Integration

Suppose that $f(x)$ and $g(x)$ are two functions such that $\frac{d}{dx} f(x) = g(x)$, then we say that the integral of $g(x)$ with respect to x is $f(x)$.

In sympol we write this as $\int g(x)\,dx = f(x)$

RESULT :

1) $\frac{d}{dx} \sin x = \cos x$

$\therefore \int \cos x \, dx = \sin x$

2) $\frac{d}{dx} \cos x = -\sin x$

$\therefore \int \sin x \, dx = -\cos x \quad \left(\int -\sin x \, dx = \cos x \right)$

3) $\frac{d}{dx} \tan x = \sec^2 x$

$\therefore \int \sec^2 x \, dx = \tan x$

4) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\therefore \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x$

5) $\frac{d}{dx} \sec x = \sec x \tan x$

$\therefore \int \sec x \tan x \, dx = \sec x$

6) $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\therefore \int \operatorname{cosec}^2 x \, dx = -\cot x$

7) $\frac{d}{dx}\log x = \frac{1}{x}$

$\therefore \int \frac{1}{x} dx = \log x$

8) $\frac{d}{dx} e^x = e^x$

$\therefore \int e^x dx = e^x$

9) $\frac{d}{dx} \frac{1}{x} = \frac{-1}{x^2}$

$\therefore \int \frac{1}{x^2} dx = -\frac{1}{x}$

10) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$\int 3^x$

$\therefore \int \frac{1}{2\sqrt{x}} dx = \sqrt{x}$

$= \int \frac{1}{\sqrt{x}} = 2\sqrt{x}$

11) $\frac{d}{dx}(x^{n+1}) = (n+1)x^{n+1-1} = (n+1)x^n$

$\therefore \int (n+1)x^n dx = x^{n+1}$

$$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$$

Note

→ $\int (f_1(x) \pm f_2(x) \pm f_3(x) \pm \dots) dx =$

$\int f_1(x) dx \pm f$

Q integrate the following with respect to x

1) $2 \sin x$

an $\int 2 \sin x \, dx = 2 \int \sin x \, dx = -2 \cos x + c$

2) $5 \sec^2 x$

an $\int 5 \sec^2 x \, dx = 5 \int \sec^2 x \, dx = 5 \tan x + c$

3) $4 \sec x \tan x$

an $\int 4 \sec x \tan x \, dx = 4 \int \sec x \tan x = 4 \sec x + c$

4) $8 \operatorname{cosec} x \cot x$

an $\int 8 \operatorname{cosec} x \cot x \, dx = 8 \int \operatorname{cosec} x \cot x \, dx = -$

$-8 \operatorname{cosec} x + c$

5) $10\cos x$

an $\int 10\cos x\,dx = 10\int \cos x\,dx = 10\sin x + C$

6) $2e^x$

an $\int 2e^x\,dx = 2\int e^x\,dx = 2e^x + C$

7) $3x^2 + 4x + 6$

an $\int (3x^2 + 4x + 6)\,dx = 3 \cdot \frac{x^{2+1}}{2+1} + 4 \cdot \frac{x^{1+1}}{1+1} + 6x + c$

$\frac{\cancel{3}x^3}{\cancel{3}} + \cancel{4}\frac{x^2}{\cancel{2}} + 6x + c = x^3 + 2x^2 + 6x + c$

8) $3\sec^2 x + 4\sin x + e^x$

an $\int (3\sec^2 x + 4\sin x + e^x)\,dx = 3\tan x - 4\cos x + e^x + c$

9) $(x+1)^2$

an $\int (x+1)^2\,dx = \int (x^2 + 1 + 2x)\,dx$

$\int (x^2 + 2x + 1)\,dx = \frac{x^{2+1}}{2+1} + 2 \cdot \frac{x^{1+1}}{1+1} + x + c$

$$= \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + x + C$$

$$= \frac{x^3}{3} + x^2 + x + C$$

10) $(x+1)(x+2)$

an $\int((x+1)(x+2))\,dx = \int (x^2+2x+x+2)\,dx$

$$= \int (x^2+3x+2)\,dx = \frac{x^{2+1}}{2+1} + 3\frac{x^{1+1}}{1+1} + 2x + C$$

$$= \frac{x^3}{3} + 3\frac{x^2}{2} + 2x + C$$

11 $(x+1)(x+2)(x+3)$

$(x^2+3x+2)(x+3)$

$= x^3 + 3x^2 + 2x + 3x^2 + 9x + 6$

an $\int (x^3+6x^2+11x+6)\,dx$

$$= \frac{x^{3+1}}{3+1} + 6\frac{x^{2+1}}{2+1} + 11\frac{x^{1+1}}{1+1} + 6x$$

$= x^3 + 6x^2 + 11x + 6$

$$= \frac{x^4}{4} + \frac{\cancel{6}x^3}{\cancel{3}} + \frac{11x^2}{2} + 6x + C$$

$$= \frac{x^4}{4} + 2x^3 + \frac{11x^2}{2} + 6x + C$$

12 $(x^2+a^2)^2$

an $\int (x^2+a^2)^2\,dx = \int (x^4+2x^2a^2+a^4)\,dx$

$= \frac{x^{4+1}}{4+1} + 2a^2\frac{x^{2+1}}{2+1} + a^4x + c$

$= \frac{x^5}{5} + 2a^2\frac{x^3}{3} + a^4x + c$

13 $\sqrt{x}(x-1)$

an $\int \sqrt{x}(x-1)\,dx = \int (x\sqrt{x} - \sqrt{x})\,dx$

$\sqrt{x}\cdot x = x^{1/2}\cdot x^1 = x^{\frac{1}{2}+1} = x^{3/2}$

$\int (x^{3/2} - x^{1/2})\,dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{3}{2}}}{3} + c$

14 $4e^x + \frac{2}{x\sqrt{x}}$

an $\int \left(4e^x + \frac{2}{x\sqrt{x}}\right)dx =$

$$\int\left(4e^x+\frac{2}{x\sqrt{x}}\right)dx = \int\left(4e^x+\frac{2}{x^{3/2}}\right)dx$$

$$\int\left(4e^x+2x^{-3/2}\right)dx = 4e^x+2\frac{x^{-3/2+1}}{\frac{-3}{2}+1}+c$$

$$= 4e^x+2\frac{x^{-1/2}}{-1/2}+c = 4e^x+\frac{2\times 2}{-1}x^{-1/2}$$

$$= 4e^x-4x^{-1/2}+c = 4e^x-\frac{4}{x^{1/2}}+c$$

$$= 4e^x-\frac{4}{\sqrt{x}}+c \qquad = 4\left(e^x-\frac{1}{\sqrt{x}}+c\right)$$

H.W 15 $(2x+1)(3x-4)$

16 $x(x-1)(x-2)$

1) find $\int\left(\frac{x^3+5x^2+x-1}{x}\right)dx$

$\int\frac{x^3}{x}dx+\int\frac{5x^2}{x}dx+\int\frac{x}{x}dx-1\int\frac{1}{x}dx$

$=\int x^2\,dx+5\int x\,dx+\int dx-1\int\frac{1}{x}\,dx$

$=\frac{x^3}{3}+5\frac{x^2}{2}+x-\log|x|+c$ //

2 find $\int\left(\frac{3x^4-1}{x}\right)dx$

an $\int\left(\frac{3x^4}{x}-\frac{1}{x}\right)dx$

$=\int\frac{3x^4}{x}dx-\int\frac{1}{x}dx$

$=3\int x^3\,dx-\log x+c$

$=3\frac{x^4}{4}-\log x+c$ //

3 $\int\left(\frac{3x-1}{x^4}\right)dx$

an $\int\left(\frac{3x}{x^4}-\frac{1}{x^4}\right)dx$

$$3\int \frac{x}{x^4}\,dx - \int \frac{1}{x^4}\,dx$$

$$= 3\int \frac{1}{x^3}\,dx - \int x^{-4}\,dx$$

$$= 3\int x^{-3}\,dx - \int x^{-4}\,dx$$

$$= 3\times \frac{x^{-3+1}}{-3+1} - \int \frac{x^{-4+1}}{-4+1}\,d$$

$$= 3\,\frac{x^{-2}}{-2} - \frac{x^{-3}}{-3}$$

$$= -3\,\frac{x^{-2}}{2} + \frac{x^{-3}}{3} + c$$

$$= \frac{-3}{2}\times\frac{1}{x^2} + \frac{1}{3x^3} + c$$

4 $$\int\left(\frac{2x^2-3x+1}{x^2}\right)dx$$

an $$\int\left(\frac{2x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2}\right)dx$$

$$= \int \frac{2x^2}{x^2}\,dx - \int \frac{3x}{x^2}\,dx + \int \frac{1}{x^2}\,dx$$

$$= 2\int 1\,dx - 3\int \frac{1}{x}\,dx + x^{-2}\,dx$$

$$= 2x - 3\log x + \frac{x^{-1}}{-1} + C$$

$$= 2x - 3\log x - \frac{1}{x} + c$$

5 $\displaystyle\int \left(\frac{x^3+4x^2-3x+1}{x}\right)dx$

an $\displaystyle\int \left(\frac{x^3}{x} + \frac{4x^2}{x} - \frac{3x}{x} + \frac{1}{x}\right) dx$

$$= \int \frac{x^3}{x}\,dx + 4\int \frac{x^2}{x}\,dx - 3\int \frac{x}{x}\,dx + \int \frac{1}{x}\,dx$$

$$= \int x^2\,dx + 4\int x\,dx - 3\int 1\,dx + \log x$$

$$= \frac{x^3}{3} + \frac{\cancel{4}^{2}x^2}{\cancel{2}} - 3x + \log x + c$$

$$= \frac{x^3}{3} + 2x^2 - 3x + \log x + c$$

① $\int \tan^2 x \, dx$

$1+\tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

apply this when integration

an $\int (\sec^2 x - 1)\, dx$

$\int \sec^2 x \, dx - \int 1 \, dx$

$= \tan x - x + c$ //

② $\int \cot^2 x$

$1+\cot^2 x = \operatorname{cosec}^2 x$

$\cot^2 x = \operatorname{cosec}^2 x -$

an $\int (\operatorname{cosec}^2 x - 1)\, dx$

$= \int \operatorname{cosec}^2 x - \int 1 \, dx$

$= -\cot x - x + c$ //

③ $\int (\tan x + \cot x)^2 dx$

$\int (\tan^2 x + \cot^2 x + 2\tan x \cot x)\, dx$

$= \int (\tan^2 x + \cot^2 x + 2)\, dx$

$= \int (\sec^2 x - 1)\, dx + \int (\operatorname{cosec}^2 x - 1)\, dx + 2\int dx$

$= \tan x - x - \cot x - x + 2x + c$

$= \tan x - \cot x - 2x + 2x + c$

$= \tan x - \cot x + c$

<u>Result</u>

→ $\int \sin(ax+b)\,dx = \dfrac{-\cos(ax+b)}{a} + c$

→ $\int \cos(ax+b)\,dx = \dfrac{\sin(ax+b)}{a} + c$

→ $\int \sec(ax+b)\tan(ax+b)\,dx = \dfrac{\sec(ax+b)}{a} + c$

→ $\int e^{ax+b}\,dx = \dfrac{e^{ax+b}}{a} + c$

→ $\int \dfrac{1}{ax+b}\,dx = \dfrac{\log|ax+b|}{a} + c$

imp $\int \sqrt{1+\sin 2x}$ with respect to x

an = $\int \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}$

$= \int \sqrt{(\sin x + \cos x)^2}\,dx = \int (\sin x + \cos x)\,dx$

$= -\cos x + \sin x + c$ //

Q $\int \sqrt{1-\sin 2x}\, dx$

an $\int \sqrt{\sin^2 x + \cos^2 x - \sin 2x}\, dx$

$\int \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x}\, dx$

$\int \sqrt{(\sin x - \cos x)^2}\, dx = \int \sin x - \cos x\, dx$

$= -\cos x - \sin x + c$ //

Q $\sqrt{1+\sin x}$

an $\int \sqrt{1+\sin x}\, dx$

$\int \sqrt{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}\, dx$

$\int \sqrt{(\sin x/2 + \cos x/2)^2}\, dx = \int \sin x/2 + \cos x/2$

$= -\dfrac{\cos x/2}{1/2} + \dfrac{\sin x/2}{1/2} + c = -2\cos \dfrac{x}{2} + 2\sin x/2 + c$ //

Q $\sqrt{1-\sin x}$

an $\int \sqrt{1-\sin x}\, dx$

$\int \sqrt{\sin^2 x/2 + \cos^2 x/2 - 2\sin x/2 \cos x/2}\, dx$

$\int \sqrt{(\sin x/2 - \cos x/2)^2}\, dx = \int \sin x/2 - \cos x/2\, dx$

$-\dfrac{\cos x/2}{1/2} - \dfrac{\sin x/2}{1/2} + c = -2\cos x/2 - 2\sin x/2 + c$

imp - $\int \frac{2+3\sin x}{\cos^2 x}$

an $\int \frac{2+3\sin x}{\cos^2 x}\,dx$

$\int\left(\frac{2}{\cos^2 x}+\frac{3\sin x}{\cos^2 x}\right)dx = \int\left(2\sec^2 x + \frac{\cancel{3}\,1}{\cos x}\cdot\frac{3\sin x}{\cos x}\right)dx$

$= \int\left(2\sec^2 x + \sec x\, \cancel{\&}\, 3\tan x\right)dx$

$\cancel{2\tan x}\ \cancel{+}\ \int\left(2\sec^2 x + 3\tan x\sec x\right)dx$

$= 2\tan x + 3\sec x + c$

imp - $\int \frac{4\cos x+5}{\sin^2 x}$

an $\int\left(\frac{4\cos x+5}{\sin^2 x}\right)dx$

$= \int\left(\frac{4\cos x}{\sin^2 x}+\frac{5}{\sin^2 x}\right)dx = \int\left(\frac{4\cos x}{\sin x}\cdot\frac{1}{\sin x}+\frac{5}{\sin^2 x}\right)dx$

$= \int\left(4\cot x\cdot\operatorname{cosec} x + 5\operatorname{cosec}^2 x\right)dx$

$= -4\operatorname{cosec} x - 5\cot x + c$

imp - $\int \frac{\cos 2x}{\cos^2 x \sin^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x$

ans = $\int \left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} \right) dx$

$= \int \left(\frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x} \sin^2 x} - \frac{\cancel{\sin^2 x}}{\cos^2 x \cancel{\sin^2 x}} \right) dx$

$= \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \right) dx = \int (\operatorname{cosec}^2 x - \sec^2 x) dx$

$= -\cot x - \tan x + c$ //

imp - $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cdot \cos^2 x}$

an $\int \left(\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \right) dx = \int \left(\frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \right) dx$

$= \int \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \right) dx =$

$= \int \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \right) dx$

$= \int (\tan x \cdot \sec x + \cot x \operatorname{cosec} x) dx$

$= \sec x - \operatorname{cosec} x$

$= \sec x + \operatorname{cosec} x + c$

exam

v. imp $\int \sin^2 x$

ans $\int (\sin^2 x)\, dx = \int \left(\frac{1-\cos 2x}{2}\right) dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$= \cancel{\int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx}$

$= \frac{1}{2}\int (1-\cos 2x)\, dx$

doubt $= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c =$

$- \int \sin^2 2x\, dx$

$\sin^2 2x = \frac{1-\cos 4x}{2}$

$\int \left(\frac{1-\cos 4x}{2}\right) dx$

$= \frac{1}{2}\int (1-\cos 4x)\, dx$

$\frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) + c$

$= x/2 - \frac{\sin 4x}{8} + c$

v.imp $\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx = \int \left(\frac{1}{2} + \frac{\cos 2x}{2}\right)dx$

doubt $= \frac{x}{2} + \frac{\sin 2x}{4} + c$

$-\int \cos^2 4x\,dx = \int \left(\frac{1+\cos 8x}{2}\right)dx = \int \left(\frac{1}{2} + \frac{\cos 8x}{2}\right)dx$

$= \frac{x}{2} + \frac{\cos 8x}{16} + c$

1. $\int \sin^2 3x$
2. $\int \sin^2 4x$
3. $\int \cos^2 2x$
4. $\int \cos^2 5x$

Q→ $\int \sin^3 x$

$\sin 3x = 3\sin x - 4\sin^3 x$

an $\int \left(\frac{3\sin x - \sin 3x}{4}\right)dx$ i.e. $4\sin^3 x = 3\sin x - \sin 3x$

$= \frac{1}{4}\int (3\sin x - \sin 3x)\,dx$

$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$

$$= \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + c$$

Q $\int \cos^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$4\cos^3 x = \cos 3x + 3\cos x$

$\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$

an $\int \cos^3 x\, dx$

$$\int \left(\frac{\cos 3x + 3\cos x}{4}\right) dx$$

$$\frac{1}{4}\int (\cos 3x + 3\cos x)\, dx$$

$$= \frac{1}{4}\left(\frac{\sin 3x}{3} + 3\sin x\right) + c \;//$$

Result

imp

1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

2) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

imp!

$$\int \cos 2x \cos x\, dx$$

an $\frac{1}{2}\int 2\cos 2x\cos x\, dx$

$$= \frac{1}{2}\int \cos(2x+x) + \cos(2x-x)\, dx$$

$= \frac{1}{2}\int(\cos 3x + \cos x)\,dx$

$= \frac{1}{2}\left(\frac{\sin 3x}{3} + \sin x\right) + c$ //

imp 2 $\int \sin 3x \sin x\,dx$

$\frac{1}{2}\int(2\cos(3x-x) - \cos(3x+x))dx$

$\frac{1}{2}\int(\cos 2x - \cos 4x)\,dx$

$= \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 4x}{4}\right) + c$ //

imp 3 - $\int \sin 2x \sin x\,dx$

$\frac{1}{2}\int 2\sin 2x \sin x\,dx$

$\frac{1}{2}\int(\cos(2x-x) - \cos(2x+x))dx$

$\frac{1}{2}\int(\cos x - \cos 3x)dx = \frac{1}{2}\left(\sin x - \frac{\sin 3x}{3}\right) + c$ //

4 $\int \cos 4x \cos 5x\,dx$

an $\frac{1}{2}\int 2\cos 4x \cos 5x\,dx$

$$= \frac{1}{2}\int \cos(4x+5x) + \cos(4x-5x)\, dx$$

$$= \frac{1}{2}\int (\cos 9x + \cos -x)\cdot dx$$

$$= \frac{1}{2}\int (\cos 9x + \cos x)\, dx$$

$$= \frac{1}{2}\left(\frac{\sin 9x}{9} + \sin x\right) + c$$

$$5 = \int \cos 5x \cos x \, dx$$

$$= \frac{1}{2}\int 2\cos 5x \cos x \, dx = \frac{1}{2}\int (\cos 6x + \cos 4x)\, dx$$

$$= \frac{1}{2}\left(\frac{\sin 6x}{6} + \frac{\sin 4x}{4}\right) + c$$

V.V. IMP Integration by substitution method

1 - integrate $x \sin(x^2)$

an $\int x \sin(x^2)\, dx$

$$\boxed{\int x^{n-1} f(x^n)\, dx}$$

Put $t = x^2$

$$\frac{dt}{dx} = 2x$$

$$dt = 2x\, dx$$

$$x\, dx = \frac{dt}{2}$$

$$\int x \sin(x^2)\, dx = \int \sin t \, \frac{dt}{2} = \frac{1}{2}\int \sin t \, dt$$

$$= \frac{1}{2} \cdot -\cos t + c = -\frac{1}{2}\cos x^2 + c$$

2 $\int x^2 \cos(x^3)$

ans $\int x^2 \cos(x^3)\, dx$

Put $t = x^3$

$$\frac{dt}{dx} = 3x^2.$$

$$dt = 3x^2\, dx$$

$$x^2\, dx = \frac{dt}{3}$$

ie $\int x^2 \cos(x^3)\, dx = \frac{1}{3}\int \cos t \, dt$

$$= \cancel{\frac{1}{3}\int x^2 \cos(x^3)\, dx} = \cancel{\frac{1}{3}\int \sin(x^3)\, dt}$$

$$= \frac{1}{3}\sin t + c = \frac{1}{3}\sin x^3 + c$$

3 $\int x^{n-1} \sec^2(x^n)$

ans $\int x^{n-1} \sec^2 x^n \, dx$

put $t = x^n$

$\frac{dt}{dx} = n x^{n-1}$

$dt = n x^{n-1} dx$

$x^{n-1} dx = \frac{dt}{n}$

ie $\int \sec^2 t \frac{dt}{n} = \frac{1}{n} \int \sec^2 t \, dt = \frac{1}{n} \tan t + c$

$= \frac{1}{n} \tan (x^n) + c$ //

4 $\int x^4 \sec(x^5) \tan(x^5) dx$

an $\int x^4 \sec(x^5) \tan(x^5) dx$

put $x^5 = t$

$\frac{dt}{dx} = 5x^4$

$dt = 5x^4 dx$

$x^4 dx = \frac{dt}{5}$

ie $\frac{1}{5} \int \sec t \tan t \, dt$

$= \frac{1}{5} \sec t = \frac{1}{5} \sec x^5 + c$ //

5 $\int$ ~~[illegible]~~ $x^2 e^{x^3} dx$

an put $t = x^3$

$$\frac{dt}{dx} = 3x^2$$

$$dt = 3x^2 dx$$

$$x^2 dx = \frac{dt}{3}$$

$$\int x^2 e^{x^3} dx = \int e^{x^3} \frac{dt}{3} = \frac{1}{3} e^{x^3} + c$$ //

6 $\int x^{n-1} e^{x^n} dx$

Put $t = x^n$

$$\frac{dt}{dx} = n x^{n-1}$$

$$dt = n x^{n-1} dx$$

$$x^{n-1} dx = \frac{dt}{n}$$

$$\int x^{n-1} e^{x^n} dx = \int e^{t} \frac{dt}{n} = \frac{1}{n} e^{x^n} + c$$ //

a $\int e^{\sin x} \cos dx$

an pt $t = \sin x$

$$\frac{dt}{dx} = \cos x$$

$$dt = \cos x\, dx$$

$$\therefore \int e^{t} \cos x\, dx = \int e^{t} \, dt = e^{\sin x} + c$$

$$= e^{\sin x} + c$$

8. imp $\int e^{\sin^2 x} \sin 2x \, dx$

put $t = \sin^2 x$. $\quad x^2 - 2x$ (1 - is the derivative of x)

$$\frac{dt}{dx} = \cancel{2\cos x \sin x} \cdot 2\sin x \cos x$$

$$dt = \sin 2x\, dx$$

$$\therefore \int e^{t} \, dt = e^{\sin^2 x} + c$$

9. $\int e^{\tan x} \sec^2 x \, dx$

10. $\int e^{\cos x} \cdot \sin x \, dx$

put $t = \cos x$

11 $\displaystyle\int \frac{2x+1}{x^2+x+1}\,dx$

2nd model

ans put $t = x^2+x+1$

$\frac{dt}{dx} = 2x+1$

$dt = 2x+1\,dx$

$\displaystyle\int \frac{2x+1}{x^2+x+1}\,dx = \int \frac{dt}{t} = \log t + c$

$\log(x^2+x+1)+c$

12 $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

ans put $t = e^x+1$

$\frac{dt}{dx} = e^x$

$dt = e^x dx$

$\displaystyle\int \frac{dt}{t} = \log t + c = \log(e^x+1)+c$

* 13 $\int \frac{\sin 2x}{\sin^2 x}\,dx$

an put $t = \sin^2 x$

$\frac{dt}{dx} = 2\sin x \cos x$ $\qquad 2\sin x\cos x = \sin 2x$

$dt = 2\sin 2x\,dx$

$\int \frac{dt}{t} = \log t + c = \log \sin^2 x + c$ //

imp 14 $\int \frac{1-2x}{x^2-x+1}\,dx$

Put $t = x^2 - x + 1$

$\frac{dt}{dx} = 2x - 1$

$dt = 2x-1\,dx = -(1-2x)\,dx$

$\therefore$ ~~$\int \frac{-(2x-1)}{x^2-x+1}dx$~~

$\int -\frac{dt}{t} = -\log t + c = -\log(x^2 - x + 1) + c$ //

15 $\int \frac{x^2+1}{x^3+3x}\,dx$

*

an Put $t = x^3 + 3x$

$\frac{dt}{dx} = 3x^2 + 3$

$$\frac{dt}{dx} = 3(x^2+1)$$

$$\frac{dt}{3} = \cancel{3}(x^2+1)\,dx$$

$$\frac{1}{3}\int \frac{\cancel{3}dt}{t} = \frac{1}{3}\log t + c = \frac{1}{3}\log(x^3+3x)+c$$

16 $\displaystyle\int \frac{1+\cos x}{x+\sin x}\,dx$

put $t = x+\sin x$

$$\frac{dt}{dx} = 1+\cos x$$

$$dt = 1+\cos x\,dx$$

$$\therefore \int \frac{dt}{t} = \log t + c = \log(1+\cos x)+c \,//$$

17 $\displaystyle\int \frac{1+\cos x}{(x+\sin x)^2}\,dx$

put $t = x+\sin x$

$$\frac{1}{x^n} = \frac{-n}{x^{n-1}}$$

$$\frac{dt}{dx} = 1+\cos x$$

$$dt = 1+\cos x\,dx$$

$$\therefore \int \frac{dt}{t^2} = \frac{-1}{t} + c = \frac{-1}{x+\sin x} + c \,//$$

18 $\int \frac{\sec^2 x}{1+\tan x} dx$

Put $1+\tan x = t$

$\frac{dt}{dx} = \sec^2 x$

$dt = \sec^2 x \, dx$

$\therefore \int \frac{dt}{t} = \log t + c = \log 1+\tan x + c$ //

19 $\int \frac{1 - \sin x}{x+\cos x} dx$

put $t = x+\cos x$

$\frac{dt}{dx} = 1-\sin x$

$dt = 1-\sin x \, dx$

$\int \frac{dt}{t} = \log t + c = \log x+\cos x + c$ //

20 $\int \frac{e^{2x}}{(e^{2x}+1)} dx$

put $e^{2x}+1 = t$

$\frac{dt}{dx} = e^{2x}$

$dt = e^{2x} dx$

$\int \frac{dt}{t} = \log t + c = \log(e^{2x}+1) + c$

21 $\int \frac{x+1}{x^2+2x-1} dx$

put $t = x^2 + 2x - 1$

$$\frac{dt}{dx} = 2x + 2$$

$$dt = 2(x+1)dx$$

$$\frac{dt}{2} = (x+1)dx$$

$$\int \frac{dt}{2t} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2} \log t + c = \frac{1}{2} \log(x^2+2x-1)+c$$

Q2 $\int \frac{x^3}{(1+x^4)^2} dx$

put $t = 1+x^4$.

$$\frac{dt}{dx} = 4x^3$$

$$dt = 4x^3 dx.$$

$$\frac{dt}{4} = x^3 dx$$

$$\int \frac{dt}{4t^2} = \frac{1}{4}\int \frac{dt}{t^2} = \frac{1}{4} \times \frac{-1}{t} = \frac{-1}{4t}$$

1 $\int (2x+1)\sqrt{x^2+x+1}\,dx$

put $t = x^2+x+1$

$\frac{dt}{dx} = 2x+1$

$dt = (2x+1)\,dx$

$\therefore \int \sqrt{t} \times dt$

$\therefore \int t^{1/2}\,dt$

$\frac{t^{1/2+1}}{1/2+1} = \frac{t^{3/2}}{3/2} + c$

$\frac{2}{3}(x^2+x+1)^{3/2} + c$

2 $\int x\sqrt{x^2+9}\,dx$

put $t = x^2+9$

$\frac{dt}{dx} = 2x.$

$\cancel{\frac{dt}{dx}}$

$dt = 2x\,dx$

$dt/2 = x\,dx$

$\therefore \int \sqrt{t}\,.\,dt/2$

$\frac{1}{2}\int t^{1/2}\,dt = \frac{1}{2}\cdot\frac{t^{3/2}}{3/2} + c = \frac{2}{3}\times\frac{1}{2}\{x^2+9\}^{3/2} + c$

$= \frac{(x^2+9^{3/2})}{3} + c$

3 $\int (x-1)\sqrt{x^2-2x+1}\, dx$

ans Put $x^2-2x+1 = t$

$$\frac{dt}{dx} = 2x-2$$

$$dt = (2x-2)\,dx$$

$$2(x-1)\,dx$$

$$dt/2 = (x-1)\,dx$$

$$\therefore \int \sqrt{t}\,.\,dt/2 = \frac{1}{2}\int t^{1/2}\,.\,dt$$

$$= \frac{1}{2}\,.\,\frac{t^{3/2}}{3/2} + c$$

$$= \frac{1}{2}(x^2-2x+1)^{3/2}\,.\,2/3 + c$$

$$= \frac{(x^2-2x+1)^{3/2}}{3} + c \;//$$

9 $\int \cos x \sqrt{1+\sin x}\, dx$

Put $t = 1+\sin x$

$$\frac{dt}{dx} = 0 + \cos x$$

$$dt = \cos x\, dx$$

$$\therefore \int \sqrt{t}\,.\,dt = \int t^{1/2}\,.\,dt = \frac{t^{3/2}}{3/2} + c$$

$= (1+\sin)^{3/2} \; 2/3 + c$ //

5 $\int (1-\sin x)\sqrt{x+\cos x}\,dx$

an put $t = x+\cos x$

$dt/dx = 1 - \sin x$

$dt = (1-\sin x)\,dx$

$\int \sqrt{t}\,.\,dt \quad = \int t^{1/2}\,.\,dt$

$= \dfrac{t^{3/2}}{3/2} + c \quad = \; (x+\cos x)^{3/2} \cdot 2/3 + c$ //

6 $\int \cos x \sqrt{\sin x}\,dx$

an put $t = \sin x$

$\dfrac{dt}{dx} = \cos x$

$dt = \cos x\,dx$

$\int \sqrt{t}\,.dt \quad = \int t^{1/2}\,.dt = \dfrac{t^{3/2}}{3/2} + c$

$= \frac{2}{3}(\sin x^{3/2}) + c$ //

HW 1 $\int (x^2+1)\sqrt{x^3+3x}\,dx$

put $t = x^3+3x$

$dt/dx = 3x^2+3$

$dt = 3(x^2+1)\,dx$

$dt/3 = (x^2+1)\,dx$

$\therefore \int \sqrt{t}\,.\,dt/3 = \frac{1}{3}\int \sqrt{t}\,dt = \frac{1}{3}\int t^{1/2}\,.\,dt$

$= \frac{1}{3}\left(\frac{t^{3/2}}{3/\cancel{2}}\right) + c \quad = \frac{1}{\cancel{3}} \times \frac{\cancel{2}}{3}(x^3+3x)^{3/2} + c$

$= \frac{2(x^3+3x)^{3/2}}{9} + c$ //

2 $\int (1+\cos x)\sqrt{x+\sin x}\,dx$

Put $t = x+\sin x$

$\frac{dt}{dx} = 1+\cos x$

$dt = (1+\cos x)\,dx$

$\therefore \int \sqrt{t}\,.\,dt = \int t^{1/2}\,.\,dt = \frac{t^{3/2}}{3/2} + c$

