Analysis of Trusses

Finite Element Methods

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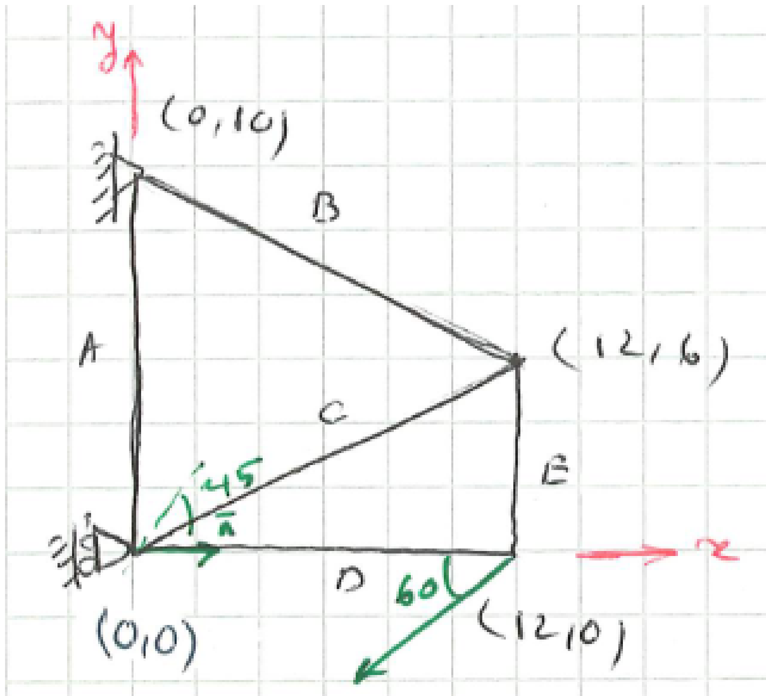
Objective

This project's goal is to analyze the following trust system experiencing load and generate code to find the displacement, reactionary forces and stresses in all the elements allowing for variable input.



We calculated all the possibilities for each element by using the following methodology.

Methodology

We begin our analysis of 2D truss structures using the direct stiffness method. The following section explains the code we used

Dependencies

NumPy

Matplotlib

Inputs

Node coordinates (x, y)

Boundary conditions for each node (horizontal constraint, vertical constraint)

Roller angle (if applicable)

Element connectivity (node indices for each element)

Material and diameter of each element (Aluminium or Steel)

External force magnitude, angle, and node number

Outputs

Displacements (dx, dy) at each node

Reactions (Rx, Ry) at each boundary node

Stress in each element

Plot of the original and displaced truss structure

Steps
Import necessary libraries and define the required constants.
Collect input values from the user, including node coordinates, boundary conditions, element connectivity, element properties, and external forces.
Define a function analyze_truss to analyze the truss structure.
Calculate the global stiffness matrix, assemble it using element stiffness matrices, and apply boundary conditions.
Solve for displacements and calculate reactions and stress for each element.
Print the results of the analysis.
Plot the original and displaced truss structures.
Code Usage
Install the necessary dependencies.
Run the Python script.
Follow the prompts to input node coordinates, boundary conditions, element connectivity, element properties, and external forces.
View the printed results and the generated plots.

The actual code itself is provided with the submission.

