Lab 3
Analysis of a Car Suspension System

Aneesh Malhotra
G00844135

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1 Introduction

Objective

The objective of this lab is to analyze the behavior of a car suspension system using a linear mass-spring model. We analyze how the position of the car behaves as a function of the road.

1.1 The Model

In this experiment we analyzed a linear model of a car suspension system by using a dampened mass-spring system. In a general mass-spring system, we use Newton's laws to find

$$my''(t) + k_d y'(t) + k_s y(t) = x(t).$$

Where $x(t)$ is the forcing input to the system. In our model, we look at $y(t)$ as the distance the wheel of the car is above level pavement, given a suspension system with a particular damping coefficient k_d and a spring constant k_s. We extend the mass-spring model to dampen the vertical velocity of the car, or rather $y'(t) - x'(t)$ and the vertical position, $y(t) - x(t)$. Using these as the velocity and position in the damped mass-spring equation and normalizing, we see

$$y''(t) + \frac{k_d}{m} y'(t) + \frac{k_s}{m} y(t) = \frac{k_d}{m} x'(t) + \frac{k_s}{m} x(t).$$

This system has a transfer function given by

$$\frac{\frac{k_d}{m}s + \frac{k_s}{m}}{s^2 + \frac{k_d}{m}s + \frac{ks}{m}}$$

. In our simulations, we set $k_s = 10^5 N/m$, $m = 250$, and let k_d vary.

1.2 Pavement

Using the model from above we analyze three cases: a curb, a pothole, and a wavy road.

1. We model a car going over a curb of height A_1 as $x(t) = A_1 u(t)$.
2. We model a car going through a pothole of depth A_2 and width T as $x(t) = -A_2 u(t) + A_2 u(t - T)$.
3. We model a car going over a wavy road of amplitude A_3 and period T as $x(t) = A_3 \cos(\frac{2\pi t}{T})$.

2 Main Body

2.1 Curb

We began by analyzing the response of a car over a curb of height 0 1m. Since we have a linear system, we use Matlab to simply find the step response, then scale it by our factor A_1. Additionally, we tested various damping coefficients k_d in order to see which would provide the smoothest ride for the passenger. We tested this for $k_d = 10^4$, $k_d = 2 \cdot 10^4$, and $k_d = 5 \cdot 10^3$. Our results are plotted below.

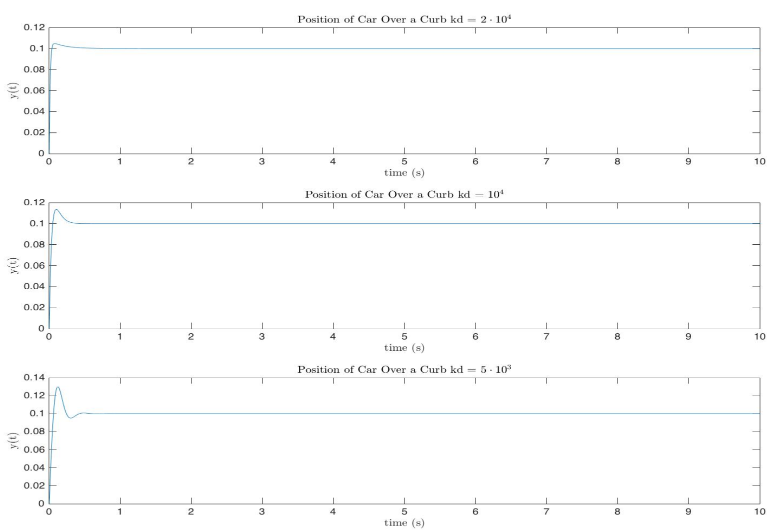

Figure 1: Response of a car going over a curb with varying damping coefficients

We see in the figure above that the higher damping constants lead to lower jumps in the vehicle, whereas the lower damping constants lead to higher jumps.

2.2 Pothole

In this section we analyze what happens when we let $x(t)$ simulate a car going over a pothole of length 1m at 5m/s. We see that in the time-domain, a wheel will traverse the pothole in $\frac{1}{5}$ s. We simulate this for k_d values of 10^4 and $2 \cdot 10^3$. The plot is shown below.

