Lab 5
Binary Data Encoding Systems

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1 Introduction

1.1 Objective

The objective of this lab is to encode bits using pulses defined by Barker codes, and analyze how we can use the properties of their matched filters to determine the original signal in the presence of noise.

1.2 Barker Codes

Barker codes are signals defined by a sequence of ± 1. Given a pulse $p(t)$, we can encode a binary signal for which at every time period T_p, the pulse $p(t)$ represents a 1 and the pulse $-p(t)$ represents a zero. Barker codes minimize the autocorrelation of a function, so that if it is convolved with its matched filter we see very little energy in the output until a complete overlap, at which case the energy is maximized, giving us a sharp spike. If Barker Codes are used to represent the pulse $p(t)$, we can take an encoded signal $s(t)$ and pass it through the Barker Codes' matched filter. This would minimize the output at every point except for complete overlaps, at which point we would see either a positive or negative spike, representing either a 1 or a 0. This would allow us to encode the input message by looking at each spike. There are only 7 known Barker Codes, corresponding to sequences of 1 and -1 of lengths 2, 3, 4, 5, 7, 11, and 13. Barker Codes are used particularly for communication, namely radar, since they allow us to easily detect pulses. Finally, Barker Codes are very good for handling Gaussian noise. If a signal defined by Barker Codes goes through a noisy channel, the matched filter will minimize the noise in the output, and maximize the spike.

2 Main Body

2.1 Part 1

First we analyzed matched filters of pulses $p_3(t)$ and $p_11(t)$, which represent the Barker Codes [1, 1, -1] and [1, 1, 1, -1, -1, -1, 1, -1, -1, 1, -1]. We passed these functions through their matched filters, whose impulse response is given by their causal time reversal $p_(T_p - t)$ The plots of p_3 and p_11 with their matched filters is shown below.

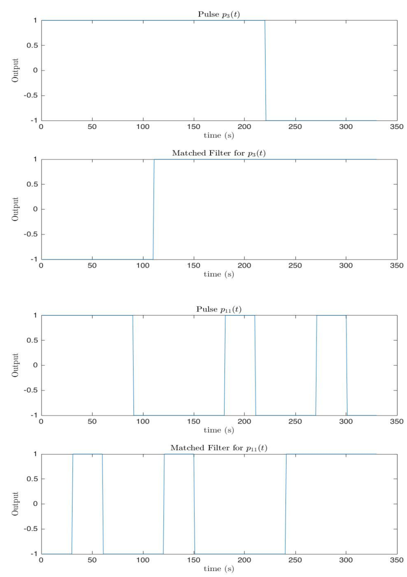

Figure 1: Pulses p_3 and p_11 with their matched filters

We then found the outputs of the matched filters by convolution and obtained the following plots:

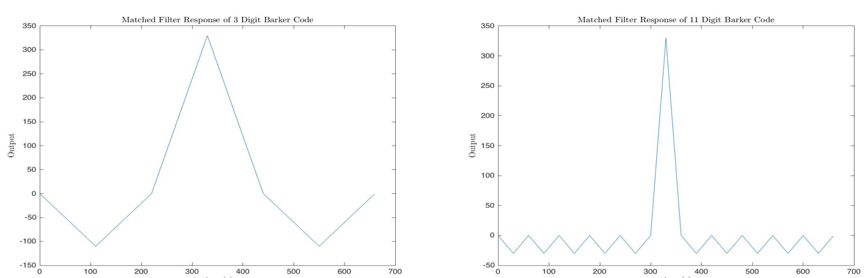

Figure 2: Output of $h_{MF3} * p_3$ and $h_{MF11} * p_{11}$

We see that the output of each of the filters is a spike at the the point where there is complete overlap. This occurs at $T_p = 330$. The maximum output is also 330.

