, $y = g(t)$ is called a parametric function.

eg:- 1) $x = \cos t$
$y = \sin t$

which is the parameteric representation of a circle $\left(x^2 + y^2 = \cos^2 t + \sin^2 t = 1 \quad \therefore x^2 + y^2 = 1 \right)$

2) $x = 2t^2 + 1$
$y = 4t$

• find $\frac{dy}{dx}$ if $x = 2t^2 + 1$, $y = 4t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dt} = \frac{d}{dt}(4t) = \underline{\underline{4}}$$

$$\frac{dx}{dt} = \frac{d}{dt}(2t^2 + 1) = 4t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4}{4t} = \underline{\underline{\frac{1}{t}}}$$

• $x = at^2$, $y = 2at$, where a is constant

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a \times \frac{dt}{dt}$$

$= 2a \times 1 = 2a$

$$\frac{d}{dt}(x) = \frac{d}{dt}(at^2) = a.2t = 2at$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{2}a}{\cancel{2}at} = \frac{1}{t}$$

- $x = a\sec\theta$

$y = b\tan\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\tan\theta) = b\sec^2\theta$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\sec\theta) = a\sec\theta\tan\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\sec^{2}\theta}{a\cancel{\sec\theta}\tan\theta}$$

$$= \frac{b\sec\theta}{a\tan\theta}$$

$$= \frac{b}{\cos\theta} \div \frac{a\sin\theta}{\cos\theta}$$

$$= \frac{b}{\cos\theta} \times \frac{\cos\theta}{a\sin\theta} = \frac{b}{a\sin\theta}$$

$x = a(\theta - \sin\theta)$

$y = a(1-\cos\theta)$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(a(1-\cos\theta)) \qquad \frac{dx}{d\theta} = \frac{d}{d\theta}(a(1-\cos\theta)$$

$$=$$

$$= a(0-\sin\theta)$$

$$= a\sin\theta$$

$$\frac{dy}{dx} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

$$\sin\theta = \sin\frac{2\theta}{2} = 2\sin\theta/2 \cdot \cos\theta/2$$

$$1-\cos\theta = 1-\cos 2\theta/2 = 2\sin^2\theta/2$$

$$\frac{dy}{dx} = \frac{\cancel{2}\cancel{\sin\theta/2}\cdot\cos\theta/2}{\cancel{2}\sin^{\cancel{2}}\theta/2}$$

$$= \frac{\cos\theta/2}{\sin\theta/2}$$

$$= \cot\theta/2$$

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- $x = e^t \cos t$

$y = e^t \sin t$

$$\frac{d}{dt}(x) = e^t \times \frac{d}{dt}\cos t + \cos t \times \frac{d}{dt}e^t$$

$$= e^t \times -\sin t + \cos t\, e^t$$

$$= e^t\cos t - e^t \sin t$$

$$\frac{d}{dt}(y) = e^t \times \frac{d}{dx}\sin t + \sin t \times \frac{d}{dt}e^t$$

$$= e^t \times \cos t + \sin t \times e^t$$

$$= e^t\cos t + e^t \sin t$$

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$$\frac{dy}{dx} = \frac{e^t \cos t + e^t \sin t}{e^t \cos t - e^t \sin t}$$

$$= \frac{\cancel{e^t}(\cos t + \sin t)}{\cancel{e^t}(\cos t - \sin t)}$$

$$\frac{dy}{dx} = \frac{\cos t + \sin t}{\cos t - \sin t}$$

x^3

$x = a\cos^3 t$

$y = b\sin^3 t$

$$\frac{d}{dt}(x) = a \times \frac{d}{dt}\cos^3 t + \cos^3 t \times \frac{d}{dt} a$$

$$= a \times 3\cos^2 t \times -\sin t + \cos^3 t \times 0$$

$$= -a\, 3\cos^2 t \sin t + 0$$

$=$

$$\frac{d}{dt}(y) = b \times \frac{d}{dt}\sin^3 t + \sin^3 t \times \frac{d}{dt} b$$

$$= b\, 3\sin^2 t \times \cos t + 0$$

$$= \cancel{b \times 3\sin^2 t \times \cos t + \sin^3 t \times 0}$$

$$= \cancel{b 3\cos^2 t \cos t} \quad 3b\sin^2 t \cos t$$

$$\frac{dy}{dx} = \frac{\cancel{3}\,\cancel{b\cos^3 t}\, b\sin^2 t \cos t}{-a\cancel{3}\cos^{\cancel{2}} t \cancel{\sin t}}$$

$$= -\frac{b}{a}\frac{\sin t}{\cos t} = -\frac{b}{a}\tan t$$

• $x = \dfrac{1-t^2}{1+t^2}$

• $y = \dfrac{2t}{1+t^2}$

$\rightarrow \quad \dfrac{dx}{dt} = \dfrac{(1+t^2)\frac{d}{dt}(1-t^2) - (1-t^2)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$

$= \dfrac{(1+t^2)\,(0-2t) - (1-t^2)(0+2t)}{1+t^4}$

$= \dfrac{(1+t^2)\,(-2t) - (1-t^2)\,(2t)}{1+t^4}$

$= \dfrac{-2t - 2t\times t^2 - (2t - 2t\,t^2)}{1+t^4}$

$= \dfrac{-2t - 2t^3 - (2t - 2t^3)}{1+t^4}$

$= \dfrac{-2t - \cancel{2t^3} - 2t + \cancel{2t^3}}{1+t^4}$

$= \dfrac{-2t-2t}{1+t^4}$

$= \underline{\underline{\dfrac{-4t}{1+t^4}}}$ or $= \dfrac{-4t}{(1+t^2)^2}$

$$\frac{dy}{dt} = \frac{(1+t^2) \times \frac{d}{dt}(2t) - 2t \times \frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2) \times (2) - 2t \times 2t}{(1+t^2)^2}$$

$$= \frac{2 + 2t^2 - 4t^2}{(1+t^2)^2}$$

$$= \frac{2 - 2t^2}{(1+t^2)^2}$$

$$= \frac{2(1-t^2)}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{\frac{2(1-t^2)}{\cancel{(1+t^2)^2}}}{\frac{-4t}{\cancel{(1+t^2)^2}}} = \frac{\cancel{2}(1-t^2)}{-\cancel{4}_{2}t}$$

$$= \frac{1-t^2}{-2t} = \underline{\underline{\frac{t^2-1}{2t}}}$$

Successive differentiation.

Given a function $y = f(x)$, then $\frac{dy}{dx}$ or y' is called the First order derivative of 'y', The IInd order derivative $\frac{d^2y}{dx^2}$ or y'' is given by ;.

