Matrices

a matrix is a rectangular arrangement of elements in different rows and columns. the horizontal arrangements are called 'rows'. the vertical arrangements are called 'columns'.

eg :

$$A = \begin{array}{c} \begin{bmatrix} 3 & -2 & 5 \\ 5 & -3 & 1 \end{bmatrix} \begin{array}{l} \text{rows } 1 \\ \text{row } 2 \end{array} \\ \begin{array}{ccc} \text{column } 1 & c_2 & c_3 \end{array} \end{array}$$

in A there are 2 rows and 3 columns then we say that the matrix A is of order 2×3 (2 by 3)

TYPES OF MATRIX

1 zero matrix (null matrix)

a matrix having all elements zero

eg:

$$N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

N is a 2×3 zero matrix

2 Sqyare matrix

a matrix with equal no. of rows and columns

eg :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 0 & 1 & 2 \end{bmatrix}$$ - square matrix of order

$[1]$ - 1x1 square matrix

3 Diagonal matrix

a square matrix where all elements except the diagonal elements are zero

(diagonal elements can be zero or not but all other elements should be zero)

eg :

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ - also a didgonal matrix

4 Identity matrix

a diagonal matrix where all diagonal elements are 1

an identity Matrix of order n. is denoted by I_n

eg $I_1 = [1]$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5 Row matrix

a matrix with a single row is called a row matrix

eg :

$$\begin{bmatrix} 1 & 3 & 2 & 5 \end{bmatrix}$$

order $= 1 \times 4$

6 column matrix

a matrix with a single column is called a column matrix

eg : $\begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$ order 3×1

7 upper triangular matrix

a square matrix is said to be upper triangular if all elements below the diagonal entries are zeros

eg :

$$U = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8 lower triangular matrix

a Square matrix is said to be if all elements above the diagonal entries are zeros

eg :

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 5 & 3 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

→ zero matrix and diagonal matrices are both upper triangular and lower triangular matrix

9) Scalar matrix

a diagonal matrix when all diagonal entries are same say a constant k

$$S = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \quad \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

EQUALITY OF MATRICES

2 matrices of the same order are said to be equal ~~b~~ if the corresponding elements are equal

1) Find x, y if

$$\begin{bmatrix} x+y & x-y \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$$

$x+y = 4$ — (1)

$x-y = 6$ — (2)

$2x = 10$

$x = \frac{10}{2} = 5$

$x = 5$

$y = 4 - 5 = -1$

$y = -1$

2) Find a, b, c, d if

$$\begin{bmatrix} 2a & a+3b \\ 5-c & d \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 1 & 0 \end{bmatrix}$$

$2a = 4$

$a + 3b = 11$

$5 - c = 7$

$d = 0$

$a = \frac{4}{2} = 2$ //

$2 + 3b = 11$

$3b = 11 - 2$

$b = \frac{9}{3} = 3$ //

$-c = 7 - 5$

$c = -2$ //

$d = 0$ //

SCALAR MULTIPLICATION

To multiply a matrix with a scalar we need to multiply each elements of that matrix by the scalar

eg :

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \qquad 2A = \begin{bmatrix} 4 & 6 \\ 2 & -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -4 & 3 \\ 5 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \quad 3A = \begin{bmatrix} 9 & -12 & 9 \\ 15 & 6 & 3 \\ 6 & 3 & 3 \end{bmatrix}$$

MATRIX ADDITION

We can add 2 matrix of the same order for that we have to add the corresponding elements

eg

1) $$\begin{bmatrix} 2 & 1 & -3 \\ 5 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 5 & 2 \\ 0 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 6 & -1 \\ 5 & -1 & -1 \end{bmatrix}$$

2) $$A = \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -2 & -1 \\ 2 & 4 \end{bmatrix}$$ find $2A + 3B$

$$2A = \begin{bmatrix} 6 & 2 \\ 4 & 10 \end{bmatrix}, \quad 3B = \begin{bmatrix} -6 & -3 \\ 6 & 12 \end{bmatrix}$$

$$2A + 3B = \begin{bmatrix} 0 & -1 \\ 10 & 22 \end{bmatrix}$$

1) Find A - B . if

$$A + B = \begin{bmatrix} 5 & 3 \\ 4 & -2 \end{bmatrix} \quad A - B = \begin{bmatrix} 7 & 5 \\ 0 & 6 \end{bmatrix}$$

$$A+B=\begin{bmatrix}5 & 3\\ 4 & -2\end{bmatrix} \quad (1)$$

$$A-B=\begin{bmatrix}7 & 5\\ 0 & 6\end{bmatrix} \text{ --- } (2)$$

$2(1)+(2)$

$$2A=\begin{bmatrix}12 & 8\\ 4 & 4\end{bmatrix}$$

$$A=\begin{bmatrix}6 & 4\\ 2 & 2\end{bmatrix}$$

$$B=\begin{bmatrix}5 & 3\\ 4 & -2\end{bmatrix}-\begin{bmatrix}6 & 4\\ 2 & 2\end{bmatrix}$$

$$B=\begin{bmatrix}-1 & -1\\ 2 & -4\end{bmatrix}$$

2 Find A, B if

$$A+B=\begin{bmatrix}7 & 5\\ 12 & 8\end{bmatrix} \text{ --- } (1)$$

$$A-B=\begin{bmatrix}-3 & 1\\ -6 & 4\end{bmatrix} \text{ --- } (2)$$

$(1)+(2)$

$$2A=\begin{bmatrix}4 & 6\\ 6 & 12\end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 4 & 6 \\ 6 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & 5 \\ 12 & 8 \end{bmatrix} - A$$

$$= \begin{bmatrix} 7 & 5 \\ 12 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 \\ 9 & 2 \end{bmatrix}$$

3) Find A, B if

$$A + 2B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix} \quad \text{—— (1)}$$

$$2A + 3B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix} \quad \text{—— (2)}$$

(1) × 2

$$2A + 4B = \begin{bmatrix} 4 & 2 & 0 \\ 2 & -2 & 4 \end{bmatrix} \quad \text{—— (3)}$$

$$2A + 3B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix} \quad \text{—— (2)}$$

(3) − (2)

$$B = \begin{bmatrix} 3 & 0 & +1 \\ 0 & -2 & 3 \end{bmatrix}$$

$$A+2B = A + 2\begin{bmatrix} 3 & 0 & +1 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 0 & +2 \\ 0 & -4 & 6 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 2-4 & 1 & -2 \\ 1 & & \end{bmatrix}}$$

$$A = \begin{bmatrix} -4 & 1 & -2 \\ 1 & 3 & -4 \end{bmatrix}$$

4) $2A+3B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ —— (1)

$3A-2B = \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix}$ —— (2)

(1) $\times 2 \Rightarrow 4A+6B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$ —— (3)

(2) $\times 3 \Rightarrow 9A-6B = \begin{bmatrix} 15 & 12 & 9 \\ 6 & 3 & 0 \end{bmatrix}$ —— (4)

$$③ + ④ \Rightarrow 13A = \begin{bmatrix} 17 & 16 & 15 \\ 14 & 13 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 17/13 & 16/13 & 15/13 \\ 14/13 & 1 & 12/13 \end{bmatrix}$$

Put A in ①

$$2A + 3B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$2\begin{bmatrix} 17/13 & 16/13 & 15/13 \\ 14/13 & 1 & 12/13 \end{bmatrix} + 3B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 34/13 & 32/13 & 30/13 \\ 28/13 & 2 & 24/13 \end{bmatrix} + 3B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$3B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} - \begin{bmatrix} 34/13 & 32/13 & 30/13 \\ 28/13 & 2 & 24/13 \end{bmatrix}$$

$$3B = \begin{bmatrix} -21/13 & \frac{-6}{13} & \frac{9}{13} \\ 24/13 & 3 & \frac{54}{13} \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{21}{39} & \frac{-6}{39} & \frac{9}{39} \\ \frac{24}{39} & \frac{3}{3} & \frac{54}{39} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{-7}{13} & \frac{-2}{13} & \frac{3}{13} \\ \frac{3}{13} & 1 & \frac{18}{13} \end{bmatrix}$$

MATRIX MULTIPLICATION

given 2 matrix A, B then we can find AB [A & B are compatible for multiplication] if no: of columns of A equals no. of rows of B

1 Find AB if $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 2 & -3 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 2 & -3 \end{bmatrix}$$

$2 \times 3 \times 3 \times 2$

2×2

$$AB = \begin{bmatrix} 1\times3 + -2\times1 + 3\times2 & 1\times2 + -2\times4 + 3\times{-3} \\ 0\times3 + 2\times1 + 5\times2 & 0\times2 + 2\times4 + 5\times{-3} \end{bmatrix}$$

$$= \begin{bmatrix} 3-2+6 & 2-8-9 \\ 0+2+10 & 6+8-15 \end{bmatrix} = \begin{bmatrix} 7 & -15 \\ 12 & -1 \end{bmatrix} // 2\times2$$

$$BA = \overset{B}{\begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 2 & -3 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & 5 \end{bmatrix}}$$

$3\times 2 \times 2 \times 3$

3×3

$$= \begin{bmatrix} 3\times1+2\times0 & 3\times-2+2\times2 & 3\times3+2\times5 \\ 1\times1+4\times0 & 1\times-2+4\times2 & 1\times3+4\times5 \\ 2\times1+-3\times0 & 2\times-2+-3\times2 & 2\times3+-3\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & -6+4 & 9+10 \\ 1+0 & -2+8 & 3+20 \\ 2-0 & -4-6 & 6-15 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & 19 \\ 1 & 6 & 23 \\ 2 & -10 & -9 \end{bmatrix} // 3\times3$$

2 $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix}$

Find AB, BC

$$AB = \overset{A}{\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \end{bmatrix}} \overset{B}{\begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & -1 \end{bmatrix}}$$

$$= \begin{bmatrix} 1\times 3 + -3\times 2 + 2\times 1 & 1\times -1 + -3\times 0 + 2\times -1 \\ 2\times 3 + 1\times 2 + 0\times 1 & 2\times -1 + 1\times 0 + 0\times -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-6+2 & -1-2 \\ 6+2 & -2+0-0 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 8 & -2 \end{bmatrix}$$

$$BC = \overset{B}{\begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & -1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 3\times 1 + -1\times 2 & 3\times -3 + -1\times 1 & 3\times 2 + -1\times 0 \\ 2\times 1 + 0\times 2 & 2\times -3 + 0\times 1 & 2\times 2 + 0\times 0 \\ \cancel{1\times 2 + -1\times 0} & 1\cdot & \\ 1\times 1 + -1\times 2 & 1\times -3 + -1\times 1 & 1\times 2 + -1\times 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{8-2} \end{bmatrix} \qquad BC = \begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix}$$

$$BC = \begin{bmatrix} 3\times 2 + -1\times 1 & 3\times 2 + -1\times 5 \\ 2\times 2 + 0\times 1 & 2\times 2 + 0\times 5 \\ 1\times 2 + -1\times 1 & 1\times 2 + -1\times 5 \end{bmatrix} = \begin{bmatrix} 6-1 & 6-5 \\ 4+0 & 4+0 \\ 2-1 & 2-5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 \\ 4 & 4 \\ 1 & -3 \end{bmatrix}$$

b) S.T $A(BC) = (AB)C$

$$A(BC) = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 4 & 4 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times5+-3\times4+2\times1 & 1\times1+-3\times4+2\times-3 \\ 2\times5+1\times4+0\times1 & 2\times1+1\times4+0\times-3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-12+2 & 1-12-6 \\ 10+4 & 2+4 \end{bmatrix} = \begin{bmatrix} -5 & -17 \\ 14 & 6 \end{bmatrix} //$$

$$(AB)C = \overset{AB}{\begin{bmatrix} -1 & -3 \\ 8 & -2 \end{bmatrix}} \overset{C}{\begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix}}$$

$$= \begin{bmatrix} -1\times2+-3\times1 & -1\times2+-3\times5 \\ 8\times2+-2\times1 & 8\times2+-2\times5 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3 & -2-15 \\ 16-2 & 16-10 \end{bmatrix} = \begin{bmatrix} -5 & -17 \\ 14 & 6 \end{bmatrix} //$$

$\therefore A(BC) = (AB)C$ //

3 $A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

ST. $A(\theta) \cdot A(\theta') = A(\theta+\theta')$

a given $A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A(\theta') = \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix}$

$A(\theta+\theta') = \begin{bmatrix} \cos(\theta+\theta') & -\sin(\theta+\theta') \\ \sin(\theta+\theta') & \cos(\theta+\theta') \end{bmatrix}$

$A(\theta) \cdot A(\theta') = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix}$

$\begin{bmatrix} \cancel{\cos\theta \times \cos\theta' - \sin\theta \times \sin\theta'} \end{bmatrix}$

$= \begin{bmatrix} \cos\theta \times \cos\theta' - \sin\theta \times \sin\theta' & \cos\theta \times -\sin\theta' + -\sin\theta \times \\ \sin\theta \times \cos\theta' + \cos\theta \times \sin\theta' & \sin\theta \times -\sin\theta' + \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta \cos\theta' - \sin\theta \sin\theta' & -(\cos\theta \sin\theta' + \sin\theta \cos\theta') \\ \sin\theta \cos\theta' + \cos\theta \sin\theta' & \cos\theta \cos\theta' - \sin\theta \sin\theta' \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\theta') & -\sin(\theta+\theta') \\ \sin(\theta+\theta') & \cos(\theta+\theta') \end{bmatrix}$$

$A(\theta+\theta')$

Transpose of a matrix

given a matrix A, then A^T is the matrix of obtained by interchanging the rows and columns

eg 1) $A = \begin{bmatrix} 4 & 3 & -2 \\ 5 & 1 & 4 \end{bmatrix}_{2\times 3}$ $\quad A^T = \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}_{3\times 2}$

$B = \begin{bmatrix} 3 & 2 & -5 \\ 6 & 4 & 7 \\ -2 & 1 & 0 \end{bmatrix}$ $\quad B^T = \begin{bmatrix} 3 & 6 & -2 \\ 2 & 4 & 1 \\ -5 & 7 & 0 \end{bmatrix}$

Properties of transpose

1) $(A^T)^T = A$

eg $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ $\quad (A^T)^T = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$

2) $(kA)^T = kA^T$ where k is a scalar

3) $(A+B)^T = A^T + B^T$

eg given 2 matrices

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 5 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

S.T $(A+B)^T = A^T + B^T$

on LHS $A + B = \begin{bmatrix} 1 & 3 & -2 \\ 5 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4 & 5 & -6 \\ 7 & -2 & 4 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 4 & 7 \\ 5 & -2 \\ -6 & 4 \end{bmatrix}$$