Calculations

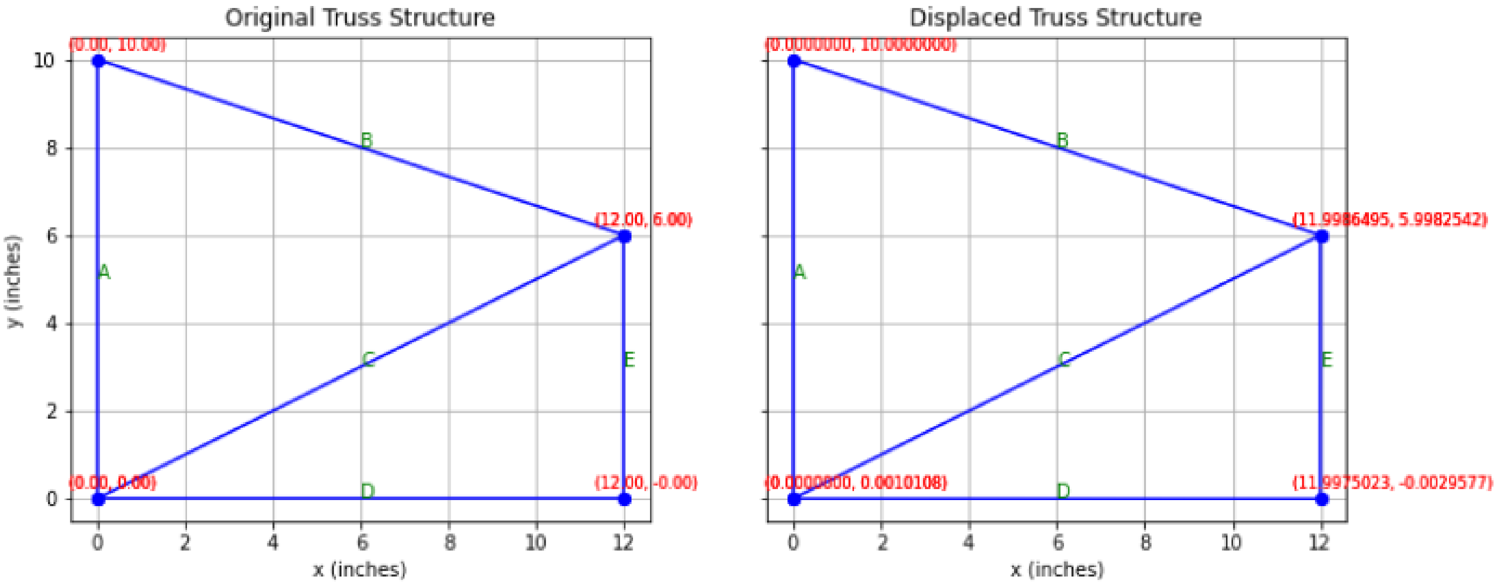
Our results are summarised in the following section.

```
|-----------Results-----------|

Displacements:
Node 0: dx = 0.00e+00, dy = 1.01e-03
Node 1: dx = 0.00e+00, dy = 0.00e+00
Node 2: dx = -1.35e-03, dy = -1.75e-03
Node 3: dx = -2.50e-03, dy = -2.96e-03

Reactions:
Node 0: Rx = 0.00e+00, Ry = 5.68e-14
Node 1: Rx = 0.00e+00, Ry = 0.00e+00
Node 2: Rx = 0.00e+00, Ry = 0.00e+00
Node 3: Rx = 0.00e+00, Ry = 0.00e+00

Stress in elements:
Element 0: stress = 3.03e+03 psi
Element 1: stress = 6.34e+02 psi
Element 2: stress = 5.46e+03 psi
Element 3: stress = 2.29e+03 psi
Element 4: stress = -6.06e+03 psi
```



These results were calculated using the data provided in the project, however, our code works with almost any arbitrary values for the roller and load angles, as well as load magnitude. The above figure shows the original truss structure compared to the displaced structure once the load is added. As can be seen, there is minimal displacement and the truss system is very secure under the assumed condition.

Conclusion

In conclusion, we achieved the objective of this project by generating a computational analysis and a finite element method for approximating the displacement, reactionary forces and stresses the truss system experiences. Our method also functions for different angles and loads and can be used to thoroughly test the truss. This information allows us to reduce testing costs as a physical prototype would be unnecessary. This experience is very valuable as the same methods can be applied to other problems, creating transferable skills.