$= \frac{2}{3}(x+\sin x)^{3/2} + c$ //

3 $\int \sec^2 x\sqrt{1+\tan x}\,dx$

an put $t = 1+\tan x$

$\frac{dt}{dx} = \sec^2 x$

$dt = \sec^2 x\,dx$

$\therefore \int \sqrt{t}\,.dt = \int t^{1/2}\,.dt = \frac{t^{3/2}}{3/2} + c$

$= 2/3 \cdot (1+\tan x)^{3/2} + c$ //

Q $\int \sin x\,(1-\cos x)^5\,dx$

an ~~put {1-cos x}~~

put $t = 1-\cos x$

$\frac{dt}{dx} = --\sin x$

$dt = \sin x\,dx$

$\therefore \int t^5\,.dt = \frac{t^{5+1}}{5+1} + c = \frac{t^6}{6} + c$

$= \frac{(1-\cos x)^6}{6} + c$ //

Q $\int \sec^2 x\,(1+\tan x)^7\,dx$

an put $t = 1+\tan x$

$\frac{dt}{dx} = \sec 2x$

$dt = \sec^2 x\,dx$

$\therefore \int t^7\,.dt = \frac{t^{7+1}}{7+1} + c = \frac{t^8}{8} + c$

$= \frac{(1+\tan x)^8}{8} + c$ //

Q $\int (x+1)(x^2+2x+1)^4 \, dx$

an put $t = x^2+2x+1$

$\frac{dt}{dx} = 2x+2$

$dt = 2(x+1)\,dx$

$dt/2 = (x+1)\,dx$

$\therefore \int t^4 \, dt/2 = \frac{1}{2} \cdot \int t^4 \cdot dt = \frac{1}{2} \cdot \frac{t^{4+1}}{4+1} + c$

$= \frac{1}{2} \cdot \frac{t^5}{5} + c = \frac{t^5}{10} + c$

$= \frac{(x^2+2x+1)^5}{10} + c$ //

H.W 1 $\int \cos x (1+\sin x)^5 \, dx$

an put $t = 1+\sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x \, dx$

$\therefore \int t^5 \cdot dt = \frac{t^{5+1}}{5+1} + c = \frac{t^6}{6} + c$

$= \frac{(1+\sin x)^6}{6} + c$ //

2 $\int (1-2x)(x^2-x+1)^7 \, dx$

an put $t = x^2 - x + 1$

$$\frac{dt}{dx} = 2x - 1$$

$$\frac{dt}{dx} = -(1-2x)$$

$$dt = -(1-2x)\,dx$$

$$-dt = (1-2x)\,dx$$

$$\therefore \int t^7 \cdot -dt = -\int t^7 \, dt = -\frac{t^{7+1}}{7+1} + c$$

$$= -\frac{t^8}{8} + c = -\frac{(x^2-x+1)^8}{8} + c \,//$$

3 $\int x\sqrt{x^2+1}\,dx$

an put $t = x^2+1$

$$\frac{dt}{dx} = 2x$$

$$dt = 2x\,dx$$

$$dt/2 = x\,dx$$

$$\therefore \int \sqrt{t} \cdot dt/2 = \frac{1}{2}\int t^{1/2}\,dt = \frac{1}{2} \cdot \frac{t^{3/2}}{3/2} + c$$

$$= \frac{1}{2} \cdot \frac{2}{3} \cdot t^{3/2} + c$$

$$= \frac{1}{2} \times \frac{2}{3}(x^2+1)^{3/2} + c = \frac{(x^2+1)^{3/2}}{3} + c \,//$$

Q $\int \sin^3 x \, dx$

an $\int \sin^2 x \cdot \sin x \, dx$

~~[illegible]~~

$\int (1-\cos^2 x) \sin x \, dx$

put $t = \cos x$

$\frac{dt}{dx} = -\sin x$

$-dt = \sin x \, dx$

$\int (1-t^2) - dt = -\int (1-t^2)\, dt$

$\int (t^2 - 1)\, dx$

$= \frac{t^{2+1}}{2+1} - 1 + c = \frac{t^3}{3} - t + c$

$= \frac{\cos^3 x}{3} - \cos x + c$ //

Q $\int \cos^3 x \, dx$

an $\int \cos^2 x \cos x \, dx$

$\int (1-\sin^2 x) \cos x \, dx$

put $t = \sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x \, dx$

$\therefore \int (1-t^2)\,.dt = t - \frac{t^{2+1}}{2+1} + c$

$= t - \frac{t^3}{3} + c \quad = \frac{\sin 3x}{3} + c$ //

$= \sin x - \frac{\sin^3 x}{3} + c$ //

$\int \sin^5 x \cos x \, dx$

put $t = \sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x \, dx$

$\therefore \int t^5 \, dt = \frac{t^{5+1}}{5+1} + c = \frac{t^6}{6} + c$

$= \frac{\sin^6 x}{6} + c$ //

$\int \cos^3 x \sin x \, dx$

put $t = \cos x$

$\frac{dt}{dx} = -\sin x$

$-dt = \sin x \, dx$

$\therefore \int t^3 . -dt = -\int t^3 . dt = -\frac{t^{3+1}}{3+1} + c$

$-\frac{t^4}{4}+c = -\frac{\cos^4 x}{4}+c$ //

Q $\int \sec^4 x \tan x \, dt$

an put $t = \sec x$

$\frac{dt}{dx} = \sec x \tan x$

$dt = \sec x \tan x \, dx$

$\therefore \int \sec^3 x . \sec x \tan x \, dx$

$= \int t^3 . dt$ $= \frac{t^{3+1}}{3+1} + c = \frac{t^4}{4}+c$

$= \frac{\sec^4 x}{4}+c$ //

Q $\int \frac{1}{x \log x} dx$

an put $t = \log x$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x} dx$

$\therefore \int \frac{1}{t} . dt$ $= \log|t| + c = \log|\log x| + c$ //

Q $\int \frac{1}{x(\log x)^2}\,dx$

An put $t = \log x$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$dt = \frac{1}{x}\,dx$$

$$\therefore \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = \frac{t^{-2+1}}{-2+1} + c$$

$$= \frac{t^{-1}}{-1} + c = -\frac{1}{t} + c = -\log x + c$$

Qns for 1st assignment

integrate the following

1. $\sqrt{1-\sin 2x}$

2. $\frac{2+3\sin x}{\cos^2 x}$

3. $2\sec^2 x + e^x + 3\sin x$

4. $x(x+1)(x+2)$

5. $\frac{\cos 2x}{\cos^2 x \sin^2 x}$.

Q $\int \frac{\sin(\log x)}{x} dx$

ans put $t = \log x$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x} dx$

$\therefore \int \sin \log x \, \frac{1}{x} dx$

$\therefore \int \sin t \, . \, dt$

$= -\cos t + c \quad = -\cos(\log x) + c$ //

Q $\int \frac{\sin(2+3\log x)}{x} dx$

ans put $t = 2+3\log x$

$\frac{dt}{dx} = 0 + 3\frac{1}{x}$

$dt = \cancel{\frac{dx}{x}} \; \frac{3}{x} dx$

~~$\therefore \int \sin(2+3t) dt$~~

~~$= \int -\cos(0+3t) + c \quad = -\cos 3x \log x$~~

$\frac{dt}{3} = \frac{dx}{x}$

$\therefore \int \sin t \, . \, \frac{dt}{3} = \frac{1}{3}\int \sin t \, dt = \frac{1}{3}(-\cos t) + c$

$= \frac{1}{3}\left(-\cos 2+3\log x\right)+c$ //

Q $\int \frac{\cos(a+b\log x)}{x}\,dx$

ans put $t = a+b\log x$

$\frac{dt}{dx} = 0+b\frac{1}{x}$

$dt = \frac{b}{x}\,dx$

$dt/b = \frac{dx}{x}$

$\therefore \int \cos t \cdot \frac{dt}{b} = \frac{1}{b}\int \cos t\,dt$

$= \frac{1}{b}(\sin t)+c = \frac{1}{b}\sin(a+b\log x)+c$ //

Q $\frac{\cos(2+3\log x)}{x}\,dx$

ans put $2+3\log x = t$

$\frac{dt}{dx} = 0+3\frac{1}{x}$

$dt = \frac{3\,dx}{x}$

$dt/3 = dx/x$

$\therefore \int \cos t \cdot \frac{dt}{3} = \frac{1}{3}\int \cos t \cdot dt$

$= \frac{1}{3}\cancel{\int\cos}(\sin t)+c = \frac{1}{3}\sin(2+3\log x)+c$ //

impQ $\displaystyle\int \frac{2x^4}{1+x^{10}}\,dx$

an

$$\int \frac{2x^4}{1+(x^5)^2}\,dx$$

Put $t = x^5$

$$\frac{dt}{dx} = 5x^4$$

$$dt = 5x^4\,dx$$

$$\frac{dt}{5} = x^4\,dx$$

$$\therefore \int \frac{2}{1+t^2} \times \frac{dt}{5} = \int 2 \quad \frac{2}{5}\int \frac{dt}{1+t^2}$$

$$= \frac{2}{5}\int \frac{1}{1+t^2}\,dt$$

$$= \frac{2}{5}\tan^{-1} t + c$$

doubt

$$\frac{2}{3}\tan^{-1}(x^5) + c \;//$$

Q $\displaystyle\int \frac{x^2}{\sqrt{1-x^6}}\,dx$

an $\displaystyle\int \frac{x^2}{\sqrt{1-(x^3)^2}}\,dx$

put $t = x^3$

$$dt/dx = 3x^2$$

$dt = 3x^2 dx$

$\frac{dt}{3} = x^2 dx$

$\therefore \int \frac{1}{\sqrt{1-t^2}} \frac{dt}{3} = \frac{1}{3} \int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{3} \int \frac{1}{\sqrt{1-t^2}} dt$

$= \frac{1}{3} \cdot \sin^{-1}(t) + c$

$= \frac{1}{3} \sin^{-1}(x^3) + c$ //

Q $\int \frac{x^{n-1}}{1+x^{2n}} dx$

an $\int \frac{x^{n-1}}{1+(x^n)^2} dx$

$= \int$

Put $t = x^n$

$dt/dx = n x^{n-1}$

$\frac{dt}{n} = x^{n-1} dx$

$\therefore \int \frac{1}{1+t^2} \cdot \frac{dt}{n} = \frac{1}{n} \int \frac{1}{1+t^2} dt$