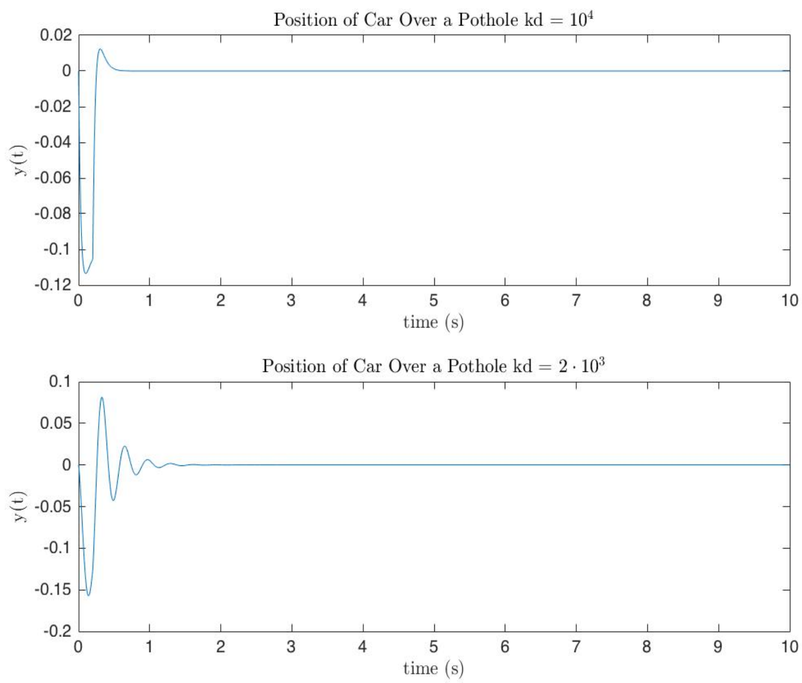

Figure 2: Response of a car going over a pothole with varying damping coefficients

In this case it we see that the car with the lower damping coefficient tends to shake more after crossing the pothole than the car with the higher damping coefficient.

2.3 Wavy Road

We finally model the case of a wavy road as a sinusoid given by $x(t) = A_3 \cos(2\pi t/T)$. In our case, the amplitude of the road is 5cm and the period of the road is 0.314s. The output is shown below for a k_d value of 10^4.

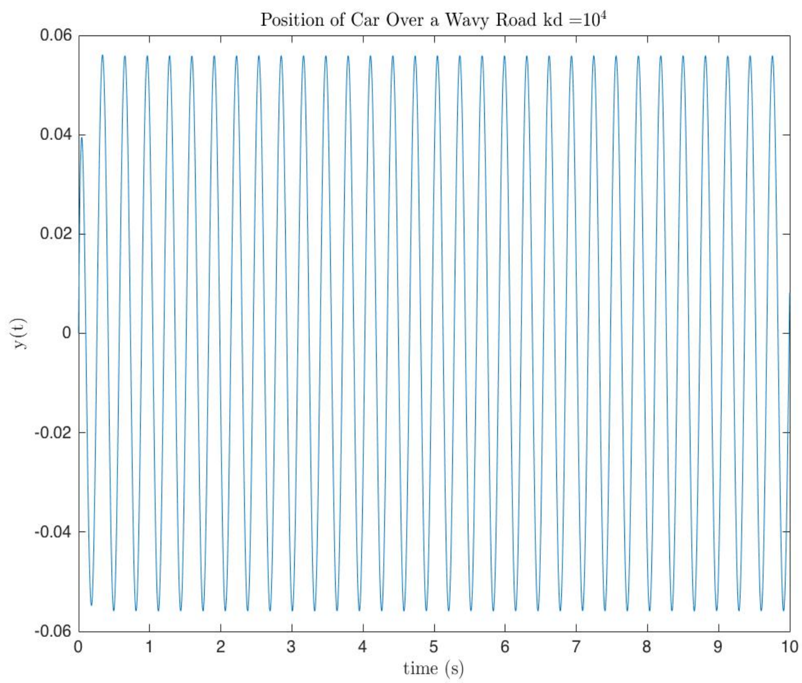

Figure 3: Response of a car going over a wavy road

In this case we initial damping when the car begins to go over the road, but the car eventually ends moving along in phase with the road.