RHS →

$$A^T = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 3 & 2 \\ 2 & -4 \\ -4 & 3 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 4 & 7 \\ 5 & 2 \\ -6 & 4 \end{bmatrix}$$

LHS = RHS//

4 $(AB)^T = B^T \cdot A^T$

$$A = \begin{bmatrix} 4 & 5 & -6 \\ 3 & 2 & -1 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}_{3\times 2} \qquad \text{S.T } (AB)^T = B^T \cdot A^T$$

LHS → $AB = \begin{bmatrix} 4 & 5 & -6 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 4\times1+5\times-2+-6\times1 & 4\times3+5\times4+-6\times0 \\ 3\times1+2\times-2+-1\times1 & 3\times3+2\times4+-1\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-10-6 & 12+20-0 \\ 3-4-1 & 9+8-0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 32 \\ -2 & 17 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} -12 & -2 \\ 32 & 17 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & -2 & 14 \\ 3 & 4 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & 3 \\ 5 & 2 \\ -6 & -1 \end{bmatrix}$$

$$B^T \cdot A^T = \begin{bmatrix} 1 & -2 & 1 \\ 3 & 4 & 0 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 5 & 2 \\ -6 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 4 + -2\times 5 + 1\times -6 & 1\times 3 + -2\times 2 + 1\times -1 \\ 3\times 4 + 4\times 5 + 0\times -6 & 3\times 3 + 4\times 2 + 0\times -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 10 - 6 & 3 - 4 - 1 \\ 12 + 20 - 0 & 9 + 8 - 1 \end{bmatrix} = \begin{bmatrix} -12 & -2 \\ 32 & 17 \end{bmatrix}$$

$$\therefore (AB)^T = B^T \cdot A^T \;//$$

Symmetric matrix

a square matrix A is said to be symmetric if $A^T = A$

eg) $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 5 & -1 \\ 2 & -1 & 8 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 5 & -1 \\ 2 & -1 & 8 \end{bmatrix} = A$

$A^T = A$ //

Skew symmetric matrix

a square matrix is said to be skew symmetric if $A^T = -A$

eg, $A = \begin{bmatrix} 0 & 7 & -8 \\ -7 & 0 & -3 \\ 8 & 3 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & -7 & 8 \\ 7 & 0 & 3 \\ -8 & 3 & 0 \end{bmatrix} = -A$

$A^T = -A$ //

$B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 5 \\ -1 & 5 & -8 \end{bmatrix}$ $B^T = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & \\ 1 & & -5 \end{bmatrix}$

write any eg.

$B = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 3 \\ -5 & 8 & 0 \end{bmatrix}$ $B^T = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}$

Note: : The diagonal elements of a skew symmetric matrix are always 0)

Proof

we have to show that $(AA^T)^T = AA^T$

$$(AA^T)^T = (A^T)^T . A^T = AA^T$$

$((AB)^T = A^T . B^T)$

1 $A = \begin{bmatrix} 3 & 4 & -5 \\ 5 & 3 & -2 \end{bmatrix}$

ST AA^T is a symetric

an

$$A^T = \begin{bmatrix} 3 & 5 \\ 4 & 3 \\ -5 & -2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 3 & 4 & -5 \\ 5 & 3 & -2 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 4 & 3 \\ -5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times3 + 4\times4 - 5\times-5 & 3\times5 + 4\times3 - 5\times-2 \\ 5\times3 + 3\times4 - 2\times-5 & 5\times5 + 3\times3 - 2\times-2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+16+25 & 15+12+10 \\ 15+12+10 & 25+9+4 \end{bmatrix} = \begin{bmatrix} 50 & 37 \\ 37 & 38 \end{bmatrix}$$

$$(AA^T)^T = \begin{bmatrix} 50 & 37 \\ 37 & 38 \end{bmatrix} = AA^T$$

$\therefore$ AA^T is symmetric

2 ST any square matrix A can be written as sum of a symmetric matrix and a skew symmetric matrix.

imp

an First we will show that $A+A^T$ is symmetric

$$(A+A^T)^T = A^T + (A^T)^T = A^T + A$$

$$= A + A^T$$

ie $A+A^T$ is symmetric

Then $\frac{1}{2}(A+A^T)$ is also symmetric

Now we'll s.t $A-A^T$ is skew symmetric

$$(A-A^T)^T = A^T - (A^T)^T = A^T - A$$

$$= -(A-A^T)$$

ie $A-A^T$ is skew symmetric

ie $\frac{1}{2}(A-A^T)$ is also skew symmetric

Now $\frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$

$$= \frac{1}{2}A + \cancel{\frac{1}{2}A^T} + \frac{1}{2}A - \cancel{\frac{1}{2}A^T}$$

$$= A$$ //

3 find $A = \begin{bmatrix} 3 & 4 & -7 \\ 6 & 2 & 8 \\ 5 & 4 & 1 \end{bmatrix}$

an $A^T = \begin{bmatrix} 3 & 6 & 5 \\ 4 & 2 & 4 \\ -7 & 8 & 1 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 6 & 10 & -2 \\ 10 & 4 & 12 \\ -2 & 12 & 2 \end{bmatrix}$

$B = \frac{1}{2}(A + A^T) = \begin{bmatrix} 3 & 5 & -1 \\ 5 & 2 & 6 \\ -1 & 6 & 1 \end{bmatrix}$

$B^T = B$ ie B is symmetric

Now. $A - A^T = \begin{bmatrix} 0 & -2 & -12 \\ 2 & 0 & 4 \\ 12 & -4 & 0 \end{bmatrix}$

$C = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -1 & -6 \\ 1 & 0 & 2 \\ 6 & -2 & 0 \end{bmatrix}$

$C^T = -C \Rightarrow C$ is skew symmetric

Now $B + C = \begin{bmatrix} 3 & 5 & -1 \\ 5 & 2 & 6 \\ -1 & 6 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & -6 \\ 1 & 0 & 2 \\ 6 & -2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & 4 & -7 \\ 6 & 2 & 8 \\ 5 & 4 & 1 \end{bmatrix} = A$ //

4) find $A = \begin{bmatrix} 6 & 4 \\ 10 & 3 \end{bmatrix}$

$$A^T = \begin{bmatrix} 6 & 10 \\ 4 & 3 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 12 & 14 \\ 14 & 6 \end{bmatrix}$$

$$B = \frac{1}{2}(A + A^T) = \begin{bmatrix} 6 & 7 \\ 7 & 3 \end{bmatrix}$$

$B^T = B$ ie B is symmetric

Now, $A - A^T = \begin{bmatrix} 0 & -4 \\ -10 & 0 \end{bmatrix}$

$$C = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -2 \\ -5 & 0 \end{bmatrix}$$

Now $B + C = \begin{bmatrix} 6 & 4 \\ 10 & 3 \end{bmatrix}$ //

Determinents

determinent of a 2×2 square matrix A is given by

$|A|$ = product of diagonal elements − product of non diagonal elements

1 find $|A|$ if $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$

$$|A| = 2 \times -1 - 3 \times 4$$

$$= -2 - 12 = -14$$

here $|A| = \begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix} = -14$

2 find $|A|$ if $A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$

$$|A| = 3 \times 2 - (-4 \times 1)$$

$$= 6 - {}^{-}4 = 10$$

3 $A = \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$ ST $|AB| = |A| \cdot |B|$

$$A \cdot B = \begin{bmatrix} 3 \times 4 + -2 \times 2 & 3 \times -3 + -2 \times 1 \\ 1 \times 4 + -1 \times 2 & 1 \times -3 + -1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 4 & -9 - 2 \\ 4 - 2 & -3 - 1 \end{bmatrix} = \begin{bmatrix} 8 & -11 \\ 2 & -4 \end{bmatrix}$$

$$|A \cdot B| = 8 \times -4 - 2 \times -11$$

$$= -32 - -22$$

$$= -32 + 22 = -10$$

$|A| = 3\times -1 - (-2\times 1) = -3 + 2 = -1$

$|B| = 4\times 1 - (-3\times 2) = 4 + 6 = 10$

$|A|.|B| = -1\times 10 = -10 = |A.B|$

$\rightarrow |A^2| = |A|.|A|$

$|AB| = |A|.|B|$

Determinent of 3×3 matrix

1) Find $|A|$ if $A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 4 & -2 & 5 \end{bmatrix} \end{array}$

$|A| = 3(3\times 5 - (-2\times 1)) - 2(2\times 5 - 1\times 4) + -1(2\times -2 - 3\times 4)$

$= 3(15+2) - 2(10-4) + -1(-4-12)$

$= 3\times 17 - 2\times 6 - 1\times -16 = 51 - 12 + 16$

$= 51 + 4 = 55$

2 Find $|B|$ if $B = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 3 & -4 & 1 \\ 2 & 3 & 5 \\ 0 & -2 & 1 \end{bmatrix} \end{array}$

$|B| = 3(3\times1 - (-2\times5)) - -4(2\times1 - 5\times0) + 1(2\times-$

$= 3(3+10) + 4(2-0) + 1(-4-0)$

$= 3\times13 + 4\times2 + 1\times-4$

$= 39 + 8 - 4 = 43$ //

3 find $|A|$ if $A = \begin{bmatrix} 2 & 3 & -5 \\ 2 & 1 & -4 \\ 4 & 6 & -10 \end{bmatrix}$

$|A| = 2(-10\times1 - -4\times6) - 3(2\times-10 - 4\times-4) + -5(2\times6 - 4\times1)$

$= 2(-10+24) - 3(-20+16) - 5(12-4)$

$= 2\times14 - 3\times-4 - 5\times8)$

$= 28 + 12 - 40. = 40-40$

$= 0$ //

4 find the value of x if $\begin{vmatrix} 2 & 1 & -3 \\ 3 & 2 & 5 \\ 4 & x & -6 \end{vmatrix} = 0$

$2(2\times-6 - 5\times x) - 1(3\times-6 - 5\times4) + -3(3x - 4$

$= 2(-12 - 5x) - 1(-18-20) - 3(3x-8) = 0$

$-\cancel{24} - 10x + 38 - 9x + \cancel{24} = 0$

$= 38 - 19x = 0$

$\cancel{+19x = 38}$

$\cancel{-19x} = \frac{38}{\cancel{-19}\,19} = 2$

$\cancel{x =}$

$38 = 19x$

$\frac{38}{19} = x$

$x = 2$ //

5 find x, $\begin{vmatrix} x & 3 \\ 5 & 2x \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ 5 & 3 \end{vmatrix}$.

an $x \times 2x - 5 \times 3 = 5 \times 3 - -4 \times 5$

$2x^2 - 15 = 15 + 20$

$2x^2 = 35 + 15$

$2x^2 = 50$

$x^2 = \frac{50}{2} = 25$

$x = 5$ //

6 $\begin{vmatrix} 2x-1 & x+1 \\ x+2 & x-2 \end{vmatrix} = 0$

an

$(2x-1)\times(x-2)-(x+2)(x+1)=0$

$2x^2-4x-x+2-(x^2+x+2x+2)=0$

$2x^2-5x+\cancel{2}-x^2-3x-\cancel{2}=0$

$2x^2-x^2-8x=0$

$x^2-8x=0$

$x(x-8)=0$

$x=0$ // or $x-8=0$

$x=8$ //

7 find x. $\begin{vmatrix} 2-x & 2 \\ 2 & 2+x \end{vmatrix}=0$

$(2-x)(2-x)-2\times2=0$

$\cancel{4}-2x-2x+x^2-\cancel{4}=0$

$x^2-4x=0$

$x(x-4)=0$

$x=0$ // or $x-4=0$

$x=4$ //

Minor & cofactor

given a square matrix $A = [a_{ij}]$ Then minor of a_{ij}, M_{ij} is the determinant of the sub matrix obtained by deleating the ith row and jth column

1 find the minors of each element of the matrix

an $A = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 3 & 0 \\ 4 & 5 & -3 \end{bmatrix}$

$M_{11} = \cancel{3\times 3} \begin{vmatrix} 3 & 0 \\ 5 & -3 \end{vmatrix} = 3\times -3 - 5\times 0 = -9$

$M_{12} = \begin{vmatrix} 2 & 0 \\ 4 & -3 \end{vmatrix} = 2\times -3 - 4\times 0 = -6$

$M_{13} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 2\times 5 - 4\times 3 = 10 - 12 = -2$

$M_{21} = \begin{vmatrix} 1 & -2 \\ 5 & -3 \end{vmatrix} = 1\times -3 - -2\times 5 = -3 + 10 = 7$

$M_{22} = \begin{vmatrix} 3 & -2 \\ 4 & -3 \end{vmatrix} = 3\times -3 - -2\times 4 = -9 + 8 = -1$

$M_{23} = \begin{vmatrix} 3 & 1 \\ 4 & 5 \end{vmatrix} = 3\times5 - 4\times1 = 15-4 = 11$

$M_{31} = \begin{vmatrix} 1 & -2 \\ 3 & 0 \end{vmatrix} = 1\times0 - {-2}\times3 = 0+6 = 6$

$M_{32} = \begin{vmatrix} 3 & -2 \\ 2 & 0 \end{vmatrix} = 0\times3 - {-2}\times2 = 0+4 = 4$

$M_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 3\times3 - 1\times2 = 9-2 = 7$

2 $C = \begin{bmatrix} 5 & -4 \\ 3 & 2 \end{bmatrix}$ find minors

$M_{11} = 2$

$M_{12} = 3$

$M_{21} = \cancel{3}\ -4$

$M_{22} = 5$

3 $A = \begin{bmatrix} 4 & -3 & 2 \\ 1 & -2 & 5 \\ 4 & 4 & 2 \end{bmatrix}$ find minors