$$\boxed{y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)}$$

- find y' and y'' if $y = x^3+4x^2+1$

$$y' = \frac{dy}{dx} = \frac{d}{dx}(x^3+4x^2+1)$$

$$= \frac{d}{dx}x^3 + \frac{d}{dx}4x^2 + \frac{d}{dx}1$$

$$y' = \underline{3x^2+8x}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(3x^2+8x)$$

$$y'' = \underline{6x+8}$$

- find y'' $y = \cos x + \sin x$

$$y' = \frac{dy}{dx} = -\sin x + \cos x$$

$$= \underline{\cos x - \sin x}$$

$$y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= -\sin x - \cos x$$

$= -(\cos x + \sin x)$

- $y = x\cos x$; $y'' = ?$

$$\frac{dy}{dx} = x \times \frac{d}{dx}\cos x + \cos x \frac{d}{dx} x$$

$$= x \times -\sin x + \cos x \times 1$$

$$= -x\sin x + \cos x$$

$$= \cos x - x\sin x$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\cos x - x\sin x)$$

$$= -\sin x - (x \times \cos x + \sin x \times 1)$$

$$= -\sin x - (x\cos x + \sin x)$$

$$= -\sin x - x\cos x - \sin x$$

$$= \cancel{-(\sin x + x\cos x + \sin x)}$$

$$= -2\sin x - x\cos x$$

- ~~$y = e^{-x}$~~

- $y = e^{-x}$

$$\frac{dy}{dx} = e^{-x} \cdot -1$$

$$= -e^{-x}$$

$$\frac{d^2y}{dx^2} = e^{-x}$$

- S.T $y'' - y = 0$ if $y = e^{x} + e^{-x}$

$$y' = e^{x} - e^{-x}$$

$$y'' = e^{x} - e^{-x} \cdot -1$$

$$= e^{x} + e^{-x}$$

$$y'' = y \Rightarrow y'' - y = 0$$

H.W. $y = a\sin x + b\cos x$, S.T $y'' + y = 0$

$y' \neq \cancel{\left(a \times \frac{d}{dx}\sin x + \sin x \times \frac{d}{dx} a\right) + \left(b \times \frac{d}{dx}\cos x + \cos x \times \frac{d}{dx}\right)}$

$y' = a \times (\cos x) + b \times (-\sin x)$

$= a\cos x - b\sin x$

$y'' = a \times -\sin x - b \times \cos x$

$= -a\sin x - b\cos x$

$= -(a\sin x + b\cos x) = \underline{\underline{-y}}$

$\cancel{= -(a\sin x + b\cos x)}$

$y'' + y = -\cancel{a\sin x} - \cancel{b\cos x} + \cancel{a\sin x} + \cancel{b\cos x}$

$= \underline{\underline{0}}$

$y'' = -y \Rightarrow y'' + y = \underline{\underline{0}}$

1) $y = ae^{x} + b^{-x}$

S.T $y'' - y = 0$

$$y' = \frac{d}{dx} ae^{x} + \frac{d}{dx} b^{-x}$$

$$= ae^{x} + -b^{-x}$$

$$y'' = \frac{d}{dx}(ae^{x}) + \frac{d}{dx}(-b^{-x})$$

$-b^{-x} \times -1$

$= +b^{-x}$

$$= \underline{\underline{ae^{x} + b^{-x}}}$$

$$y'' - y = ae^{x} + b^{-x} - (ae^{x} - b^{-x})$$

$$= a\cancel{e^{x}} + b^{-x} - a\cancel{e^{x}} + b^{-x}$$

$$= 0$$

$$y = y'' \Rightarrow y'' - y = \underline{\underline{0}}$$

2) $y = a\cos 2x + b\sin 2x$

$y'' + 4y = 0$

$$y' = \frac{d}{dx} a(\cos 2x) + \frac{d}{dx}(b\sin 2x)$$

$$= -2a\sin 2x + 2b\cos 2x$$

$$y'' = -4a\cos 2x - 4b\sin 2x$$

$$y'' = -4(a\cos 2x + b\sin 2x)$$

$$= -4y$$

$$y'' - 4y = \underline{\underline{0}}$$

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① $y = x^2 \sin x$. S·T $x^2 y'' - 4xy' + (x^2+6\cancel{x}) y = 0$

$y = x^2 \sin x$

$y' = \frac{dy}{dx} = \frac{d}{dx}(x^2 \sin x)$

$= \cancel{\frac{d}{dx}}\ x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2$

$= x^2 \times \cos x + \sin x \cdot 2x.$

$y'' = \frac{d^2y}{dx^2} = d$

$= \frac{d}{dx}(\cancel{x^2}\ x^2 \cos x + \sin x\, 2x)$

$= x^2 \times -\sin x + \cos x \cdot 2x + 2x\cos x + \sin x \cdot 2$

$= -x^2 \sin x + 4x \cos x + 2\sin x.$

$x^2 y'' - 4xy' + (x^2+6\cancel{x})y = x^2 \times (-x^2 \sin x + 4x\cos x + 2\sin x) - 4x(x^2 \cos x + 2x \sin x) + (x^2+6\cancel{x})x^2 \sin x)$

- $x^2 y'' = -x^4 \cancel{\sin x} + 4x^3 \cancel{\cos x} + 2x^2 \sin x.$
- $-4xy' = -4x^3 \cancel{\cos x} - 8x^2 \sin x$
- $(x^2+6)y = (x^2+6)\, x^2 \sin x$

$= x^4 \cancel{\sin x} + 6x^2 \cancel{\sin x}$

$2x^2 \sin x$
$-8x^2 \sin x$
$-6x^2 \sin x$
$+6x^2 \sin x$

$x^2 y'' - 4xy' + (x^2+6)y) = 0$

3.(i) $y = a\cos(\log x) + b\sin(\log x)$

S.T $x^2 y'' + xy' + y = 0$

$$y = a\cos(\log x) + b\sin(\log x)$$

$$y' = a\,{-\sin}(\log x) \times \frac{1}{x} + b\cos(\log x) \times \frac{1}{x}$$

$$= \cancel{-\frac{a\sin\log x}{x} + \frac{b\cos\log x}{x}} = \cancel{\frac{b\cos\log x - a\sin\log x}{x}}$$

$$\cancel{y'' = x}$$

multiplying by x.

$$xy' = -a\sin(\log x) + b\cos(\log x)$$

Diff. w.r.t. x.

x, x cancel

So ÷ x

$$x \cdot y'' + y' \cdot 1 = -a\cos(\log x) \times \frac{1}{x} + b \times -\sin(\log x) \times \frac{1}{x}$$

multiply by x

$$x^2 y'' + xy' = -(a\cos(\log x) + b\sin(\log x))$$

$$x^2 y'' + xy' = -y$$

$$\Rightarrow x^2 y'' + xy' + y = 0$$

- $y = \sin^{-1} x$, $(1-x^2)y'' - xy' = 0$

$$y = \sin^{-1} x$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

$$y'' = \frac{(\sqrt{1-x^2})^2 \times \frac{d}{dx}1 - 1 \times \frac{d}{dx}\sqrt{1-x^2}}{(\sqrt{1-x^2})^2}$$

$$= \frac{\sqrt{1-x^2} \times 0 - 1 \times \frac{1}{2\sqrt{1-x^2}} \times -2x}{(\sqrt{1-x^2})^2}$$

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$$y' = \frac{\cancel{\sqrt{1-x^2} \times 0 - 1 \times \frac{1}{2\sqrt{1-x^2}} \times -2x}}{\cancel{\left(\sqrt{1-x^2}\right)^2}}$$

$$= \cancel{\sqrt{1-x^2}}$$

$\frac{d}{dx}(y') = y''$

$\frac{d}{dx} x^2 = 2x$

$\frac{d}{dx}(y')^2$

$= 2y' \times y''$

$=$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\, y' = 1$$

squaring, $(1-x^2)(y')^2 = 1$

diff. w.r.to product rule

$$= (1-x^2)\frac{d}{dx}(y')^2 + (y')^2 \frac{d}{dx}(1-x^2) = 0$$

$$= \cancel{0 - 2x} \times$$

$$= (1-x^2)\, 2y' \times y'' + (y')^2 \times -2x = 0$$

$$\div \text{ by } 2y' \Rightarrow (1-x^2) \times y'' - y' x = 0$$

$$= (1-x)^2 \times y'' - y' x = 0$$

$$= (1-x^2)\, y'' - xy' = 0$$

$$\frac{(y')^2 x - \cancel{2}x}{\cancel{2y'}}$$

$$= -y'x$$

HW: $y = \cos^{-1}x$, $(1-x^2)y'' - xy' = 0$

$$y' = \frac{-1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\, . y' = -1$$

squaring $(1-x^2)(y')^2 = 1$

diff. w.r.to. pdt rule

$$(1-x)^2 \frac{d}{dx}(y')^2 + (y')^2 \frac{d}{dx}(1-x^2) = 0$$

$$= (1-x^2)\, 2y' \times y'' + (y')^2 \times -2x = 0$$

$$\div \text{ by } 2y' \Rightarrow (1-x^2) \times y'' \mp y'x = 0$$

$$= (1-x)^2 \times y'' - y'x = 0$$

$$= (1-x^2) \times y'' - xy' = 0$$

$$= \underline{\underline{(1-x^2) \times y''}}$$

$y = x + \frac{1}{x}$, S.T $x^2 y'' + x y' = y$.

