Lab 1
Manipulating Sinusoids with Matlab

Aneesh Malhotra
G00844135

January 24, 2017

1 Introduction

The objective of this lab was to gain comfort plotting and analyzing sinusoids in Matlab. Initially we created two sinusoids and plotted them, their sum, and their product. We then analyzed the Amplitude and phase of the sum of them using Matlab, and phasor addition. Finally, we created a harmonic function that adds sinusoids whose frequencies are multiples of a given fundamental frequency f_0

2 Main Body

2.1 Plotting Sinusoids

We began by plotting two sinusoids given by

$$x_1(t) = A_1 cos(2\pi(440)(t - t_{m1}))$$
$$x_1(t) = A_2 cos(2\pi(440)(t - t_{m2}))$$

where $A_2 = 42$ and $A_1 = 0.8A_2$. The time shifts were given by $t_{m1} = ?(37.2/8)T$ and $t_{m2} = (41.3/23)T$, where $T = \frac{1}{440}$. We also defined the following:

$$x_3(t) = x_1(t) + x_2(t)$$
$$x_4(t) = x_1(t)x_2(t)$$

The following plot was obtained
By taking the maximum value of x_3 in Matlab we approximated the amplitude and phase shift x_3. The amplitude was approximately 15.28 and the phase shift was .0068.

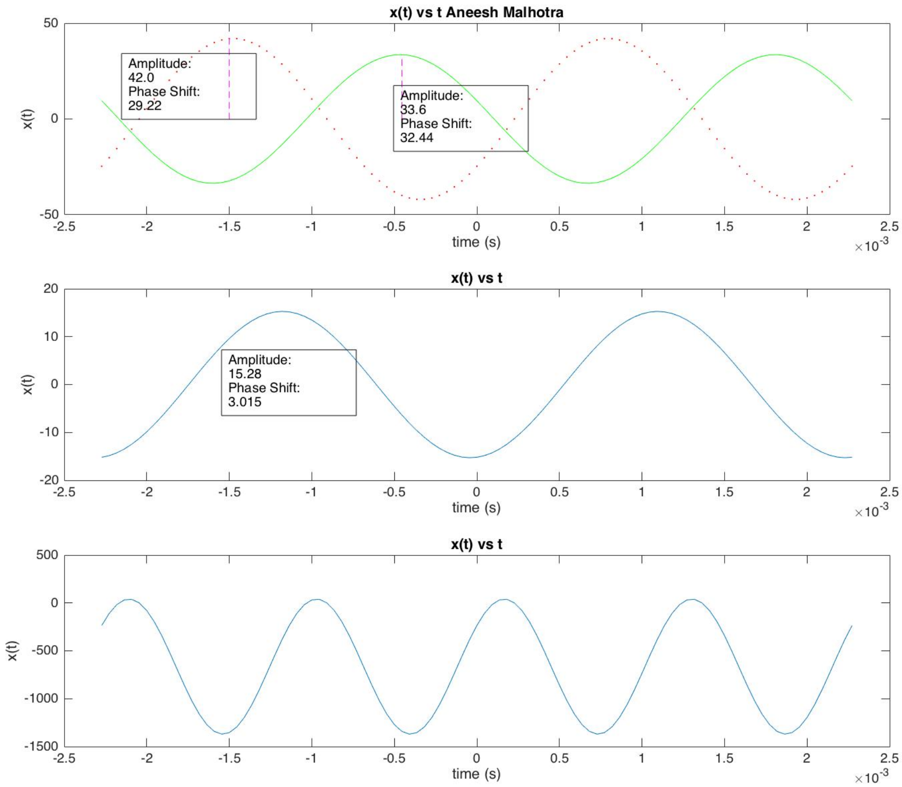

Figure 1: Plots of $x_1(t), x_2(t), x_3(t), x_4(t)$ containing approximations of phase and amplitude. Plots x_1 and x_2 are in the top subfigure, followed by x_3 and x_4 respectively.

2.2 Theoretical Calculation of Phase and Amplitude

We can represent signals $x_1(t)$ and $x_2(t)$ as the real parts of complex exponentials. Therefore, we can represent $x_i(t)$ as the real part of $X_i e^{j2\pi 440}$ where $\mathrm{X}_i = \mathrm{A}_i e^{j\phi_i}$. Therefore

$$x_1(t) + x_2(t) = \Re\{A_1 e^{j\phi_1} e^{j2\pi 440} + A_2 e^{j\phi_2} e^{j2\pi 440}\}$$

If we factor out the $e^{j2\pi 440}$, we can simply do the addition of the phasors to find the new phasor. Using Matlab, we added the two phasors and found that $|x_3(t)| \approx 15.28145$ and the phase shift was approximately -3.015. Phasor addition yielded a phase shift of -3.0267 and a magnitude of 15.28145. These values are consistent with the values obtained using the graph and vector values.

2.3 Harmonics

In this section we created a function that adds sinusoids that are harmonically related. The function takes in values of amplitudes, phases, maximum time, sample period, and fundamental frequency and returns back a sum of sinusoids in the form

$$r(t) = \sum_{n=0}^{N} C_n cos(n\omega_0 t + \theta_n).$$

The following plots were obtained for each case,

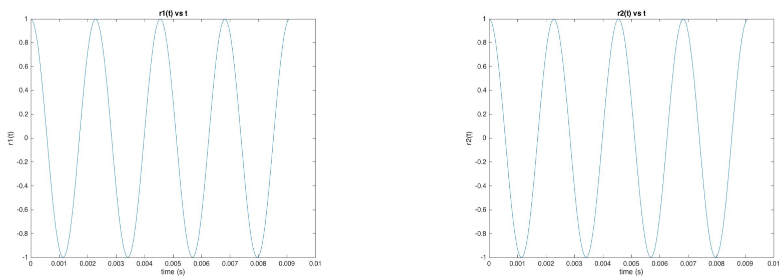

Figure 2: Plots of $r_1(t)$ and $r_2(t)$ with the following paramters: $f_0 = 440$, $C_n = [0, 1]$, $\theta_n = [0, 0]$ Both had a sample period of 50

.

The sounds produced by r_1 and r_2 were identical and is characterized by a short, medium pitched ring. Upon adding a phase shift, however, we obtain the following plot:
The phase shift in r_3 causes the graph of r_3 to look slightly different, although the sound is exactly the same.

3 Conclusion

In this lab we began by plotting sinusoids. We found that when two sinusoids have the same frequency, the amplitude and phase shift can be found easily by adding the phasors of their corresponding complex exponentials. We saw this when the phase shift and amplitude calculated from the x_3 vector were very similar to those calculated by adding phasors. Additionally we looked into the addition of harmonically related sinusoids. By simply adding a phase shift to one of these signals, we found no noticeable difference in sound. This is because the phase of a signal is not noticeable to the human ear. Therefore, chords played in sync sound the same as they would if they were arpeggiated.

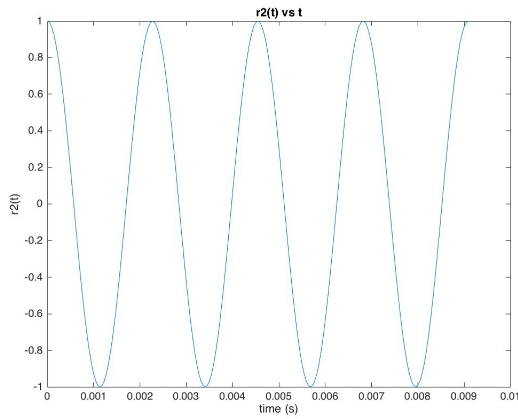

Figure 3: The plot of r_3 is the same as r_2 except with a phase shift

.