$= \frac{1}{n} \tan^{-1} t + c = \frac{1}{n} \tan^{-1} x^n + c$ //

Q $\displaystyle\int \frac{x^{n-1}}{\sqrt{1-x^{2n}}}\,dx$

$= \displaystyle\int \frac{x^{n-1}}{\sqrt{1-(x^n)^2}}\,dx$

Put $t = x^n$

$\frac{dt}{dx} = nx^{n-1}$

$\frac{dt}{n} = x^{n-1}\,dx$

$\therefore \displaystyle\int \frac{1}{\sqrt{1-t^2}}\,\frac{dt}{n} = \frac{1}{n}\int \frac{1}{\sqrt{1-t^2}}\,dt$

$= \frac{1}{n}\sin^{-1} t + c = \frac{1}{n}\sin^{-1} x^n + c$ //

Q $\displaystyle\int \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}\,dx$

ans Put $t = \sin^{-1}x$

$\frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dt = \frac{1}{\sqrt{1-x^2}}\,dx$

$\therefore \displaystyle\int t^2 \cdot dt$

$\frac{t^3}{3} + c = \frac{(\sin^{-1}x)^3}{3} + c$ //

$$\int \frac{(\tan^{-1} x)^3}{1+x^2} dx$$

Ans put $t = \tan^{-1} x$

$$\frac{dt}{dx} = \frac{1}{1+x^2}$$

$$dt = \frac{1}{1+x^2} dx$$

$$\therefore \int t^3 \, dt$$

$$\therefore \frac{t^4}{4} + c = \frac{(\tan^{-1} x)^4}{4} + c \; //$$

Q $$\int \frac{(\tan^{-1} 5x)^2}{1+25x^2} dx$$

put $t = \tan^{-1} 5x$

$$\frac{dt}{dx} = \frac{1}{1+(5x)^2} \times 5$$

$$\frac{dt}{dx} = \frac{5}{1+25x^2} dx$$

$$\frac{dt}{5} = \frac{dx}{1+25x^2}$$

$$\frac{1}{5}\int t^2 \, dt$$

$$\frac{1}{5} \frac{t^3}{3} + c = \frac{(\tan^{-1} 5x)^3}{15} + C \; //$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$$

put $t = \sin^{-1} x$

$$\frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$dt = \frac{1}{\sqrt{1-x^2}}\, dx$$

$$\therefore \int t \cdot dt$$

$$\frac{t^2}{2} + c = \frac{(\sin^{-1} x)^2}{2} + c$$

Q $$\int \frac{\tan^{-1} x}{1+x^2}\, dx$$

put $t = \tan^{-1} x$

$$\frac{dt}{dx} = \frac{1}{1+x^2}$$

$$dt = \frac{dx}{1+x^2}$$

$$\int t\, dt$$

$$\frac{t^2}{2} + c = \frac{(\tan^{-1} x)^2}{2} + c$$

Q $$\int \frac{(\tan^{-1} 3x)^2}{1+9x^2}\, dx$$

put $t = \tan^{-1} 3x$

$$\frac{dt}{dx} = \frac{1}{1+9x^2}$$

$$dt = \frac{dx}{1+9x^2}$$

$$\int t^2 . dt = \frac{t^3}{3} + c = \frac{(\tan^{-1}3x)^3}{3} + c$$ //

Q $\int e^{\tan^{-1}x} . \frac{1}{1+x^2} \, dx$

Put $t = \tan^{-1}x$

$$\frac{dt}{dx} = \frac{1}{1+x^2}$$

$$dt = \frac{1}{1+x^2} \, dx$$

$$\int dt . e^t$$

$$e^t + c = e^{\tan^{-1}x} + c$$ //

Q $\int \tan x \, dx$

an $\int \frac{\sin x}{\cos x} \, dx$

Put $t = \cos x$

$$-\log|\cos x| = \log(\sec x)$$

$$\frac{dt}{dx} = -\sin x$$

$$dt = -\sin x \, dx$$

$$\int \frac{1}{t} \times - dt$$

$$-\log|t| + c = -\log|\cos x| + c$$ //

Q $\int \cot x \, dx$

Q $\int \cot x \, dx$

an $\int \frac{\cos x}{\sin x} dx$

Put $t = \sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x \, dx$

$\int \frac{1}{t} \times dt$

$\log|t| + c = \log|\sin x| + c$ //

$$\boxed{\int \cot x \, dx = \log|\sin x|}$$

Q $\int \operatorname{cosec} x \, dx$

$= \int \frac{\operatorname{cosec} x (\operatorname{cosec} x - \cot x)}{\operatorname{cosec} x - \cot x} dx$

put $t = \operatorname{cosec} x - \cot x$

$\frac{dt}{dx} = -\operatorname{cosec} x \cot x - -\operatorname{cosec}^2 x$

$= -\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x$

$= \operatorname{cosec} x (\operatorname{cosec} x - \cot x)$

$\therefore dt = \operatorname{cosec} x (\operatorname{cosec} x - \cot x) dx$

$\int \frac{1}{t} \times dt$

$\log|t| + c$

$\log|\operatorname{cosec} x - \cot x| + c$ //

Q $\int \sec x \, dx$

an $\int \frac{\sec x (\sec x + \tan x)}{\sec x + \tan x} dx$

Put $t = \sec x + \tan x$

$\frac{dt}{dx} = \sec x \tan x + \sec^2 x$

$= \sec x (\tan x + \sec x)$

$dt = \sec x (\sec x + \tan x) dx$

$\therefore \int \frac{1}{t} \times dt$

$\log|t| + c$

$\log|\sec x + \tan x| + c$

$\int \sec x = \log|\sec x + \tan x|$

Integration by parts

If u & v are 2 integrable functions in x, then

$$\boxed{\int u\,v\,dx = u\int v\,dx - \int\left\{\frac{d}{dx}(u) \cdot \int v\,dx\right\}dx}$$

1. Q integrate xe^x w.r.t x

An $\int x \cdot e^x\,dx = x\int e^x dx - \int\left\{\frac{d}{dx}x \cdot \int e^x\,dx\right\}dx$

$= x \times e^x - \int 1 \cdot e^x\,dx$

$= x \cdot e^x - \int e^x\,dx$

$= x\,e^x - e^x + c$ // $= e^x(x-1)+c$ //

2. Q $\int x \cdot e^{-x}\,dx$

An $x\int e^{-x}\,dx - \int\left\{\frac{d}{dx}x \cdot \int e^{-x}\,dx\right\}dx$

$= x\,\frac{e^{-x}}{-1} - \int 1 \cdot \frac{e^{-x}}{-1}\,dx$

$-xe^{-x} - - \int e^{-x}\,dx$

$= -xe^{-x} + \frac{e^{-x}}{-1} + c$

$= -xe^{-x} - e^{-x} + c$

$= -e^{-x}(x+1) + c$ //

Q 3 $\int x e^{2x} dx$

$= x\int e^{2x}dx - \int\left\{\frac{d}{dx}x \cdot \int e^{2x}dx\right\}dx$

$= \frac{x e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} dx$

$= \frac{x e^{2x}}{2} - \frac{1}{2}\int e^{2x} dx = \frac{x e^{2x}}{2} - \frac{1}{2} \cdot \frac{e^{2x}}{2} + c$

$= \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + c \quad = \frac{e^{2x}}{2}\left(x - \frac{1}{2}\right) + c$ //

Q 4 $\int x e^{-3x} dx$

an $x\int e^{-3x}dx - \int\left\{\frac{d}{dx}x \cdot \int e^{-3x}dx\right\}dx$

$= \frac{x e^{-3x}}{-3} - \int 1 \frac{e^{-3x}}{-3} dx$

$= \frac{x e^{-3x}}{-3} - \frac{1}{-3}\int e^{-3x} dx$

$= \frac{x e^{-3x}}{-3} + \frac{1}{3} \cdot \frac{e^{-3x}}{-3} + c$

$= \frac{x e^{-3x}}{-3} + \frac{e^{-3x}}{-9} + c$

$= \frac{e^{-3x}}{-3}\left(x + \frac{1}{3}\right) + c$ //

Imp Q $\int x^2 e^x$

ans $= \int x^2 e^x dx = x^2 \int e^x dx - \int \{ \frac{d}{dx}(x^2) \cdot \int e^x dx \} dx$

$= x^2 e^x - 2\int x \, e^x dx$

$= x^2 e^x - 2 \int x \int e^x - \int \{ \frac{d}{dx} x \cdot \int e^x dx \} dx$

$= x^2 e^x - 2 \int x e^x - 1 . e^x dx$

$= x^2 e^x - 2 (x e^x - e^x] + c$ //

Q $x^2 e^{-x}$

a $= \int x^2 e^{-x} dx = x^2 \int e^{-x} dx - \int \{ \frac{d}{dx}(x^2) \cdot \int e^{-x} dx \} dx$

$= -x^2 e^{-x} + 2\int x \, e^{-x} dx$

$= -x^2 e^{-x} + 2\int x \int e^{-x} - \int \{ \frac{d}{dx} x \cdot \int e^{-x} dx \} dx$

$= -x^2 e^{-x} + 2\int x e^{-x} - 1 . e^{-x} dx$

$= -x^2 e^{-x} + 2 (-x e^{-x} - e^{-x}] + c$ //

2nd section

v.imp <u>algebraic and trignometric function</u>

1 $\int x \sin x \, dx$

an $x \int \sin x - \int \left\{ \frac{d}{dx} x \cdot \int \sin x \, dx \right\} dx$

$x \cdot -\cos x - \int 1 \cdot -\cos x \, dx$

$x - \cos x - - \sin x + c$

$= -x \cos x + \sin x + c$ //

v.imp $\int x \cos x \, dx$

an $x \int \cos x - \int \left\{ \frac{d}{dx} x \cdot \int \cos x \, dx \right\} dx$

$= x \sin x - \int 1 \cdot \sin x \, dx$

$= x \sin x - - \cos x + c$

$= x \sin x + \cos x + c$ //

3 $\int x \sin 2x \, dx$

an $x \int \sin 2x - \int \left\{ \frac{d}{dx} (x) \cdot \int \sin 2x \, dx \right\} dx$

$= x \frac{\cos 2x}{2} - \int 1 \cdot \frac{-\cos 2x}{2} \, dx$

$\left[\frac{x \cos 2x}{2} - \frac{1}{2} \int \cos 2x \, dx \right.$

$$-\frac{x\cos 2x}{2} + \frac{1}{2}\times\frac{\sin 2x}{2} + c$$

$$= -\frac{x\cos 2x}{2} + \frac{\sin 2x}{4} + c$$

4. $x\cos 3x$

ans $x\int \cos 3x - \int\left\{\frac{d}{dx}(x)\cdot\int \cos 3x\,dx\right\}dx$

$$= x\frac{\sin 3x}{3} - \int 1\cdot\frac{\sin 3x}{3}\,dx$$

$$= x\frac{\sin 3x}{3} - \frac{1}{3}\times\int \sin 3x\,dx$$

$$= x\frac{\sin 3x}{3} - \frac{1}{3}\cdot\cancel{\sin 3}\ -\frac{\cos 3x}{3}\ \cancel{dx} + c$$

$$= x\frac{\sin 3x}{3} + \frac{\cos 3x}{9} + c$$

5. $x^2 \sin x$

an $x^2\int \sin x - \int\left\{\frac{d}{dx}(x^2)\int \sin x\,dx\right\}dx$

$$= x^2 - \cos x - 2\int x\ -\cos x\,dx$$

$$= -x^2\cos x + \cancel{2\int x}\ 2\int x\cos x\,dx$$

$$= -x^2\cos x + 2\left[x\int \cos x - \int\left\{\frac{d}{dx}x\cdot\int \cos x\,dx\right\}dx\right]$$

$$= -x^2\cos x + 2\left[x\sin x - \int 1\cdot\sin x\,dx\right]$$

$= -x^2 \cos x + 2 \left[x \sin x - \cos x + c \right]$

$= -x^2 \cos x + 2 \left[x \sin x + \cos x + c \right]$

5 $\int x^2 \cos x \, dx$

an $x^2 \int \cos x - \int \left\{ \frac{d}{dx}(x^2) \cdot \int \cos x \, dx \right\} dx$

$= x^2 \sin x - 2 \int x \sin x \, dx$

$= x^2 \sin x - 2 \left[x \int \sin x - \int \left\{ \frac{d}{dx} x \cdot \int \sin x \, dx \right\} dx \right]$

$= x^2 \sin x - 2 \left[x - \cos x - \int 1 \int -\cos x \, dx \right]$

$= x^2 \sin x - 2 \left[x \cos x - - \sin x + c \right]$

$= x^2 \sin x - 2 \left[- x \cos x + \sin x + c \right]$

$= x^2 \sin x - 2 \left[- x \cos x + \sin x + c \right]$

spl qu 6 $\int x \sec x \tan x \, dx$

an $\int x (\sec x \tan x) \, dx$

$= x \int \sec x \tan x - \int \left\{ \frac{d}{dx}(x) \cdot \int \sec x \tan x \, dx \right\} dx$

$= x \sec x - \int 1 \sec x \, dx$

$= x \sec x - \log |\sec x + \tan x| + c$

$\int$ Third section

$$x^1 = \frac{x^{1+1}}{1+1}$$

Q $\int x \log x$

an $\int \log x \; x \, dx = \log x \int x - \int \left\{ \frac{d}{dx}(\log x) \cdot \int x \, dx \right\} dx$

$= \log x \; \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx$

$= \log x \; \frac{x^2}{2} - \frac{1}{2} \int x \, dx = \log x \; \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + c$

$= \log x \frac{x^2}{2} - \frac{x^2}{4} + c$ // $\frac{x^2}{2}\left(\log x - \frac{1}{2}\right) + c$ //

4th section

Q $\int x^2 \log x \, dx$

an $\int \log x \; x^2 \, dx = \log x \int x^2 \, dx - \int \left\{ \frac{d}{dx}(\log x) \cdot \int x^2 \, dx \right\} dx$

$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} \, dx$

$= \log x \; \frac{x^3}{3} - \frac{1}{3} \int x^2 \, dx$

$= \log x \; \frac{x^3}{3} - \frac{1}{3} \times \frac{x^3}{3} + c = \log x \; \frac{x^3}{3} - \frac{x^3}{9} + c$ //

$\frac{x^3}{3}\left(\log x - \frac{1}{3}\right) + c$ //

Q $\int x^n \log x \, dx$

ans = $\log x \int x^n dx - \int \left\{ \frac{d}{dx} (\log x) \cdot \int x^n dx \right\} dx$

$= \log x \frac{x^{n+1}}{n+1} - \int \frac{1}{x} \cdot \frac{x^{n+1}}{n+1} dx$

$= \log x \frac{x^{n+1}}{n+1} - \frac{1}{n+1} \int x^n dx$

$= \log x \frac{x^{n+1}}{n+1} - \frac{1}{n+1} \times \frac{x^{n+1}}{n+1} + c$

$= \log x \frac{x^{n+1}}{n+1} - \frac{x^{n+1}}{(n+1)^2} + c$

$= \frac{x^{n+1}}{n+1} \left(\log x - \frac{1}{(n+1)} \right) + c$

Q
v.imp $\int \log x$ with respect to x

ans $\int \log x \times 1 \, dx = \log x \int 1 \, dx - \int \left\{ \frac{d}{dx} \log x \right\} \cdot \int 1 \, dx \} dx$

$= \log x \; x - \int \frac{1}{x} \cdot x \, dx$

$= \log x \; x - \int 1 \, dx = x \log x - x + c$

$x (\log x - 1) + c$ //

Q $\int (\log x)^2 \, dx$

an $\int \log x^2 \cdot 1 \, dx = (\log x)^2 \int 1 \, dx - \int \left\{ \frac{d}{dx} (\log x)^2 \cdot \int 1 \, dx \right\} dx$

$= (\log x)^2 \cdot x - \int 2 \log x \cdot \frac{1}{x} \cdot x \, dx$

$= (\log x)^2 \cdot x - 2 \int \log x \, dx$

$= (\log x)^2 \cdot x - 2 \cdot x(\log x - 1) + c$

$= \cancel{(\log x)^2 \cdot x - \dots x \log x - x + c}$

$(\log x)^2 x - 2 \, x(\log x - 1) + c$ //

Q $\int \tan^{-1} x$ with respect to x

an $\int \tan^{-1} x \cdot 1 \, dx = \tan^{-1} x \int 1 \, dx - \int \left\{ \frac{d}{dx} \tan^{-1} x \cdot \int 1 \, dx \right\} dx$

$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$

$\cancel{\tan^{-1} x \cdot x - \frac{x}{x + \frac{x^3}{3}}}$

$\tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$

put $t = 1 + x^2$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x \, dx$

$\tan^{-1} x \quad x - \int \frac{dt}{2} \cdot \frac{1}{t}$

$\tan^{-1} x \cdot x - \frac{1}{2} \log |t| + c$ //

Q $\int e^x \sin x$ with respect to x

$\int e^x \sin x \, dx$

$I = e^x \sin x \, dx$

$I = \sin x \int e^x dx - \int \left\{ \frac{d}{dx} (\sin x) \cdot \int e^x dx \right\} dx$

$I = \sin x \, e^x - \int \cos x \cdot e^x \, dx$

$I = \sin x \, e^x - \left(\cos x \int e^x dx - \int \left\{ \frac{d}{dx} \cos x \cdot \int e^x dx \right\} dx \right.$

$I = \cancel{\sin x \, e^x} - \cancel{(\cos x \, e^x} - \int \sin x \, e^x \, dx$

$I = \cancel{\sin x \, e^x} - \cancel{(} \cos x \, e^x$.

$I = \sin x \, e^x - \left(e^x \cos x + \int e^x \sin x \, dx \right)$

$e^x \sin - e^x \cos x - \sin x \, dx$

$I = e^x \sin x - e^x \cos x - I$

$(I + I = 2I)$

$I + I = e^x (\sin x - \cos x)$

$2I = e^x (\sin x - \cos x) + c$

$I = \frac{e^x (\sin x - \cos x)}{2} + c$ //

defenite integral

let $\int f(x)\,dx = F(x)+c$

Then the value of the integral when $x=a$ is $F(a)+c$. The value of the integral when $x=b$ is $F(b)+c$.

$\therefore$ (the value of the integral when $x=b$) $-$ (the value of the integral when $x=a$)

$$F(b)+c-[F(a)+c] = F(b)-F(a).$$

This value $F(b)-F(a)$ is called the definite integral of ~~f(x)~~ ~~small~~ ~~f(x)~~ $f(x)$ b/w the limits a & b

In symbol we write it as $\int_a^b f(x)\,dx = F(b)-F(a)$

here a & b are called the limits of integration a is called the lower limit and b is called upper limit

Note :

$$\int_a^b f(x)\,dx = \int_a^b f(y)\,dy$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

1 Q evaluate $\int_0^1 (x^2+1)dx$

an $\frac{x^3}{3}+x$ $= \left[\frac{x^3}{3}+x\right]_0^1$

$= \frac{1^3}{3}+1 - \frac{0^3}{3}+0 = \frac{1}{3}+1-0$

$\frac{4}{3}$ //

2 $\int_0^1 (x^2+2x-3)dx$

$= \left[\frac{x^3}{3}+\frac{2x^2}{2}-3x\right]_0^1 = \left(\frac{1^3}{3}+\frac{2\times1^2}{2}-3\times1\right)-(0)$

$= \frac{1}{3}+\frac{2}{2}-1\times3 = \frac{1}{3}+1-3 =$

$=$

$\frac{4}{3}-3 = \frac{4-9}{3} = -\frac{5}{3}$ //

3 $\int_0^1 x(x+2)dx$

$\int_0^1 (x^2+2x)dx$

$= \left(\frac{x^3}{3}+\frac{2x^2}{2}\right)_0^1 = \frac{1^3}{3}+\frac{2\times1^2}{2} = \frac{1}{3}+1 =$

4 $\int_0^{\pi/2} \sin x \, dx$

$= \left[-\cos x\right]_0^{\pi/2} = -\left[\cos x\right]_0^{\pi/2} =$

$-\left(\cos \pi/2 - \cos 0\right) = -(0-1) = --1 = 1$ //

5 $\int_0^{\pi/2} \cos x \, dx$

an $= \left[\sin x\right]_0^{\pi/2} = \left(\sin \pi/2 - \sin 0\right) = 1-0 = 1$ //

6 $\int_0^{\pi/2} (\sin x + \cos x) \, dx$

an $= \left(-\cos x + \sin x\right)_0^{\pi/2} = \left(-\cos \pi/2 + \sin \cancel{\pi}\, \pi/2\right) -$

$\left(-\cos 0 + \sin 0\right]$

$= (-0+1) - (-1+0) = 1 - -1 = +2$ //

7 $\int_0^{\pi/4} \tan x \, dx$

an $\left[\log|\sec x|\right]_0^{\pi/4}$

$\log|\sec \pi/4| - \log|\sec 0|$

$= \log|\sqrt{2}| - \log|1| = \log|\sqrt{2}| - 0$

$= \sqrt{2}$ //

8 $\int_{\pi/4}^{\pi/2} \cot x \, dx$

an $\left[\log \sin x\right]_{\pi/4}^{\pi/2}$

$\log \sin \pi/2 - \log \sin \pi/4$

$\log 1 - \log \frac{1}{\sqrt{2}}$

$- \log (1/\sqrt{2})$ //

imp
9 $\int_0^1 \frac{1}{1+x^2} dx = (\tan^{-1} x) = \tan^{-1}(1) - \tan^{-1}(0)$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$ //

imp 10 $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx = (\sin^{-1} x) = \sin^{-1}(1) - \sin^{-1}(0)$

$= \pi/2 - 0 = \pi/2$ //

imp 11 $\int_0^{\pi/2} \sqrt{1+\sin 2x}\, dx$

An $\int \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}\, dx$

$= \int \sqrt{(\sin x+\cos x)^2}\, dx$

$= \int \sin x+\cos x = -\cos x+\sin x$

$\int_0^{\pi/2} \sqrt{1+\sin 2x} = \left[-\cos x+\sin x\right]_0^{\pi/2}$

$= (-\cos \pi/2+\sin \pi/2)-(-\cos 0+\sin 0)$

$= (-0+1) - (-1+0)$

$= 1 - -1 = 2$ //

12 $\int_0^{\pi/2} \sin^2 x\, dx$

$\int \sin^2 x = \int \left(\frac{1-\cos 2x}{2}\right) = \frac{1}{2}\int 1-\cos 2x\, dx$

$= \frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c = \frac{x}{2}-\frac{\sin 2x}{4}$ //

$\int_0^{\pi/2} \sin^2 x\, dx = \left(\frac{x}{2}-\frac{\sin 2x}{4}\right)_0^{\pi/2}$

$$\left(\frac{\pi/2}{2} - \frac{\sin 2\times\frac{\pi}{2}}{4}\right) - \left(\frac{0}{2} - \frac{\sin 0}{4}\right)$$

$$= \left(\frac{\pi}{4} - \frac{\sin \pi}{4}\right) - (0 - 0)$$

$$= \left(\frac{\pi}{4} - 0 - 0\right) = \frac{\pi}{4}$$

* $\sin \pi = \sin 2\pi = \sin 3\pi = \sin 4\pi = \cdots\cdots = 0$
* $\cos \frac{\pi}{2} = \cos 3\frac{\pi}{2} = \cos 5\frac{\pi}{2} = \cdots\cdots = 0$
* $\cos \cdot \pi = -1$, $\cos 2\pi = 1$, $\cos 3\pi = -1$
* $\log|1| = 0$, $\log e = 1$,

  $e^0 = 1$ $\left(e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1\right)$

13 $\int_0^{\pi/2} (\sin^2 2x)$

an $\int \sin^2 2x \, dx = \int \left(\frac{1 - \cos 4x}{2}\right) = \frac{1}{2}\int 1 - \cos 4x \, dx$

$$= \frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) = \frac{x}{2} - \frac{\sin 4x}{8}$$

$$\int_0^{\pi/2} (\sin^2 2x) = \left(\frac{x}{2} - \frac{\sin 4x}{8}\right)_0^{\pi/2}$$

$$= \left(\frac{\pi/2}{2} - \frac{\sin 2\frac{\pi}{2}}{8}\right) - \left(\frac{0}{2} - \frac{\sin 0}{8}\right)$$

$$= \frac{\pi}{4} - \frac{\sin 2\pi}{8} - 0$$

$$= \frac{\pi}{4} - 0 - 0 = \pi/4 //$$

14 $\int_0^{\pi/2} \cos^2 x \, dx$

$$\int \cos^2 x \, dx = \int \left(\frac{1 + \cos 2x}{2}\right) dx = \frac{1}{2}\int \left(1 + \cos 2x\right)$$

$$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int_0^{\pi/2} \cos^2 x \, dx = \left(\frac{x}{2} + \frac{\sin 2x}{4}\right)_0^{\pi/2}$$

$$= \left(\frac{\frac{\pi}{2}}{2} + \frac{\sin 2\frac{\pi}{2}}{4}\right) - \left(\frac{0}{2} - \frac{\sin 0}{4}\right)$$

$$= \left(\frac{\pi}{4} + \frac{\sin \pi}{4} - 0\right) = \frac{\pi}{4} //$$

15 $\int_0^1 x(x+1)(x+2) \, dx$

16 $\int_0^{\pi/2} \cos^2 2x \, dx$

$$\int_0^1 x(x+1)(x+2)\,dx$$

$(x^2+x)(x+2)$

$x^3+2x^2+x^2+2x$

$= x^3+3x^2+2x$

$$\int_0^1 (x^3+3x^2+2x)\,dx$$

$$\left(\frac{x^4}{4}+3\frac{x^3}{3}+2\frac{x^2}{2}\right)_0^1$$

$$= \left(\frac{1^4}{4}+\frac{3\times1^3}{3}+\frac{2\times1^2}{2}\right)-0 = \frac{1}{4}+\frac{3}{3}+\frac{2}{2}$$

$$\frac{1}{4}+1+1 = \frac{1}{4}+2 = \frac{9}{4}$$

10 $$\int_0^{\pi/2} \cos^2 2x\,dx$$

$$\int \cos^2 2x\,dx = \int\left(\frac{1+\cos 4x}{2}\right)dx$$

$$= \frac{1}{2}\int(1+\cos 4x)\,dx$$

$$= \frac{1}{2}\left(x+\frac{\sin 4x}{4}\right) = \frac{x}{2}+\frac{\sin 4x}{8}$$

$= \quad x$

$$\int_0^{\pi/2} \cos^2 2x\,dx = \left(\frac{x}{2}+\frac{\sin 4x}{8}\right)_0^{\pi/2}$$

$$= \left(\frac{\frac{\pi}{2}}{2}+\frac{\sin 4\frac{\pi}{2}}{8}\right)-\left(\frac{0}{2}+\frac{\sin 0}{8}\right)$$

$\frac{\pi}{4} + 0 - 0 = \frac{\pi}{4}$ //

Q $\int_0^1 \frac{2x+1}{x^2+x+1} \, dx$

put $t = x^2+x+1$

$\frac{dt}{dx} = 2x+1$ $\qquad \int \frac{dt}{t} = \log(t)$

$dt = 2x+1 \, dx$

$\int_0^1 \frac{2x+1 \, dx}{x^2+x+1} = \log (x^2+x+1)_0^1$ $\qquad \log 1 = 0$

$= \log 1^2+1+1 - \log 1$

$= \log 3 - \log 1$

$= \log 3$ //

Q $\int_0^3 \frac{x^2+1}{x^3+3x} \, dx$

put $t = x^3+3x$

$\frac{dt}{dx} = 3x^2+3$

$dt = 3(x^2+1)dx$

$\frac{dt}{3} = (x^2+1)\,dx$

$\int_0^3 \frac{1}{t} \cdot \frac{dt}{3} = \frac{1}{3}\int \frac{1}{t} \, dt = \frac{1}{3}\left[\log|t|\right]_0^3$

$= \frac{1}{3} \log\left[x^3+3x\right]_0^3 = \frac{1}{3} \log 3^3+3\times3 - \log 0^3 ...$

$= \frac{1}{3}\left[\log 36 - \log 4\right]$

$\log a - \log b = \log a/b$

$= \frac{1}{3} \log (36/4) = \frac{1}{3} \log 9$ //

Imp Q $\int_0^{\pi/4} \frac{\sec^2 x}{1+\tan x}\, dx$

an Put $t = 1+\tan x$

$\frac{dt}{dx} = \sec^2 x$

$dt = \sec^2 x \, dx$

$\int \frac{dt}{t} = \log |t|$

$\int_0^{\pi/4} \frac{\sec^2 x}{1+\tan x}\, dx = \left[\log (1+\tan x)\right]_0^{\pi/4}$

$\log (1+\tan \pi/4) - \log (1+\tan 0)$

$= \log 2 - \log 1 = \log 2$ //

a $\int_0^{\pi} \frac{1-\sin x}{x+\cos x}\, dx$

put $t = x+\cos x$

$\frac{dt}{dx} = 1-\sin x$

a $\int \frac{1-\sin x}{x+\cos x}\, dx = \left[\log(x+\cos x)\right]_0^{\pi}$

$dt = 1-\sin x\, dx$

$\int \frac{dt}{t} = \log(t)$

$\log \pi + \cos\pi - \log(0+\cos 0)$

$= \log(\pi-1) - \log(1)$

$\log \frac{(\pi-1)}{(1)}$ //

(mi) $\int_0^4 x\sqrt{x^2+9}\, dx$

put $t = (x^2+9)^{1/2}$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x\, dx$

an $\int_0^4 x\sqrt{x^2+9}\, dx = \frac{1}{3}(x^2+9)^{3/2}$

$\int_0^4 x\sqrt{x^2+9}\, dx = \frac{1}{3}\left[(x^2+9)^{3/2}\right]_0^4$

$\cancel{\int \frac{dt}{2t} = \frac{1}{2}\int\frac{dt}{t}} =$

$\frac{1}{2}\log(t)$

$= \frac{1}{3}\left[25^{3/2} + 9^{3/2}\right]$

$\int \frac{1}{2} t^{1/2} \frac{dt}{2}$

$= \frac{1}{3}(125-27) = 98/3$ //

$\frac{1}{2}\left(\frac{t^{3/2}}{3/2}\, dt\right)$

$\left[\frac{1}{2}\times\frac{2}{3}\left(t^{3/2}\right]_0^4 = \frac{1}{2}\left(\frac{t^{3/2}}{3/2}\right]_0^4$

a) $\int_0^{\pi/2} \sin x (1-\cos x)^5 \, dx$

put $t = 1-\cos x$

$\frac{dt}{dx} = \sin x$.