$y' = \frac{d}{dx} \cdot 1 + \frac{-1}{x^2}$

$= 1 - \frac{1}{x^2}$

$= \cancel{\frac{x^2 - 1}{x^2}}$

~~multiplying by x^2~~

~~$x^2 y' = \frac{\cancel{x^2}(x^2 - 1)}{\cancel{x^2}}$~~

~~$x^2 y' = x^2 - 1$~~

~~$x^2 \times \frac{d}{dx}(y') + y' \times \frac{d}{dx} x^2 = 2x$~~

~~$x^2 \times y'' + y' \times 2x = 2x$~~

$1/x = \frac{-n}{x^{n+1}}$

$\cancel{y'' = 1 - \frac{1}{x}}$

$y' = 1 - \frac{1}{x^2}$

$\frac{1}{n}$

$y'' = \cancel{\frac{-d \times -2}{x^3}} \quad 0 - \frac{-2}{x^3}$

$y'' = \frac{2}{x^3}$

$x^2 y'' + x y' = x^2 \left(\frac{2}{x^3}\right) + x\left(1 - \frac{1}{x^2}\right)$

$= \frac{2x^{\cancel{2}}}{\cancel{x^3}} + x - \frac{\cancel{x}}{x^{\cancel{2}}}$

$= \frac{2}{x} + x - \frac{1}{x}$

$= \frac{1}{x} + x$

$= y$ //

3/9/19 4) APPLICATIONS OF DIFFERENTIAL CALCULAS

(Module)

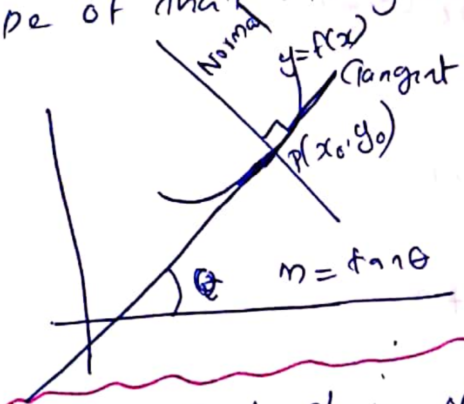
~~Tangents & Normals~~

TANGENTS & NORMALS

(Chapter)

- Given a curve $y=f(x)$ and let $p(x_0, y_0)$ be a given point on this curve, then tangent to the curve $y=f(x)$ at $p(x_0,y_0)$ is the straight line passing through $p(x_0, y_0)$ in the direction of the curve at that point. And normal is the straight line passing through $p(x_0,y_0)$ $\perp^r$ to the tangent.

If a straightline makes an angle 'θ' with 'x' axis then Slope of that straightline is given by, $m = \tan\theta$



* We can find slope of a tangent at a given point using derivatives. slope of the tangent to the curve, $y=f(x)$ $\boxed{\text{at } p(x_0,y_0) = \left(\frac{dy}{dx}\right)_{(at)}(x_0,y_0)}$

- Find slope of tangent to the following curves at given points

1) $y = x^2$ at $(3,9)$

slope of tangent $= \left(\frac{dy}{dx}\right)_{(3,9)}$

$\frac{dy}{dx} = 2x$ at $(3,9)$

$= 2 \times 3$

$= 6$

- $x^2+y^2=25$ at $(-3,4)$

slope of tangent $= \frac{dy}{dx}(-3,4)$

Diff w.r.to x $\quad 2x+2y\frac{dy}{dx}=0$

$$\Rightarrow \cancel{2}y\frac{dy}{dx} = -\cancel{2}x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{dy}{dx}(-3,4) = \frac{-(-3)}{4} = 3/4$$

- $y^2=4ax$ at $(a,2a)$

slope of tangent $= \frac{dy}{dx}(a,2a)$

Diff. w.r.to x

$$2y \times \frac{dy}{dx} = 4a\times 1$$

$$2y \times \frac{dy}{dx} = 4a$$

$$y\frac{dy}{dx} =$$

$$\frac{dy}{dx} = \frac{\cancel{4}^{2}a}{\cancel{2}y}$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

At $(a,2a)$

$$\frac{dy}{dx} = \frac{2a}{2a} = 1$$

• $xy = 2$ at $(1,2)$

Slope of tangent $= \frac{dy}{dx}(1,2)$

Diff. w.r. to x

$$\cancel{x} \cdot 1 \cancel{\frac{dy}{dx}} = 0$$

$$\cancel{\frac{dy}{dx}} =$$

$$x\frac{dy}{dx} + y = 0$$

$$x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\left(\frac{dy}{dx}\right)(1,2) = \frac{-2}{1} = \underline{\underline{-2}}$$

• Find slope of tangent to the curve $x = t^2 + 1$

$y = t^3 - 3t^2 + 3$, at $t = 1$

slope of tangent at $t = 1 = \left(\frac{dy}{dx}\right)_{t=1}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dx}{dt} = 2t$$

$$\frac{dy}{dt} = 3t^2 - 6t$$

$$\frac{dy}{dx} = \frac{3t^2 - 6t}{2t} = \frac{3\cancel{t}(t-2)}{2\cancel{t}}$$

$$= \underline{\underline{\frac{3(t-2)}{2}}}$$

let $t=1$

$$\frac{dy}{dx} = \frac{3(1-2)}{2}$$

$$= \frac{-3}{2}$$

$x = 1^2+1 = 2$

$y = 1^3 - 3\times1^2 + 3 = 1$

$t = 1 \longrightarrow (2, 1)$

$\rightarrow$ $y = \sqrt{25-x^2}$

at $(-3, 4)$

$$\frac{dy}{dx}_{(-3,4)} = \frac{1}{\cancel{2}\sqrt{25-x^2}} \cdot \cancel{2}x$$

$$= \frac{-1}{\sqrt{25-x^2}} \cdot x = \frac{-x}{\sqrt{25-x^2}}$$

$$= \frac{-(-3)}{\sqrt{25-(-3)^2}}$$

$$= \frac{3}{\sqrt{25-9}}$$

$$= \frac{3}{\sqrt{16}}$$

$$= \frac{3}{4}$$

Equation of a tangent a given point

Given a curve $y = F(x)$ let (x_0, y_0) be a given point on this curve then the equation of tangent (x_0, y_0) is given by

$$\boxed{y - y_0 = \left(\frac{dy}{dx}\right)_{(x_0, y_0)} (x - x_0)}$$

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• find slope of tangent to the curve

$$x = \cos 2t$$
$$y = \sin 2t$$

at $t = \pi/8$.