$M_{11} = \begin{vmatrix} -2 & 5 \\ 4 & 2 \end{vmatrix} = -2\times2 - 5\times4 = -4 - 20 = -24$

$$M_{12} = \begin{vmatrix} -3 & 2 \\ 4 & 2 \end{vmatrix} = -3\times 2 - 4\times 2 = -6 - 8 = -14$$

$$M_{12} = \begin{vmatrix} 1 & 5 \\ 4 & 2 \end{vmatrix} = 1\times 2 - 5\times 4 = 2 - 20 = -18$$

$$M_{13} = \begin{vmatrix} 1 & -2 \\ 4 & 4 \end{vmatrix} = 1\times 4 - {-2}\times 4 = 4 + 8 = 12$$

$$M_{21} = \begin{vmatrix} -3 & 2 \\ 4 & 2 \end{vmatrix} = -3\times 2 - 4\times 2 = -6 - 8 = -14$$

$$M_{22} = \begin{vmatrix} \cancel{1} & \cancel{5} \\ 4 & 2 \\ 4 & 2 \end{vmatrix} = 8 - 8 = 0$$

$$M_{23} = \begin{vmatrix} 4 & -3 \\ 4 & 4 \end{vmatrix} = 16 - {-12} = 28$$

$$M_{31} = \begin{vmatrix} -3 & 2 \\ -2 & 5 \end{vmatrix} = -15 - {-4} = -11$$

$$M_{32} = \begin{vmatrix} 4 & 2 \\ 1 & 5 \end{vmatrix} = 20 - 2 = 18$$

$$M_{33} = \begin{vmatrix} 4 & -3 \\ 1 & -2 \end{vmatrix} = -8 - {-3} = -5$$

Cofactor

Signed minor is known as cofactor

sign is decided by the rule

$$C_{ij} = (-1)^{i+j} \, M_{ij}$$

1 Write the co-factor of the matrix $A = \begin{bmatrix} 4 & -3 & 2 \\ 1 & -2 & 5 \\ 4 & 4 & 2 \end{bmatrix}$

$M_{11} = 24$, $C_{11} = (-1)^{1+1} \; -24$

$= -1^{2} \times -24$

$= 1 \times -24 = -24$ //

$M_{12} = -18$, $C_{12} = (-1)^{1+2} \cdot -18$

$= -1^{3} \times -18 = -1 \times -18 = 18$ //

$M_{13} = 12$, $C_{13} = (-1)^{1+3} \times 12$

$= -1^{4} \times 12 = 12$ //

$M_{21} = -14$, $C_{21} = (-1)^{2+1} \times -14$

$= -1^{3} \times -14 = -1 \times -14$

$= 14$ //

$M_{22} = 0 \qquad C_{22} = (-1)^{2+2} \times 0 = 0$

$M_{23} = 28, \qquad C_{23} = (-1)^{2+3} \times 28 = -1^{5} \times 28 = -28$

$M_{31} = -11, \qquad C_{31} = (-1)^{3+1} \times -11 = -1^{4} \times -11 = 11$

$M_{32} = 18, \qquad C_{32} = (-1)^{3+2} \times 18 = -1^{5} \times 18 = -18$

$M_{33} = -5 \qquad C_{33} = (-1)^{3+3} \times -\cancel{1}5 = -1^{6} \times -5 = 5$

The matrix of cofactor is known as cofactor matrix ie

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -24 & 18 & 12 \\ 14 & 0 & -28 \\ 11 & -18 & 5 \end{bmatrix}$$

Q write the cofactor matrix of A

$$A = \begin{bmatrix} 5 & 6 & -4 \\ 0 & -2 & 3 \\ 1 & 1 & -2 \end{bmatrix}$$

3 Find the cofactor matrix of A

another method

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 0 & 3 \\ 2 & -2 & 4 \end{bmatrix}$$

an

$$\begin{array}{cccccc} 3 & 2 & -1 & 3 & 2 & -1 \\ 1 & 0 & 3 & 1 & 0 & 3 \\ 2 & -2 & 4 & 2 & -2 & 4 \\ 3 & 2 & -1 & 3 & 2 & -1 \\ 1 & 0 & 3 & 1 & 0 & 3 \\ 2 & -2 & 4 & 2 & -2 & 4 \end{array}$$

$C_{11} = 0 \times 4 - 3 \times -2 = 0 + 6 = 6$

$C_{12} = 3 \times 2 - 1 \times 4 = 6 - 4 = 2$

$C_{13} = 1 \times -2 - 0 \times 2 = -2$

$C_{21} = -2 \times -1 - 4 \times 2 = 2 - 8 = -6$

$C_{22} = 4 \times 3 - -1 \times 2 = 12 + 2 = 14$

$C_{23} = 2 \times 2 - -2 \times 3 = 4 + 6 = 10$

$C_{31} = 2 \times 3 - -1 \times 0 = 6 + 0 = 6$

$C_{32} = -1 \times 1 - 3 \times 3 = -1 - 9 = -10$

$C_{33} = 3 \times 0 - 1 \times 2 = -2$

$$C = \begin{bmatrix} 6 & 2 & -2 \\ -6 & 14 & 10 \\ 6 & -10 & -2 \end{bmatrix}$$

Adjoint of a matrix

Transpose of cofactor matrix is known as adjoint matrix

1 Q a) find adjoint of A. if $A = \begin{bmatrix} 4 & 5 & -1 \\ 2 & 0 & 3 \\ 1 & 1 & -2 \end{bmatrix}$

$$C = \begin{array}{cccccc} 4 & 5 & -1 & 4 & 5 & -1 \\ 2 & 0 & 3 & 2 & 0 & 3 \\ 1 & 1 & -2 & 1 & 1 & -2 \\ 4 & 5 & -1 & 4 & 5 & -1 \\ 2 & 0 & 3 & 2 & 0 & 3 \\ 1 & 1 & -2 & 1 & 1 & -2 \end{array}$$

$C_{11} = 0\times-2 - 3\times1 = 0-3 = \bar{3}$

$C_{12} = 3\times1 - 2\times-2 = 3+4 = 7$

$C_{13} = 2\times1 - 1\times0 = 2-0 = 2$

$C_{21} = 1\times-1 - \bar{2}\times5 = -1+10 = 9$

$C_{22} = -2\times4 - 1\times-1 = -8+1 = -7$

$C_{23} = 5-4 = 1$

$C_{31} = 15-0 = 15$

$C_{32} = -2-12 = -14$

$C_{33} = 0-10 = -10$

$$C = \begin{bmatrix} -3 & 7 & 2 \\ 9 & -7 & 1 \\ 15 & -14 & -10 \end{bmatrix} \qquad \text{adj } A = \begin{bmatrix} -3 & 9 & 15 \\ 7 & -7 & -14 \\ 2 & 1 & -10 \end{bmatrix}$$

b) Show that $A \times \text{adj } A = |A| \cdot I_3$

$-14+35$
14
21

$$A \cdot \text{adj } A = \begin{bmatrix} 4 & 5 & -1 \\ 2 & 0 & 3 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} -3 & 9 & 15 \\ 7 & -7 & -14 \\ 2 & 1 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times-3+5\times7+-1\times2 & 4\times9+5\times-7-1\times1 & 4\times15+5\times-14+-1\times-10 \\ 2\times-3+0\times7+3\times2 & 2\times9+0\times-7+3\times1 & 2\times15+0\times-14\times3\times-10 \\ 1\times-3+1\times7+-2\times2 & 1\times9+1\times-7+-2\times1 & 1\times15+1\times-14+-2\times-10 \end{bmatrix}$$

$$= \begin{bmatrix} -12+35-2 & 36-35-1 & 60-70+10 \\ -6+0+6 & 18-0+3 & 30-0-30 \\ -3+7-4 & 9-7-2 & 15-14+20 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$|A| = 4(\cancel{80}\ 0\times -2 - 3\times 1) - 5(2\times -2 - 3\times 1) +$$

$$-1(2\times 1 - 1\times 0)$$

$$= 4\times -3 - 5(-4-3) + -1(2)$$

$$= -12 + 35 - 2 = -14 + 35 = 21$$

$$|A|\cdot I_3 = 21 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix} = A \cdot \text{adj } A$$

2 show that $A \cdot adj = |A| \cdot I_3$

$$A = \begin{array}{ccc} + & - & + \end{array}$$
$$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$

$|A| = 3(2\times3 - -1\times2) - -2(2\cdot3 - -1\cdot0) + 1(2\cdot2 - 2\cdot0)$

$= 3\times(6+2) + 2(6+0) + 1(4-0)$

$= 3\times8 + 2\times6 + 1\times4$

$= 24 + 12 + 4 = 40$

cofactor

$$\begin{array}{cccccc} 3 & -2 & 1 & 3 & -2 & 1 \\ 2 & 2 & -1 & 2 & 2 & - \\ 0 & 2 & 3 & 0 & 2 & 3 \\ 3 & -2 & 1 & 3 & -2 & 1 \\ 2 & 2 & -1 & 2 & 2 & -1 \\ 0 & 2 & 3 & 0 & 2 & 3 \end{array}$$

$C_{11} = 2\times3 - -1\times2$ $\qquad$ $C_{21} = 2\times1 - 3\times-2$

$C_{12} = -1\times0 - 3\times2$ $\qquad$ $C_{22} = 3\times3 - 1\times0$

$C_{13} = 2\times2 - 2\times0$ $\qquad$ $C_{23} = 0\times-2 - 3\times2$

$C_{31} = \cancel{2\times8-1\times2}\ -2\times-1-1\cdot2$

$C_{32} = 1\times2-3\times-1$

$C_{33} = 3\cdot2--2\times2$

$C_{11} = 6+2=8$ $C_{21} = 2+6=8$

$C_{12} = -0-6 = -6$ $C_{22} = 9-0 = 9$

$C_{13} = 4+0=4$ $C_{23} = -0-6 = -6$

$C_{31} = \cancel{6+2=8}\ 2-2=0$

$C_{32} = 2+3=5$

$C_{33} = 6+4 = \cancel{8}\ 10$

$$C = \begin{bmatrix} 8 & -6 & 4 \\ 8 & 9 & -6 \\ 0 & 5 & 10 \end{bmatrix}$$

$$\text{adj}^{\circ} = C^{T} = \begin{bmatrix} 8 & 8 & 0 \\ -6 & 9 & 5 \\ 4 & -6 & 10 \end{bmatrix}$$

$adj(A) \cdot A =$

$$\begin{bmatrix} 8 & 8 & 0 \\ -6 & 9 & 5 \\ 4 & -6 & 10 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 2 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8\times3 + 8\times2 + 0\times0 & 8\times-2 + 8\times2 + 0\times2 & 8\times1 + 8\times-1 + 0\times3 \\ -6\times3 + 9\times2 + 5\times0 & -6\times-2 + 9\times2 + 5\times2 & -6\times1 + 9\times-1 + 5\times3 \\ 4\times3 + -6\times2 + 10\times0 & 4\times-2 + -6\times2 + 10\times2 & 4\times1 + -6\times-1 + 10\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 24+16 & -16+16 & 8-8 \\ -18+18 & 12+18+10 & -6-9+15 \\ 12-12 & -8-12+20 & 4+6+30 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

adj(A)

$$|A| \cdot I_3 = 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$\therefore adj(A) \cdot A = |A| \cdot I_3$ //

9 $A = \begin{bmatrix} 7 & 5 \\ 2 & 2 \end{bmatrix}$ S.T $A \cdot adj(A) = |A| \cdot I_2$

$adj\, A = \begin{bmatrix} 2 & -5 \\ -2 & 7 \end{bmatrix}$

To find adjoint of 2x2 matrix
i) interchange the diagonal elements
ii) change the sign of non diagonal elements

$|A| = 7\times2 - 5\times2$

$= 14 - 10 = 4$ //

$A \cdot adj A = \begin{bmatrix} 7 & 5 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 7\times2+5\times-2 & 7\times-5+5\times7 \\ 2\times2+2\times-2 & 2\times-5+2\times7 \end{bmatrix}$

$= \begin{bmatrix} 14-10 & -35+35 \\ 4-4 & -10+14 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ //

$|A| \cdot I_2 = 4 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = A \cdot adj\, A$ //

4 $A = \begin{bmatrix} 2 & -1 & -1 \\ 3 & -1 & 1 \\ 0 & 2 & -2 \end{bmatrix}$ find Adj (A)

$$\begin{array}{cccccc} 2 & -1 & -1 & 2 & -1 & -1 \\ 3 & -1 & 1 & 3 & -1 & 1 \\ 0 & 2 & -2 & 0 & 2 & -2 \\ 2 & -1 & -1 & 2 & -1 & -1 \\ 3 & -1 & 1 & 3 & -1 & 1 \\ 0 & 2 & -2 & 0 & 2 & -2 \end{array}$$

$C_{11} = -1\times-2 - 1\times2 = -2-2 = 0$

$C_{12} = 0\times1 - 3\times-2 = 0+6 = 6$

$C_{13} = 3\times2 - -1\times0 = 6+0 = 6$

$C_{21} = 2\times-1 - (-2\times-1) = -2-2 = -4$

$C_{22} = -2\times2 - -1\times0 = -4+0 = -4$

$C_{23} = -1\times0 - 2\times2 = -0-4 = -4$

$C_{31} = -1\times1 - (-1\times-1) = -1-1 = -2$

$C_{32} = 3\times-1 - 2\times1 = -3-2 = -5$

$C_{33} = 2\times-1 - -1\times3 = -2+3 = 1$

$$C = \begin{bmatrix} 0 & 6 & 6 \\ -4 & -4 & -4 \\ -2 & -5 & 1 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 0 & -4 & -2 \\ 6 & -4 & -5 \\ 6 & -4 & 1 \end{bmatrix}$$

5 find $|A|$. $|\text{Adj } A|$

$$A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 2 & -1 & -1 \\ 3 & -1 & 1 \\ 0 & 2 & -2 \end{bmatrix} \end{array}$$

$$|A| = 2(+2-2) - -1(-6-0) + -1(6-0)$$

$$= 2 \times 0 + 1 \times -6 - 1 \times 6$$

$$= -6 + -6 = -12$$

$$|\text{adj } A| = 0(-4\cancel{8}-20) - 4(6-30) - -2(-24-$$

$$= 0 + 4 \times 36 + 2 \times 0$$

$$= 144$$

$n = 3$

$$\boxed{|\operatorname{adj}(A)| = |A|^{3-1} = |A|^2}$$

Inverse of a matrix

given a non-singular matrix A ie $|A| \neq 0$ then A^{-1} is given by

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

A^{-1} is that matrix which satisfies.

$$AA^{-1} = A^{-1}A = I_n$$

A^{-1} is does not exist for singular matrix

$(|A| = 0)$

1) find A^{-1} if $A = \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$|A| = 4\times2 - 3\times2 = 8-6 = 2$ //

$$\operatorname{adj} A = \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix} //$$

2) $$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 4 & -1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$|A| = 1(8 - {}^{+}3) - -3(4 - {}^{-}4) + 2(-6 - 16)$$

$$= 1 \times 5 + 3 \times 8 + 2 \times -22$$

$$= 5 + 24 - 44$$

$$= -15 //$$

$$C = \begin{array}{cccccc} 1 & -3 & 2 & 1 & -3 & 2 \\ 2 & 4 & -1 & 2 & 4 & -1 \\ 4 & -3 & 2 & 4 & -3 & 2 \\ 1 & -3 & 2 & 1 & -3 & 2 \\ 2 & 4 & -1 & 2 & 4 & -1 \\ 4 & -3 & 2 & 4 & -3 & 2 \end{array}$$

