

# AFAC ENGLISH SCHOOL & JUNIOR COLLEGE

## Homework 2020

### MATHEMATICS & STATISTICS

DATE 07/11/2020

XII SCIENCE

Time- 2 ½ hrs

### General Instructions:

#### (1) Section A:

Q.No. 1. contains multiple choice type of questions

Q.No. 2. contains very short answer type of questions

#### (2) Section B:

Q.No. 3.to 13 are short answer type of questions

#### (3) Section C:

Q.No. 14.to 19 are short answer type of questions

#### (4) Section D:

Q.No. 20.to 23 are long answer type of questions

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Use of graph paper is not necessary. Only rough sketch is expected

(7) For each MCQ, correct answer must be written along with its alphabet.

e. g. a) ...../ (b)...../ (c) ...../(d) .....

### SECTION A

1.

(i) If a line makes angles  $\alpha, \beta, \gamma$  with co-ordinate axes ,then

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$$

(a) 1 (b)-1 (c)2 (d)-2

(ii) The direction cosines of the normal to the plane  $2x-y+2z=3$  are

(a)  $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$  (b)  $-\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$  (c)  $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$  (d)  $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$

(iii) The acceleration of a moving particle whose space time equation is given by  $s=3t^2+2t-5$  is

(a)6 (b)5 (c)0 (d)1

(iv) If the vectors  $-3\hat{i}+4\hat{j}-2\hat{k}$ ,  $\hat{i}+2\hat{k}$  and  $\hat{i}-p\hat{j}$  are coplanar, then value of p is

(a) 2 (b) 1 (c) 3 (d) 4

(v) The proposition  $p \wedge \sim p$  is

(a) a contradiction

(b) a tautology

(c) tautology and contradiction

(d) contingency.

2. Answer the following.

(i) Find the vector equation of the line passing through the point having position vector  $-2\hat{i} + \hat{j} + \hat{k}$  and parallel to vector  $4\hat{i} - \hat{j} + 2\hat{k}$ .

(ii) If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $A(\text{adj } A) = kI$ , then find the value of k

(iii) Write the negation of  $p \wedge (q \rightarrow r)$

(iv) If  $y = \log \{ \log (\log x) \}$  find  $\frac{dy}{dx}$ .

Section B (Attempt any 8)

3. Find the Cartesian equation of the line passing through A(2,2,1) and B(1,3,0).

4. Find the principal solution of the equation  $\cos \theta = \frac{1}{2}$

5. Find the matrix of cofactors for the matrix

$$\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$$

6. Find the vector equation of the line passing through the point having position vector  $\hat{i} + 2\hat{j} + 3\hat{k}$ . and perpendicular to vectors

$$\hat{i} + \hat{j} + \hat{k} \text{ and } 2\hat{i} - \hat{j} + \hat{k}.$$

7. Evaluate  $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ .

8.  $\int \frac{\tan x}{\sec x + \tan x} dx.$

9. Find the vector equation of a plane which is at 42 unit distance from the origin and which is normal to the vector  $2\hat{i} + \hat{j} - 2\hat{k}$ .

10. If  $e^x + e^y = e^{x+y}$ , then show that  $\frac{dy}{dx} = -e^{y-x}$

11. Construct the truth table for the following:

$$P \rightarrow (q \rightarrow p)$$

12. Differentiate w.r.t x  $(\sin x)^x$

13. Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \text{ and}$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$$

**Section C (Attempt any four)**

14. Find the area of the parallelogram whose adjacent sides are the vectors  $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 3\hat{j} - 3\hat{k}$ .

15. Find the approximate value of  $\tan^{-1}(1.001)$ .

16. Write converse, inverse and contrapositive of the statement:

If surface area decreases then pressure increases.

17. Find the polar co-ordinates of the point whose Cartesian co-ordinates are  $(\sqrt{2}, \sqrt{2})$ .

18. Find the inverse of  $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$  by elementary row transformations.

19. Prove that  $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$ .

**Section D (Attempt any two)**

20. If  $A(\vec{a})$  and  $B(\vec{b})$  be any two points in the space and  $R(\vec{r})$  be a point on the line segment AB dividing it internally in the ratio  $m:n$  then prove that  $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$ .

21. P.T  $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$ .

22. If  $x = \frac{2bt}{1+t^2}$ ,  $y = a\left(\frac{1-t^2}{1+t^2}\right)$  show that  $\frac{dx}{dy} = -\frac{yb^2}{a^2x}$

23. Find the maximum and minimum of the function

$f(x) = 2x^3 - 21x^2 + 36x - 20$ .