Slope of tangent :-

$$= \frac{dy}{dx} (t = \pi/8)$$

$\tan \pi/4 = 1$
$\cot \pi/4 = 1$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(\sin 2t)}{\frac{d}{dt}(\cos 2t)} = \frac{\cos 2t \cdot 2}{-\sin 2t \cdot 2}$$

$$= \frac{\cancel{2}\cos 2t}{-\cancel{2}\sin 2t}$$

$$= \underline{\underline{-\cot 2t}}$$

At $t = \pi/8$, $\frac{dy}{dx} = -\cot \cancel{2} \times \frac{\pi}{\cancel{8}_4}$

$$= -\cot \frac{\pi}{4} = -1$$

Note: -

at a give point (x_0, y_0),

$$\boxed{\text{Slope of Normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_0, y_0)}}}$$

d Tangent Normal

$m_1 \cdot m_2 = -1$

$\tan g \cdot \text{Normal} = -1$

• find slope of Normal to the curve $xy = 2$ at $(1, 2)$

$$\text{slope of Normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_0, y_0)}}$$

$$= \frac{-1}{\left(\frac{dy}{dx}\right)_{(1,2)}}$$

$\frac{d}{d}$

$\frac{dy}{dx} =$

$xy = 2$

diff. w.r.to x

$$x \cdot \frac{dy}{dx} + y \cdot 1 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

$$\text{Slope of Normal} = \frac{-1}{\left(-y/x\right)_{(1,2)}}$$

$$= \frac{-1}{\left(\frac{-2}{1}\right)}$$

$$= \frac{+1}{+2} = \frac{1}{2}$$

• find Slope of tangent and Normal to the curve $y = t^2+1$

$x = t^3 - 3t^2 + 3$ at $t = 1$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2 - 6t}$$

$$= \frac{2\cancel{t}}{3\cancel{t}(t-2)}$$

$$= \frac{2}{3(t-2)}$$

Slope of Normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{(t=1)}}$

$$= \frac{-1}{\left(\frac{2}{3(t-2)}\right)_{t=1}}$$

BODMAS

$$= \frac{-1}{\frac{2}{3(1-2)}}$$

$$= \frac{-1}{\frac{2}{3-6}}$$

$$= \frac{-1}{\frac{2}{-3}}$$

$$= -1 \times \frac{-3}{2}$$

$$= \frac{3}{2}$$

Slop Norm . Slope. tang = -1

$\frac{3}{2}$. Slop. tangent $= -1$

slope of tangent $= -1 \times \frac{2}{3}$

$= \frac{-2}{3}$

* find equation of tangent & Normal to the curve.

$x^2 + y^2 = 100$ at $(-6, 8)$

$\frac{dy}{dx}(x^2+y^2)$

eqn of tangent $= \quad y - y_0 = \left(\frac{dy}{dx}\right)_{(x_0, y_0)} (x - x_0)$

$y - 8 = \left(\frac{dy}{dx}\right)_{(-6, 8)} (x - -6)$

Now finding $\frac{dy}{dx}$

$\frac{dy}{dx} = x^2 + y^2 = 100$

$2x + 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}y}$

$= \frac{-x}{y}$

at $(-6, 8)$

$\frac{dy}{dx} = \frac{--6}{8}$

$= \frac{6}{8}$

$= 3/4$

substitute

eqn of tangent is

$$y-8 = \frac{3}{4}(x+6)$$

$$4(y-8) = 3(x+6)$$

$$4y-32 = 3x+18$$

$$3x-4y+50=0$$

- in eqn of tangent $y-8 = \frac{3}{4}(x+6)$

take reciprocal and -ve.

eqn of Normal $= y-8 = \frac{-4}{3}(x+6)$

$$3y-24 = -4x-24 \Rightarrow 4x+3y=0$$

- $y = \frac{1}{3+x}$ at $(-4,-1)$ find eqn of tangent & Normal.

eqn of tangent $= y-y_0 = \left(\frac{dy}{dx}\right)_{(x_0,y_0)}(x-x_0)$

$$= y--1 = \left(\frac{dy}{dx}\right)_{(-4,-1)}(x-4)$$

$$= y+1 = \left(\frac{dy}{dx}\right)_{(-4,-1)}(x+4)$$

Now finding $\frac{dy}{dx}$

$$y = \frac{1}{3+x}$$

$$\frac{dy}{dx} = \frac{-1}{(3+x)^2}\cdot 1$$

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

$\frac{d}{dx}(3+x) = 1$

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$\frac{3}{2}$. Slop. tangent $= -1$

slope of tangent $= -1 \times \frac{2}{3}$

$= \frac{-2}{3}$

- Find eqaction of tangent & Normal to the curve.

$x^2 + y^2 = 100$ at $(-6, 8)$

$\frac{dy}{dx}(x^2 + y^2)$

eqn of tangent $= \quad y - y_0 = \left(\frac{dy}{dx}\right)_{(x_0, y_0)} (x - x_0)$

$y - 8 = \left(\frac{dy}{dx}\right)_{(-6, 8)} (x - -6)$

Now finding $\frac{dy}{dx}$

$\frac{dy}{dx} = x^2 + y^2 = 100$

$2x + 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}y}$

$= \frac{-x}{y}$

at $(-6, 8)$

$\frac{dy}{dx} = \frac{--6}{8}$

$= \frac{6}{8}$

$= 3/4$

substitute

eqn of tangent is

$$y-8 = \frac{3}{4}(x+6)$$

$$4(y-8) = 3(x+6)$$

$$4y-32 = 3x+18$$

$$\underline{\underline{3x-4y+50=0}}$$

- in eqn of tangent $y-8 = \left(\frac{3}{4}\right)(x+6)$

take reciprocal and -ve.

eqn of Normal = $y-8 = \frac{-4}{3}(x+6)$

$$3y-24 = -4x-24 \Rightarrow \underline{\underline{4x+3y=0}}$$

- $y = \frac{1}{3+x}$ at $(-4,-1)$ find eqn of tangent & Normal.

eqn of tangent = $y-y_0 = \left(\frac{dy}{dx}\right)_{(x_0,y_0)}(x-x_0)$

$$= y--1 = \left(\frac{dy}{dx}\right)_{(-4,-1)}(x-4)$$

$$= y+1 = \left(\frac{dy}{dx}\right)_{(-4,-1)}(x+4)$$

Now finding $\frac{dy}{dx}$

$$y = \frac{1}{3+x}$$

$$\frac{dy}{dx} = \frac{-1}{(3+x)^2}\cdot 1$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

$$\frac{d}{dx}(3+x) = 1$$

at $(-4,-1)$, $\frac{dy}{dx} = \frac{-1}{(3+-4)^2} = \frac{-1}{(-1)^2} = \underline{\underline{-1}}$

eqn tangent $y+1 = -1(x+4)$

$$\underline{\underline{x+y+5=0}}$$

eqn Normal $y+1 = 1(x+4)$

$\Rightarrow \underline{x-y+3=0}$

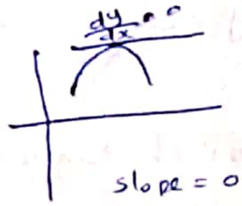
ഈ point substitute satisfy ആവുന്നുണ്ടെങ്കിൽ Ans correct.

