UNIVERSITY OF LJUBLJANA
FACULTY OF MATHEMATICS AND PHYSICS

Financial mathematics – 1st cycle

Anej Rozman, Tanja Luštrek

Rich-Neighbor Edge Colorings

Term Paper in Finance Lab

Short Presentation

Advisers: Assistant Professor Janoš Vidali, Professor Riste Škrekovski

Ljubljana, 2023

1. Introduction

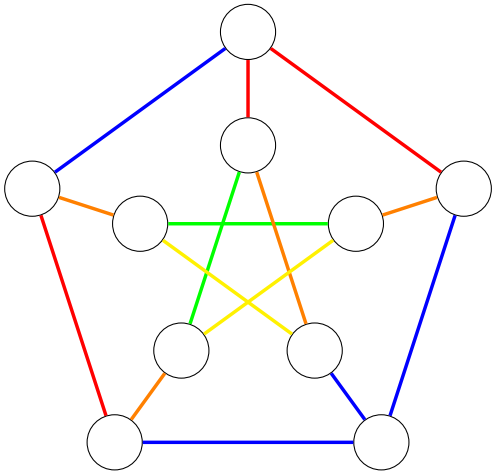
In this paper we set out to analyse an open conjecture in a modern graph theory problem known as rich-neighbor edge coloring.

Definition 1.1. In an edge coloring, an edge e is called *rich* if all edges adjacent to e have different colors. An edge coloring is called a *rich-neighbor edge coloring* if every edge is adjacent to some rich edge.

Definition 1.2. $X'_{rn}(G)$ denotes the smallest number of colors for which there exists a rich-neighbor edge coloring.

Conjecture 1.3. *For every graph G of maximum degree Δ, $X'_{rn}(G) \leq 2\Delta - 1$ holds.*

Example 1.4. Let's take a look at the Petersen graph and an example of a rich-neighbor edge coloring.



We can see that for the Petersen graph (which is 3-regular) $X'_{rn} = 5 \leq 5$. ♢

2. Plan

Our assingnment is to create an algorthm that proves the conjecture for graphs of maximum degree 4 (So it finds a rich-neighbor edge coloring for every graph, for example for all graphs with 5 veriticies and a maximum degree of 5), and to make a random search algorythm for graphs of maximum degree ≥ 5.