$C_{11} = 8 - {}^{+}3 = 5$

$C_{12} = -4 - 4 = -8$

$C_{13} = -6 - 16 =$ [illegible]

$C_{21} = \cancel{-8 - {}^{-}3} = \cancel{-8}$

$-6 - {}^{-}6 = 0$

$$C_{22} = 2 - 8 = -6$$

$$C_{23} = {}^{-}12 - {}^{-}3 = -9$$

$$C_{31} = 3 - 8 = -5$$

$$C_{32} = 4 - {}^{-}1 = 5$$

$$C_{33} = 4 - {}^{-}6 = 10$$

$$C = \begin{bmatrix} 5 & -8 & -22 \\ 0 & -6 & -9 \\ -5 & 5 & 10 \end{bmatrix}$$

$$\text{adj} A = \begin{bmatrix} 5 & 0 & -5 \\ -8 & -6 & 5 \\ -22 & -9 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$= \frac{1}{15} \begin{bmatrix} 5 & 0 & -5 \\ -8 & -6 & 5 \\ -22 & -9 & 10 \end{bmatrix}$$

-22

Solving system of linear equations

1) $x + 2y = 5$

$x - y = -1$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$$

coefficient matrix

$$X = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \quad B = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\boxed{X = A^{-1} B}$$

$$A^{-1} = \frac{1}{|A|} \cdot adj(A)$$

$$|A| = 1 \times -1 - 1 \times 2 = -1 - 2 = -3$$

$$adj(A) = \begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{-1}{3} \begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$$

$X = A^{-1} B$

$= \frac{-1}{3}\begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$= -\frac{1}{3}\begin{bmatrix} -1\times5+-2\times-1 \\ -1\times5+1\times-1 \end{bmatrix}$

$= -\frac{1}{3}\begin{bmatrix} -5+2 \\ -5-1 \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \cancel{\frac{y}{3}}$ $x=1,\ y=2$ //

2 $2x-3y=-5$

$x+2y=8$

an $A = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$

coefficient matrix

$X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} -5 \\ 8 \end{bmatrix}$

$X = A^{-1} B$

$A^{-1} = \frac{1}{|A|} \cdot adj(A)$

$$|A| = 2\times 2 - -3\times 1 \qquad = 4 + 3 = 7$$

$$adj(A) = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -5 \\ 8 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 2\times -5 + 3\times 8 \\ -1\times -5 + 2\times 8 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -10+24 \\ 5+16 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 14 \\ 21 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$x = 2$ // $y = 3$ //

3 $\quad 3x + y - z = 3$

$-x + y + z = 1$

$x + y + z = 3$

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

coefficient matrix $X = \begin{vmatrix} x \\ y \\ z \end{vmatrix}$, $B = \begin{vmatrix} 3 \\ 1 \\ 3 \end{vmatrix}$

$X = A^{-1} B$

$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$

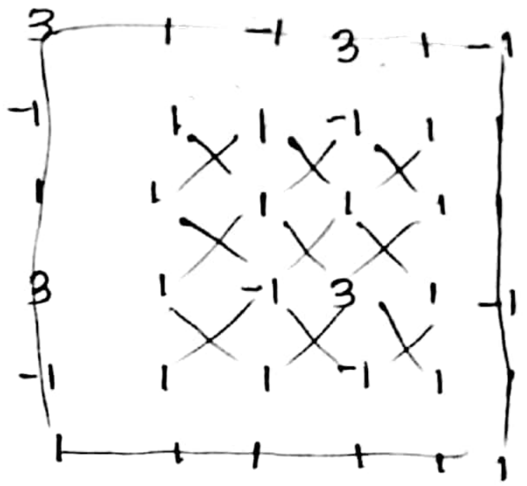
$|A| = 3(1-1) - 1(-1-1) + {}^{-}1(-1-1)$

$= 3 \times 0 - {}^{-}1 \times 2 - 1 \times 2$

$=$ ~~$-1 \ldots 2$~~

$2 - {}^{-}2 = 4$

$\operatorname{adj}(A) =$



$C_{11} = 1 - 1 = 0$

$C_{12} = 1 - {}^{-}1 = 2$

$C_{13} = -1 - 1 = -2$

$C_{21} = -1 - 1 = -2$

$C_{22} = 3 - {}^{-}1 = 4$

$C_{23} = 1 - 3 = -2$

$C_{31} = 1 - {}^{-}1 = 2$

$C_{32} = 1 - 3 = -2$

$C_{33} = 3 - {}^{-}1 = 4$

~~adj(A)~~ =

$$C = \begin{bmatrix} 0 & 2 & -2 \\ -2 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

~~$A^{-1} = \frac{1}{4}$~~ $\Big[$

$$\text{adj } A = \begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}$$

$$X = \frac{1}{4}\begin{bmatrix} 0 & -2 & 2 \\ 2 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0-2+6 \\ 6+4-6 \\ -6-2+12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

4- $x + 2y + z = 7$

$x + 3z = 11$

$2x - 3y - 1 = 0$

the given eqn can be written as

$x + 2y + z = 7$

$x + 0y + 3z = 11$

$2x - 3y + 0z = 1$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$X = A^{-1} B$

$A^{-1} = \dfrac{1}{|A|} \operatorname{adj}(A)$

$|A| = 1(0-9) - 2(0-6) + 1(-3-0)$

$= 1 \times 9 - 2 \times -6 + 1 \times \bar{3}$

$= 9 + 12 - 3$

$= 18$

$adj(A) = \begin{vmatrix} & 2 & 1 & 1 & 2 & \\ & 0 & 3 & 1 & 0 & 3 \\ 2 & -3 & 0 & 2 & -3 & 0 \\ 1 & 2 & 1 & 1 & 2 & \\ & 0 & 3 & 1 & 0 & 3 \\ 2 & -3 & 0 & 2 & -3 & 0 \end{vmatrix}$

$c_{11} = 0 - -9 = 9$

$c_{12} = 6 - 0 = 6$

$c_{13} = -3 - 0 = -3$

$c_{21} = -3 - 0 = -3$

$c_{22} = 0 - 2 = -2$

$c_{23} = 4 - {-3} = 7$

$c_{31} = 6 - 0 = 6$

$c_{32} = 1 - 3 = -2$

$c_{33} = 0 - 2 = -2$

$$C = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}$$

$$adj(A) = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{18} \times \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$$X = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$$= \frac{1}{18}\begin{bmatrix} 9\times 7 + -3\times 11 + 6\times 1 \\ 6\times 7 + -2\times 11 - 2\times 1 \\ -3\times 7 + 7\times 11 - 2\times 1 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 63 - 33 + 6 \\ 42 - 22 - 2 \\ -21 + 77 - 2 \end{bmatrix}$$

$$= \frac{1}{18}\begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad x = 2,\ y = 1,\ z = 3$$

Cramer's rule / determinent method

Q1 $x + 2y + z = 7$

$x + 3z = 11$

$2x - 3y - 1 = 0$

Δ = determinent of coefficient matrix

Δ_1 = determinent of coefficient matrix where first column is replaced by constant

Δ_2 = the 2nd column is replaced

Δ_3 = the 3rd column is replaced

$x + 2y + z = 7$

$x + 0y + 3z = 11$

$2x - 3y + 0z = 1$

$x = \frac{\Delta_1}{\Delta},\ y = \frac{\Delta_2}{\Delta},\ z = \frac{\Delta_3}{\Delta}$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix} = 1(0-9) - 2(0-6) + 1(-3-0)$$

$$= 1\times 9 + 12 - 3 = 18$$

$$\Delta_1 = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & 3 \\ 1 & -3 & 0 \end{vmatrix} = 7(0-9) - 2(0-3) + 1(-33-0)$$

$= 7\times 9 - 2\times -3 + 1\times -33$

$= 63 - 6 - 33 = 36$

$$\Delta_2 = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & 3 \\ 2 & 1 & 0 \end{vmatrix} = 1(0-3) - 7(0-6) + 1(1-22)$$

$= 1\times -3 - 7\times -6 + 1\times -21$

$= -3 + 42 - 21 = 18$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & 1 \end{vmatrix} = 1(0-33) - 2(1-22) + 7(-3-0)$$

$= 1\times 33 - 2\times -21 + 7\times 3$

$= 33 - 42 - 21 = 54$ //

$x = \dfrac{\Delta_1}{\Delta} = \dfrac{36}{18} = 2$ //

$y = \dfrac{\Delta_2}{\Delta} = \dfrac{18}{18} = 1$ //

$z = \dfrac{\Delta_3}{\Delta} = \dfrac{54}{18} = 3$ //

H.W

2 $3x + y - z = 3$

$-x + y + z = 1$

$x + y + z = 3$

$$\Delta = \begin{vmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 3(1-1) - 1(-1-1) + -1(-1-1)$$

$= 3\times 0 - 1\times -2 + -1\times -2$

$= 2 + 2 = 4$

$$\Delta_1 = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 3(1-1) - 1(1-3) + -1(1-3)$$

$$= 3 \times 0 - 1 \times -2 + -1 \times -2$$

$$= 2 + 2 = 4$$

$$\Delta_2 = \begin{vmatrix} 3 & 3 & -1 \\ -1 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 3(1-3) - 3(-1-1) - 1(-3-1)$$

$$= 3 \times -2 - 3 \times -2 - 1 \times -4$$

$$= -6 - -6 + 4$$

$$= -6 + 6 + 4 = 4$$

$$\Delta_3 = \begin{vmatrix} 3 & 1 & 3 \\ -1 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 3(3-1) - 1(-3-1) + 3(-1-1)$$

$$= 3 \times 2 - 1 \times -4 + 3 \times -2$$

$$= 6 + 4 - 6 = 4$$

$$x = \frac{\Delta_1}{\Delta} = \frac{4}{4} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{4}{4} = 1$$

$$z = \frac{\Delta_3}{\Delta} = \frac{4}{4} = 1$$

3 solve the system of eq^n using Cramer's rule

$2x+3y=13$

$2x+y=7$

$\Delta = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2\times 1 - 3\times 2 = 2-6 = -4$

$\Delta_1 = \begin{vmatrix} 13 & 3 \\ 7 & 1 \end{vmatrix} = 13-21 = -8$

$\Delta_2 = \begin{vmatrix} 2 & 13 \\ 2 & 7 \end{vmatrix} = 14-26 = -12$

$x = \frac{\Delta_1}{\Delta} = \frac{-8}{-4} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3$

4 $x+2y=5$

$2x-3y=-4$

$\Delta = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = -3-4 = -7$

$\Delta_1 = \begin{vmatrix} 5 & 2 \\ -4 & -3 \end{vmatrix} = -15-8 = -7$

$$\Delta_2 = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = -4-10 = -14$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-7}{-7} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-14}{-7} = 2$$

5

$$\frac{5}{x} + \frac{2}{y} = 4$$

$$\frac{2}{x} - \frac{1}{y} = 7$$

let $X = \frac{1}{x}$ $Y = \frac{1}{y}$

$$5X + 2Y = 4$$

$$2X - Y = 7$$

$$\Delta = \begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = -5-4 = -9$$

$$\Delta_1 = \begin{vmatrix} 4 & 2 \\ 7 & -1 \end{vmatrix} = 4 - 14 = -18$$

$$\Delta_2 = \begin{vmatrix} 5 & 4 \\ 2 & 7 \end{vmatrix} = 35 - 8 = 27$$

$$X = \frac{\Delta_1}{\Delta} = \frac{-18}{-9} = 2$$

$$Y = \frac{\Delta_2}{\Delta} = \frac{27}{-9} = -3$$

$$x = \frac{1}{2} \qquad y = \frac{-1}{3}$$

5 $\frac{6}{x} + \frac{7}{y} = 5$

$$\frac{2}{x} + \frac{5}{y} - 3 = 0$$

ab $\frac{6}{x} + \frac{7}{y} = 5$

$$\frac{2}{x} + \frac{5}{y} = -3$$

let $X = \frac{1}{x} \quad Y = \frac{1}{y}$

$$\begin{cases} 6X + 7Y = 5 \\ 2X + 5Y = 3 \end{cases}$$

an

$$\Delta = \begin{vmatrix} 6 & 7 \\ 2 & 5 \end{vmatrix} = 30 - 14 = 16$$

$$\Delta_1 = \begin{vmatrix} 5 & 7 \\ 3 & 5 \end{vmatrix} = 25 - 21 = 4$$

$$\Delta_2 = \begin{vmatrix} 6 & 5 \\ 2 & 3 \end{vmatrix} = 18 - 10 = 8$$

$$x = \frac{\Delta_1}{\Delta} = \frac{4}{16} = \frac{1}{4}$$

$$Y = \frac{\Delta_2}{\Delta} = \frac{8}{16} = \frac{1}{2}$$

$x = 4 \quad y = 2$

Q in a circuit, The currents are given by I_A, I_B, I_C

$I_A + I_B + I_C = 0$

$7 I_A - 10 I_B - 3 = 0$

$-10 I_B + 5 I_C - 6 = 0$

an $I_A + I_B + I_C = 0$

$4 I_A - 10 I_B + 0 I_C = 3$

$10 I_A - 10 I_B + 5 I_C = 6$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & -10 & 0 \\ 0 & -10 & 5 \end{vmatrix} = 1(-50-0) - 1(20-0) + 1(-40-0)$$

$$= 1 \times -50 - 1 \times 20 + 1 \times -40$$

$$= -50 - 20 - 40 = -110$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ 3 & -10 & 0 \\ 6 & -10 & 5 \end{vmatrix} = 0 - 1(15-0) + 1(-30-60)$$

$$= -1 \times 15 + 1 \times +30$$

$$= -15 + 30 = 15$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ 4 & 3 & 0 \\ 0 & 6 & 5 \end{vmatrix} = 1(15-0) - 0 + 1(24-0)$$

$$= 1 \times 15 + 24 = 39$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ 4 & -10 & 3 \\ 0 & -10 & 6 \end{vmatrix} = 1(-60-30) - 1(24-0) + 0$$