HOT ⊙ For what values of x is the tangent to the curve $y = x^3 - 2x^2 + 1$ $\parallel^{l}$ to x axis

→ here
We have to find the values of x where tangent is $\parallel^{l}$ to x-axis
ie, the values of x where slope of tangent $=0$, ie the values of x where $\frac{dy}{dx} = 0$

• x axis ന് $\parallel^{l}$ means slope $=0$

• ഏതൊക്കെ point ൽ $\frac{dy}{dx} = 0$?



slope $= 0$
ie $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 4x$$

Now $3x^2 - 4x = 0$

$\Rightarrow x(3x-4) = 0$

either $x=0$ or $3x-4=0$

$x=0$ or $3x=4 \Rightarrow x = 4/3$

$$\underline{\underline{x = 0,\ 4/3}}$$

$x=0$ ആയ case കൾ ഒരിക്കലും ഒഴിവാക്കരുത്.
eg:- $3x^2 = 4x$
$= 3x = 4$
$x = \frac{4}{3}$
Not correct
also take 0

Q) for what values of x is the tangent to the curve $y = 2x^3 - 9x^2 + 12x - 3$ $\parallel$ to x-axis

→ here, we have to find the values of x where

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 6x^2 - 18x + 12$$

Now $6x^2 - 18x + 12 = 0$

$$= 6(x^2 - 3x + 2) = 0$$

~~either 6 = 0~~ $\Rightarrow x^2 - 3x + 2 = 0$

-3

$-2 - 1$

$$\Rightarrow (x-2)(x-1) = 0$$

$$\Rightarrow (x-1) = 0 \text{ or } (x-2) = 0$$

$$\Rightarrow x = 1, 2$$

• for what values of x is the tangent to the curve $y = \frac{x}{x^2+1}$ parallel to x-axis

→ here, we have to find the values of x where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}(x) - x\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

~~$(a-b)^2 = a^2 - 2ab + b^2$~~

$$= \frac{(x^2+1)\times 1 - x \times 2x}{(x^2+1)^2}$$

$$= \frac{x^2+1 - 2x^2}{(x^2+1)^2}$$

$$\cancel{\frac{x^2+1}{x^2+1}} - \frac{2x^2}{x^2+1}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow \frac{1-x^2}{(x^2+1)^2} = 0$$

$$\Rightarrow 1-x^2=0 \Rightarrow x^2=1$$

$$\Rightarrow x = \pm 1$$

Find values of x where the tangent to the curve $y = \frac{x}{1+x^2}$

• show that all points on the curve $x^3+y^3=3axy$ at which tangents are parallel to x axis lie on the curve $ay=x^2$

Here we have to show that

$$\frac{dy}{dx}=0 \Rightarrow ay=x^2$$

$$x^3+y^3=3axy.$$

$$3x^2+3y^2\frac{dy}{dx} = 3a\left[x \times \frac{dy}{dx} + y \times 1\right]$$

$$3x^2+3y^2\frac{dy}{dx} = 3ax\frac{dy}{dx} + 3ay$$

$$3x^2-3ay = 3ax\frac{dy}{dx} + 3y^2\frac{dy}{dx}$$

$$3x^2-3ay = \frac{dy}{dx}(3ax-3y^2)$$

$$\frac{3x^2-3ay}{3ax-3y^2} = 0$$

$$\frac{\cancel{3}(x^2-ay)}{\cancel{3}(ax-y^2)} = 0$$

$$x^2-ay=0$$

$$x^2=ay$$

- find eqn of tangent and normal to the curve $x = t^2+1$
$y = t^3 - 3t^2 + 3$ at $t = 1$

$$= \frac{dx}{dt} - 2t$$

$$\frac{dy}{dt} = 3t^2 - 6t$$

$$\frac{dx}{dt}(t=1) = 2 \times 1 = \underline{2}$$

$$\frac{dy}{dt}(t=1) = 3 \times 1^2 - 6 \times 1$$
$$= 3-6$$
$$= -3$$

$$\frac{dy}{dx} = -3/2$$

when $t = 1$ $x = 1^2 + 1 = 2$

the $t = 1$ $y = 1^3 - 3 \times 1^2 + 3 = 1 - 3 + 3 = \underline{1}$

$\therefore (x_0 y_0) = (2, 1)$

$\therefore$ eqn of tangent

$$y - y_0 = \frac{dy}{dx}(x - x_0)$$

$$(y-1) = -3/2 (x-2)$$

$$2y - 2 = -3x + 6$$

$$3x + 2y - 2 - 6 = 0$$

eqn of Normal

$$y - y_0 = -\frac{1}{dy/dx}(x - x_0)$$

$$y - 1 = \frac{2}{3}(x-2)$$

$$3y - 3 = 2x - 4$$

$$2x - 3y + 3 - 4 = 0$$

$$2x - 3y - 1 = \underline{\underline{0}}$$

$\rightarrow$ $x = 2t$ $y = 2/t$ at $t = 1$

$$\frac{dx}{dt} = 2 \quad \frac{dy}{dt} = 2 \times \frac{-1}{t^2} = \frac{-2}{t^2} = -2$$

$$\frac{dy}{dx} = \frac{-\cancel{2}}{t^2} \times \frac{1}{\cancel{2}} = \frac{-1}{-2} = 1/2$$

at $t = 1$, $x = 2 \times 1 = 2$

$$y = \frac{2}{1} = 2$$

$$(x_0, y_0) = (2, 2)$$

eqn of tangent

$$y - y_0 = \frac{dy}{dx}(x - x_0)$$

$$y - 2 = \frac{1}{2}(x - 2)$$

$$2y - 4 = -x + 2$$

$$x^2 + y^2 - 4 - 2 = 0$$

$$x + 2y - 4 - 2 = 0$$

$$x + 2y - 6 = \underline{\underline{0}}$$

f.d.li

eqn of Normal

$$(y-2) = 2(x-2)$$

$$y - 2 = x^2$$

19/9/19

~~Rates and motion~~

RATES & MOTION

⊙ Volume of a cube with side 'a' is given by $v = a^3$, then rate of change of volume w.r.to the side is given by

$$\frac{dv}{dA} = 3a^2$$

⊙ area of a circle of radius r, $a = \pi r^2$

rate of change of area w.r.to r $\frac{dA}{dr} = \pi \cdot 2r$
$= 2\pi r$

⊙ find the rate of change of volume of a sphere w.r.to its radius.

Volume of a sphere $v = \frac{4}{3}\pi r^3$

rate of change of volume w.r.to the radius

$$\frac{dv}{dr} = \frac{4}{\cancel{3}}\pi \cancel{3}r^2$$
$$= 4\pi r^2$$

• work done by a moving body is expressed as $w = 2t - \frac{3}{t}$ over an interval of 3 seconds. Find the power of the force creating the motion, at $t = 2$ seconds

→ $w = 2t - \frac{3}{t}$

$$P = \frac{dw}{dt}$$

$$P = 2 + \frac{3}{t^2}$$

power when $t = 2sec$ $= 2 + \frac{3}{2^2}$

$= 2 + \frac{3}{4}$

$= \frac{11}{4}$

Displacement, velocity and acceleration

let 'S' be the displacement of a moving particle in time 't', then it's velocity

$$v = \frac{ds}{dt}$$

it's acceleration $a = \frac{dv}{dt}$

$$= \frac{d}{dt}\left(\frac{ds}{dt}\right)$$

$$= \frac{d^2 s}{dt^2}$$

- The displacement of a moving particle over a interval of time 't' second is given by $S = 3t^2 - 2t + 1$ find it's velocity and acceleration, when $t = 2sec$.

