# Geometric Flows

Orton Babb, Aneesh Malhotra, Ryan Vaughan

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## 1 Constructing the Normalized Graph Laplacian on a Manifold

## 2 Heat Flow

Suppose $\frac{\partial}{\partial t}\tilde{f} = -\Delta\tilde{f}$ with $\tilde{f}(0,\theta) = f_0(\theta)$. Note that if $\tilde{f}$ is an eigenfunction of $\Delta$, then $\Delta\tilde{f} = \lambda\tilde{f}$. Since $\frac{\partial}{\partial t}\tilde{f} = -\lambda\tilde{f}$, this has the solution $\tilde{f}(t,\theta) = e^{-\lambda t} f_0(\theta)$. More generally, we want to expand $\tilde{f}$ to its generalized fourier series:

$$\tilde{f}(t,\theta) = \sum_{n=0}^{\infty} c_n(t)\phi_n(\theta)$$

Let $\phi$, so that $\Delta\phi_n = \lambda_n\phi_n$, be the eigenfunctions of $\Delta$. Then,

$$\frac{\partial}{\partial t}\tilde{f} = \sum_{n=0}^{\infty} \dot{c}_n(t)\phi_n(\theta) \qquad = \Delta\tilde{f} = \sum_{n=0}^{\infty} c_n(t)\Delta\phi_n(\theta)$$