3. Lagrange Interpolation

(a) Define the function $p(i, xs, x)$ which for an integer i, list xs, and a real variable x returns

$$\prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

where x_k is the k'th element of the list xs.

(b) Build a polynomial interpolation of the data:

x_i	0.1	0.2	0.3	0.4
y_i	10	5	-4	-7

.

Evaluate your interpolation at $x = 0.2$ and $x = 0.25$ and print the result.

4. Monte Carlo Method

(a) Find the volume inside the intersection of the ellipsoid $x^2 + y^2/4 + z^2 = 1$ and the cube $|x| < 1$ and $|y| < 1$ and $|z| < 1$ using the Monte Carlo method with 1000 points.

(b) Compare your result with the analytic result for the area $11\pi/6$. Print the actual error and the theoretical estimate for the error of the Monte Carlo integration.

5. Euler Method

(a) Use the Euler method to solve the equation $y'(x) = y^2(x) + \cos(20x)$ in the interval $x \in [0, 1]$ with the initial data $y(0) = 1/2$. Do not use more than 100 intermediate points.

(b) Plot the resulting function $y(x)$ for $x \in [0, 1]$.

6. Simpson's Integration Method

(a) Implement the Simpson's integration method.
Hint: you can use that for one segment

$$\int_{x_i}^{x_{i+1}} f(x)dx \simeq \frac{x_{i+1} - x_i}{6} \left(f(x_i) + 4f\left(\frac{x_i + x_{i+1}}{2}\right) + f(x_{i+1}) \right) .$$

(b) Use it to compute $\int\limits_{-1}^{1} \frac{2}{1+x^2}\, dx$. Do not use more than 100 intermediate points.

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