→ $v = \frac{ds}{dt}$

$v = \frac{d}{dt}(3t^2 - 2t + 1)$

$= 6t - 2$

$\frac{dv}{dt} = 6t - 2$

when $t = 2sec$

$v = 6 \times 2 - 2$

$= 12 - 2$ $= 10 \ m/s.$

acceleration $a = \frac{dv}{dt} = \frac{d}{dt}(6t - 2)$

$= \underline{\underline{6}} \ m/s^2$

- find velocity & acceleration when t = 3 seconds $S = 4t^2 - 3t + 2$

$v = \frac{ds}{dt}$

$= \frac{d}{dt}(4t^2 - 3t + 2)$

$= 4 \times 2t - 3 \times 1$

$= 8t - 3$

when t = 3 seconds

$= 8 \times 3 - 3$

$= 24 - 3$

$v = \underline{\underline{21}} \ m/s$

$a = \frac{dv}{dt}$

$= \frac{d}{dt}(8t - 3)$

$= 8 \ m/s^2$

Qn The displacement of a moving particle is given by

$s = t^3 - 6t^2 + 8t - 4$

a) find the time 't' when acceleration a = 12 (m/s²)

b) velocity at this time

→ $a = \frac{d^2s}{dt^2}$ $v = \frac{ds}{dt}$

$= 3t^2 - 12t + 8$

Now, $a = \frac{dv}{dt} = 6t - 12$

when a = 12, ie $6t - 12 = 12$

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$\Rightarrow 12+12 = 6t$

$\Rightarrow 24 = 6t$

$t = \frac{24}{6}$

$= 4$.

b) $v = \frac{ds}{dt}$

$= 3t^2 - 12t + 8$

$= 3 \times (4)^2 - 12 \times 4 + 8$

$= 3 \times 16 - 48 + 8$

$= 48 - 48 + 8$

$= 8 \text{ m/sec}$

- A particle move such that displacement from a fixed point 'O' is always given by . $S = 5\cos nt + 4\sin nt$ where 'n' is a constant. P.T acceleration varies as it's displacement

→ here, we have to S.T acceleration is directly propotional to the displacement or $a \propto s$

or $a = ks$

where k is a constant

$\sin 2x$

$2\cos 2x$

$S = 5\cos nt + 4\sin nt$

$v = \frac{ds}{dt}$

$= \frac{d}{dt}(5\cos nt + 4\sin nt)$

$= 5n \cdot -\sin nt + 4n \cos nt$.

$\Rightarrow -5n \cdot \sin nt + 4n \cos nt$.

~~$= 5n \times -\sin t + 4n \times \cos t$~~

~~$= -5n\sin t + 4n\cos t$~~

~~$= 4n\cos t - 5n\sin t$~~

$$a = \frac{d^2 s}{dt^2}$$

$$= \frac{dv}{dt}$$

$$= \frac{d}{dt}(4n\cos t - 5n\sin t)$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(-5n \cdot \sin nt + 4n \cos nt).$$

$$\Rightarrow (-5n \cdot \cos nt \cdot n - 4n \cdot \sin nt \cdot n)$$

$$\Rightarrow -5n^2 \cdot \cos nt - 4n^2 \sin nt$$

$$\Rightarrow -n^2(5\cos nt - 4\sin nt).$$

$$\Rightarrow \underline{\underline{-n^2 \cdot s}}$$

- Distance 's' travelled by a particle in t 'seconds' is given by. $s = ae^{nt} + be^{-nt}$. S.T acceleration varies as it's displacement

→ $s = ae^{nt} + be^{-nt}$ $\qquad n^2 = k$

$$v = \frac{ds}{dt}$$

$$= \frac{d}{dt}(ae^{nt} + be^{-nt})$$

$$= ane^{nt} - bne^{-nt}$$

$$\frac{dv}{dt} = \frac{d}{t}\, ane^{nt} - bne^{-nt}$$

$$= an \cdot ne^{nt} - bn \times ne^{-nt}$$

$$\Rightarrow an^2 e^{nt} + bn^2 e^{-nt}$$

$$\Rightarrow n^2(ae^{nt} + be^{-nt})$$

$$\Rightarrow \underline{\underline{n^2 s}}$$

$a \qquad a = ks$

Related Rates

If 2 variables are related then their rates of change are also related. If the rate of change of one variable is given, then, we can find the rate of change of the other variable. For example; consider the area of a circle of radius 'r'

$$a = \pi r^2$$

$$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$$

$$\boxed{\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}}$$

if $\frac{dr}{dt}$ is known then we can find $\frac{dA}{dt}$ and vice versa.

- $v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$= \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$$

$$\boxed{= 4\pi r^2 \frac{dr}{dt}}$$

6/6/19.

- The air pumbed into a spherical rubber bladder of 3 inches. If the radius rises at a uniform of 1 inch/minute find the rate at which the volume rises at the end of 3 minutes

$$v = 4/3\, \pi r^3$$

$$\Rightarrow \frac{dv}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$$

$$= 4\pi r^2 \frac{dr}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

given that $\frac{dr}{dt} = 1$

the radius r = 3 by the end of 3 minutes the radius will be $3+3\times1 = 6$

minute
1 - 3
~~3 minute~~

$$\frac{dv}{dt} = 4\pi \cdot 6^2 \cdot 1$$

$$= 4\pi \times 36$$

$$= 144\pi$$

- A baloon is spherical in shape gas is escaping from it at the rate of 20 c c persecond. How fast is the surface area shrinking, when the radius is 15cm.

$$\frac{dv}{dt} = -20\ c.c/s$$

−ve becaus of volume ↓ses

$r = 15$ cm.

$S = 4\pi r^2$

$$\frac{ds}{dt} = 4\pi \times 2r \frac{dr}{dt}$$

$$\Rightarrow \frac{ds}{dt} = 8\pi r \frac{dr}{dt}$$

To find $\frac{dr}{dt}$, $\frac{dv}{dt} = -20$

$V = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-20 = 4\pi \cdot 15^2 \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-20}{4\pi \cdot 15^2} \quad — (1)$$

put (1) in $\frac{ds}{dt} = 8\pi r \frac{dr}{dt}$

$$\frac{ds}{dt} = \cancel{8}^{2}\cancel{\pi} \cdot \cancel{15} \cdot \frac{-20}{\cancel{4}\cancel{\pi}\, 15^2} = \frac{-40}{15} = \frac{-8}{3}$$

hence the surface area is shrinking at the rate of $\frac{8}{3}$ cm^2/sec

• A spherical baloon is ~~insulated~~ inflated with air, so that it's volume rises at the rate of 5 c.c/sec. find the rate at which it's curved surface area is ~~rising~~ rises when the radius is 7cm.

$$\frac{dv}{dt} = 5 \text{ cc/sec}$$

$$r = 7cm$$

$$S = 4\pi r^2$$

$$\frac{ds}{dt} = 4\pi \times 2r \frac{dr}{dt}$$

$$\Rightarrow \frac{ds}{dt} = 8\pi r \frac{dr}{dt}$$

to find $\frac{dr}{dt}$, $\frac{dv}{dt} = 5$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4\pi \times 7^2 \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{5}{4\pi \times 7^2}$$

$$\frac{ds}{dt} = \cancel{8}^{2}\cancel{\pi}\cdot\cancel{7} \cdot \frac{5}{\cancel{4}\cancel{\pi} \times 7^{\cancel{2}}}$$

$$= \frac{2 \times 5}{7} = \frac{10}{7} \text{ cm}^2/\text{sec}$$

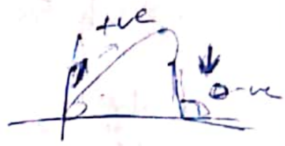
hence the surface Area is ~~shr.~~ enlarged at the rate of $\frac{10}{7}$ cm^2/sec

MAXIMA AND MINIMA

(chapter)

- given a function $y = f(x)$ and let (x_0, y_0) be a point on this curve $y = f(x)$.