3 Conclusion

In this lab we studied three major cases. In the first two cases, we saw that higher damping coefficients were key in keep the car steady when running across disruptions the road. In the case of the curb, we saw that the car with the lowest damping coefficient depressed down before coming back up, creating a more bumpy ride whereas the car with the higher damping coefficients only rose and came back down. In the pothole case, the car with the lowest damping coefficient in the suspension kept bouncing for over a second after passing the pothole, whereas the one with higher damping went to equilibrium almost immediately. In the wavy road case, we only studied the $k_d = 10^4$ case and saw that there was only initial damping, but the car only moved along with the road. Overall, the model used was not entirely accurate, as you would expect in the case of curved motion, like in the sinusoidal case, there would be a force of gravity that would play a role in the net centripetal force felt by the car. Additionally, if the pothole or curb were too high, to resemble a wall or a ditch, we would not expect the car to keep moving with time.

3.1 Code

```
%Lab 3

%3.2 Response to a curb

%a kd = 2*10^4
kd = 2*10^4; %We will Change kd
m = 250; %Parameter Values
ks = 10^5;
A1 = 0.1;

b = [kd/m, ks/m];
a = [1, kd/m, ks/m];

SYS = tf(b,a); %System

t = linspace(0,10,10000);
ya = A1*step(SYS,t);
subplot(3,1,1)
plot(t,ya)
xlabel('time (s)','Interpreter','latex')
ylabel('y(t)','Interpreter','latex')
title('Position of Car Over a Curb kd = $2 \cdot
    10^4$','Interpreter','latex')
```

```
%b. kd = 10^4
kd = 10^4;
b = [kd/m, ks/m];
a = [1, kd/m, ks/m];
SYS = tf(b,a);
ya = A1*step(SYS,t);
subplot(3,1,2)
plot(t,ya)
xlabel('time (s)','Interpreter','latex')
ylabel('y(t)','Interpreter','latex')
title('Position of Car Over a Curb kd = $10^4$','Interpreter','latex')

%c. kd = 5000
kd = 5000;
b = [kd/m, ks/m];
a = [1, kd/m, ks/m];
SYS = tf(b,a);
ya = A1*step(SYS,t);
subplot(3,1,3)
plot(t,ya)
xlabel('time (s)','Interpreter','latex')
ylabel('y(t)','Interpreter','latex')
title('Position of Car Over a Curb kd = $5 \cdot
    10^3$','Interpreter','latex')


%Position over a pothole
figure
%a. kd = 10^4
A2 = 0.1;
kd = 10^4;
b = [kd/m, ks/m];
a = [1, kd/m, ks/m];
SYS = tf(b,a);
x = zeros(size(t));
x(t<(1/5)) = -A2;
ya = lsim(SYS,x,t);
subplot(2,1,1)
plot(t,ya)
xlabel('time (s)','Interpreter','latex')
ylabel('y(t)','Interpreter','latex')
title('Position of Car Over a Pothole kd = $10^4$','Interpreter','latex')


%b. kd = 2000
A2 = 0.1;
kd = 2000;
b = [kd/m, ks/m];
a = [1, kd/m, ks/m];
```

```
SYS = tf(b,a);
x = zeros(size(t));
x(t<(1/5)) = -A2;
ya = lsim(SYS,x,t);
subplot(2,1,2)
plot(t,ya)
xlabel('time (s)','Interpreter','latex')
ylabel('y(t)','Interpreter','latex')
title('Position of Car Over a Pothole kd = $2\cdot
    10^3$','Interpreter','latex')

figure
% Position over wavy pavement
kd = 10^4;
A3 = .05;
T0 = 0.314;
p = @(t) A3*cos(2*pi*t/T0);
b = [kd/m, ks/m];
a = [1, kd/m, ks/m];
SYS = tf(b,a);
yc =lsim(SYS,p(t),t);
plot(t,yc)
xlabel('time (s)','Interpreter','latex')
ylabel('y(t)','Interpreter','latex')
title('Position of Car Over a Wavy Road kd
    =$10^4$','Interpreter','latex')
```

4 References

Lathi, B. P. Linear Systems and Signals. New York: Oxford UP, 2005. Print.