4 Appendix

4.1 Code to Obtain Plots

```
%3.1 Plotting Sinusoids

tt = -(1/440):(1/(440*50)): 1/440; %Creates about 51 samples per period

A1 = 2*21; %Twice my age
A2 = A1*0.8;
T = 1/440;
M = 8; % Birth Month
D = 23; % Birth Day
tm1 = -(37.2/M)*T; % Constructing the time shifts
tm2 = (41.3/D)*T;


x1 = @ (t) A1*cos(2*pi*440*(t - tm1));
x2 = @ (t) A2*cos(2*pi*440*(t - tm2));
x3 = x1(tt) + x2(tt); % x3 and x4 are not function handles
x4 = x1(tt) .* x2(tt);
[max1,I1] = max(x1(tt));
[max2,I2] = max(x2(tt));


figure
subplot(3,1,1); plot (tt,x1(tt),'.r'); hold on; plot (tt , x2 (tt),'g');
```

```
plot(tt(I1)*ones(1000,1), linspace(0, max(x1(tt)), 1000),'--m');
plot(tt(I2)*ones(1000,1), linspace(0, max(x2(tt)), 1000),'--m');
hold off
title('x(t) vs t Aneesh Malhotra')
xlabel('time (s)');
ylabel('x(t)');
subplot(3,1,2); plot (tt,x3);
title('x(t) vs t')
xlabel('time (s)'); % This block of code does the subplots
ylabel('x(t)');
subplot(3,1,3); plot (tt,x4);
title('x(t) vs t')
xlabel('time (s)');
ylabel('x(t)');

%Part 3.3

X = 2.0;
w = 2*pi*440; % period of this waveform is (1/440)
t = linspace(-1/440,  1/440, 100);
x1 = real(X*exp(j*w*t));


% 3.4: Harmonics (function defined in separate file)

w0 = 2*pi*440;
Tsample = 1/(50*440); % defines 50 samples per period
Cn = [0 1];
thetan = [0 0];
tmax = 100/440;


[t,r1] = harmonics(w0,Cn,thetan,tmax,Tsample);
figure
plot(t,r1)
%soundsc(r1,1/Tsample) % Comment this line to listen to other sounds
plot(t(1:201),r1(1:201)); % Index 201 is where t = 4/440 (4 periods)
xlabel('time (s)')
ylabel('r1(t)')
title('r1(t) vs t')
clear w0 Tsample Cn thetan

load paramFall2016.mat % Loads new variables

figure
```

```
tmax = 100/440;
[t,r2] = harmonics(w0,Cn,thetan,tmax,Tsample);
soundsc(r2,1/Tsample) % Ran this separately as Matlab attempts to play
                      % this and r1 simultaenously
plot(t(1:201),r2(1:201));
xlabel('time (s)')
ylabel('r2(t)')
title('r2(t) vs t')

thetan = 2*pi*(rand(length(r2),1)-1);

[t,r3] = harmonics(w0,Cn,thetan,tmax,Tsample);
figure
plot(t(1:201),r3(1:201));
xlabel('time (s)')
ylabel('r3(t)')
title('r3(t) vs t')
```

4.2 Harmonic Function

```
function [t,r] = harmonics(w0,Cn,thetan,tmax,Tsample)

t = 0:Tsample:tmax;

r = zeros(size(t)); % ensures r is the correct size
for i = 1:length(Cn)
    r = r + Cn(i)*cos((i-1)*w0.*t + thetan(i));
end

end
```

4.3 Matlab Diary for Calculating Phase

```
x3;
P = A1*exp(j*2*pi*440*(-tm1));
Q = A2*exp(j*2*pi*440*(-tm2))

Q =

         9.50623889096877 +        32.2271845209574i

JJ = P+Q;
abs(JJ)

ans =
```

```
        15.2814519405665

atan(imag(JJ) / real(JJ))

ans =

       0.114870442697732

diary off
```

5 References

Lathi, B. P. Linear Systems and Signals. New York: Oxford UP, 2005. Print.