In [6]:

```
import numpy as np
import math
import matplotlib.pyplot as plt

# Constants
E_Steel = 30 * 10**6  # psi
E_Alum = 11 * 10**6   # psi

# Node coordinates
node_coordinates = []
boundaries = []
for i in range(4):
    coords = input(f"Enter the coordinates of node {i} (comma-separated): ")
    x, y = [float(value) for value in coords.split(',')]
    node_coordinates.append([x, y])
    bound = input(f"Enter the Boundary Conditions of node {i} (comma-separated) (horizontal constraint, vertical contraint) 0 = Free, 1 = Fixed: ")
    h,v = [int(value) for value in bound.split(',')]
    if (h == 1 and v == 0) or (h==0 and v == 1):
        roller_angle = float(input("Enter the angle of the roller (degrees): "))
    boundaries.append([int(i),int(h),int(v)])

nodes = np.array(node_coordinates)
Elements_Name = ['A','B','C','D','E']
# Element connectivity
element_connectivity = []
element_property = []
for i in Elements_Name:
    conn = input(f"Enter the connectivity for element {i} (comma-separated node indices): ")
    node1, node2 = [int(value) for value in conn.split(',')]
    element_connectivity.append([node1, node2])
    prop = input(f"Enter the Material (A for Aluminium and S for Steel) for element {i} and Diameter of Rod (comma-separated node indices): ")
    prop1, prop2 = [value for value in prop.split(',')]
    if prop1 == 'A':
        prop1 = E_Alum
    else:
            prop1 = E_Steel
    prop2 = ((float(prop2)/2)**2)*math.pi
    element_property.append([prop1,prop2])
```

```
elements = np.array(element_connectivity)


# Update boundary conditions based on the roller angle
boundary_conditions = np.array(boundaries)


# Material properties (element, E, A)
properties = np.array(element_property)
print(properties)
# External force (node, force_x, force_y)
forces = np.array([
    [0, 0, 0],
    [1, 0, 0],
    [2, 0, 0],
    [3, 0, 0],

])

force_magnitude = float(input(f"Enter the Force Magnitude on Truss: ")) #
arbitrary force magnitude, modify as needed
force_angle = float(input(f"Enter the Force Angle in Degrees (Counter-
Clockwise from x-plane) on Truss: ")) # arbitrary force magnitude, modify as
needed  # degrees
force_node = int(input(f"Enter the node number (0-3) the Force is applied on:
"))
forces[force_node, 1] = force_magnitude * math.cos(math.radians(force_angle))
forces[force_node, 2] = force_magnitude * math.sin(math.radians(force_angle))


# Truss analysis function
def analyze_truss(nodes, elements, properties, forces, boundary_conditions):
    # Calculate number of nodes and elements
    num_nodes = nodes.shape[0]
    num_elements = elements.shape[0]

    # Calculate element lengths and unit vectors
    lengths = np.zeros(num_elements)
    unit_vectors = np.zeros((num_elements, 2))
    for i, element in enumerate(elements):
        node1 = nodes[element[0]]
        node2 = nodes[element[1]]
        dx = node2[0] - node1[0]
```

```
        dy = node2[1] - node1[1]
        length = np.sqrt(dx**2 + dy**2)
        unit_vectors[i] = [dx/length, dy/length]
        lengths[i] = length


    # Global stiffness matrix
    K = np.zeros((2*num_nodes, 2*num_nodes))
    for i, element in enumerate(elements):
        E, A = properties[i]
        length = lengths[i]
        unit_vector = unit_vectors[i]
        k = E*A/length

        # Local stiffness matrix
        k_local = np.array([
                    [1, 0, -1, 0],
                    [0, 1, 0, -1],
                    [-1, 0, 1, 0],
                    [0, -1, 0, 1]
        ]) * k
        # Local to global transformation matrix
        T = np.array([
            [unit_vector[0], 0, unit_vector[1], 0],