2.2 Part 2

In this section we created a discrete Gaussian noise signal of length 100, called SIG by taking the sign of a random noise signal. We then defined the signals s_3 to be p_3 at every nT_p seconds if the nth value of SIG is 1 and $-p_3(t)$ if the nth value of SIG is positive. This creates a signal of $33,000$ samples containing either p_3 or $-p_3$ at every 330 samples. We repeated this for p_{11} to create s_{11}. The plots of the first 4 bits of each is shown below.

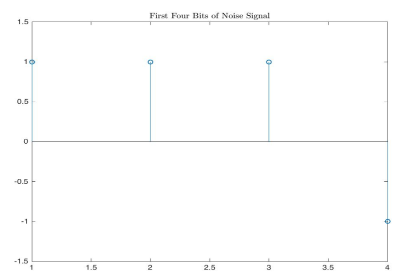

Figure 3: First 4 bits of SIG

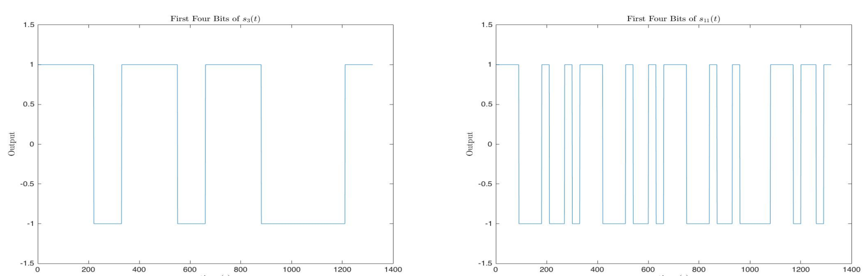

Figure 4: First 4 time periods of $s_1 1$ and s_3

2.3 Part 3

In this section we defined a Gaussian Noise signal nse and added it to the signals defined in the previous section

$$x_1(t) = s_3(t) + nse$$
$$x_3(t) = s_3(t) + 3nse$$
$$x_5(t) = s_5(t) + 5nse.$$

We then found the output of these signals with both matched filter h_{MF3} and h_{MF11}. This was repeated for $s_{11}(t)$. The plots are shown below:

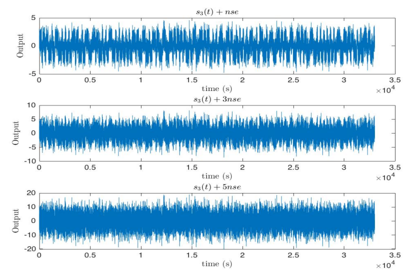

Figure 5: Noisy s_3 signal

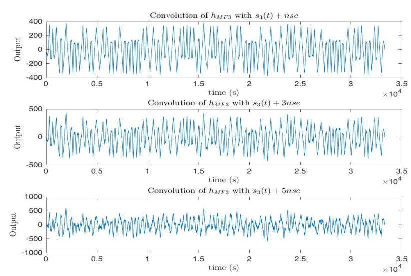

Figure 6: Noisy s_3 through the correct matched filter

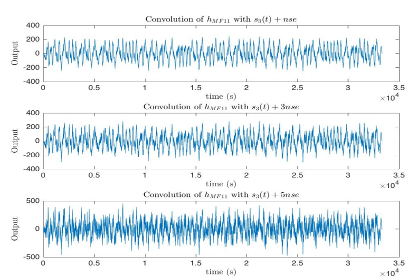

Figure 7: Noisy s_3 through the incorrect matched filter

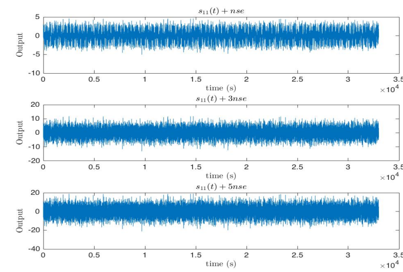

Figure 8: Noisy s_{11} signal

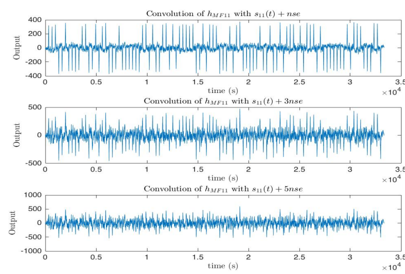

Figure 9: s_{11} through the correct matched filter

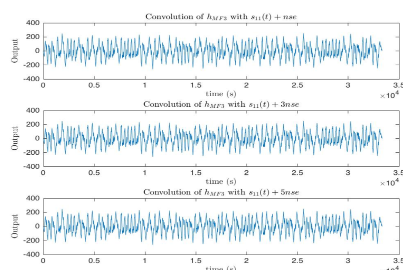

Figure 10: s_{11} through incorrect matched filter

We see that in each case, the correct matched filter clearly shows the bits, whereas using the other matched filter makes it more difficult to identify the bits

3 Conclusion

In this lab we saw how to send binary signals using continuous waveforms by using Barker Codes. Barker Codes allowed us to create pulses that are orthogonal when they overlap, creating an output signal with far more energy at their overlap point T_p than at every other point. By sending messages encoded with Barker Codes, we can identify the spikes, and therefore the Boolean value of the signal at each time period, allowing us to decode messages easily. We saw that this was even true for signals with Gaussian noise. The signals with low noise were easily detectable. While the signal with the highest noise $s_1 1 + 5nse$ was still mostly readable, there were still some ambiguities about where spikes were. This was likely due to the noise being in high enough magnitude to swamp the original signal, making the output from the matched filter far less readable. Finally, we saw that by using the incorrect matched filter, there were many more ambiguities about the input signal, as the spikes were very poorly defined and many spikes corresponded to incorrect bits. This shows the precision of the spikes in the convolution of the Barker Codes with their matched filter and how using the incorrect filter can make the input unreadable.

