UNIVERSITY OF LJUBLJANA
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Financial mathematics – 1st cycle

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Rich-Neighbor Edge Colorings

Term Paper in Finance Lab

Short Presentation

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1. Introduction

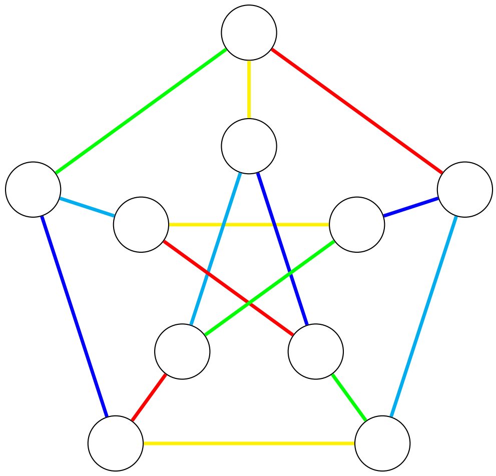
In this paper we set out to analyse an open conjecture in a modern graph theory problem known as rich-neighbor edge coloring.

Definition 1.1. In an edge coloring, an edge e is called *rich* if all edges adjacent to e have different colors. An edge coloring is called a *rich-neighbor edge coloring* if every edge is adjacent to some rich edge.

Definition 1.2. $X'_{rn}(G)$ denotes the smallest number of colors for which there exists a rich-neighbor edge coloring.

Conjecture 1.3. *For every graph G of maximum degree Δ, $X'_{rn}(G) \leq 2\Delta - 1$ holds.*

Example 1.4. Let's take a look at the Petersen graph and an example of a rich-neighbor edge coloring.



We can see that for the Petersen graph (which is 3-regular) $X'_{rn} \leq 5$. ◇

2. Plan

Our assingnment is to create an algorthm that 'proves' the conjecture for regular graphs of degree 4$\geq$ (So it finds a rich-neighbor edge coloring for every k-regular graph on n verticies), and to make a random search algorythm for checking classes of graphs that are too large to be checked individually.

2.1. **Whole number integer programming.** Using sagemath we plan to create an integer programming model for the problem, that checks all smaller graphs if there exists a rich-neighbor coloring with $\leq 2\Delta - 1$ colors. Our interger program looks like this:

$$\min t$$
$$\forall e: \sum_{i=1}^{k} x_{ei} = 1$$
$$\forall i \forall u \forall v, w \sim u, v \neq u : x_{uv,i} + x_{uw,i} \leq 1$$
$$\forall e \forall i : x_{ei} \cdot i \leq 1$$
$$\forall uv \forall w \sim u, w \neq v \forall z \; v, z \neq u, w \forall i : x_{uq,i} + x_{vz,i} + y_{uv} \leq 1$$
$$\forall e : \sum_{f \sim e} y_f \geq 1$$
$$\forall e \forall i : x_{ei} \in \{0,1\}, y_e \in \{0,1\}$$

where

$$x_{ei} = \begin{cases} 1, & \text{if edge } e \text{ has color } i \\ 0, & \text{otherwise} \end{cases} \qquad y_e = \begin{cases} 1, & \text{if edge } e \text{ is rich} \\ 0, & \text{otherwise} \end{cases}$$

We will determine at what point does the computation of rich-neighor coloring become too intense for this technique and will use the next random search algorythm.

2.2. **Random search.** We will create a random search algorythm that will check if the conjecture holds for a class of graphs that are too large to be checked individually. Our plan is to create a random graph generator that will generate a graph with a given number of verticies and edges. Then we will check if the conjecture holds for the generated graph. We will repeat this process for a given number of times and then we will check if the conjecture holds for the class of graphs. We will repeat this process for different classes of graphs and different number of verticies and edges. The algorythm will be implemented in sagemath, and its pseudocode is as follows: