

AFAC ENGLISH SCHOOL & JUNIOR COLLEGE

Prelims Examination 2020

MATHEMATICS & STATISTICS

DATE 10/2/2021

XII SCIENCE

Marks 80

Time- 3 hrs

General Instructions:

(1) Section A:

Q.No. 1. contains multiple choice type of questions carrying 2 mark each

Q.No. 2. contains very short answer type of questions carrying 1 mark each

(2) Section B:

Q.No. 3.to 14 are short answer type of questions carrying 2 mark each

(3) Section C:

Q.No. 15.to 26 are short answer type of questions carrying 3 mark each

(4) Section D:

Q.No. 27.to 34 are long answer type of questions carrying 4 mark each

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Use of graph paper is not necessary. Only rough sketch is expected

(7) For each **MCQ**, correct answer must be written along with its alphabet.

e. g. a)/ (b)...../ (c)/(d)

SECTION A

1. Select and write the correct answer (2 marks each)

(i)The logically equivalent statement of $p \rightarrow q$ is

(a) $\sim q \vee \sim p$ (b) $q \rightarrow \sim p$ (c) $\sim q \vee p$ (d) $\sim p \vee q$

(ii)The principal solution of the equation $\sin \theta = -\frac{1}{2}$ are

(a) $\frac{5\pi}{6}, \frac{\pi}{6}$ (b) $\frac{7\pi}{6}, 11\frac{\pi}{6}$ (c) $\frac{\pi}{6}, 7\frac{\pi}{6}$ (d) $7\frac{\pi}{6}, \frac{\pi}{6}$

(iii) If $f(x)=e^x \cdot g(x)$, $g(0)=2$, $g'(0)=1$, then $f'(0)$ is

(a)1 (b) 2 (c) 3 (d)0

(iv) The order and degree of $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 5y=0$ is

**(a) order 2, degree 2 (b) order 1, degree 2 (c) order 1, degree 1
(d) order 2, degree 1.**

(v) $\int \frac{dx}{9x^2+1} =$

**(a) $\frac{1}{3} \tan^{-1} 2x+c$ (b) $\frac{1}{3} \tan^{-1} x+c$ (c) $\frac{1}{3} \tan^{-1} 3x+c$
(d) $\frac{1}{3} \tan^{-1} 6x+c$**

(vi) The law of motion of a particle is given by $s=3t^2-4t+5$, where s is the displacement in centimetres and t is the time in seconds. The velocity at $t=2$ seconds is

(a) 3 (b) 5 (c) 4 (d) 8

(vii) The foot of the perpendicular drawn from the point $(0,0,0)$ to the plane is $(4,-2,-5)$, then the equation of the plane is

(a) $4x+y+5z=14$ (b) $4x-2y-5z=45$ (c) $x-2y-5z=10$ (d) $4x+y+6z=11$

(viii) If $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, k\right)$ represent direction cosines of a line, then $k=$

(a) $\pm \frac{1}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) k can take any value.

**2. Answer the following questions
each)**

(1 mark

(i) write the converse of the statement

If voltage increases then current decreases.

(ii) If two sides of a triangle are $\hat{i} + 2\hat{j}$ and $\hat{i} + \hat{k}$, then find the length of the third side.

(iii) Differentiate $\sin(x^2 + x)$ w.r.t. x .

(iv) Evaluate $\int e^x [\cot x + \log(\sin x)] dx$

SECTION B

3. Find the matrix of cofactors for the matrix $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$

4. Find the angle between planes $\vec{r} \cdot (-2\hat{i} + \hat{j} + 2\hat{k}) = 17$ and $\vec{r} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 71$

5. Given that $X \sim B(n, p)$. If $n=10$ and $p=0.4$, find $E(x)$ and $Var(x)$.

6. Using rules of negation write the negation of the following with justification.

$$p \wedge \sim q$$

7. The probability distribution of discrete r.v X is as follows

$X=x$	1	2	3	4	5	6
$P(X=x)$	k	$2k$	$3k$	$4k$	$5k$	$6k$

Determine the value of k .

8. For the following differential equation find the particular solution satisfying the given condition.

$$3e^x \cdot \tan y \cdot dx + (1 + e^x) \sec^2 y \cdot dy = 0, \text{ when } x=0, y=\pi.$$

9. Find the points on the curve $y = x^3 - 2x^2 - x$, where the tangents are parallel to $3x - y + 1 = 0$.

10. A ball is thrown in the air. Its height at any time t is given by $h=3+14t-5t^2$. Find the maximum height it can reach

11. If $y=Ae^{mx}+Be^{nx}$, show that $y_2-(m+n)y_1+(mn)y=0$

12. Find the vector equation of the plane which makes equal non-zero intercepts on the coordinate axes and passes through $(1,1,1)$.

13. If D, E, F are the midpoints of the sides BC, CA, AB of a triangle ABC , prove that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \overrightarrow{0}$.

14. Find the polar coordinates of the point whose cartesian coordinates are $(\sqrt{2}, \sqrt{2})$

SECTION C

15. $P.T \tan^{-1} \left[\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right] = \frac{\pi}{4} + \theta$ if $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$

16. Find the joint equation of the pair of lines through the origin and perpendicular to the lines given by $x^2 + 4xy - 5y^2 = 0$.

17. If $A(\vec{a})$ and $B(\vec{b})$ are any two points in the space and $R(\vec{r})$ be a point on the line segment AB dividing it internally in the ratio $m:n$ then prove that $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$

18. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0.

19. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as number greater than 4 appears on at least one die.

$$20. \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$$

$$21. P.T \sin^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13} = \sin^{-1} \frac{56}{65}$$

$$22. P.T \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

$$23. \int \frac{10x^9 + 10^x}{10^x + x^{10}} \cdot \log 10$$

24 Find the maximum and minimum of the function

$$f(x) = 2x^3 - 21x^2 + 36x - 20.$$

25. If θ is the measure of acute angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then prove that

$$\tan \theta = \left| 2 \frac{\sqrt{h^2 - ab}}{a+b} \right|$$

26. Using truth table prove the following logical equivalence.

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

27. A horticulturist wishes to mix two brands of fertilizers that will provide a minimum of 15 units of potash, 20 units of nitrate and 24 units of phosphate. One unit of brand 1 provides 3 units of potash, 1 unit of nitrate, 3 units of phosphate. One unit of brand II provides 1 unit of potash, 5 units of nitrate and 2 units of phosphates. One unit of brand I costs Rs. 120 and 1 unit of brand II

costs Rs.60 per unit. Formulate this problem as L.P.P to minimize the cost.

28. Find the vector equation of the plane which bisects the segment joining A(2,3,6) and B(4,3,-2) at right angles.

29. Solve the differential equation $\frac{dy}{dx} + y \sec x = \tan x$.

30. Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ by elementary row transformations.

31. Find the area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum.

32. P.T $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + c$.

33. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

34. Prove that two vectors whose direction cosines are given by the relations $al+bm+cn=0$ and $fmn+gnl+hlm=0$ are perpendicular

$$i\left[f \frac{f}{a} + \frac{g}{b} + \frac{h}{c}\right] = 0$$