⊙ The function is increasing at (x_0, y_0) if $\frac{dy}{dx}_{(x_0, y_0)} > 0$



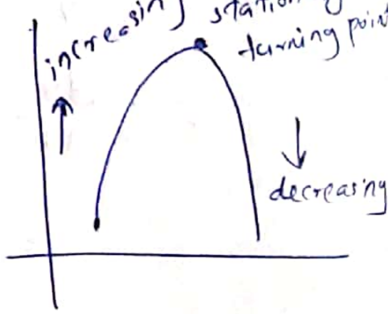
⊙ The function is ↓sing at (x_0, y_0)

if $\frac{dy}{dx}_{(x_0, y_0)} < 0$

⊙ if $\frac{dy}{dx}_{(x_0, y_0)} = 0$ then (x_0, y_0) is a stationary point or turning point.

eg:- check whether the functions are ↑sing or ↓sing at given point.

a) $y = x^2 + 2x + 1$, at $x = -4$



$\frac{dy}{dx} = 2x + 2$

at $x = -4$

$\frac{dy}{dx} = 2 \times -4 + 2$

$= -6 + 2$

$= -6 \quad < 0$

so, it is a ↓sing function.

b) $y = 3x^3 + 5x^2 - 7$

at $x = 2, -2$

$\frac{dy}{dx} = 9x^2 + 10x$

at $x = 2$ $\quad \frac{dy}{dx} = 9 \times (2)^2 + 10 \times 2$

$= 9 \times 4 + 20 \quad = 36 + 20 \quad = 56 > 0$ ↑sing

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at $x=-2$

$$\frac{dy}{dx} = 9\times(-2)^2 + 10\times -2$$

$$= 9\times 4 - 20$$

$$= 36-20$$

$$= \underline{16 > 0}$$ ↑sing function.

- find the range of values of 'x' for which the function $y = x^2 - 3x + 4$ is ↑sing.

→ we have to find the range of values of x for which

$$\frac{dy}{dx} > 0$$

$$\frac{dy}{dx} = 2x-3$$

$$\frac{dy}{dx} > 0 \Rightarrow (2x-3) > 0$$

$$\Rightarrow 2x > 3$$

$$\Rightarrow x > \frac{3}{2}$$

3/2 2...

$(3/2, \infty)$

~~Find the turning values of the given functions.~~

Stationary point

★ if $\frac{dy}{dx} = 0$ at $x=a$, then a is called a turning point or stationary point

The value of the function at $x=a$ ie $f(a)$ is called turning value.

• find durning points and durning values of the given functions.

$y = 2x^3 - 9x^2 + 12x + 2$

$x^2 - 3x + 2$

$\frac{dy}{dx} = 6x^2 - 18x + 12$

$-1, -2 = 2$

$(x-1)(x-1)$

$\frac{dy}{dx} = 0$

ie $6x^2 - 18x + 12$

24×12

$= 0$

$\Rightarrow 6(x^2 - 3x + 2) = 0$

~~$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$~~

$\Rightarrow x^2 - 3x + 2 = 0$

~~$= \frac{-18 \pm \sqrt{(-18)^2 - 4 \times 6 \times 12}}{2 \times 6}$~~

$\Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x - 1 = 0, \; x - 2 = 0$

~~$= +18 \pm \sqrt{\;} -$~~

$\Rightarrow x = 1, 2$

Turning points are 1, 2

Turning values

$f(x) = 2x^3 - 9x^2 + 12x + 2$

$f(2) = 2 \times 2^3 - 9 \times 2^2 + 12 \times 2 + 2$

$= 16 - 36 + 24 + 2$

$= \underline{\underline{6}}$

$f(1) = 2 \times 1^3 - 9 \times 1^2 + 12 \times 1 + 2$

$= 7$

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• $y = x^3 - 3x^2 - 9x + 5$

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

$$\frac{dy}{dx} = 0$$

ie $3x^2 - 6x - 9 = 0$

$$3(x^2 - 2x - 3) = 0$$

$$x^2 - 2x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(-2)^2 - 4 \times 1 \times -3}}{2 \times 1}$$

$$= \frac{+2 \pm \sqrt{4 - (4 \times -3)}}{2}$$

-4×-3
$= 12$

$$= \frac{+2 \pm \sqrt{16}}{2}$$

$$= \frac{+2 \pm \sqrt{\cancel{16}}\,4}{2}$$

$=$ $x = \frac{6}{2}$ or $\frac{-2}{2}$

~~$x = 8$ or 4~~ $x = 3$ or -1

~~$x = 8$ or $x = 4$~~

$f(-1) = \cancel{(-1)^2 - 2\times -1 - 3} = 10$

$f(3) = \cancel{(3)^2 - 2\times 3 - 3 = 9 - 6 - 3}$ -22

$= 9-$

1/10/19.

Maximum point and minimum point

*maximum point

1 → $\frac{dy}{dx} = 0$

2 → $\frac{d^2y}{dx^2} < 0$

* Minimum point

1 → $\frac{dy}{dx} = 0$

2 → $\frac{d^2y}{dx^2} > 0$

• find max. value of the function $y = 2x^3 - 9x^2 + 12x + 2$

~~put~~ $\frac{dy}{dx} = 0$?

$\frac{dy}{dx} = 6x^2 - 18x + 12$

finding the stationary points

$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 18x + 12 = 0$

$\Rightarrow 6(x^2 - 3x + 2) = 0$

$\Rightarrow (x^2 - 3x + 2) = 0$

$\Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x = 1, 2$

Which are the stationary points.

$$\frac{d^2y}{dx^2} = 12x - 18$$

At $x = 1$, $\frac{d^2y}{dx^2} = 12 \times 1 - 18$

$= 12 - 18$

$= -16 < 0$

ie, $x = 1$ is a maximum point.

~~and $x = 2$ is the min~~

When $x = 1$ $f(1) = 2 \times 1^3 - 9 \times 1^2 + 12 \times 1 + 2$

$= 2 - 9 + 12 + 2$

$= 16 - 9$

$= \underline{\underline{7}}$

- find the minimum value of $x^3 - 3x^2 - 9x + 5$

$\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

finding stationary points

$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow 3(x^2 - 2x - 3) = 0$

$\Rightarrow x^2 - 2x - 3$

$\Rightarrow (x + 3)(x - 1)$

$\Rightarrow x = 3, -1$

which are the stationary points

$$\frac{d^2y}{dx^2} = 6x - 6$$

At $x = -1$, $\frac{d^2y}{dx^2} = 6 \times -1 - 6$

$= -6 - 6$

$= -12 < 0$

$x = -1$ is a max. point

When $x = 3$, $\frac{d^2y}{dx^2} = 6 \times 3 - 6$

$= 18 - 6$

$= 12 > 0$

$x = 3$ is a min·point

mini. value of $f(x) = f(3)$

$$3^3 - 3\times3^2 - 9\times3 + 5$$

$$-27 + 5 \over -22$$

$$= 27 - 3\times9 - 27 + 5$$

$$= 27 - 27 - \cancel{27} + 5$$

$$= -22$$

• Deflection of a beam is given by $y = 4x^3 + 9x^2 - 12x + 2$

find max. Deflection.

$$\frac{dy}{dx} = 12x^2 + 18x - 12.$$

finding the stationary points

$$\frac{dy}{dx} = 0 \Rightarrow 12x^2 + 18x - 12 = 0$$

6+

$$\Rightarrow 6(2x^2 + 3x - 2) = 0$$

$$\Rightarrow 2x^2 + 3x - 2 = 0$$

-1×2

$$x = \frac{-3 \pm \sqrt{3^2 - 4\times2\times-2}}{2\times2}$$

$$x = \frac{-3 \pm \sqrt{9 - -16}}{4} = \frac{-3 \pm \sqrt{25}}{4}$$

$$x = \frac{-3+5}{4} \text{ or } \frac{-3-5}{4}$$

$$x = \frac{2}{4} \text{ or } \frac{-8}{4}$$

$$x = \frac{1}{2} \text{ or } -2$$

These are the stationary points.

$24\times2 = 48$, $48 - 18 = 30$

$$\frac{d^2y}{dx^2} = 24x + 18$$

at $x = -2$, $\frac{d^2y}{dx^2} = 24\times-2 + 18$

$$= -48 + 18 = -30 < 0$$

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$x = -2$ is the maximum point

when $x = -2$, $f(-2) = 12 \times (-2)^2 + 18 \times -2 - 12$

$12 \times 4 + -36 - 12$

$= 48 - 36 - 12$

$= 48 - 48$

$= 36.$

$$\begin{array}{r} 18\times \\ 2 \\ \hline 36 \end{array} \quad \begin{array}{r} 12\times \\ 4 \\ \hline 48 \end{array} \quad \begin{array}{r} -36+ \\ -12 \\ \hline 48 \end{array}$$

$f(-2) = 4x^3 + 9x^2 - 12x + 2$

$= 4(-2)^3 + 9(-2)^2 - 12(-2) + 2$

$= 4 \times -8 + 9 \times 4 - -24 + 2$

$= -32 + 36 + 24 + 2$

$= 62 - 32$

$= \underline{\underline{30}}$

$$\begin{array}{r} 36+ \\ 24 \\ \hline 60 \\ 62- \\ 32 \\ \hline 30 \end{array}$$

- The bending moment of a rode of length 10m with weight 40kg and resting at it's ends at a distance of x m from one end is given by $M = 2(10x - x^2)$ kgm. find the max. bending moment.

$M = 20x - 2x^2$

$M = 2(10x - x^2)$

$\frac{dM}{dx} = \cancel{20x - 2x}$

$= 20 - 4x$

$\frac{d}{dx} = 0 \rightarrow$ stationary points

finding stationary point,

$\frac{dM}{dx} = 0 \Rightarrow 20 - 4x = 0$

$\Rightarrow 4x = 20 \Rightarrow x = 5$

whis the stationary point.

$\frac{d^2m}{dx^2} < 0$, $x = 5 \rightarrow$ (is the maximum point)

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max value of $M = 20 \times 5 - 2 \times 5^2$

$= 100 - 50$

$= 50$ kgm

- S.P a rectangle of fixed perimeter as its max. Area when it becomes ~~are~~ a square.

let 'x' be length

y be the breadth

then it's area, $A = xy$ —(1)

given that perimeter is a constant

$$2x + 2y = l$$

perimeter, $2x + 2y = l$

$\Rightarrow 2y = l - 2x$

$\Rightarrow y = l/2 - x$ sub in (1)

$\Rightarrow A = x(l/2 - x)$

$$A = \frac{l}{2}x - x^2$$

we have to find the values of x so that A is maximum finding stationary points

$$\frac{dA}{dx} = \frac{l}{2} - 2x$$

$$\frac{dA}{dx} = 0 \Rightarrow l/2 - 2x = 0$$

$\Rightarrow 2x = l/2 \Rightarrow x = l/2$

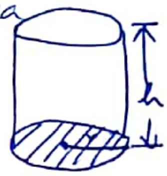
$$\frac{d^2A}{dx^2} = -2 < 0 \text{ when } x = l/4$$

ie, $x = l/4$ is a max. point

Area is max. when $x = l/4$

- A hollow cylindrical cann 100 c.c of water is to be made so that the area of the metal used is minimum. P.T the radius which will give minimum area is ~~cuberoot~~ $\sqrt[3]{\frac{100}{\pi}}$ cm

Total surface area = Base Area + curved surface area



$$S = \pi r^2 + 2\pi r h \quad \text{---} (1)$$

Now, given that, volume, $V = 100$

ie $\pi r^2 h = 100$

$$\Rightarrow h = \frac{100}{\pi r^2} \quad \text{sub in } (1)$$

$$\text{ie. } S = \pi r^2 + 2\cancel{\pi}\cancel{r}.\frac{100}{\cancel{\pi} r^{\cancel{2}}}$$

$$S = \pi r^2 + \frac{200}{r}$$

finding stationary points

$$\frac{ds}{dr} = 2\pi r - \frac{200}{r^2}$$

$$\frac{ds}{dr} = 0 \Rightarrow 2\pi r - \frac{200}{r^2} = 0$$

$$\Rightarrow \cancel{2}\pi r = \frac{\cancel{200}\,100}{r^2}$$

$$\Rightarrow \pi r = \frac{100}{r^2}$$

$$\Rightarrow r^3 = \frac{100}{\pi}$$

$$\text{or} \quad r = \sqrt[3]{\frac{100}{\pi}}$$

Which is the stationary point.

$$\frac{d^2s}{dr^2} = 2\pi - 200 \times \frac{-2}{r^3}$$

$$= \cancel{2\pi + \frac{400}{r^3}} = 2\pi + \frac{400}{r^3}$$

when $r = \sqrt[3]{\frac{100}{\pi}}$, ie, $r^3 = \frac{100}{\pi}$

$$\frac{d^2s}{dr^2} = 2\pi + \frac{4\cancel{00}}{\frac{1\cancel{00}}{\pi}}$$

$$= 2\pi + 4\pi$$

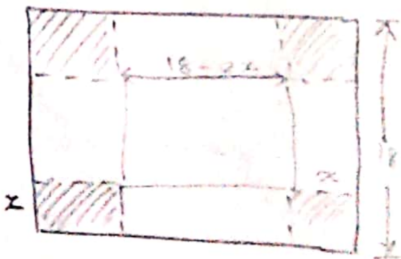
$$= 6\pi > 0$$

ie, $r = \sqrt[3]{\frac{100}{\pi}}$ is the minimum point

$\Rightarrow$ s is minimum, when $r = \sqrt[3]{\frac{100}{\pi}}$

An open box is to be made out of a Square sheet of side 18cm by cutting of equal squares at each corner and turning up the sides. What size of the squares should be cut in order that volume of the box may be maximum

$\rightarrow$ volume of the box = Base Area × height



Base Area = $(18-2x)^2$

height = x

$v = (18-2x)^2 \cdot x$

finding the stationary points

$v = x(18-2x)^2$

$$\frac{dv}{dx} = \cancel{x(18-2x)}^2$$

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$$\frac{dv}{dx} = x(324 - 72x + 4x^2)$$

$$= 324x - 72x^2 + 4x^3$$

$$\frac{dv}{dx} = 324 - 144x + 12x^2$$

$$\frac{dv}{dx} = 0$$

ie $(324 - 144x + 12x^2) = 0$

$12(27 - 12x + x^2) = 0$

ie $27 - 12x + x^2 = 0$

$x^2 - 12x + 27 = 0$

$\Rightarrow (x-3)(x-9) = 0$

$\Rightarrow x = +3, +9$ which are the stationary point

if $x = 9$ is not possible, ie $x = 3$

Now $\frac{d^2v}{dx^2} = 24x - 144$

at $x = 3$,

$$\frac{d^2v}{dx^2} = 24 \times 3 - 144$$

$$= 72 - 144$$

$$= \underline{\underline{-72}}$$

ie $x = 3 < 0$ is the maximum point

Hence, the volume of the box is maximum when $x=3$, if we cutoff squares of side 3

when $x = 3$

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