Code No: EC3508   **SRGEC-R20**

II B.Tech I Semester Regular/Supplementary Examinations, January 2024
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Electronics and Communication Engineering)

Time: 3 Hours   **Max. Marks: 70**

Note: Answer one question from each unit.

       All questions carry equal marks.

                                               5 × 14 = 70M

UNIT-I

1. a) Explain conditional probability with examples. (6M)

   b) A missile can be accidentally launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.01 and 0.03 respectively. It is also known that B is more likely to fail (probability 0.06) if A has failed (i) What is the probability of an accidental missile launch (ii) What is the probability that A will fail if B has failed (iii) Check the independency of the events 'A fails' and 'B fails' (8M)

(OR)

2. a) Let A, B, and C be independent events, with P(C) > 0. Prove that A and B are conditionally independent given C. (7M)

   b) Let A and B be events. Show that P(A ∩ B | B) = P(A | B), assuming that P(B) > 0. (7M)

UNIT-II

3. a) State and prove any four properties of joint probability density function. (8M)

   b) Show that X and Y are identically distributed and not necessarily independent, then Cov (X + Y, X − Y) = 0. (6M)

(OR)

4. a) Define (i) Random variable (ii) PDF (iii) CDF (iii) Expected value (8M)

   b) Assume that the height of clouds above the ground at some location is a Gaussian random variable X having $a_X = 1830\,m$ and $\sigma_X = 460\,m$, find the probability that clouds will be higher than 2750 m. (6M)

UNIT-III

5. a) Determine the time average and time auto correlation function of the random process $X(t) = A\cos(\omega_0 t + \theta),$ where A and ω_0 are constants and θ is a uniformly distributed random variable on $(0, 2\pi)$. (6M)

   b) Consider a random process X(t) defined by X(t) = Y cos(wt + Θ) where Y and Θ are independent r.v's and are uniformly distributed over (-A, A) and (-π, π), respectively. (i) Find the mean of X(t). (ii) Find the autocorrelation function $R_{xx}(t_1, t_2)$ of X(t). (8M)

(OR)

6. a) State and prove any four properties of auto correlation function. (8M)

   b) Find the variance of the stationary ergodic process X(t) whose auto correlation function is given by (6M)

$$R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$$

UNIT-IV

7. a) What is the significance of Wiener-Khintchine theorem in computing power spectral density? Prove it for an WSS random process. (8M)

   b) The cross spectral density of two random processes X(t) and Y(t) is given as

   $S_{XY}(\omega) = \frac{1}{-\omega^2 + j4\omega + 4}$. Find out the cross correlation function. (6M)

(OR)

8. a) Let X(t) = A cos(w_0 t + Θ), where A and w, are constants, Θ is a uniform r.v. over (-π, π) . Find the power spectral density of X(t). (8M)

   b) Prove that the power density spectrum of a random process is an even function. (6M)

UNIT-V

9. Let Y(t) be the output of an LTI system with impulse response h(t) when a WSS random process X(t) is applied as input. Derive the expressions for mean value of the output and power spectral density of output. (14M)

(OR)

10. a) A white Gaussian noise X(t) with zero mean and spectral density $N_0/2$ is applied to a low pass RC filter. Determine the auto correlation of the output. (7M)

    b) If a system is connected by a convolution integral, where X(t) is the input and Y(t) is the output then prove that the system is a linear time invariant system. (7M)

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