$$= 1 \times -30 - 1 \times 24 = -30 - 24$$

$$= -54$$

$$I_A = \frac{\Delta_1}{\Delta} = \frac{15}{-110}$$

$$I_B = \frac{\Delta_2}{\Delta} = \frac{39}{-110}$$

$$I_C = \frac{\Delta_3}{\Delta} = \frac{-54}{-110}$$

Binomial Theorm

for a natural number n. $n!$ is defined as

$n! = n(n-1)(n-2)\ldots\ldots 1$

$1! = 1$

$2! = 2\times 1 = 2$

$3! = 3\times 2\times 1 = 6$

$4! = 4\times 3\times 2\times 1 = 24$

$5! = 5\times 4\times 3\times 2\times 1 = 120$

$\ldots\ldots\ldots$

$\ldots\ldots\ldots$

$10! = 10\times 9\times 8\ldots\ldots\ldots\times 1$

Note that $0! = 1$

Properties

1) $n! = n(n-1)!$

eg $12! = 12\times 11!$

combinations$({}^nC_r)$

${}^nC_r$ represents the no. of ways r objects can be selected from a group of n objects

$$nC_r = \frac{n!}{r!(n-r)!}$$

<u>properties</u>

1) $nC_0 = nC_n = 1$

eg. $50C_0 = 1$

$14C_{14} = 1$

$3C_3 = 1$

2) $nC_1 = \frac{n!}{1!(n-1)!} = \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = n$

$$nC_{n-1} = \frac{n!}{(n-1)!(n-(n-1))!} = \frac{n(n-1)!}{(n-1)!} = n$$

$nC_1 = nC_{n-1} = 1$

$nC_0 = nC_n = 1$

$nC_1 = nC_{n-1} = n$

eg: $4C_0 = 1$

$5C_1 = 5$

$19C_{18} = 19$

$20C_1 = 20$

$${}^{10}C_3 = \frac{10\times 9\times 8}{1\times 2\times 3}$$

$${}^{20}C_4 = \frac{20\times 19\times 18\times 17}{1\times 2\times 3\times 4}$$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots\dots + {}^nC_r a^{n-r} b^r + \dots\ b^n$$

eg: $(a+b)^5 = a^5 + 5C_1 a^4 b + 5C_2 a^3 b^2 + 5C_3 a^2 b^3 + 5C_4 a^1 b^4 + b^5$

$$= a^5 + 5a^4 b + \frac{5\cdot 4}{1\cdot 2} a^3 b^2 + \frac{5\times 4\times 3}{1\cdot 2\cdot 3} a^2 b^3 + \frac{5\cdot 4\cdot 3\cdot 2}{1\cdot 2\cdot 3\cdot 4} ab^4 + b^5$$

$$= a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5$$

Properties

in the binomial expansion of $(a+b)^n$

1 → there will $n+1$ terms

2 → The first term will be a^n and last term will be b^n

3. The sum of powers of a and b will be n

- $(r+1)^{th}$ term $= {}^nC_r a^{n-r} b^r$

1 find the binomial expansion of $(x+a)^4$.

$$(x+a)^4 = x^4 + 4C_1 x^3 a + 4C_2 x^2 a^2 + 4C_3 x a^3 + a^4$$

$$= x^4 + 4x^3 a + \frac{4\times 3}{1\times 2} x^2 a^2 + \frac{4\times 3\times 2}{1\times 2\times 3} x a^3 + b^4$$

$$= x^4 + 4x^3 a + 6x^2 a^2 + 4x a^3 + b^4 \,//$$

2 $(2x+3y)^5$

$$= (2x)^5 + 5C_1 (2x)^4 (3y) + 5C_2 (2x)^3 (3y)^2 + 5C_3 (2x)^2 (3y)^3 + 5C_4\, 2x (3y)^4$$

$$+ 3y^5$$

$$= (2x)^5 + 5(2x)^4 3y + \frac{5\times 4}{1\times 2}(2x)^3 (3y)^2 + \frac{5\times 4\times 3}{1\times 2\times 3}(2x)^2 (3y)^3 +$$

$$\frac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4} 2x (3y)^4 + 3y^5$$

$$= (2x)^5 + 5(2x)^4 3y + 10(2x)^3 (3y)^2 + 10(2x)^2 (3y)^3 + 5\, 2x (3y)^4$$
$$+ (3y)^5$$

$$= 32x^5 + 5\cdot 16\, x^4\, 3y + 10\cdot 8x^3 \cdot 9y^2 + 10\cdot 4x^2 \cdot 27y^3$$

$$+ 5\cdot 2x\cdot 81\cdot y^4 + 243\cdot y^5$$

$$= 32x^5 + 240x^4 3y + 720x^3 y^2 + 1080 x^2 y^3 + 810 x y^4 +$$
$$243y^5 \,//$$

3 $(2x-3y)^5$

an $(2x+^{-}3y)^5$

$= (2x)^5 + {}^5C_1(2x)^4(-3y) + {}^5C_2(2x)^3(-3y)^2 + {}^5C_3(2x)^2(-3y)^3 +$
${}^5C_4\, 2x\,(-3y)^4 + (-3y)^5$

$= (2x)^5 + (5\times 2x)^4(-3y) + \frac{5\times 4}{1\times 2}(2x)^3(-3y)^2 + \frac{5\times4\times3}{1\times2\times3}(2x)^2(-3y)^3$
$+ \frac{5\times4\times3\times2}{1\times2\times3\times4}\, 2x\,(-3y)^4 + (-3y)^5$

$= 32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4$
ans $-243y^5$

4 $(x-\frac{1}{x})^4$

ans $= x^4 + {}^4C_1\, x^3(\frac{-1}{x}) + {}^4C_2\, x^2(-\frac{1}{x})^2 + {}^4C_3\, x(\frac{-1}{x})^3 + (\frac{-1}{x})^4$

$= x^4 - 4x^3 \cdot \frac{1}{x} + \frac{4\times3}{1\times2}\, x^2 \frac{1}{x^2} + \frac{4\times3\times2}{1\times2\times3}\, x\frac{-1}{x^3} + \frac{1}{x}$

$= x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$

$= x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$

5 $\left(x^2 - \frac{3}{x}\right)^5$

$= (x^2)^5 + {}^5C_1 (x^2)^4 \left(\frac{-3}{x}\right) + {}^5C_2 (x^2)^3 \left(\frac{-3}{x}\right)^2 + {}^5C_3 (x^2)^2 \left(-\frac{3}{x}\right)^3$

$+ {}^5C_4\, x^2 \left(-\frac{3}{x}\right)^4 + \left(\frac{-3}{x}\right)^5$

$= x^{10} - 5x^8 \frac{3}{\cancel{x}} + 10x^6 \frac{9}{x^2} + 10x^4 \cdot \frac{-27}{x^3} + 5x^2 \frac{81}{x^4}$

$- \frac{243}{x^5}$

$= x^{10} - 15x^7 + 90x^4 - 270x + \frac{405}{x^2} - \frac{243}{x^5}$ //

6 $\left(2a - \frac{b}{3}\right)^4$

Ans $= (2a)^4 + {}^4C_1 (2a)^3 \left(\frac{-b}{3}\right) + {}^4C_2 (2a)^2 \left(\frac{-b}{3}\right)^2 + {}^4C_3 (2a) \left(\frac{-b}{3}\right)^3$

$+ \left(\frac{-b}{3}\right)^4$

$= (2a)^4 + 4(2a)^3\left(-\frac{b}{3}\right) + 6(2a)^2\left(-\frac{b}{3}\right)^2 + 4(2a)\left(-\frac{b}{3}\right)^3 + \left(\frac{b^4}{81}\right)$

$= 16a^4 - 32\frac{a^3b}{3} + \frac{24a^2b^2}{9} - \frac{8ab^3}{27} + \frac{b^4}{81}$ /

- finding a particular term in a binomial expansion

$$\boxed{\text{The } (r+1)^{th} \text{ term in } (a+b)^n = {}^nC_r\, a^{n-r} b^r}$$

1) Find the 7 th term in $(x+a)^{10}$

an $(r+1)^{th}$ term in $(x+a)^{10} = 10C_r\, x^{10-r} a^r$

Now take $(r+1) = 7 \Rightarrow r = 6$

7^{th} term $= 10C_6\, x^{10-6} a^6$

$= 10C_6\, x^4 a^6$

2. 5^{th} term in $(2x+3y)^7$?

$(r+1)^{th}$ term in $(2x+3y)^7 = 7C_r (2x)^{7-r} (3y)^r$

$(r+1) = 5 \Rightarrow r = 4$

5th term $= 7C_4 (2x)^{7-4} (3y)^4$

$= 7C_4 (2x^3) (3y)^4$

$= 7C_4\, 2^3 x^3\, 3^4 y^4$

$= 7C_4\, 2^3 3^4 x^3 y^4$

$= \frac{7.6.5.4}{1.2.3.4} \times 8 \times 81\, x^3 y^4$

$= 22680\, x^3 y^4$

3) 6^{th} term in $(x+\frac{1}{x})^{10}$

$(r+1)^{th}$ term $= 10C_r \, x^{10-r} (\frac{1}{x})^r$

$r+1 = 6 \Rightarrow r = 5$

6^{th} term $= 10C_5 \, x^{10-5} (\frac{1}{x})^5$

$= 10C_5 \, x^5 \, \frac{1}{x^5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 252$ //

4) 5^{th} term in $(2a - b/3)^7$

$(2a - b/3)^7 \Rightarrow (2a + -b/3)^7$

$(r+1)^{th}$ term $= 7C_r (2a)^{7-r} (-b/3)^r$

$r+1 = 5 \Rightarrow r = 4$

5^{th} term $= 7C_4 (2a)^{7-4} (-b/3)^4$

$= 7C_4 (2a)^3 (-b/3)^4$

$= \frac{7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4} \cdot 2^3 \times (-\frac{1}{3})^4 \cdot a^3 b^4$

$= 35 \times 8 \times \frac{1}{81} a^3 b^4$

$= \frac{280}{81} a^3 b^4$ //

5 10^{th} term in $(x^2 - \frac{1}{x^2})^{20}$

$(r+1)^{th}$ term $= 20C_r \, (x^2)^{20-r} \, (\frac{-1}{x^2})^r$

$(r+1) = 10 \Rightarrow r = 9$

10th term $= 20C_9 \, (x^2)^{20-9} \, (-\frac{1}{x^2})^9$

$= 20C_9 \, (x^2)^{11} \, (-\frac{1}{x^2})^9$

$= 20C_9 \, x^{22} \, \frac{-1}{x^{18}} = -20C_9 \, x^4$ //

Middle term

- In a binomial expansion the middle term (s) is given by

if n = even $(\frac{n}{2}+1)^{th}$ term

n = odd $\frac{n+1}{2}, +1$

1 Find the middle term in the binomial expansion at $(2x+3y)^8$.

a middle term $= 8/2 + 1 = 5^{th}$ term

$(r+1)^{th}$ term $= 8C_r \, (2x)^{8-r} \, (3y)^r$

$r+1 = 5 \Rightarrow r = 4$

5^{th} term $= 8C_4 \, (2x)^{8-4} \, (3y)^4$

$= 8C_4 \cdot 2^4 x^4 \cdot 3^4 y^4$

$= \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \cdot 16 \cdot 81 \cdot x^4 \cdot y^4$

$= 90720 \, x^4 y^4$ //

$(x+2y)^6$

Middle term $= 6/2 + 1 = 4$

$(r+1)^{th}$ term $= 6C_r \, x^{6-r} (2y)^r$

$r+1 = 4 \Rightarrow r = 3$

4^{th} term $= 6C_{\cancel{4}3} \, x^{6-3} (2y)^3$

$= 6C_3 \, x^3 \, 2^3 y^3$

$= \dfrac{\cancel{6}^{2} \times 5 \times 4}{1 \times 2 \times \cancel{3}} \cdot 8 \cdot x^3 y^3$

$= 160 x^3 y^3$ //

3) $(2x + 3/x)^9$

Middle terms $\dfrac{9+1}{2} = 5$, $\dfrac{9+1}{2} + 1 = 5+1 = 6$

5^{th} & 6^{th} terms are the middle terms

$(r+1)^{th}$ term $= 9C_r (2x)^{9-r} \left(\frac{3}{x}\right)^r$

$r+1 = 5 \Rightarrow r = 4$

5^{th} term $= 9C_4 \, (2x)^{9-4} \left(\frac{3}{x}\right)^4$

$= \dfrac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6}}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4}} \cdot 2^5 \, x^{\cancel{5}} \cdot \dfrac{3^4}{\cancel{x^4}}$

$= 126 \cdot 32 \cdot x \cdot 81$

$= 326592x$ //

$r+1 = 6 \Rightarrow r = 5$

6^{th} term $= 9C_5 \, (2x)^{9-5} \left(\frac{3}{x}\right)^5$

$$= \frac{9 \times \cancel{8} \times \cancel{7} \times \cancel{6} \times 5}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4} \times 5} \cdot 2^4 \cancel{x^4} \frac{3^9}{x^6}$$

$$= 126 \cdot 16 \cdot \frac{249}{x} = \frac{489888}{x}$$ //

4 $\left(2a - \frac{b}{3}\right)^{12}$

an middle term

$(r+1)^{th}$ term $= \frac{12}{2} + 1 = 7^{th}$ term

$$= 12C_r \times (2a)^{12-r} \times \left(\frac{-b}{3}\right)^r$$

$r+1 = 7 \Rightarrow r = 6$

7^{th} term $= 12C_6 (2a)^{12-6} \left(\frac{-b}{3}\right)^6$

$$= 12C_6 (2a)^6 \left(\frac{-b}{3}\right)^6$$

$$= 924 \times 64 \times a^6 \times \frac{b^6}{729}$$

$$= \frac{59136 a^6 b^6}{729}$$ //

finding the coefficient of x^r in a binomial expansion

1) find the coefficient of x^{12} in $\left(x^2 + \frac{1}{x^2}\right)^{10}$

$(r+1)^{th}$ term $= 10C_r (x^2)^{10-r} \left(\frac{1}{x^2}\right)^r$

$$= 10C_r \, x^{2(10-r)} \times \frac{1^r}{x^{2r}}$$

$$= 10C_r \, 1^r \times x^{20-2r} \times x^{-2r}$$

$10C_r\,(1)^r \times x^{20-2r-2r}$

$10C_r\,(1)^r \times x^{20-4r}$

Take $20-4r=12$

$4r=20-12$

$=8$

$r=8/4=2$

Now

$Ce.$ of $x^{12}=10C_2 = \frac{\overset{5}{10}\times 9}{1\cdot 2} = 45$ //

2 Cc of x^{11} in $\left(x^4-\frac{1}{x^3}\right)^{15}$

$(r+1)^{th}$ term $=15C_r\,(x^4)^{15-r}\left(\frac{-1}{x^3}\right)^r$

$=15C_r\,(x^4)^{15-r}\,\frac{-1^r}{x^{3r}}$

$=15C_r\,x^{60-4r}\,\frac{(-1^r)}{x^{3r}}$

$=15C_r\,(-1)^r\,x^{60-4r}\,x^{-3r}$

$=15C_r\,(-1)^r\,x^{60-4r-3r}$

$=15C_r\,(-1)^r\,x^{60-7r}$

Take $60-7r=11$

$7r=11-60$

$r=\frac{11-60}{7}=7$

cc of $x^{11} = 15C_7\,(-1)^7 = -6435$ //

3 C.C of x^{10} in $\left(2x^2 - \frac{3}{x}\right)^{11}$

$(r+1)^{th}$ term $= {}^{11}C_r \, (2x^2)^{11-r} \left(\frac{-3}{x}\right)^r$

$= {}^{11}C_r \; 2 \, x^{22-2r} \; (-3)^r \cdot x^{-r}$

$= {}^{11}C_r \; (-3)^r \cdot 2^{11-r} \; x^{22-2r} \cdot x^{-r}$

$= {}^{11}C_r \; (-3)^r \cdot 2^{11-r} \; x^{22-2r-r}$

$= {}^{11}C_r \; (-3)^r \cdot 2^{11-r} \; x^{22-3r}$

Take $22 - 3r = 10$

$3r = 22 - 10$

$r = \frac{22-10}{3}$ 4

co.c of x^{10} $= {}^{11}C_4 \; (-3)^4 \cdot 2^{11-4}$

$=$ 9421440//

4 coc of x^{-17} in $\left(x^4 - \frac{1}{x^3}\right)^{15}$

$(r+1)^{th}$ term $= {}^{15}C_r \, (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r$

$= {}^{15}C_r \; x^{60-4r} \; (-1)^r \; x^{-3r}$

$= {}^{15}C_r \; (-1)^r \; x^{60-4r} \; x^{-3r}$

$= {}^{15}C_r \; (-1)^r \; x^{60-4r-3r}$

$= {}^{15}C_r \; (-1)^r \; x^{60-7r}$

Take $60 - 7r = -17$

$7r = 60 + 17$

$r = \frac{60+17}{7} = 11$

c.c of x^{-17} = $15C_{11} \; (-1)^{11}$

$= -1365$ //

Q find the term independent of x in $(x+\frac{3}{x})^{10}$

$(r+1)^{th}$ term $= 10C_r \; x^{10-r} \left(\frac{3}{x}\right)^r$

$= 10C_r \; x^{10-r} \; 3^r \; x^{-r}$

$= 10C_r \cdot 3^r \cdot x^{10-r} \; x^{-r}$

$= 10C_r \cdot 3^r \; x^{10-r-r}$

$= 10C_r \cdot 3^r \; x^{10-2r}$

Take $10-2r = 0$

$2r = 10$

$r = 5$

independent of x = 3^5

$= 495$

(i) Finding the term independent of x

i) $\left(x^2 - \frac{2}{3x}\right)^9$

$(r+1)^{th}$ term $= 9C_r (x^2)^{9-r} \left(\frac{-2}{3x}\right)^r$

$x^{18-2r} \times x^{-r}$
$= x^{18-2r-r}$
$= x^{18-3r}$

$= 9C_r\, x^{18-2r} (-2)^r (3x)^{-r}$

$= 9C_r \times (-2)^r \times (3)^{-r} \times x^{18-2r}\, x^{-r}$

$= 9C_r \times (-2)^r (3)^{-r} \times x^{18-3r}$

$18 - 3r = 0$

$3r = 18$

$r = \frac{18}{3} = 6$

Term independent of $x =$ ~~$9C_6 (-2)^6 (3x)^6$~~.

$=$ ~~[illegible]~~ $9C_6 (-2)^6 (3)^{-6}$

$= 9C_6 \left(\frac{-2}{3}\right)^6$

$= 9C_6 \times \frac{64}{729}$

$= 84 \times \frac{64}{729} = 7.3748$ //

(ii) $\left(\sqrt{x} - \frac{2}{x^2}\right)^{10}$

as $\left(x^{1/2} - \frac{2}{x^2}\right)^{10}$

$(r+1)^{th}$ term $= {}^{10}C_r (x^{1/2})^{10-r} \left(\frac{-2}{x^2}\right)^r$

$= {}^{10}C_r \, x^{5-\frac{r}{2}} (-2)^r (x^2)^{-r}$

$= {}^{10}C_r \, x^{5-\frac{r}{2}} (2)^r x^{-2r}$

$8 - \frac{r}{2} - \frac{2r}{1}$

$\frac{-r-4r}{2} = \frac{-5r}{2}$

$= {}^{10}C_r (-2)^r \, x^{5-\frac{r}{2}} \, x^{-2r}$

$= {}^{10}C_r (-2)^r \, x^{5-\frac{5r}{r}}$

$5 - \frac{5r}{r} = 0$

$-\left(\frac{5r}{2}\right) = 5$

$-5r = -10$

$r = \frac{-10}{-5} = 2$

Term independent of $x = {}^{10}C_2 \, 2^2$

$= 180$

H.W. $\left(\frac{4x^3}{3} - \frac{3}{2x}\right)^8$

$(r+1)^{th}$ term $= {}^8C_r \left(\frac{4x^3}{3}\right)^{8-r} \left(\frac{-3}{2x}\right)^r$

$= {}^8C_r (4x^3)^{8-r} \, 3^{-(8-r)} \quad -3^r \quad (2x)^{-r}$

$= {}^8C_r \, 4^{8-r} \, x^{24-3r} \, 3^{-8+r} \, (-3)^r \, (2)^{-r} \, x^{-r}$

$= {}^8C_r\, 4^{8-r}\, 3^{-8+r} (-3)^r (2)^{-r}\, x^{24-3r}\, x^{-r}$

$= {}^8C_r\, 4^{8-r}\, 3^{-8+r} (-3)^r (2)^{-r}\, x^{24-4r}$

$24 - 4r = 0$

$24 = 4r$

$r = \frac{24}{4} = 6$

Term independent of $x = {}^8C_6\, 4^2\, 3^{-2} (-3)^6\, 2^{-6}$

$= \frac{{}^8C_6 \times 4^2 \times (-3^6)}{3^2 \times 2^6} = 567$ //

Exam Qns

Q1 find the middle term in the expansion of $(x+2y)^7$

Q2 expand $\left(x^3 - \frac{1}{x^2}\right)^5$ binomially

3 . c.c of x^{18} in $\left(x^4 - \frac{1}{x^3}\right)^{15}$

ans . 2 terms in middle $\frac{n+1}{2}$ and $\frac{n+1}{2}+1$

$= \frac{7+1}{2}$ and $\frac{7+1}{2}+1$

$= 4$ // and 5 //

$(r+1)$ term $= {}^7C_r\, x^{7-r} (2y)^r$

$r+1 = 4 \Rightarrow r = 3$

$= {}^7C_3\, x^{7-3}\, 2^3 y^3 = {}^7C_3\, x^4\, 2^3 y^3$

$= \dfrac{7\times 6\times 5}{1\times 2\times 3} \cdot 8\, x^4 y^3 = 280\, x^4 y^3$ //

$r+1 = 5 \Rightarrow r = 4$

5th term $= {}^7C_4\, x^{7-4} (2y)^4$

$= {}^7C_4\, x^3\, 2^4 y^4$

$= \dfrac{7\cdot 6\cdot 5\cdot 4}{1\cdot 2\cdot 3\cdot 4} \cdot 16\, x^3 y^4$

$= 560\, x^3 y^4$ //

2 $\left(x^3 + \left(\frac{-1}{x^2}\right)\right)^5 = (x^3)^5 + {}^5C_1 (x^3)^4 \left(\frac{-1}{x^2}\right) + {}^5C_2 (x^3)^3 \left(\frac{-1}{x^2}\right)^2$

$+ {}^5C_3 (x^3)^2 \left(\frac{-1}{x^2}\right)^3 + {}^5C_4\, x^3 \left(\frac{-1}{x^2}\right)^4 + \left(\frac{-1}{x^2}\right)^5$

$= x^{15} - 5x^{12} \times \frac{1}{x^2} + \frac{5\times 4}{1\times 2} x^9 \frac{1}{x^4} - \frac{5\times 4\times 3}{1\times 2\times 3} x^6 \frac{1}{x^6}$

$+ \frac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4} x^3 \frac{1}{x^8} - \frac{1}{x^{10}}$

$= x^{15} - 5x^{10} + 10x^5 - 10 + \frac{5}{x^5} - \frac{1}{x^{10}}$ //

$$\left(x^4 - \frac{1}{x^3}\right)^{15}$$

$$(r+1)^{th} \text{ term} = 15C_r (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r$$

$$= 15C_r \; x^{60-4r} \; \frac{-1^r}{x^{3r}}.$$

$$= 15C_r \; x^{60-4r} \; (-1)^r \; (x)^{-3r}$$

$$= 15C_r \; (-1)^r \; x^{60-4r} \; x^{-3r}$$

$$= 15C_r \; (-1)^r \; x^{60-7r}$$

$$60 - 7r = 18$$

$$60 - 18 = 7r$$

$$42 = 7r$$

$$\frac{42}{7} = r = 6.$$

$$r+1 \Rightarrow 6 \; , \; r = 5$$

i.e. of $x^{15} = 15C_5 \; (-1)^5 \; + x^{60-35}$

$$= 15C_5 \times x^{25} \times (-1)$$

$$= 3003 - x^{25}.$$

VECTOR ALGEBRA

- Quantities can be classified into 2 vectors & scalar quantities having only magnitude are called scalar eg: mass, distance, temperature

→ quantities having both magnitude & direction are known as vectors eg: displacement, velocity, ~~vector~~ force

<u>Types of vector</u>

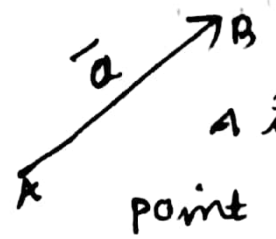
1) zero/null vector

a vector having zero magnitude
it is denoted by $\vec{0}$
eg: $\vec{AA}$ is a zero vector

2 <u>unit vector</u>

a vector having magnitude one is known as unit vector
we can find a unit vector in the direction of any given vector

A vector is always represented by a direct line segment as follows



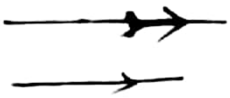
here $\vec{a} = \vec{AB}$

A is the initial point & B is terminal point. the magnitude of $\vec{a}$ is denoted by $|\vec{a}|$ or a here $|\vec{a}|$ is the length of of the line segment

LIKE VECTORS & UNLIKE VECTORS

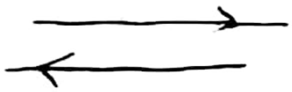
Vectors in the same direction are called like vectors

eg:



Vectors in opposite direction are called unlike vectors

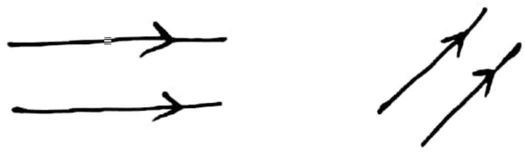
eg



EQUAL VECTORS

2 Vectors are said to be equal if they have same magnitude and direction

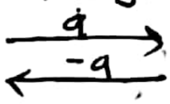
eg:



NEGATIVE VECTORS

negative of a vector is the vector having same magnitude but in the opposite direction

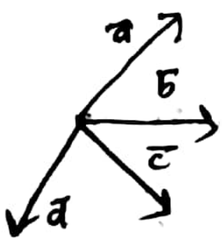
eg:



CO-INITIAL POINT VECTORS

Vectors having same initial point

eg:



COLLINEAR VECTORS

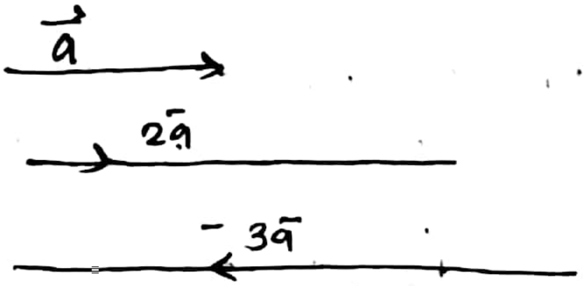
2 vectors are collinear if they have same line of action or lines of action parallel to each other

eg :

Two vectors are collinear if and only if one is a scalar multiple of other

SCALAR MULTIPLICATION

we can multiply a vector by a given scalar in the diagram the scalar multiples of $\vec{a}$ is given

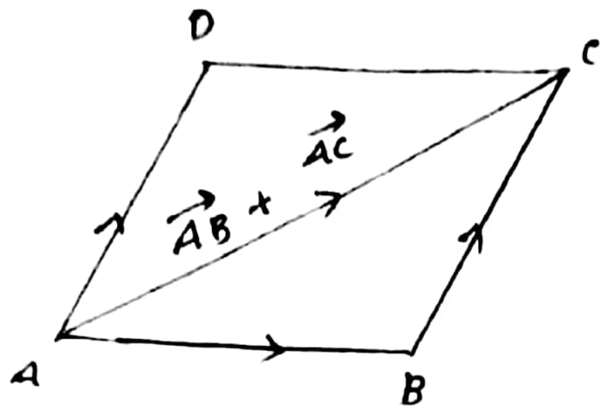


Result: 2 vectors are collinear if and only if (iff) one is the scalar multiple of other

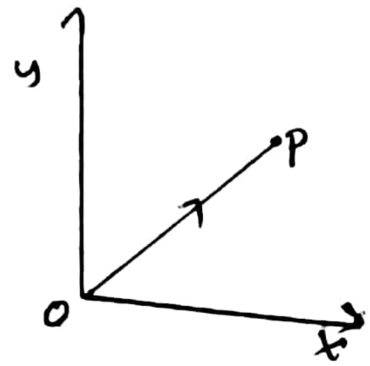
<u>Vector addition</u>

Triangle law of vector addition

parallelogram law of vector addition

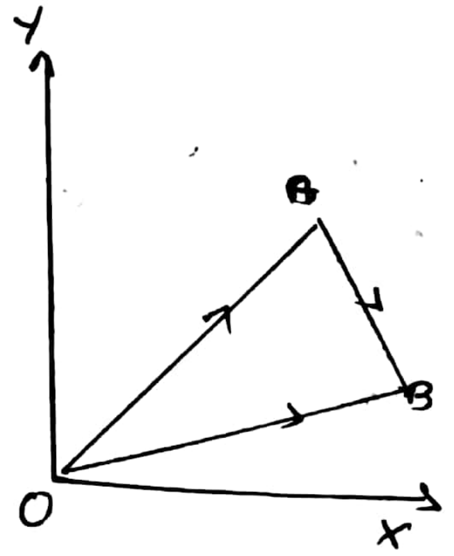


position vectors of a point



Position vector of point P, $PV(P) = \vec{OP}$

we usually represent position vector by $\vec{r}$



by Δ^{le} law

$\vec{OA} + \vec{AB} = \vec{OB}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\boxed{\vec{AB} = P.V(B) - P.V(A)}$

1) P.V (A) = $a + 2b + c$

P.V (B) = $2\bar{a} - 3\bar{b} + 4\bar{c}$

find $\bar{AB}$?

an $A\bar{B}$ = P.V (B) − P.V (A)

$= (2\bar{a} - 3\bar{b} + 4c) - (\bar{a} + 2\bar{b} + c)$

$= 2\bar{a} - 3\bar{b} + 4c - \bar{a} - 2\bar{b} - c$

$= \bar{a} - 5\bar{b} + 3c$ //

2 P.V (A) = $2\bar{a} + 3\bar{b} - \bar{c}$, P.V (B) = $3\bar{a} + 2\bar{b} + \bar{c}$

P.V (c) = $\bar{a} - \bar{b} + 3\bar{c}$ find $A\bar{B}$, $\bar{BC}$, $\bar{AC}$?

an P.V $A\bar{B}$ = P.V (B) − P.V (A)

$= (3\bar{a} + 2\bar{b} + c) - (2\bar{a} + 3\bar{b} - c)$

$= 3\bar{a} + 2\bar{b} + c - 2\bar{a} - 3\bar{b} + c$

$= \bar{a} - \bar{b} + 2c$ //

$B\bar{C}$ = P.V (c) − P.V (B)

$= \bar{a} - \bar{b} + 3\bar{c} - (3\bar{a} + 2\bar{b} + \bar{c})$

$= \bar{a} - \bar{b} + 3\bar{c} - 3\bar{a} - 2\bar{b} - \bar{c}$

$= -2\bar{a} - 3\bar{b} + 2\bar{c}$ ///

$A\bar{C}$ = P.V (c) − P.V (A)

$= \bar{a} - \bar{b} + 3\bar{c} - (2\bar{a} + 3\bar{b} - \bar{c})$

$\bar{a} - \bar{b} + 3\bar{c} - 2\bar{a} - 3\bar{b} + \bar{c}$

$= -\bar{a} - 4\bar{b} + 4\bar{c}$ //

3) P.V (A) = $2\bar{a} + 3\bar{b} - \bar{c}$, P.V (B) = $\bar{a} - 2\bar{b} + 3\bar{c}$

P.V (C) = $3\bar{a} + 4\bar{b} - 2\bar{c}$ P.V (D) = $\bar{a} - 6\bar{b} + 6\bar{c}$

S.t $\bar{AB} \parallel \bar{CD}$ or $\bar{AB}$ and $\bar{CD}$ are collinear

$\bar{AB}$ = P.V (B) − P.V (A)

$= \bar{a} - 2\bar{b} + 3\bar{c} - (2\bar{a} + 3\bar{b} - \bar{c})$

$= \bar{a} - 2\bar{b} + 3\bar{c} - 2\bar{a} - 3\bar{b} + \bar{c}$

$= -\bar{a} - 5\bar{b} + 4\bar{c}$ //

$\bar{CD}$ = P.V (D) − P.V (C)

$\bar{a} - 6\bar{b} + 6\bar{c} - (3\bar{a} + 4\bar{b} - 2\bar{c})$

$= \bar{a} - 6\bar{b} + 6\bar{c} - 3\bar{a} - 4\bar{b} + 2\bar{c}$

$= 2\bar{a} - 10\bar{b} + 8\bar{c}$

$\bar{CD} = 2\bar{AB}$

ie $\bar{AB} \parallel \bar{CD}$ or $\bar{AB}$ & $\bar{CD}$ are collinear note * The points A, B, C are collinear if vectors $\bar{AB}$ and $\bar{AC}$ are collinear

4 The position vectors of A, B, C are given by the $-2\bar{a}+3\bar{b}+5\bar{c}$, $\bar{a}+2\bar{b}+3\bar{c}$, $7\bar{a}-\bar{c}$ respectively show that A, B, C are collinear

an position vector of A $= -2\bar{a}+3\bar{b}+5\bar{c}$

(P.V) of B $= \bar{a}+2\bar{b}+3\bar{c}$

P.V of C $= 7\bar{a}-\bar{c}$

$\bar{AB} = P.V(B) - P.V(A)$

$= (\bar{a}+2\bar{b}+3\bar{c}) - (-2\bar{a}+3\bar{b}+5c)$

$= \bar{a}+2\bar{b}+3\bar{c}+2\bar{a}-3\bar{b}-5c$

$\bar{AB} = 3\bar{a}-\bar{b}-\bar{2c}$

$\bar{Ac} = P.V(C) - P.V(A)$

$= (7\bar{a}-\bar{c}) - (-2\bar{a}+3\bar{b}+5\bar{c})$

$= 7\bar{a}-\bar{c}+2\bar{a}-3\bar{b}-5\bar{c}$

$= 9\bar{a}-3\bar{b}-6\bar{c}$

so $\bar{AB} = 3(\bar{AC})$

$\Rightarrow$ $\bar{AB}$ & $\bar{AC}$ are collinear hence the points A, B, C are collinear

5 The points A, B, C have p.v

$\bar{a} - 2\bar{b} + 3\bar{c}$, $2\bar{a} + 3\bar{b} - 4\bar{c}$ & $-7\bar{b} + 10\bar{c}$.

st the points A, B, C are collinear

an $\bar{AB} = PV(B) - P.V(A)$

$= (2\bar{a} + 3\bar{b} - 4\bar{c}) - (\bar{a} - 2\bar{b} + 3\bar{c})$

$= 2\bar{a} + 3\bar{b} - 4\bar{c} - \bar{a} + 2\bar{b} - 3c$

$= \bar{a} + 5\bar{b} - 7\bar{c}$

$\bar{AC} = P.V(C) - P.V(A)$

$= (-7\bar{b} + 10\bar{c}) - (\bar{a} - 2\bar{b} + 3\bar{c})$

$= -7\bar{b} + 10\bar{c} - \bar{a} + 2\bar{b} - 3\bar{c}$

$=$

$= -\bar{a} - 5\bar{b} + 7\bar{c}$

$\bar{AC} = -(\bar{AB})$

which implies $\bar{AB}$ & $\bar{AC}$ are collinear

hence points A, B, C are collinear

components of a vector in plane

position vector of point p is

defined $\hat{i}$ = unit vector along x-axis

$\hat{j}$ = unit vector along y-axis

then $\bar{OA} = x\hat{i}$

$\bar{AP} = y\hat{j}$

ie. $\bar{OP} = x\hat{i} + y\hat{j}$

p.v of a point $p(x, y) = x\hat{i} + y\hat{j}$

$|\bar{OP}| = \sqrt{x^2 + y^2}$

if $P(x, y, z)$ is a point in space then position

$p.v(p) = x\hat{i} + y\hat{j} + z\hat{k}$

where $\hat{k}$ is unit vector along z axis

$|\bar{OP}| = \sqrt{x^2+y^2+z^2}$

p.v of $p(2,4,3)$

$\bar{OP} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$|\bar{OP}| = \sqrt{2^2+4^2+3^2}$

$= \sqrt{29}$ //

given 2 points $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$

$\vec{PQ} = P.V(Q) - P.V(P)$

$=$ [illegible]

$=$ [illegible]

$=$ [illegible]

$= x_2\hat{i} + y_2\hat{j} + z_2\hat{k} - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$

$= x_2\hat{i} + y_2\hat{j} + z_2\hat{k} - x_1\hat{i} - y_1\hat{j} - z_1\hat{k}$

$$\boxed{\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}$$

$$\boxed{|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

1) $P(1, 5, 4)$
$Q(3, 2, -1)$ find $\vec{PQ}$, $|\vec{PQ}|$

A $\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (3 - 1)\hat{i} + (2 - 5)\hat{j} + (-1 - 4)\hat{k}$

$\vec{PQ} = 2\hat{i} - 3\hat{j} - 5\hat{k}$

$|PQ| = \sqrt{2^2 + (3^2) + (-5)^2} = \sqrt{4 + 4 + 25} = \sqrt{38}$

2 find the unit vector in the direction of $\vec{AB}$ where the points

A $(1, -2, 3)$

B $(3, -3, 1)$

an $\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (3-1)\hat{i} + (-3-2)\hat{j} + (1-3)\hat{k}$

$= 2\hat{i} - \hat{j} - 2\hat{k}$

$|AB| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4+1+4} = \sqrt{9}$

$= 3$

Unit vector in the direction of $\vec{AB}$

$\frac{1}{|\vec{AB}|}\vec{AB} = \frac{1}{3}(2i - j - 2k)$

$= \frac{2}{3}i - \frac{1}{3}j - \frac{2}{3}k.$

3 given 2 points P $(2, 4, -1)$ Q $(3, 2, 1)$ find the unit vector in the direction of $\vec{PQ}$

an $\vec{PQ} = (3-2)\hat{i} + (2-4)\hat{j} + (1-{-1})\hat{k}$

$= 1\hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{PQ}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

unit vector in the direction of $\vec{AB}$

$\frac{1}{|\vec{AB}|} \cdot \vec{AB} = \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k}) = \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}k$

Q give the vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 4k$

$\bar{b} = -\hat{i} + 3\hat{j} + 2k$

find the unit vector in the direction of $3\vec{a} + 4\bar{b}$

or the unit vector in the direction of $3\bar{a} + 4b$

is given by $x\hat{i} + y\hat{j} + z\hat{k}$ find x, y, z

A $3\bar{a} + 4\bar{b} = 3(2i + 3j + 4k) + 4(-i + 3j + 2k)$

$= 6i + 9j + 12k - 4i + 12j + 8k$.

$3\bar{a} + 4\bar{b} = 2i + 21j + 20k$.

$(3\bar{a} + 4\bar{b}) = \sqrt{2^2 + 21^2 + 20^2}$

$= \sqrt{4 + 441 + 400}$

$= \sqrt{845}$

Unit vector in the direction of $3\bar{a} + 4\bar{b}$

$= \frac{1}{|3\bar{a} + 4\bar{b}|} \; 3\bar{a} + 4\bar{b}$

$= \frac{1}{\sqrt{845}} (2\hat{i} + 21j + 20k)$

$= (\sqrt{169 \times 5} = 13\sqrt{5})$

$= \frac{2}{13\sqrt{5}} \hat{i} + \frac{21}{13\sqrt{5}} \hat{j} + \frac{20}{13\sqrt{5}} k$

Thus $x = \frac{2}{13\sqrt{5}}$ $y = \frac{21}{13\sqrt{5}}$, $z = \frac{20}{13\sqrt{5}} k$

Q show that the points $(-2, 3, 5)$, $(1, 2, 3)$ & $(7, 0, -1)$ are collinear

a $A(-2, 3, 5)$, $B(1, 2, 3)$ $C(7, 0, -1)$

To prove A, B, C are collinear we have to S.T

$\bar{AB}$ and $\bar{AC}$ are collinear

$\bar{AB} = (1-{}^{-}2) + (2-3)j + (3-5)k$

$= 3i - j - 2k$

$\bar{AC} = (7-2)i + (0-3)j + (-1-5)k$

$= 9i - 3j - 6k$

$\bar{AC} = 3\bar{AB}$

$\bar{AB}$ & $\bar{AC}$ are collinear, hence the points A, B, C are collinear points

Q4 position vectors of vertices of a triangle are given by $i+2j+3k$, $2i+3j+k$, $3i+j+2k$ show that the Δle is equilateral

$P.V(A) = i+2j+3k$

$P.V(B) = 2i+3j+k$

$P.V(C) = 3i+j+2k$

we have to S.T $|\bar{AB}| = |\bar{BC}| = |\bar{AC}|$

$\bar{AB} = P.V(B) - P.V(A)$

$= (2-1)i + (3-2)j + (1-3)k = i + j - 2k$

$|\bar{AB}| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{1+1+4} = \sqrt{6}$

$\vec{BC} = PV(C) - PV(A)$

$= 3\hat{i} + \hat{j} + 2k - 2\hat{i} + 3\hat{j} + k$

$=$ ~~[illegible]~~

$=$ ~~[illegible]~~

$(3-2)\hat{i} + (1-3)\hat{j} + (2-1)k$

$= \hat{i} - 2\hat{j} + k$

$|BC| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{1+4+1} = \sqrt{6}$

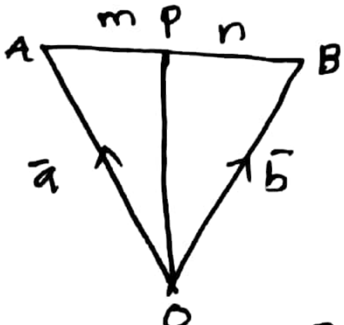
$\vec{AC} = P.V(C) - P.V(A)$

$= (3\hat{i} + \hat{j} + 2k) - (\hat{i} + 2\hat{j} + 3k)$

$= (3-1)\hat{i} + (1-2)\hat{j} + (2-3)k$

$= 2\hat{i} + \hat{j} - k$

$|AC| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{4+1+1} = \sqrt{6}$



The point P devides AB in the ratio $m:n$

$P.V(A) = \bar{a}$, $P.V(B) = \bar{b}$

then $P.V(P) = \dfrac{m\bar{b} + n\bar{a}}{m+n}$

If P is the midpoint of $\overline{AB}$ ie. $m:n = 1:1$

then $PV(P) = \dfrac{\bar{a}+\bar{b}}{2}$

Q given two points A & B with position vectors $2\hat{i}+3\hat{j}-4k$ - $4\hat{i}+\hat{j}-2k$. find the p.v(P)

if i) P is the midpoint of $\overline{AB}$

ii) P devides $\overline{AB}$ in the ratio, 3 : 2

an $\bar{a} = 2\hat{i}+3\hat{j}-4k$

$\bar{b} = 4\hat{i}+\hat{j}-2k$

i) $p.v(P) = \frac{\bar{a}+\bar{b}}{2} = \frac{2\hat{i}+3\hat{j}-4k + 4\hat{i}+\hat{j}-2k}{2}$

$= \frac{6\hat{i}+4\hat{j}-6k}{2} = 3\hat{i}+2\hat{j}-3k$

ii) $p.v(P) = \frac{m\bar{b}+n\bar{a}}{m+n}$ $\quad m:n = 3:2$

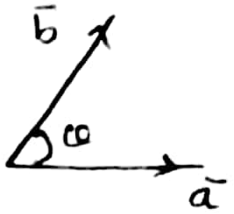
$= \frac{3(4\hat{i}+\hat{j}-2k)+2(2\hat{i}+3\hat{j}-4k)}{3+2}$

$= \frac{12\hat{i}+3\hat{j}-6k+4\hat{i}+6\hat{j}-8k}{5}$

$= \frac{16\hat{i}+9\hat{j}-14k}{5} = \frac{16}{5}\hat{i}+\frac{9}{5}\hat{j}-\frac{14}{5}k$

Dot product (scalar product)

given 2 vectors $\bar{a}$ & $\bar{b}$ with intermediate angle.



The dot product of $\bar{a}$ & $\bar{b}$ is defined as

$$\bar{a}\ \bar{b} = |\bar{a}|\ |\bar{b}| \cos\theta$$

properties

1) $\bar{a}\cdot\bar{a} = |\bar{a}|\,|\bar{a}|\cos\theta = |\bar{a}|^2$

2) $\hat{i}\cdot\hat{i} = |\hat{i}|^2 =$ similarly ||ly $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = k\cdot k = 1$

3) $\bar{a}\ \bar{o} = 0$

4) $\bar{a}\ \bar{b} = \bar{b}\ \bar{a}$

5) $\bar{a} \perp^r \bar{b}$ $\quad \bar{a}\ \bar{b} = |\bar{a}|\cdot|\bar{b}| \cos 90 = 0$

$\bar{a} \perp^r \bar{b} \iff \bar{a}\cdot\bar{b} = 0$

6) $\hat{i}\cdot\hat{j} = \hat{j}\cdot k = \hat{i}\cdot k = 0$

($\because$ $\hat{i}, \hat{j}, \hat{k}$ are mutually $\perp^r$)

let $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3k$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3k$

Then $\boxed{\bar{a}\cdot\bar{b} = a_1b_1 + a_2b_2 + a_3b_3}$

Q1 find $\bar{a}\cdot\bar{b}$ if $\bar{a} = 2i+3j-2k$. $\bar{b} = 3i-4j+3k$

ans $\bar{a}\cdot\bar{b} = 2\cdot 3 + 3\times -4 + -2\times 3$

$= 6-12-6 = -12$ //

2) $\bar{a} = 2i+3j-4k$

$\bar{b} = i+j+3k$

a $\bar{a}\cdot\bar{b} = 2\times 1 + 3\times 1 + -4\times 3$

$= 2+3-12 = 5-12 = -7$ //

3) $\bar{a} = 2i+3j-2k$

$\bar{b} = i+2j-3k$

$\bar{c} = 2i+4j-3k$

S.T $\bar{a}\cdot(\bar{b}+\bar{c}) = \bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c}$

ans $\bar{b}+\bar{c} = 3i+6j-6k$

$\bar{a}\cdot(\bar{b}+\bar{c}) = (2i+3j-2k)\cdot(3i+6j-6k)$

$= 2\times 3 + 3\times 6 + -2\times -6$

$= 6+18+12 = 36$ /

$\bar{a}\cdot\bar{b} = (2i+3j-2k)\cdot(i+2j-3k)$

$= 2\times 1 + 3\times 2 + -2\times -3 = 2+6+6$

$= 14$ //

~~that is the field is maximum~~

$\bar{a}\cdot\bar{c} = (2i+3j-2k)\cdot(2i+4j-3k)$

$= 4+12+6 = 22$

$\bar{a}\,\bar{b}+\bar{a}\,\bar{c} = 14+22 = 36$ //

$\therefore \bar{a}\,(\bar{b}+\bar{c}) = \bar{a}\cdot\bar{b}+\bar{a}\,\bar{c}$ /

4 given 2 vectors $\bar{a} = 5i-j+3k$,

$\bar{b} = i+3j-5k$. S.T $\bar{a}+\bar{b}$ $\perp^r$ to $\bar{a}-\bar{b}$

an $\bar{a}+\bar{b} = 6i+2j-2k$

$\bar{a}-\bar{b} = 4i-4j+8k$

$(\bar{a}+\bar{b})\cdot(\bar{a}-\bar{b}) = 6\times4+2\times-4+-2\times8$

$= 24-8-16$

$= 24-24 = 0$ //

$\therefore \bar{a}+\bar{b}$ $\perp^r$ to $\bar{a}-\bar{b}$ //

5 find the value of λ so that the vectors $2i+3j-k$ and $4i+6j-\lambda k$ are

a) $\parallel^{el}$ b) $\perp^r$

a) $\bar{a} = 2i+3j-k$

$\bar{b} = 4i+6j-\lambda k$

b) $\bar{a}\cdot\bar{b} = 0.$

$(2\hat{i}+3\hat{j}-k)\cdot(4\hat{i}+6\hat{j}-\lambda k)$

$8 + 18 + \lambda = 0$

$26 + \lambda = 0.$

$\lambda = -26$ //

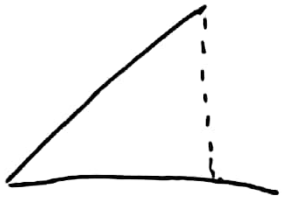
a) $\bar{a} \parallel \bar{b} \Rightarrow a$ is a scalar multiple of $\bar{b}$

ie $\frac{4}{2} = \frac{6}{3} = \frac{-\lambda}{-1} \Rightarrow 2 = \lambda$

$\lambda = 2$ //

Projection of $\bar{b}$ on $\bar{a}$

From the diagram it is clear that projection of $\bar{b}$ on $\bar{a}$ is $|\bar{b}| \times \cos\theta$



we know that

$$\bar{a} \cdot \bar{b} = |\bar{a}| \cdot |\bar{b}| \cos\theta$$

$$\Rightarrow |\bar{b}| \cos\theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}|}$$

Hence we get

$$\boxed{\begin{array}{l} \text{Projection of } \bar{b} \text{ on } \bar{a} = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}|} \\ \\ \text{Projection of } \bar{a} \text{ on } \bar{b} = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{b}|} \end{array}}$$

Q1 Find the projection of $\hat{i} - 2\hat{j} + 3\hat{k}$ on $2\hat{i} + 3\hat{j} - \hat{k}$

as $\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

projection of $\bar{a}$ on $\bar{b} = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{b}|}$

$\bar{a} \cdot \bar{b} = (\hat{i} - 2\hat{j} + 3\hat{k})(2\hat{i} + 3\hat{j} - \hat{k})$

$2 - 6 - 3 = -7$

$|\bar{b}| = \sqrt{2^2 + 3^2 + (-1^2)} = \sqrt{4+9+1} = \sqrt{14}$

progition of $\bar{a}$ on $\bar{b}$ $= \dfrac{-7}{\sqrt{14}}$ //

a
2 $2\hat{i} - 3\hat{j} + 5k$ on $\hat{i} + 2\hat{j} - 2k$

an $\bar{a} = 2\hat{i} - 3\hat{j} + 5k$

$\bar{b} = \hat{i} + 2\hat{j} - 2k$

$\bar{a} \cdot \bar{b} = (2\hat{i} - 3\hat{j} + 5k)(\hat{i} + 2\hat{j} - 2k)$

$= 2 - 6 - 10 = -14$

$|\bar{b}| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

prgition of $\bar{a}$ on $\bar{b}$ $= \dfrac{\bar{a} \cdot \bar{b}}{|b|}$

$= \dfrac{-14}{3}$ //

- Finding the angle b/w 2 vectors

we know that $\bar{a} \cdot \bar{b} = |a| \cdot |b| \cos \theta$.

$\Rightarrow \boxed{\theta = \cos^{-1}\left(\dfrac{\bar{a} \cdot \bar{b}}{|a| \cdot |b|}\right)}$

1 Find the angle b/w the vectors $i-2j+3k$ &

$3i-2j+k$

an $\bar{a} = i-2j+3k$

$\bar{b} = 3i-2j+k$

$\bar{a}\,\bar{b} = (i-2j+3k)(3i-2j+k)$

$= 3+4+3 = 10$

$|\bar{a}| = \sqrt{1^2+(-2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$

$|\bar{b}| = \sqrt{3^2+(-2)^2+1^2} = \sqrt{9+4+1} = \sqrt{14}$

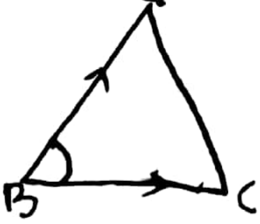
$\cos^{-1} = \dfrac{\bar{a}\cdot\bar{b}}{|a||b|} = \dfrac{10}{\sqrt{14}\times\sqrt{14}}$

$\cos^{-1} = \dfrac{10}{14}$

$\cos^{-1} = \dfrac{5}{7}$

2 given three points $A(1,0,-3)$ $B(2,1,4)$ $C(1,2,3)$

find the angle B in triangle ABC



$BC = (1-2)i + (2-1)j + (3-4)k$

$= i - j - k$

$BA = (1-2)i + (0-1)j + (-3-4)k$

$= -i - j - 7k$

$\angle B = \cos^{-1}\left(\dfrac{\bar{BC} \cdot \bar{BA}}{|BC| . |\bar{BA}|} \right)$

$\bar{BC} \cdot \bar{BA} = (i - j - k)(-i - j - 7k)$

$= -1 + 1 + 7 = 7$ //

$|\bar{BC}| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3}$

$|\bar{BA}| = \sqrt{(1)^2 + (-1)^2 + (-7)^2} = \sqrt{1+1+49} = \sqrt{51}$

$\angle B = \cos^{-1}\left(\dfrac{7}{\sqrt{3} \times \sqrt{51}} \right)$ //

3 Prove that the points with position vectors $2i - j + k$, $i - 3j - 5k$, $3i - 4j - 4k$ are the vertices right angled triangle

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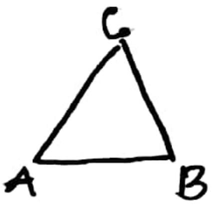

$\bar{AB} = P.V\ of(B) - P.V(A)$

$\bar{BC} = P.V(C) - P.V(B)$

$\bar{AC} = P.V(C) - P.V(A)$

$P.V(A) = 2i - j + k$

$P.V(B) = i - 3j - 5k$

P.V (c) = $3\hat{i} - 2\hat{j} - 4k$

$$\bar{AB} = (\hat{i} - 3\hat{j} - 5k)(2\hat{i} - \hat{j} + k)$$

$$-\hat{i} - 2\hat{j} - 6k$$

$$\bar{BC} = (3\hat{i} - 2\hat{j} - 4k)(\hat{i} - 3\hat{j} - 5k)$$

$$= 2\hat{i} - \hat{j} + k$$

$$\bar{AC} = (3\hat{i} - 2\hat{j} - 4k)(2\hat{i} - \hat{j} + k)$$

$$= \hat{i} - \hat{j} - \cancel{3k}\ 5k$$

$$\bar{BC} \cdot \bar{AC} = 2 + 1 - 5$$

$2 - 1$

$-2 + 1$

work done by a force

A force $\vec{F}$ is acting on a body to displace it from the point A to point B Then work done by the force is given by

$$\boxed{W = \vec{F} \; \vec{AB}}$$

A force $\vec{F} = 3\hat{i}+2\hat{j}+4k$ is acting on an object to displace it from A (2, 3, −1) to B (3, 6, 2) find work done by the force

an $\bar{F} = 3\hat{i}+2\hat{j}+4k$ $\bar{A} = 2\hat{i}+3\hat{j} - k$ $B = 3\hat{i}+6\hat{j}+2k$

$\bar{AB} = (3-2)\hat{i} + (6-3)k + (2-1)k$

$= \hat{i} + 3k + 3k$

$W = \vec{F} \cdot \bar{AB} = (3\hat{i}+2\hat{j}+4k) \cdot (\hat{i}+3k+3k)$

$3\times1 + 2\times3 + 4\times3 = 3+6+12 = 21$ //

Q find work done by a force $\vec{F} = \hat{i}+2\hat{j}+k$ which is acting on a particle to displace it from a point with position vector

$2\hat{i}+\hat{j}+k$ to the PV $3\hat{i}+2\hat{j}+4k$

an

$F = \hat{i} + 2\hat{j} + k$

$\bar{A} = 2\hat{i} + \hat{j} + k \quad \bar{B} = 3\hat{i} + 2\hat{j} + 4k$

$AB = PV(B) - PV(A)$

$= (3-2)\hat{i} + (2-1)\hat{j} + (4-1)k$

$= \hat{i} + \hat{j} + 3k$

$\vec{F} \cdot AB = W = (\hat{i} + 2\hat{j} + k)(\hat{i} + \hat{j} + 3k)$

$= 1 \times 1 + 2 \times 1 + 1 \times 3 = 1 + 2 + 3 = 6$

- is the constant force to displace it from A to B then work done by the force $W = \vec{F} \cdot \vec{AB}$ where $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

Q the constant forces $2\hat{i} - 5\hat{j} + 6k$, $-\hat{i} + 2\hat{j} - k$ & $2\hat{i} + 7\hat{j}$ - act on a particle to displace it from a point with PV $4\hat{i} - 3\hat{j} - 2k$ to a a point with PV $6\hat{i} + \hat{j} - 3k$ - find the total workdone

ans $\vec{F_1} = 2\hat{i} - 5\hat{j} + 6k$

$F_2 = -\hat{i} + 2\hat{j} - k$

$F_3 = 2\hat{i} + 7\hat{j}$

$\bar{F} = F_1 + F_2 + F_3$

$= (2 - 1 + 2)\hat{i} + (-5 + 2 + 4)\hat{j} + (6 - 1 + 0)k$

$= 3\hat{i} + 4\hat{j} + 5k$

$\bar{A} = 4\hat{i} - 3\hat{j} - 2k \qquad \bar{B} = 6\hat{i} + \hat{j} - 3k$

$\overline{AB} = P.V(B) - P.V(A)$

$= (6 - 4)\hat{i} + (1 - {}^-3)\hat{j} + (-3 - {}^-2)k$

$= 2\hat{i} + 4\hat{j} - k$

$W = \bar{F} \cdot \overline{AB} = 3 \times 2 + 4 \times 4 + 5 \times -1$

$= 6 + 16 - 5 = 17$

Q The constant forces $2\hat{i} + 3\hat{j} + k$, $\hat{i} + \hat{j} + 2k$, $2\hat{i} - \hat{j} + k$ act on a particle to displace it from the point A (1, -2, 3) to B (3, -1, 4) find the total workdone