4 Appendix

4.1 Code

```
Tp = 330;
np = 100;
N = 33000;
p_3 = [ones(1,220), -1*ones(1,110)];
xlabel('time (s)','Interpreter','latex')
h_MF3 = [-1*ones(1,110), ones(1,220)];
out3 = conv(h_MF3,p_3); %Expect 330*2 -1 points
%Max occurs at 330 since largest overlap

subplot(2,1,1)
plot(p_3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Pulse $p_3(t)$','Interpreter','latex')
subplot(2,1,2)
plot(h_MF3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Matched Filter for $p_3(t)$','Interpreter','latex')


figure
plot(out3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Matched Filter Response of 3 Digit Barker
    Code','Interpreter','latex')
```

```
p_11 = [];
code11 = [1 1 1 -1 -1 -1 1 -1 -1 1 -1];
for i = 1:length(code11)
   p_11 = [p_11, ones(1,30)*code11(i)];
end
h_MF11 = fliplr(p_11);
out11 = conv(h_MF11,p_11);

subplot(2,1,1)
plot(p_11) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Pulse $p_{11}(t)$','Interpreter','latex')
subplot(2,1,2)
plot(h_MF11) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Matched Filter for $p_{11}(t)$','Interpreter','latex')

figure
plot(out11) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Matched Filter Response of 11 Digit Barker
    Code','Interpreter','latex')



%Section 2: Binary Data Encoding Systems

 N = 100;
 fs = 330;
 samples = 33000;

 %SIG = sign(randn(1,100));

load('SIG_Values.mat') %loads random values

 s_3 = [];
 s_11 = [];

 figure
 stem(SIG(1:4))
 title('First Four Bits of Noise Signal','Interpreter','latex')
 ylim([-1.5,1.5])
 for i = 1:length(SIG)
    s_3 = [s_3 p_3*SIG(i)]; %first 4 has 330*4 samples
    s_11 = [s_11 p_11*SIG(i)];
 end
 figure
```

```matlab
plot(s_3(1:4*330));
ylim([-1.5,1.5])
xlabel('time (s)','Interpreter','latex')
title('First Four Bits of $s_{3}(t)$','Interpreter','latex')
ylabel('Output','Interpreter','latex')

figure
plot(s_11(1:(4*330)))
xlabel('time (s)','Interpreter','latex')
ylim([-1.5,1.5])
ylabel('Output','Interpreter','latex')
title('First Four Bits of $s_{11}(t)$','Interpreter','latex')
%Section 3:

nse = randn(1,33000);
x_1 = s_3 + nse;
x_3 = s_3 + 2*nse; % Adding Noise to s_3
x_5 = s_3 + 5*nse;

y_1 = conv(h_MF3,x_1); %correct matched filter
y_3 = conv(h_MF3,x_3);
y_5 = conv(h_MF3,x_5);

y_e1 = conv(h_MF11,x_1); % wrong matched filter cases
y_e3 = conv(h_MF11,x_3);
y_e5 = conv(h_MF11,x_5);


f_1 = s_11 + nse;
f_3 = s_11 + 3*nse; %Adding Noise to s_11
f_5 = s_11 + 5*nse;

g_1 = conv(h_MF11,f_1);
g_3 = conv(h_MF11,f_3); %Correct Matched Filter
g_5 = conv(h_MF11,f_5);

g_e1 = conv(h_MF3,f_1);
g_e3 = conv(h_MF3,f_1); %Incorrect Matched Filter
g_e5 = conv(h_MF3,f_1);


%Plotting

figure
subplot(3,1,1)
plot(x_1) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('$s_3(t) + nse$','Interpreter','latex')
subplot(3,1,2)
```

```
plot(x_3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('$s_3(t) + 3nse$','Interpreter','latex')
subplot(3,1,3)
plot(x_5)
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('$s_3(t) + 5nse$','Interpreter','latex')

figure
subplot(3,1,1)
plot(y_1) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF3}$ with $s_3(t) +
    nse$','Interpreter','latex')
subplot(3,1,2)
plot(y_3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF3}$ with $s_3(t) +
    3nse$','Interpreter','latex')
subplot(3,1,3)
plot(y_5)
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF3}$ with $s_3(t) +
    5nse$','Interpreter','latex')

figure
subplot(3,1,1)
plot(y_e1) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF11}$ with $s_3(t) +
    nse$','Interpreter','latex')
subplot(3,1,2)
plot(y_e3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF11}$ with $s_3(t) +
    3nse$','Interpreter','latex')
subplot(3,1,3)
plot(y_e5)
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF11}$ with $s_3(t)
    +5nse$','Interpreter','latex')
```

```
figure
subplot(3,1,1)
plot(f_1) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('$s_{11}(t) + nse$','Interpreter','latex')
subplot(3,1,2)
plot(f_3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('$s_{11}(t) + 3nse$','Interpreter','latex')
subplot(3,1,3)
plot(f_5)
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('$s_{11}(t) + 5nse$','Interpreter','latex')

figure
subplot(3,1,1)
plot(g_1) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF11}$ with $s_{11}(t) +
    nse$','Interpreter','latex')
subplot(3,1,2)
plot(g_3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF11}$ with $s_{11}(t) +
    3nse$','Interpreter','latex')
subplot(3,1,3)
plot(g_5)
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF11}$ with $s_{11}(t) +
    5nse$','Interpreter','latex')


figure
subplot(3,1,1)
plot(g_e1) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF3}$ with $s_{11}(t) +
    nse$','Interpreter','latex')
subplot(3,1,2)
plot(g_e3) %max still occurs at 330
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
```

```
title('Convolution of $h_{MF3}$ with $s_{11}(t) +
    3nse$','Interpreter','latex')
subplot(3,1,3)
plot(g_e5)
xlabel('time (s)','Interpreter','latex')
ylabel('Output','Interpreter','latex')
title('Convolution of $h_{MF3}$ with $s_{11}(t)
    +5nse$','Interpreter','latex')
```

5 References

1. Lathi, B. P. Linear Systems and Signals. New York: Oxford UP, 2005. Print.

2. Ranganath, P., Rao, S. (2014). Effect of Pulse Shaping on Autocorrelation Function of Barker and Frank Phase Codes. Journal of Advanced Electrical and Computer Engineering, 1(1), 22-31.