            [0, unit_vector[0], 0, unit_vector[1]],
            [unit_vector[1], 0, -unit_vector[0], 0],
            [0, unit_vector[1], 0, -unit_vector[0]]
        ])

        # Global stiffness matrix for element
        k_global = np.dot(T.T, np.dot(k_local, T))

        # Assemble global stiffness matrix
        for a in range(2):
            for b in range(2):
                K[2*element[a]:2*element[a]+2, 2*element[b]:2*element[b]+2]
+= k_global[a*2:a*2+2, b*2:b*2+2]

    # Apply boundary conditions
    for bc in boundary_conditions:
        node, constraint_x, constraint_y = bc
        if constraint_x:
            K[2*node, :] = 0
            K[:, 2*node] = 0
```

```
            K[2*node, 2*node] = 1
        if constraint_y:
            K[2*node+1, :] = 0
            K[:, 2*node+1] = 0
            K[2*node+1, 2*node+1] = 1

    # Solve for displacements
    F = np.zeros(2*num_nodes)
    for f in forces:
        node, force_x, force_y = f
        F[2*node] = force_x
        F[2*node+1] = force_y

    displacements = np.linalg.solve(K, F)

    # Calculate reactions
    reactions = np.zeros((boundary_conditions.shape[0], 2))
    for i, bc in enumerate(boundary_conditions):
        node, constraint_x, constraint_y = bc
        if constraint_x or constraint_y:
            reaction = np.dot(K[2*node:2*node+2, :], displacements)
            reactions[i] = reaction

    # Calculate stress in all elements
    stress = np.zeros(num_elements)
    for i, element in enumerate(elements):
        E, _ = properties[i]
        length = lengths[i]
        unit_vector = unit_vectors[i]
        displacement = displacements[2*element[0]:2*element[0]+2] -
displacements[2*element[1]:2*element[1]+2]
        local_displacement = np.dot(unit_vector, displacement)
        stress[i] = E * local_displacement / length

    return displacements, reactions, stress

# Analyze truss and print results
displacements, reactions, stress = analyze_truss(nodes, elements, properties,
forces, boundary_conditions)
print("\n|-----------Results-----------|")
print("\nDisplacements:")
for i, displacement in enumerate(displacements.reshape(-1, 2)):
    print(f"Node {i}: dx = {displacement[0]:.2e}, dy =
{displacement[1]:.2e}")
```

```
print("\nReactions:")
for i, reaction in enumerate(reactions):
    print(f"Node {i}: Rx = {reaction[0]:.2e}, Ry = {reaction[1]:.2e}")

print("\nStress in elements:")
for i, stress_value in enumerate(stress):
    print(f"Element {i}: stress = {stress_value:.2e} psi")

  # Calculate the displaced nodes
displaced_nodes = nodes + displacements.reshape(-1, 2)

# Plot trusses in a single figure with two subplots
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 6), sharey=True)

for i, element in enumerate(elements):
    node1, node2 = nodes[element[0]], nodes[element[1]]
    displaced_node1, displaced_node2 = displaced_nodes[element[0]],
displaced_nodes[element[1]]

    # Plot truss in the first subplot
    ax1.plot([node1[0], node2[0]], [node1[1], node2[1]], 'bo-')
    ax1.annotate(f"({displaced_node1[0]:.2f}, {displaced_node1[1]:.2f})",
(displaced_node1[0], displaced_node1[1]), textcoords="offset points", xytext=
(-15, 5), fontsize=8, color='red')
    ax1.annotate(f"({displaced_node2[0]:.2f}, {displaced_node2[1]:.2f})",
(displaced_node2[0], displaced_node2[1]), textcoords="offset points", xytext=
(-15, 5), fontsize=8, color='red')
    ax1.annotate(f"{Elements_Name[i]}", (0.5 * (node1[0] + node2[0]), 0.5 *
(node1[1] + node2[1])), fontsize=10, color='green')

    # Plot truss in the second subplot with displaced nodes
    ax2.plot([displaced_node1[0], displaced_node2[0]], [displaced_node1[1],