$dt = \sin x \, dx$

$\int t^5 \, dt = \frac{t^6}{6}$

$\int_0^{\pi/2} \sin x (1-\cos x)^5 dx = \left(\frac{t^6}{6}\right]_0^{\pi/2} = \left[\frac{(1-\cos x)^6}{6}\right]_0^{\pi/2}$

$\frac{1}{6}\left[1-\cos x^6\right]_0^{\pi/2} = \frac{1}{6}\left((1-\cos \frac{\pi}{2})^6 - (1-\cos 0)^6\right)$

$= \frac{1}{6}\left(1-0)^6 - (1-1)^6\right] = \frac{1}{6} \times 1^6 - 0$

$= \frac{1}{6}$

c) $\int_0^{\sqrt{\pi}/2} x \sin (x^2) dx$

put $t = x^2$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x dx$

$\int_0^{\sqrt{\pi}/2} \sin t \cdot \frac{dt}{2} = \frac{1}{2}\int_0^{\sqrt{\pi}/2} \sin t \cdot dt$

$$\frac{1}{2}\left[\cos t\right]_0^{\sqrt{\pi/2}}$$

$$\int_0^{\sqrt{\pi/2}} x \sin(x^2)\,dx = \frac{1}{2}\left[-\cos x^2\right]_0^{\sqrt{\pi/2}}$$

$$= \frac{1}{2}\left[-\cos \sqrt{\pi/2}^{\,2} - -\cos 0^2\right]$$

$$= -\frac{1}{2}\left[\cos \pi/2 - \cos 0^2\right] = -\frac{1}{2}(0-1) = -\frac{1}{2} \times -1$$

$$= 1/2 \;//$$

Q $\int_0^{\pi/2} x \sin x \, dx$

$$\int x \int \sin x\,dx - \int\left\{\frac{d}{dx} x\right\} \cdot \int \sin x\} \,dx$$

$$x \cdot -\cos x - \int 1 \cdot -\cos x \, dx$$

$$-x \cos x - (-\sin x + C)$$

$$\int_0^{\pi/2} x \sin x\,dx = \left[-x\cos x + \sin x\right]_0^{\pi/2}$$

$$\left(-\pi/2 \cos \pi/2 + \sin \pi/2\right) - \left[-0\cos 0 + \sin 0\right]$$

$$= (-0+1) + (0+0) = 1 \;//.$$

Q $\int_0^{\pi/2} n \cos n \, dn$

$n \int \cos n \, dn - \int \left\{ \frac{d}{dn} n \right\} \times \int \cos n \cdot dn$

$n \cdot \sin n - \int 1 \times \sin n \, dn$

$n \sin n - (-\cos n) = n \sin n + \cos n$

$\int_0^{\pi/2} n \cos n \, dn = \Big[n \sin n + \cos n \Big]_0^{\pi/2}$

$= \left(\pi/2 \times \sin \pi/2 + \cos \pi/2 \right) - \left(0 \sin 0 + \cos 0 \right)$

$= \left(1 \times \pi/2 + 0 \right) - \left(0 + 1 \right)$

$= \pi/2 - 1$

Q $\int_1^3 n^2 \log n \, dn$

an $\int \log n \int n^2 dn - \int \left\{ \frac{d}{dn} \log n \right\} \times \int n^2 dn$

$\log n \frac{n^3}{3} - \int \frac{1}{\cancel{n}} \times \frac{n^{\cancel{3}}}{3} dn$

$\log n \frac{n^3}{3} - \frac{1}{3} \int n^2 \, dn$

$\log n \frac{n^3}{3} - \frac{1}{3} \cdot \frac{n^3}{3}$

$\log x \quad \frac{x^3}{3} - \frac{1}{3}\frac{1}{3}x^3$

$\frac{x^3}{3}\left(\log x - \frac{1}{3}\right)$

$\int_1^3 x^2 \log x \, dx = \frac{x^3}{3}\left(\log x - \frac{1}{3}\right)_1^3$

$= \frac{3^3}{3} \cdot \left(\log 3 - \frac{1}{3}\right)$ ~~$-$~~

$= \frac{1^3}{3}\left(\log 1 - \frac{1}{3}\right)$

$= 9\left(\log 3 - \frac{1}{3}\right) - \frac{1}{3}\left(0 - \frac{1}{3}\right)$

$9\left(\log 3 - \frac{1}{3}\right) + \frac{1}{9}$ //

Q $\int_1^e \log x \, dx$

an $\int_1^e 1 \cdot \log x \, dx$

$\log x \int 1 \, dx - \int \left\{\frac{d}{dx}\right\} \log x \int 1 \, dx$

$\log x \cdot x - \int \frac{1}{x} \cdot x \, dx$

$\log x \cdot x - x$

$x(\log x - 1)$

$\int_1^e \log x \, dx = \left(x(\log x - 1)\right)_0^e =$

$e(\log e - 1) - 1(\log 1 - 1)$

$e(1-1) - 1(0-1) = 1$ //

Q $\int_0^{\pi/2} \sin 3x \cos x \, dx = \frac{1}{2}\int 2\sin 3x \cos x \, dx$

$= \frac{1}{2}\int \sin(3x+x) + \sin(3x-x) \, dx$

$= \frac{1}{2}\int (\sin 4x + \sin 2x) \, dx$

$= \frac{1}{2}\left(\frac{-\cos 4x}{4} - \cos 2x\right)$

$= -\frac{1}{2}\left(\frac{\cos 4x}{4} + \cos\frac{2x}{2}\right)_0^{\pi/2}$

$\int_0^{\pi/2} \sin 3x \cos x \, dx = -\frac{1}{8}\left(\cos\right.$

$= \frac{-1}{2}\left(\frac{\cos 2\times\frac{\pi}{2}}{4} + \cos\frac{2\times\pi/4}{2}\right) - \frac{1}{2}\left(\frac{\cos 4\times 0}{4} + \cos\frac{0}{2}\right)$

$= -\frac{1}{2}\left(\frac{1}{4} - \frac{1}{2}\right) - \frac{1}{2}\left(\frac{1}{4} + \frac{1}{2}\right)$

$= -\frac{1}{8} + \frac{1}{4} - -\frac{1}{8} = \frac{1}{4}$

$= \frac{-4+8}{32} - \frac{-4-8}{32} = \frac{-4}{32} - \frac{-12}{32}$

$-4+12$ $\qquad \frac{\cancel{8}}{32} =$

$\frac{-1}{2}\left(\frac{-1}{4}-\frac{3}{4}\right)=\frac{1}{2}$ //

Q $\int_0^{\pi/3} \cos 4x \cos x \, dx$

an $\frac{1}{2}\int 2\cos 4x \cos x \, dx = \frac{1}{2}\int(\cos 5x + \cos 3x)\,dx$

imp

$\frac{1}{2}\int\left(\frac{\sin 5x}{5}+\frac{\sin 3x}{3}\right)+c$

$\frac{1}{2}\left(\frac{\sin 5x}{5}+\frac{\sin 3x}{3}\right)_0^{\pi/3}$

$=\frac{1}{2}\left[\frac{\sin 5\times\pi/3}{5}+\frac{\sin 3\times\pi/3}{3}\right)-\left(\frac{\sin 0}{5}+\frac{\sin 0}{2}\right]$

$=\frac{1}{2}\left(\frac{\sin 300}{5}+\frac{\sin \pi}{3}\right)-(0+0)$

$\frac{1}{2}\left(\frac{-\sqrt{3}/2}{5}+\frac{0}{3}\right)-0$

$\sin 300 =$

$\sin(360-60)$

$-\sin 60 = -\sqrt{3}/2$

$= \frac{1}{2}\times\frac{-\sqrt{3}/2}{5} = \sqrt{3}/20$ //

Q $\int_0^{\pi}\frac{1}{1+\sin x}\,dx$

an $\int\frac{1-\sin x}{(1+\sin x)(1-\sin x)}\,dx$

$$\int \frac{1-\sin x}{1^2-\sin^2 x}\,dx$$

$$= \int \frac{1-\sin x}{\cos^2 x}\,dx$$

$$= \int \cancel{\frac{1-\sin x}{\cos x} \times \frac{1}{\cos x}}\,dx$$

$$= \int \cancel{\frac{1}{\cos x}} - \sin x$$

$$\int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\right) dx$$

$$= \int \sec^2 x - \tan x \cdot \sec x \; dx$$

$$= \tan x - \sec x + c$$

$$= \left[\tan x - \sec x\right]_0^{\pi} = (\tan\pi - \sec\pi) - (\tan 0 - \sec 0)$$

$$= (0 - -1) - (0 - 1)$$

$$= 1 + 1 = 2$$

Q1 $\int_0^{\pi/2} \sin 3x \, dx$

3) $\int_0^{\pi/2} \frac{\cos x}{2+\sin x}\, dx$

2 $\int_0^{\pi/2} \cos^3 x \, dx$

4, $\int_0^{\pi/2} \cos x \sqrt{1+\sin x}\, dx$

area

area bounded (enclosed) by the

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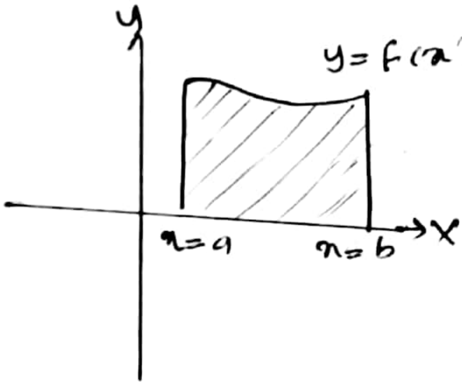
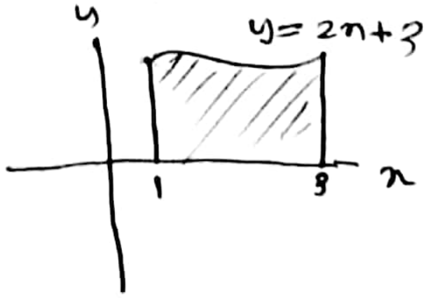
area

area bounded (enclosed under) by the curve $y = f(x)$, the x axis, the lines (ordinates) at $x = a$ & $x = b$



$$\boxed{\text{area} = \int_a^b y\,dx}$$

Q find the area under the straight line $y = 2x+3$ bounded by the x axis and ordinates at $x = 1$ & $x = 3$



$$\text{area} = \int_a^b y\,dx$$

$$\int_1^3 2x+3\,dx$$

dx

$$= \int_1^3 \frac{\cancel{2}x^2}{\cancel{2}} + 3x$$

$$= \left(x^2 + 3x\right]_1^3 = 3^2 + 3\times 3 - 1^2 + 3\times 1$$

$$= (9+9) - (1+3)$$

$$= 18 - 4 = 14 \text{ Square unit}$$

Q find the area bounded by the curve $y = x + \sin x$ the x axis and the lines at $x = 0$, $x = \pi/2$

$$\text{area} = \int_a^b y\,dx$$

$$= \int_0^{\pi/2} x + \sin x \, dx$$

$$= \left[\frac{x^2}{2} - \cos x\right]_0^{\pi/2}$$

$$\left(\frac{(\pi/2)^2}{2} - \cos \pi/2\right) - \left(\frac{0}{2} - \cos 0\right)$$

$$= \left(\frac{\pi^2}{8} - 0\right) - (0-1)$$

$$= \frac{\pi^2}{8} + 1 \text{ square unit}$$

Parabola