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Rich-Neighbor Edge Colorings

Term Paper in Finance Lab

Short Presentation

1. Introduction

In this paper we set out to analyse an open conjecture in a modern graph theory problem known as rich-neighbor edge coloring.

Definition 1.1. In an edge coloring, an edge e is called *rich* if all edges adjacent to e have different colors. An edge coloring is called a *rich-neighbor edge coloring* if every edge is adjacent to some rich edge.

Definition 1.2. $X'_{rn}(G)$ denotes the smallest number of colors for which there exists a rich-neighbor edge coloring.

Conjecture 1.3. *For every graph G of maximum degree Δ, $X'_{rn}(G) \leq 2\Delta - 1$ holds.*

Example 1.4. Let's take a look at the Petersen graph and an example of a rich-neighbor edge coloring.
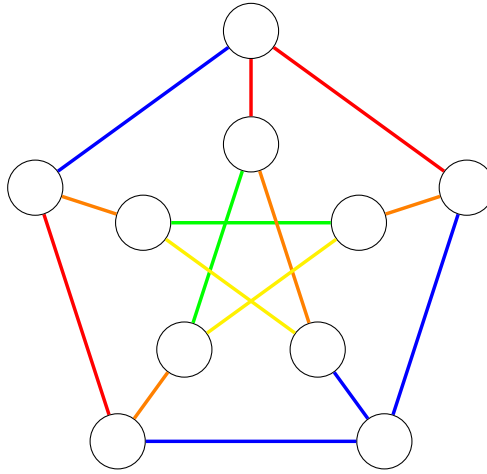
We can see that for the Petersen graph (which is 3-regular) $X'_{rn} \leq 5$. $\qquad \diamondsuit$

2. Plan

Our assingnment is to create an algorthm that 'proves' the conjecture for regular graphs of degree $4\geq$ (So it finds a rich-neighbor edge coloring for every graph, for example for all 5-regular graphs with 5 veriticies), and to make a random search algorythm at the point where our algorithm is too slow.