displaced_node2[1]], 'bo-')
    ax2.annotate(f"({displaced_node1[0]:.7f}, {displaced_node1[1]:.7f})",
(displaced_node1[0], displaced_node1[1]), textcoords="offset points", xytext=
(-15, 5), fontsize=8, color='red')
    ax2.annotate(f"({displaced_node2[0]:.7f}, {displaced_node2[1]:.7f})",
(displaced_node2[0], displaced_node2[1]), textcoords="offset points", xytext=
(-15, 5), fontsize=8, color='red')
    ax2.annotate(f"{Elements_Name[i]}", (0.5 * (displaced_node1[0] +
displaced_node2[0]), 0.5 * (displaced_node1[1] + displaced_node2[1])),
```

```python
fontsize=10, color='green')

ax1.set_aspect('equal', 'box')
ax1.set_title("Original Truss Structure")
ax1.set_xlabel('x (inches)')
ax1.set_ylabel('y (inches)')
ax1.grid(True)

ax2.set_aspect('equal', 'box')
ax2.set_title("Displaced Truss Structure")
ax2.set_xlabel('x (inches)')
ax2.grid(True)

plt.show()



ax2.set_aspect('equal', 'box')
ax2.set_title("Displaced Truss Structure")
ax2.set_xlabel('x (inches)')
ax2.grid(True)

plt.show()
```

```
Enter the coordinates of node 0 (comma-separated): 0,0
Enter the Boundary Conditions of node 0 (comma-separated) (horizontal constraint, vertical
contraint) 0 = Free, 1 = Fixed: 1,0
Enter the angle of the roller (degrees): 45
Enter the coordinates of node 1 (comma-separated): 0,10
Enter the Boundary Conditions of node 1 (comma-separated) (horizontal constraint, vertical
contraint) 0 = Free, 1 = Fixed: 1,1
Enter the coordinates of node 2 (comma-separated): 12,6
Enter the Boundary Conditions of node 2 (comma-separated) (horizontal constraint, vertical
contraint) 0 = Free, 1 = Fixed: 0,0
Enter the coordinates of node 3 (comma-separated): 12,0
Enter the Boundary Conditions of node 3 (comma-separated) (horizontal constraint, vertical
contraint) 0 = Free, 1 = Fixed: 0,0
Enter the connectivity for element A (comma-separated node indices): 0,1
Enter the Material (A for Aluminium and S for Steel) for element A and Diameter of Rod (co
mma-separated node indices): S,0.5
Enter the connectivity for element B (comma-separated node indices): 1,2
Enter the Material (A for Aluminium and S for Steel) for element B and Diameter of Rod (co
mma-separated node indices): A,0.4
Enter the connectivity for element C (comma-separated node indices): 0,2
Enter the Material (A for Aluminium and S for Steel) for element C and Diameter of Rod (co
mma-separated node indices): S,0.5
Enter the connectivity for element D (comma-separated node indices): 0,3
Enter the Material (A for Aluminium and S for Steel) for element D and Diameter of Rod (co
mma-separated node indices): A,0.4
Enter the connectivity for element E (comma-separated node indices): 2,3
Enter the Material (A for Aluminium and S for Steel) for element E and Diameter of Rod (co
mma-separated node indices): S,0.5
```

```
[[3.00000000e+07 1.96349541e-01]
 [1.10000000e+07 1.25663706e-01]
 [3.00000000e+07 1.96349541e-01]
 [1.10000000e+07 1.25663706e-01]
 [3.00000000e+07 1.96349541e-01]]
Enter the Force Magnitude on Truss2000
Enter the Force Angle in Degrees (Counter-Clockwise from x-plane) on Truss225
Enter the node number (0-3) the Force is applied on3

|-----------Results-----------|

Displacements:
Node 0: dx = 0.00e+00, dy = 1.01e-03
Node 1: dx = 0.00e+00, dy = 0.00e+00
Node 2: dx = -1.35e-03, dy = -1.75e-03
Node 3: dx = -2.50e-03, dy = -2.96e-03

Reactions:
Node 0: Rx = 0.00e+00, Ry = 5.68e-14
Node 1: Rx = 0.00e+00, Ry = 0.00e+00
Node 2: Rx = 0.00e+00, Ry = 0.00e+00
Node 3: Rx = 0.00e+00, Ry = 0.00e+00

Stress in elements:
Element 0: stress = 3.03e+03 psi
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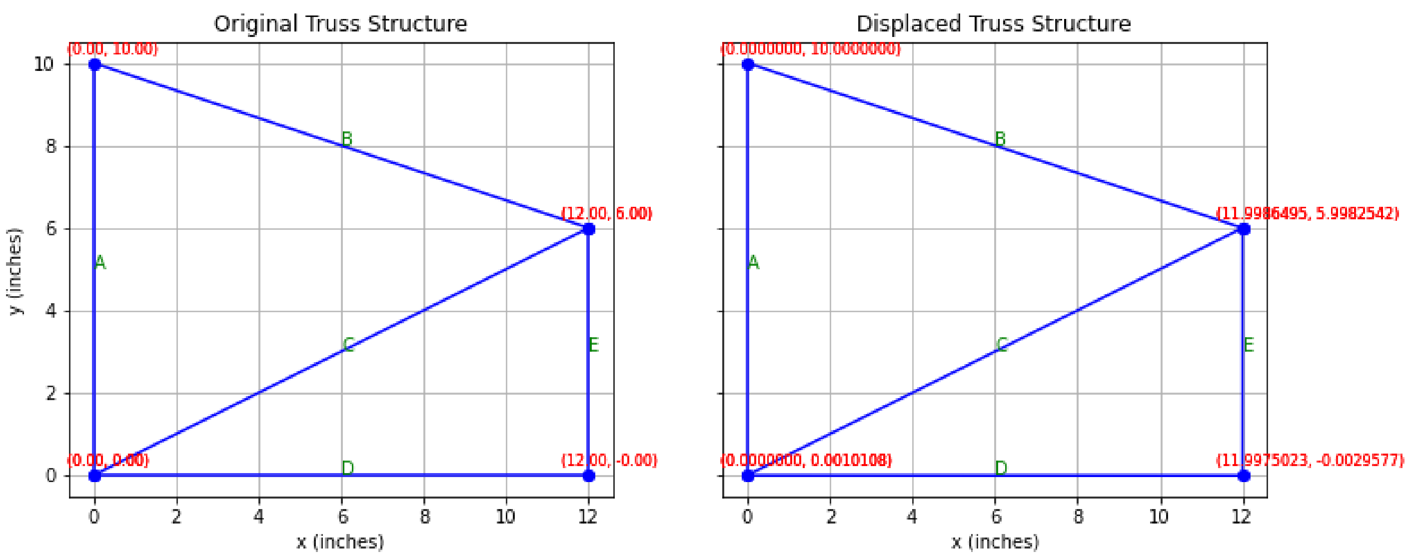

In [ ]: